STAT 521: Assignment 2

Make sure to show your computation and/or attach appropriate output.

Problem 1

Do Example 3 of Lecture 2 note (page 5).

Problem 2

Use SAS, R or Python to produce the following probability histograms (See Lecture 2 code files):

a) For a binomial random variable X with $n = 14$ and $p = 0.1$. How does the probability histogram look like? Is it symmetric? Skewed? Attach the output. Comment on it.

b) Do the same for $X \sim Bin(14, 0.9)$ and also for $X \sim Bin(14, 0.5)$. Attach the output. How do they look?

c) Describe how the success probability p affects the shape of the binomial distribution.

Problem 3

Tay-Sachs is a metabolic disorder that is inherited as an autosomal recessive trait. Both recessive alleles are necessary for expression of the disease. Therefore, when each parent is a carrier, there is a one in four chance of transmitting the genetic disorder to each offspring.

a) Let X be the number of offspring affected in three consecutive conceptions from Tay-Sachs carrier parents. Is X a binomial random variable? Explain your response.

b) Suppose a carrier couple plans on having three children ($n = 3$). Build the probability distribution (probability mass function) for the number of conceptions that will receive Tay-Sachs genes from both parents. Do this by hand and also use SAS, R, or Python to verify your answer. Attach appropriate outputs.

c) What are the expected value and variance of X?

Problem 4

The prevalence of a trait is 76.8%.

a) How many would you expect to see with this characteristic in a random sample of $n = 10$?

b) What is the probability of seeing nine or more individuals with this characteristic in a random sample of $n = 10$?

Problem 5

In a study of the effectiveness of an insecticide against a certain insect, a large area of land was sprayed. Later the area was examined for live insects by randomly selecting squares and counting the number of live insects per square. Past experience has shown the average number of live insects per square after spraying to be 0.5. If the number of live insects per square follows a Poisson distribution, find the probability that a selected square will contain:

a) No live insects

b) One or more live insects

Problem 6

For a particular age group of adult males the distribution of cholesterol readings, in mg/dl, is normally distributed with a mean of 210 and a standard deviation of 15.

a) What percentage of this population would have readings exceeding 250?

b) What percentage would have readings less than 180?

c) What percentage would have readings between 190 and 220?

Problem 7 (Biostat students only)

Let X be a discrete random variable with the probability mass function $P(X = x) = p(x)$. The expected value of X is defined as (See Lecture 2, page 3):

$$E(X) = \mu = \sum_x xp(x)$$

Suppose instead we are interested in a discrete random variable Y which is some real-valued function of X, i.e., $Y = g(X)$. The expected value of Y is then defined as:

$$E(Y) = E\left(g(X)\right) = \sum_x g(x)p(x)$$

Also, there are some properties of the expected value. Let c be a constant and X and Y are both discrete random variables.

- $E(c) = c$
- $E(cX) = cE(X)$
- $E(X + Y) = E(X) + E(Y)$

More in general, if $a_1, a_2, \cdots, a_k$ are constants, then:

$$E\left(\sum_x^k a_i X_i\right) = E\left(a_1X_1 + a_2X_2 + \cdots + a_kX_k\right) = \sum_x^k a_i E(X_i)$$

Question: Suppose that discrete random variables X and Y have the following probability distributions:

x	1.0	2.0	3.0	4.0
p(x)	0.2	0.4	0.3	0.1

y	1.0	2.0	3.0
p(y)	0.5	0.3	0.2

Use the properties of the expected value and calculate $E(X)$, $E\left(\frac{1}{X}\right)$, and $E(X^2 + 2Y)$.

Problem 8 (Biostat students only)

Let X be a random variable with the mean $E(X) = \mu$. The variance of X is defined as $Var(X) = E[(X-\mu)^2]$. Use the properties of the expected value in Problem 7 and show that:

a) $Var(X) = E[(X-\mu)^2] = E\left(X^2\right) - \mu^2$

b) $Var(X + c) = Var(X)$ where c is a constant

c) $Var(cX) = c^2 Var(X)$ where c is a constant