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# Local recurrence based performance prediction and prognostics in the nonlinear and nonstationary systems

Hui Yang [a,*], Satish T.S. Bukkapatnam [b], Leandro G. Barajas [c]

[a] Department of Industrial & Management Systems Engineering, University of South Florida, Tampa, FL 33620, USA
[b] Department of Industrial Engineering & Management, Oklahoma State University, Stillwater, OK 74078, USA
[c] Manufacturing Systems Research Laboratory, General Motors R&D Center, Warren, MI 48090, USA

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ABSTRACT

This paper presents a local recurrence modeling approach for state and performance predictions in complex nonlinear and nonstationary systems. Nonstationarity is treated as the switching force between different stationary systems, which is shown as a series of finite time detours of system dynamics from the vicinity of a nonlinear attractor. Recurrence patterns are used to partition the system trajectory into multiple near-stationary segments. Consequently, piecewise eigen analysis of ensembles in each near-stationary segment can capture both nonlinear stochastic dynamics and nonstationarity. The experimental studies using simulated and real-world datasets demonstrate significant prediction performance improvements in comparison with other alternative methods.



## 1. Introduction

Prediction of state and performance is critical to the assurance of integrity and sustainability of many complex natural and engineering physical systems [1–4]. In the modern manufacturing enterprises, timely and frequent prediction of manufacturing system Key Performance Indicators (KPI), including throughput rates, throughput losses due to breakdowns, blocking and starving, Work In Process (WIP) levels, etc., is essential to support day-to-day manufacturing planning and control decisions. As products move through various stages of a manufacturing supply chain, from material procurement to the transportation of finished goods, internal and external disruptions that require adjustments occur in all areas of the process.

In order to compensate for system perturbations and stay competitive, a variety of sensors and plant floor information systems have been invested to improve information visibility and predictability. The large amount of datasets offer an unprecedented opportunity to develop data driven models for the prediction and estimation of system state and performance. However, current prediction applications largely use stationary models or those based on local ensembles. Due to the presence of nonstationarity, near-term predictability of these complex systems deteriorates significantly overtime [5,6].

The present approach exploits the inherent nonlinear stochastic dynamics of complex systems to improve predictive capability under nonlinear and nonstationary conditions. Central to our approach is the segmentation of global performance signature (time series) into multiple stationary segments using recurrence pattern analysis. Stationary segmentation will lead to reduced order models that can capture the local evolution patterns (including the nonlinearity and nonstationarity) better than any global model.

The remainder of this paper is organized as follows: Section 2 reviews the relevant work of state space prediction and recurrence analysis. The proposed research methodology is presented in Section 3. Section 4 demonstrates experimental results of local recurrence prediction model; and Section 5 concludes the reported research.

## 2. Background

This paper addresses the problems of making predictions in the nonlinear system under highly nonstationary conditions. Weak sense stationarity requires the statistical moments up to second order to be constant and all parameters of relevant system dynamics are fixed. The stochastic process $Y_t$ is said to be stationary when the cumulative distribution functions $F(.)$ of the joint distribution of $Y_t$ at times $t_1$, $t_2$, …, $t_k$ do not depend on the shift parameter $\tau$ [7], i.e., for all $k$, for all $\tau$, and for all $t_1$, $t_2$, …, $t_k$,

$$F(y_{t1},y_{t2},\ldots,y_{tk};Y_{t1},Y_{t2},\ldots,Y_{tk}) = F(y_{t1},y_{t2},\ldots,y_{tk};Y_{t1+\tau},Y_{t2+\tau},\ldots,Y_{tk+\tau})$$

But natural phenomena, like behaviors of manufacturing systems, are usually nonstationary. The exhibited signals always

* Corresponding author. Tel.: +1 813 974 5579; fax: +1 813 974 5953.
*E-mail address:* huiyang@eng.usf.edu (H. Yang).

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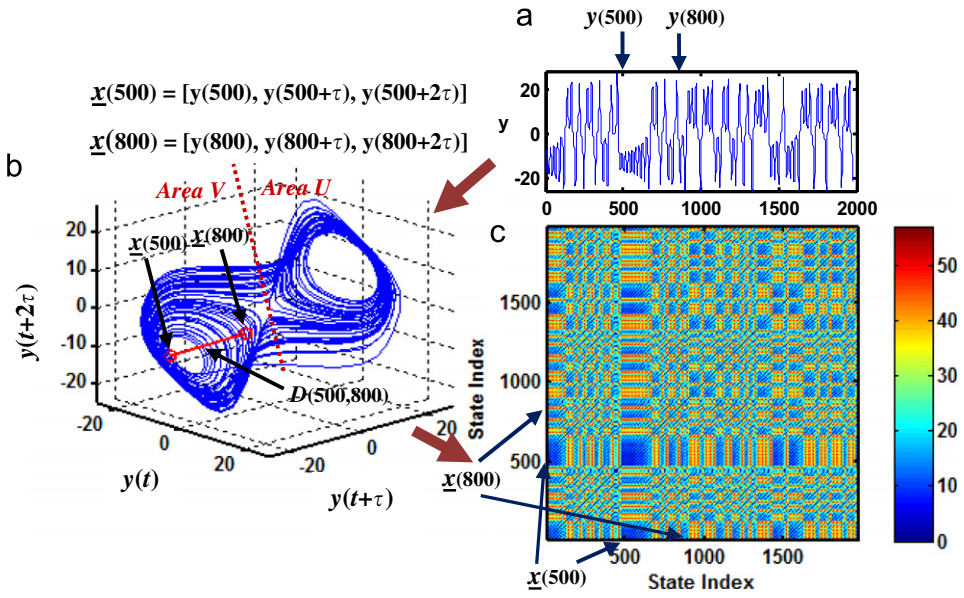



**Fig. 1.** Graphical illustration of the relationship among the Lorenz time series, attractor and recurrence plot. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

alternate between periodic (regular, and laminar) and chaotic (irregular, and turbulent) behaviors in an irregular fashion.

However, much of the complexity in real-world systems is known to emerge from the underlying nonlinear stochastic dynamics. Dynamics often manifests in the vicinity of an attractor A (e.g., a Lorenz attractor [8] shown in Fig. 1), an invariant set defined in an $m$-dimensional state space. Takens' delay embedding theorem [9] shows that system dynamics can be adequately reconstructed using the time-delay coordinates of the individual measurements because of the high dynamic coupling existing in physical system. For discrete time series $y$ (see Fig. 1(a) for illustration), state vector $\underline{x}$ (Fig. 1(b)) is reconstructed using a delay sequence of points in $\{y(t_n)\}$ as,

$$\underline{x}(t_n) = [y(t_n), y(t_n+\tau), y(t_n+2\tau), \ldots, y(t_n+(m-1)\tau)]$$

where $m$ is the embedding dimension and $\tau$ is the time delay. The optimal sufficient embedding dimension $m$ to unfold the attractor is determined by false nearest neighbor method [10]. Mutual information function [11] is used to minimize both nonlinear and linear correlations for the choice of optimal time delay $\tau$.

This present approach is built on a premise, heretofore unexplored, that despite high levels of nonstationarity, system dynamics are structurally stable. Both nonlinearity and nonstationarity are treated to cause finite time detours of system dynamics from the attractor's vicinity in the stochastic processes. Here, finite time detours in system dynamics include not only system frequency variations, drifts in 1st and 2nd statistical moments overtime, but also intermittent low and high dimensional chaotic behaviors resulting from the random fluctuations of the model parameters. While Poincare recurrence theorem implies that if one has a measure preserving transformation, the trajectories eventually reappear at neighborhood $\Omega$ of former points. Thus recurrence dynamics (see Fig. 1(c)) can be used to identify the aforementioned finite time detour structures in the state space. **Poincare recurrence theorem** [12]**:** *Let* T *be a measure-preserving transformation of a probability space* ($\boldsymbol{A}$, $\mu$), *and let* $\Omega \subset \mathbf{A}$ *be a measurable set. Then for any* $J \in N$, $\mu(\{\underline{x} \in \Omega \,|\, \{T^j(\underline{x})\}_{j \geq J} \subset \mathbf{A} \backslash \Omega\}) = 0$.

Recurrence plots delineate the distances of every point $\underline{x}(t_i)$, the state vector realized at time $t_i$, to all the others in the reconstructed state space, i.e., $D(t_i,t_j) : \Theta(\|\underline{x}(t_i) - \underline{x}(t_j)\|)$, where $\|\cdot\|$ is a distance measurement (e.g., the Euclidean norm) and $\Theta(\cdot)$ is the color code that maps the distance to a color scale [13,14]. As shown in Fig. 1, the distance between the 500th and the 800th points in the state space is shown as a color code at the coordinates (500, 800) and (800, 500) of the recurrence plot. If the color code at the recurrence plot is blue, then the points are located close to each other in the state space, and if the color code is red, the points are located farther apart. Thus, a recurrence plot represents the topological relationships existing in the $m$-dimensional state space in the form of 2D images. The ridges locate the nonstationarities and/or the switchings between local behaviors [15], for e.g., the system evolves from one behavior (area $\boldsymbol{U}$) to the other (area $\boldsymbol{V}$) (see Fig. 1). The separations between dark diagonal lines (along 45°) indicate the time periods between the recurring system behaviors over certain time segments.

Recurrence-based methods have of late shown potential for representation and de-noising of measurements from complex systems [16,17]. However, it is necessary to determine the time intervals and state space subsets in which the stationary assumptions hold. Recent nonlinear dynamic forecasting approaches applied to weather and financial market predictions generally proposed to address the nonstationary issues with either in a piecewise quasi-stationary fashion or by means of adaptive recursive algorithms that track the time-varying parameters of the system [1,2,4,18–22]. Local modeling approaches in previous investigations were also shown to yield better performances than the global models for predicting the nonlinear chaotic time series [21,23–25]. This present work is one of the first investigations into the use of recurrence pattern recognition for dividing nonlinear and nonstationary system into piecewise linear and stationary segments. Within each linear and stationary segment, local recurrence characteristics of the present system state $\underline{x}$ are used to predict the future events in the $m$-dimensional space.

A number of previous prediction approaches model the one dimensional process outputs $y$ as a linear or nonlinear function of $p$ past values of $y$ and $q$ previous realizations of independent noise (shock) events $\varepsilon_t$. Such models include AR($p$) ($y(t_{n+1}) = a_0 + \sum_{i=1}^{p} a_i y(t_{n-p+i}) + \varepsilon_n$), ARMA($p,q$) ($y(t_{n+1}) = a_0 +$

$\sum_{i=1}^{p} a_i y(t_{n-p+i}) + \sum_{j=1}^{q} b_j \varepsilon_{n-q+j}$), ARIMA($p,q$) ($\tilde{y}(t_n) = y(t_n) - y(t_n - \tau)$, $\tilde{y}(t_{n+1}) = a_0 + \sum_{i=1}^{p} a_i \tilde{y}(t_{n-p+i}) + \sum_{j=1}^{q} b_j \varepsilon_{n-q+j}$), or neural network ($y(t_n) = f(\sum_{i=1}^{p} w_i \emptyset_i(y(t_{n-p+i})) + b)$). ARMA models assume stationarity and linearity, and ARIMA models incorporate a simple case of nonstationarity, namely low frequency components [26]. These classical time series models cannot capture the complexities in nonlinear chaotic processes. The prediction accuracy of these models deteriorates significantly overtime because of the nonlinear and nonstationary nature of these systems [6]. Neural network models are known to need prohibitively large datasets for prediction under stochastic and highly nonstationary conditions [27]. Polynomial models with global or arbitrary local support tend to be unstable and not adequate for extrapolation [28]. Delay differential equations $\dot{\underline{x}}(t_n) = f(\underline{x}(t_n), \underline{x}(t_n - \tau_1), \cdots, \underline{x}(t_n - \tau_k))$ work well under deterministic conditions, but they pose acute stability issues due to possible trajectory crossing under nonlinear conditions, that is, same future states for different pasts.

## 3. Research methodology

This paper leverages local recurrence characteristics, exhibited over certain time segments, to delineate the various local evolution patterns and nonstationary regimes from attractor $A$ and develop compact local models in the piecewise near stationary segments. Finite time detours represent the deviations from one local area to the other in the state space. Recurrent patterns are distorted from finite time detours and distinguished local topologies. As summarized in Fig. 2, first, a measured time series is embedded in the $m$-dimensional state space. Then finite time detours and local topologies are analyzed using recurrence methods to partition system state space into multiple linear and near-stationary segments. System trajectory strands within each segment bear a similar evolution pattern and stationary assumptions hold therein.

This proposed approach leads to reduced order models that can effectively capture the local evolutions of dynamic system's state space, including nonlinearity and nonstationarity, better than with any stationary model. For illustration purposes, let us consider a Lorenz attractor shown in Fig. 1(b). All trajectories emerging from a region (e.g., $\boldsymbol{U}$) have a similar local evolution pattern. However, these local evolution patterns can be vastly dissimilar for the trajectories emerging from two arbitrary regions, say, $\boldsymbol{U}$ and $\boldsymbol{V}$. Local dynamics within each of these regions can be simplified as unstable periodic orbits (UPOs) defined about certain fixed points, and the global dynamics can be derived from concatenating the trajectories from each of these regions as stated in the following.

**Proposition 1.** *Dynamics about an attractor of complex nonlinear system can be approximated by linear affine systems linked by switching laws,* i.e.,

$$\dot{\underline{x}} = F(\underline{x}) \approx \sum_i f_i[s(\underline{x}) - \gamma_i] A_i(\underline{x} - \underline{x}_i) \tag{1}$$

*where $A_i$ defines the local linear dynamics of affine subsystems, $s(\bullet)$ defines the switching surface, $f_i[\bullet]$ is a Boolean switching function that transits the dynamics among multiple linear or affine systems, each defined over fixed points $\underline{x}_i$ , and $\gamma_i$ determine the locations of the switching surface* [29].

It has recently been shown that the system trajectories within a region $i$ locally evolve as unstable periodic orbits defined about certain saddle or foci type fixed points $\underline{x}_i$ [29]. Such local dynamics for region $i$ can be captured at a specified parameter setting using a linear system of the form $\dot{\underline{x}}^{(i)} = A_i(\underline{x} - \underline{x}_i)$.

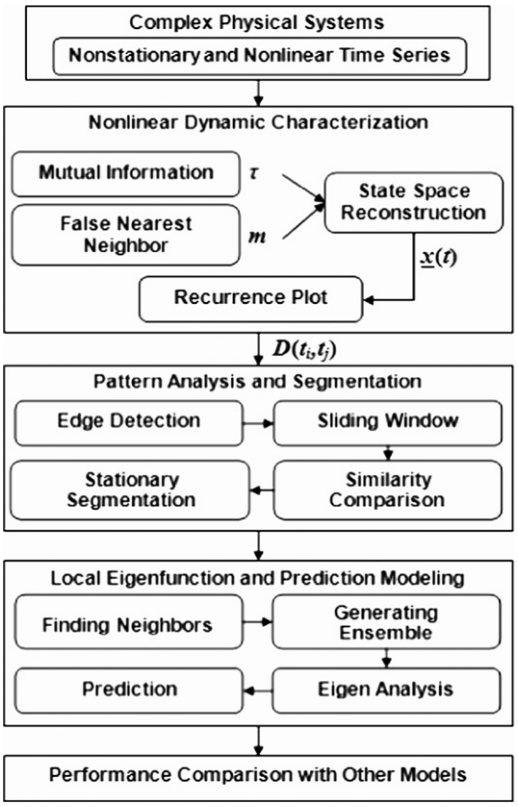



**Fig. 2.** Overall framework of local recurrence prediction model.

The system trajectories in an attractor can be derived from solving this piecewise affine model. The global trajectories switch between the multiple local patterns as they evolve about saddle type fixed points, i.e., $\underline{x} = \underline{x}^{(1)} \oplus \underline{x}^{(2)} \oplus \cdots \oplus \underline{x}^{(Z)}$. For instance, a Lorenz attractor may be approximated using piecewise affine model consisting of two linear components. The Jacobians of the respective linear/affine part are evaluated about the corresponding saddles or foci type fixed points [29]. The saddle type fixed points help determine the boundaries of the orbits defined about the foci.

Switching from one piecewise affine system to another takes place whenever the system trajectories shift from exhibiting an unstable periodic behavior about a fixed point to the other. The Boolean functions $f_i[\bullet]$ is defined as following:

$$f_i[s(\underline{x}) - \gamma_i] = \begin{cases} 1, s(\underline{x}) - \gamma_i \geq 0 \\ 0, s(\underline{x}) - \gamma_i < 0 \end{cases} \tag{2}$$

This function is designed to make sure that only one of them equals unity at a given location $\gamma_i$. A very minor perturbation in the states can switch the evolution path from one region to the other. The points forming the transition states are rare or far from the local regions $\boldsymbol{U}$ and $\boldsymbol{V}$, i.e., $\{\underline{x}(b_i)\}_{i=1}^{B} \subseteq \partial\mathbf{U} \cap \partial\mathbf{V}$. Within regions $\boldsymbol{U}$ and $\boldsymbol{V}$, the trajectory flow lines are laminar, and the divergence rate between closeby trajectories is close to zero. The transition points are marked by significant changes in the divergence rates.

Recurrence patterns can be exploited to systematically identify the transitions. A recurrence plot is symmetric about the diagonal, and patterns are equally distributed along horizontal and vertical directions. If the distribution of states $\underline{x}(t)$ in the embedding space is homogeneous, then the recurrence plot is also homogeneous. The presence of such homogenous recurrence patterns implies a typically stationary process which has short relaxation times in comparison with time spanned by recurrence plot. Interruptions in the homogenous patterns indicate that some states over these times $\{\underline{x}(b_i)\}_{i=1}^{B}$ deviate from the local area of state space and are

no longer interior points of local sets $\boldsymbol{U}$ or $\boldsymbol{V}$. Those states correspond to transition points between various local homogeneous sets [15]. Hence, the disruptions in the recurrence plot can be used to detect finite time detours from local vicinity of attractors, namely nonlinearity and nonstationarity.

The transition points between various local homogeneous sets, resulting from the underlying chaotic dynamics and/or nonstationarities, are detected by applying the image edge detection filters, the Sobel operators $I_u$ and $I_v$, on recurrence plot $D(t_i, t_j)$, where $I_u$ and $I_v$ are defined in terms of a $3\times 3$ matrix pair

$$I_u = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix}, \quad I_v = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \tag{3}$$

The intensity gradient magnitude $G_{u,v}$ at a point $(t_i, t_j)$ in the recurrence plot is given by $G_{u,v} = \sqrt{G_u^2 + G_v^2}$, and $G_u$ and $G_v$ are the horizontal and vertical gradient estimations, respectively, defined as following:

$$G_u = I_u * D \quad \text{and} \quad G_v = I_v * D \tag{4}$$

where $*$ is the convolution or vector product operation.

Thus, we delineate and preserve the important nonhomogeneous structural patterns by applying the Sobel operators to recurrence plots. Boundaries of linear and stationary segments are shown as horizontal and vertical edges in the $G_u(t_i, t_j)$ image. Histogram similarities of adjacent sliding windows are compared to locate these boundaries. Such edge detection methods give prominence to high frequency components in the image, for e.g., the boundaries of objects, the boundaries of surface markings or curves that correspond to discontinuities in surface orientation. The length of each segment depends on the local divergence rate of trajectories emanating from a given neighborhood. As shown in Fig. 3, the transition states or saddle points $\{\underline{x}(b_i)\}_{i=1}^{B}$ are identified in the Lorenz recurrence plot (also refer to Fig. 1). The enlarged portion in Fig. 3 shows the homogeneity of point distributions in each segment.

In this investigation, ensembles are generated within a linear and near-stationary segment to provide a more homogeneous set for eigen analysis. The boundary of each segment $\underline{x}(b_i)$, $i=1,\ldots,B$, is identified from the transition states or saddle points by the edge detection of recurrence plots. The length of each near-stationary segment $L_i$ will be the time interval between $b_i$ and $b_{i+1}$. The $K$ nearest neighbors $\underline{x}^i(t_k)$ $(k=1,\ldots,K)$ of the $i$th segment's starting boundary $\underline{x}(b_i)$ can be discovered by thresholding the recurrence plot with a small distance $r$ as $\tilde{D}(t_i,t_j) : \Theta(\|\underline{x}(t_i)-\underline{x}(t_j)\| < r)$. Thus, we can extract $K$ ensembles as $\{\underline{x}^i(t_k), x^i(t_k+1), \ldots, \underline{x}^i(t_k+L_i)\}_{k=1}^{K}$ from the nearest neighbors $\underline{x}^i(t_k)(k=1,\ldots,K)$ in the $i$th segment. The ensembles of segment length $L_i$ are only collected from the historical data such that $t_K+L_i < N$, where $N$ is the length of time series, and consequently, the model is causal. The eigen representation of the stochastic process $\underline{x}(t)$ in the $i$th segment can be given by

$$\underline{x}(t) = \sum_j \vartheta_j \alpha_j(t) \tag{5}$$

Here, $\vartheta_j$'s are eigen representation coefficients, and the basis function $\alpha_j(t)$ are the linearly independent solutions of

$$\int_R C(t,\tau)\alpha_j(\tau)d\tau = e_j\alpha_j(t) \tag{6}$$

where $C(t,\tau)$ is the autocovariance function calculated from a set of $K$ ensembles. It is evident that $e_j$ are eigenvalues, and $\alpha_j(t)$ are eigenfunctions of $C(t,\tau)$, and are therefore orthogonal. The order of eigenvalues $e_j$, highest to lowest, indicates the components in order of significance [30,31].

As shown in Fig. 4, local eigen representation in the $i$th segment can hence be constructed using the $K$ ensembles of the segment starting boundary $\underline{x}(b_i)$. The system state is extrapolated along the leading eigen directions $\vartheta_j$ to predict the future states $\tilde{\underline{x}}(t) = \sum_j \vartheta_j \alpha_j(t)$ in the last segment. Whenever the system evolves (due to chaos) or drifts (due to nonstationarity) into a different regime, a new local eigen representation will be established to capture the new patterns. Thus, the local eigen representation captures major variations of the ensembles as well as nuances of the ensembles in each stationary segment.

**Proposition 2.** *Local eigen ($L_2$-optimal) representations obtained from segmenting a chaotic attractor are more parsimonious than a global ensemble representation.*

More often than not, the operators $A_i$ defined in Eq. (1) either do not have a full rank, or have a highly ill-conditioned eigen

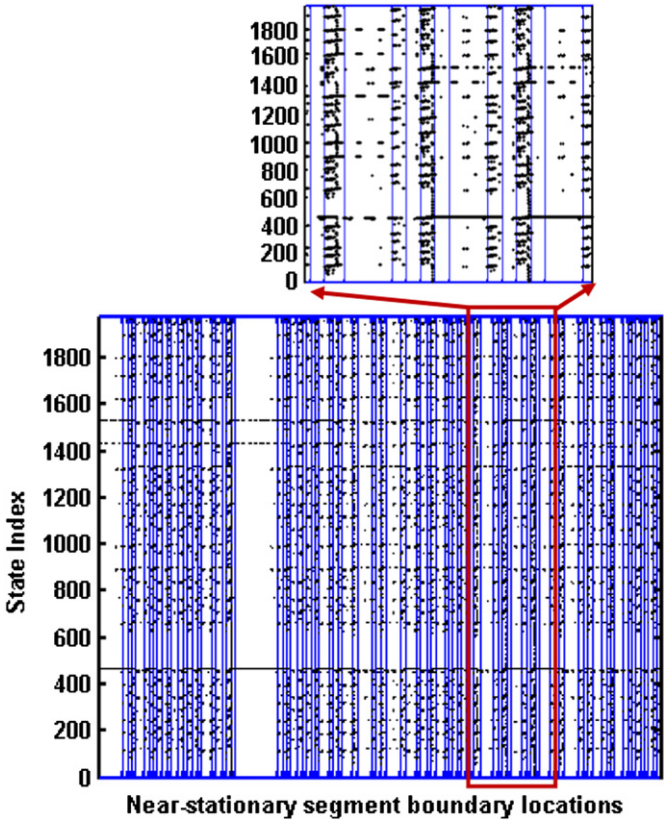



**Fig. 3.** Edge detection in a Lorenz recurrence plot.

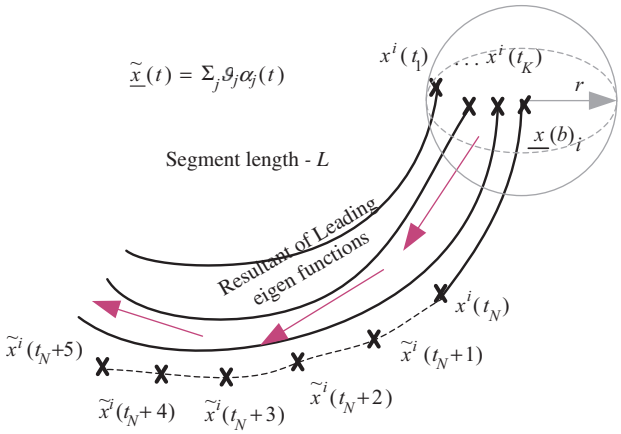



**Fig. 4.** Local eigen prediction model from ensembles within a stationary segment.

system where one or more eigenvalues are significantly lower than the rest. For example, in affine approximation of a Lorenz-like systems [29], the local eigenvectors of $A_i$'s tend to be significantly smaller along the directions normal to the planes of the "butterfly wings," and generally speaking, along the so-called nullclinic directions [29], of a Lorenz attractor compared to the other orthogonal directions. The eigen directions that lie along the nullclinic directions, and hence the signal components projected along these directions, can be ignored. Therefore much of the salient evolutions about a certain fixed point on the attractor can be captured by considering very few eigen basis functions. Consequently, the local representations, defined between two successive "switching points," will be more compact than a representation based on global ensembles. From a noise reduction standpoint, consideration of fewer basis functions implies that signal energy is concentrated along limited number of directions.

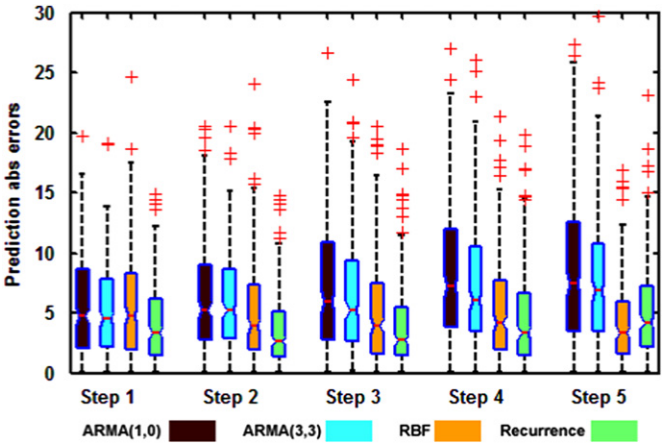



**Fig. 5.** Five step prediction RMS error comparisons for noise contaminated Lorenz time series with various prediction models.

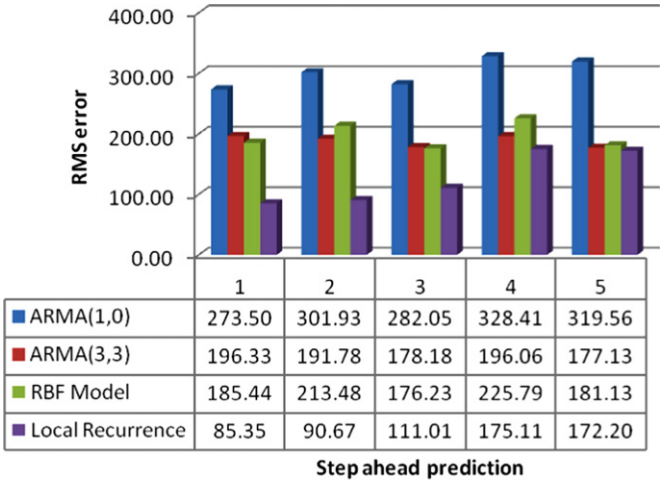
## 4. Results

In this section, we evaluate the performance of local recurrence model using noise contaminated Lorenz and real-world manufacturing system throughput datasets. The prediction results are compared to traditional time series models (e.g. ARMA) and some of the well known nonlinear models (e.g. polynomial, neural network, radial basis function (RBF)). It may be noted that few if any of the previous approaches is suitable for prediction under nonlinear and highly nonstationary conditions. Polynomial models and delay differential equation models are not included for comparison because the experimental results are observed to easily diverge from or fail to predict the actual system behaviors.

The noisy Lorenz time series is generated by contaminating the first component of Lorenz attractor with five different level Gaussian noises (signal noise ratios (SNR) 0–21.35 dB) towards evaluating the local recurrence modeling approach. The prediction results are compared at multiple locations to those from RBF and two classical time series models: ARMA(1,0) – a prediction model currently used in the practice, and ARMA(3,3) – the "optimal" model form emerged from a classical system identification standpoint.

Fig. 5 shows the comparisons of prediction error statistics (in terms of root mean square (RMS) errors) for the noisy Lorenz time series. Since linear models cannot capture the chaotic dynamics underlying the Lorenz time series, we can find that ARMA model predictions progressively deteriorate with prediction look-ahead. RBF model is found to yield lower average prediction errors than two ARMA models. It indicates that RBF model can effectively capture system nonlinearity. But in Fig. 5, local recurrence model has smallest mean and standard deviation of prediction errors among all the models considered, especially in steps 1–4 ahead. It may also be noted that prediction errors from the local recurrence model do not increase with prediction steps, which is because eigen analysis of ensembles in a near-stationary segment guarantees that predictions always lie within the bounds of the reconstructed attractor.

The proposed local recurrence modeling approach was also used for predicting the outputs of a real-world automotive power train manufacturing system [32,33], which consists of 18 stations of which 17 are allocated in tandem. One pair of stations is located in a parallel arrangement in the assembly line. The system is known to be highly nonstationary and nonlinear, and previous prediction attempts only have limited success. The throughput data considered for the present investigation is approximately one-year long period. Fig. 6 shows the local recurrence model




| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| ARMA(1,0) | 273.50 | 301.93 | 282.05 | 328.41 | 319.56 |
| ARMA(3,3) | 196.33 | 191.78 | 178.18 | 196.06 | 177.13 |
| RBF Model | 185.44 | 213.48 | 176.23 | 225.79 | 181.13 |
| Local Recurrence | 85.35 | 90.67 | 111.01 | 175.11 | 172.20 |

**Fig. 6.** Five step ahead prediction RMS error comparisons for a real-world manufacturing station throughput time series with various prediction models (ARMA(1,0), ARMA(3,3), RBF model and local recurrence model).

prediction RMS errors are significantly lower than all the other three models. The step one prediction RMS errors are reduced by 105% compared to ARMA(1,0), 79% to ARMA(3,3), and 74% to RBF which are calculated in the percent difference form ($\%\text{Diff} = 2 \times |p_1 - p_2|/(p_1 + p_2)$, where $p_1$, $p_2$ are prediction RMS errors for two models and *%Diff* is percent difference). As shown in Fig. 7, ARMA(1,0) is excessively sensitive to more noise effects, and ARMA(3,3) and RBF model smooth out underlying nonlinear dynamics for the real-world throughput time series in a manufacturing system. The local recurrence model is able to capture the system nonlinear dynamics and nonstationarity, avoid the influences of extraneous phenomena, and thus enhance prediction accuracies. It is conceivable that the present approach is applicable to a variety of complex system prediction scenarios where consideration of nonstationarity becomes necessary.

## 5. Conclusions

This paper proposes a local recurrence model for making predictions in the nonlinear systems under highly nonstationary conditions, which is one of the first attempts to use localized

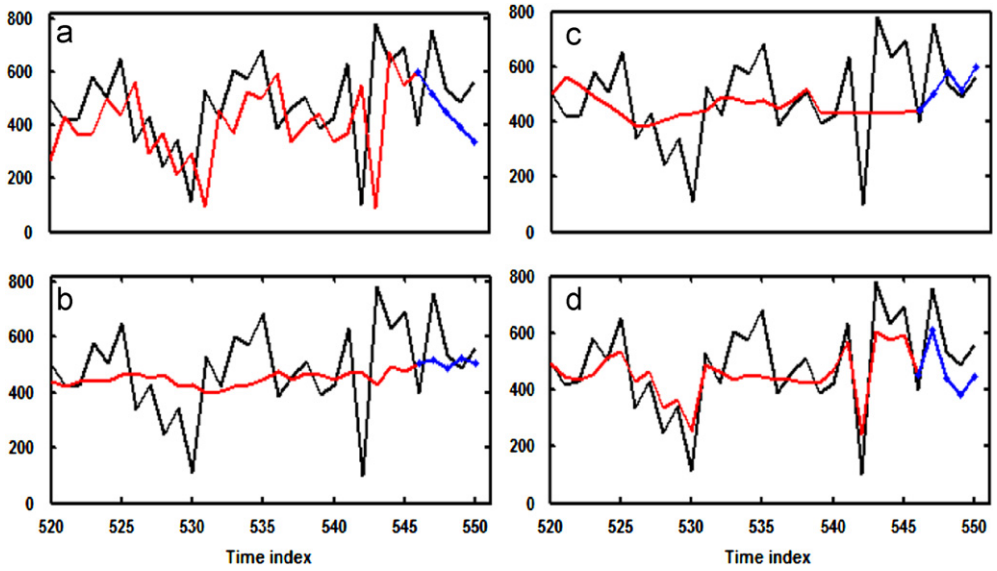



**Fig. 7.** Given throughput measured signals (black solid line) vs. estimates (red line—model outputs, blue star—predictions from (a) ARMA(1,0); (b) ARMA(3,3); (c) RBF model; and (d) local recurrence model). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

recurrence pattern analysis for prediction purposes. It combines the principles of statistical pattern estimation with dynamic systems theory. Most of current recurrence analysis studied the patterns of nearest neighbors $\tilde{D}(t_i,t_j) : \Theta(\|\underline{x}(t_i)-\underline{x}(t_j)\| < r)$. In contrast, the proposed approach exploits the topological irregularity information contained in recurrence plots, and uses pattern recognition methods for stationary segmentation. The transition states that occur when the system behaviors are switched have found to be effectively captured. Experimentations using simulated and real-world datasets reveal the superiority of local recurrence model over other alternative models under nonlinear and nonstationary conditions. In particular, more than 70% accuracy improvements in one step ahead prediction RMS errors are shown for the real-world case study. It is conceivable that such a method can be applied to improve predictability of similar complex physical systems under nonstationary conditions.

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**Hui Yang** was born in Nanjing, Jiangsu province, PR China. He received his Ph.D. degree from the School of Industrial Engineering and Management at Oklahoma State University. He is an Assistant Professor in the Department of Industrial and Management Systems Engineering at the University of South Florida. His research interests are sensor based computational modeling and analysis of complex systems with special focus on nonlinear stochastic dynamics, and the resulting chaos, multifractal, self-organization, long range dependence behaviors.

He is a member of IEEE (Institute of Electrical and Electronics Engineers), INFORMS (Institute for Operations Research and Management Science), and IIE (Institute of Industrial Engineers).

**Satish T.S. Bukkapatnam** was born in Tirupati, India. He received the Ph.D. degree in industrial and manufacturing engineering from Pennsylvania State University, University Park. He is a Professor in the School of Industrial Engineering and Management, Oklahoma State University.

He has previously served as a summer Faculty Consultant at the US Naval Research Laboratory and an Assistant Professor of Industrial and Systems Engineering at the University of Southern California (USC). His research in sensor-based modeling is a fundamentally new approach to improve quality and performance of manufacturing machines and processes, and other real-world complex systems. These systems include structural systems, the Internet, and a variety of infrastructure systems of interest to IT, transportation, medical, and defense enterprises. This approach is based on augmenting the statistical and intelligent systems foundations of modern monitoring technologies with nonlinear dynamics principles.

Dr. Bukkapatnam was a recipient of Alpha Pi Mu/Omega Rho Outstanding Teacher of the Year in Industrial Systems Engineering, USC, in 2002, and the SME Dougherty Outstanding Young Manufacturing Engineer Award.

**Leandro G. Barajas** (S'95–M'99–SM'06) was born in Bogotá, Colombia, in 1973. He received the Honor degree in electronics engineering as Valedictorian from the Universidad Distrital F.J.C., Bogotá, Colombia, in 1998, and the M.S. and Ph.D. degrees in electrical and computer engineering from the Georgia Institute of Technology (Georgia Tech.), Atlanta, in 2000 and 2003, respectively.

He is currently a Staff Researcher at the General Motors R&D Center, Manufacturing Systems Research Laboratory, Warren, MI, where he focuses on the area of robotics and intelligent perception. During his graduate studies, he worked on surface mount technology manufacturing control at Georgia Tech's Manufacturing Research Center (MARC).

Dr. Barajas is a Senior Member and former Chapter 069 Programs Chair of the Society of Manufacturing Engineers (SME), a Member of SAE International, and Elected Full Member & GM Chapter President of SIGMA XI (The Scientific Research Society). In 2000 and 2003, he received M.S. and Ph.D. OMED Tower Awards from Georgia Tech. During his tenure at GM he has been distinguished with the 2005 GM R&D "Spark-Plug" Award, two (2) 2006 GM "Boss" Kettering Awards (GM's highest technical honor), the 2006 GM Chairman's Honors Award, the 2006 GM R&D Center People's Choice Award, the 2006 and 2008 GM R&D Charles L. McCuen Special Achievement Innovation Awards, the 2007 SME Kuo K. Wang Outstanding Young Manufacturing Engineer Award, the 2008 GM Inventor Milestone Award, and with the 2008 GM Intellectual Property Gold Award.