第7週レポート

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(解答)

$$K_1 = \frac{1}{2} m_1 \dot{s}_1^2 + \frac{1}{2} \hat{I}_1 \dot{\theta}_1^2$$

$$K_2 = \frac{1}{2} m_2 \dot{s}_2^2 + \frac{1}{2} \hat{I}_2 (\dot{\theta}_1 + \dot{\theta}_2)^2$$

$$s_1 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} Lg_1 \cos\theta_1 \\ Lg_1 \sin\theta_1 \end{bmatrix}, \quad \dot{s}_1 = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -Lg_1 \sin\theta_1 \dot{\theta}_1 \\ Lg_1 \cos\theta_1 \dot{\theta}_1 \end{bmatrix}$$

$$s_2 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} L_1 \cos\theta_1 + Lg_2 \cos(\theta_1 + \theta_2) \\ L_1 \sin\theta_1 + Lg_2 \sin(\theta_1 + \theta_2) \end{bmatrix}$$

$$\dot{s}_2 = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -L_1 \sin\theta_1 - Lg_2 \sin(\theta_1 + \theta_2) & -Lg_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos\theta_1 + Lg_2 \cos(\theta_1 + \theta_2) & Lg_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$K_1 = \frac{1}{2} (m_1 Lg_1^2 + \hat{I}_1) \dot{\theta}_1^2, \quad K_2 = \frac{1}{2} \Big(m_2 \big((L_1^2 + Lg_2^2 + 2 L_1 Lg_2 \cos\theta_2) \dot{\theta}_1^2 + Lg_2^2 \dot{\theta}_2^2 + 2 (Lg_2^2 + L_1 Lg_2 \cos\theta_2) \dot{\theta}_1 \dot{\theta}_2 \big) + \hat{I}_2 (\dot{\theta}_1 + \dot{\theta}_2)^2 \Big)$$

$$P_1 = m_1 g Lg_1 \sin\theta_1, \quad P_2 = m_2 g \big(L_1 \sin\theta_1 + Lg_2 \sin(\theta_1 + \theta_2) \big)$$

Lagrange 関数 $L = K - P = (K_1 + K_2) - (P_1 + P_2)$

$$\tau_1 = \frac{d}{dt} \left[\frac{\partial L}{\partial \dot{q}_1} \right] - \frac{\partial L}{\partial q_1} = \frac{d}{dt} \left(\frac{\partial K}{\partial \dot{q}_1} \right) - \left(\frac{\partial K}{\partial q_1} - \frac{\partial P}{\partial q_1} \right)$$

$$= (\hat{I}_1 + \hat{I}_2 + 2 m_2 L_1 Lg_2 \cos\theta_2 + m_2 L_1^2 + m_1 Lg_1^2 + m_2 Lg_2^2) \ddot{\theta}_1 + (\hat{I}_2 + m_2 L_1 Lg_2 \cos\theta_2 + m_2 Lg_2^2) \ddot{\theta}_2$$
$$- m_2 L_1 Lg_2 \sin\theta_2 (2 \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) + \{ (m_1 Lg_1 + m_2 L_1) \cos\theta_1 + m_2 Lg_2 \cos(\theta_1 + \theta_2) \} g$$

$$\tau_2 = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_2} \right) - \frac{\partial L}{\partial q_2} = \frac{d}{dt} \left(\frac{\partial K}{\partial \dot{q}_2} \right) - \left(\frac{\partial K}{\partial q_2} - \frac{\partial P}{\partial q_2} \right)$$

$$= (\hat{I}_2 + m_2 L_1 Lg_2 \cos\theta_2 + m_2 Lg_2^2) \ddot{\theta}_1 + (\hat{I}_2 + m_2 Lg_2^2) \ddot{\theta}_2 + m_2 L_1 Lg_2 \sin\theta_2 \dot{\theta}_1^2 + m_2 Lg_2 \cos(\theta_1 + \theta_2) g$$

$I_1 = m_1 Lg_1^2 + \hat{I}_1$, $I_2 = m_2 Lg_2^2 + \hat{I}_2$ とおく.

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} I_1 + I_2 + 2 m_2 L_1 Lg_2 \cos\theta_2 + m_2 L_1^2 & I_2 + m_2 L_1 Lg_2 \cos\theta_2 \\ I_2 + m_2 L_1 Lg_2 \cos\theta_2 & I_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$
$$+ \begin{bmatrix} -m_2 L_1 Lg_2 \sin\theta_2 (2 \dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) \\ m_2 L_1 Lg_2 \sin\theta_2 \dot{\theta}_1^2 \end{bmatrix}$$
$$+ \begin{bmatrix} (m_1 Lg_1 + m_2 L_1) \cos\theta_1 + m_2 Lg_2 \cos(\theta_1 + \theta_2) \\ m_2 Lg_2 \cos(\theta_1 + \theta_2) \end{bmatrix} g$$