ロボットアームの運動学: 理論
Robot Arm Kinematics: Theory

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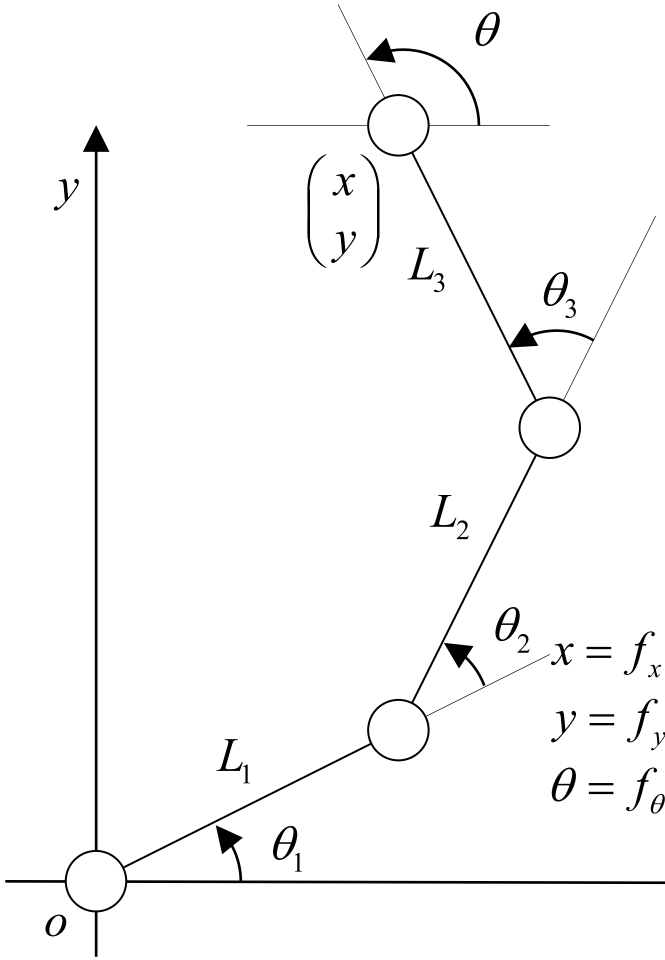
ロボットアームの順運動学
Forward Kinematics of Robot Arm

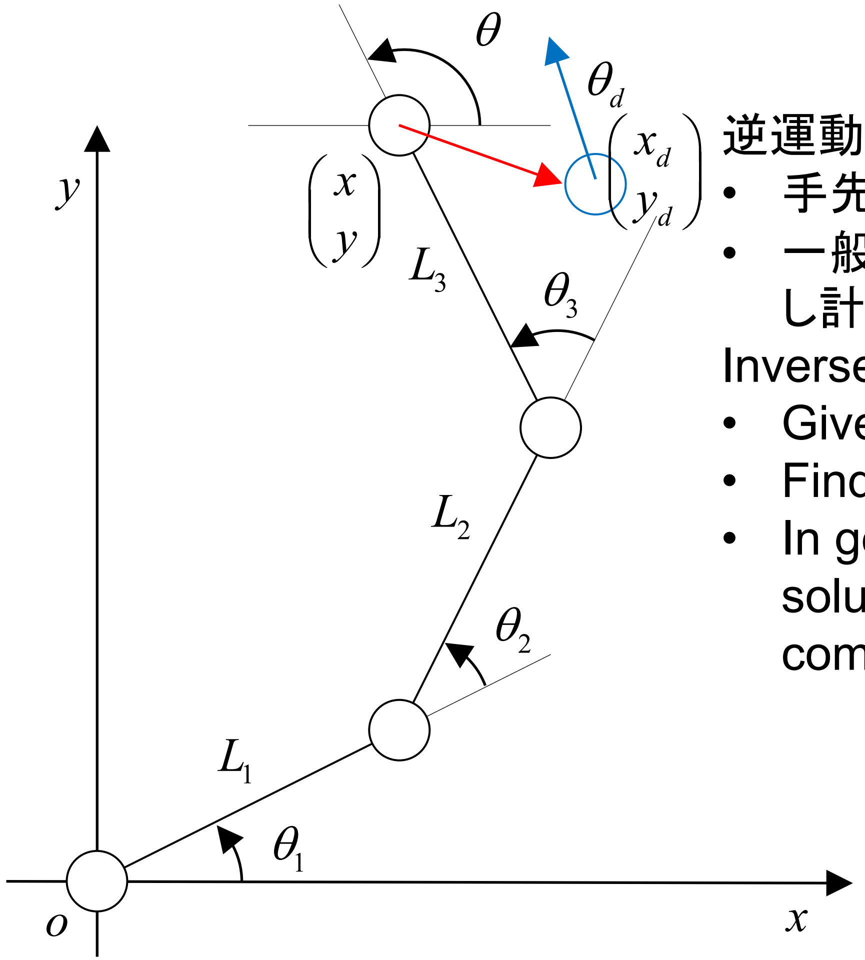


順運動学
- 関節角度から手先の姿勢(位置と向き)を求める
- 閉じた形式の解が求められる

Forward kinematics
- Given: Joint angles
- Find: A hand pose (position and orientation)
- We can find a closed form solution

$$x = f_x(\theta_1, \theta_2, \theta_3) = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y = f_y(\theta_1, \theta_2, \theta_3) = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$
$$\theta = f_\theta(\theta_1, \theta_2, \theta_3) = \theta_1 + \theta_2 + \theta_3$$

ロボットアームの逆運動学
Inverse Kinematics of Robot Arm



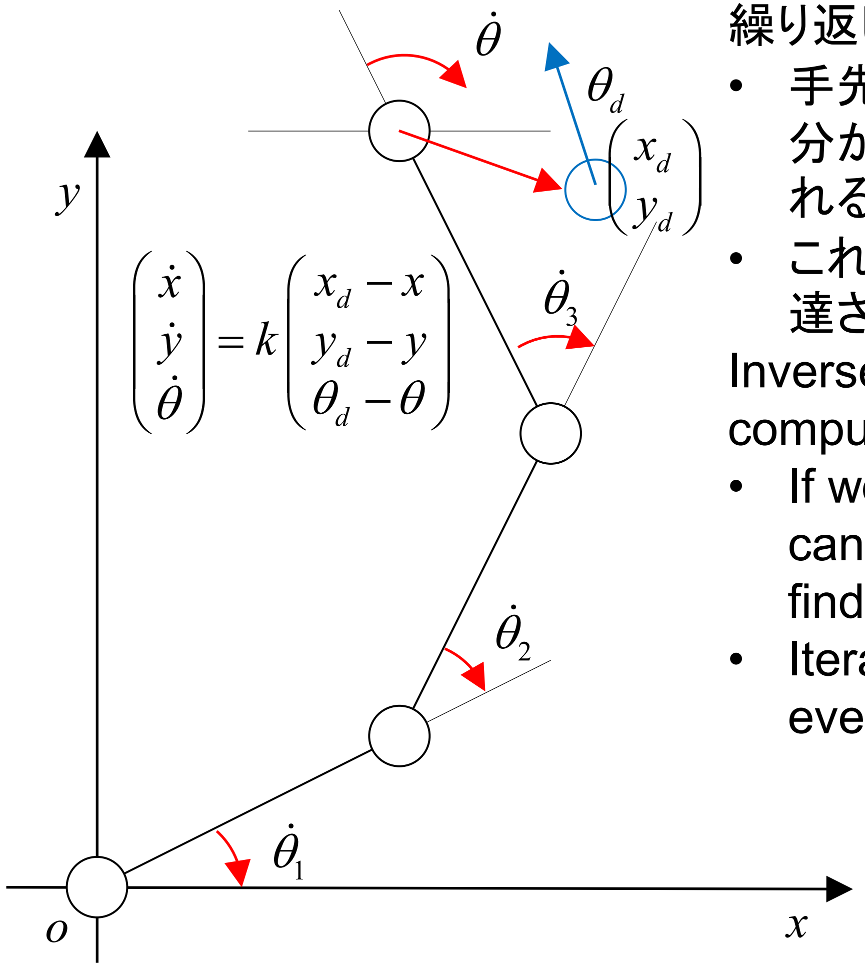
逆運動学
- 手先の位置と向きから関節角度を求める
- 一般に閉じた形式の解は存在しない．繰り返し計算で求める

Inverse kinematics
- Given: A hand position and orientation
- Find: Joint angles
- In general, we cannot find a closed form solution. We solve it by iterative computations

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} f_1(x, y, \theta) \\ f_2(x, y, \theta) \\ f_3(x, y, \theta) \end{pmatrix} ?$$

繰り返し計算による逆運動学解法

Inverse Kinematics Solution by Iterative Computation



繰り返し計算による逆運動学解法

- 手先を動かしたい方向，すなわち手先速度が分かれば，それを実現する関節速度が求められる
- これの繰り返しにより，手先の目標位置に到達させることができる

Inverse kinematics solution by iterative computation

- If we know a hand direction to move, we can interpret it as a hand velocity, and can find joint angle velocity
- Iterating it, a hand can be reached a target eventually

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = J \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} \longrightarrow \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} = J^{-1} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}$$

手先速度と関節速度とヤコビアン
Velocity and Jacobian

逆運動学解を求めるために，手先速度と関節速度の関係を考察する
Let us consider a relation between hand velocity and joint angle velocity

全微分の原理より次が成り立つ
From the principle of total derivative, the following holds

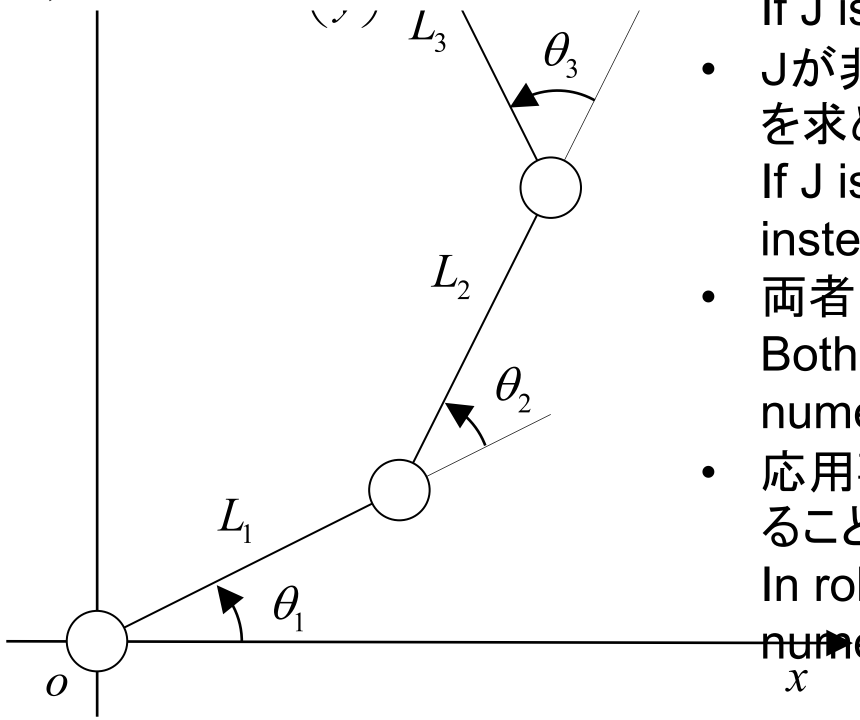
$$\frac{dx}{dt}=\frac{\partial x}{\partial \theta_1}\frac{d\theta_1}{dt}+\frac{\partial x}{\partial \theta_2}\frac{d\theta_2}{dt}+\frac{\partial x}{\partial \theta_3}\frac{d\theta_3}{dt}$$
$$\frac{dy}{dt}=\frac{\partial y}{\partial \theta_1}\frac{d\theta_1}{dt}+\frac{\partial y}{\partial \theta_2}\frac{d\theta_2}{dt}+\frac{\partial y}{\partial \theta_3}\frac{d\theta_3}{dt}$$
$$\frac{d\theta}{dt}=\frac{\partial \theta}{\partial \theta_1}\frac{d\theta_1}{dt}+\frac{\partial \theta}{\partial \theta_2}\frac{d\theta_2}{dt}+\frac{\partial \theta}{\partial \theta_3}\frac{d\theta_3}{dt}$$

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{pmatrix}=\begin{pmatrix}\frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3}\\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3}\\ \frac{\partial \theta}{\partial \theta_1} & \frac{\partial \theta}{\partial \theta_2} & \frac{\partial \theta}{\partial \theta_3}\end{pmatrix}\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{pmatrix}=J\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{pmatrix}$$

手先速度と関節速度が行列Jで関係付けられた．このJはヤコビアンと呼ばれる
Now, hand and joint velocity is associated with J, which is called Jacobian

記号的ヤコビアンの例
Instance of Symbolic Jacobian

$$J=\begin{pmatrix}\frac{\partial x}{\partial\theta_1} & \frac{\partial x}{\partial\theta_2} & \frac{\partial x}{\partial\theta_3}\\ \frac{\partial y}{\partial\theta_1} & \frac{\partial y}{\partial\theta_2} & \frac{\partial y}{\partial\theta_3}\\ \frac{\partial\theta}{\partial\theta_1} & \frac{\partial\theta}{\partial\theta_2} & \frac{\partial\theta}{\partial\theta_3}\end{pmatrix}=\begin{pmatrix}-L_1\sin\theta_1-L_2\sin(\theta_1+\theta_2)-L_3\sin(\theta_1+\theta_2+\theta_3) & -L_2\sin(\theta_1+\theta_2)-L_3\sin(\theta_1+\theta_2+\theta_3) & -L_3\sin(\theta_1+\theta_2+\theta_3)\\ L_1\cos\theta_1+L_2\cos(\theta_1+\theta_2)+L_3\cos(\theta_1+\theta_2+\theta_3) & L_2\cos(\theta_1+\theta_2)+L_3\cos(\theta_1+\theta_2+\theta_3) & L_3\cos(\theta_1+\theta_2+\theta_3)\\ 1 & 1 & 1\end{pmatrix}$$



- Jが正方行列なら逆行列を求めることができる
  If J is square, we can find an inverse of J
- Jが非正方行列なら，その代わりに疑似逆行列を求めることができる
  If J is not, we can find a pseudo inverse of J, instead.
- 両者は記号的にも数値的に求めることができる
  Both can be found symbolically and numerically
- 応用事例では順運動学を用いて数値的に求めることが多い
  In robotics applications, we frequently find J numerically with forward kinematics function.

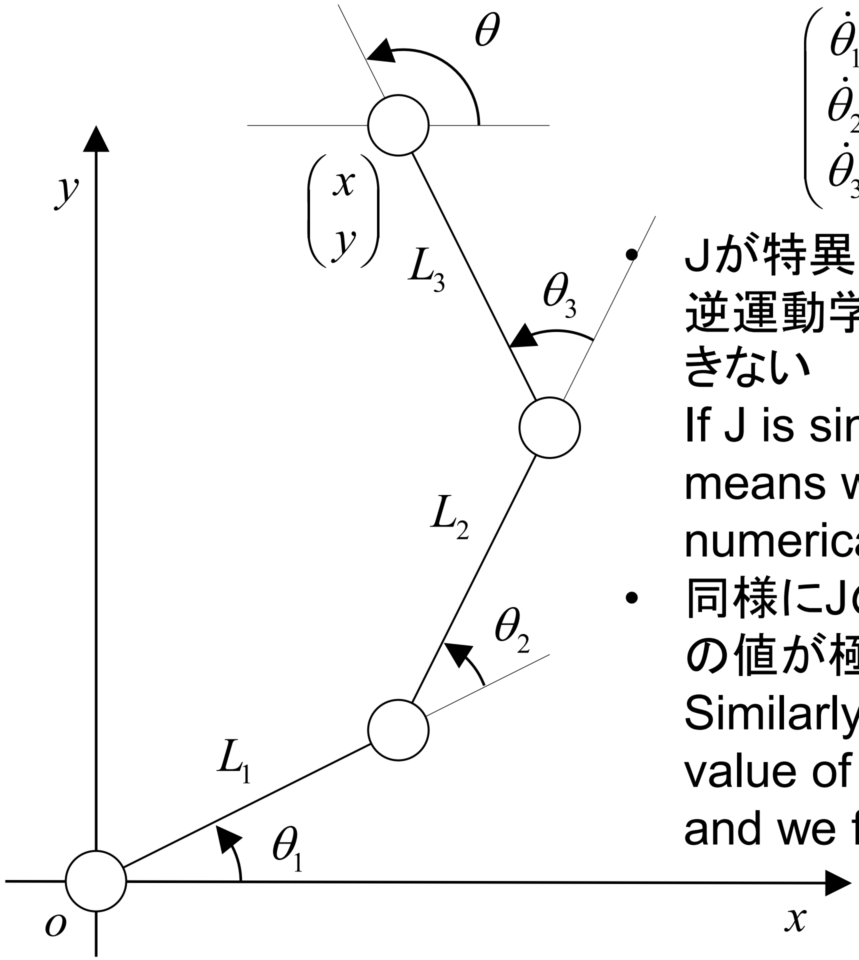
数値的ヤコビアンの求め方

Method to find Numerical Jacobian

$$J = \begin{pmatrix} \dfrac{f_x(\theta_1+\Delta\theta_1,\theta_2,\theta_3)-f_x(\theta_1,\theta_2,\theta_3)}{\Delta\theta_1} & \dfrac{f_x(\theta_1,\theta_2+\Delta\theta_2,\theta_3)-f_x(\theta_1,\theta_2,\theta_3)}{\Delta\theta_2} & \dfrac{f_x(\theta_1,\theta_2,\theta_3+\Delta\theta_3)-f_x(\theta_1,\theta_2,\theta_3)}{\Delta\theta_3} \\ \dfrac{f_y(\theta_1+\Delta\theta_1,\theta_2,\theta_3)-f_y(\theta_1,\theta_2,\theta_3)}{\Delta\theta_1} & \dfrac{f_y(\theta_1,\theta_2+\Delta\theta_2,\theta_3)-f_y(\theta_1,\theta_2,\theta_3)}{\Delta\theta_2} & \dfrac{f_y(\theta_1,\theta_2,\theta_3+\Delta\theta_3)-f_y(\theta_1,\theta_2,\theta_3)}{\Delta\theta_3} \\ \dfrac{f_\theta(\theta_1+\Delta\theta_1,\theta_2,\theta_3)-f_\theta(\theta_1,\theta_2,\theta_3)}{\Delta\theta_1} & \dfrac{f_\theta(\theta_1,\theta_2+\Delta\theta_2,\theta_3)-f_\theta(\theta_1,\theta_2,\theta_3)}{\Delta\theta_2} & \dfrac{f_\theta(\theta_1,\theta_2,\theta_3+\Delta\theta_3)-f_\theta(\theta_1,\theta_2,\theta_3)}{\Delta\theta_3} \end{pmatrix}$$

$$\frac{\partial x}{\partial\theta_1} = \lim_{\Delta\theta_1\to 0}\frac{f_x\left(\theta_1+\Delta\theta_1,\theta_2,\theta_3\right)-f_x\left(\theta_1,\theta_2,\theta_3\right)}{\Delta\theta_1}$$

数値計算の不安定性
Unstable Numerical Solution



$$\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} = J^{-1} \begin{pmatrix} \dot{x} \\ y \\ \dot{\theta} \end{pmatrix}$$

- Jが特異行列のときは逆行列は求められないため，逆運動学解は存在するにも関わらず求めることができない
  If J is singular, an inverse does not exist, which means we cannot solve inverse kinematics numerically even if a solution exists
- 同様にJの固有値が非常に小さいときはJの逆行列の値が極端に大きくなり，数値計算が不安定になる
  Similarly, if an eigenvalue of J is very small, a value of J inverse becomes significantly large, and we find unstable numerical solution