MAE 541 / APC 571: Applied Dynamical Systems

Homework 2

Assigned: 5 Feb 2026
Due: 12 Feb 2026

1. (42 points) (Modification of Ex. 1.3.2 in GH) For each of the following systems, find all of the fixed points and classify their local stability using linearization. At each fixed point, compute eigenvalues and eigenvectors of the linearization and sketch (by hand) the trajectories of the local flow near the fixed points. Then use a computer to plot sample trajectories (e.g., by using `matplotlib.pyplot.streamplot`, or `scipy.integrate.solve_ivp` in Python) and sketch the global phase portrait. [*Hint:* Be sure to first rewrite second-order equations as first-order systems as we did in lecture, and in each case consider $\varepsilon > 0, \varepsilon = 0$, and $\varepsilon < 0$.]
   (a) $\ddot{x} + \varepsilon\dot{x} - x + x^3 = 0$
   (b) $\ddot{x} + \varepsilon\dot{x}^3 + \sin x = 0$
   (c) $\dot{x} = -x + x^2,\ \dot{y} = x + y$

2. (20 points) *Linear vector fields and maps.*
   (a) For the following linear ODE, compute the stable, unstable, and center subspaces E^s, E^u, E^c of the origin. Sketch these subspaces and some representative trajectories of the system in the phase space.
   $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$
   (b) For the following linear map on $\mathbb{R}^2$, compute the stable, unstable, and center subspaces E^s, E^u, E^c of the origin. What are the qualitative differences in the dynamics between $\lambda < 0$, $\lambda = 0$, and $\lambda > 0$? Sketch the subspaces and some representative trajectories of the system in the phase space.
   $$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} 1 & -1 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad |\lambda| < 1.$$

3. (38 points) For the following dynamical systems, find the fixed point(s) and compute the associated stable and unstable manifolds, up to fourth order (that is, if you consider an approximation $y = h(x)$, let h be a quartic polynomial). Use these results to sketch the global phase portraits.
   (a) For the following ODE, assume $\lambda > 0$.
   $$\begin{aligned} \dot{x} &= x(1 - x^2) \\ \dot{y} &= -\lambda(y - x^2). \end{aligned}$$

(b) For the following map, assume $|\lambda| < 1$ and $|\mu| > 1$.

$$\begin{aligned} x &\mapsto \lambda x, \\ y &\mapsto \mu y - x^3. \end{aligned}$$