Problem

(30 points) Do Exercise 3.2.4 on page 134 of Guckenheimer & Holmes. Sketch graphs of the center and stable subspaces and of the center manifold and indicate the stability type of the equilibrium at the origin. Use numerical software (e.g., Python, MATLAB) to plot your center manifold approximation on a phase portrait of the system dynamics.

(Exercise 3.2.4) Find the approximation $\dot{x} = Bx + f(x, h(x))$ for the reduced system on W^c near the origin, in the following case:

$$\dot{x} = \alpha x^2 - y^2,$$
$$\dot{y} = -y + x^2 + xy;$$

First suppose $\alpha \neq 0$, and then let $\alpha = 0$. Carry the series approximations for h sufficiently far to determine the stability of the reduced system.

Notes

Solution