Problem

(20 points) Consider the system of ODEs given by

$$\dot{x} = -y + ax(x^2 + y^2)$$
$$\dot{y} = x + ay(x^2 + y^2)$$

(a) Find all the equilibrium points for this system (for any value of the parameter a).

(b) Find the equations linearized about $(x, y) = (0, 0)$. What does the linearization let you conclude about the stability of the equilibrium point (for the nonlinear system)?

(c) Write the system (1) in polar coordinates (r, θ), with

$$x = r\cos\theta$$
$$y = r\sin\theta$$

and determine the stability type of the equilibrium at the origin. Consider separately the cases $\alpha < 0$, $\alpha = 0$, and $\alpha > 0$.

Notes

Solution
