MAE 541 / APC 571: Applied Dynamical Systems

Homework 4

Assigned: 19 Feb 2026
Due: 26 Feb 2026

1. (40 points) Consider the ODE

$$\frac{dx}{dt} = \nu_0 + \nu_1 x + \nu_2 x^2 + \nu_3 x^3$$

where $\nu_3 \neq 0$. Show that this system can be transformed into

$$\frac{dy}{d\tau} = \mu_0 + \mu_1 y - y^3$$

by setting $x = \alpha y + \beta$ and $t = \gamma\tau$ (affine rescaling of state space, and time scale change). Thus, the second system may be regarded as a "universal unfolding" of the degenerate system $\dot{x} = -x^3$. Find suitable α, β, γ and derive relationships between the new (μ_j) and old (ν_j) parameters. Can the number of parameters be further reduced?

Analyze the second (simpler) system and sketch bifurcation diagrams in the form of a surface of equilibria over the (μ_0, μ_1)-parameter space, as well as branches of equilibria in (μ_1, y) space for "well-chosen" constant μ_0 slices. Find and indicate stability types of all equilibria.

[*Hint:* Start by setting $\mu_0 = 0$; then move on to small $\mu_0 \neq 0$.]

2. (30 points) Do Exercise 3.2.4(a) on page 134 of Guckenheimer & Holmes. Sketch graphs of the center and stable subspaces and of the center manifold and indicate the stability type of the equilibrium at the origin. Use numerical software (e.g., Python, MATLAB) to plot your center manifold approximation on a phase portrait of the system dynamics.

3. (30 points) Do Exercise 3.2.8(c) on page 137 of Guckenheimer & Holmes (ignore the "describe their bifurcations" phrase). As in the previous problem, sketch graphs of the center and stable subspaces and the center manifold, and indicate the stability type of the equilibrium. Use numerical software to plot the center manifold approximation along with some representative trajectories of the system's dynamics.