Problem

(38 points) For the following dynamical systems, find the fixed point(s) and compute the associated stable and unstable manifolds, up to fourth order (that is, if you consider an approximation $y = h(x)$, let h be a quartic polynomial). Use these results to sketch the global phase portraits.

(a) For the following ODE, assume $\lambda > 0$.

$$\dot{x} = x(1 - x^2)$$
$$\dot{y} = -\lambda(y - x^2).$$

(b) For the following map, assume $|\lambda| < 1$ and $|\mu| > 1$.

$$x \mapsto \lambda x,$$
$$y \mapsto \mu y - x^3.$$

Notes

Solution