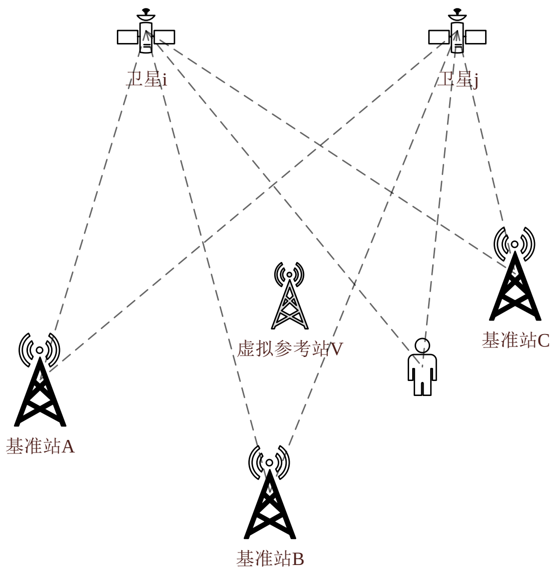
本文简要推导 VRS 虚拟参考站观测值生成的基本算法[1];

假设在 t 时刻，流动站 u 和基准站 A、B、C 几乎同时观测到 n 颗卫星播发的 L1 C/A 卫星信号，L1 C/A 信号波长为λ，则伪距观测值为

$$
\begin{aligned}
\rho_u^i &= r_u^i + cdt_u - cdt^i + \mathrm{I}_u^i + \mathrm{T}_u^i + \epsilon_u^i \\
\rho_A^i &= r_A^i + cdt_u - cdt^i + \mathrm{I}_A^i + \mathrm{T}_A^i + \epsilon_A^i \\
\rho_B^i &= r_B^i + cdt_u - cdt^i + \mathrm{I}_B^i + \mathrm{T}_B^i + \epsilon_B^i \\
\rho_C^i &= r_C^i + cdt_u - cdt^i + \mathrm{I}_C^i + \mathrm{T}_C^i + \epsilon_C^i
\end{aligned}
$$

式中，$i = 1,2, \dots \dots, n$;



以卫星 j 为参考卫星，A 为主参考站;
假设

$$
\begin{cases}
\mathrm{I}_u^i = \mathrm{F}_\mathrm{I}^i(r_u, r^i) \\
\mathrm{I}_A^i = \mathrm{F}_\mathrm{I}^i(r_A, r^i) \\
\mathrm{I}_B^i = \mathrm{F}_\mathrm{I}^i(r_B, r^i) \\
\mathrm{I}_C^i = \mathrm{F}_\mathrm{I}^i(r_C, r^i)
\end{cases},
\quad
\begin{cases}
\mathrm{I}_u^j = \mathrm{F}_\mathrm{I}^j(r_u, r^j) \\
\mathrm{I}_A^j = \mathrm{F}_\mathrm{I}^j(r_A, r^j) \\
\mathrm{I}_B^j = \mathrm{F}_\mathrm{I}^j(r_B, r^j) \\
\mathrm{I}_C^j = \mathrm{F}_\mathrm{I}^j(r_C, r^j)
\end{cases}
$$

$$
\begin{cases}
\mathrm{T}_u^i = \mathrm{F}_\mathrm{T}^i(r_u, r^i) \\
\mathrm{T}_A^i = \mathrm{F}_\mathrm{T}^i(r_A, r^i) \\
\mathrm{T}_B^i = \mathrm{F}_\mathrm{T}^i(r_B, r^i) \\
\mathrm{T}_C^i = \mathrm{F}_\mathrm{T}^i(r_C, r^i)
\end{cases},
\quad
\begin{cases}
\mathrm{T}_u^j = \mathrm{F}_\mathrm{T}^j(r_u, r^j) \\
\mathrm{T}_A^j = \mathrm{F}_\mathrm{T}^j(r_A, r^j) \\
\mathrm{T}_B^j = \mathrm{F}_\mathrm{T}^j(r_B, r^j) \\
\mathrm{T}_C^j = \mathrm{F}_\mathrm{T}^j(r_C, r^j)
\end{cases}
$$

构建基准站和流动站的伪距双差观测方程为

$$
\begin{aligned}
\rho_{uA}^{ij} &= r_{uA}^{ij} + \mathrm{I}_{uA}^{ij} + \mathrm{T}_{uA}^{ij} + \epsilon_{uA}^{ij} \\
\rho_{BA}^{ij} &= r_{BA}^{ij} + \mathrm{I}_{BA}^{ij} + \mathrm{T}_{BA}^{ij} + \epsilon_{BA}^{ij}
\end{aligned}
$$

[1]苏景岚

$$\rho_{CA}^{ij}=r_{CA}^{ij}+\mathrm{I}_{CA}^{ij}+\mathrm{T}_{CA}^{ij}+\epsilon_{CA}^{ij}$$

若已知I_{BA}^{ij}和I_{CA}^{ij}，估计I_{uA}^{ij}：

$$\mathrm{I}_{uA}^{ij}=\mathrm{I}_{u}^{i}-\mathrm{I}_{A}^{i}-\left(\mathrm{I}_{u}^{j}-\mathrm{I}_{A}^{j}\right)=\mathrm{F}_{\mathrm{I}}^{i}\left(r_u,r^i\right)-\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)-\left(\mathrm{F}_{\mathrm{I}}^{j}\left(r_u,r^j\right)-\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)\right)$$

$$\mathrm{I}_{BA}^{ij}=\mathrm{I}_{B}^{i}-\mathrm{I}_{A}^{i}-\left(\mathrm{I}_{B}^{j}-\mathrm{I}_{A}^{j}\right)=\mathrm{F}_{\mathrm{I}}^{i}\left(r_B,r^i\right)-\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)-\left(\mathrm{F}_{\mathrm{I}}^{j}\left(r_B,r^j\right)-\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)\right)$$

$$\mathrm{I}_{CA}^{ij}=\mathrm{I}_{C}^{i}-\mathrm{I}_{A}^{i}-\left(\mathrm{I}_{C}^{j}-\mathrm{I}_{A}^{j}\right)=\mathrm{F}_{\mathrm{I}}^{i}\left(r_C,r^i\right)-\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)-\left(\mathrm{F}_{\mathrm{I}}^{j}\left(r_C,r^j\right)-\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)\right)$$

将$\mathrm{F}_{\mathrm{I}}^{i}\left(r_B,r^i\right)$和$\mathrm{F}_{\mathrm{I}}^{i}\left(r_C,r^i\right)$在$r_A$处进行泰勒级数展开并忽略高阶项，即

$$\mathrm{F}_{\mathrm{I}}^{i}\left(r_B,r^i\right)=\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)+\frac{\partial\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)}{\partial r_A^T}\cdot(r_B-r_A)$$

$$\mathrm{F}_{\mathrm{I}}^{i}\left(r_C,r^i\right)=\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)+\frac{\partial\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)}{\partial r_A^T}\cdot(r_C-r_A)$$

将$\mathrm{F}_{\mathrm{I}}^{j}\left(r_B,r^j\right)$和$\mathrm{F}_{\mathrm{I}}^{j}\left(r_C,r^j\right)$在$r_A$处进行泰勒级数展开并忽略高阶项，即

$$\mathrm{F}_{\mathrm{I}}^{j}\left(r_B,r^j\right)=\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)+\frac{\partial\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)}{\partial r_A^T}\cdot(r_B-r_A)$$

$$\mathrm{F}_{\mathrm{I}}^{j}\left(r_C,r^j\right)=\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)+\frac{\partial\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)}{\partial r_A^T}\cdot(r_C-r_A)$$

由此可得

$$\mathrm{I}_{BA}^{ij}=\left(\frac{\partial\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)}{\partial r_A^T}-\frac{\partial\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)}{\partial r_A^T}\right)\cdot(r_B-r_A)$$

$$\mathrm{I}_{CA}^{ij}=\left(\frac{\partial\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)}{\partial r_A^T}-\frac{\partial\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)}{\partial r_A^T}\right)\cdot(r_C-r_A)$$

即有

$$\begin{bmatrix}\mathrm{I}_{BA}^{ij}\\ \mathrm{I}_{CA}^{ij}\end{bmatrix}=\begin{bmatrix}(r_B-r_A)^T\\ (r_C-r_A)^T\end{bmatrix}\cdot\underbrace{\left(\frac{\partial\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)}{\partial r_A^T}-\frac{\partial\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)}{\partial r_A^T}\right)^T}_{\chi_{\mathrm{I}}^{ij}}$$

将$\mathrm{F}_{\mathrm{I}}^{i}\left(r_u,r^i\right)$和$\mathrm{F}_{\mathrm{I}}^{j}\left(r_u,r^j\right)$在$r_A$处进行泰勒级数展开并忽略高阶项，即

$$\mathrm{F}_{\mathrm{I}}^{i}\left(r_u,r^i\right)=\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)+\frac{\partial\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)}{\partial r_A^T}\cdot(r_u-r_A)$$

$$\mathrm{F}_{\mathrm{I}}^{j}\left(r_u,r^j\right)=\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)+\frac{\partial\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)}{\partial r_A^T}\cdot(r_u-r_A)$$

可得

$$\mathrm{I}_{uA}^{ij}=\left(\frac{\partial\mathrm{F}_{\mathrm{I}}^{i}\left(r_A,r^i\right)}{\partial r_A^T}-\frac{\partial\mathrm{F}_{\mathrm{I}}^{j}\left(r_A,r^j\right)}{\partial r_A^T}\right)\cdot(r_u-r_A)=(r_u-r_A)^T\cdot\chi_{\mathrm{I}}^{ij}$$

对对流层延迟做同样的处理，也有

$$\begin{bmatrix} \mathrm{T}_{BA}^{ij} \\ \mathrm{T}_{CA}^{ij} \end{bmatrix} = \begin{bmatrix} (r_B - r_A)^T \\ (r_C - r_A)^T \end{bmatrix} \cdot \underbrace{\left(\frac{\partial \mathrm{F}_{\mathrm{T}}^{i}\left(r_A, r^i\right)}{\partial r_A^T} - \frac{\partial \mathrm{F}_{\mathrm{T}}^{j}\left(r_A, r^j\right)}{\partial r_A^T} \right)^T}_{\chi_{\mathrm{T}}^{ij}}$$

$$\mathrm{T}_{uA}^{ij} = \left(\frac{\partial \mathrm{F}_{\mathrm{T}}^{i}\left(r_A, r^i\right)}{\partial r_A} - \frac{\partial \mathrm{F}_{\mathrm{T}}^{j}\left(r_A, r^j\right)}{\partial r_A} \right) \cdot (r_u - r_A) = (r_u - r_A)^T \cdot \chi_{\mathrm{T}}^{ij}$$

假设在流动站附近有一个虚拟参考站 V，以基准站 A 为主参考站，则有

$$\mathrm{I}_{VA}^{ij} = (r_V - r_A)^T \cdot \chi_{\mathrm{I}}^{ij}$$
$$\mathrm{T}_{VA}^{ij} = (r_V - r_A)^T \cdot \chi_{\mathrm{T}}^{ij}$$

$$\rho_{VA}^{ij} = r_{VA}^{ij} + \mathrm{I}_{VA}^{ij} + \mathrm{T}_{VA}^{ij} + \epsilon_{VA}^{ij}$$

$$\rho_V^i - \rho_A^i - \left(\rho_V^j - \rho_A^j\right) = r_V^i - r_A^i - \left(r_V^j - r_A^j\right) + \mathrm{I}_{VA}^{ij} + \mathrm{T}_{VA}^{ij} + \epsilon_{VA}^{ij}$$
$$\rho_u^i - \rho_A^i - \left(\rho_u^j - \rho_A^j\right) = r_u^i - r_A^i - \left(r_u^j - r_A^j\right) + \mathrm{I}_{uA}^{ij} + \mathrm{T}_{uA}^{ij} + \epsilon_{uA}^{ij}$$

$$\begin{aligned} &\rho_u^i - \rho_A^i - \left(\rho_u^j - \rho_A^j\right) \\ &\qquad = r_u^i - \left(r_V^i + \left(r_A^i - r_V^i\right)\right) - \left(r_u^j - \left(r_V^j + \left(r_A^j - r_V^j\right)\right)\right) + \left(\mathrm{I}_{uV}^{ij} + \mathrm{I}_{uA}^{ij} - \mathrm{I}_{uV}^{ij}\right) \\ &\qquad + \left(\mathrm{T}_{uV}^{ij} + \mathrm{T}_{uA}^{ij} - \mathrm{T}_{uV}^{ij}\right) + \epsilon_{uV}^{ij} \end{aligned}$$

$$\begin{aligned} \rho_u^i - \underbrace{\left(\rho_A^i + \left(r_V^i - r_A^i\right) + \mathrm{I}_{VA}^{ij} + \mathrm{T}_{VA}^{ij}\right)}_{\rho_V^i} - \left(\rho_u^j - \underbrace{\left(\rho_A^j + \left(r_V^j - r_A^j\right)\right)}_{\rho_V^j}\right) \\ = r_u^i - r_V^i - \left(r_u^j - r_V^j\right) + \mathrm{I}_{uV}^{ij} + \mathrm{T}_{uV}^{ij} + \epsilon_{uV}^{ij} \end{aligned}$$

由上可得，虚拟伪距观测值为

$$\rho_V^i = \rho_A^i + \left(r_V^i - r_A^i\right) + \mathrm{I}_{VA}^{ij} + \mathrm{T}_{VA}^{ij}$$
$$\rho_V^j = \rho_A^j + \left(r_V^j - r_A^j\right)$$

假设以卫星 1 为参考卫星，则虚拟参考站的虚拟观测值为

$$\begin{bmatrix} \rho_V^1 \\ \rho_V^2 \\ \vdots \\ \rho_V^n \end{bmatrix} = \begin{bmatrix} \rho_A^1 + (r_V^1 - r_A^1) \\ \rho_A^2 + (r_V^2 - r_A^2) + \mathrm{I}_{VA}^{21} + \mathrm{T}_{VA}^{21} \\ \vdots \\ \rho_A^n + (r_V^n - r_A^n) + \mathrm{I}_{VA}^{n1} + \mathrm{T}_{VA}^{n1} \end{bmatrix}$$

假设在流动站是以卫星 2 为参考卫星，则其与虚拟参考站组成的伪距双差观测方程为

$$\begin{aligned} \rho_u^1 - \left(\rho_A^1 + (r_V^1 - r_A^1)\right) - \left(\rho_u^2 - (\rho_A^2 + (r_V^2 - r_A^2) + \mathrm{I}_{VA}^{21} + \mathrm{T}_{VA}^{21})\right) \\ = r_u^1 - r_V^1 - (r_u^2 - r_V^2) + \mathrm{I}_{uV}^{12} + \mathrm{T}_{uV}^{12} + \epsilon_{uV}^{12} \end{aligned}$$

$$\begin{aligned}
&\rho_u^1-\rho_A^1-(\rho_u^2-\rho_A^2)\\
&\qquad\qquad=r_u^1-r_V^1-(r_u^2-r_V^2)+(r_V^1-r_A^1)-(r_V^2-r_A^2)+\mathrm{I}_{uV}^{12}-\mathrm{I}_{VA}^{21}+\mathrm{T}_{uV}^{12}-\mathrm{T}_{VA}^{21}+\epsilon_{uV}^{12}\\
&\qquad\qquad=r_u^1-r_A^1-(r_u^2-r_A^2)+\mathrm{I}_{uA}^{12}+\mathrm{T}_{uA}^{12}+\epsilon_{uA}^{12}
\end{aligned}$$

$$\rho_u^3-(\rho_A^3+(r_V^3-r_A^3)+\mathrm{I}_{VA}^{31}+\mathrm{T}_{VA}^{31})-\left(\rho_u^2-(\rho_A^2+(r_V^2-r_A^2)+\mathrm{I}_{VA}^{21}+\mathrm{T}_{VA}^{21})\right)$$
$$=r_u^3-r_V^3-(r_u^2-r_V^2)+\mathrm{I}_{uV}^{32}+\mathrm{T}_{uV}^{32}+\epsilon_{uV}^{32}$$

$$\begin{aligned}
&\rho_u^3-\rho_A^3-(\rho_u^2-\rho_A^2)\\
&\qquad\qquad=r_u^3-r_V^3-(r_u^2-r_V^2)+\mathrm{I}_{uV}^{32}+\mathrm{T}_{uV}^{32}+(r_V^3-r_A^3)+\mathrm{I}_{VA}^{31}+\mathrm{T}_{VA}^{31}-(r_V^2-r_A^2)-\mathrm{I}_{VA}^{21}\\
&\qquad\qquad-\mathrm{T}_{VA}^{21}+\epsilon_{uV}^{32}\\
&\qquad\qquad=r_u^3-r_A^3-(r_u^2-r_A^2)+\mathrm{I}_{uV}^{32}+\mathrm{I}_{VA}^{31}-\mathrm{I}_{VA}^{21}+\mathrm{T}_{uV}^{32}+\mathrm{T}_{VA}^{31}-\mathrm{T}_{VA}^{21}+\epsilon_{uV}^{32}\\
&\qquad\qquad=r_u^3-r_A^3-(r_u^2-r_A^2)+\mathrm{I}_{uA}^{32}+\mathrm{T}_{uA}^{32}
\end{aligned}$$

$$\mathrm{I}_{uV}^{32}+\mathrm{I}_{VA}^{31}-\mathrm{I}_{VA}^{21}=\mathrm{I}_u^3-\mathrm{I}_V^3-(\mathrm{I}_u^2-\mathrm{I}_V^2)+\left(\mathrm{I}_V^3-\mathrm{I}_A^3-(\mathrm{I}_V^1-\mathrm{I}_A^1)\right)-\left(\mathrm{I}_V^2-\mathrm{I}_A^2-(\mathrm{I}_V^1-\mathrm{I}_A^1)\right)$$
$$=\mathrm{I}_u^3-\mathrm{I}_A^3-(\mathrm{I}_u^2-\mathrm{I}_A^2)=\mathrm{I}_{uA}^{32}$$

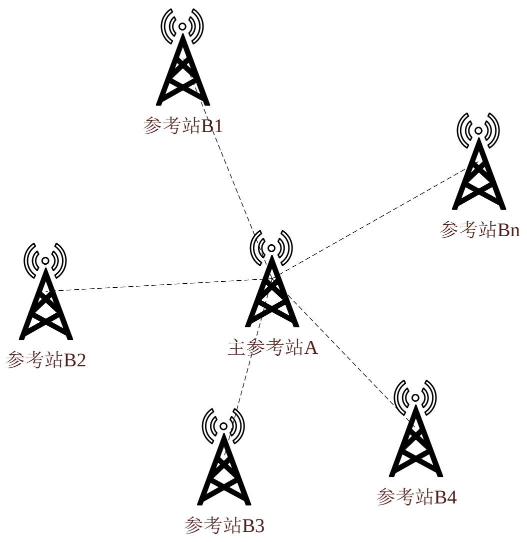
以此类推，即有

$$\begin{bmatrix}\rho_{uV}^{12}=r_{uV}^{12}+\mathrm{I}_{uV}^{12}+\mathrm{T}_{uV}^{12}+\epsilon_{uV}^{12}\\ \rho_{uV}^{32}=r_{uV}^{32}+\mathrm{I}_{uV}^{32}+\mathrm{T}_{uV}^{32}+\epsilon_{uV}^{32}\\ \vdots\\ \rho_{uV}^{n2}=r_{uV}^{n2}+\mathrm{I}_{uV}^{n2}+\mathrm{T}_{uV}^{n2}+\epsilon_{uV}^{n2}\end{bmatrix}\equiv\begin{bmatrix}\rho_{uA}^{21}=r_{uA}^{12}+\mathrm{I}_{uA}^{12}+\mathrm{T}_{uA}^{12}+\epsilon_{uA}^{12}\\ \rho_{uA}^{32}=r_{uA}^{32}+\mathrm{I}_{uA}^{32}+\mathrm{T}_{uA}^{32}+\epsilon_{uA}^{32}\\ \vdots\\ \rho_{uA}^{n2}=r_{uA}^{n2}+\mathrm{I}_{uA}^{n2}+\mathrm{T}_{uA}^{n2}+\epsilon_{uA}^{n2}\end{bmatrix}$$

上式表明两者是等价的;

假设主参考站 A 附近有 n 个参考站:

$$B_1,B_2,B_3,\dots\dots,B_n$$



主参考站 A 与基准站$B_1,B_2,B_3,\dots\dots,B_n$分别组成基线，即有

$$AB_1,AB_2,AB_3,\dots\dots,AB_n$$

对于双差电离层有

$$\begin{bmatrix} \mathrm{I}_{B_1A}^{ij} \\ \mathrm{I}_{B_2A}^{ij} \\ \mathrm{I}_{B_3A}^{ij} \\ \vdots \\ \mathrm{I}_{B_nA}^{ij} \end{bmatrix} = \begin{bmatrix} (r_{B_1}-r_A)^T \\ (r_{B_2}-r_A)^T \\ (r_{B_3}-r_A)^T \\ \vdots \\ (r_{B_n}-r_A)^T \end{bmatrix} \cdot \underbrace{\left(\frac{\partial \mathrm{F}_{\mathrm{I}}^{i}(r_A, r^i)}{\partial r_A^T} - \frac{\partial \mathrm{F}_{\mathrm{I}}^{j}(r_A, r^j)}{\partial r_A^T} \right)^T}_{\chi_{\mathrm{I}}^{ij}}$$

将χ_{I}^{ij}表示在站心坐标系下，假设主参考站 A 由 ECEF 坐标系转换至站心坐标系的转换矩阵为S，则有

$$\begin{bmatrix} \mathrm{I}_{B_1A}^{ij} \\ \mathrm{I}_{B_2A}^{ij} \\ \mathrm{I}_{B_3A}^{ij} \\ \vdots \\ \mathrm{I}_{B_nA}^{ij} \end{bmatrix} = \begin{bmatrix} (r_{B_1}-r_A)^T \\ (r_{B_2}-r_A)^T \\ (r_{B_3}-r_A)^T \\ \vdots \\ (r_{B_n}-r_A)^T \end{bmatrix} \cdot \underbrace{S^T \cdot S}_{I} \cdot \underbrace{\left(\frac{\partial \mathrm{F}_{\mathrm{I}}^{i}(r_A, r^i)}{\partial r_A^T} - \frac{\partial \mathrm{F}_{\mathrm{I}}^{j}(r_A, r^j)}{\partial r_A^T} \right)}_{\chi_{\mathrm{I}}^{ij}}$$

$$\underbrace{\begin{bmatrix} \mathrm{I}_{B_1A}^{ij} \\ \mathrm{I}_{B_2A}^{ij} \\ \mathrm{I}_{B_3A}^{ij} \\ \vdots \\ \mathrm{I}_{B_nA}^{ij} \end{bmatrix}}_{L_I^{ij}} = \underbrace{\begin{bmatrix} \left(S \cdot (r_{B_1}-r_A)\right)^T \\ \left(S \cdot (r_{B_2}-r_A)\right)^T \\ \left(S \cdot (r_{B_3}-r_A)\right)^T \\ \vdots \\ \left(S \cdot (r_{B_n}-r_A)\right)^T \end{bmatrix}}_{H} \cdot \underbrace{S \cdot \left(\frac{\partial \mathrm{F}_{\mathrm{I}}^{i}(r_A, r^i)}{\partial r_A^T} - \frac{\partial \mathrm{F}_{\mathrm{I}}^{j}(r_A, r^j)}{\partial r_A^T} \right)^T}_{\chi_{\mathrm{I,ENU}}^{ij}}$$

$$L_I^{ij} = H \cdot \chi_{\mathrm{I,ENU}}^{ij}$$

$\chi_{\mathrm{I,ENU}}^{ij}$表示双差电离层在 ENU 方向的梯度值；

对上式进行最小二乘估计，可得

$$\chi_{\mathrm{I,ENU}}^{ij} = (H^T \cdot H)^{-1} \cdot H^T \cdot L_I^{ij}$$

由上式计算虚拟站 V 的双差电离层，即

$$\mathrm{I}_{VA}^{ij} = (r_V - r_A)^T \cdot \chi_{\mathrm{I}}^{ij} = \underbrace{\left(S \cdot (r_V - r_A)\right)^T}_{r_{ENU}^{VA}} \cdot (H^T \cdot H)^{-1} \cdot H^T \cdot L_I^{ij} = \left((H^T \cdot H)^{-1} \cdot H^T \cdot L_I^{ij}\right)^T \cdot r_{ENU}^{VA}$$

同样地，对于双差对流层也有

$$\mathrm{T}_{VA}^{ij} = (r_V - r_A)^T \cdot \chi_{\mathrm{T}}^{ij} = \left((H^T \cdot H)^{-1} \cdot H^T \cdot L_T^{ij}\right)^T \cdot r_{ENU}^{VA}$$

下面推导 VRS 虚拟参考站载波相位观测值生成的基本算法：
载波相位观测值为

$$\begin{gathered}
\varphi_u^i = r_u^i + cdt_u - cdt^i - \mathrm{I}_u^i + \mathrm{T}_u^i + N_u^i + \varepsilon_u^i \\
\varphi_A^i = r_A^i + cdt_u - cdt^i - \mathrm{I}_A^i + \mathrm{T}_A^i + N_A^i + \varepsilon_A^i \\
\varphi_B^i = r_B^i + cdt_u - cdt^i - \mathrm{I}_B^i + \mathrm{T}_B^i + N_B^i + \varepsilon_B^i \\
\varphi_C^i = r_C^i + cdt_u - cdt^i - \mathrm{I}_C^i + \mathrm{T}_C^i + N_C^i + \varepsilon_C^i
\end{gathered}$$

以卫星 j 为参考卫星，A 为主参考站，构建基准站和流动站的伪距双差观测方程为

$$\begin{gathered}
\varphi_{uA}^{ij} = r_{uA}^{ij} - \mathrm{I}_{uA}^{ij} + \mathrm{T}_{uA}^{ij} + N_{uA}^{ij} + \epsilon_{uA}^{ij} \\
\varphi_{BA}^{ij} = r_{BA}^{ij} - \mathrm{I}_{BA}^{ij} + \mathrm{T}_{BA}^{ij} + N_{BA}^{ij} + \epsilon_{BA}^{ij} \\
\varphi_{CA}^{ij} = r_{CA}^{ij} - \mathrm{I}_{CA}^{ij} + \mathrm{T}_{CA}^{ij} + N_{CA}^{ij} + \epsilon_{CA}^{ij}
\end{gathered}$$

与伪距虚拟观测值的生成同理，载波相位虚拟观测值为

$$\begin{gathered}
\varphi_V^i = \varphi_A^i + \left(r_V^i - r_A^i\right) - \mathrm{I}_{VA}^{ij} + \mathrm{T}_{VA}^{ij} \\
\varphi_V^j = \varphi_A^j + \left(r_V^j - r_A^j\right)
\end{gathered}$$

假设以卫星 1 为参考卫星，则虚拟参考站的载波相位虚拟观测值为

$$\begin{bmatrix} \varphi_V^1 \\ \varphi_V^2 \\ \vdots \\ \varphi_V^n \end{bmatrix} = \begin{bmatrix} \varphi_A^1 + (r_V^1 - r_A^1) \\ \varphi_A^2 + (r_V^2 - r_A^2) - \mathrm{I}_{VA}^{21} + \mathrm{T}_{VA}^{21} \\ \vdots \\ \varphi_A^n + (r_V^n - r_A^n) - \mathrm{I}_{VA}^{n1} + \mathrm{T}_{VA}^{n1} \end{bmatrix}$$

需要的话，可将$\mathrm{F}_{\mathrm{I}}^i\left(r_u, r^i\right)$和$\mathrm{F}_{\mathrm{I}}^j\left(r_u, r^j\right)$进行泰勒级数展开至二阶，则有

$$\mathrm{F}_{\mathrm{I}}^i\left(r_u, r^i\right) = \mathrm{F}_{\mathrm{I}}^i\left(r_A, r^i\right) + \frac{\partial \mathrm{F}_{\mathrm{I}}^i\left(r_A, r^i\right)}{\partial r_A^T} \cdot (r_u - r_A) + (r_u - r_A)^T \cdot \frac{\partial^2 \mathrm{F}_{\mathrm{I}}^i\left(r_A, r^i\right)}{\partial r_A \partial r_A} \cdot (r_u - r_A)$$

$$\mathrm{F}_{\mathrm{I}}^j\left(r_u, r^j\right) = \mathrm{F}_{\mathrm{I}}^j\left(r_A, r^j\right) + \frac{\partial \mathrm{F}_{\mathrm{I}}^j\left(r_A, r^j\right)}{\partial r_A^T} \cdot (r_u - r_A) + (r_u - r_A)^T \cdot \frac{\partial^2 \mathrm{F}_{\mathrm{I}}^j\left(r_A, r^j\right)}{\partial r_A \partial r_A} \cdot (r_u - r_A)$$

$$\begin{aligned}
\mathrm{I}_{uA}^{ij} &= \left(\frac{\partial \mathrm{F}_{\mathrm{I}}^i\left(r_A, r^i\right)}{\partial r_A^T} - \frac{\partial \mathrm{F}_{\mathrm{I}}^j\left(r_A, r^j\right)}{\partial r_A^T}\right) \cdot (r_u - r_A) + (r_u - r_A)^T \cdot \frac{\partial^2 \mathrm{F}_{\mathrm{I}}^i\left(r_A, r^i\right)}{\partial r_A \partial r_A} \cdot (r_u - r_A) - (r_u - r_A)^T \\
&\quad \cdot \frac{\partial^2 \mathrm{F}_{\mathrm{I}}^j\left(r_A, r^j\right)}{\partial r_A \partial r_A} \cdot (r_u - r_A) \\
&= (r_u - r_A)^T \cdot \underbrace{\left(\frac{\partial \mathrm{F}_{\mathrm{I}}^i\left(r_A, r^i\right)}{\partial r_A^T} - \frac{\partial \mathrm{F}_{\mathrm{I}}^j\left(r_A, r^j\right)}{\partial r_A^T}\right)^T}_{\chi_{\mathrm{I}}^{ij}} + (r_u - r_A)^T \\
&\quad \cdot \underbrace{\left(\frac{\partial^2 \mathrm{F}_{\mathrm{I}}^i\left(r_A, r^i\right)}{\partial r_A \partial r_A} - \frac{\partial^2 \mathrm{F}_{\mathrm{I}}^j\left(r_A, r^j\right)}{\partial r_A \partial r_A}\right)}_{\psi_{\mathrm{I}}^{ij}} \cdot (r_u - r_A)
\end{aligned}$$

忽略梯度变化在方向的相关性，则有

$$\psi_{\mathrm{I}}^{ij} = \frac{\partial^2 \mathrm{F}_{\mathrm{I}}^i\left(r_A, r^i\right)}{\partial r_A \partial r_A} - \frac{\partial^2 \mathrm{F}_{\mathrm{I}}^j\left(r_A, r^j\right)}{\partial r_A \partial r_A} \approx \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\mathrm{I}_{uA}^{ij} = (r_u - r_A)^T \cdot \chi_{\mathrm{I}}^{ij} + \underbrace{\left[(r_u - r_A)_x^2 \quad (r_u - r_A)_y^2 \quad (r_u - r_A)_z^2\right]}_{\tau_u^A} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

主参考站 A 与基准站$B_1, B_2, B_3, \dots\dots, B_n$分别组成基线，即有

$$AB_1, AB_2, AB_3, \dots\dots, AB_n$$

对于双差电离层有

$$\underbrace{\begin{bmatrix} \mathrm{I}_{B_1A}^{ij} \\ \mathrm{I}_{B_2A}^{ij} \\ \mathrm{I}_{B_3A}^{ij} \\ \vdots \\ \mathrm{I}_{B_nA}^{ij} \end{bmatrix}}_{L_I^{ij}} = \underbrace{\begin{bmatrix} \left(S \cdot \left(r_{B_1} - r_A\right)\right)^T & \tau_{B_1}^A \\ \left(S \cdot \left(r_{B_2} - r_A\right)\right)^T & \tau_{B_2}^A \\ \left(S \cdot \left(r_{B_3} - r_A\right)\right)^T & \tau_{B_3}^A \\ \vdots & \\ \left(S \cdot \left(r_{B_n} - r_A\right)\right)^T & \tau_{B_n}^A \end{bmatrix}}_{\Phi} \cdot \begin{bmatrix} \chi_{\mathrm{I,ENU}}^{ij} \\ a \\ b \\ c \end{bmatrix}$$

$$L_I^{ij} = \Phi \cdot \begin{bmatrix} \chi_{\mathrm{I,ENU}}^{ij} \\ a \\ b \\ c \end{bmatrix}$$

对上式进行最小二乘估计，可得

$$\begin{bmatrix} \chi_{\mathrm{I,ENU}}^{ij} \\ a \\ b \\ c \end{bmatrix} = (\Phi^T \cdot \Phi)^{-1} \cdot \Phi^T \cdot L_I^{ij}$$

由上式计算虚拟站 V 的双差电离层，即

$$\mathrm{I}_{VA}^{ij} = (r_V - r_A)^T \cdot \chi_{\mathrm{I}}^{ij} + (r_u - r_A)^T \cdot (r_u - r_A) \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \left(S \cdot (r_V - r_A)\right)^T \cdot \chi_{\mathrm{I,ENU}}^{ij} + \tau_V^A \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$= \left[\left(S \cdot (r_V - r_A)\right)^T \quad \tau_V^A\right] \cdot (\Phi^T \cdot \Phi)^{-1} \cdot \Phi^T \cdot L_I^{ij}$$

对于双差对流层同理。