Mark-to-market Credit Index Option Pricing and Credit Volatility Index

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Abstract

Credit index options cannot be priced as simple vanilla options. In this paper we derive the pricing formula using both replication method and linear annuity mapping. We develop a model-free process to calculate a VIX-like credit volatility index based on observable credit index options market. We imply all model parameters from market traded data.

Credit Index Options (CDXO) are options that give the investors the right to enter credit index position at given spreads or prices. They are analogue to interest rate swaptions, but there are some special features, e.g., the front end protection payment (FEP) and catastrophe state, making the valuation complicated [1, 2, 3].

To make exercise decision of CDXO, the market value of the underlying contract will be computed on the exercise date. If the exercise payoff can compensate for FEP, the option will be exercised. A common practice employed to simplify this process is to adjust the forward spread or price and use the adjusted spread/price to make exercise decision [1]. However, there is no standard way to calculate FEP. It can be argued that FEP shall be calculated under either homogeneous recovery assumptions or non-homogeneous assumptions. An index credit curve is usually nothing different from a single name curve. It is used to calculate the expected loss up to the option expiry, but the curve could

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be equally set as a flat or not. In the former case, the calculation involes calculating theoretical values of the index basket curve term structure from the constituents curves. Due to improved liquidity and other added economic values, the theoretical value of a basket of single name CDS contracts is different from the trading level of the corresponding index. This index-basket basis has to be addressed in the FEP calculation.

More importantly there exist two radically different conventions for quoting credit indices contracts. For high yield (HY) and emerging markets (EM) families, credit indices are quoted in prices. For other indices, they are quoted in spreads. This convention has been applied to CDXO market. The strikes of the options written on HY and EM indices are expressed in terms of prices, whereas the strikes of the options written on other indices are expresed in terms of spreads. For price-strike options, the exercise values are determined by the contract. Therefore, one can directly apply Blacks Model to value the price-strike options. For a spread-strike option, it is a fixed spread that are determined by the contract for exercise. On the exercise date, the exercise values will be converted from the exercise spread using the standard ISDA upfront converter [4]. Unfortunately in the case of spread quoted options, the exercise value as expressed in dollars cannot be determined until the exercise date when the yield curve becomes available.

This special quotation convention of spread-strike options makes the valuation of this type of CDXO non-trivial. Some existing pricing formulas have totally ignored this special exercise feature [5], while the others make assumptions that the execise values are fixed to values observed at valuation date. In this paper, we recognize the exercise feature and propose alternative pricing methods. Furthermore, we resolve the difficulties of calculating the FEP adjusted forward spread/price by implying them from market quotes exclusively.

The market traded CDXO are not vanilla options due to the special exercise treatment and its second order derivative with respect to strike are not clearly defined. Therefore, it cannot be directly used in the replication method for constructing VIX-like index as Mele and Obayashi have proposed [5]. In this paper, we derive the vanilla options from the market traded non-vanilla options and use the usual replication method to construct the VIX-like credit volatility index.

The paper is organized as following, in Section 1, we describe how to price price-strike options using regular Blacks Model and how to imply the FEP adjusted forward price from market data; in Section 2, we evaluate the spread-strike CDXO using a static replication method along with a linear annuity mapping method. We discuss how to imply the model parameters from market quotes; in Section 3, we derive a more tractable spread-strike CDXO pricing formula using the linear annuity mapping; in Section 4, we discuss how to construct the VIX-like index properly using market traded CDXOs; we summarize in the last section.

1 Options with Price Strikes

Price-strike options have determined exercise values. Ignoring accrued interest and assuming unit notional, we have $Index\ Price = (1 - Settlement\ Value) \cdot 100$. The payoff of the options can be written as $\{\gamma[F_a(T) - K,\ 0]\}^+$ where $\gamma = -1$ for payers and 1 for receivers; K is the strike price; and $F_a(T)$ is the FEP adjusted index price observed at exercise date T.

Notice that the price-strike payers are similar to put options and receivers are similar to call options, due to the relation between the index price and the settlement value.

1.1 Price-Strike Options Pricing Formula

It's straightforward to apply Black's Model to value for price-strike CDXO's at valuation date t,

$$
\begin{aligned}
V(K,t) &= \gamma \cdot [F_a(t)N(\gamma d_1) - KN(\gamma d_2)] \cdot D(t,T) \qquad (1)\\
d_1 &= \frac{\ln\left(\frac{F_a}{K}\right)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2}\\
d_2 &= d_1 - \sigma\sqrt{T-t},
\end{aligned}
$$

where $D(t,T)$ is the risky discount factor to T as seen at date t; $N(\cdot)$ is the standard normal cumulative distribution function; σ is the volatitliy of price dynamics.

1.2 Implying Parameters from Market Data

There are many different method proposed to compute FEP and the FEP adjusted forward price F_a. The computation of $D(t,T)$ has the same complication. $D(t,T)$ is a risky discount factor which contains a factor of survival probability at exercise. A full term structure of the index survival curve has to be used to calculate this survival probability. The survival curve term structure can be computed by theoretical calculation. However, the credit curves of all underlying constituents are required. The calculation also involves adjustment for the basis between index trading level and theoretical basket value. Fortunately, it is possible to imply these quantities (D and $F_a(t)$) directly from CDXO market data. In this way, our valuation is consistent with the CDXO market itself. There is no ambiguity in the timing of the interest rate curve or making any assumption of the basis adjustment.

From the pricing formula, it's easy to derive the put-call parity for price-strike credit index options,

$$
\begin{aligned}
V_{Receiver}(K) - V_{Payer}(K) =& [F_a(t) - K] \cdot D(t,T) \qquad (2)\\
=& D(t,T) \cdot F_a(t) - D(t,T) \cdot K
\end{aligned}
$$

As long as we have two or more pairs of payer and receiver quotes at different strikes, we are able to calculate D and $F_a(t)$. Usually, we have more than two

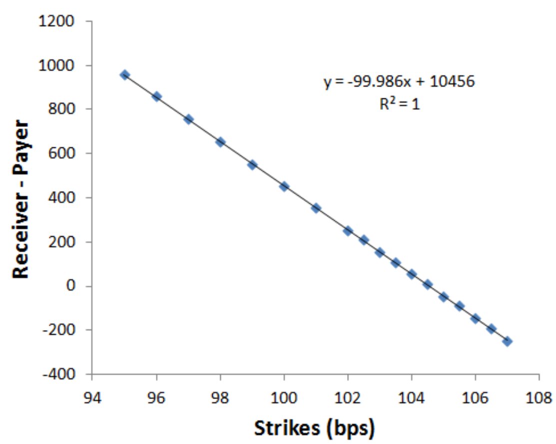

Figure 1: Price Strike Option Parity Fit

pair of receiver-payer market quotes. Therefore we can run a regression to determine the two quantities with more confidence,

$$D(t,T) = -\beta_1 \tag{3}$$

$$F_a(t) = \frac{\beta_0}{D(t,T)}, \tag{4}$$

where β_0 and β_1 are the constant intercept and the slope coefficient from the linear regression, respectively.

Figure 1 shows an example of the parity regression. Vertical axis is the difference of the receiver and payer for the same strike. The underlying index of the options is CDX.NA.HY Series 20 Version 1. Valuation Date is July 15th, 2013. Option expiration is September 2013. The implied adjusted forward price is 104.57 while the provided reference index level is 105.5.

2 Options with Spread Strikes

While price-strike pricing is relatively straightforward, it is much more complicated for spread-strike CDXOs. To determine the payoff, we have to calculate two risky annuities, with one derived from realized forward spread (adjusted by FEP) and the other from strike spread. The payoff can be written as,

$$g(S) = \{\gamma[(S-C)A(S,T,T+\tau|T) - (K-C)A(K,T,T+\tau|T), 0]\}^+ \tag{5}$$

where $\gamma = 1$ for payers and -1 for receivers; S is the forward observed FEP adjusted spread; $A(x,T,T+\tau|T)$ is the risky annuity converted from spread x conditional on the information observed on time T. Notice that γ has a opossite sign with respect to price-strike options. For spread-strike options, payers are similar to call options and receivers are similar to call options.

More importantly, the strike value has to be evaluated at the exercise date. This requires the information of the yield curve at the exercise date. Most literatures assume a deterministic yield curve to fix the exercise values and apply Blacks Formula under a risky annuity measure. In this and next sections, we propose alternative methods without ignoring the dynamics of interest rates.

2.1 Replication with Vanilla Options

An option with any payoff $g(S)$ can be replicated by the strip of vanilla options. The undiscounted option price is given by:

$$E[g(S)] = g(F) + \int_0^F dx\tilde{P}(x)g''(x) + \int_F^{+\infty} dx\tilde{C}(x)g''(x), \tag{6}$$

where $\tilde{C}(x)$ and $\tilde{P}(x)$ are undiscounted vanilla options,

$$\begin{aligned}\tilde{C}(K) &= \int_K^{+\infty}(S-K)\phi(S)dS \\ \tilde{P}(K) &= \int_0^K (K-S)\phi(S)dS.\end{aligned}$$

First we need to find out the second order derivative of the payoff function Eq. (5),

$$\begin{aligned}g''(S) =& [A(S) + (S-C)A'(S)]\delta(S-K) \\ &+ \gamma[2A'(S) + (S-C)A''(S)]\theta(\gamma(S-K))\end{aligned} \tag{7}$$

where, for brevity, we set $A(x) = A(x, T, T+\tau|T)$. Pluging Eq. (7) into the

Eq. (6) we obtain:

$$
\begin{aligned}
V^{ITM}_{Payer}(K) =& (F_a - C)A(F_a) - (K - C)A(K) \\
&+ [A(K) + (K - C)A'(K)]\,\tilde{P}(K) \\
&+ \int_K^{F_a} dx\tilde{P}(x)[2A'(x) + (x - C)A''(x)] \\
&+ \int_{F_a}^{+\infty} dx\tilde{C}(x)[2A'(x) + (x - C)A''(x)] && (8)\\
V^{OTM}_{Payer}(K) =& [A(K) + (K - C)A'(K)]\tilde{C}(K) \\
&+ \int_K^{+\infty} dx\tilde{C}(x)[2A'(x) + (x - C)A''(x)] && (9)\\
V^{ITM}_{Receiver}(K) =& (K - C)A(K) - (F_a - C)A(F_a) \\
&+ [A(K) + (K - C)A'(K)]\tilde{C}(K) \\
&- \int_0^{F_a} dx\tilde{P}(x)[2A'(x) + (x - C)A''(x)] \\
&- \int_{F_a}^{K} dx\tilde{C}(x)[2A'(x) + (x - C)A''(x)] && (10)\\
V^{OTM}_{Receiver}(K) =& [A(K) + (K - C)A'(K)]\tilde{P}(K) \\
&- \int_0^{K} dx\tilde{P}(x)[2A'(x) + (x - C)A''(x)]. && (11)
\end{aligned}
$$

where F_a has been redefined as FEP adjusted forward index spread.

2.2 Linear Annuity Mapping

When pricing interest rate swaptions, it is common practice to chose the measure whith risk free annuity as numeraire[6]. This simplifies the valuation model. Therefore, only the par swap rate dynamics needs to be modeled under the associated measure. To value a spread-strike CDXO, the interest rate dynamics can be factored out, too. Liu and Jackel [7] have applied a linear annuity mapping to formulate the dependency of forward annuity on forward spread. We adopt similar methodology. Assume the risky annuity can be decomposed into two factors,

$$A(K, T, T + \tau|t) = f(K, t)A(0, T, T + \tau|t) \tag{12}$$

The first factor purely depends on spread and the second factor is an annuity which accounts for interest rate dynamics. The second annuity can be understood as a risk free annuity from T to $T + \tau$ but risky discounted back to time t. Furthermore, we assume a linear form of the spread factor,

$$f(K, t) = 1 + bK \tag{13}$$

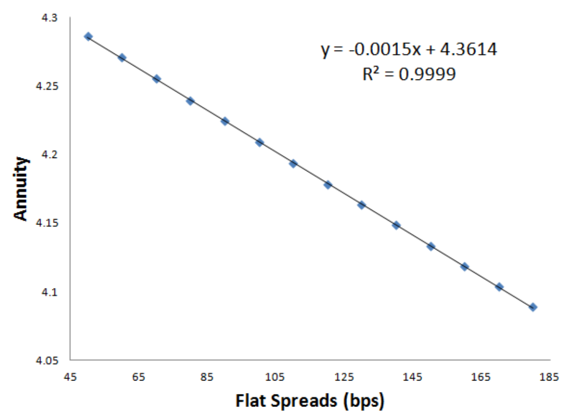

Figure 2: Risky Annuity Profile

Notice that, we also assume this form is time-independent. Combining Eq. (12) and Eq. (13), we get

$$A(K, T, T + \tau|t) = (1 + bK)A(0, T, T + \tau|t) \tag{14}$$

The difference between the approaches in this paper and Liu and Jackel's is we apply the linear mapping and variable separation on both forward annuity and strike annuity whereas Liu and Jackel assume strike annuity is predetermined.

To justify the linear annuity mapping method, we computed the risky annuities assuming different flat spreads as of January 21st of 2015 maturing at June 20th of 2019, as we can see in Figure 2. The risky annuity also is known to be a nonliear function of flat spread. However, linear mapping provided a very good approximation especially in the region where options are usually quoted. From Figure 2 we can imply $A(K, T, T + \tau|t) = 4.3614$ and $b = -3.48 \times 10^{-4}$.

Plug in Eq. (14) into the Eqs. (8 - 11),

$$\begin{aligned}
V_{Payer}^{ITM}(K) =& A(0,T,T+\tau|t)\Bigg\{\Bigg[(F_a-C)(1+bF_a)-(K-C)(1+bK)\Bigg] \\
&+\Bigg[(1+bK)+(K-C)b\Bigg]\tilde{P}(K)+2b\Bigg[\int_K^{F_a}dx\tilde{P}(x) \\
&+\int_{F_a}^{+\infty}dx\tilde{C}(x)\Bigg]\Bigg\} \qquad (15) \\
V_{Payer}^{OTM}(K) =& A(0,T,T+\tau|T)\Bigg\{\Bigg[(1+bK)+(K-C)b\Bigg]\tilde{C}(K) \\
&+2b\int_K^{+\infty}dx\tilde{C}(x)\Bigg\} \qquad (16) \\
V_{Receiver}^{ITM}(K) =& A(0,T,T+\tau|T)\Bigg\{\Bigg[(K-C)(1+bK)-(F_a-C)(1+bF_a)\Bigg] \\
&+\Bigg[(1+bK)+(K-C)b\Bigg]\tilde{C}(K)-2b\Bigg[\int_0^{F_a}dx\tilde{P}(x) \\
&+\int_{F_a}^{K}dx\tilde{C}(x)\Bigg]\Bigg\} \qquad (17) \\
V_{Receiver}^{OTM}(K) =& A(0,T,T+\tau|T)\Bigg\{\Bigg[(1+bK)+(K-C)b\Bigg]\tilde{P}(K) \\
&-2b\int_0^{K}dx\tilde{P}(x)\Bigg\} \qquad (18)
\end{aligned}$$

Eqs. (15 - 18) can be used as pricing formulas for market traded spread-strike CDXOs. However, we have to determine $A(0,T,T+\tau|T)$, b and all the vanilla options $\tilde{C}(K)$ and $\tilde{P}(K)$ first. The next two subsections will solve these problems.

2.3 Implying Parameters from Market Data

Combining Eqs. (15 - 18), the put-call parity of the spread-strike CDXOs can be written as,

$$\begin{aligned}
V_{Payer}(K)-V_{Receiver}(K) =& A(0,T,T+\tau|T)\Bigg\{(F_a-C)(1+bF_a) \\
&-(K-C)(1+bK)+2b\Bigg[\int_0^{F_a}dx\tilde{P}(x) \\
&+\int_{F_a}^{+\infty}dx\tilde{C}(x)\Bigg]\Bigg\} \qquad (19)
\end{aligned}$$

It is interesting to see that the put-call parity contains a correction term which is the integral over all OTM vanilla options. This correction term will complicate the process of implying the parameters and we will take a closer look at it later.

Rearranging the parity

$$\begin{aligned} V_{Payer}(K) - V_{Receiver}(K) =& A(0,T,T+\tau|t)\Bigg\{ -bK^2 - (1-Cb)K \\ & + bF_a^2 + (1-Cb)F_a + 2b\left[\int_0^{F_a} dx\tilde{P}(x) \right. \\ & \left. + \int_{F_a}^{+\infty} dx\tilde{C}(x)\right]\Bigg\}, \end{aligned} \tag{20}$$

Eq. (20) can be regarded as a regression formula just like we have done in Section 1.2. The difference between payers and receivers can be regressed against K and K^2 so that the model parameters can be implied,

$$\begin{aligned} bA(0,T,T+\tau|t) &= -\beta_2 \\ (1-Cb)A(0,T,T+\tau|t) &= -\beta_1 \end{aligned}$$

$$A(0,T,T+\tau|t)\left\{bF_a^2 + (1-Cb)F_a + 2b\left[\int_0^{F_a} dx\tilde{P}(x) + \int_{F_a}^{+\infty} dx\tilde{C}(x)\right]\right\} = \beta_0$$

where β_0, β_1 and β_2 are the constant intercept, the linear slope coefficient and the second order curvature coefficient of the two-factor linear regression, respectively. From the first two equations, we directly obtain b and $A(0,T,T+\tau|t)$.

$$A(0,T,T+\tau|t) = -\left(\beta_1 + C\cdot\beta_2\right) \tag{21}$$

$$b = -\frac{\beta_2}{A(0,T,T+\tau|t)} \tag{22}$$

However, implying the FEP adjusted forward spread F_a, needs special treatment because one needs to estimate the correction term by integrating OTM vanilla options. The OTM vanilla options can be derived from market quoted non-vanilla options.

Figure 3 shows an example of the spread option parity regression where we fit the difference between payers and receivers to a second order polynomial of strikes. The underlying index is CDX.NA.IG Series 22 Version 1. Valuation date is July 18, 2014. Options expire in January 2015. From the regression we can directly imply the annuity factor $A(0,T,T+\tau|t) = 4.3239$ and annuity mapping coefficient $b = -3.47\times 10^{-4}$. These implied coefficients are very close to the realized ones shown in Figure 2.

Even though Figure 3 shows very linear relation of the price differences and strikes, the non linear regression coefficient is actually statistically significant. If we ignore the nonlinearity, assuming $b = 0$, we can imply risky annuity to be

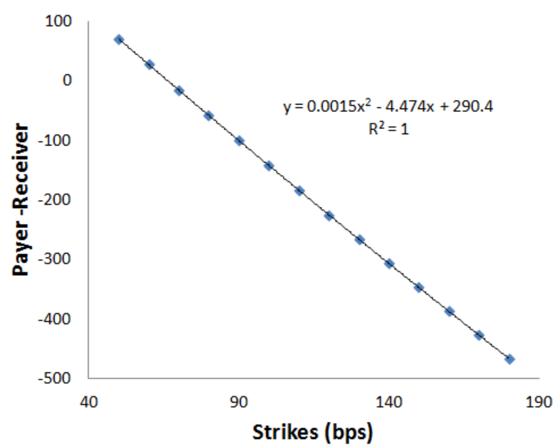

Figure 3: Spread Strike Option Parity Fit

4.1287 and adjusted forward spread to be 66.12 bps. As we will see in Figure 4, this combination of parameter will easily generate problem when we try to imply volatilies.

2.4 Bootstrapping Vanilla Options

Equations Eq. (16) and Eq. (18) can be approximated as,

$$\begin{aligned} V^{OTM}_{Payer}(K) =& A(0,T,T+\tau|t)[(1+bK)+(K-C)b]\tilde{C}(K) \\ &+ 2bA(0,T,T+\tau|t)\sum_{L\geq K}\tilde{C}(L)\Delta L \end{aligned} \tag{23}$$

$$\begin{aligned} V^{OTM}_{Receiver}(K) =& A(0,T,T+\tau|t)[(1+bK)+(K-C)b]\tilde{P}(K) \\ &- 2bA(0,T,T+\tau|t)\sum_{L\leq K}\tilde{P}(L)\Delta L \end{aligned} \tag{24}$$

These equations implicitly depend on the adjusted forward, F_a, which is only used to determine level of moneyness. As we will see later, the correction term in the put-call parity is relatively small, therefore, the market traded ATM options can be approximately identified by finding the payer and receiver with the smallest difference in price.

In Eq. (23) and Eq. (24), for N observed OTM market options, we have exactly N unknowns (vanilla option prices). Therefore, we are able to bootstrap their values. Notice that in both linear systems for payers and receivers, the matrices are triangular, making the solving process straight forward.

Once we have bootstrapped the OTM vanilla options, the correction integral

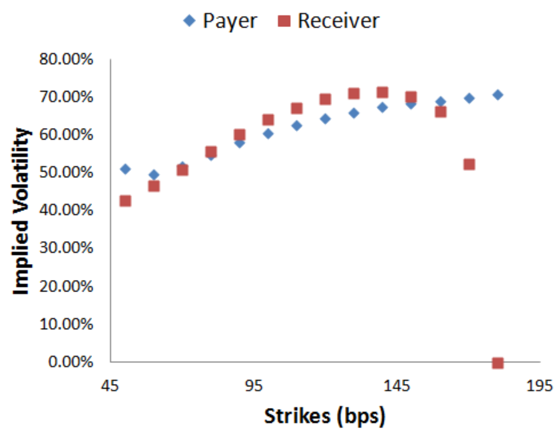

Figure 4: Black Model Implied Volalities

in the intercept can be approximately discretized as,

$$\int_0^{F_a} dx\tilde{P}(x) + \int_{F_a}^{+\infty} dx\tilde{C}(x) = \sum_{k\leq F_a} \tilde{P}(K)\Delta K + \sum_{k\geq F_a} \tilde{C}(K)\Delta K \tag{25}$$

Therefore, F_a can be solved from,

$$bF_a^2 + (1 - Cb)F_a = \beta_0 - 2b\left[\sum_{k<F_a} \tilde{P}(K)\Delta K + \sum_{k>F_a} \tilde{C}(K)\Delta K\right] \tag{26}$$

For the example we have shown in Figure 3, the adjust forward spread is 66.65.

It is possible to obtain a more granular vanilla option chain if interpolation between market quoted stirkes is allowed. ITM vanilla options can be calculated from OTM options with corresponding strikes using regular put-call parity for vanilla options.

3 Spread-Strike Options Pricing Formula

After solving all model parameters, Eqs. (15 - 18) allow us to price spread-strike CDXOs consistently with market traded options. This could be understood as a mark-to-market pricing. Even though a linear annuity mapping approximation has been applied, all pricing parameters are determined purely from credit index options market itself. Other pricing methodologies not only approximate strike values but also depend on interest rate market and/or single name CDS market.

The replication method does have a drawback. It is less tractable than a simple Black's Formula. In this section, we propose an approximate closed form formula for spread-strike CDXO pricing.

Applying Eq. (14) to Eq. (5),

$$\begin{aligned} g(S) =& A(0,T,T+\tau|T)\left\{\gamma[(S-C)(1+bS)-(K-C)(1+bK),0]\right\}^+ \\ =& A(0,T,T+\tau|T)\left\{\gamma[b(S^2-K^2)+(1-Cb)(S-K),0]\right\}^+ \end{aligned} \tag{27}$$

Under the annuity measure,

$$\frac{V(K)}{A(0,T,T+\tau|t)} = E^A\left[\frac{g(S)}{A(0,T,T+\tau|T)}\right]$$

Notice that $A(0,T,T+\tau|T)$ is a risk free annuity, thus it will never be zero.

Assume the adjusted par spread follow the driftless lognormal dynamics, the equation

$$\frac{dS}{S} = \sigma dW^A$$

Then the expectation can be calculated. Strictly speaking, $b(S^2-K^2)+(1-Cb)(S-K)=0$ has two roots, thus the integral for payer(call) options should be done in the interval $[0,\ -K-\frac{1}{b}(1-Cb)]$. However, we notice that the upper bound will not be reached until the probability density of the spread becomes negligible. Therefore, for simplicity, the upper bound is replaced by positive infinity and the pricing formula can be derived as,

$$\begin{aligned} V(K) =& \gamma A(0,T,T+\tau|t)\left\{(1-cb)[F_a N(\gamma d_1)-KN(\gamma d_2)]\right. \\ & \left.+b[F_a^2\exp(\sigma^2(T-t))N(\gamma d_3)-K^2N(\gamma d_2)]\right\} \end{aligned} \tag{28}$$

where

$$\begin{aligned} F_a =& E^A[S] \\ d_1 =& \frac{\ln\frac{S}{K}+\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}} \\ d_2 =& d_1-\sigma\sqrt{T-t} \\ d_3 =& d_1+\sigma\sqrt{T-t} \end{aligned}$$

As we can see, the approximate pricing formula of the market traded spread-strike CDXO is composed of a major component of a vanilla option and a small correction from a power option. All the required parameters can be determined from market quotes using the approaches described in previous sections.

This simple formula allows us to easily compute CDXO prices and implying volatilities from market quoted premia. The implied volatilities can be used to calculate vanilla option prices because the underlying dynamics is the same. Thus, the correction integral in Eq. (19) can be calculated without bootstrapping the vanilla options. First, an F_a is guessed, probably by assuming the correction integral is zero; then use Eq. (28) to imply volatilities and calculate the vanilla option prices; compute the correction integral to reach a better F_a; repeat these steps iteratively until F_a converges. The convergence speed is very

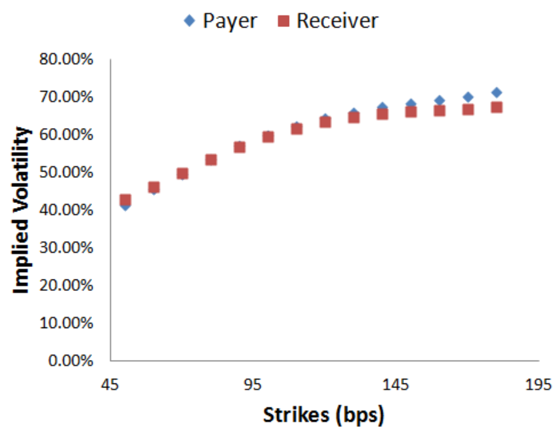

Figure 5: Markit Model Implied Volalities

fast, because the contribution of the correction term is very small. Therefore, even though we may have large discretization error when replacing the integral with summation, the error can be safelty ignored.

Figure 5 shows how the annuity mapping method as well as the modified pricing formula can help to improve the implied volatility calculation. We used the same data set as in Figure 3 for Figure 5.

4 Credit Volatility Index

Mele and Obayashi have laid out the formulas of constructing a VIX-like credit volatility index using spread-strike credit index options. The authors have chosen a forward risky annuity as the numeraire in their formulation. Unfortunately, considering the special exercise value conversion, the forward risky annuity is not a natural numeraire in this market. As we have shown in previous sections, market traded spread-strike CDXOs are not vanilla options. Therefore, its second derivative with respect to strike is not the probability density and the regular replication method for the variance swap breaks down.

However, we have also shown that the market traded options themselves can be replicated by vanilla options and these underlying implicit vanilla options can be derived from the market traded options. In section 2.4 the derived vanilla options are used to calculate the correction term for determining the forward spread. They can be used to construct the usual VIX-like credit volatility index as well.

Following the VIX approach, the credit volatility index can be defined as

$$\sigma_I = 100 \times \sqrt{\frac{2}{T-t} \sum_i \frac{\Delta K_i}{K_i^2} Q(K_i)} \tag{29}$$

where σ_I is the volatility index; K_i is the strike spread or price of the i-th OTM option, as seen in the market quotes; ΔK_i are the intervals between quoted strikes; and $Q(K)$ are OTM vanilla options.

Please note that Eq. (29) is suitable for both price-strike options and spread-strike options. For price-strike options, the market traded options are vanilla options and the market quotes can be directly plugged in. For spread-strike options, vanilla options have to be bootstrapped from the market traded options, see Section 2.4.

Unlike the spread-strike option pricing formula, the construction of the volatility index does not require any assumption of the underlying distribution and dynamics. The calculation is carried out purely based on credit index option market quotes. There is no interest rate dynamics or index-basket basis assumption either. The only requirement is market traded data.

5 Summary

We have proposed a new approach to price CDXOs of which strikes are spreads. This type of options require spread-upfront conversion to determine the exercise value which is random number viewed as of today. We have applied the replication and linear annuity mapping methodologies and derived pricing formulas that takes no assumption on interest rate dynamics. We have also provided an approach to imply the model parameters from the market traded CDXO data. Therefore, we have removed the ambiguity in calculating the FEP adjusted forward spread and the discount factor(risky annuity). By deriving vanilla options from the market traded non-vanilla options, regular replication methodology can be applied to construct the VIX-like credit volatility index.

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