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Journal of Derivatives; Spring 2011; 18, 3; ProQuest Central
pg. 61

Force-Fitting CDS Spreads to CDS Index Swaps

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Issues of contemporaneity, liquidity, different restructuring clauses, and market supply/demand contribute to the fact that the market-quoted term structure of CDS index spreads does not always agree with the term structure of CDS index spreads implied by the CDS term structures of the constituent credits. This can cause problems for those who wish to calibrate no-arbitrage pricing models in order to price and hedge index derivatives. This is true even if bid–offer spreads prevent the arbitrage from being tradable. This problem is especially relevant for single-tranche index derivatives for which the model must calibrate to the index tranche prices, but the output hedges are expressed in terms of single-name CDS. In this article, O'Kane presents an approach that can be used to overcome this problem and that is also simple, intuitive, and fast.

Credit default swap (CDS) portfolio indices make it fast, easy, and relatively inexpensive for credit market participants to assume or hedge an exposure to an index of, typically, 125 individual credits. Their high liquidity has also meant that they have become the underlying for a number of advanced credit derivative structures, including index options and single-tranche CDOs.

In the case of single-tranche CDOs, whose pricing is discussed by Hull and White [2004], it is not possible to hedge single-name risk using an index. This is because the sensitivity of the value of a single-tranche CDO to a particular credit, known as its *delta*, depends on the CDS spread curve of that credit name and the degree of default correlation it has with other credits in the index, as shown by Greenberg and Schloegl [2003]. As a result, different credits have different deltas, and hedging with an index of equally weighted credits does not work. Instead, dealers hedge the credit-specific risks of a single-tranche CDO using a single-name CDS contract.

Care must be taken, however, because of a theoretical link between the index spreads and the individual CDS spreads. Because the protection leg of a CDS index can be exactly replicated by the protection legs of a portfolio of underlying CDSs, no-arbitrage considerations require that the expected present value of the index upfront payment plus subsequent spread payments must equal the total expected present value of the premium legs of the underlying CDSs.

In practice, the higher-liquidity, more-frequent quotation and the differences in conventions that may exist between the index and the single-name CDS market mean that these no-arbitrage requirements are not always obeyed exactly. This can cause problems since trading desk analytics for single-tranche CDOs attempt to calibrate to the single-tranche CDO market quotes that are at an index level using a model that captures

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the individual spread curves of the credits in the reference portfolio. Such a model will fail if the expected loss of the reference portfolio implied by the index spreads is not consistent with the expected loss of the reference portfolio implied by the individual CDS spreads at different maturity points. The most liquid maturity points are at 3, 5, 7, and 10 years.

One way to overcome this problem is to force-fit the CDS spreads. In other words, we assume that the index spreads, with their higher liquidity, are correct, and we then adjust the individual-name CDS curves to conform to the no-arbitrage relationship. A number of ways for doing this exist. In this article, we present an approach that is simple, intuitive, and computationally efficient.

The type of arbitrage that we are attempting to eliminate here is a *theoretical arbitrage*. We define a theoretical arbitrage as one that assumes a bid–offer spread of zero. Whether it is a *tradable arbitrage* depends on the size of the bid–offer spread in the CDS index market and the single-name CDS market. The spread is currently about 1 basis point (bp) for the index swap, but can be much wider for a single-name credit. For example, Citibank was trading at 155–168 bps, a bid–offer spread of 13 bps, in January 2010. Due to the wide bid–offer spreads in the market, we find that the arbitrages are usually not tradable. They are certainly large enough, however, to create problems when it comes to calibrating and pricing single-tranche index derivatives.

The structure of this article is as follows. We first introduce the CDS index and then set out a valuation approach for both single-name CDSs and the CDS index. We show how to use this to determine the no-arbitrage relationship. We then describe reasons for the existence of the basis between the index and underlying CDS. After this we describe the mathematics of the adjustment methodology, and we propose three algorithms. We then test these algorithms using a real-world example.

THE CDS INDEX

A CDS index is an over-the-counter bilateral contract. The buyer of the CDS index is a buyer of credit risk (and is a seller of protection) who receives the coupon but takes losses if there are defaults in the index portfolio.[1] The seller of the CDS index is short the credit index (and is a buyer of protection) and takes the opposite position of the seller. The index upfront value is therefore quoted as the cash amount paid by an index buyer (protection seller) to enter into a CDS contract.

The most liquid CDS portfolio indices are the investment-grade indices of iTraxx and CDX. Each contains 125 investment-grade reference credits chosen to represent the most liquid investment-grade credits satisfying the index criteria. For iTraxx, these require that all the issuers in the index be domiciled in Europe. For CDX, these require that the 125 credits be all domiciled in North America. Both the CDX and the iTraxx families of indices follow the same rules in terms of how they work and so can be treated within the same modelling framework.

CDS indices are issued semi-annually, each with a series number to denote when it was issued and with a coupon that is fixed for the lifetime of that series. For the investment-grade iTraxx and CDX indices, the issue dates are March 20 and September 20, respectively. Indices are issued with nominal maturities of 3, 5, 7, and 10 years. Each series of an index shares the same underlying list of reference credits. Note also that the so-called 3-, 5-, 7-, and 10-year indices do not have exactly 3, 5, 7, and 10 years to maturity, respectively, on their issue date. At issuance, the time to maturity of a T-year index is typically $T + 3$ months, declining to $T - 3$ months when the next index is issued, and it is no longer considered to be "on-the-run." For example, a 5-year iTraxx index issued on March 20, 2009, matures on June 20, 2014, and has 63 months to maturity on the issue date.

MECHANICS OF SINGLE-NAME CDS

Unlike CDS indices, which are issued semi-annually, single-name CDS contracts are issued quarterly. Depending upon when it was traded, a T-year CDS contract will mature on whichever of the four standard maturity dates—March 20, June 20, September 20, and December 20[2]—follows the T-year anniversary of the initial trade date. For example, a five-year CDS contract traded on March 16, 2009, matures on March 20, 2014, and one traded 10 days later on March 26, 2009, matures on June 20, 2014.

Prior to 2009, the coupon of a CDS contract was set at contract initiation in order to ensure that the initial value of the CDS contract was equal to zero. However, in early 2009, the International Swaps and Derivatives Association (ISDA) began the "recouponing" of North

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American CDS as part of the "Big Bang" revision of the CDS market. This change in market convention was intended to facilitate the process of contract netting within the CDS market and thus reduce the risk of a systemic contagion event due to counterparty risk. This change was also intended to facilitate the centralized clearing of CDS contracts.

Prior to this recouponing, a purchase of CDS protection by party A from party B, and followed by a later sale of CDS protection by party A to the same party B to the same contract maturity date, resulted in a risky annuity with a cash flow equal to the difference in the value of the coupons.[3] This stream of coupons was exposed to the risk of default because it terminated either at default or at contract maturity, whichever occurred first. The main disadvantage of the ongoing annuity was that the two offsetting positions would continue to sit on the books of the counterparties to these two contracts until default or until contract maturity, exposing each party to the other party's credit risk. Furthermore, if these trades were cleared via a centralized clearing house, the ongoing existence of these payment legs would require the warehousing of many tens of thousands of offsetting contracts and the administration of all of the contractual annuity payments. The technological, administrative, and operational burden associated with this would have been significant.

To avoid this problem, the mechanics of the standard CDS were amended. By fixing the coupon on all contracts linked to each issuer, the offsetting sale of CDS protection occurs at the exact same coupon as the earlier purchase of protection, and consequently, the net cash flows on the resulting annuity will be zero. As a result, both offsetting contracts can be terminated. The fixing of the coupon means, however, that a new contract can no longer have a zero initial cost,[4] so there must be an initial upfront exchange of cash.

The recouponing of legacy contracts took place in Europe and the U.S. during 2009. The mechanics of the recouponing are discussed in the Appendix. The coupon assigned to the CDS contracts of each issuer depends on the issuer's credit quality[5] and region of domicile.[6]

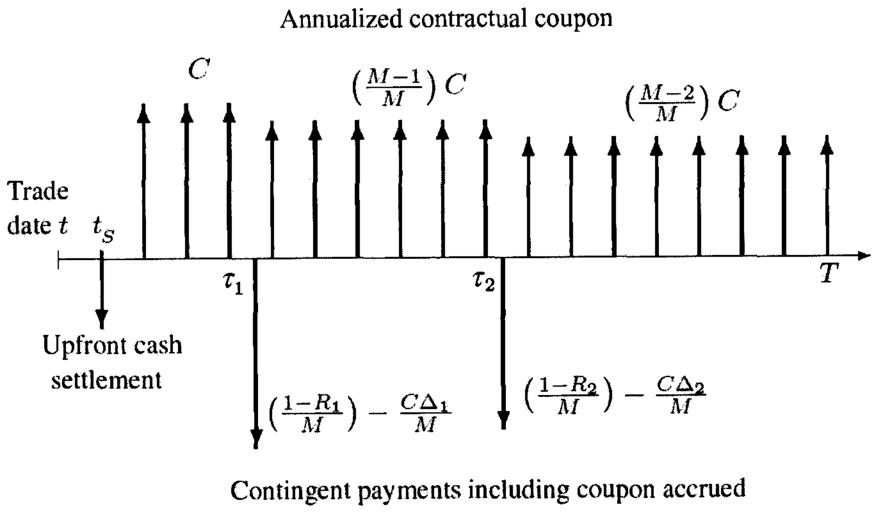
MECHANICS OF A CDS INDEX

To determine the value of a standard T-maturity CDS index, we need to know what payments are made, under what conditions, and when. Exhibit 1 shows the payment mechanics of a typical CDS index. The contract is entered into on trade date t and is cash-settled three days later. On this date, the buyer of the index contract (seller of protection) makes an upfront payment to the seller of the index contract (buyer of protection) given by $U_I(t,T)$.

On the issue date of an index this value is typically close to zero, because at inception the index coupon, $C_I(T)$, is set close to the fair value spread of the index. It is not set exactly equal to the fair value spread because the index coupon is usually chosen to be a round multiple of 5 bps. Following the issue of the index, the upfront value of the index, $U_I(t,T)$, can become positive or negative depending on the evolution of the index spread.

Following settlement, the index buyer has a contract that pays the index coupon. This is usually paid

EXHIBIT 1
The Mechanics of a CDS Index



Note: The exhibit shows the cash flows on a CDS index of M credits with a face value of \$1 initiated at time t and settlement at time t_S. The index coupon is C and the upward arrows denote the incoming payments for a protection seller. The scenario depicts two defaults in the index at times τ_1 and τ_2. Following each default, a contingent payment is made from the protection seller to the protection buyer, and a reduction is made in the notional on which the index coupon is paid.

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quarterly according to an actual/360 basis convention. This is the same as the standard CDS premium-leg convention. If there are no credit events on the underlying portfolio, then the same coupon (ignoring variations in the day count fraction) is paid until the contract matures at time T. If there is a credit event, however, and assuming that the portfolio consists of M credits, the following happens:

1. The index buyer pays $1/M$ of the face value of the contract to the seller in return for delivery of a defaulted asset also on $1/M$ of the contract notional. In practice, an ISDA auction is used to determine a cash settlement price for the recovery value of the credit in the CDS index that experienced a credit event.
2. The index buyer receives the fraction of coupon that has accrued from the previous coupon date on the defaulted credit.
3. The notional of the contract is reduced by a factor of $1/M$. As a result, the absolute amount of the index coupon received on the premium leg is reduced.

Exhibit 1 shows what happens in the case of two credit events over the life of a portfolio index. In both cases we show the default loss and the payment of the accrued coupon at default. We also show how a default reduces the size of the subsequent coupon payments.

SPREAD QUOTATION CONVENTIONS AND NOTATION

The recouponing of CDS contracts has made them mechanically more similar to CDS indices, because now all contracts linked to reference credit m have the same fixed coupon, C_m. Individual CDS contracts are also now being quoted using the same conventions as CDS indices. The most direct quotation convention is simply the upfront price paid by a protection seller to enter into a T-maturity CDS contract on reference credit m, which is given by $U_m(t,T)$. Although this is the actual value of the contract, market participants prefer to use a spread-based measure to quote prices because it makes it easier to compare the pricing of contracts at different maturities and to compare the pricing of CDS contracts on different issuers who may have different fixed coupons.

There are two different spread measures. The first is the par spread, $S_m(t,T)$. This is the spread that has been traditionally used in the CDS market prior to recouponing because it used to represent the coupon paid by a newly traded CDS contract with an initial value of zero. Although the contract no longer works in this way, the spread measure is still calculated and used for quotation.[7] Using this spread measure, the upfront value of a CDS contract is given by

$$U_m(t,T)=(C_m(T)-S_m(t,T))\cdot A_m(t,T) \tag{1}$$

where the annuity term $A_m(t,T)$ is defined as the expected present value of a $1 annuity paid on the premium (coupon) leg of the CDS contract taking into account that the coupon payments stop at a credit event or the contract maturity, whichever occurs sooner. It is a function of the entire term structure of CDS par spreads out to maturity time T and the issuer expected recovery rate, R_m. Given an upfront value, knowledge of the CDS par spread and of the CDS coupon, the market value of $A_m(t,T)$ can be implied from Equation (1). If we are given just a term structure of CDS spreads, then it must be calculated using a CDS valuation model; see Equation (5).

The second spread measure is the flat spread, $\overline{S}_m(t,T)$. This is defined as the level at which a flat CDS par spread curve used with the standard CDS valuation model would reprice the contract to its market-quoted upfront value, $U_m(t,T)$. The advantage of the flat spread is that it permits a one-to-one mapping between price and spread. But the cost of doing this is that we must ignore the shape of the term structure of spreads in exactly the same way that a bond's yield to maturity ignores the shape of the yield curve. Formally, the upfront value of a CDS contract is given by

$$U_m(t,T)=(C_m(T)-\overline{S}_m(t,T))\cdot\overline{A}_m(t,T) \tag{2}$$

where the annuity term is different from the one in Equation (2) because it assumes a flat spread curve (i.e., it is only a function of $\overline{S}_m(t,T)$).

A model is needed to determine the value of the annuity term in both cases. The standard CDS valuation model was set out by O'Kane and Turnbull [2003] and is now provided formally by the ISDA.[8] In order to clarify the different quotation conventions, a list of notations and corresponding descriptions is provided in Exhibit 2.

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EXHIBIT 2
Notations Used with Descriptions

Symbol	Description
$Z(t, T)$	Price at time t of \$1 paid at time T discounted on the LIBOR curve
Δ_n	Year fraction in the market standard actual/360 basis between t_{n-1} and t_n
	Index
$C_I(T)$	Fixed coupon of the CDS index, which can depend on the contract maturity T
$T_1, T_2, \ldots, T_N$	Index term structure maturity dates
$U_I(t, T)$	Market-quoted upfront value of the T-maturity CDS index
$V_I(t, T)$	Intrinsic (CDS-implied) upfront value of the CDS index
$\bar{S}_I(t, T)$	Flat CDS index spread for a contract on credit m with maturity time T
R_I	Market convention recovery rate used to value index
$\bar{A}_I(t, T)$	Present value at time t of a \$1 risky annuity on the index with maturity T. The spread curve used to calculate $\bar{A}_I(t, T)$ is flat at a par spread, $\bar{S}_I(t, T)$
	Individual CDS
C_m	Fixed coupon paid on the premium leg of a CDS contract on credit m
$\hat{T}_1, \hat{T}_2, \ldots, \hat{T}_N$	CDS term structure maturity dates
$U_m(t, T)$	Upfront market value of the T-maturity CDS contract on credit m
$S_m(t, T)$	Par CDS spread for a contract on credit m with maturity T
$S_m(t, T)$	Flat CDS spread for a contract on credit m with maturity T
$Q_m(t, T)$	Survival probability of credit m from time t to T
$A_m(t, T)$	Time-t value of a \$1 risky annuity on credit m with maturity T. This depends on the full term structure of the par CDS spreads
$\bar{A}_m(t, T)$	Time-t value of a \$1 risky annuity on credit m with maturity T calculated by assuming a flat CDS par spread curve equal to $\bar{S}_m(t, T)$
R_m	Expected recovery rate for reference credit m
τ_m	Default time of reference credit m
$R_m(\tau_m)$	Realized recovery rate for reference credit m at the time of default, τ_m

THE MARKET-QUOTED VALUE OF A CDS INDEX

CDS indices are quoted using the index flat spread. The relationship between the index coupon, $C_I(T)$; upfront index value, $U_I(t,T)$; and index flat spread, $\bar{S}_I(t,T)$, is given by

$$U_I(t,T) = (C_I(T) - \bar{S}_I(t,T)) \cdot \bar{A}_I(t,T) \qquad (3)$$

where $\bar{A}_I(t,T)$ is the index risky annuity that is calculated using the standard CDS valuation model with a flat spread curve at a spread, $\bar{S}_I(t,T)$, and an assumption about the index expected recovery rate, R_I. The value of the recovery rate is set according to market convention, which is currently 40% for the investment-grade CDS indices CDX and iTraxx.

Exhibit 3 shows the maturity dates, coupons, index spreads, and upfront values of the 3-, 5-, 7-, and 10-year

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EXHIBIT 3

Term Structure of Quotes for the CDX NA IG Series 13 Investment-Grade Indices as of January 4, 2010

Maturity Term (years)	Maturity Date	Coupon (bps)	Index Spread (bps)	Upfront Cost (%)
3	Dec. 20, 2012	100	79	+0.600
5	Dec. 20, 2014	100	82	+0.825
7	Dec. 20, 2016	100	90	+0.609
10	Dec. 20, 2019	100	105	−0.400

Note: The coupon is the contractual coupon on the premium leg of the CDS index and the index spread is the market-quoted flat par spread. The upfront cost is the initial payment that must be made by a protection seller to enter into the respective CDS index contract. The relationship between the coupon, index spread, and upfront cost is given in Equation (3).

Source: Markit.

maturity issues of Series 13 of the North American CDX investment-grade index, which is the on-the-run series of the index in January 2010. The index spread is defined as the level of the flat spread curve at which the index, valued as a CDS contract, would re-price the upfront value of the index. According to our notation, it is $\bar{S}_I(t,T)$. If the index spread is greater than the actual fixed coupon on the index, as it is in the case of the 10-year index, then the index has a negative upfront value, $U_I(t,T)$, from the perspective of an index buyer. As a result, an investor who wishes to buy the 10-year index would actually receive a cash payment equal to 0.40% of the index face value.

THE INTRINSIC VALUE OF A CDS INDEX

At time t, an investor buys (sells protection on) a CDS index with maturity time T. The contract pays the investor a fixed coupon that we denote as $C_I(T)$. To enter into this contract, the investor has to make an initial upfront payment of $U_I(t,T)$, which may be positive or negative. This is the quoted market value of the index.

We now introduce another value for the index. This is the *intrinsic* value of the index based on the underlying constituent CDS spread curves. We denote it as $V_I(t,T)$. If $V_I(t,T) = U_I(t,T)$, then the intrinsic value calculated from the underlying CDS equals the upfront value quoted in the market and there is no arbitrage. If $V_I(t,T) \neq U_I(t,T)$, there is a theoretical arbitrage.[9] To determine the intrinsic value of the CDS index in terms of the CDS spreads, we consider the protection and premium (coupon) legs separately. All of the discussion until this point has been model independent and based on no-arbitrage relationships. The calculation of the intrinsic value of a CDS index, however, requires the use of a specific pricing model, and for this we use the market standard valuation model.

Protection Leg

We index the M reference credits in the CDS index with $m = 1, \ldots, M$. The default of a reference credit m at time τ_m results in an immediate loss of $(1 - R_m(\tau_m))$ to the index buyer. We can thus write the expected present value of the protection leg of the index at valuation time t as

$$\text{Protection PV}_I(t,T) = E_t^Q\left[\frac{1}{M}\sum_{m=1}^{M}(1-R(\tau_m)) \times \exp\left(-\int_t^{\tau_m} r(s)ds\right)1_{\tau_m \le T}\right]$$

where $1_{\tau \le T} = 1$ if $\tau \le T$, and zero otherwise. The expectation is taken in the risk-neutral measure. We assume independence between the default time, interest rate, and recovery rate for each credit in the index. We can then write the value of the protection leg in terms of the reference entity survival curve, $Q_m(t,T)$, and the LIBOR discount factor, $Z(t,T)$, as follows:

$$\text{Protection PV}_I(t,T) = \frac{1}{M}\sum_{m=1}^{M}(1-R_m)\int_t^T Z(t,s)(-dQ_m(t,s)) \tag{4}$$

where $Q_m(t,T) = E_t^Q[1_{\tau m > T}]$, and R_m is the expected recovery rate of credit m at default (i.e., $R_m = E_t^Q[R(\tau_m)]$). CDS contracts on a specific credit, m, with a fixed coupon, C_m, have an initial upfront value, $U_m(t,T)$. According to the standard CDS valuation model, this is given by

$$U_m(t,T) = C_m \cdot A_m(t,T) - (1-R_m)\int_t^T Z(t,s)(-dQ_m(t,s))$$

The first term is the present value of the coupon payment leg. This is given by the fixed coupon multiplied by the risky annuity $A_m(t,T)$. This is the value of an annualized payment of \$1 paid on the coupon

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leg. Including the effects of the accrued coupon paid following a credit event, O'Kane [2008] showed this to be very well approximated by

$$A_m(t,T) = \frac{1}{2}\sum_{n=1}^{N}\Delta_n Z(t,t_n)(Q_m(t,t_{n-1}) + Q_m(t,t_n)) \quad (5)$$

The index n enumerates the coupon payment dates between today and the contract maturity, and Δ_n is the accrual factor for the period t_{n-1} to t_n in the market standard actual/360 basis.

The CDS par spreads, $S_m(t,T)$, are defined as the coupon on a time T-maturity CDS on reference credit m that would make the initial CDS value equal zero, that is,

$$S_m(t,T)\cdot A_m(t,T) - (1-R_m)\int_t^T Z(t,s)(-dQ_m(t,s)) = 0 \quad (6)$$

Substituting this into Equation (4) allows us to write the present value of the index protection in terms of the individual-name par spreads and risky annuities,

$$\text{Protection PV}_I(t,T) = \frac{1}{M}\sum_{m=1}^{M} S_m(t,T)\cdot A_m(t,T) \quad (7)$$

Premium (Coupon) Leg

Each credit event in the CDS index portfolio causes a reduction in the contractual notional on which the coupon is paid by a factor of $1/M$. Thus, the value of the CDS index premium leg is a sum over the reference credits and payment dates—with a payment only being made if it occurs before the default time—to give

$$\text{Premium PV}_I(t,T) = C_I(T) \cdot E_t^Q\left[\frac{1}{M}\sum_{m=1}^{M}\sum_{n=1}^{N}\exp\left(-\int_t^{t_n} r(s)ds\right)1_{\tau_m > t_n}\right]$$

Assuming independence of the default time and interest rate process for each credit, we can write this as the discounted coupons weighted by their corresponding survival probability,

$$\text{Premium PV}_I(t,T) = \frac{C_I(T)}{M}\sum_{m=1}^{M}\sum_{n=1}^{N} Q_m(t,t_n)Z(t,t_n)$$

To fully model the premium leg, we also need to add on the fraction of the accrued premium that is paid at default. We can then write the index premium leg value as a sum over all M premium legs of the individual CDSs with all of them paying the same contractual coupon, $C_I(T)$,

$$\text{Premium PV}_I(t,T) = \frac{C_I(T)}{M}\sum_{m=1}^{M} A_m(t,T) \quad (8)$$

where $A_m(t,T)$ is the risky annuity term defined in terms of the issuer's survival probability curve in Equation (5).

The Intrinsic Value

The intrinsic value at time t of a long position in the index swap is the expected present value of the premium leg minus the expected present value of the protection leg. This is given by

$$V_I(t,T) = \frac{1}{M}\sum_{m=1}^{M}(C_I(T) - S_m(t,T))\cdot A_m(t,T) \quad (9)$$

For reasons discussed in the following section, this may not be the same as the market-quoted index upfront value, $U_I(t,T)$. No-arbitrage requires that this intrinsic value, $V_I(t,T)$, equal $U_I(t,T)$, the upfront value of the index. Because the upfront value of the index is determined as though it were a CDS with a flat par spread curve at the index spread, no-arbitrage requires the following relationship among the market-quoted index upfront, the CDS spreads, and the index flat par spread, $\overline{S}_I(t,T)$,

$$\begin{aligned} U_I(t,T) &= (C_I(T) - \overline{S}_I(t,T))\cdot \overline{A}_I(t,T) \\ &= \frac{1}{M}\sum_{m=1}^{M}(C_I(T) - S_m(t,T))\cdot A_m(t,T) \quad (10) \end{aligned}$$

This relationship may not be observed in practice for reasons discussed in the next section.

THE CDS INDEX BASIS

The CDS index basis is the observed difference between the market-quoted index spread and the implied spread given by the CDS term structures of the constituent credits in the index. The theoretical relationship

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is set out in Equation (10). In practice, this relationship can break down for the following reasons:

1. *Restructuring clauses.* In the case of the North American CDX index, the payment of protection on the index protection leg is only triggered when the credit event is a bankruptcy or failure to pay. Restructuring is not included as a credit event. This type of contract is therefore known as a No-Re contract. However, until mid-2009 the market standard for single-name CDS contracts in the U.S. was based on the use of the Mod-Re restructuring clause in which restructuring is included as a credit event. The designation Mod-Re refers to the fact that there are certain restrictions on what can be delivered by the protection buyer in order to settle a CDS contract following a restructuring credit event. Because Mod-Re contracts include an additional credit-event trigger, they trade at a wider spread to No-Re CDS contracts. Where both trade, the Mod-Re CDS spreads are typically about 5–10% higher than the No-Re spreads. Before mid-2009, when the North American CDS market switched to No-Re, this created an immediate basis between market-quoted single-name CDS spreads and the CDS index spread. Since the switch, differences in restructuring clause only affect the CDS index basis if non-standard Mod-Re CDS quotes are used. In Europe both the iTraxx indices and the standard underlying single-name CDS contracts trade according to the same Mod-Mod-Re convention, so restructuring is not a driver of the index basis.
2. *Market technicals.* Mid-market spreads may also violate Equation (10) for reasons due to market technicals. For example, the CDS index tends to be the preferred instrument used by market participants to express a changing view on the credit market as a whole. As a result, the CDS index may be considered to lead the CDS market wider (tighter) in times of negative (positive) news that does not relate to a specific credit. This market technical can drive apart the theoretical relationship in Equation (10).
3. *Mid-market spreads.* Another reason why Equation (10) is not always obeyed by market prices is that it is based on contemporaneous mid-market spreads. The CDS index market is extremely liquid and presents bid–offer spreads of less than 1 bp in the investment-grade indices, rising to a few basis points for the less liquid indices such as the CDX high-yield index. By contrast, CDS spreads are much less liquid and may trade with bid–offer spreads equal to between 5% and 10% of the spread. It may also be difficult to obtain good quality contemporaneous quotes for the 100 or so CDSs in the index. Assuming an average bid–offer spread of 10 bps, the index spread and the intrinsic mid-market spread would need to differ by an amount greater than this in order for an arbitrage to be tradable, and this is unlikely. The position would also have to be held to maturity to avoid re-crossing the bid–offer spread. If such an opportunity did appear, trades would be placed by dealers or hedge funds that would push any mispricing back inside the bid–offer limits.

There are two ways to ensure that Equation (10) is obeyed. The first is to adjust for any difference in restructuring clause between the CDS index and the underlying single-name CDS contracts, and to then adjust for no-arbitrage using one of the methods described in the next section. The second way is to simply correct for both the restructuring clause and the theoretical arbitrage in one adjustment. This is the approach we use in the next section.

THE PORTFOLIO SWAP ADJUSTMENT

One way to ensure that the index swap spread equals the intrinsic swap spread is to adjust the individual CDS curves in such a way that the adjusted CDS spreads obey Equation (10). The exact nature of the adjustment is somewhat subjective and arbitrary, however, we would like the adjustment to possess some desirable properties:

1. We would prefer a proportional adjustment of the spread because this seems more realistic when applied across the broad range of spread levels found in an index. One downside is that it can change the slope of the CDS curve—while a flat curve remains flat, an upwardly sloped curve can become flatter or steeper if the adjustment factor is less than or greater than one. We believe that it is preferable to an additive adjustment, which may require a narrowing of all the spreads in the index by the same amount, which can cause the spreads of high-quality credits to go negative. This is not permitted for reasons of no-arbitrage.

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2. We require that the proportional adjustment not induce any arbitrage effects. For example, it should ensure that each adjusted issuer survival curve, $Q_m(t, T)$, remains a monotonically decreasing function of T.
3. Speed is another very important consideration because the adjustment of CDS spread curves to agree with CDS index market quotes is an essential pre-processing step to the pricing and risk management of all index-based correlation products. For example, if we are calculating the sensitivity of the price of a synthetic tranche to changes in the CDS curve of each of the credits in the index at each of the four curve points used here, then we need to run the spread adjustment algorithm for each of the N curve points and M credits in the portfolio. For a standard investment-grade CDS index with $M = 125$ and $N = 4$, this means 500 repeats of the spread adjustment algorithm. The speed of the algorithm can therefore have a significant impact on the time taken to calculate the numbers needed to risk manage a correlation book.

MATHEMATICS OF THE ADJUSTMENT

We wish to adjust the single-name par spreads, $S_m(t, \hat{T}_n)$, to fit the index spread quotes, $S_I(t, T_n)$, at the $n = 1, \ldots, N$ quoted index maturity points, $T_1, T_2, \ldots, T_N$. We have assumed that we have CDS market spread quotes to the CDS maturity dates, $\hat{T}_1, \hat{T}_2, \ldots, \hat{T}_N$. The quarterly rolling of the CDS maturity dates compared to the semi-annual rolling of the index maturities means that the maturity dates of the 3-, 5-, 7-, and 10-year CDS do not always coincide with the maturity dates of the 3-, 5-, 7-, and 10-year CDS indices.

When the CDS and index maturity dates do not coincide,[10] we need to begin by creating a new set of CDS curve par spread quotes for each of the M issuers with the same maturity dates as the indices. To calculate these we can simply interpolate the spreads from the market-quoted curve, $S_m(t, \hat{T}_n)$, using the standard CDS valuation model, which has its own implicit interpolation scheme to give $S_m(t, T_n)$. We do this because having the same maturity dates for the CDS curve quotes as we have for the CDS index curves mean that there is an exact correspondence between changing the CDS curve and changing the index curve. We denote the adjusted issuer spreads, which eliminate the theoretical arbitrage with $S^*_m(t, T_n)$. Across the term structure of index spreads, we have N equations, as follows:

$$U_I(t,T_1) = \frac{1}{M}\sum_{m=1}^{M}(C_I(T_1) - S^*_m(t,T_1)) \cdot A_m(t,T_1)$$
$$U_I(t,T_2) = \frac{1}{M}\sum_{m=1}^{M}(C_I(T_2) - S^*_m(t,T_2)) \cdot A_m(t,T_2)$$
$$\cdots$$
$$U_I(t,T_N) = \frac{1}{M}\sum_{m=1}^{M}(C_I(T_N) - S^*_m(t,T_N)) \cdot A_m(t,T_N) \qquad (11)$$

These equations are nonlinear in $S^*_m(t,T)$ due to the spread dependence of the risky annuity terms $A_m(t,T)$.

FITTING ALGORITHMS

We set out three algorithms that can be used to adjust the individual CDS curves to obey Equation (11). They all have in common the use of a bootstrap approach to solve the N-coupled equations. According to this methodology, we successively adjust the single-name CDS spread curves to fit the CDS index–implied spreads at 3-, 5-, 7-, and 10-year spreads. Because we have adjusted the CDS spread quotes to have the same maturity dates as the CDS indices, adjusting the 3-year CDS spreads only changes the 3-year index spread. Then, we can hold the adjusted 3-year CDS quotes fixed and adjust the 5-year CDS quotes to fit the 5-year CDS index quotes, and so on.

The second common feature of the algorithms is that they apply the adjustment methodology to all of the credits in the index using a single parameter for each maturity. This means that we have one unknown for each maturity date, and given that we have just one condition to satisfy, we should be able to solve uniquely for a solution, provided one exists.

Method I: Spread Adjustment

The first method consists of a fitting adjustment of the CDS spreads of the form

$$S^*_m(t,T_n) = \alpha(n) \cdot S_m(t,T_n) \qquad (12)$$

At each index maturity date, each of the credit spreads in the index is scaled by the same factor.

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If we adjust the spreads directly using Equation (12), then testing the relationship in Equation (11) will involve a full reconstruction of each issuer survival curve for each realization of the adjusted spreads. This can be time consuming because the standard CDS curve calibration procedure requires N nonlinear root searches, one for each value of $Q_m(t, T_n)$ on the CDS term structure. We must also check to ensure that the adjustments do not create any internal arbitrage in the CDS curve because this adjustment is not guaranteed to preserve the no-arbitrage properties of the unadjusted spread curve. This test involves checking that each of the resulting CDS survival curves, $Q_m(t, T)$, is a monotonically decreasing function of T.

Method II: Default Rate Multiplier

The second method is a fitting adjustment applied directly to the survival probabilities and takes the form

$$Q_m^*(t,T_n) = Q_m^*(t,T_{n-1}) \cdot Q_m(T_{n-1},T_n)^{\alpha(n)} \tag{13}$$

We raise the unadjusted forward survival probability to the power of $\alpha(n)$. To better understand this choice of adjustment, we write the survival probability curve for issuer m as a function of its forward continuously compounded default rate, $h_m(t)$,

$$Q_m(t,T_n) = Q_m(t,T_{n-1})\exp\left(-\int_{T_{n-1}}^{T_n} h_m(s)ds\right)$$

As a result, we can write the adjustment as a simple linear multiplier of $\alpha(n)$ on the forward default rate,

$$Q_m^*(t,T_n) = Q_m^*(t,T_{n-1})\exp\left(-\alpha(n)\int_{T_{n-1}}^{T_n} h_m(s)ds\right)$$

This adjustment preserves the no-arbitrage properties of the unadjusted curve provided $\alpha(n)$ is positive. Given that the adjustment is made directly to survival probabilities rather than to spreads, this is straightforward and fast. For each maturity, we determine the value of $\alpha(n)$ using a fixed-point iteration scheme. The algorithm is as follows:

1. Build all M credit curves using the market-quoted spreads to CDS maturity dates, $\hat{T}_1, \hat{T}_2, \ldots, \hat{T}_N$.
2. From these curves, interpolate the CDS spreads at the index maturity dates, $T_1, T_2, \ldots, T_N$, and using these quotes and the appropriate recovery rates, R_m, construct CDS survival curves, $Q_m(t,T_n)$.
3. Set $n = 1$.
4. Set $k = 0$, and $Q_m^{(k)}(t,T_n) = Q_m(t,T_n)$, $\alpha_n(k) = 1.0$ and $f_n(k) = 1.0$.
5. For all M issuers, adjust the survival probabilities using $Q_m^{(k+1)}(t,T_n) = Q_m^*(t,T_{n-1})Q_m^k(T_{n-1},T_n)^{\alpha(n)}$ where $T_0 = t$ so $Q_m(t,T_0) = 1.0$.
6. Calculate g using the equation

$$g = \frac{-U_I(t,T) + C_I(T)\frac{1}{M}\sum_{m=1}^{M} A_m(t, T_n, Q_m^{(k+1)})}{\frac{1}{M}\sum_{m=1}^{M} S_m^{(k+1)}(t, T_n, Q_m^{(k+1)}) \cdot A_m(t, T_n, Q_m^{(k+1)})} \tag{14}$$

7. Set $\alpha_n = \alpha_n \times g$.
8. If $|g - 1| \leq \varepsilon$ where ε is the required tolerance, continue to step 9. Otherwise, set $k = k + 1$ and return to step 5.
9. Set $Q_m^*(t,T_n) = Q_m^{(k+1)}(t,T_n)$ for each of the M issuers.
10. Set $n = n + 1$. If $n > N$, then stop. Otherwise return to step 4.

Method III: Survival Probability Exponentiation

We also consider an adjustment of the form

$$Q_m^*(t,T_n) = Q_m(t,T_n)^{\alpha(n)} \tag{15}$$

in which the full survival probability is raised to the power of $\alpha(n)$. This adjustment is very simple to implement using the iterative scheme described in the previous section: simply replace the adjustment equation in step 5 of the algorithm with this new equation.

Once again we must check to ensure that the adjustments do not create any internal arbitrage in the individual CDS curves. In all of the algorithms we used Brent's method as described by Press et al. [2007] to perform the one-dimensional root search.

NUMERICAL TEST

We tested all three fitting algorithms on the CDX IG Series 12 index using index prices and CDS spread

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curves obtained on August 7, 2009. The single-name CDS quotes used here are based on the Mod-Re convention, while the CDX index trades on the No-Re convention. Note that the North American market switched conventions to quote CDS on the basis of No-Re in mid-2009. The spreads used in this example, however, are based on Mod-Re. Although no longer the standard quotation convention for North American CDS, we use these because they create a larger CDS index basis and this combined with the resulting switch in the sign of the index basis between the five and seven-year point provides a more robust test for the adjustment algorithm.

Because CDS spreads with the Mod-Re convention should have a higher spread than the equivalent No-Re CDS, a difference should be created between the index value and the intrinsic value implied by the single-name CDS. We would expect this difference to bias the CDS-implied intrinsic index spread to be wider than that of the actual index. In upfront value terms, this should mean that the intrinsic upfront index value should be less than the market-quoted upfront index value.

The index details used in our test are shown in Exhibit 4. The intrinsic upfront index values based on the CDS spread curves are clearly not equal to the upfront value of the quoted in the market. As expected, the intrinsic value is less than the quoted market value of the index at the 3- and 5-year maturity points. Specifically, the 3-year index has a positive market value of 0.018%, but its intrinsic value is –0.4060%. To adjust for this we need to reduce the 3-year CDS spreads. This pattern of the intrinsic being lower than the market value also occurs for the 5-year index. The effect reverses at the longer 7- and 10-year maturities, reflecting market technical effects.

Exhibit 5 shows the input CDS spread curves for a sample of the credits in the index. The CDX IG Series 12 index contains a number of financial institutions that were trading at distressed spread levels in August 2009.[11] As a result, the range of spreads in what was initially an index of investment-grade credits is unusually broad.[12] For example, we calculated that across the index portfolio the five-year CDS spreads had a range of 15.2 bps to 4,407 bps with a mean of 146.6 bps and a standard deviation of 415 bps. This very high spread dispersion reinforces our claim that this example is a particularly robust test of the spread adjustment algorithm.

EXHIBIT 4

CDX IG Series 12 Market Prices and Index Indicative Information on August 7, 2009

Index Tenor (years)	Index Maturity	Index Coupon (bps)	Index Spread (bps)	Upfront Cost (%)	Intrinsic Value (%)
3	June 20, 2012	100	100	0.0180	–0.4060
5	June 20, 2014	100	111	–0.5006	–0.5488
7	June 20, 2016	100	108	–0.4816	–0.2153
10	June 20, 2019	100	105	–0.3938	0.3022

Note: The index spread is the flat par spread of the index to the corresponding maturity. The upfront cost is the mid-market cost of entering into this CDS index. The intrinsic value is the mid-market value of the contract implied by the spreads of the 125 underlying single-name CDS contracts.

Source: Markit.

EXHIBIT 5

Term Structure of Single-Name CDS Curves for a Selection of the Credits in the Index

	Contract Maturity			
Ticker	3-Year	5-Year	7-Year	10-Year
AA	239.29	258.67	249.70	249.78
ABX	70.06	80.08	80.05	80.03
ACE	75.03	85.05	80.01	74.94
AEP	33.08	43.19	42.11	40.01
AET	48.08	60.16	60.11	60.07
AIG	1315.29	1156.35	1030.73	935.09

RESULTS

We used each of the three adjustment algorithms to correct for both the basis due to the different restructuring clauses and any residual theoretical arbitrages in one go. Exhibit 6 shows the effect of the adjustments on the CDS spreads, which can be compared to their pre-adjustment values in Exhibit 5.

Since all of the adjustment algorithms are multiplicative, some of the absolute changes in spread appear high, especially for high-spread credits such as AIG.

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EXHIBIT 6

Term Structure of Adjusted CDS Curves for a Selection of Credits in the Index Portfolio for Each Adjustment Method

Ticker	3-Year	5-Year	7-Year	10-Year
Method I: Spread Multiplier				
AA	203.92	251.54	260.36	274.07
ABX	59.70	77.58	83.20	87.81
ACE	63.94	82.44	83.63	82.54
AEP	28.19	41.51	43.87	44.03
AET	40.97	57.97	62.48	65.91
AIG	1121.05	1141.29	1083.00	1031.92
Method II: Forward Default Rate Multiplier				
AA	203.72	253.47	262.07	277.77
ABX	59.63	78.79	85.42	90.22
ACE	63.86	83.69	84.57	82.49
AEP	28.15	42.34	44.91	44.39
AET	40.92	59.03	64.25	67.87
AIG	1122.51	1102.63	1013.46	944.88
Method III: Exponentiated Q				
AA	203.72	253.30	260.84	273.11
ABX	59.63	78.68	84.24	88.83
ACE	63.86	83.59	84.59	83.36
AEP	28.15	42.21	44.54	44.68
AET	40.92	58.89	63.38	66.83
AIG	1122.51	1105.46	1031.98	971.33

If we calculate the percentage adjustment, however, we find it is similar from reference credit to reference credit with the 3-, 5-, 7-, and 10-year spreads being adjusted by approximately –17%, –3%, +6%, and +10%, respectively. The large adjustment at the short end of the curve is due to differences in the restructuring clause and to the high spread dispersion of the portfolio. The spread dispersion causes the spread-weighted annuity to be lower than it would otherwise be if the spreads were all at the same mean level, so a larger spread adjustment is required to compensate for the difference between the intrinsic and quoted index upfront values. Because the large adjustments at 3 years and 5 years are mostly due to the difference in restructuring clauses between the single-name CDS and the CDS index, they are not arbitrageable, and neither are the differences at 7 years and 10 years, which fall within the bid–offer spread.

The range of spreads and the nature of the spread changes shown in Exhibit 6 make it even clearer that a multiplicative rather than an additive adjustment is preferable. At the three-year maturity point, applying the same additive adjustment to all spreads to the extent that the adjusted CDS spreads of tighter credits would have gone negative. At this point the algorithm would have failed since this implies an arbitrage.

The main difference in the methods is that the percentage spread changes from the spread multiplier methodology are more consistent across the different credits than when using the other methods. This is to be expected because the adjustment applies directly to spreads. Overall, there is no material difference among the curves output by the different approaches.

The adjustment parameters, $\alpha(n)$, are shown in Exhibit 6. At the three-year maturity point they are almost identical because a linear multiplier on the spread is almost identical to a linear multiplier of the hazard rate. Beyond three years, differences in the nature of the adjustments occur because, although methods I and III adjust the entire term structure, method II only adjusts the forward default rate, which means that bigger adjustments are needed to obtain the same effect.

The computation time of each algorithm is shown in Exhibit 7. The algorithm was coded in C# and run on a PC with an AMD Phenom 8450 Triple Core processor running at 2.11 Ghz. Different languages and hardware could change these numbers. Our purpose in presenting the computation times is to give an estimate of relative computation times across the different adjustment methodologies.

We observe that method III is the fastest and it beats method I—the spread multiplier approach—which required a computationally expensive recalibration of all 125 CDS curves for each change in the value of $\alpha(n)$. We find that adjustment method III takes fewer iterations than method II to converge to a solution of

EXHIBIT 7

Term Structure of the Adjustment Factor $\alpha(n)$ for Each Adjustment Methodology

Method	3-Year	5-Year	7-Year	10-Year
I: Spread Multiplier	0.85522	0.98820	1.04464	1.10057
II: Forward Default Rate Multiplier	0.85397	1.03026	1.35921	1.72061
III: Exponentiated Q	0.85397	1.01479	1.07489	1.14123

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EXHIBIT 8

Computation Time for Each Adjustment Method to Calculate the Adjusted Spreads for the Entire Index Portfolio

Adjustment Method	Computation Time (in Seconds)
I: Spread Multiplier	7.094
II: Forward Default Rate Multiplier	2.235
III: Exponentiated Q	0.687

Equation (14), because it acts on the full survival probability rather than just a forward survival probability. As a result, it has a higher sensitivity to changes in the value of $\alpha(n)$, which helps the iterative approach to converge. The convergence speed of method II may be improved with the use of a more adaptive one-dimensional root search algorithm. We tested the relative computation times across a range of portfolios and found that they are fairly consistent.

CONCLUSIONS

We have set out the fundamental no-arbitrage relationship between CDS index spreads and the spread curves of the underlying single-name CDS. We have explained why CDS index spreads are not always consistent with the underlying single-name CDS spread curves and why this can be a problem for the pricing and risk management of index derivatives, especially index tranches. To overcome this problem, we have suggested three approaches to adjust for this arbitrage, all of which behave quite similarly. Of these approaches, we prefer method III because it performs fastest and thus is the recommended approach.

APPENDIX

RECOUPONING OF LEGACY CDS CONTRACTS

The aim of recouponing legacy CDS contracts was to force all the contracts of each issuer to have as few as possible coupon values in order to facilitate contract netting. In the ideal situation, all contracts would be recouponed with the same coupon. This is not practicable, however, as it would have required a market-wide revaluation of every single legacy CDS contract followed by a corresponding cash payment from one party to the other.

Instead, a simpler route was chosen in which every existing CDS contract linked to a specific issuer was split into two new CDS contracts, each with its own fixed coupon in a way that replicated the economics of the initial contract. Only two contracts are needed because the economics of a CDS contract require the replication of the premium leg and the contingent payment leg, and these are both linear in the contract notionals. To see this, consider a legacy contract with notional N and coupon S, and replace it with two contracts with notionals N_1 and N_2 with coupons S_1 and S_2, respectively. We then have

$$N_1S_1 + N_2S_1 = NS$$

for the premium leg and

$$N_1(1 - R) + N_2(1 - R) = N(1 - R)$$

for the protection leg where R is the assumed recovery rate if there is a credit event. This gives

$$N_1 = N\frac{(S_2 - S)}{(S_2 - S_1)}$$

Due to the asymmetry of CDS contracts caused by the existence of the protection buyer's delivery option, an additional criterion is that one long (short) protection position must be replaced by two long (short) protection positions. For this to be guaranteed, we require $S_1 \leq S \leq S_2$. The two coupons, therefore, need to bracket the range of spreads at which the issuer traded over the past five or more years for which legacy contracts remain existent.

ENDNOTES

The author would like to thank Lutz Schloegl for his comments on an earlier version of this article. Helpful comments from two anonymous referees are also acknowledged.

[1]This reverses the convention used in the single-name CDS market that considers the buyer to be a buyer of protection rather than a buyer of credit risk. It reflects the fact that the single-name CDS market is primarily a market used by banks to hedge their loan book credit risk, so buying a contract means buying protection. But the CDS index market is an investor-driven market in which investors seek exposure to a diversified portfolio of credits, so "buying an index" means assuming credit risk in order to get paid a coupon.

[2]If these dates fall on weekends or holidays, any scheduled payments will roll to the next business day.

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[3]For simplicity, we consider a purchase and sale of protection between the same counterparties, A and B. In practice, the offsetting transaction may be done with a third counterparty.

[4]Except if, by chance, the fixed coupon exactly equals the CDS spread.

[5]The reason for assigning different coupons is to keep the initial entry cost of the contract close to zero. For example, a sale of protection on a AA-rated credit with a fixed coupon of 500 bps would have a large up-front cost because 500 bps is not commensurate with its risk. A coupon of 50 bps or 100 bps would be more appropriate.

[6]The standard for North American corporate-linked CDSs is to have fixed coupons of either 100 bps or 500 bps. The standard for European corporate-linked CDSs is to have a coupon of 25, 100, 300, 500, 750, or 1,000 bps.

[7]By itself it is not sufficient to recover the upfront value, $U_m(t,T)$, of a contract since the upfront value is a function of the full term structure of par spreads to time T.

[8]See www.cdsmodel.com for the source code of the standard CDS valuation model.

[9]Note that all input prices used in pricing models for calibration are midprices. Adjustments for bid- or offer-side pricing are made to the output price by the trader.

[10]For example, on January 20, 2010, the five-year on-the-run CDS matures on March 20, 2015, and the current on-the-run five-year CDS index matures on June 20, 2015.

[11]The financial institutions include AIG, CIT Group, International Lease Finance Company, and Textron Financial Corporation.

[12]The rules of the CDX investment-grade index mean that on its issue date all of the credits included will be investment grade. Thereafter, a credit is only removed from that series of the index if it incurs a credit event. For this reason, highly distressed junk credits can be found in an investment-grade index.

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to capture these important features of real-world returns. Here, Markose and Alentorn propose the use of the generalized extreme value (GEV) distribution in place of the lognormal. Tail shape in the GEV distribution is governed by a single parameter, and it can be proven that the extreme tails of any plausible return density resemble those from a GEV. The authors develop a pricing model under GEV returns, whose form is very like Black–Scholes, along with equations for its Greek letter risks. They then demonstrate its superior ability to fit FTSE option prices from 1997 to 2009 and explore its behavior around critical market events, such as the 1997 Asian crisis, the 9/11 attack in 2001, and the collapse of Lehman Brothers in 2008.

FORCE-FITTING CDS SPREADS TO CDS INDEX SWAPS 61

DOMINIC O'KANE

CDOs present some of the thorniest valuation problems in derivatives these days. The most widely used approaches are based on the strong assumption that the individual securities in the underlying pool are homogeneous. The most popular portfolios used as underliers are constructed from credit default swaps to replicate standard credit indexes, like CDX.NA.IG, or iTraxx, each of which contains CDS on 125 names. The CDO model assumes they can be treated as if they were identical, but in reality, their spreads in the market can be quite diverse. The result can be an internal discrepancy between the pricing of the CDO tranches and the 125 CDS they are composed of, especially with regard to the term structure of credit spreads. Although O'Kane shows that transactions costs are too large for the apparent arbitrage trades to be profitably implemented, it is still undesirable for model values for closely related instruments to be inconsistent with one another. In this article, O'Kane first gives a detailed description of the pricing conventions in the two markets and explains how differences in the way features like restructuring clauses are treated give rise to different valuations. He then offers a way to harmonize the two by appropriately modifying CDS quotes.

VALUATION OF LONG-TERM FLEXIBLE GAS CONTRACTS 75

LARS HOLDEN, ANDERS LØLAND, AND OLA LINDQVIST

Energy derivatives allow consumers and producers of natural gas and electricity to manage what are very complicated fluctuations in demand and supply. In particular, energy demand depends heavily on weather, which is impossible to predict accurately more than a week or two into the future. Long-term supply contracts allow some of this variability to be managed, but it is common to incorporate extensive optionality into them. Maximum and minimum values may be set for the amounts to be purchased in a given time period (day, week, month), but with options to exceed the bounds on a finite number of occasions and possible carry-forward provisions for unused options. Determining the optimal exercise strategy for these options and the appropriate valuation of the contracts that takes option exercise into account are challenging problems. In this article, the authors describe the most common optional features found in the European market for long-term natural gas contracts and offer a useful approach for valuing them, based on least squares Monte Carlo simulation.

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