CS161 HW # 5 Due: 11/9/17

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1) Use truth tables to show the following sentence pairs are equivalent:

• $P \Rightarrow \neg Q,\ Q \Rightarrow \neg P$

Apply implication elimination

$= \neg P \vee \neg Q,\ \neg Q \vee \neg P$

Also equivalent by commutativity of $\vee$

P	Q	$\neg P \vee \neg Q$	$\neg Q \vee \neg P$
F	F	T	T
F	T	T	T
T	F	T	T
T	T	F	F

• $P \Leftrightarrow \neg Q,\ ((P \vee \neg Q) \vee (\neg P \wedge Q))$

Apply biconditional elimination

$= ((P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P)),\ ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

Equivalent

P	Q	$((\neg P \vee \neg Q) \wedge (Q \vee P))$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
F	F	F	F
F	T	T	T
T	F	T	T
T	T	F	F

2) Consider the following sentences & decide whether it is valid, unsatisfiable, or neither:

• $(\text{Smoke} \Rightarrow \text{Fire}) \Rightarrow (\neg\text{Smoke} \Rightarrow \neg\text{Fire})$ Apply implication elimination

$= (\neg S \vee F) \Rightarrow (S \vee \neg F)$ Apply implication elimination

$= \neg(\neg S \vee F) \vee (S \vee \neg F)$ Apply De Morgans law

$= (S \wedge \neg F) \vee (S \vee \neg F)$

Satisfiable

S	F	$(S \wedge \neg F) \vee (S \vee \neg F)$
F	F	T
F	T	F
T	F	T
T	T	T

• $(\text{SMOKE} \Rightarrow \text{FIRE}) \Rightarrow ((\text{SMOKE} \lor \text{HEAT}) \Rightarrow \text{FIRE})$ APPLY IMPLICATION ELIMINATION

$= (\neg S \lor F) \Rightarrow ((S \lor H) \Rightarrow F)$ " "

$= \neg(\neg S \lor F) \lor (\neg(S \lor H) \lor F)$ DE MORGANS LAW

$= (S \land \neg F) \lor ((\neg S \land \neg H) \lor F)$

SATISFIABLE

S	F	H	$(S \land \neg F) \lor ((\neg S \land \neg H) \lor F)$
F	F	F	T
F	F	T	F
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

• $((\text{SMOKE} \land \text{HEAT}) \Rightarrow \text{FIRE}) \Leftrightarrow ((\text{SMOKE} \Rightarrow \text{FIRE}) \lor (\text{HEAT} \Rightarrow \text{FIRE}))$

$= ((S \land H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \lor (H \Rightarrow F))$

$= (\neg(S \land H) \lor F) \Leftrightarrow ((\neg S \lor F) \lor (\neg H \lor F))$

$= ((\neg S \lor \neg H) \lor F) \Leftrightarrow (\neg S \lor F \lor \neg H \lor F)$

$= (\neg(\neg S \lor \neg H \lor F) \lor (\neg S \lor F \lor \neg H)) \land \overbrace{(\neg(\neg S \lor F \lor \neg H) \lor (\neg S \lor \neg H \lor F))}^{\text{SAME AS OTHER SIDE!}}$

SATISFIABLE

S	F	H	$(\neg(\neg S \lor \neg H \lor F) \lor (\neg S \lor F \lor \neg H)) \land$ "DISREGARD SINCE SAME"
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

3 CONSIDER THE FOLLOWING: Object of interest UNICORN

a → If the unicorn is mythical, then it is immortal, but if it is not mythical then it is a mortal mammal.

b → If the unicorn is either immortal or a mammal, then it is horned.

c → The unicorn is magical if it is horned.

FUNCTIONS: MYTHICAL( ), IMMORTAL( ), MAMMAL( ), HORNED( ), MAGICAL( )
VARIABLES: MYT, IMM, MAM, HOR, MAG (ALL BINARY), UNICORN

} variables represent a call to their function for a unicorn

PREPOSITIONS TO CLAUSES | PART A

a → i) $MYT \Rightarrow IMM : (\neg MYT \vee IMM)$ ①

ii) $(\neg MYT \Rightarrow MAM) \wedge (\neg MYT \Rightarrow \neg IMM) :$ $(MYT \vee MAM)$ ②, $(MYT \vee IMM)$ ③

b → i) $(IMM \vee MAM) \Rightarrow (HOR) : (\neg IMM \wedge \neg MAM) \vee HOR$ → $(\neg IMM \vee HOR)$ ④, $(\neg MAM \vee HOR)$ ⑤

c → i) $(HOR) \Rightarrow (MAG) : (\neg HOR \vee MAG)$ ⑥

CNF | PART B

CNF: $(\neg MYT \vee IMM) \wedge (MYT \vee MAM) \wedge (MYT \vee IMM) \wedge (\neg IMM \vee HOR) \wedge (\neg MAM \vee HOR) \wedge (\neg HOR \vee MAG)$

USE RESOLUTION TO PROVE UNICORN IS: HORNED, MYTHICAL, MAGICAL

HORNED?

① $\neg MYT \vee IMM$
② $MYT \vee MAM$
③ $MYT \vee IMM$
④ $\neg IMM \vee HOR$
⑤ $\neg MAM \vee HOR$
⑥ $\neg HOR \vee MAG$
⑦ $\neg HOR$

⑧ $\neg MAM$
⑨ $\neg IMM$
⑩ IMM
⑪ NIL

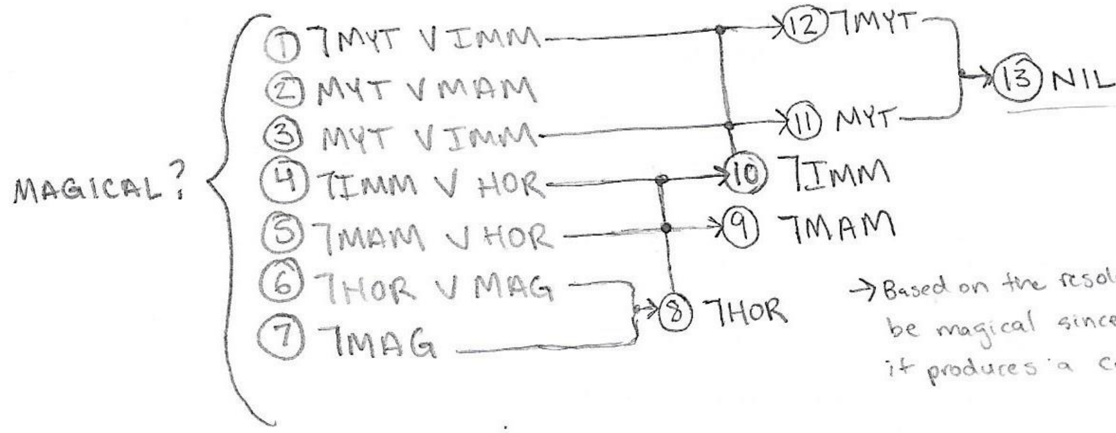
→ Given the statement 'not horned' $\neg H$, we see when this is applied for some unicorn there is a contradiction. Therefore unicorns are horned.

MYTHICAL?

① $\neg MYT \vee IMM$
② $MYT \vee MAM$
③ $MYT \vee IMM$
④ $\neg IMM \vee HOR$
⑤ $\neg MAM \vee HOR$
⑥ $\neg HOR \vee MAG$
⑦ $\neg MYT$

⑧ MAM
⑨ HOR
⑩ MAG

→ Given the statement 'not mythical' there is no contradiction, therefore not all unicorns are mythical



→ Based on the resolution all unicorns must be magical since if one is not then it produces a contradiction.