HW#4

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Partial code credit to Prof Li (UC Davis), Weiping Zhang(USTC)

Number 2

Part 1

two-sample Hotelling's T2 test ———-

```
# now we perform the two-sample Hotelling T^2-test
n<-c(18,18)
p<-2
xmean1<-c(85,83)
xmean2<-c(85,87)
d<-xmean1-xmean2
S1<-matrix(c(16, 8, 8, 16), nrow=2)
S2<-matrix(c(16, 8, 8, 16), nrow=2)
Sp<-((n[1]-1)*S1+(n[2]-1)*S2)/(sum(n)-2)
t2 <- t(d)%*%solve(sum(1/n)*Sp)%*%d
t2
```

```
##      [,1]
## [1,]   12
```

```
alpha<-0.05
cval <- (sum(n)-2)*p/(sum(n)-p-1)*qf(1-alpha,p,sum(n)-p-1)
cval
```

```
## [1] 6.768921
```

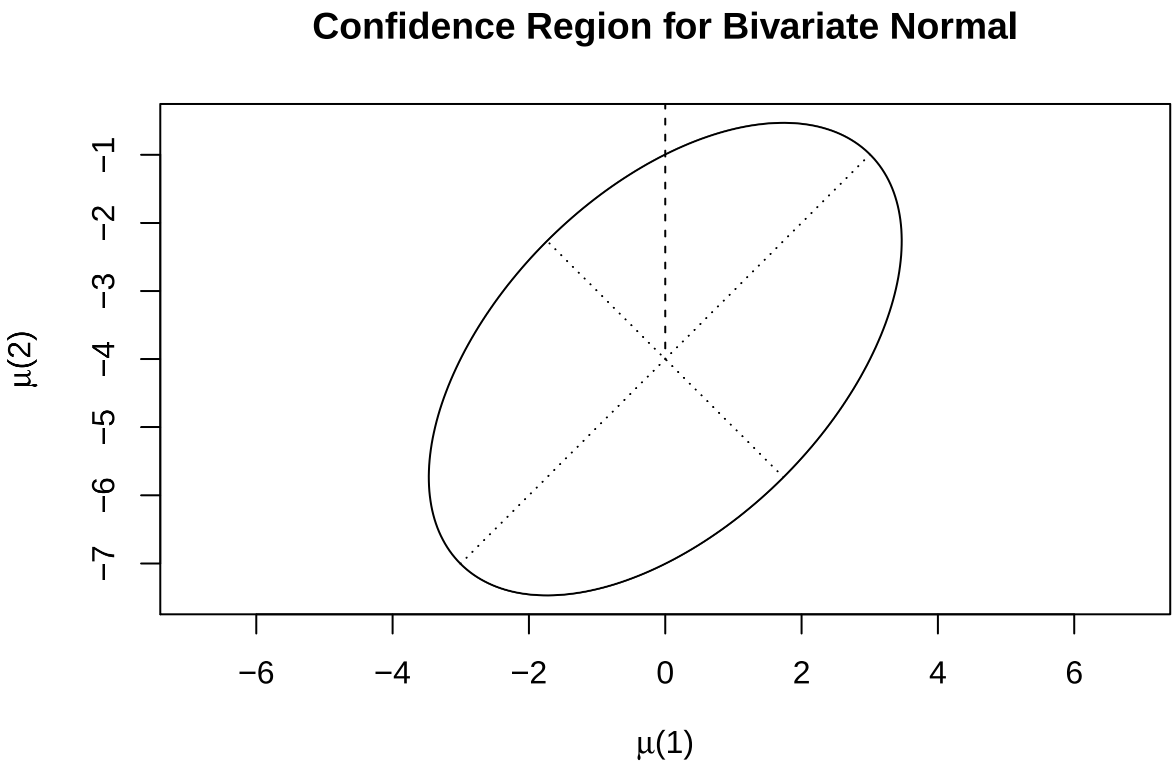
Since T^2 = 12 > 6.76 the null hypothesis is rejected at 5% level of significance.

Part 2

Confidence Region

```
es<-eigen(sum(1/n)*Sp)
e1<-es$vec %*% diag(sqrt(es$val))
r1<-sqrt(cval)
theta<-seq(0,2*pi,len=250)
v1<-cbind(r1*cos(theta), r1*sin(theta))
pts<-t(d-(e1%*%t(v1)))
plot(pts,type="l",main="Confidence Region for Bivariate Normal",xlab=expression(paste(mu, "(1)")), ylab
segments(0,d[2],d[1],d[2],lty=2) # highlight the center
segments(d[1],0,d[1],d[2],lty=2)
```

```r
th2<-c(0,pi/2,pi,3*pi/2,2*pi)   #adding the axis
v2<-cbind(r1*cos(th2), r1*sin(th2))
pts2<-t(d-(e1%*%t(v2)))
segments(pts2[3,1],pts2[3,2],pts2[1,1],pts2[1,2],lty=3)
segments(pts2[2,1],pts2[2,2],pts2[4,1],pts2[4,2],lty=3)
```




```r
# since we reject the null, we use the simultaneous confidence intervals
# to check the significant components
```

Part 3

Simultaneous confidence intervals

```r
wd<-sqrt(cval*diag(Sp)*sum(1/n))
Cis<-cbind(d-wd,d+wd)

# 95% simultaneous confidence interval
Cis
```

```
##           [,1]       [,2]
## [1,] -3.468953  3.4689534
## [2,] -7.468953 -0.5310466
```

```r
#plot(Cis[1][1]:0, 1:10, type="l", lty=2)
```

Part 4

Bonferroni simultaneous confidence intervals

```r
wd.b<- qt(1-alpha/(2*p),n[1]+n[2]-2) *sqrt(diag(Sp)*sum(1/n))
Cis.b<-cbind(d-wd.b,d+wd.b)
```

```
# 95% Bonferroni simultaneous confidence interval
Cis.b
```

```
##            [,1]       [,2]
## [1,] -3.126742  3.1267415
## [2,] -7.126742 -0.8732585
```

```
# both component-wise simultaneous confidence intervals do not contain 0, so they have significant diff
```

Number 3

Part 1

two-sample Hotelling's T2 test ——-

```
c <- matrix(c(-1,0,1,-1,0,1), nrow=2)
tran_c <- t(c)
s_1 <- matrix(c(16,8,8,8,16,8,8,8,16), nrow=3)
corrected_s <- c%*%s_1%*%tran_c
```

```
# now we perform the two-sample Hotelling T^2-test
n<-c(18,18)
p<-2
xmean1<-c(0,0)
xmean2<-c(2,2)
d<-xmean1-xmean2
S1<-corrected_s
S2<- corrected_s
Sp<-((n[1]-1)*S1+(n[2]-1)*S2)/(sum(n)-2)
t2 <- t(d)%*%solve(sum(1/n)*Sp)%*%d
t2
```

```
##      [,1]
## [1,]    9
```

```
alpha<-0.05
cval <- (sum(n)-2)*p/(sum(n)-p-1)*qf(1-alpha,p,sum(n)-p-1)
cval
```

```
## [1] 6.768921
```

Since T^2 = 9 > 6.76 the null hypothesis is rejected at 5% level of significance.

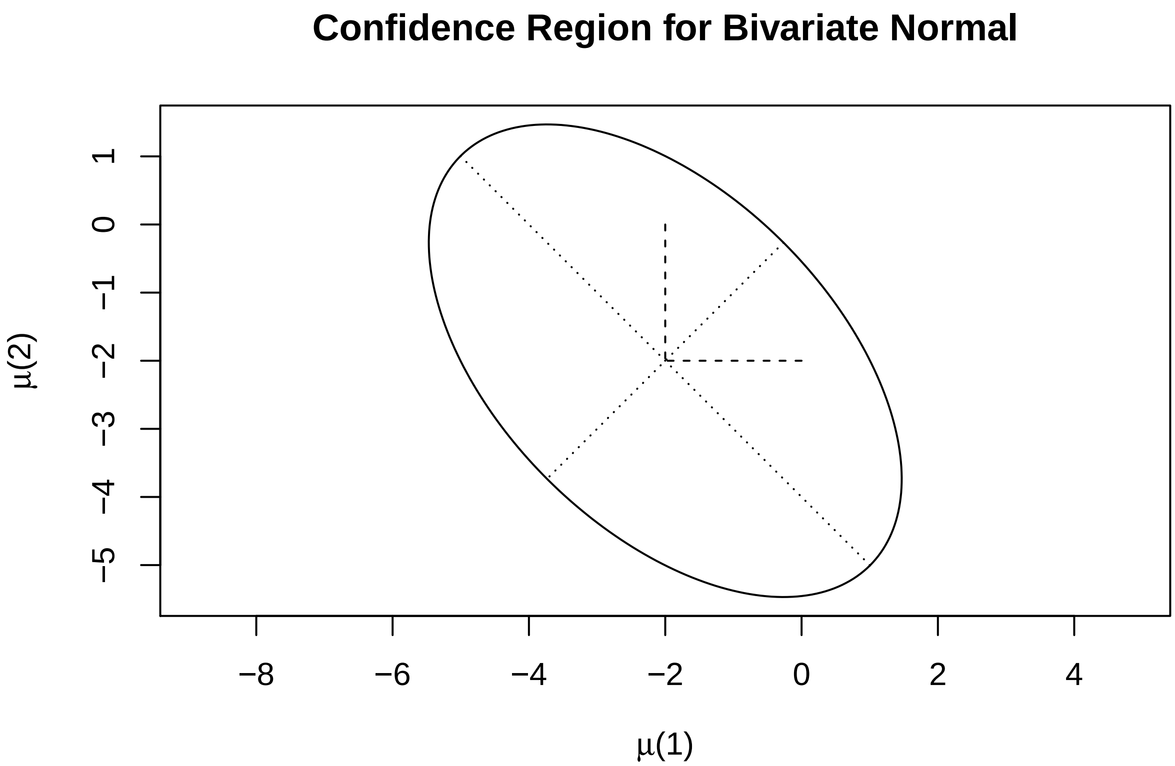
Part 2

Confidence Region

```
es<-eigen(sum(1/n)*Sp)
e1<-es$vec %*% diag(sqrt(es$val))
r1<-sqrt(cval)
theta<-seq(0,2*pi,len=250)
v1<-cbind(r1*cos(theta), r1*sin(theta))
pts<-t(d-(e1%*%t(v1)))
plot(pts,type="l",main="Confidence Region for Bivariate Normal",xlab=expression(paste(mu, "(1)")), ylab
segments(0,d[2],d[1],d[2],lty=2) # highlight the center
segments(d[1],0,d[1],d[2],lty=2)

th2<-c(0,pi/2,pi,3*pi/2,2*pi)   #adding the axis
```

```r
v2<-cbind(r1*cos(th2), r1*sin(th2))
pts2<-t(d-(e1%*%t(v2)))
segments(pts2[3,1],pts2[3,2],pts2[1,1],pts2[1,2],lty=3)
segments(pts2[2,1],pts2[2,2],pts2[4,1],pts2[4,2],lty=3)
```




```r
# since we reject the null, we use the simultaneous confidence intervals
# to check the significant components
```

Part 3

Simultaneous confidence intervals

```r
wd<-sqrt(cval*diag(Sp)*sum(1/n))
Cis<-cbind(d-wd,d+wd)

# 95% simultaneous confidence interval
Cis
```

```
##           [,1]     [,2]
## [1,] -5.468953 1.468953
## [2,] -5.468953 1.468953
```

```r
#plot(Cis[1][1]:0, 1:10, type="l", lty=2)
```

Part 4

Bonferroni simultaneous confidence intervals

```r
wd.b<- qt(1-alpha/(2*p),n[1]+n[2]-2) *sqrt(diag(Sp)*sum(1/n))
Cis.b<-cbind(d-wd.b,d+wd.b)
# 95% Bonferroni simultaneous confidence interval
Cis.b
```

```
##            [,1]     [,2]
## [1,] -5.126742 1.126742
## [2,] -5.126742 1.126742
```

```
# both component-wise simultaneous confidence intervals do not contain 0, so they have significant diff
```

Number 4

two-sample Hotelling's T2 test

```
# now we perform the two-sample Hotelling T^2-test
n<-c(45,55)
p<-2
xmean1<-c(204.4,556.6)
xmean2<-c(130,355)
d<-xmean1-xmean2
S1 <- matrix(c(13825.3, 23823.4, 23823.4, 73107.4), nrow=2)
S2 <- matrix(c(8632.0, 19616.7, 19616.7, 55964.5), nrow=2)

Sp<-((n[1]-1)*S1+(n[2]-1)*S2)/(sum(n)-2)
t2 <- t(d)%*%solve(sum(1/n)*Sp)%*%d
t2
```

```
##          [,1]
## [1,] 16.06622
```

```
alpha<-0.05
cval <- (sum(n)-2)*p/(sum(n)-p-1)*qf(1-alpha,p,sum(n)-p-1)
cval
```

```
## [1] 6.244089
```

Since T^2 = 16.06 > 6.24 the null hypothesis is rejected at 5% level of significance.

```
a <- solve(Sp)%*%d
a
```

```
##             [,1]
## [1,] 0.00170252
## [2,] 0.00259163
```

Where a is the linear combination of mean components most responsbile for the rejection of the null hypothesis.