Assignment 2: Multiple Regression Analysis

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The Data Set

The “Boston Housing” dataset recorded properties of 506 housing zones in the Greater Boston area. Typically, one is interested in predicting `MEDV` (median home value) based on other attributes.

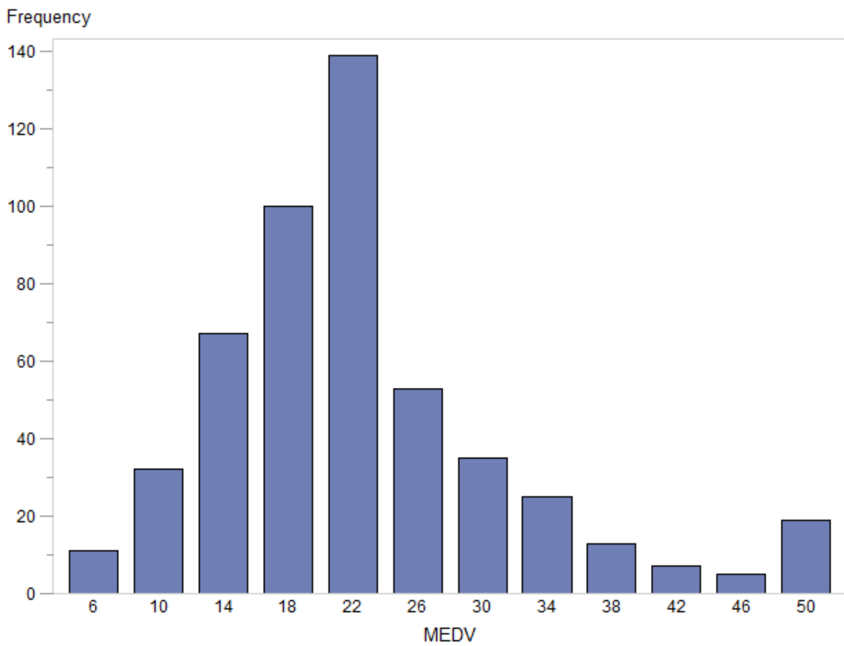
Here is a list of attribute information:

1. `CRIM`: per capita crime rate by town
2. `ZN`: proportion of residential land zoned for lots over 25,000 ft^2
3. `INDUS`: proportion of non-retail business acres per town
4. `CHAS`: Charles River dummy variable (=1 if tract bounds river; 0 otherwise)
5. `NOX`: nitric oxides concentration (parts per 10 million)
6. `RM`: average number of rooms per dwelling
7. `AGE`: proportion of owner-occupied units built prior to 1940
8. `DIS`: weighted distances to five Boston employment centers
9. `RAD`: index of accessibility to radial highways
10. `TAX`: full-value property-tax rate per $10,000
11. `PTRATIO`: pupil-teacher ratio by town
12. `LSTAT`: % lower status of the population
13. `MEDV`: median value of owner-occupied homes in $1000’s

Pre-processing

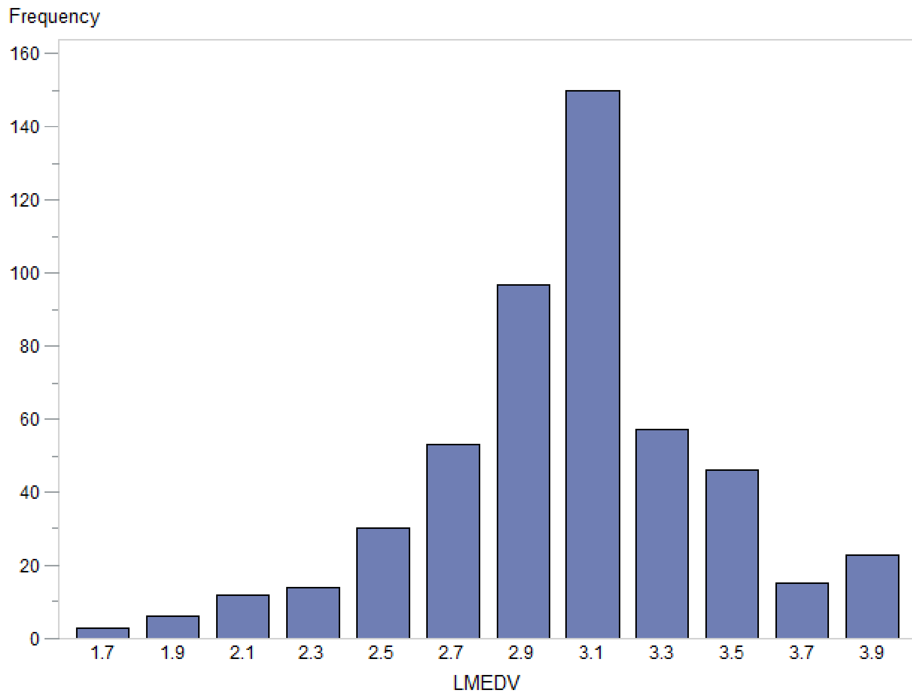
MEDV has somewhat longish tail and is not so normally distributed, so we will take the log transform, and then predict LMEDV instead.

Histogram of MEDV




```
PROC SQL;
   CREATE TABLE WORK.QUERY_FOR_BOSTON_HOUSING AS
   SELECT t1.CRIM,
          t1.ZN,
          t1.INDUS,
          t1.CHAS,
          t1.NOX,
          t1.RM,
          t1.AGE,
          t1.DIS,
          t1.RAD,
          t1.TAX,
          t1.PTRATIO,
          t1.LSTAT,
          t1.MEDV,
          /* LMEDV */
            (LOG(t1.MEDV)) AS LMEDV
      FROM WORK.'BOSTON HOUSING'n t1;
QUIT;
```

Histogram of log(MEDV)



Questions

1. **Please perform the multicollinearity diagnosis based on the VIF calculation results. Do we need to drop any variables that might have multicollinearity concerns?**

The REG Procedure
Model: Linear_Regression_Model
Dependent Variable: LMEDV

Number of Observations Read	506
Number of Observations Used	506

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	12	66.09326	5.50777	148.51	<.0001
Error	493	18.28323	0.03709		
Corrected Total	505	84.37649			

Root MSE	0.19258	R-Square	0.7833
Dependent Mean	3.03451	Adj R-Sq	0.7780
Coeff Var	6.34620		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	1	4.33112	0.19812	21.86	<.0001	0
CRIM	1	-0.01087	0.00132	-8.20	<.0001	1.76749
ZN	1	0.00120	0.00055706	2.15	0.0322	2.29846
INDUS	1	0.00215	0.00249	0.86	0.3887	3.98718
CHAS	1	0.10769	0.03492	3.08	0.0022	1.07117
NOX	1	-0.82243	0.15458	-5.32	<.0001	4.36909
RM	1	0.08409	0.01687	4.99	<.0001	1.91253
AGE	1	0.00034023	0.00053500	0.64	0.5251	3.08823
DIS	1	-0.04976	0.00809	-6.15	<.0001	3.95404
RAD	1	0.01353	0.00269	5.04	<.0001	7.44530
TAX	1	-0.00064120	0.00015256	-4.20	<.0001	9.00216
PTRATIO	1	-0.03760	0.00531	-7.09	<.0001	1.79706
LSTAT	1	-0.03025	0.00203	-14.88	<.0001	2.87078

The `TAX` variable has a high variance inflation factor, and can be seen to be highly correlated with `RAD`; since this violates the assumption of no multicollinearity, it will be removed from the model.

Pearson Correlation Coefficients, N = 506
Prob > |r| under H0: Rho=0

	RAD	TAX
RAD	1.00000	0.91023
		<.0001
TAX	0.91023	1.00000
	<.0001	

2. Please run the linear regression analyses.

- **Use the stepwise model selection approach to determine the final model. Drop variables based on their significance.**
- **Report summary output for *each step*, including ANOVA, R^2, and parameter estimates.**

Linear Regression Results

The REG Procedure
Model: Linear_Regression_Model
Dependent Variable: LMEDV

Number of Observations Read	506
Number of Observations Used	506

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	11	65.43814	5.94892	155.18	<.0001
Error	494	18.93835	0.03834		
Corrected Total	505	84.37649			

Root MSE	0.19580	R-Square	0.7755
Dependent Mean	3.03451	Adj R-Sq	0.7706
Coeff Var	6.45236		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	1	4.24264	0.20029	21.18	<.0001	0
CRIM	1	-0.01082	0.00135	-8.03	<.0001	1.76735
ZN	1	0.00067450	0.00055212	1.22	0.2224	2.18417
INDUS	1	-0.00245	0.00228	-1.08	0.2822	3.21795
CHAS	1	0.12571	0.03523	3.57	0.0004	1.05502
NOX	1	-0.87235	0.15670	-5.57	<.0001	4.34330
RM	1	0.08919	0.01710	5.21	<.0001	1.90264
AGE	1	0.00027655	0.00054373	0.51	0.6112	3.08576
DIS	1	-0.05044	0.00823	-6.13	<.0001	3.95244
RAD	1	0.00459	0.00167	2.75	0.0061	2.77221
PTRATIO	1	-0.03926	0.00538	-7.30	<.0001	1.78705
LSTAT	1	-0.03015	0.00207	-14.59	<.0001	2.87041

AGE has the highest p-value at p = 0.61, so it will be dropped.

Linear Regression Results

The REG Procedure
Model: Linear_Regression_Model
Dependent Variable: LMEDV

Number of Observations Read	506
Number of Observations Used	506

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	10	65.42823	6.54282	170.92	<.0001
Error	495	18.94827	0.03828		
Corrected Total	505	84.37649			

Root MSE	0.19565	R-Square	0.7754
Dependent Mean	3.03451	Adj R-Sq	0.7709
Coeff Var	6.44753		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	1	4.23589	0.19970	21.21	<.0001	0
CRIM	1	-0.01082	0.00135	-8.04	<.0001	1.76731
ZN	1	0.00064168	0.00054792	1.17	0.2421	2.15434
INDUS	1	-0.00244	0.00228	-1.07	0.2847	3.21743
CHAS	1	0.12663	0.03516	3.60	0.0003	1.05226
NOX	1	-0.85125	0.15100	-5.64	<.0001	4.03899
RM	1	0.09093	0.01675	5.43	<.0001	1.82640
DIS	1	-0.05167	0.00786	-6.57	<.0001	3.61317
RAD	1	0.00450	0.00166	2.72	0.0068	2.74684
PTRATIO	1	-0.03903	0.00536	-7.29	<.0001	1.77476
LSTAT	1	-0.02980	0.00195	-15.31	<.0001	2.54673

`INDUS` has the next highest p-value at p = 0.28, so it will be dropped.

Linear Regression Results

The REG Procedure
Model: Linear_Regression_Model
Dependent Variable: LMEDV

Number of Observations Read	506
Number of Observations Used	506

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	9	65.38432	7.26492	189.73	<.0001
Error	496	18.99218	0.03829		
Corrected Total	505	84.37649			

Root MSE	0.19568	R-Square	0.7749
Dependent Mean	3.03451	Adj R-Sq	0.7708
Coeff Var	6.44848		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	1	4.24137	0.19966	21.24	<.0001	0
CRIM	1	-0.01076	0.00134	-8.00	<.0001	1.76426
ZN	1	0.00064853	0.00054797	1.18	0.2372	2.15405
CHAS	1	0.12459	0.03512	3.55	0.0004	1.04919
NOX	1	-0.90820	0.14135	-6.43	<.0001	3.53822
RM	1	0.09335	0.01660	5.62	<.0001	1.79324
DIS	1	-0.04966	0.00763	-6.51	<.0001	3.40711
RAD	1	0.00428	0.00164	2.60	0.0096	2.70103
PTRATIO	1	-0.04008	0.00527	-7.61	<.0001	1.71584
LSTAT	1	-0.03000	0.00194	-15.49	<.0001	2.52325

ZN has the next highest p-value at $p = 0.24$, and is the final predictor above the $\alpha = 0.05$ threshold, so it will be dropped to produce the final model.

Final model

Linear Regression Results

The REG Procedure
Model: Linear_Regression_Model
Dependent Variable: LMEDV

Number of Observations Read	506
Number of Observations Used	506

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	65.33068	8.16634	213.10	<.0001
Error	497	19.04581	0.03832		
Corrected Total	505	84.37649			

Root MSE	0.19576	R-Square	0.7743
Dependent Mean	3.03451	Adj R-Sq	0.7706
Coeff Var	6.45108		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	4.25599	0.19936	21.35	<.0001
CRIM	1	-0.01062	0.00134	-7.93	<.0001
CHAS	1	0.12379	0.03512	3.52	0.0005
NOX	1	-0.91847	0.14114	-6.51	<.0001
RM	1	0.09583	0.01647	5.82	<.0001
DIS	1	-0.04521	0.00664	-6.80	<.0001
RAD	1	0.00453	0.00163	2.78	0.0057
PTRATIO	1	-0.04209	0.00499	-8.44	<.0001
LSTAT	1	-0.02997	0.00194	-15.47	<.0001

$$\mathrm{LMEDV} = 4.26 - 0.011 * \mathrm{CRIM} + 0.124 * \mathrm{CHAS} - 0.918 * \mathrm{NOX} + 0.096 * \mathrm{RM} - 0.045 * \mathrm{DIS} + 0.004 * \mathrm{RAD} - 0.042 * \mathrm{PTRATIO} - 0.030 * \mathrm{LSTAT} + e$$

Interpretation of Coefficients:

The natural log of median home value ($1000) changes by the estimated coefficient for each predictor for each unit change in that predictor, while all other predictors are held constant.

CRIM:

The log median home value ($1000) decreases on average by 0.011 ± 0.003 for each unit increase in the per capita crime rate, while all other predictors are held constant.

CHAS:

The log median home value ($1000) is on average 0.124 ± 0.069 higher for homes that bound the Charles River versus those that don't, while all other predictors are held constant.

NOX:

The log median home value ($1000) decreases on average by 0.918 ± 0.277 for each parts per 10 million increase in nitric oxide concentration, while all other predictors are held constant.

RM:

The log median home value ($1000) increases on average by 0.096 ± 0.032 for each 1 room increase in the average number of rooms per dwelling, while all other predictors are held constant.

DIS:

The log median home value ($1000) decreases on average by 0.045 ± 0.013 for each 1 unit increase in the weighted distances to five Boston employment centers, while all other predictors are held constant.

RAD:

The log median home value ($1000) increases on average by 0.004 ± 0.003 for each 1 unit increase in the index of accessibility to radial highways, while all other predictors are held constant.

PTRATIO:

The log median home value ($1000) decreases on average by 0.042 ± 0.010 for each 1 pupil increase in the pupil-teacher ratio, while all other predictors are held constant.

LSTAT:

The log median home value ($1000) decreases on average by 0.030 ± 0.004 for each 1% increase in the percent lower status of the population, while all other predictors are held constant.

The 95% confidence interval for each coefficient is calculated by obtaining the 0.975 t-quantile on 497 degrees of freedom multiplied by the standard error for that coefficient. This value is added and subtracted from the fitted value at each X_i to get the upper and lower bounds.
Finally the R^2 value of 0.77 means that 77% of the variance in the data is explained by the model.