Answers (Problem set 1)

Online S520

1. (a) Randomized experiment — we want to know if the vaccine has a causal effect on flu.

   (b) Observational study — we want to observe the beliefs of the population.

   (c) Randomized experiment — we want to know if an intervention (banning laptops) has a causal effect on exam scores.

   (d) Observational study — we want to observe two populations (voters with and without college degrees) and compare them.

   (e) Arguable. A randomized experiment would be good because we want to study the causal effect of bacon. However, it may not be ethical to force or encourage people to eat bacon, so we might have to rely on observational data instead.

2. (a) An observational study. (A blind experiment to study the pill would lead to a lot of pregnant women filing lawsuits.)

   (b) No. For this to prove the pill causes cervical cancer, all relevant differences between the pill and no-pill groups would need to be correctly adjusted for, and there are important variables that haven't been considered (see part (c).)

   (c) There are many candidates, but sexual activity would seem to be a major one. Cervical cancer is caused by HPV viruses, which can be sexually transmitted.

3. (a) No, the survey was biased. We can see that the sample demographics don't represent "people" in general, e.g. only about 18% of the sample are over 44, whereas if the population is U.S. adults, this should be close to half. Young people are overrepresented, and young people tend to be more socially liberal, so perhaps the percentage of the sample who agree with the statement is higher than in the general population.

   (b) We would want to take a fairly large random sample of some kind to answer the question. Hire a reputable survey phone with a big list of phone numbers, including cell phones, and call people randomly until you get 1000 responses. (In practice, this would still require adjustments after the data was collected because of nonresponse.)

4. (a) $A^c = \{2, 4, 6, 8, 10\}$.

   (b) $B^c = \{4, 6, 8, 9, 10\}$.

   (c) $A \cup B = \{1, 2, 3, 5, 7, 9\}$. $(A \cup B)^c = \{4, 6, 8, 10\}$.

   (d) $A \cap B = \{1, 3, 5, 7\}$. $(A \cap B)^c = \{2, 4, 6, 8, 9, 10\}$.

5. (a) This is choosing 6 tosses from 15, or

$$\binom{15}{6} = \frac{15!}{6!(15-6)!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 5005$$

   (b) There are

$$\binom{10}{2} = \frac{10!}{2!(10-2)!} = \frac{10 \cdot 9}{2 \cdot 1} = 45$$

sequences resulting in exactly 2 heads in 10 tosses of the penny and

$$\left(\begin{array}{c} 5 \\ 4 \end{array} \right) = \frac{5!}{4!(5-4)!} = 5$$

sequences resulting in exactly 4 heads in 5 tosses of the quarter; hence $45 \cdot 5 = 225$ sequences result in exactly 2 heads in 10 tosses of the penny and exactly 4 heads in 5 tosses of the quarter.

6. (a) $\phi(6) = 2^6 = 64$.
   (b) $\phi(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$.
   (c) $\phi(\Re) = (0, \infty)$
   (d) $\phi(4) = 2^4 = 16$, so $\phi^{-1}(16) = \{4\}$.
   (e) $\phi(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$, so $\phi^{-1}(1/4) = -2$.