

Midterm 1 Practice Questions

Online S520

1. (a) I toss six fair coins. What is the probability exactly four of the coins show heads?
- (b) I toss six fair coins. What is the probability that there are more heads than tails?
- (c) I toss six fair coins. Given that there are more heads than tails, what is the conditional probability exactly four of the coins show heads?
- (d) I toss six fair coins. What is the expected value and variance of number of coins that show heads?

Notes:

- You can use either counting or Binomial distribution to calculate the probabilities.
- Also review the concept of “independence”. How do you use conditional probability to determine whether two events are independent.

2. Let X be a discrete random variable with probability mass function

$$P(X = x) = \begin{cases} 0.2 & x = -1 \\ 0.3 & x = 2 \\ 0.3 & x = 4 \\ 0.2 & x = 7 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Write down an expression for $F(y)$, the cumulative distribution function (CDF) of X , for all y -values from $-\infty$ to ∞ .
- (b) Find the expected value and variance of X .
- (c) Let X_1 and X_2 be independent random variables with the same distribution as X . Compare the expected value and variance of $X_1 + X_2$ and $2X$.
- (d) Let X_1 and X_2 be independent random variables with the same distribution as X . What is the probability that $X_1 + X_2$ is positive?

Notes:

- To write a complete CDF, include all real numbers, even though $f(x) = 0$ on some intervals.
- Also notice which points are open/closed (included/excluded) in each interval.
- Review the properties of expected values: what are the expected value and variance of a sum of two random variables and a linear transformation of a random variable?

3. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 0.1 & 0 \leq x < 2 \\ 0.2 & 2 \leq x < 6 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find F , the cumulative distribution function of X .
- (b) The *median* of X is the value m such that $F(m) = 0.5$. Find the median of X .
- (c) Find the expected value of X .

Notes:

- To find F , either use integration or geometry.
- To write a complete CDF, include all real numbers, even though $f(x) = 0$ on some intervals. Also notice which points are included/excluded in each interval.
- To find the median, either use $F(x)$ to solve for median or use geometry to calculate areas. Refer to the problem set 5 solutions, 1(a).
- There are several methods to find expected value. Refer to the problem set 5 solutions, 1(b).

4. Let X be a random variable with probability density function (PDF)

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leq x < 1 \\ \frac{1}{2} & 1 \leq x < 2 \\ \frac{1}{2}(3-x) & 2 \leq x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

Calculus shows that $E(X) = 3/2$ and $E(X^2) = 8/3$. You may use these results below.

- (a) Is X discrete, continuous, or neither? Explain.
- (b) Find F , the cumulative distribution function of X .
- (c) Find the standard deviation of X . (Hint: Use the results above from calculus.)

Notes:

- To find F , either use integration or geometry.
- To write a complete CDF, include all real numbers, even though $f(x) = 0$ on some intervals. Also notice which points are included/excluded in each interval.
- Integration of $f(x)$ is not required in this course. Please don't attempt it if you are not familiar with it.

5. Let X be a standard normal random variable. Let $Y = |X|$.

- (a) Find $P(-1.5 < X < 2.5)$.
- (b) Find $P(Y > 1)$.
- (c) Find the 0.9-quantile of Y .

Notes:

- You are not expected to and don't need to figure out the distribution of Y . Just convert the probability for Y to a probability involving X .

- Use R to find the final results
6. (a) Let X be a normal random variable with mean -5 and standard deviation 10 . Find $P(X > 0)$.
- (b) Let Z_1, Z_2, Z_3, Z_4 , and Z_5 be independent standard normal random variables. What is the probability that at least two of the five variables are greater than 1 ?

Notes:

- For a discrete random variable X taking only integer values, $P(X < 3) = P(X \leq 2)$. But for a continuous random variable X , $P(X < 3) = P(X \leq 3)$.
7. Let X, Y , and Z be independent standard normal random variables.
- (a) Find $P(1 < X < 2)$.
- (b) Find $P(1 < X^2 < 2)$.
- (c) Find $P(1 < X + Y + Z < 2)$.

Notes:

- `pnorm` can be used to calculate the cumulative probability for any normal distribution. But when it is not for standard normal distribution, you will need specify the mean and/or standard deviation.
 - Review the distribution of the sum of independent normal random variables.
8. I give a ten question true/false statistics test to a class of ten chimpanzees. The chimpanzees, who do not know any statistics, randomly guess true or false, independently for each question and independently of each other. A score of at least 8 out of 10 is required to pass the test. What is the probability that at least one of the chimpanzees passes the test?

Notes:

- Consider how many experiments you are having in this question.
- Consider what kind of random variables you should use for the experiment(s).
- For a discrete random variable X taking only integer values, $P(X \geq 3) = 1 - P(X \leq 2)$. But for a continuous random variable X , $P(X \geq 3) = 1 - P(X < 3)$.