# Mathematical Probability

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*An Introduction to Statistical Inference and Its Applications with R*

Michael W. Trosset
Department of Statistics
Indiana University, Bloomington

# The Probability Model

While the concept of probability may be interpreted in various ways, probabilists and statisticians usually adopt a common set of rules that determine the mathematical properties of probability. These rules are often called the *Kolmogorov probability model*. A thorough explication of these rules is not needed for this course; a brief survey of the highlights will suffice.

The Kolmogorov probability model has three components:

$S$ A *sample space*, a universe of "possible" outcomes for the experiment in question.

$\mathcal{C}$ A designated collection of "observable" subsets (called *events*) of the sample space.

$P$ A *probability measure*, a function that assigns real numbers (called *probabilities*) to events.

# Experimental Outcomes

The sample space is a model of what might occur when the experiment of interest is performed. Mathematical models often compromise fidelity for convenience. In the words of the famous statistician G.E.P. Box, "all models are wrong, but some are useful."

Example: When tossing a coin, we generally ignore the possibility that it might land perfectly balanced on edge and declare the sample space to be $S = \{\texttt{Heads}, \texttt{Tails}\}$.

Example: When measuring the time it takes an Olympic sprinter to run 100 meters, we generally treat time as a continuous phenomenon, ignoring the fact that fully automatic timing only records times to one thousandth of a second. Furthermore, although the time in question will certainly be greater than a second and less than an hour, we generally do not worry about imposing realistic lower and upper bounds on what we might observe. Thus, we might declare the sample space to be $S = (0, +\infty)$, or even $S = (-\infty, +\infty)$.

# Events

Informally, an event is a subset of the sample space.

The Kolmogorov probability model was designed to accommodate the possibility that one might not be able to determine whether or not a certain experimental outcome lies in a certain subset of $S$. This is a nuance that we shall ignore.

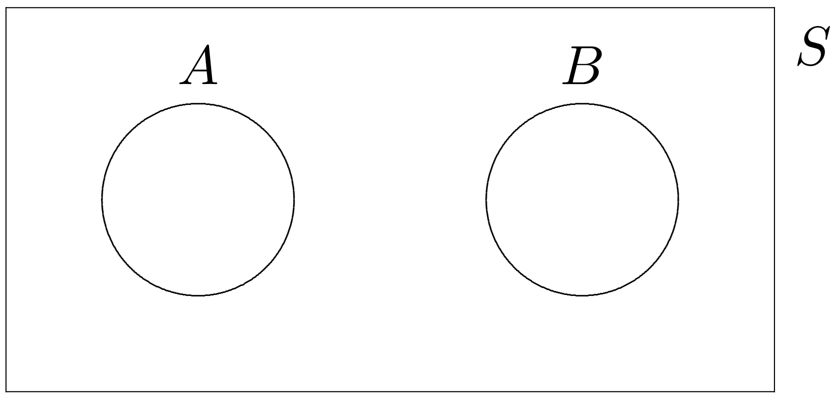
Events are the objects to which probabilities are assigned. For technical reasons, it is often impossible to assign a probability to every subset of the sample space. This is another nuance that we shall ignore. No difficulties arise in the study of elementary statistics if we simply assume that every subset of $S$ that interests us can be assigned a probability.

# Probability Measure

A probability measure is a function that assigns real numbers to events, i.e., $P : \mathcal{C} \to \Re$. The labels are called probabilities.

By convention, probabilities lie in $[0, 1]$ and $P(S) = 1$.

Key Property: If events $A$ and $B$ are disjoint, then $P(A \cup B) = P(A) + P(B)$.




The above property is called finite additivity. The Kolmogorov probability model actually assumes a stronger property called countable additivity, yet another nuance that we shall ignore.