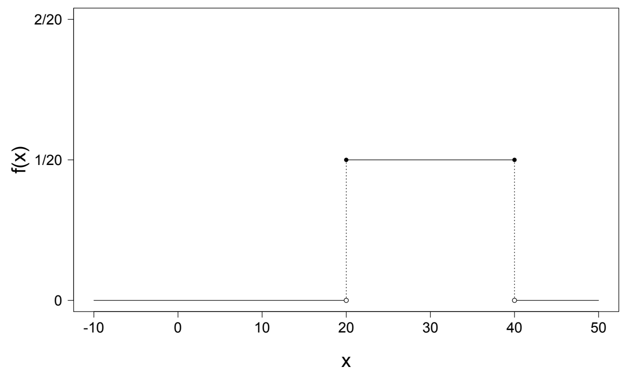
Answers (Problem set 4)

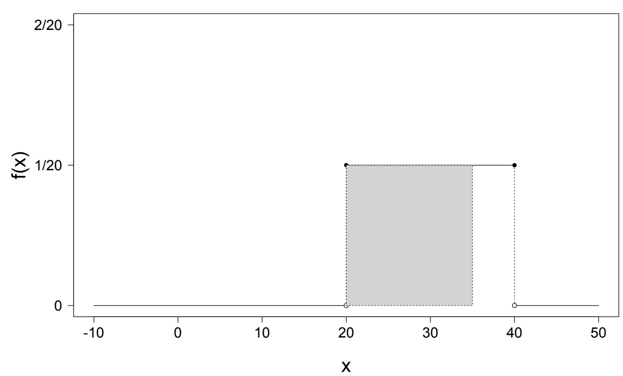
Online S520

1. (a) The graph of the pdf is:



(b) $f(x) \geq 0$ for all x and the area of the rectangle is $20 \times \frac{1}{20} = 1$, so f is a pdf.

(c) This is the areas of the shaded region, which is a rectangle with base 15 and height $\frac{1}{20}$.



$$\begin{aligned} P(0 < X < 35) &= P(X < 35) - P(X \leq 0) \\ &= P(20 < X < 35) \\ &= 15 \times \frac{1}{20} \\ &= 0.75. \end{aligned}$$

(d) *Find the CDF of X, $F(y)$, for all y.*

$$F(y) = \begin{cases} 0 & y < 20 \\ \frac{y-20}{20} & 20 \leq y < 40 \\ 1 & y \geq 40. \end{cases}$$

2. *Trosset exercise 5.6.2*

   (a) The graph of the pdf is:

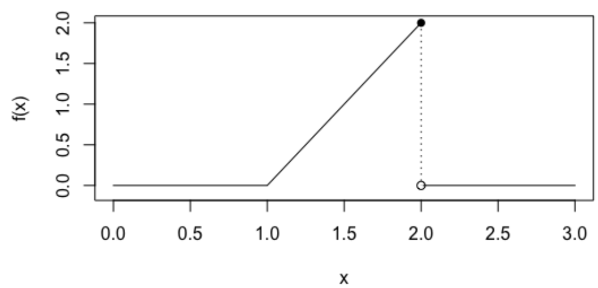


Figure 1: pdf for Exercise 5.6.2.

   (b) $f(x) \geq 0$ for all x and the area of the triangle is 1, so f is a pdf.

   (c) This is the difference between the areas of two triangles, one with base 0.5 and height 1, and one with base 0.75 and height 1.5.

$$\begin{aligned} P(1.5 < X < 1.75) &= P(X < 1.75) - P(X < 1.5) \\ &= (0.5 \times 0.75 \times 1.5) - (0.5 \times 0.5 \times 1) \\ &= \frac{5}{16} = 0.3125. \end{aligned}$$

3. *Let X be a random variable with PDF*

$$f(x) = \begin{cases} \frac{1}{30} & 0 \le x < 20 \\ \frac{1}{60} & 20 \le x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

To find the expected value, you could either use integration,

$$\begin{aligned} EX &= \int_0^{40} x \cdot f(x)\,dx \\ &= \int_0^{20} \frac{1}{30} x\,dx + \int_{20}^{40} \frac{1}{60} x\,dx \\ &= \left[x^2/60\right]_0^{20} + \left[x^2/120\right]_{20}^{40} \\ &= (20/3 - 0) + (40/3 - 10/3) \\ &= 50/3 \end{aligned}$$

or treat X as a weighted average of 2 uniform random variables.

$$E(X) = 20/30 \cdot \frac{0+20}{2} + 20/60 \cdot \frac{20+40}{2} = 50/3.$$

4. *Trosset exercise 5.6.7*

   (a) $P(X < 0) =$ `pnorm(0, mean=-5, sd=10)` $= 69\%$

   (b) $P(X > 5) = 1 - P(X \le 5) =$ `1 - pnorm(5, mean=-5, sd=10)` $= 16\%$

   (c) $P(-3 < X < 7) = P(X < 7) - P(X \le -3) = P(X < 7)$
   $=$ `pnorm(7, mean=-5, sd=10) - pnorm(-3, mean=-5, sd=10)` $= 31\%$

   (d) $P(|X + 5| < 10) = P(-10 < X + 5 < 10) = P(-15 < X < 5)$
   $=$ `pnorm(5, mean=-5, sd=10) - pnorm(-15, mean=-5, sd=10)` $= 68\%$

   (e) $P(|X - 3| > 2) = P(X - 3 > 2) + P(X - 3 < -2) = P(X > 5) + P(X < 1)$
   $=$ `1 - pnorm(5, mean=-5, sd=10) + pnorm(1, mean=-5, sd=10)` $= 88\%$

5. *Trosset exercise 5.6.8*

   (a) Expected value 4, variance 25
   (b) Expected value -3, variance 16
   (c) Expected value -2, variance 25
   (d) Expected value 2, variance 36
   (e) Expected value -4, variance 100

6. Let X be the demand for regular gasoline during the lead-time period, then $X \sim N(\mu, \sigma^2)$ where $\mu = 930$ and $\sigma = 140$. If the demand during the lead time exceeds the inventory of 1200 gals, the station runs out of gas. So we want to calculate the probability that X assumes a value greater than 1200.
   $P(X > 1200) = 1 - P(X \leq 1200) = 1 -$ `pnorm(1200, mean=930, sd=140)` $= 1 - 0.9731 = 0.0269$

7. Let X be a standard normal random variable. Let $Y = X^2$.

   (a) *Find* $P(-1.5 < X < 2.5)$.
   We can calculate this as the difference between the CDF of the standard normal at 2.5 and the CDF at -1.5:

   ```
   > pnorm(2.5) - pnorm(-1.5)
   [1] 0.9269831
   ```

   This gives about 93%.

   (b) *Find* $P(Y > 1)$.

$$
\begin{aligned}
P(Y > 1) &= P(X^2 > 1) \\
&= P(X > 1 \text{ or } X < -1) \\
&= P(X > 1) + P(X < -1) \\
&= 1 - P(X \leq 1) + P(X < -1)
\end{aligned}
$$

   $=$ `(1 - pnorm(1)) + pnorm(-1)` $= 31.7\%$.