Midterm 1 Applied Data Mining

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November 14, 2017

Problem 1

```
## install.packages("data.table")
library(data.table)
library(ggplot2)
mydata <-
read.csv("C:/Users/khickman/Desktop/Personal/IUMSDS/AppliedDataMining/Midterm
/mydata.csv", sep=",")

summary(mydata)

##        V1                     V2             V3
##  Min.   :   1.0   ?            :   4   Min.   :-6.7749
##  1st Qu.: 500.8   -0.001405791:   1   1st Qu.:-2.2878
##  Median :1000.5   -0.002235545:   1   Median :-0.4438
##  Mean   :1000.5   -0.003699072:   1   Mean   :-0.9815
##  3rd Qu.:1500.2   -0.006583953:   1   3rd Qu.: 0.4354
##  Max.   :2000.0   -0.006972429:   1   Max.   : 3.1754
##                   (Other)     :1991   NA's   :8
##             V4              V5                X
##  ?           :   6   Min.   :-12.342   Min.   :1.00
##  -0.00303014 :   1   1st Qu.: -9.420   1st Qu.:1.75
##  -0.012157336:   1   Median : -8.628   Median :2.00
##  -0.017954776:   1   Mean   : -6.775   Mean   :1.75
##  -0.027248905:   1   3rd Qu.: -4.728   3rd Qu.:2.00
##  -0.031789989:   1   Max.   :  3.355   Max.   :2.00
##  (Other)     :1989

str(mydata)

## 'data.frame':    2000 obs. of  6 variables:
##  $ V1: int  1 2 3 4 5 6 7 8 9 10 ...
##  $ V2: Factor w/ 1997 levels "-0.001405791",..: 1987 1330 1766 1850 1817
1768 1462 1870 1583 1809 ...
##  $ V3: num  -3.27 -3.94 -4.92 -2.79 -4.66 ...
##  $ V4: Factor w/ 1995 levels "-0.00303014",..: 745 746 942 889 662 742 809
1855 681 764 ...
##  $ V5: num  -9.21 -9.92 -8.66 -10.35 -10.58 ...
##  $ X : int  1 1 1 1 1 1 1 1 1 1 ...

mydata[100:110,]
```

```
##       V1           V2        V3           V4         V5 X
## 100 100 2.106898626 -3.673282            ? -10.551039 1
## 101 101           ? -2.746142 9.093913921  -5.315355 1
## 102 102 3.326490963 -6.774908 9.702177633  -9.461351 1
## 103 103  2.33460081 -3.738696 10.36410186 -11.209379 1
## 104 104 4.167999798        NA 8.183001977  -9.487888 1
## 105 105 2.836025413 -3.413888 8.467948059  -8.823351 1
## 106 106 3.672512903 -3.648048 10.37116448  -8.959097 1
## 107 107 4.265924166 -3.897882  8.79822484  -9.610169 1
## 108 108 3.288826248 -2.518422 8.479622918  -8.470363 1
## 109 109 2.860199674 -3.332852 11.25037736  -8.391735 1
## 110 110 3.748196493 -2.918405 7.787542705  -8.632427 1
```

1.

How many entries are in the data set? There are 2000 observations of 6 variables.

2.

How many unknown or missing data are in the data set? I noticed 8 missing values here, as well as some values as ? which is the same here as NA. Additionally, I've got two of the variables that should be continuous listed as factors.

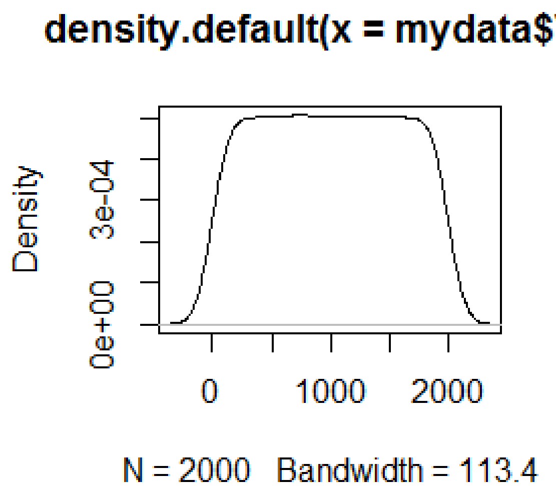
Starting with setting the datatypes factors:

```
mydata$V2 <- as.numeric(mydata$V2)
mydata$V4 <- as.numeric(mydata$V4)
str(mydata)
```

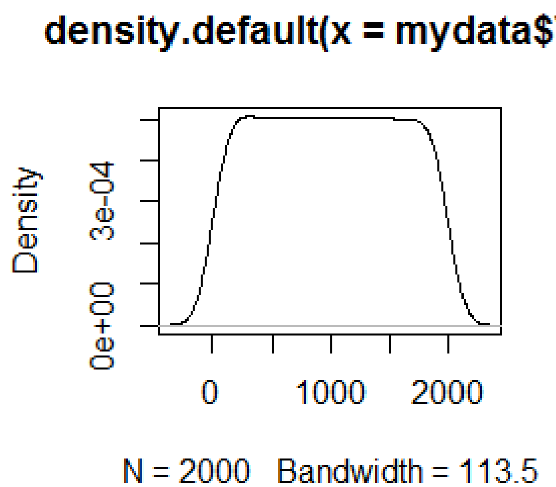
```
## 'data.frame':    2000 obs. of  6 variables:
##  $ V1: int  1 2 3 4 5 6 7 8 9 10 ...
##  $ V2: num  1987 1330 1766 1850 1817 ...
##  $ V3: num  -3.27 -3.94 -4.92 -2.79 -4.66 ...
##  $ V4: num  745 746 942 889 662 ...
##  $ V5: num  -9.21 -9.92 -8.66 -10.35 -10.58 ...
##  $ X : int  1 1 1 1 1 1 1 1 1 1 ...
```

Now that the data columns are of the correct type, we can deal with missing or incorrect values. To impute missing or incorrect values, I'll start with examining the distribution of each variable with missing values. Interestingly, this step also looks like it took care of reverting the "?" to actual values. Not sure why this happened, or whether the values are correct.

```
plot(density(mydata$V2))
```




```
## plot(density(mydata$V3))
plot(density(mydata$V4))
```



I learned that only V3 is non-normal, and that V2 and V4 are almost uniformly distributed and symmetric. I'll use median for replacing missing values of V3.

```
## v3na <- is.na(mydata$V3)
v3na <- mydata[rowSums(is.na(mydata)) > 0,]
v3na

##       V1   V2 V3   V4         V5 X
## 50    50 1855 NA  908  -8.659472 1
## 70    70 1843 NA  653 -10.469044 1
## 104 104 1938 NA 1078  -9.487888 1
## 201 201 1912 NA 1923  -9.402905 1
```

```
## 301 301 1586 NA 1179  -9.106679 1
## 401 401 1631 NA 1658  -9.761055 1
## 800 800  619 NA  834  -9.125540 2
## 900 900 1588 NA 1639  -9.802796 2
```

Great - 8 rows of the V3 variable have NA values. I'll also check again for question marks a bit later on. For now, let's impute the missing values. We'll use the mean because the shape of the variable indicates that thus will be a good representation of the values.

```
mydata[50, "V3"] <- median(mydata$V3, na.rm = TRUE)
mydata[70, "V3"] <- median(mydata$V3, na.rm = TRUE)
mydata[104, "V3"] <- median(mydata$V3, na.rm = TRUE)
mydata[201, "V3"] <- median(mydata$V3, na.rm = TRUE)
mydata[301, "V3"] <- median(mydata$V3, na.rm = TRUE)
mydata[401, "V3"] <- median(mydata$V3, na.rm = TRUE)
mydata[800, "V3"] <- median(mydata$V3, na.rm = TRUE)
mydata[900, "V3"] <- median(mydata$V3, na.rm = TRUE)

summary(mydata$V3)

##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## -6.7750 -2.2680 -0.4438 -0.9793  0.4299  3.1750
```

Great - looks like that did the trick. It imputed all values to the same number, however, so that's something that we may have to come back to later on.

Let's continue with problem 1.

3.

Calculate mean and median of variable V2.

```
mean(mydata$V2)

## [1] 998.6115

median(mydata$V2)

## [1] 997.5
```

The values are very close together, especially considering the scale. This looks good for a normally distributed variable.

4.

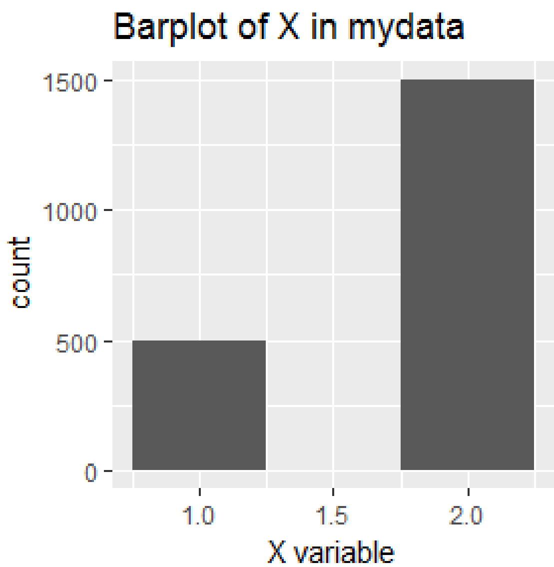
Find variance, standard deviation and interquartile range of variable V4.

```
var(mydata$V4)

## [1] 332438.5

sd(mydata$V4)
```

```
## [1] 576.5748
IQR(mydata$V4)
## [1] 999.5
```

Moving on to the barplot.

```
qplot(mydata$X, bins=3, xlab="X variable", main = "Barplot of X in mydata")
```

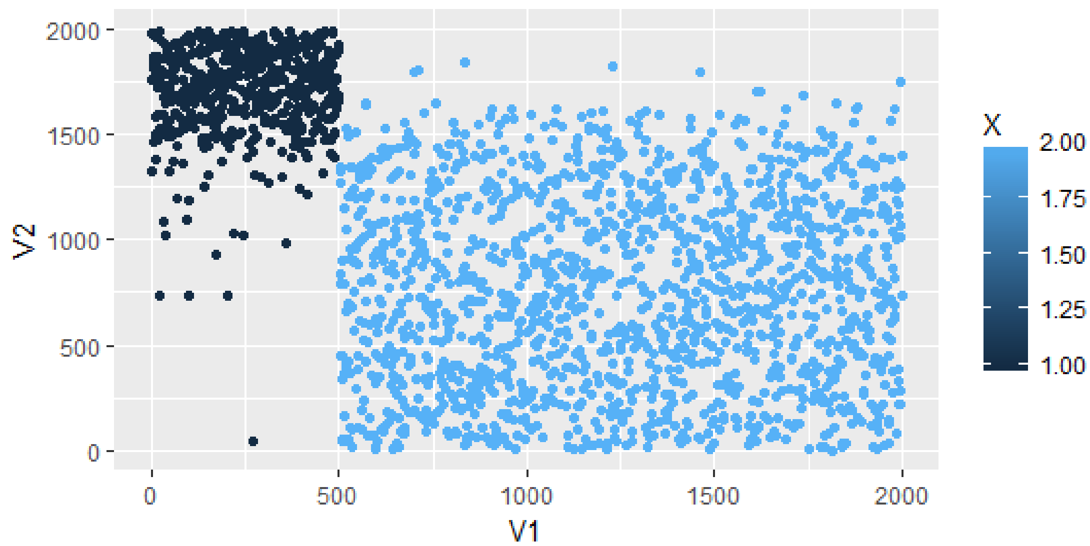


Looks like we have an uneven class distribution between class 1 and 2 in the X variable. This will likely be a factor in fitting models later on.

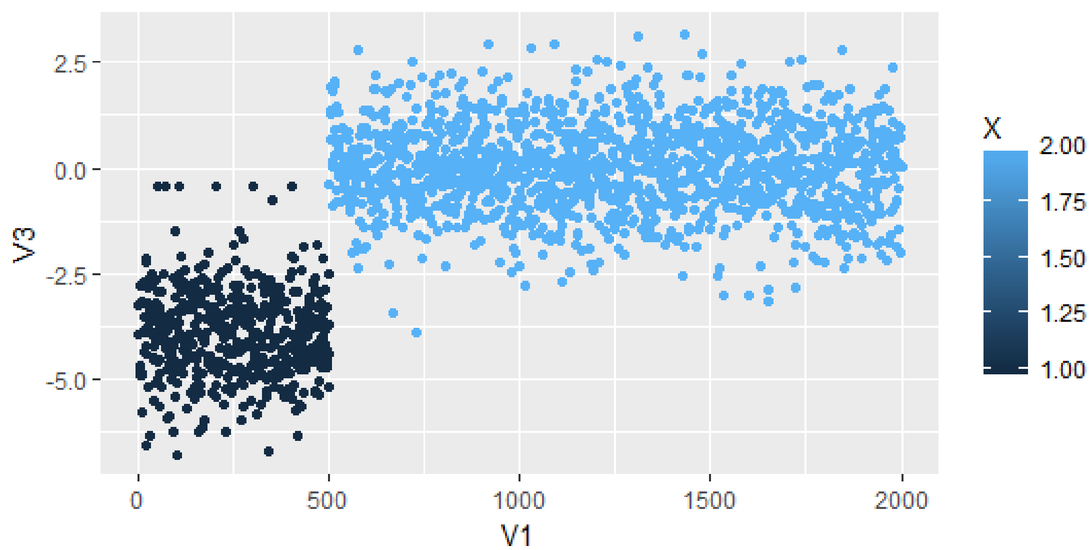
5.

Create a bar plot that shows count of data points for classes " and " (variable 5). Is the data skewed?

```
qplot(V1, V2, data=mydata, colour=X)
```




```
qplot(V1, V3, data=mydata, colour=X)
```



#Problem 2.

1.

How many principal components explain 90% of the variance? PC1 and PC2 will explain 90% of the variance. In plot 1 (`V1` and `V2`), I get the sense that V1 is going to be a good predictor of class. There is a clear pattern in both of these plots. From observation alone, some initial rules emerge for assigning target variables based on V1 that will handle a large majority of our cases. Where the observations have V1 <500, assign to class X=1. Where V1 > 500, assign to class 2.

In the second plot (`V1` and `V3`) it's apparent that both V1 and V3 are well correlated with the class variable. Let's see whether this bears out in the PCA analysis. We'll use the `prcomp` function.

```r
mydata.pca <- mydata[,1:4]
summary(mydata.pca)
```

```
##        V1               V2               V3                 V4
##  Min.   :   1.0   Min.   :   1.0   Min.   :-6.7749   Min.   :   1.0
##  1st Qu.: 500.8   1st Qu.: 500.8   1st Qu.:-2.2679   1st Qu.: 495.8
##  Median :1000.5   Median : 997.5   Median :-0.4438   Median : 995.5
##  Mean   :1000.5   Mean   : 998.6   Mean   :-0.9793   Mean   : 996.1
##  3rd Qu.:1500.2   3rd Qu.:1497.2   3rd Qu.: 0.4299   3rd Qu.:1495.2
##  Max.   :2000.0   Max.   :1997.0   Max.   : 3.1754   Max.   :1995.0
```

```r
princa <- princomp(mydata.pca)
prca <- prcomp(mydata.pca)
summary(princa)
```

```
## Importance of components:
##                             Comp.1      Comp.2      Comp.3       Comp.4
## Standard deviation     780.6856279 531.8591332 324.3447227 1.337746e+00
## Proportion of Variance   0.6109697   0.2835702   0.1054583 1.793968e-06
## Cumulative Proportion    0.6109697   0.8945399   0.9999982 1.000000e+00
```

```r
summary(prca)
```

```
## Importance of components:
##                            PC1      PC2      PC3   PC4
## Standard deviation     780.881 531.9921 324.4258 1.338
## Proportion of Variance   0.611   0.2836   0.1055 0.000
## Cumulative Proportion    0.611   0.8945   1.0000 1.000
```

```r
print(princa)
```

```
## Call:
## princomp(x = mydata.pca)
##
## Standard deviations:
##     Comp.1     Comp.2     Comp.3     Comp.4
## 780.685628 531.859133 324.344723   1.337746
##
##  4  variables and  2000 observations.
```

```r
print(prca)
```

```
## Standard deviations:
## [1] 780.880872 531.992148 324.425839   1.338081
##
## Rotation:
##             PC1          PC2          PC3           PC4
## V1  0.67259513 -0.007526584 -0.739970162  0.0018174482
## V2 -0.51586849 -0.721692291 -0.461560664 -0.0012991176
## V3  0.00160239  0.001302478 -0.001012861 -0.9999973550
## V4 -0.53055907  0.692171865 -0.489290187  0.0005469607
```

```
## loadings(princa)
## loadings(prca)
```

2. Loadings in PCA.

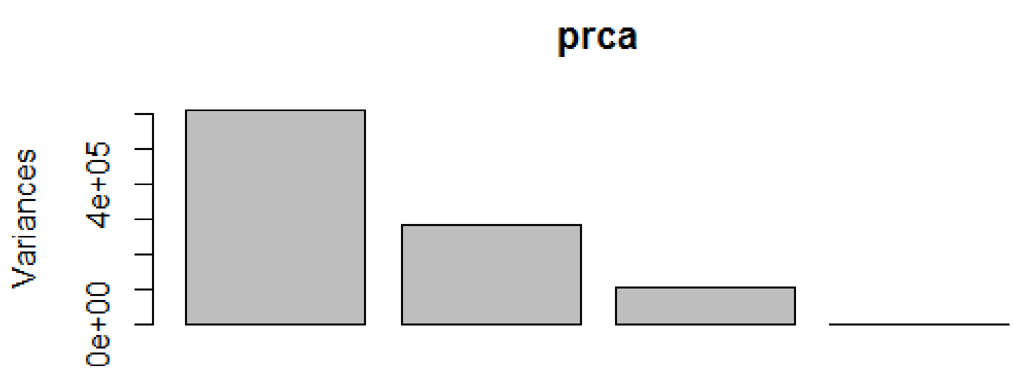
What are loadings in PCA? Observe loadings and express the principal components using the original variables. Loadings are the breakdown of how the pricinpal component variables were arrived at (e.g. what linear function was used), and what percentage of the variance each one explains. It appears that the first two components explain a 90% of the variance. PC1 was created by the function `V1 * .673 + V2 * -.516 + V3 * .001 + V4 * -.530.` Interestingly, the 4th PC in the PCA did not explain any of the variance. I initially didn't understand the difference betweeen `princomp` and `prcomp` but it appears that the use of `eigen` is the main difference, as well as the output. I can't call the `loadings` function on the variable transformed with `prcomp`, as it returns null.

3. Scree Plot.

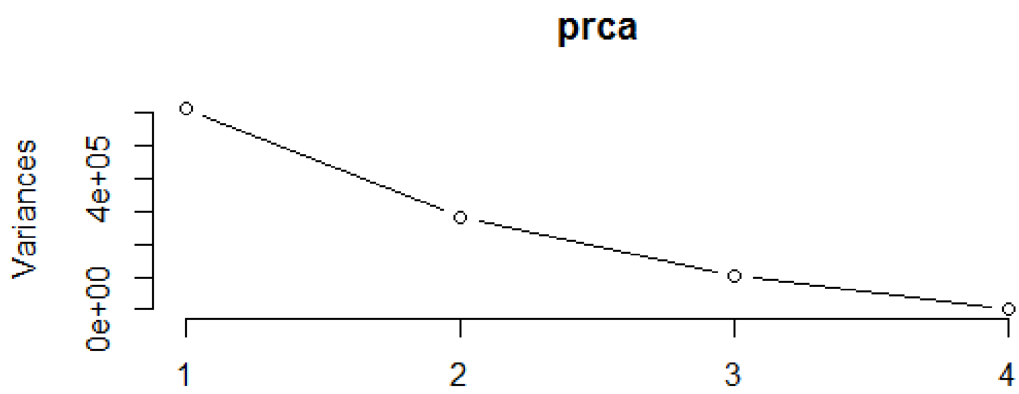
Make a scree plot. Discuss the plot, i.e., what is a scree plot? What is the optimal number of dimensions based on the plot?

Let's look at the scree and line plots:

```
screeplot(prca)
```




```
plot(prca,type="l")
```



If we were concerned with computer performance, we would likely select only the first two variables. Since it doesn't cost of anything, we can select the first three, as the 4th doesn't offer any added benefit.

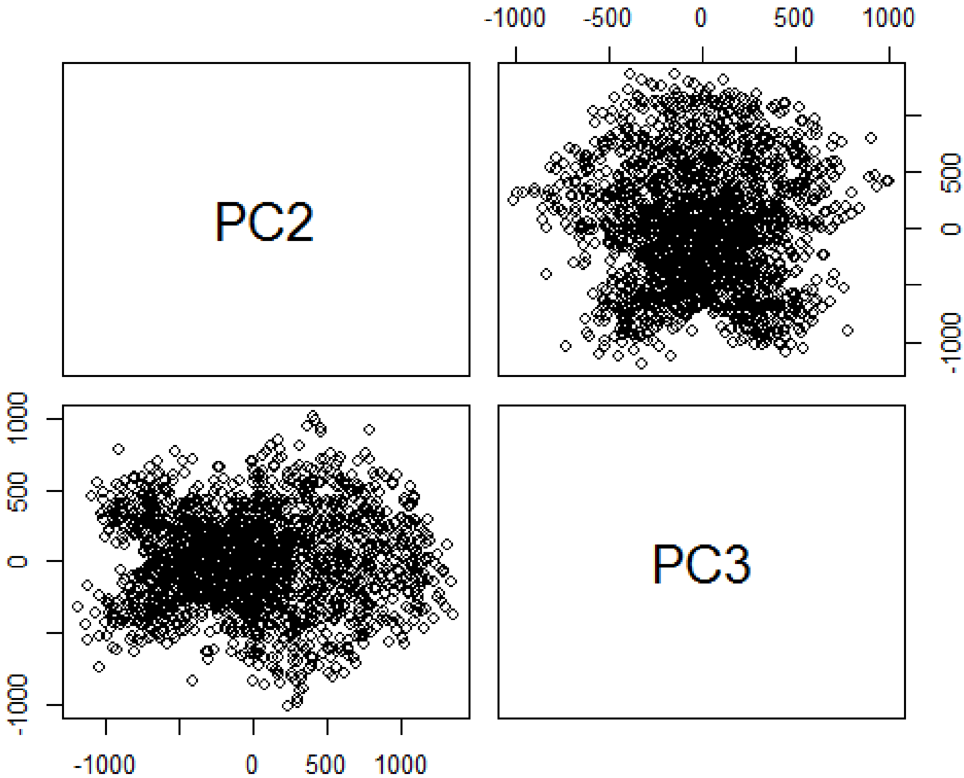
4. Scatter plot of PCs

Make a scatter plot of PC2 and PC3. Do you observe any relationship? i.e., Calculate the correlation between PC2 and PC3? What does it show?

The transformed variables are normally distributed, but since the underlying variables are closer to uniform distributions, I will try other correlation methods Spearman and Kendall.

Examining a scatter plot and correlation coefficient of PC2 and PC3:

```
## hist(prca$x)
pairs(prca$x[,2:3])
```




```r
cor(prca$x, prca$x)
```

```
##               PC1           PC2           PC3          PC4
## PC1  1.000000e+00 -7.698859e-16 -4.415020e-15 4.553976e-15
## PC2 -7.698859e-16  1.000000e+00 -3.415184e-16 6.208564e-14
## PC3 -4.415020e-15 -3.415184e-16  1.000000e+00 2.049505e-15
## PC4  4.553976e-15  6.208564e-14  2.049505e-15 1.000000e+00
```

```r
cor(prca$x, prca$x, method= "kendall")
```

```
##             PC1          PC2         PC3          PC4
## PC1 1.000000000  0.008620310 0.001432716  0.002543272
## PC2 0.008620310  1.000000000 0.001158579 -0.014747374
## PC3 0.001432716  0.001158579 1.000000000  0.002243122
## PC4 0.002543272 -0.014747374 0.002243122  1.000000000
```

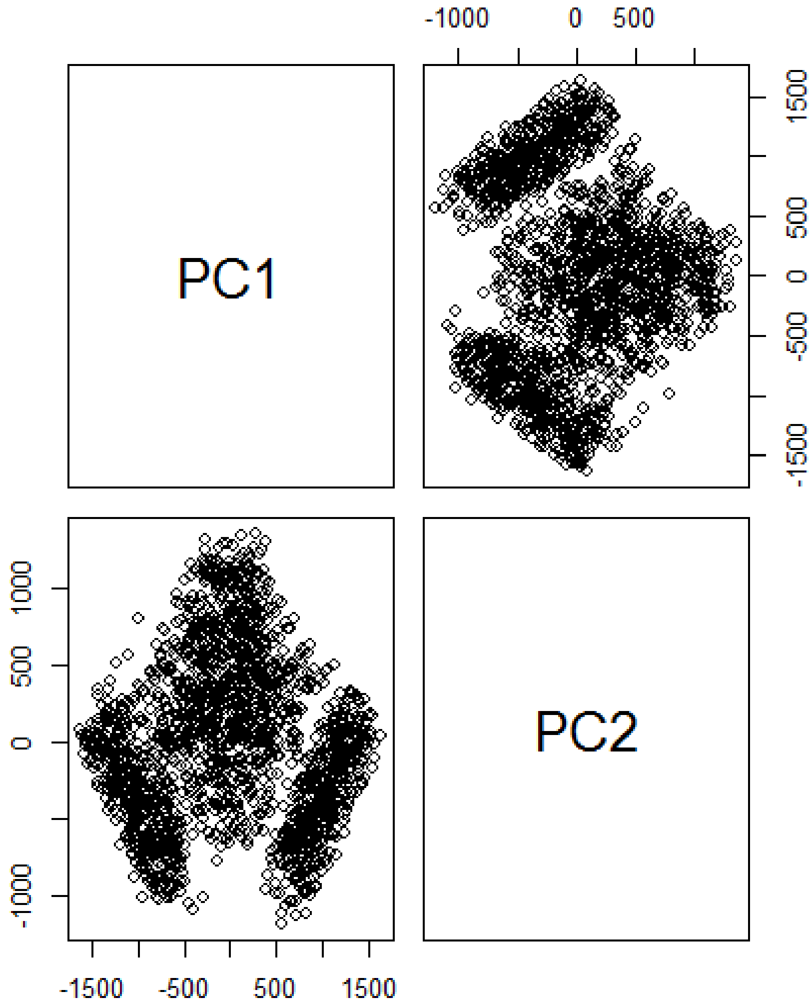
```r
cor(prca$x, prca$x, method= "spearman")
```

```
##              PC1          PC2          PC3          PC4
## PC1  1.000000000  0.009710051 -0.012794202 -0.009230396
## PC2  0.009710051  1.000000000  0.001295745 -0.024909630
## PC3 -0.012794202  0.001295745  1.000000000  0.004654344
## PC4 -0.009230396 -0.024909630  0.004654344  1.000000000
```

```
??cor

## starting httpd help server ... done
```

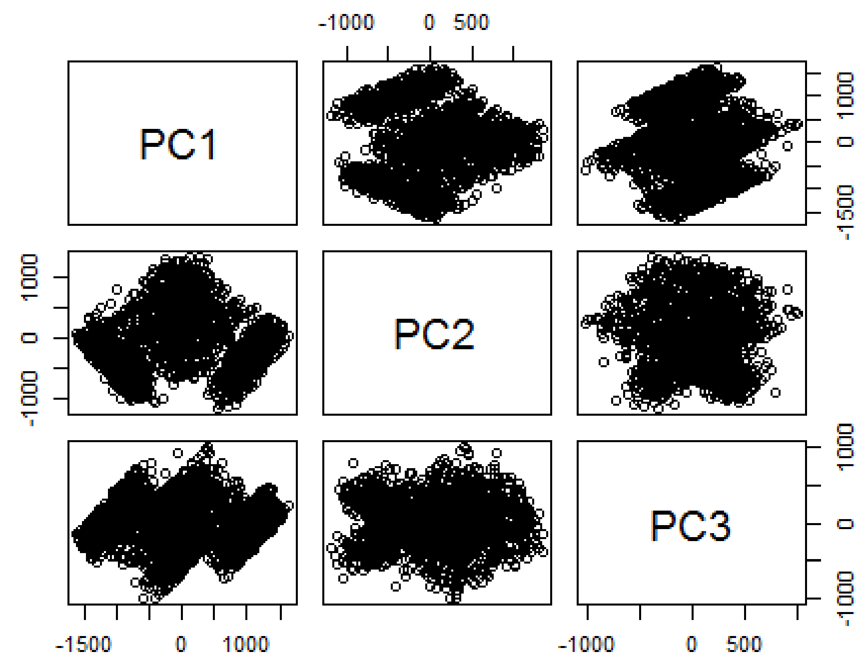
Interesting. There aren't any strong linear correlations in the data, but there are definite clusters present. The correlation matrix shows very small Pearson correlation coefficients. As for the scatter plots, there are two prongs on the left side of the scale, a cluster in the middle, and less dense values of PC2 above 250. I was also curious about the correlation between PC1 and PC2, which is plotted here:

```
pairs(prca$x[,1:2])
```



I found this correlation to be very compelling, and likely better suited to a clustering algorithm we might perform, because the clusters would probably be much clearer, and individual instance values would be easier to classify. However, I don't know whether k-means will perform exceptionally well, as the apparent clusters tend to be oblong and irregularly shaped vs. circular, where k-means performs best. Additionally, we can capture more variance of the original dataset by using PC1 and PC2 as well. What about PC1 and PC3?

```
pairs(prca$x[,1:3])
```



Looks like all three pairs of variables have interesting correlations! PC1 and PC3 align into three or possibly four neat clusters.

Problem 3.

1.

Randomly sample without replacement 300 data points from kmeans.mydata. (call the sampled data mysample). Cluster mysample with K-means. Include the R code and answer the questions below:

Code:

```
#Creating the vector
kmeans.mydata <- mydata[,c(1,2)]
## V1 appears to be an id variable - it just iterates as n+1 for every
observation. Should this be included in the k-means?

kmeans.mydata <- mydata[,c(1,2)]
kmeans.mydata
```

```
##         V1   V2
## 1        1 1987
## 2        2 1330
## 3        3 1766
## 4        4 1850
## 5        5 1817
## 6        6 1768
## 7        7 1462
## 8        8 1870
## 9        9 1583
## 10      10 1809
## 11      11 1762
## 12      12 1380
## 13      13 1506
## 14      14 1970
## 15      15 1599
## 16      16 1851
## 17      17 1729
## 18      18 1474
## 19      19 1871
## 20      20 1997
## 21      21 1812
## 22      22  740
## 23      23 1530
## 24      24 1792
## 25      25 1744
## 26      26 1813
## 27      27 1951
## 28      28 1943
## 29      29 1864
## 30      30 1677
## 31      31 1597
## 32      32 1087
## 33      33 1722
## 34      34 1732
## 35      35 1933
## 36      36 1027
## 37      37 1652
## 38      38 1789
## 39      39 1505
## 40      40 1493
## 41      41 1723
## 42      42 1897
## 43      43 1571
## 44      44 1547
## 45      45 1724
## 46      46 1733
## 47      47 1568
## 48      48 1327
## 49      49 1705
```

```
## 50      50 1855
## 51      51 1659
## 52      52 1785
## 53      53 1497
## 54      54 1702
## 55      55 1526
## 56      56 1839
## 57      57 1869
## 58      58 1373
## 59      59 1835
## 60      60 1811
## 61      61 1940
## 62      62 1761
## 63      63 1605
## 64      64 1820
## 65      65 1930
## 66      66 1449
## 67      67 1201
## 68      68 1690
## 69      69 1917
## 70      70 1843
## 71      71 1834
## 72      72 1513
## 73      73 1662
## 74      74 1868
## 75      75 1590
## 76      76 1934
## 77      77 1937
## 78      78 1673
## 79      79 1885
## 80      80 1701
## 81      81 1900
## 82      82 1763
## 83      83 1546
## 84      84 1707
## 85      85 1368
## 86      86 1698
## 87      87 1844
## 88      88 1535
## 89      89 1630
## 90      90 1952
## 91      91 1664
## 92      92 1565
## 93      93 1814
## 94      94 1726
## 95      95 1955
## 96      96 1095
## 97      97 1694
## 98      98 1188
## 99      99 1975
```

```
## 100   100 1541
## 101   101  740
## 102   102 1815
## 103   103 1602
## 104   104 1938
## 105   105 1713
## 106   106 1882
## 107   107 1948
## 108   108 1804
## 109   109 1721
## 110   110 1893
## 111   111 1564
## 112   112 1671
## 113   113 1668
## 114   114 1764
## 115   115 1529
## 116   116 1647
## 117   117 1651
## 118   118 1681
## 119   119 1740
## 120   120 1972
## 121   121 1770
## 122   122 1667
## 123   123 1574
## 124   124 1697
## 125   125 1985
## 126   126 1731
## 127   127 1993
## 128   128 1743
## 129   129 1787
## 130   130 1755
## 131   131 1582
## 132   132 1909
## 133   133 1878
## 134   134 1458
## 135   135 1899
## 136   136 1683
## 137   137 1484
## 138   138 1956
## 139   139 1703
## 140   140 1894
## 141   141 1566
## 142   142 1254
## 143   143 1437
## 144   144 1672
## 145   145 1769
## 146   146 1655
## 147   147 1716
## 148   148 1895
## 149   149 1489
```

```
## 150   150 1994
## 151   151 1949
## 152   152 1451
## 153   153 1310
## 154   154 1591
## 155   155 1739
## 156   156 1914
## 157   157 1841
## 158   158 1644
## 159   159 1772
## 160   160 1969
## 161   161 1798
## 162   162 1492
## 163   163 1559
## 164   164 1790
## 165   165 1616
## 166   166 1824
## 167   167 1944
## 168   168 1537
## 169   169 1680
## 170   170  930
## 171   171 1485
## 172   172 1780
## 173   173 1524
## 174   174 1539
## 175   175 1920
## 176   176 1747
## 177   177 1615
## 178   178 1699
## 179   179 1791
## 180   180 1575
## 181   181 1494
## 182   182 1799
## 183   183 1788
## 184   184 1584
## 185   185 1637
## 186   186 1376
## 187   187 1954
## 188   188 1587
## 189   189 1968
## 190   190 1793
## 191   191 1715
## 192   192 1856
## 193   193 1982
## 194   194 1781
## 195   195 1669
## 196   196 1471
## 197   197 1734
## 198   198 1881
## 199   199 1988
```

```
## 200   200 1964
## 201   201 1912
## 202   202  740
## 203   203 1866
## 204   204 1876
## 205   205 1941
## 206   206 1666
## 207   207 1797
## 208   208 1440
## 209   209 1500
## 210   210 1854
## 211   211 1760
## 212   212 1908
## 213   213 1991
## 214   214 1522
## 215   215 1487
## 216   216 1617
## 217   217 1657
## 218   218 1636
## 219   219 1737
## 220   220 1853
## 221   221 1036
## 222   222 1983
## 223   223 1891
## 224   224 1911
## 225   225 1816
## 226   226 1974
## 227   227 1932
## 228   228 1939
## 229   229 1883
## 230   230 1802
## 231   231 1613
## 232   232 1502
## 233   233 1959
## 234   234 1777
## 235   235 1965
## 236   236 1960
## 237   237 1612
## 238   238 1510
## 239   239 1826
## 240   240 1718
## 241   241 1625
## 242   242 1704
## 243   243 1984
## 244   244 1852
## 245   245 1992
## 246   246 1978
## 247   247 1023
## 248   248 1823
## 249   249 1633
```

```
## 250   250 1942
## 251   251 1867
## 252   252 1598
## 253   253 1682
## 254   254 1921
## 255   255 1782
## 256   256 1849
## 257   257 1392
## 258   258 1896
## 259   259 1862
## 260   260 1455
## 261   261 1831
## 262   262 1860
## 263   263 1910
## 264   264 1469
## 265   265 1947
## 266   266 1736
## 267   267 1689
## 268   268 1417
## 269   269 1794
## 270   270 1480
## 271   271 1527
## 272   272 1693
## 273   273   43
## 274   274 1847
## 275   275 1311
## 276   276 1746
## 277   277 1827
## 278   278 1604
## 279   279 1927
## 280   280 1957
## 281   281 1634
## 282   282 1490
## 283   283 1661
## 284   284 1622
## 285   285 1886
## 286   286 1656
## 287   287 1846
## 288   288 1749
## 289   289 1298
## 290   290 1696
## 291   291 1840
## 292   292 1695
## 293   293 1822
## 294   294 1858
## 295   295 1946
## 296   296 1977
## 297   297 1916
## 298   298 1646
## 299   299 1931
```

```
## 300   300 1735
## 301   301 1586
## 302   302 1727
## 303   303 1892
## 304   304 1561
## 305   305 1692
## 306   306 1879
## 307   307 1775
## 308   308 1650
## 309   309 1848
## 310   310 1756
## 311   311 1268
## 312   312 1875
## 313   313 1684
## 314   314 1784
## 315   315 1907
## 316   316 1670
## 317   317 1863
## 318   318 1632
## 319   319 1464
## 320   320 1902
## 321   321 1643
## 322   322 1548
## 323   323 1821
## 324   324 1771
## 325   325 1578
## 326   326 1980
## 327   327 1810
## 328   328 1989
## 329   329 1874
## 330   330 1750
## 331   331 1663
## 332   332 1563
## 333   333 1753
## 334   334 1962
## 335   335 1658
## 336   336 1973
## 337   337 1976
## 338   338 1880
## 339   339 1966
## 340   340 1687
## 341   341 1889
## 342   342 1778
## 343   343 1915
## 344   344 1806
## 345   345 1700
## 346   346 1416
## 347   347 1302
## 348   348 1606
## 349   349 1926
```

```
## 350   350 1554
## 351   351 1758
## 352   352 1877
## 353   353 1950
## 354   354 1888
## 355   355 1609
## 356   356 1828
## 357   357  987
## 358   358 1519
## 359   359 1837
## 360   360 1945
## 361   361 1865
## 362   362 1935
## 363   363 1645
## 364   364 1861
## 365   365 1709
## 366   366 1923
## 367   367 1712
## 368   368 1901
## 369   369 1990
## 370   370 1558
## 371   371 1728
## 372   372 1394
## 373   373 1553
## 374   374 1686
## 375   375 1961
## 376   376 1711
## 377   377 1600
## 378   378 1808
## 379   379 1515
## 380   380 1452
## 381   381 1953
## 382   382 1478
## 383   383 1675
## 384   384 1498
## 385   385 1725
## 386   386 1905
## 387   387 1929
## 388   388 1742
## 389   389 1629
## 390   390 1570
## 391   391 1603
## 392   392 1857
## 393   393 1247
## 394   394 1685
## 395   395 1748
## 396   396 1443
## 397   397 1751
## 398   398 1825
## 399   399 1520
```

```
## 400   400 1786
## 401   401 1631
## 402   402 1544
## 403   403 1465
## 404   404 1918
## 405   405 1714
## 406   406 1638
## 407   407 1967
## 408   408 1678
## 409   409 1979
## 410   410 1679
## 411   411 1408
## 412   412 1936
## 413   413 1884
## 414   414 1665
## 415   415 1218
## 416   416 1767
## 417   417 1922
## 418   418 1396
## 419   419 1642
## 420   420 1818
## 421   421 1466
## 422   422 1608
## 423   423 1717
## 424   424 1610
## 425   425 1783
## 426   426 1924
## 427   427 1838
## 428   428 1468
## 429   429 1958
## 430   430 1730
## 431   431 1639
## 432   432 1873
## 433   433 1516
## 434   434 1819
## 435   435 1963
## 436   436 1836
## 437   437 1626
## 438   438 1557
## 439   439 1830
## 440   440 1919
## 441   441 1773
## 442   442 1913
## 443   443 1801
## 444   444 1776
## 445   445 1754
## 446   446 1986
## 447   447 1796
## 448   448 1538
## 449   449 1540
```

```
## 450   450 1720
## 451   451 1928
## 452   452 1585
## 453   453 1805
## 454   454 1618
## 455   455 1323
## 456   456 1779
## 457   457 1525
## 458   458 1832
## 459   459 1971
## 460   460 1996
## 461   461 1741
## 462   462 1641
## 463   463 1745
## 464   464 1473
## 465   465 1757
## 466   466 1859
## 467   467 1842
## 468   468 1738
## 469   469 1581
## 470   470 1872
## 471   471 1674
## 472   472 1890
## 473   473 1795
## 474   474 1903
## 475   475 1488
## 476   476 1476
## 477   477 1649
## 478   478 1573
## 479   479 1409
## 480   480 1759
## 481   481 1981
## 482   482 1708
## 483   483 1833
## 484   484 1995
## 485   485 1676
## 486   486 1596
## 487   487 1504
## 488   488 1514
## 489   489 1719
## 490   490 1393
## 491   491 1620
## 492   492 1765
## 493   493 1887
## 494   494 1774
## 495   495 1898
## 496   496 1925
## 497   497 1906
## 498   498 1660
## 499   499 1904
```

```
## 500   500 1691
## 501   501 1345
## 502   502  445
## 503   503 1332
## 504   504 1273
## 505   505  842
## 506   506  788
## 507   507  899
## 508   508  453
## 509   509  343
## 510   510   49
## 511   511   39
## 512   512  793
## 513   513  366
## 514   514  164
## 515   515   42
## 516   516 1155
## 517   517 1055
## 518   518  660
## 519   519  420
## 520   520  401
## 521   521 1534
## 522   522 1204
## 523   523  426
## 524   524 1495
## 525   525   56
## 526   526  702
## 527   527  363
## 528   528  955
## 529   529  105
## 530   530  908
## 531   531 1348
## 532   532 1211
## 533   533 1344
## 534   534  677
## 535   535   19
## 536   536  101
## 537   537 1235
## 538   538  566
## 539   539  755
## 540   540  901
## 541   541 1242
## 542   542  891
## 543   543 1512
## 544   544 1199
## 545   545  294
## 546   546  961
## 547   547  973
## 548   548  880
## 549   549  452
```

```
## 550   550  387
## 551   551 1119
## 552   552  110
## 553   553 1320
## 554   554  993
## 555   555 1291
## 556   556  614
## 557   557 1295
## 558   558 1205
## 559   559  797
## 560   560 1354
## 561   561  195
## 562   562 1147
## 563   563   95
## 564   564  239
## 565   565  525
## 566   566 1015
## 567   567 1333
## 568   568  605
## 569   569  422
## 570   570 1654
## 571   571 1195
## 572   572 1640
## 573   573  705
## 574   574 1079
## 575   575  518
## 576   576   73
## 577   577 1371
## 578   578 1307
## 579   579  597
## 580   580  158
## 581   581 1035
## 582   582  818
## 583   583 1223
## 584   584 1335
## 585   585  360
## 586   586 1033
## 587   587 1030
## 588   588  801
## 589   589 1388
## 590   590  918
## 591   591   71
## 592   592  607
## 593   593  882
## 594   594 1071
## 595   595   15
## 596   596    5
## 597   597  324
## 598   598  627
## 599   599  456
```

```
## 600   600  556
## 601   601   41
## 602   602  598
## 603   603  341
## 604   604  945
## 605   605 1383
## 606   606  348
## 607   607 1077
## 608   608  438
## 609   609  717
## 610   610  639
## 611   611  701
## 612   612 1430
## 613   613   97
## 614   614  936
## 615   615  906
## 616   616 1436
## 617   617  984
## 618   618  964
## 619   619 1088
## 620   620   85
## 621   621  417
## 622   622 1086
## 623   623 1532
## 624   624 1198
## 625   625  121
## 626   626  455
## 627   627 1399
## 628   628  855
## 629   629  186
## 630   630  477
## 631   631  581
## 632   632  134
## 633   633  494
## 634   634  474
## 635   635  776
## 636   636  315
## 637   637 1292
## 638   638  168
## 639   639 1425
## 640   640  405
## 641   641  989
## 642   642  613
## 643   643  467
## 644   644    3
## 645   645  888
## 646   646  354
## 647   647 1309
## 648   648  839
## 649   649  440
```

```
## 650   650  284
## 651   651 1054
## 652   652 1099
## 653   653  269
## 654   654   35
## 655   655 1017
## 656   656  654
## 657   657  150
## 658   658  339
## 659   659   11
## 660   660 1329
## 661   661 1435
## 662   662  649
## 663   663   91
## 664   664 1189
## 665   665  699
## 666   666  939
## 667   667  236
## 668   668   84
## 669   669  571
## 670   670  314
## 671   671  468
## 672   672  104
## 673   673  207
## 674   674   77
## 675   675 1562
## 676   676  530
## 677   677  781
## 678   678 1483
## 679   679 1101
## 680   680 1116
## 681   681  858
## 682   682 1444
## 683   683  184
## 684   684  404
## 685   685  729
## 686   686 1509
## 687   687  594
## 688   688  416
## 689   689  809
## 690   690 1098
## 691   691  443
## 692   692 1313
## 693   693 1040
## 694   694 1475
## 695   695  896
## 696   696  551
## 697   697  181
## 698   698  995
## 699   699  192
```

```
## 700   700 1106
## 701   701 1803
## 702   702 1193
## 703   703 1413
## 704   704  338
## 705   705 1037
## 706   706  926
## 707   707  999
## 708   708 1459
## 709   709 1807
## 710   710  527
## 711   711 1080
## 712   712 1128
## 713   713  243
## 714   714 1081
## 715   715  592
## 716   716  962
## 717   717  167
## 718   718  731
## 719   719  725
## 720   720  953
## 721   721  723
## 722   722  986
## 723   723  874
## 724   724  140
## 725   725  722
## 726   726  337
## 727   727  948
## 728   728 1607
## 729   729  446
## 730   730   90
## 731   731  187
## 732   732  913
## 733   733  138
## 734   734  683
## 735   735 1183
## 736   736  166
## 737   737  434
## 738   738 1161
## 739   739  311
## 740   740  350
## 741   741 1285
## 742   742 1237
## 743   743  661
## 744   744  769
## 745   745  165
## 746   746 1229
## 747   747  659
## 748   748  693
## 749   749 1325
```

```
## 750   750  593
## 751   751  128
## 752   752  524
## 753   753  641
## 754   754 1140
## 755   755 1277
## 756   756 1246
## 757   757  894
## 758   758 1653
## 759   759 1083
## 760   760  823
## 761   761  180
## 762   762  700
## 763   763 1026
## 764   764 1442
## 765   765 1131
## 766   766 1251
## 767   767  193
## 768   768  870
## 769   769  252
## 770   770  370
## 771   771  325
## 772   772  866
## 773   773 1412
## 774   774  515
## 775   775  398
## 776   776  220
## 777   777  313
## 778   778  310
## 779   779  419
## 780   780 1342
## 781   781  837
## 782   782  388
## 783   783  291
## 784   784  656
## 785   785 1018
## 786   786  942
## 787   787   93
## 788   788  535
## 789   789  611
## 790   790 1401
## 791   791 1178
## 792   792 1461
## 793   793  573
## 794   794  789
## 795   795   74
## 796   796 1283
## 797   797  356
## 798   798  347
## 799   799  266
```

```
## 800   800  619
## 801   801 1457
## 802   802  250
## 803   803  403
## 804   804  626
## 805   805 1094
## 806   806 1555
## 807   807 1340
## 808   808  203
## 809   809 1517
## 810   810  202
## 811   811   89
## 812   812  208
## 813   813   81
## 814   814 1358
## 815   815 1149
## 816   816 1363
## 817   817  328
## 818   818  800
## 819   819 1460
## 820   820 1194
## 821   821 1148
## 822   822  406
## 823   823  277
## 824   824  197
## 825   825  188
## 826   826  813
## 827   827  542
## 828   828 1378
## 829   829 1418
## 830   830  856
## 831   831  376
## 832   832  688
## 833   833   21
## 834   834 1375
## 835   835 1503
## 836   836 1845
## 837   837  879
## 838   838 1208
## 839   839   60
## 840   840  808
## 841   841  115
## 842   842   30
## 843   843  655
## 844   844  213
## 845   845  259
## 846   846  828
## 847   847  716
## 848   848  950
## 849   849  708
```

```
## 850   850  300
## 851   851   69
## 852   852  719
## 853   853  368
## 854   854 1000
## 855   855  273
## 856   856  523
## 857   857  832
## 858   858  270
## 859   859  814
## 860   860 1400
## 861   861  805
## 862   862  176
## 863   863  242
## 864   864  704
## 865   865  342
## 866   866   63
## 867   867 1143
## 868   868  979
## 869   869  355
## 870   870  301
## 871   871  251
## 872   872 1165
## 873   873 1551
## 874   874 1454
## 875   875  770
## 876   876  256
## 877   877  217
## 878   878  539
## 879   879  391
## 880   880  802
## 881   881 1381
## 882   882  990
## 883   883  709
## 884   884  111
## 885   885  333
## 886   886  502
## 887   887  385
## 888   888  365
## 889   889 1244
## 890   890  517
## 891   891  451
## 892   892 1410
## 893   893   62
## 894   894 1232
## 895   895  629
## 896   896  759
## 897   897  588
## 898   898 1282
## 899   899  316
```

```
## 900   900 1588
## 901   901  364
## 902   902  329
## 903   903  296
## 904   904  919
## 905   905   36
## 906   906 1415
## 907   907 1096
## 908   908 1191
## 909   909 1628
## 910   910 1264
## 911   911  120
## 912   912  276
## 913   913  909
## 914   914  965
## 915   915   22
## 916   916   96
## 917   917 1250
## 918   918  450
## 919   919 1219
## 920   920   72
## 921   921  475
## 922   922  648
## 923   923   40
## 924   924 1507
## 925   925 1230
## 926   926  635
## 927   927  637
## 928   928  344
## 929   929 1550
## 930   930 1111
## 931   931  742
## 932   932  487
## 933   933 1321
## 934   934 1156
## 935   935  890
## 936   936  173
## 937   937   58
## 938   938 1433
## 939   939  630
## 940   940 1176
## 941   941  696
## 942   942  935
## 943   943  367
## 944   944  431
## 945   945 1580
## 946   946  441
## 947   947  174
## 948   948  707
## 949   949   23
```

```
## 950   950  332
## 951   951  157
## 952   952  493
## 953   953  302
## 954   954  227
## 955   955  732
## 956   956  636
## 957   957  674
## 958   958 1374
## 959   959 1624
## 960   960  409
## 961   961 1196
## 962   962  400
## 963   963  695
## 964   964 1286
## 965   965  146
## 966   966 1576
## 967   967  237
## 968   968  997
## 969   969   10
## 970   970  959
## 971   971  974
## 972   972  712
## 973   973  928
## 974   974   70
## 975   975  510
## 976   976 1215
## 977   977 1241
## 978   978  604
## 979   979  565
## 980   980  785
## 981   981  949
## 982   982  784
## 983   983  318
## 984   984  615
## 985   985  214
## 986   986  933
## 987   987  143
## 988   988 1404
## 989   989 1024
## 990   990  827
## 991   991  288
## 992   992  260
## 993   993  873
## 994   994  131
## 995   995   54
## 996   996  153
## 997   997 1253
## 998   998 1011
## 999   999  238
```

```
## 1000 1000  835
## 1001 1001  845
## 1002 1002  563
## 1003 1003 1352
## 1004 1004 1222
## 1005 1005 1481
## 1006 1006  663
## 1007 1007  106
## 1008 1008 1227
## 1009 1009 1221
## 1010 1010  579
## 1011 1011  253
## 1012 1012 1594
## 1013 1013  102
## 1014 1014  651
## 1015 1015  603
## 1016 1016 1041
## 1017 1017  424
## 1018 1018  694
## 1019 1019  286
## 1020 1020  509
## 1021 1021  200
## 1022 1022  912
## 1023 1023  885
## 1024 1024 1406
## 1025 1025  116
## 1026 1026  730
## 1027 1027  664
## 1028 1028 1346
## 1029 1029  497
## 1030 1030  522
## 1031 1031  429
## 1032 1032 1181
## 1033 1033 1339
## 1034 1034 1365
## 1035 1035  529
## 1036 1036 1062
## 1037 1037 1206
## 1038 1038  617
## 1039 1039 1357
## 1040 1040   64
## 1041 1041 1252
## 1042 1042  883
## 1043 1043  897
## 1044 1044  686
## 1045 1045  736
## 1046 1046 1238
## 1047 1047  245
## 1048 1048  210
## 1049 1049  399
```

```
## 1050 1050 1543
## 1051 1051  834
## 1052 1052  601
## 1053 1053  844
## 1054 1054  850
## 1055 1055  893
## 1056 1056 1006
## 1057 1057  645
## 1058 1058 1284
## 1059 1059  796
## 1060 1060  586
## 1061 1061  358
## 1062 1062  851
## 1063 1063 1118
## 1064 1064  449
## 1065 1065  248
## 1066 1066  931
## 1067 1067 1104
## 1068 1068  240
## 1069 1069  290
## 1070 1070 1477
## 1071 1071 1556
## 1072 1072  706
## 1073 1073  830
## 1074 1074  230
## 1075 1075  371
## 1076 1076  516
## 1077 1077  691
## 1078 1078 1623
## 1079 1079 1288
## 1080 1080 1174
## 1081 1081 1262
## 1082 1082  713
## 1083 1083  938
## 1084 1084  460
## 1085 1085  557
## 1086 1086  773
## 1087 1087 1175
## 1088 1088 1110
## 1089 1089 1063
## 1090 1090  219
## 1091 1091  622
## 1092 1092  745
## 1093 1093 1179
## 1094 1094  872
## 1095 1095 1536
## 1096 1096  351
## 1097 1097  754
## 1098 1098 1173
## 1099 1099 1141
```

```
## 1100 1100 1317
## 1101 1101  482
## 1102 1102  209
## 1103 1103  570
## 1104 1104  631
## 1105 1105  852
## 1106 1106  392
## 1107 1107  390
## 1108 1108  541
## 1109 1109  924
## 1110 1110 1390
## 1111 1111   67
## 1112 1112   13
## 1113 1113  393
## 1114 1114  752
## 1115 1115  287
## 1116 1116 1299
## 1117 1117  689
## 1118 1118  840
## 1119 1119 1138
## 1120 1120  774
## 1121 1121  867
## 1122 1122  922
## 1123 1123  151
## 1124 1124    7
## 1125 1125  727
## 1126 1126  384
## 1127 1127  905
## 1128 1128  662
## 1129 1129  458
## 1130 1130  911
## 1131 1131  862
## 1132 1132 1446
## 1133 1133 1377
## 1134 1134   98
## 1135 1135  212
## 1136 1136 1482
## 1137 1137 1407
## 1138 1138 1085
## 1139 1139  481
## 1140 1140  679
## 1141 1141  678
## 1142 1142 1316
## 1143 1143  211
## 1144 1144  436
## 1145 1145    8
## 1146 1146 1296
## 1147 1147  231
## 1148 1148 1589
## 1149 1149  293
```

```
## 1150 1150  681
## 1151 1151  379
## 1152 1152  826
## 1153 1153  599
## 1154 1154  407
## 1155 1155  892
## 1156 1156 1084
## 1157 1157   51
## 1158 1158 1303
## 1159 1159  744
## 1160 1160 1214
## 1161 1161  846
## 1162 1162  317
## 1163 1163   25
## 1164 1164 1002
## 1165 1165  735
## 1166 1166  998
## 1167 1167 1542
## 1168 1168 1523
## 1169 1169 1032
## 1170 1170  268
## 1171 1171  533
## 1172 1172  780
## 1173 1173  547
## 1174 1174 1601
## 1175 1175   28
## 1176 1176 1337
## 1177 1177  806
## 1178 1178 1359
## 1179 1179 1042
## 1180 1180 1224
## 1181 1181 1531
## 1182 1182  838
## 1183 1183 1203
## 1184 1184  132
## 1185 1185 1012
## 1186 1186 1240
## 1187 1187 1234
## 1188 1188  353
## 1189 1189 1334
## 1190 1190   31
## 1191 1191  650
## 1192 1192   94
## 1193 1193 1133
## 1194 1194 1261
## 1195 1195  319
## 1196 1196  471
## 1197 1197  860
## 1198 1198 1508
## 1199 1199  190
```

```
## 1200 1200 1434
## 1201 1201  819
## 1202 1202 1614
## 1203 1203 1322
## 1204 1204 1364
## 1205 1205 1427
## 1206 1206 1045
## 1207 1207  958
## 1208 1208 1090
## 1209 1209 1312
## 1210 1210  470
## 1211 1211 1486
## 1212 1212 1528
## 1213 1213  499
## 1214 1214    4
## 1215 1215   37
## 1216 1216  836
## 1217 1217  552
## 1218 1218  478
## 1219 1219 1091
## 1220 1220  257
## 1221 1221  345
## 1222 1222  162
## 1223 1223 1225
## 1224 1224 1351
## 1225 1225  183
## 1226 1226 1157
## 1227 1227 1829
## 1228 1228  205
## 1229 1229 1428
## 1230 1230  408
## 1231 1231  330
## 1232 1232 1068
## 1233 1233  100
## 1234 1234  335
## 1235 1235  191
## 1236 1236   59
## 1237 1237  574
## 1238 1238  562
## 1239 1239   44
## 1240 1240 1467
## 1241 1241  737
## 1242 1242 1431
## 1243 1243 1389
## 1244 1244  307
## 1245 1245  171
## 1246 1246  228
## 1247 1247  795
## 1248 1248  505
## 1249 1249  558
```

```
## 1250 1250  479
## 1251 1251 1010
## 1252 1252  585
## 1253 1253  137
## 1254 1254 1414
## 1255 1255  322
## 1256 1256 1123
## 1257 1257 1028
## 1258 1258  977
## 1259 1259  170
## 1260 1260  804
## 1261 1261  568
## 1262 1262 1306
## 1263 1263  531
## 1264 1264   24
## 1265 1265 1021
## 1266 1266  934
## 1267 1267   26
## 1268 1268 1269
## 1269 1269 1075
## 1270 1270  177
## 1271 1271  375
## 1272 1272  423
## 1273 1273 1289
## 1274 1274  272
## 1275 1275 1171
## 1276 1276 1567
## 1277 1277  932
## 1278 1278 1217
## 1279 1279   46
## 1280 1280   87
## 1281 1281  602
## 1282 1282  810
## 1283 1283  241
## 1284 1284  490
## 1285 1285 1151
## 1286 1286  944
## 1287 1287  491
## 1288 1288  576
## 1289 1289 1105
## 1290 1290 1146
## 1291 1291  863
## 1292 1292  536
## 1293 1293  292
## 1294 1294 1499
## 1295 1295 1167
## 1296 1296  549
## 1297 1297 1552
## 1298 1298  503
## 1299 1299  871
```

```
## 1300 1300  575
## 1301 1301 1595
## 1302 1302  994
## 1303 1303  996
## 1304 1304 1048
## 1305 1305 1265
## 1306 1306 1270
## 1307 1307  782
## 1308 1308 1007
## 1309 1309  670
## 1310 1310  751
## 1311 1311  760
## 1312 1312 1422
## 1313 1313 1272
## 1314 1314  513
## 1315 1315  112
## 1316 1316 1611
## 1317 1317 1129
## 1318 1318  306
## 1319 1319 1479
## 1320 1320  484
## 1321 1321   14
## 1322 1322   76
## 1323 1323  687
## 1324 1324 1047
## 1325 1325  960
## 1326 1326 1239
## 1327 1327   61
## 1328 1328  414
## 1329 1329 1331
## 1330 1330  249
## 1331 1331 1164
## 1332 1332  172
## 1333 1333 1126
## 1334 1334  129
## 1335 1335  734
## 1336 1336   68
## 1337 1337 1019
## 1338 1338  334
## 1339 1339  282
## 1340 1340  459
## 1341 1341  600
## 1342 1342 1163
## 1343 1343 1152
## 1344 1344  298
## 1345 1345 1243
## 1346 1346 1361
## 1347 1347  386
## 1348 1348 1260
## 1349 1349 1139
```

```
## 1350 1350 1185
## 1351 1351 1429
## 1352 1352  136
## 1353 1353 1379
## 1354 1354 1593
## 1355 1355  572
## 1356 1356  638
## 1357 1357 1210
## 1358 1358  537
## 1359 1359 1158
## 1360 1360 1142
## 1361 1361  519
## 1362 1362  480
## 1363 1363 1070
## 1364 1364  486
## 1365 1365  532
## 1366 1366  761
## 1367 1367  261
## 1368 1368 1092
## 1369 1369   79
## 1370 1370  555
## 1371 1371  179
## 1372 1372  790
## 1373 1373  685
## 1374 1374 1315
## 1375 1375  149
## 1376 1376  169
## 1377 1377  199
## 1378 1378  682
## 1379 1379  382
## 1380 1380 1278
## 1381 1381 1398
## 1382 1382  267
## 1383 1383  684
## 1384 1384  223
## 1385 1385  457
## 1386 1386  623
## 1387 1387  224
## 1388 1388 1275
## 1389 1389 1073
## 1390 1390 1150
## 1391 1391  595
## 1392 1392  340
## 1393 1393 1328
## 1394 1394  326
## 1395 1395  117
## 1396 1396  771
## 1397 1397  442
## 1398 1398  946
## 1399 1399 1290
```

```
## 1400 1400  444
## 1401 1401 1305
## 1402 1402  889
## 1403 1403   53
## 1404 1404 1020
## 1405 1405 1039
## 1406 1406  483
## 1407 1407  383
## 1408 1408  550
## 1409 1409   38
## 1410 1410  743
## 1411 1411 1102
## 1412 1412  538
## 1413 1413  492
## 1414 1414   80
## 1415 1415 1350
## 1416 1416  618
## 1417 1417  321
## 1418 1418  485
## 1419 1419  816
## 1420 1420  747
## 1421 1421  625
## 1422 1422  983
## 1423 1423  559
## 1424 1424  427
## 1425 1425  675
## 1426 1426  247
## 1427 1427   50
## 1428 1428  139
## 1429 1429  632
## 1430 1430  859
## 1431 1431  710
## 1432 1432  119
## 1433 1433 1013
## 1434 1434  943
## 1435 1435 1533
## 1436 1436 1577
## 1437 1437  609
## 1438 1438 1197
## 1439 1439  265
## 1440 1440  411
## 1441 1441 1419
## 1442 1442  463
## 1443 1443  907
## 1444 1444  546
## 1445 1445  703
## 1446 1446  512
## 1447 1447  876
## 1448 1448 1200
## 1449 1449 1287
```

```
## 1450 1450  750
## 1451 1451 1001
## 1452 1452 1100
## 1453 1453 1319
## 1454 1454  589
## 1455 1455  857
## 1456 1456  869
## 1457 1457  884
## 1458 1458   65
## 1459 1459 1281
## 1460 1460  564
## 1461 1461   52
## 1462 1462 1441
## 1463 1463 1800
## 1464 1464 1249
## 1465 1465 1403
## 1466 1466 1439
## 1467 1467  799
## 1468 1468  410
## 1469 1469 1426
## 1470 1470 1082
## 1471 1471  175
## 1472 1472 1360
## 1473 1473 1060
## 1474 1474  720
## 1475 1475  591
## 1476 1476  672
## 1477 1477 1463
## 1478 1478 1059
## 1479 1479  642
## 1480 1480  511
## 1481 1481  425
## 1482 1482  975
## 1483 1483 1372
## 1484 1484   82
## 1485 1485  787
## 1486 1486  783
## 1487 1487    6
## 1488 1488  971
## 1489 1489 1326
## 1490 1490  923
## 1491 1491 1511
## 1492 1492  496
## 1493 1493  196
## 1494 1494 1127
## 1495 1495  130
## 1496 1496  878
## 1497 1497   34
## 1498 1498  951
## 1499 1499 1355
```

```
## 1500 1500  141
## 1501 1501  766
## 1502 1502 1135
## 1503 1503  606
## 1504 1504 1066
## 1505 1505  421
## 1506 1506 1058
## 1507 1507  308
## 1508 1508 1231
## 1509 1509  232
## 1510 1510  952
## 1511 1511 1044
## 1512 1512  824
## 1513 1513  577
## 1514 1514 1592
## 1515 1515  465
## 1516 1516   99
## 1517 1517  312
## 1518 1518  222
## 1519 1519  758
## 1520 1520  587
## 1521 1521 1016
## 1522 1522  887
## 1523 1523  147
## 1524 1524  161
## 1525 1525 1137
## 1526 1526  698
## 1527 1527  811
## 1528 1528  954
## 1529 1529  514
## 1530 1530  976
## 1531 1531 1300
## 1532 1532 1267
## 1533 1533  346
## 1534 1534  580
## 1535 1535  665
## 1536 1536  667
## 1537 1537  937
## 1538 1538  280
## 1539 1539 1184
## 1540 1540  323
## 1541 1541  763
## 1542 1542 1053
## 1543 1543 1153
## 1544 1544 1008
## 1545 1545 1226
## 1546 1546  898
## 1547 1547  985
## 1548 1548  652
## 1549 1549 1216
```

```
## 1550 1550  561
## 1551 1551  271
## 1552 1552  733
## 1553 1553 1545
## 1554 1554  362
## 1555 1555  263
## 1556 1556  155
## 1557 1557  305
## 1558 1558  145
## 1559 1559  303
## 1560 1560  956
## 1561 1561  488
## 1562 1562  109
## 1563 1563 1579
## 1564 1564  658
## 1565 1565  778
## 1566 1566 1470
## 1567 1567  297
## 1568 1568  299
## 1569 1569 1074
## 1570 1570  917
## 1571 1571 1125
## 1572 1572 1022
## 1573 1573  738
## 1574 1574 1014
## 1575 1575 1447
## 1576 1576  921
## 1577 1577  982
## 1578 1578  608
## 1579 1579  992
## 1580 1580 1052
## 1581 1581 1078
## 1582 1582 1572
## 1583 1583 1109
## 1584 1584  621
## 1585 1585  680
## 1586 1586  775
## 1587 1587  466
## 1588 1588  501
## 1589 1589 1271
## 1590 1590  476
## 1591 1591  118
## 1592 1592   27
## 1593 1593  473
## 1594 1594  548
## 1595 1595   55
## 1596 1596  653
## 1597 1597  159
## 1598 1598 1336
## 1599 1599  847
```

```
## 1600 1600 1107
## 1601 1601  972
## 1602 1602  233
## 1603 1603  620
## 1604 1604 1341
## 1605 1605   45
## 1606 1606 1169
## 1607 1607   88
## 1608 1608  432
## 1609 1609  469
## 1610 1610 1706
## 1611 1611  669
## 1612 1612 1182
## 1613 1613  767
## 1614 1614   47
## 1615 1615  229
## 1616 1616  113
## 1617 1617  877
## 1618 1618  108
## 1619 1619  791
## 1620 1620  970
## 1621 1621 1005
## 1622 1622  895
## 1623 1623  553
## 1624 1624 1276
## 1625 1625 1710
## 1626 1626 1212
## 1627 1627 1391
## 1628 1628  543
## 1629 1629 1518
## 1630 1630  941
## 1631 1631  657
## 1632 1632  765
## 1633 1633  278
## 1634 1634  160
## 1635 1635 1166
## 1636 1636 1162
## 1637 1637  903
## 1638 1638 1280
## 1639 1639  668
## 1640 1640  154
## 1641 1641  643
## 1642 1642  148
## 1643 1643 1301
## 1644 1644  914
## 1645 1645 1213
## 1646 1646 1236
## 1647 1647  753
## 1648 1648  349
## 1649 1649 1382
```

```
## 1650 1650 1347
## 1651 1651   57
## 1652 1652  544
## 1653 1653  798
## 1654 1654 1065
## 1655 1655  380
## 1656 1656 1170
## 1657 1657   92
## 1658 1658  462
## 1659 1659  590
## 1660 1660  690
## 1661 1661  739
## 1662 1662 1064
## 1663 1663 1456
## 1664 1664  402
## 1665 1665  258
## 1666 1666   75
## 1667 1667 1501
## 1668 1668  506
## 1669 1669  624
## 1670 1670 1114
## 1671 1671  156
## 1672 1672  394
## 1673 1673 1069
## 1674 1674  560
## 1675 1675 1384
## 1676 1676  352
## 1677 1677  854
## 1678 1678  281
## 1679 1679  714
## 1680 1680  772
## 1681 1681 1220
## 1682 1682 1266
## 1683 1683 1136
## 1684 1684  724
## 1685 1685 1180
## 1686 1686 1549
## 1687 1687 1308
## 1688 1688 1076
## 1689 1689  762
## 1690 1690   48
## 1691 1691 1619
## 1692 1692  963
## 1693 1693 1314
## 1694 1694  123
## 1695 1695  794
## 1696 1696 1450
## 1697 1697 1049
## 1698 1698  289
## 1699 1699  940
```

```
## 1700 1700  792
## 1701 1701  875
## 1702 1702 1370
## 1703 1703  430
## 1704 1704  437
## 1705 1705  969
## 1706 1706  966
## 1707 1707 1113
## 1708 1708  644
## 1709 1709  711
## 1710 1710  107
## 1711 1711 1362
## 1712 1712  981
## 1713 1713  495
## 1714 1714  927
## 1715 1715 1192
## 1716 1716  498
## 1717 1717  126
## 1718 1718  822
## 1719 1719  279
## 1720 1720  534
## 1721 1721 1154
## 1722 1722 1438
## 1723 1723  978
## 1724 1724 1304
## 1725 1725  125
## 1726 1726 1445
## 1727 1727 1046
## 1728 1728 1112
## 1729 1729  369
## 1730 1730  749
## 1731 1731  295
## 1732 1732 1029
## 1733 1733  201
## 1734 1734 1688
## 1735 1735 1397
## 1736 1736  244
## 1737 1737 1293
## 1738 1738 1294
## 1739 1739  841
## 1740 1740 1202
## 1741 1741 1411
## 1742 1742  395
## 1743 1743 1187
## 1744 1744 1259
## 1745 1745  647
## 1746 1746 1117
## 1747 1747 1421
## 1748 1748 1089
## 1749 1749  359
```

```
## 1750 1750    9
## 1751 1751  554
## 1752 1752  807
## 1753 1753  886
## 1754 1754  980
## 1755 1755 1025
## 1756 1756  904
## 1757 1757  361
## 1758 1758 1003
## 1759 1759 1072
## 1760 1760 1177
## 1761 1761  578
## 1762 1762  715
## 1763 1763  235
## 1764 1764  418
## 1765 1765 1256
## 1766 1766  868
## 1767 1767  264
## 1768 1768  374
## 1769 1769  634
## 1770 1770  320
## 1771 1771  988
## 1772 1772 1274
## 1773 1773 1432
## 1774 1774  947
## 1775 1775 1420
## 1776 1776  504
## 1777 1777  843
## 1778 1778  275
## 1779 1779 1056
## 1780 1780  396
## 1781 1781 1061
## 1782 1782  746
## 1783 1783 1521
## 1784 1784   12
## 1785 1785  929
## 1786 1786  815
## 1787 1787  226
## 1788 1788  234
## 1789 1789  849
## 1790 1790  246
## 1791 1791  829
## 1792 1792 1424
## 1793 1793   66
## 1794 1794 1207
## 1795 1795   16
## 1796 1796 1491
## 1797 1797  472
## 1798 1798  304
## 1799 1799  671
```

```
## 1800 1800 1324
## 1801 1801  507
## 1802 1802   78
## 1803 1803  825
## 1804 1804 1160
## 1805 1805  372
## 1806 1806  225
## 1807 1807  526
## 1808 1808 1297
## 1809 1809  331
## 1810 1810  178
## 1811 1811 1050
## 1812 1812 1343
## 1813 1813 1453
## 1814 1814 1496
## 1815 1815    1
## 1816 1816 1124
## 1817 1817  255
## 1818 1818  336
## 1819 1819  381
## 1820 1820  103
## 1821 1821 1145
## 1822 1822 1057
## 1823 1823  821
## 1824 1824  373
## 1825 1825 1648
## 1826 1826  915
## 1827 1827  861
## 1828 1828  500
## 1829 1829  428
## 1830 1830 1387
## 1831 1831  786
## 1832 1832  991
## 1833 1833  721
## 1834 1834 1402
## 1835 1835  831
## 1836 1836  198
## 1837 1837 1172
## 1838 1838  412
## 1839 1839  569
## 1840 1840  666
## 1841 1841   17
## 1842 1842   29
## 1843 1843  254
## 1844 1844  182
## 1845 1845 1031
## 1846 1846 1338
## 1847 1847 1356
## 1848 1848  820
## 1849 1849  520
```

```
## 1850 1850   33
## 1851 1851 1423
## 1852 1852 1097
## 1853 1853  902
## 1854 1854  916
## 1855 1855  583
## 1856 1856 1448
## 1857 1857  848
## 1858 1858  812
## 1859 1859  768
## 1860 1860  206
## 1861 1861  582
## 1862 1862 1132
## 1863 1863  612
## 1864 1864  489
## 1865 1865  124
## 1866 1866  194
## 1867 1867 1318
## 1868 1868  777
## 1869 1869 1108
## 1870 1870 1560
## 1871 1871  508
## 1872 1872 1263
## 1873 1873  633
## 1874 1874 1130
## 1875 1875  764
## 1876 1876  616
## 1877 1877 1621
## 1878 1878 1122
## 1879 1879  448
## 1880 1880 1635
## 1881 1881  461
## 1882 1882  152
## 1883 1883 1051
## 1884 1884  144
## 1885 1885  726
## 1886 1886  728
## 1887 1887 1043
## 1888 1888  464
## 1889 1889  900
## 1890 1890  925
## 1891 1891  584
## 1892 1892  756
## 1893 1893  133
## 1894 1894 1367
## 1895 1895  447
## 1896 1896 1134
## 1897 1897  262
## 1898 1898  881
## 1899 1899  285
```

```
## 1900 1900  357
## 1901 1901  910
## 1902 1902 1190
## 1903 1903 1386
## 1904 1904  596
## 1905 1905 1233
## 1906 1906  646
## 1907 1907  545
## 1908 1908  957
## 1909 1909 1369
## 1910 1910  864
## 1911 1911  528
## 1912 1912 1121
## 1913 1913  567
## 1914 1914 1228
## 1915 1915  853
## 1916 1916 1472
## 1917 1917 1366
## 1918 1918  833
## 1919 1919  218
## 1920 1920  114
## 1921 1921  413
## 1922 1922 1034
## 1923 1923   18
## 1924 1924 1209
## 1925 1925 1245
## 1926 1926  439
## 1927 1927 1349
## 1928 1928 1353
## 1929 1929  189
## 1930 1930  757
## 1931 1931    2
## 1932 1932  433
## 1933 1933  122
## 1934 1934  135
## 1935 1935  185
## 1936 1936  163
## 1937 1937  748
## 1938 1938  676
## 1939 1939   86
## 1940 1940  204
## 1941 1941  435
## 1942 1942  673
## 1943 1943 1120
## 1944 1944  378
## 1945 1945  803
## 1946 1946  397
## 1947 1947  415
## 1948 1948 1159
## 1949 1949 1395
```

```
## 1950 1950  741
## 1951 1951  221
## 1952 1952  309
## 1953 1953  967
## 1954 1954 1257
## 1955 1955  697
## 1956 1956   20
## 1957 1957  920
## 1958 1958 1144
## 1959 1959  540
## 1960 1960  968
## 1961 1961  779
## 1962 1962  127
## 1963 1963 1093
## 1964 1964   32
## 1965 1965  454
## 1966 1966  142
## 1967 1967 1103
## 1968 1968  718
## 1969 1969  521
## 1970 1970 1569
## 1971 1971  610
## 1972 1972 1385
## 1973 1973  865
## 1974 1974  377
## 1975 1975 1038
## 1976 1976  274
## 1977 1977 1258
## 1978 1978  640
## 1979 1979  817
## 1980 1980  692
## 1981 1981 1627
## 1982 1982  628
## 1983 1983   83
## 1984 1984 1279
## 1985 1985 1004
## 1986 1986 1248
## 1987 1987 1186
## 1988 1988  389
## 1989 1989 1115
## 1990 1990  215
## 1991 1991 1168
## 1992 1992  327
## 1993 1993 1255
## 1994 1994 1067
## 1995 1995 1752
## 1996 1996  283
## 1997 1997  216
## 1998 1998 1009
```

```
## 1999 1999 1405
## 2000 2000  740
```

```r
summary(kmeans.mydata)
```

```
##        V1               V2        
##  Min.   :   1.0   Min.   :   1.0  
##  1st Qu.: 500.8   1st Qu.: 500.8  
##  Median :1000.5   Median : 997.5  
##  Mean   :1000.5   Mean   : 998.6  
##  3rd Qu.:1500.2   3rd Qu.:1497.2  
##  Max.   :2000.0   Max.   :1997.0
```

```r
#Creating the sample
mysample <- sample(nrow(kmeans.mydata),300,replace = FALSE)
summary(mysample)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##     1.0   579.5  1074.0  1044.0  1557.0  1999.0
```

```r
km.outHW <- kmeans(mysample,centers=2,nstart=20, algorithm = "Hartigan-Wong")
km.outLl <- kmeans(mysample,centers=2,nstart=20, algorithm = "Lloyd")
```

Hartigan-Wong and Lloyd appear to give the same results, paying attention to Within Sum and Between/Total ratio, as well as the areas in the center of the graph where observations might slip from one class to another between algorithms. I'll use the default Hartigan Wong.

2.

Explain iter:max and algorithm parameters of kmeans function in R and run k-means on mysample data set where nstart = 35 and k = 2. Report total within squares error and within squares error for each cluster.

```r
set.seed(1234)
km.out2 <- kmeans(mysample,centers=2,nstart=35)
km.out2$cluster
```

```
##   [1] 1 2 2 2 1 1 1 2 1 1 1 2 2 2 2 2 2 2 1 1 2 1 2 2 2 1 1 2 1 2 2 2 2 1
1
##  [36] 2 2 1 1 1 2 1 2 2 1 1 1 1 1 2 2 2 2 2 1 2 2 2 2 2 1 1 1 2 1 2 2 1 2
2
##  [71] 2 2 1 1 1 1 1 1 1 2 1 1 1 2 1 2 1 1 2 1 1 1 1 2 2 2 2 2 2 2 1 2 2 2
1
## [106] 2 2 2 2 2 1 1 2 2 1 2 1 2 2 1 1 2 1 2 2 2 1 1 1 2 2 1 2 1 2 2 1 2 2
1
## [141] 2 2 2 1 2 2 2 2 2 2 2 1 2 2 1 2 1 1 1 1 1 1 2 1 2 2 1 1 1 1 2 1 2 2
1
## [176] 1 1 1 1 2 2 1 1 2 1 2 1 1 1 2 2 2 1 1 1 1 2 2 1 1 1 2 2 2 2 1 2 1 2
2
## [211] 1 1 2 1 2 1 2 1 2 2 2 2 1 1 1 2 2 2 1 1 2 2 2 1 1 1 2 1 1 2 1 1 1 2
2
```

```
## [246] 1 2 1 2 2 2 1 1 2 2 1 2 2 1 2 2 1 1 2 1 1 2 1 1 1 2 1 1 1 2 2 1 2 1
1
## [281] 2 1 2 2 2 2 2 1 1 1 2 1 2 1 1 1 2 2 1 2
```

```
#Total within-cluster sum of squares
km.out2$tot.withinss
```

```
## [1] 24318617
```

```
# within-cluster sum of squares for each cluster
km.out2$withinss
```

```
## [1] 11826520 12492097
```

```
km.out2
```

```
## K-means clustering with 2 clusters of sizes 144, 156
##
## Cluster means:
##        [,1]
## 1  518.4792
## 2 1530.0321
##
## Clustering vector:
##   [1] 1 2 2 2 1 1 1 2 1 1 1 2 2 2 2 2 2 2 1 1 2 1 2 2 2 1 1 2 1 2 2 2 2 1
1
##  [36] 2 2 1 1 1 2 1 2 2 1 1 1 1 1 2 2 2 2 2 1 2 2 2 2 2 1 1 1 2 1 2 2 1 2
2
##  [71] 2 2 1 1 1 1 1 1 1 2 1 1 1 2 1 2 1 1 2 1 1 1 1 2 2 2 2 2 2 2 1 2 2 2
1
## [106] 2 2 2 2 2 1 1 2 2 1 2 1 2 2 1 1 2 1 2 2 2 1 1 1 2 2 1 2 1 2 2 1 2 2
1
## [141] 2 2 2 1 2 2 2 2 2 2 2 1 2 2 1 2 1 1 1 1 1 1 2 1 2 2 1 1 1 1 2 1 2 2
1
## [176] 1 1 1 1 2 2 1 1 2 1 2 1 1 1 2 2 2 1 1 1 1 2 2 1 1 1 2 2 2 2 1 2 1 2
2
## [211] 1 1 2 1 2 1 2 1 2 2 2 2 1 1 1 2 2 2 1 1 2 2 2 1 1 1 2 1 1 2 1 1 1 2
2
## [246] 1 2 1 2 2 2 1 1 2 2 1 2 2 1 2 2 1 1 2 1 1 2 1 1 1 2 1 1 1 2 2 1 2 1
1
## [281] 2 1 2 2 2 2 2 1 1 1 2 1 2 1 1 1 2 2 1 2
##
## Within cluster sum of squares by cluster:
## [1] 11826520 12492097
##  (between_SS / total_SS =  75.9 %)
##
## Available components:
##
## [1] "cluster"      "centers"      "totss"        "withinss"
## [5] "tot.withinss" "betweenss"    "size"         "iter"
## [9] "ifault"
```

```
## km.out3
```

The itermax parameter sets the number of iterations the algorithm will perform. The algorithm selects one of at least four different algorithms to use. H The total within squares error and total squares errors are included for both clusters. Within sum of squares error indicates the total distance between each point in a variable and the center point of that variable. The between sum of squares indicates how far the two variables are from each other.

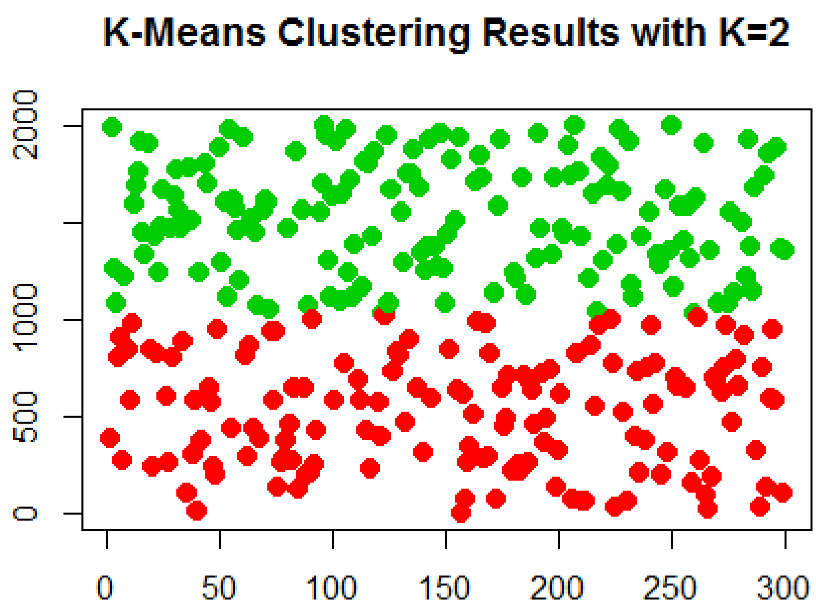
Total Within SS = 25,101,542

Within SS for each cluster = 13454956 11646587

3.

Make a plot of data points and color the observation according to the cluster labels obtained.

```
plot(mysample, col=(km.out2$cluster+1), main="K-Means Clustering Results with K=2",
     xlab="", ylab="", pch=20, cex=2)
```



4.

Run k-means on mysample data set where nstart = 35 and k = 4. Report total within squares error and within squares error for each cluster.

```
km.out4 <- kmeans(mysample,centers=4,nstart=35)
km.out4

## K-means clustering with 4 clusters of sizes 81, 76, 68, 75
##
## Cluster means:
##        [,1]
## 1 1765.0370
## 2 1272.8158
## 3  254.1324
## 4  751.5067
##
## Clustering vector:
##   [1] 3 1 2 2 4 4 3 2 4 4 4 1 1 1 1 2 2 1 4 3 2 4 2 2 1 4 3 2 4 1 1 1 2 4
3
##  [36] 1 2 3 4 3 2 3 1 1 4 4 3 3 4 1 2 1 2 1 3 1 1 2 2 1 4 3 4 1 3 2 2 3 1
1
##  [71] 1 2 4 4 4 3 3 3 3 2 3 3 4 1 3 1 4 3 2 3 4 3 3 1 1 1 1 2 2 1 4 1 2 1
4
## [106] 1 2 1 2 2 4 4 2 1 3 1 3 2 1 4 3 2 2 1 2 1 4 4 4 1 2 3 1 4 1 1 4 1 2
3
## [141] 2 2 1 4 2 2 1 1 2 2 2 4 1 2 4 1 3 4 3 3 3 4 1 4 1 1 3 4 3 4 2 3 1 1
4
## [176] 3 3 4 3 2 2 3 3 1 4 2 3 4 3 2 1 2 4 3 3 4 2 1 3 3 4 2 2 1 1 3 1 4 1
2
## [211] 3 3 2 4 1 4 2 4 1 2 1 1 4 4 3 2 1 1 4 3 1 2 2 3 4 3 2 3 4 1 4 4 4 2
2
## [246] 3 1 3 2 1 2 4 4 1 2 4 1 2 3 2 1 4 3 1 3 3 2 3 4 4 2 4 4 4 2 1 3 2 4
4
## [281] 2 4 2 1 2 2 1 3 3 4 1 3 1 4 4 4 1 2 3 2
##
## Within cluster sum of squares by cluster:
## [1] 1674599 1579109 1255556 1498017
##  (between_SS / total_SS =  94.0 %)
##
## Available components:
##
## [1] "cluster"      "centers"      "totss"        "withinss"
## [5] "tot.withinss" "betweenss"    "size"         "iter"
## [9] "ifault"

km.out4$tot.withinss

## [1] 6007281

km.out4$withinss

## [1] 1674599 1579109 1255556 1498017
```

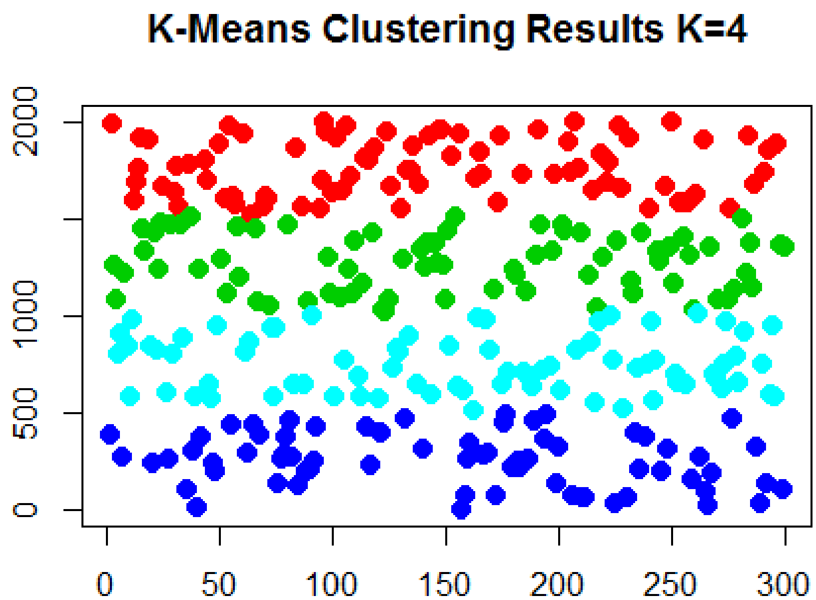
The Within sum of squares by cluster is as follows: - C1: 1245029 - C2: 1343345 - C3: 980807 - C4: 2702241

Total within squares error is 6,271,425

5.

Make a plot of data points and color the observation according to the cluster labels obtained.

```
plot(mysample, col=(km.out4$cluster+1), main="K-Means Clustering Results 
K=4",
     xlab="", ylab="", pch=20, cex=2)
```



##6. Compare (2) and (4). With a higher number of clusters, we obviously have a lower total within sum of square error, as there are more centroids, thus a shorter distance and less overall error. There is an interesting function in the text that describes how to determine the optimum number of clusters between two and six:

```
library(cluster)

set.seed(1234)
d <- dist(mydata[,-5])
avgS <- c()

for(k in 2:6) {
  cl <- kmeans(mydata[,-5],centers=k,iter.max=200)
  s <- silhouette(cl$cluster,d)
  avgS <- c(avgS,mean(s[,3]))
```

```
}
data.frame(nClus=2:6,Silh=avgS)
```

```
##   nClus      Silh
## 1     2 0.3724321
## 2     3 0.4216897
## 3     4 0.3795938
## 4     5 0.3583321
## 5     6 0.3645822
```

This appears to indicate that the optimum number of clusters is 3, and it appears that four clusters is slightly better than two clusters in this case.

4

1.

In the listing below, which line number eectively reads the data into an R data.frame?

This is the correct method for reading this data into R: `teach <???? read.table( f i l e ="c:/R/rainfalldataraw.txt", header=TRUE, sep=",")`

```
teach <-
read.table("C:\\Users\\khickman\\Desktop\\Personal\\IUMSDS\\AppliedDataMining
\\Midterm\\rainfalldataraw.txt", header = TRUE, sep=",")
teach
```

```
##    SEEDED SEASON    A      B    C      D    E
## 1       S AUTUMN 1.69  3.730 1.65   1.80 3.33
## 2       U AUTUMN 0.74  0.780 1.09   0.79 1.59
## 3       S WINTER 0.81  0.860 2.39   0.36 2.06
## 4       U WINTER 1.44  2.010 2.96   1.27 4.05
## 5       S WINTER 2.48  4.610 4.16   2.16 6.00
## 6       U WINTER 0.84  2.390 2.76   0.87 4.17
## 7       U WINTER 0.37  1.370 1.08   0.85 3.45
## 8       S WINTER 0.37  0.840 0.26   0.47 0.90
## 9       U SPRING 1.33  2.310 2.53   1.08 3.65
## 10      S SPRING 3.38  5.560 2.76   3.10 5.06
## 11      S SPRING 0.69  1.460 1.07   0.64 1.95
## 12      U SPRING 1.42  2.790 1.42   1.08 1.22
## 13      S SPRING 0.44  1.050 0.24   0.44 0.94
## 14      U SPRING 0.76  1.240 0.70   0.67 0.94
## 15      S SUMMER 1.13  2.280 0.97   1.66 2.21
## 16      U SUMMER 0.88  1.580 1.06   1.13 1.46
## 17      S SUMMER 0.17  0.550 0.13   0.27 0.35
## 18      U SUMMER 0.25  0.770 0.10   0.30 0.34
## 19      U SUMMER 0.78  1.450 0.38   0.58 0.67
## 20      S SUMMER 0.40  0.340 0.45   0.43 0.44
## 21      S AUTUMN 0.52  0.790 0.42   0.47 0.53
```

```
## 22        U AUTUMN 2.73  2.090 2.24   4.02 2.52
## 23        U AUTUMN 0.90  2.450 0.52   1.32 2.18
## 24        S AUTUMN 1.62  2.540 0.94   1.59 1.73
## 25        U AUTUMN 0.93  2.110 1.19   0.85 2.31
## 26        S AUTUMN 0.63  1.310 0.76   0.71 1.28
## 27        S WINTER 0.42  1.230 0.13   0.59 0.91
## 28        U WINTER 0.64  0.430 1.50   0.24 1.15
## 29        U WINTER 0.30  0.690 1.03   0.22 1.88
## 30        S WINTER 0.88  1.320 1.87   0.58 2.97
## 31          WINTER 0.76  1.250 1.85   1.36 2.17
## 32        S WINTER 1.25  1.000 2.04   0.71 2.22
## 33        U WINTER 1.08  0.990 1.44   1.00 1.64
## 34        S WINTER 1.11  0.800 1.46   1.48 0.40
## 35        S SPRING 3.43  2.550 5.08   1.77 4.20
## 36        U SPRING 0.54  0.430 0.66   0.73 0.91
## 37        S SPRING 0.39  0.440 0.49   0.55 0.51
## 38        U SPRING 2.53  3.180 3.27   2.68 3.60
## 39        U SPRING 0.81  0.890 1.33   0.43 2.18
## 40        S SPRING 0.39  1.220 0.25   0.46 0.89
## 41        S SUMMER 0.86  1.240 0.69   0.49 0.69
## 42        U SUMMER 2.16  2.290 2.12   0.95 1.82
## 43        U SPRING 1.70  2.180 1.45   1.47 2.20
## 44        S SPRING 1.22  2.000 2.13   1.13 2.33
## 45        S SPRING 0.07  0.220 0.02   0.08 0.24
## 46        U SPRING 0.49  1.070 0.36   0.87 0.57
## 47        U SPRING 0.71  1.730 0.72   0.99 0.98
## 48        S SPRING 1.67  3.460 1.02   1.89 2.47
## 49        U SUMMER 0.73  1.510 0.18   1.42 0.71
## 50        S SUMMER 1.79  3.130 1.83   1.82 3.11
## 51        U SUMMER 0.19  1.050 0.08   0.40 0.57
## 52        S SUMMER 0.00  0.150 0.00   0.04 0.04
## 53        S SUMMER 0.44  0.890 0.83   0.38 0.70
## 54        U SUMMER 0.31  1.150 0.01   0.44 0.66
## 55        S SUMMER 0.96  0.880 2.65   0.85 1.48
## 56        U SUMMER 1.04  1.200 1.27   1.39 1.20
## 57        S AUTUMN 0.05  0.060 0.01 200.30 0.10
## 58        U AUTUMN 0.04  0.200 0.35   0.75 0.20
## 59        S AUTUMN 1.83  2.930 1.80   1.62 3.02
## 60        U AUTUMN 2.24  2.170 4.44   1.05 3.59
## 61        S AUTUMN 2.50  3.990 2.84   2.44 4.48
## 62        U AUTUMN 1.10  1.710 2.05   1.30 4.04
## 63        S AUTUMN 1.83  3.870 3.01   1.66 4.56
## 64        U AUTUMN 1.41 -0.034 2.58   1.21 3.95
## 65        U WINTER 0.74  1.360 2.22   0.61 2.68
## 66        S WINTER 1.09  3.560 0.07   2.26 2.08
## 67        S WINTER 0.79  1.430 1.62   1.16 2.87
## 68        U WINTER 4.06  6.710 4.34   3.29 6.40
## 69        U WINTER 0.40  0.640 1.03   0.58 1.77
## 70        S WINTER 0.76  1.830 1.50   0.41 2.56
## 71        S SPRING 1.53  3.620 1.52   1.62 2.86
```

```
## 72       U SPRING 0.56  2.880 0.37   1.25 1.74
## 73       U SPRING 1.74  3.450 2.14   1.00 4.39
## 74       S SPRING 1.59  3.190 2.36   1.53 3.03
## 75       U SPRING 1.91  4.740 1.71   2.03 3.24
## 76       S SPRING 2.09  5.230 2.12   2.77 4.44
## 77       U SUMMER 1.59  3.920 1.38   2.11 3.01
## 78       S SUMMER 0.66  2.220 0.21   1.41 0.80
## 79       U SUMMER 0.68  0.420 0.48   0.59 0.68
## 80       S SUMMER 0.46  1.080 0.01   0.65 0.48
## 81       S SUMMER 0.22  0.620 0.15   0.13 0.42
## 82       U SUMMER 1.11  1.700 1.32   0.57 1.54
## 83       S SUMMER 1.76  1.190 2.26   1.04 1.27
## 84       U SUMMER 5.12  5.250 5.95   3.97 5.37
## 85       U AUTUMN 0.12  0.600 0.19   0.28 0.70
## 86       S AUTUMN 0.37     NA 0.31   0.23 0.83
## 87       S AUTUMN 4.97  3.030 1.44   3.14 0.86
## 88       U AUTUMN 0.57  1.530 0.30   0.72 1.38
## 89       S AUTUMN 0.13  0.540 0.11   0.14 0.58
## 90       U AUTUMN 2.47  4.700 3.66   1.84 5.36
## 91       U AUTUMN 1.01  2.320 1.14   0.81 2.09
## 92       S AUTUMN 0.55  1.130 1.30     NA 2.45
## 93       S WINTER 0.24  0.610 0.05   0.38 0.90
## 94       U WINTER 2.36  1.150 1.84   1.73 2.33
## 95       S WINTER 2.35  4.290 4.24   1.67 5.48
## 96       U WINTER 2.23  4.300 1.99   1.90 3.67
## 97       U WINTER 1.16  3.060 2.44   1.52 4.01
## 98       S WINTER 1.63  3.310 2.21   2.36 3.25
## 99       S WINTER 1.08  3.170 0.80   2.25 2.79
## 100      U WINTER 6.00  6.150 9.42   3.60 7.84
## 101      S SPRING 2.67  6.930   NA   3.03 6.39
## 102      U SPRING 0.36  0.150 0.00   0.19 0.06
## 103      S SPRING 0.58  1.410 0.96   0.64 1.24
## 104      U SPRING 1.36  3.430 1.38   1.86 2.91
## 105      S SPRING 1.17  1.650 1.22   2.28 1.58
## 106      U SPRING 2.37  1.940 2.46   2.47 2.39
## 107      S        0.02  0.080 0.05   0.02 0.09
## 108      U SPRING 0.92  2.090 0.61   0.87 1.35
## 109      S SPRING 3.43  2.550 5.08   1.77 4.20
```

```r
str(teach)
```

```
## 'data.frame':    109 obs. of  7 variables:
##  $ SEEDED: Factor w/ 3 levels "","S","U": 2 3 2 3 2 3 3 2 3 2 ...
##  $ SEASON: Factor w/ 5 levels "","AUTUMN","SPRING",..: 2 2 5 5 5 5 5 5 3 3
...
##  $ A     : num  1.69 0.74 0.81 1.44 2.48 0.84 0.37 0.37 1.33 3.38 ...
##  $ B     : num  3.73 0.78 0.86 2.01 4.61 2.39 1.37 0.84 2.31 5.56 ...
##  $ C     : num  1.65 1.09 2.39 2.96 4.16 2.76 1.08 0.26 2.53 2.76 ...
##  $ D     : num  1.8 0.79 0.36 1.27 2.16 0.87 0.85 0.47 1.08 3.1 ...
##  $ E     : num  3.33 1.59 2.06 4.05 6 4.17 3.45 0.9 3.65 5.06 ...
```

2.

Give a select operation on the data.frame that gives the rows whose E variable values are greater than 4, but less than 5.

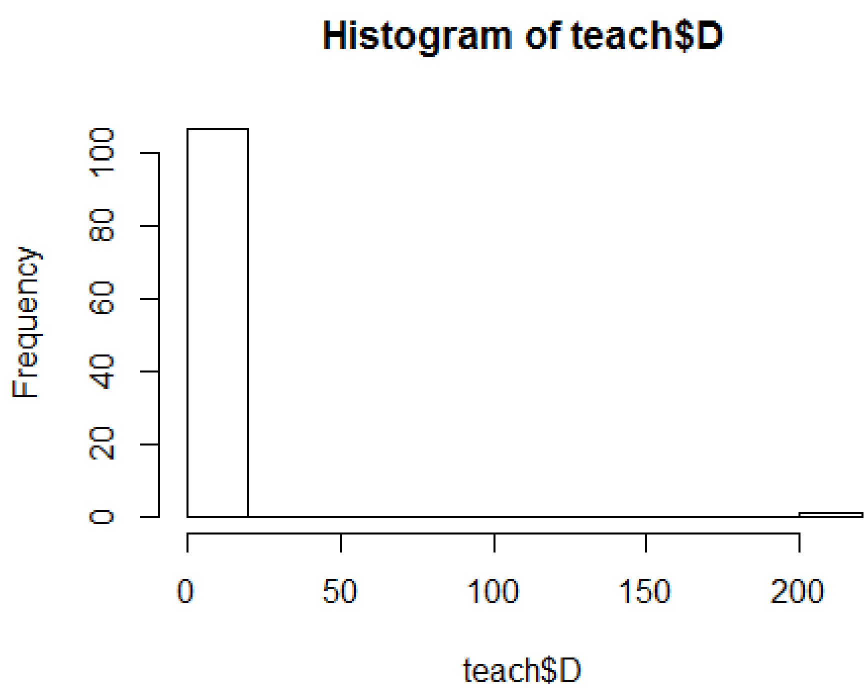
```
??subset
teach.sub <- teach[which(teach$E > 4 & teach$E < 5),]
teach.sub
```

```
##     SEEDED SEASON    A    B    C    D    E
## 4        U WINTER 1.44 2.01 2.96 1.27 4.05
## 6        U WINTER 0.84 2.39 2.76 0.87 4.17
## 35       S SPRING 3.43 2.55 5.08 1.77 4.20
## 61       S AUTUMN 2.50 3.99 2.84 2.44 4.48
## 62       U AUTUMN 1.10 1.71 2.05 1.30 4.04
## 63       S AUTUMN 1.83 3.87 3.01 1.66 4.56
## 73       U SPRING 1.74 3.45 2.14 1.00 4.39
## 76       S SPRING 2.09 5.23 2.12 2.77 4.44
## 97       U WINTER 1.16 3.06 2.44 1.52 4.01
## 109      S SPRING 3.43 2.55 5.08 1.77 4.20
```

3.

Give the code that produces the histogram of variable D.

```
hist(teach$D)
```



4.

How many tuples (or records) are in the data?

```
summary(teach)
```

```
##  SEEDED    SEASON         A                B                C
##   : 1          : 1   Min.   :0.000   Min.   :-0.034   Min.   :0.000
##  S:55   AUTUMN:24   1st Qu.:0.520   1st Qu.: 0.890   1st Qu.:0.410
##  U:53   SPRING:32   Median :0.920   Median : 1.555   Median :1.285
##         SUMMER:24   Mean   :1.253   Mean   : 2.036   Mean   :1.528
##         WINTER:28   3rd Qu.:1.690   3rd Qu.: 2.955   3rd Qu.:2.132
##                     Max.   :6.000   Max.   : 6.930   Max.   :9.420
##                                     NA's   :1        NA's   :1
##       D                  E
##  Min.   :  0.0200   Min.   :0.040
##  1st Qu.:  0.5775   1st Qu.:0.890
##  Median :  1.0200   Median :1.950
##  Mean   :  3.0679   Mean   :2.211
##  3rd Qu.:  1.7400   3rd Qu.:3.110
##  Max.   :200.3000   Max.   :7.840
##  NA's   :1
```

There are 109 observations of 7 variables.

5.

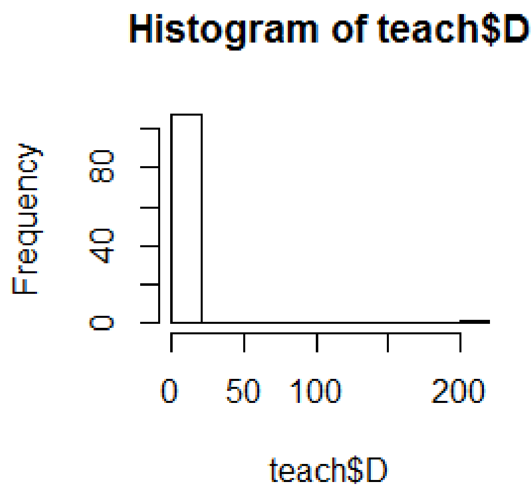
Identify the data that is either missing or likely corrupted: Most of the variables here have at least one missing or corrupted value. `Seeded, Season, B, C, and D` all have missing or NA values.

6.

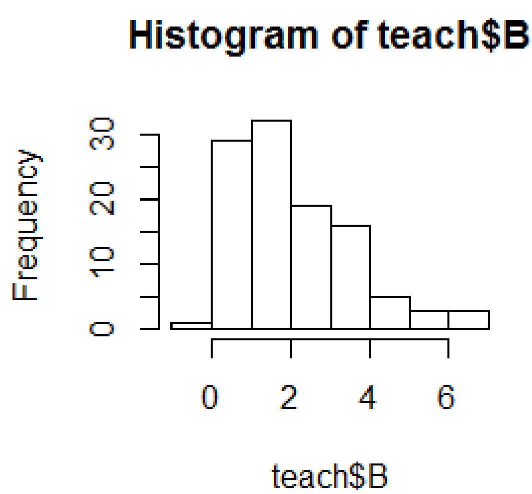
Preprocess the data, addressing the problems above and save the file as rainfixed.txt as a .csv file. Explain explicitly what you have done in preprocessing this file.

Since the number of missing values are relatively small, we can either remove the cases with NA, or we can impute the values using a statistic of centrality like mean. We're dealing with two types of variables here as well so we might use two different methods. Examining the distribution of each variable:

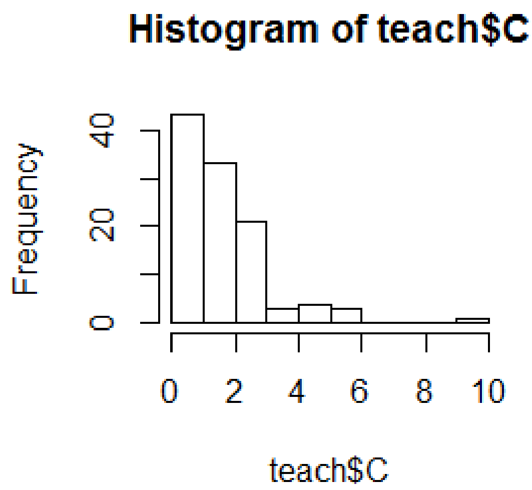
```
hist(teach$D)
```




```r
hist(teach$B)
```




```r
hist(teach$C)
```



Since none of the three variables is normally distributed, the median is the preferred imputation statistic.

```
na.var <- teach[rowSums(is.na(teach)) > 0,]
na.var

##     SEEDED SEASON    A    B    C    D    E
## 86       S AUTUMN 0.37   NA 0.31 0.23 0.83
## 92       S AUTUMN 0.55 1.13 1.30   NA 2.45
## 101      S SPRING 2.67 6.93   NA 3.03 6.39

teach[86, "B"] <- median(teach$B, na.rm = TRUE)
na.var

##     SEEDED SEASON    A    B    C    D    E
## 86       S AUTUMN 0.37   NA 0.31 0.23 0.83
## 92       S AUTUMN 0.55 1.13 1.30   NA 2.45
## 101      S SPRING 2.67 6.93   NA 3.03 6.39

teach[92,"D"] <- median(teach$D, na.rm = TRUE)
na.var

##     SEEDED SEASON    A    B    C    D    E
## 86       S AUTUMN 0.37   NA 0.31 0.23 0.83
## 92       S AUTUMN 0.55 1.13 1.30   NA 2.45
## 101      S SPRING 2.67 6.93   NA 3.03 6.39

teach[101,"C"] <- median(teach$C, na.rm = TRUE)

na.var

##     SEEDED SEASON    A    B    C    D    E
## 86       S AUTUMN 0.37   NA 0.31 0.23 0.83
```

```
## 92       S AUTUMN 0.55 1.13 1.30   NA 2.45
## 101      S SPRING 2.67 6.93   NA 3.03 6.39
```

```
summary(teach)
```

```
##  SEEDED    SEASON          A                B                 C        
##   : 1         : 1   Min.   :0.000   Min.   :-0.034   Min.   :0.000  
##  S:55   AUTUMN:24   1st Qu.:0.520   1st Qu.: 0.890   1st Qu.:0.420  
##  U:53   SPRING:32   Median :0.920   Median : 1.555   Median :1.285  
##         SUMMER:24   Mean   :1.253   Mean   : 2.032   Mean   :1.526  
##         WINTER:28   3rd Qu.:1.690   3rd Qu.: 2.930   3rd Qu.:2.130  
##                     Max.   :6.000   Max.   : 6.930   Max.   :9.420  
##       D                  E        
##  Min.   :  0.020   Min.   :0.040  
##  1st Qu.:  0.580   1st Qu.:0.890  
##  Median :  1.020   Median :1.950  
##  Mean   :  3.049   Mean   :2.211  
##  3rd Qu.:  1.730   3rd Qu.:3.110  
##  Max.   :200.300   Max.   :7.840
```

It appears that all of our unknown numeric variables have been replaced. Now on to the categorical variables.

```
teach$SEEDED
```

```
##   [1] S U S U S U U S U S S U S U S U S U U S S U U S U S S U U S   S U S
S
##  [36] U S U U S S U U S S U U S U S U S S U S U S U S U S U S U U S S U U
S
##  [71] S U U S U S U S U S S U S U U S S U S U U S S U S U U S S U S U S U
S
## [106] U S U S
## Levels:  S U
```

```
teach$SEASON
```

```
##   [1] AUTUMN AUTUMN WINTER WINTER WINTER WINTER WINTER WINTER SPRING
SPRING
##  [11] SPRING SPRING SPRING SPRING SUMMER SUMMER SUMMER SUMMER SUMMER
SUMMER
##  [21] AUTUMN AUTUMN AUTUMN AUTUMN AUTUMN AUTUMN WINTER WINTER WINTER
WINTER
##  [31] WINTER WINTER WINTER WINTER SPRING SPRING SPRING SPRING SPRING
SPRING
##  [41] SUMMER SUMMER SPRING SPRING SPRING SPRING SPRING SPRING SUMMER
SUMMER
##  [51] SUMMER SUMMER SUMMER SUMMER SUMMER SUMMER AUTUMN AUTUMN AUTUMN
AUTUMN
##  [61] AUTUMN AUTUMN AUTUMN AUTUMN WINTER WINTER WINTER WINTER WINTER
WINTER
##  [71] SPRING SPRING SPRING SPRING SPRING SPRING SUMMER SUMMER SUMMER
```

```
SUMMER
##  [81] SUMMER SUMMER SUMMER SUMMER AUTUMN AUTUMN AUTUMN AUTUMN AUTUMN
AUTUMN
##  [91] AUTUMN AUTUMN WINTER WINTER WINTER WINTER WINTER WINTER WINTER
WINTER
## [101] SPRING SPRING SPRING SPRING SPRING SPRING        SPRING SPRING
## Levels:  AUTUMN SPRING SUMMER WINTER
```

```
teach.clean <- teach[-c(31, 107), ]
summary(teach.clean)
```

```
##  SEEDED    SEASON         A                B                 C
##   : 0          : 0   Min.   :0.000   Min.   :-0.034   Min.   :0.000
##  S:54   AUTUMN:24   1st Qu.:0.530   1st Qu.: 0.940   1st Qu.:0.435
##  U:53   SPRING:32   Median :0.930   Median : 1.580   Median :1.285
##         SUMMER:24   Mean   :1.269   Mean   : 2.057   Mean   :1.537
##         WINTER:27   3rd Qu.:1.695   3rd Qu.: 2.980   3rd Qu.:2.135
##                     Max.   :6.000   Max.   : 6.930   Max.   :9.420
##        D                 E
##  Min.   :  0.040   Min.   :0.040
##  1st Qu.:  0.580   1st Qu.:0.895
##  Median :  1.020   Median :1.950
##  Mean   :  3.093   Mean   :2.231
##  3rd Qu.:  1.750   3rd Qu.:3.175
##  Max.   :200.300   Max.   :7.840
```

We only eliminated two rows of data where the `SEEDED` and `SEASON` variables were missing, while imputing values to three other rows using median.

Now, we can write the resulting matrix to a .csv file.

```
write.csv(teach.clean, file = "rainfixed.txt")
```

7.

Using any techniques you've learned, answer this question to a policy maker... I think we can dive in and explore the average rainfall for Seeded and Unseeded areas both as a group and individually. I want to examine this alternative hypothesis first - "there is a difference in rainfall bewteen seeded and unseeded areas" (null is that that there is no difference).

First, Let's create a new variable that averages the area rainfall for each row Then I can set up my variables Seeded and Unseeded, which is our variable of interest.

```
teach$avg <- rowMeans(teach[,3:7], na.rm = FALSE, dims = 1)
teach$avg
```

```
##   [1]  2.4400  0.9980  1.2960  2.3460  3.8820  2.2060  1.4240  0.5680
##   [9]  2.1800  3.9720  1.1620  1.5860  0.6220  0.8620  1.6500  1.2220
##  [17]  0.2940  0.3520  0.7720  0.4120  0.5460  2.7200  1.4740  1.6840
##  [25]  1.4780  0.9380  0.6560  0.7920  0.8240  1.5240  1.4780  1.4440
##  [33]  1.2300  1.0500  3.4060  0.6540  0.4760  3.0520  1.1280  0.6420
```

```
##  [41]  0.7940  1.8680  1.8000  1.7620  0.1260  0.6720  1.0260  2.1020
##  [49]  0.9100  2.3360  0.4580  0.0460  0.6480  0.5140  1.3640  1.2200
##  [57] 40.1040  0.3080  2.2400  2.6980  3.2500  2.0400  2.9860  1.8232
##  [65]  1.5220  1.8120  1.5740  4.9600  0.8840  1.4120  2.2300  1.3600
##  [73]  2.5440  2.3400  2.7260  3.3300  2.4020  1.0600  0.5700  0.5360
##  [81]  0.3080  1.2480  1.5040  5.1320  0.3780  0.6590  2.6880  0.9000
##  [89]  0.3000  3.6060  1.4740  1.2900  0.4360  1.8820  3.6060  2.8180
##  [97]  2.4380  2.5520  2.0180  6.6020  4.0610  0.1520  0.9660  2.1880
## [105]  1.5800  2.3260  0.0520  1.1680  3.4060
```

```
seeded <- subset(teach, SEEDED=="S")
seeded
```

```
##     SEEDED SEASON    A     B     C      D    E    avg
## 1        S AUTUMN 1.69 3.730 1.650   1.80 3.33  2.440
## 3        S WINTER 0.81 0.860 2.390   0.36 2.06  1.296
## 5        S WINTER 2.48 4.610 4.160   2.16 6.00  3.882
## 8        S WINTER 0.37 0.840 0.260   0.47 0.90  0.568
## 10       S SPRING 3.38 5.560 2.760   3.10 5.06  3.972
## 11       S SPRING 0.69 1.460 1.070   0.64 1.95  1.162
## 13       S SPRING 0.44 1.050 0.240   0.44 0.94  0.622
## 15       S SUMMER 1.13 2.280 0.970   1.66 2.21  1.650
## 17       S SUMMER 0.17 0.550 0.130   0.27 0.35  0.294
## 20       S SUMMER 0.40 0.340 0.450   0.43 0.44  0.412
## 21       S AUTUMN 0.52 0.790 0.420   0.47 0.53  0.546
## 24       S AUTUMN 1.62 2.540 0.940   1.59 1.73  1.684
## 26       S AUTUMN 0.63 1.310 0.760   0.71 1.28  0.938
## 27       S WINTER 0.42 1.230 0.130   0.59 0.91  0.656
## 30       S WINTER 0.88 1.320 1.870   0.58 2.97  1.524
## 32       S WINTER 1.25 1.000 2.040   0.71 2.22  1.444
## 34       S WINTER 1.11 0.800 1.460   1.48 0.40  1.050
## 35       S SPRING 3.43 2.550 5.080   1.77 4.20  3.406
## 37       S SPRING 0.39 0.440 0.490   0.55 0.51  0.476
## 40       S SPRING 0.39 1.220 0.250   0.46 0.89  0.642
## 41       S SUMMER 0.86 1.240 0.690   0.49 0.69  0.794
## 44       S SPRING 1.22 2.000 2.130   1.13 2.33  1.762
## 45       S SPRING 0.07 0.220 0.020   0.08 0.24  0.126
## 48       S SPRING 1.67 3.460 1.020   1.89 2.47  2.102
## 50       S SUMMER 1.79 3.130 1.830   1.82 3.11  2.336
## 52       S SUMMER 0.00 0.150 0.000   0.04 0.04  0.046
## 53       S SUMMER 0.44 0.890 0.830   0.38 0.70  0.648
## 55       S SUMMER 0.96 0.880 2.650   0.85 1.48  1.364
## 57       S AUTUMN 0.05 0.060 0.010 200.30 0.10 40.104
## 59       S AUTUMN 1.83 2.930 1.800   1.62 3.02  2.240
## 61       S AUTUMN 2.50 3.990 2.840   2.44 4.48  3.250
## 63       S AUTUMN 1.83 3.870 3.010   1.66 4.56  2.986
## 66       S WINTER 1.09 3.560 0.070   2.26 2.08  1.812
## 67       S WINTER 0.79 1.430 1.620   1.16 2.87  1.574
## 70       S WINTER 0.76 1.830 1.500   0.41 2.56  1.412
## 71       S SPRING 1.53 3.620 1.520   1.62 2.86  2.230
```

```
## 74       S SPRING 1.59 3.190 2.360   1.53 3.03  2.340
## 76       S SPRING 2.09 5.230 2.120   2.77 4.44  3.330
## 78       S SUMMER 0.66 2.220 0.210   1.41 0.80  1.060
## 80       S SUMMER 0.46 1.080 0.010   0.65 0.48  0.536
## 81       S SUMMER 0.22 0.620 0.150   0.13 0.42  0.308
## 83       S SUMMER 1.76 1.190 2.260   1.04 1.27  1.504
## 86       S AUTUMN 0.37 1.555 0.310   0.23 0.83  0.659
## 87       S AUTUMN 4.97 3.030 1.440   3.14 0.86  2.688
## 89       S AUTUMN 0.13 0.540 0.110   0.14 0.58  0.300
## 92       S AUTUMN 0.55 1.130 1.300   1.02 2.45  1.290
## 93       S WINTER 0.24 0.610 0.050   0.38 0.90  0.436
## 95       S WINTER 2.35 4.290 4.240   1.67 5.48  3.606
## 98       S WINTER 1.63 3.310 2.210   2.36 3.25  2.552
## 99       S WINTER 1.08 3.170 0.800   2.25 2.79  2.018
## 101      S SPRING 2.67 6.930 1.285   3.03 6.39  4.061
## 103      S SPRING 0.58 1.410 0.960   0.64 1.24  0.966
## 105      S SPRING 1.17 1.650 1.220   2.28 1.58  1.580
## 107      S        0.02 0.080 0.050   0.02 0.09  0.052
## 109      S SPRING 3.43 2.550 5.080   1.77 4.20  3.406
```

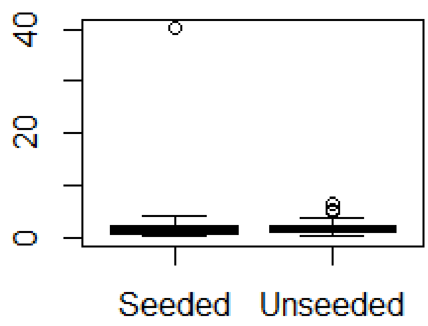
```
unseeded <- subset(teach, SEEDED=="U")
unseeded
```

```
##     SEEDED SEASON    A      B    C    D    E    avg
## 2        U AUTUMN 0.74  0.780 1.09 0.79 1.59 0.9980
## 4        U WINTER 1.44  2.010 2.96 1.27 4.05 2.3460
## 6        U WINTER 0.84  2.390 2.76 0.87 4.17 2.2060
## 7        U WINTER 0.37  1.370 1.08 0.85 3.45 1.4240
## 9        U SPRING 1.33  2.310 2.53 1.08 3.65 2.1800
## 12       U SPRING 1.42  2.790 1.42 1.08 1.22 1.5860
## 14       U SPRING 0.76  1.240 0.70 0.67 0.94 0.8620
## 16       U SUMMER 0.88  1.580 1.06 1.13 1.46 1.2220
## 18       U SUMMER 0.25  0.770 0.10 0.30 0.34 0.3520
## 19       U SUMMER 0.78  1.450 0.38 0.58 0.67 0.7720
## 22       U AUTUMN 2.73  2.090 2.24 4.02 2.52 2.7200
## 23       U AUTUMN 0.90  2.450 0.52 1.32 2.18 1.4740
## 25       U AUTUMN 0.93  2.110 1.19 0.85 2.31 1.4780
## 28       U WINTER 0.64  0.430 1.50 0.24 1.15 0.7920
## 29       U WINTER 0.30  0.690 1.03 0.22 1.88 0.8240
## 33       U WINTER 1.08  0.990 1.44 1.00 1.64 1.2300
## 36       U SPRING 0.54  0.430 0.66 0.73 0.91 0.6540
## 38       U SPRING 2.53  3.180 3.27 2.68 3.60 3.0520
## 39       U SPRING 0.81  0.890 1.33 0.43 2.18 1.1280
## 42       U SUMMER 2.16  2.290 2.12 0.95 1.82 1.8680
## 43       U SPRING 1.70  2.180 1.45 1.47 2.20 1.8000
## 46       U SPRING 0.49  1.070 0.36 0.87 0.57 0.6720
## 47       U SPRING 0.71  1.730 0.72 0.99 0.98 1.0260
## 49       U SUMMER 0.73  1.510 0.18 1.42 0.71 0.9100
## 51       U SUMMER 0.19  1.050 0.08 0.40 0.57 0.4580
## 54       U SUMMER 0.31  1.150 0.01 0.44 0.66 0.5140
```

```
## 56       U SUMMER 1.04  1.200 1.27 1.39 1.20 1.2200
## 58       U AUTUMN 0.04  0.200 0.35 0.75 0.20 0.3080
## 60       U AUTUMN 2.24  2.170 4.44 1.05 3.59 2.6980
## 62       U AUTUMN 1.10  1.710 2.05 1.30 4.04 2.0400
## 64       U AUTUMN 1.41 -0.034 2.58 1.21 3.95 1.8232
## 65       U WINTER 0.74  1.360 2.22 0.61 2.68 1.5220
## 68       U WINTER 4.06  6.710 4.34 3.29 6.40 4.9600
## 69       U WINTER 0.40  0.640 1.03 0.58 1.77 0.8840
## 72       U SPRING 0.56  2.880 0.37 1.25 1.74 1.3600
## 73       U SPRING 1.74  3.450 2.14 1.00 4.39 2.5440
## 75       U SPRING 1.91  4.740 1.71 2.03 3.24 2.7260
## 77       U SUMMER 1.59  3.920 1.38 2.11 3.01 2.4020
## 79       U SUMMER 0.68  0.420 0.48 0.59 0.68 0.5700
## 82       U SUMMER 1.11  1.700 1.32 0.57 1.54 1.2480
## 84       U SUMMER 5.12  5.250 5.95 3.97 5.37 5.1320
## 85       U AUTUMN 0.12  0.600 0.19 0.28 0.70 0.3780
## 88       U AUTUMN 0.57  1.530 0.30 0.72 1.38 0.9000
## 90       U AUTUMN 2.47  4.700 3.66 1.84 5.36 3.6060
## 91       U AUTUMN 1.01  2.320 1.14 0.81 2.09 1.4740
## 94       U WINTER 2.36  1.150 1.84 1.73 2.33 1.8820
## 96       U WINTER 2.23  4.300 1.99 1.90 3.67 2.8180
## 97       U WINTER 1.16  3.060 2.44 1.52 4.01 2.4380
## 100      U WINTER 6.00  6.150 9.42 3.60 7.84 6.6020
## 102      U SPRING 0.36  0.150 0.00 0.19 0.06 0.1520
## 104      U SPRING 1.36  3.430 1.38 1.86 2.91 2.1880
## 106      U SPRING 2.37  1.940 2.46 2.47 2.39 2.3260
## 108      U SPRING 0.92  2.090 0.61 0.87 1.35 1.1680
```

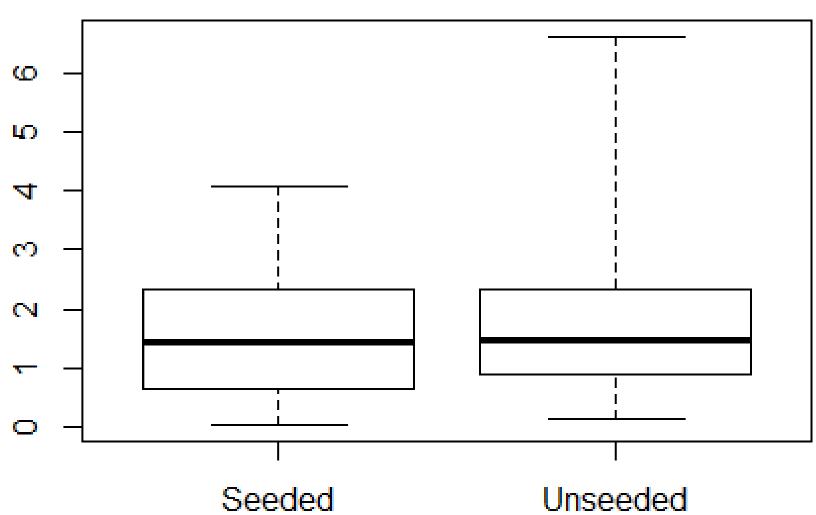
Seeded has 52 observations and Unseeded has 53. This is good, as we have a fair class balance between the two. Let's compare the averages for Seeded A and Unseeded A using a boxplot.

```r
boxplot(seeded$avg, unseeded$avg, names=c("Seeded","Unseeded"))
```



Looks like there's one outlier in the seeded that could be raising the average of all the data for that category. Let's filter it out and continue on. Since all but one of the values in our variables are below 10, we'll set the filter at less than 10.

```
seededleq10 <- subset(seeded, avg<10)
unseededleq10 <- subset(unseeded, avg<10)
boxplot(seededleq10$avg, unseededleq10$avg, range=0, names=c("Seeded",
"Unseeded"))
```



There appears to be no significant difference in the median rainfall, especially when controlling for outliers, which can greatly skew a statistic of centrality like the mean. Thus, we could not reject the null hypothesis here. I would advise the policy maker that cloud-seeding does not appear to work based on the available data. In fact, unseeded areas seem to have slightly higher rainfall on average.

We could further explore statistical measures such as Welch's t-test, compare the differences in mean, and create p-values, and actually test the null vs. alternative. We would probably get the same result.

We could additionally explore and compare average rainfall between areas, between seasons, etc. to find any interesting trends or correlations. I would use k-means, or random forest.

Problem 5

1.

Assume four pieces of data x1 = (.5; 2000;????100); x2 = (:2; 3000;????200); x3 = (4; 4000;????100); x4 = (:14; 4400;????140). You've been hired to datamine this data using Euclidean distance. How would you preprocess this before datamining and explain why. What are the two closest data points?

Let's create the variables:

```
x1 <- c(.5, 2000, 100)
x2 <- c(.2, 300, 200)
x3 <- c(4, 4000, 100)
x4 <- c(.14, 4400, 140)
```

If we are comparing our variables, we need them to be on the same scale. Variable `x3` is not, with a value or 4 that will make comparison difficult. The variables need to be combined into a dataframe and normalized first.

```
df <- data.frame(x1, x2, x3, x4)
df

##       x1    x2   x3      x4
## 1 5e-01   0.2    4    0.14
## 2 2e+03 300.0 4000 4400.00
## 3 1e+02 200.0  100  140.00

df.scale <- scale(df)
df.scale

##               x1         x2         x3         x4
## [1,] -0.6209393 -1.0909958 -0.5982760 -0.6050868
## [2,]  1.1535745  0.8730587  1.1544446  1.1542490
## [3,] -0.5326352  0.2179371 -0.5561686 -0.5491622
```

```
## attr(,"scaled:center")
##        x1        x2        x3        x4
##  700.1667  166.7333 1368.0000 1513.3800
## attr(,"scaled:scale")
##        x1        x2        x3        x4
## 1126.7875  152.6434 2279.8842 2500.8641
```

We can now measure the Euclidian Distance between the points.

```
dist(df.scale)

##          1        2
## 2 3.629559
## 3 1.313775 3.016669
```

Thus, x1 and x3 are the closest points. We could compare the unscaled matrix as well.

Problem 6

You're given a sample of data: 15,2,44,21,40,20,19,18. Calculate the sample mean and sample variance.

```
x <- c(15,2,44,21,40,20,19,18)
mean(x)

## [1] 22.375

var(x)

## [1] 183.6964
```

The sample mean is 22.375 and the sample variance is 183.7.

Problem 7

Choose all that apply. Which of the following statistical measures can be observed on a box plot?

(c) Median
(d) Outliers
(e) Maximum element
(f) Minimum element
(g) Variance or covariance (No actual number, but the var or cov is depicted by the distance between the upper and lower whiskers and box edges.)

Problem 8

Choose all that apply. The most common methods of removing outliers are: (a) Removing tuples with missing values. (c) Observing the probability of existing values in (?).

I have used the following techniques in this paper (from the text):

Remove the cases with unknowns. Fill in the unknown values with the most frequent values. Fill in the unknown values by exploring the correlations between variables. Fill in the unknown values by exploring the similarity between cases.

Problem 9

Swiss bank data contains various lengths measurements on 200 Swiss bank notes. Load the Swiss bank data as follows:

```
## install.packages("alr3")
library("alr3")

## Loading required package: car

head(banknote)

##   Length  Left Right Bottom  Top Diagonal Y
## 1  214.8 131.0 131.1    9.0  9.7    141.0 0
## 2  214.6 129.7 129.7    8.1  9.5    141.7 0
## 3  214.8 129.7 129.7    8.7  9.6    142.2 0
## 4  214.8 129.7 129.6    7.5 10.4    142.0 0
## 5  215.0 129.6 129.7   10.4  7.7    141.8 0
## 6  215.7 130.8 130.5    9.0 10.1    141.4 0
```

First, summary exploration.

```
str(banknote)

## 'data.frame':    200 obs. of  7 variables:
##  $ Length  : num  215 215 215 215 215 ...
##  $ Left    : num  131 130 130 130 130 ...
##  $ Right   : num  131 130 130 130 130 ...
##  $ Bottom  : num  9 8.1 8.7 7.5 10.4 9 7.9 7.2 8.2 9.2 ...
##  $ Top     : num  9.7 9.5 9.6 10.4 7.7 10.1 9.6 10.7 11 10 ...
##  $ Diagonal: num  141 142 142 142 142 ...
##  $ Y       : int  0 0 0 0 0 0 0 0 0 0 ...
```

We have 200 observations of 7 variables. 6 of the variables are numbers, but Y is listed as an integer. Let's explore Y more thoroughly.

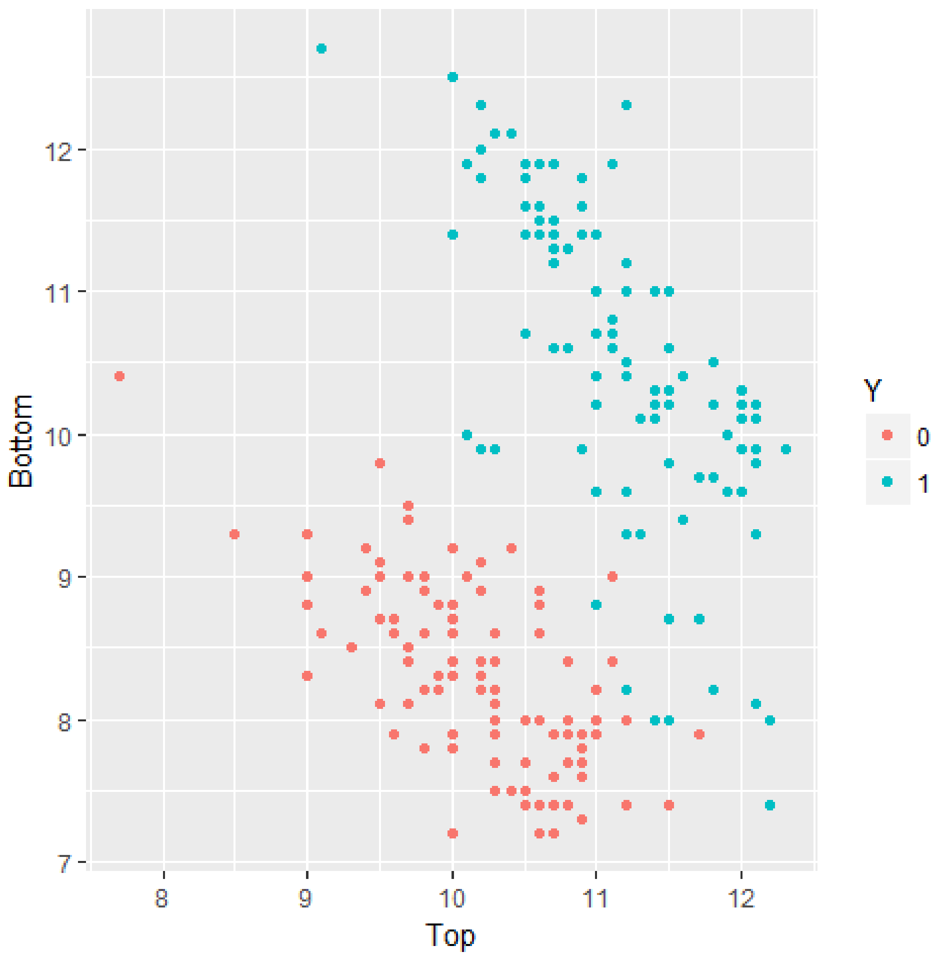
```
banknote$Y

##   [1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0
```

```
##  [36] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0
##  [71] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1
1
## [106] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1
## [141] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1
## [176] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

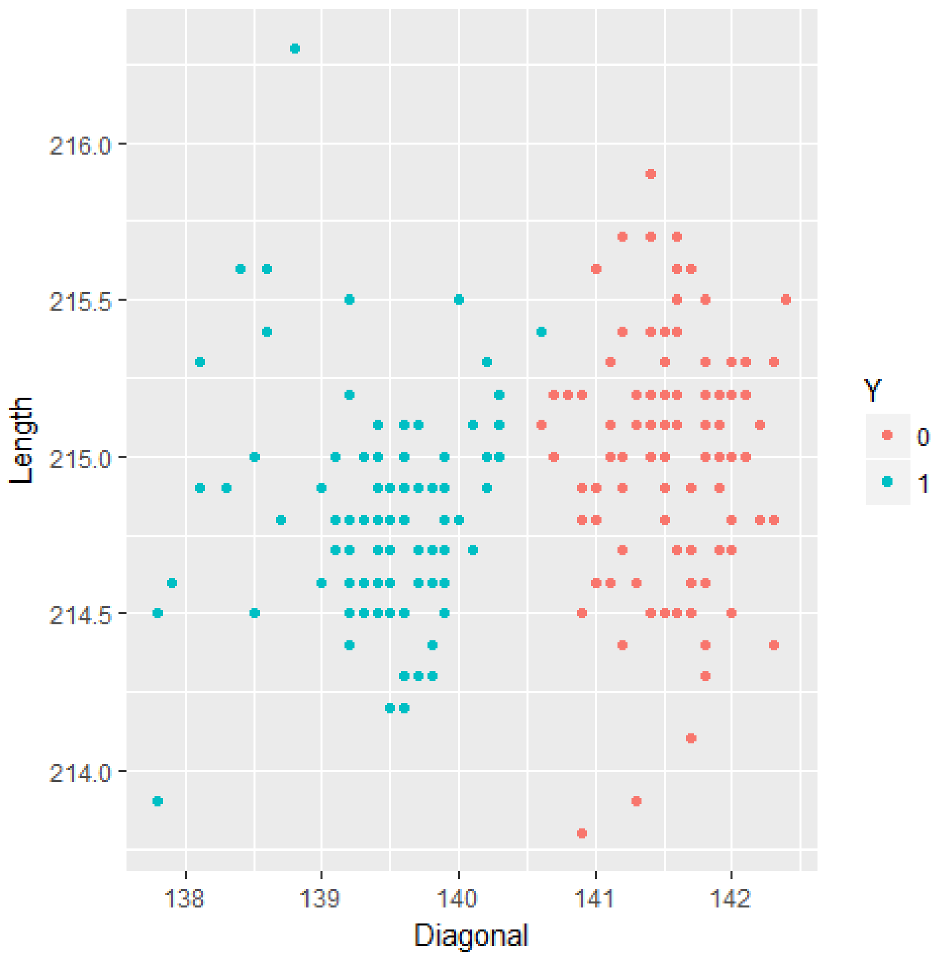
It's listed as an integer, but appears to be a factor, and possibly a class indicator. Let's recode the datatype as a factor. My initial hunch is that the Y variable is denoting whether the note is counterfeit or not, as the remaining variables are all measurements of a physical note.

```
banknote$Y <- as.factor(banknote$Y)
str(banknote)

## 'data.frame':    200 obs. of  7 variables:
##  $ Length  : num  215 215 215 215 215 ...
##  $ Left    : num  131 130 130 130 130 ...
##  $ Right   : num  131 130 130 130 130 ...
##  $ Bottom  : num  9 8.1 8.7 7.5 10.4 9 7.9 7.2 8.2 9.2 ...
##  $ Top     : num  9.7 9.5 9.6 10.4 7.7 10.1 9.6 10.7 11 10 ...
##  $ Diagonal: num  141 142 142 142 142 ...
##  $ Y       : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 1 ...

qplot(Left, Right, data=banknote, color=Y)
```


```
qplot(Top, Bottom, data=banknote, color=Y)
```




```r
qplot(Diagonal, Length, data=banknote, color=Y)
```



There's a pretty clear pattern here in the relationship between Length, Diagonal, and Y variables. This pattern looks suitable for k-means clustering. We could further explore the relationships between the variables, but the plot above shows strong evidence that the Length and Diagonal variables conditioned by `Y` will make a decent plot.

```
bn3 <- kmeans(banknote[,c(1,6)], centers=3, iter.max=200)
bn3

## K-means clustering with 3 clusters of sizes 13, 100, 87
##
## Cluster means:
##     Length Diagonal
## 1 215.0231 138.3154
## 2 214.9740 141.5270
## 3 214.7874 139.6080
##
## Clustering vector:
##   [1] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
2
##  [36] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
3
```

```
##  [71] 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3
3
## [106] 3 3 3 3 3 3 3 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 1 3
3
## [141] 3 3 3 3 3 3 3 1 3 3 3 3 3 3 3 3 3 3 3 1 1 1 3 3 3 3 1 1 3 3 1 3 3 3
3
## [176] 3 3 3 3 1 3 1 3 3 3 3 1 3 3 3 3 1 3 1 3 3 3 3 3 3
##
## Within cluster sum of squares by cluster:
## [1]  6.00000 31.98950 15.92046
##  (between_SS / total_SS =  81.6 %)
##
## Available components:
##
## [1] "cluster"      "centers"      "totss"        "withinss"
## [5] "tot.withinss" "betweenss"    "size"         "iter"
## [9] "ifault"
```

```
bn3$tot.withinss
```

```
## [1] 53.90996
```

```
bn3$withinss
```

```
## [1]  6.00000 31.98950 15.92046
```

```
table(bn3$cluster, banknote$Y)
```

```
##
##      0  1
##   1  0 13
##   2 99  1
##   3  1 86
```

We have pretty good performance with three clusters. Only two notes were mis-classified. A Type 1 error is the more egregious error, as we wouldn't want any counterfeit notes to be passed as legitimate notes. Let's experiment with two and four clusters and compare.

```
bn2 <- kmeans(banknote[,c(1,6)], centers=2, iter.max=200)
bn2$tot.withinss
```

```
## [1] 73.4371
```

```
table(bn2$cluster, banknote$Y)
```

```
##
##      0  1
##   1 99  1
##   2  1 99
```

Again, we get decent performance and misclassify only two instances, but our total sum of square errors is higher. Let's look at our 4 clusters and move on after that.

```
bn4 <- kmeans(banknote[,c(1,6)], centers=4, iter.max = 200)
table(bn4$cluster, banknote$Y)

##
##      0  1
##   1  4 29
##   2  0 13
##   3  1 58
##   4 95  0

bn3$tot.withinss

## [1] 53.90996
```

The lowest Total SS we've seen so far, but we're not getting any increased performance on classifying our two instances. Other methods we could use would involve splitting the data into train and test sets, possibly scaling the data, etc. Neural nets, random forests, etc... might give better results.

10. Bonus question

```
## install.packages("CORElearn")
library(CORElearn)

LocData <-
read.csv("C:\\Users\\khickman\\Desktop\\Personal\\IUMSDS\\AppliedDataMining\\
Midterm\\entropy.csv", header = TRUE)
LocData$User <- as.factor(LocData$User)
LocData

##    User Location Clicks
## 1     1       UL      3
## 2     1       LR      1
## 3     1        M      2
## 4     1       LL      0
## 5     1       UR      0
## 6     2       UL      1
## 7     2       LR      1
## 8     2        M      2
## 9     2       LL      0
## 10    2       UR      0
## 11    3       UL      0
## 12    3       LR      0
## 13    3        M      2
## 14    3       LL      1
## 15    3       UR      1

attrEval(Clicks ~., LocData, estimator = "GainRatio")

## Changing dependent variable to factor with levels: 0 1 2 3
```

```
## Warning in attrEval(Clicks ~ ., LocData, estimator = "GainRatio"):
Possibly
## this is an error caused by regression formula and classification attribute
## estimator or vice versa.

##       User   Location
## 0.07992343 0.39362419

attrEval(Clicks ~., LocData, estimator = "Gini")

## Changing dependent variable to factor with levels: 0 1 2 3

## Warning in attrEval(Clicks ~ ., LocData, estimator = "Gini"): Possibly
## this is an error caused by regression formula and classification attribute
## estimator or vice versa.

##       User   Location
## 0.01777778 0.28444444

attrEval(Clicks ~., LocData, estimator = "InfGain")

## Changing dependent variable to factor with levels: 0 1 2 3

## Warning in attrEval(Clicks ~ ., LocData, estimator = "InfGain"): Possibly
## this is an error caused by regression formula and classification attribute
## estimator or vice versa.

##      User  Location
## 0.1266756 0.9139671

attrEval(Clicks ~., LocData, estimator = "MDL")

## Changing dependent variable to factor with levels: 0 1 2 3

## Warning in attrEval(Clicks ~ ., LocData, estimator = "MDL"): Possibly
## this is an error caused by regression formula and classification attribute
## estimator or vice versa.

##       User   Location
## -0.1469251  0.1326246
```

The Information Gain estimator indicates that 91% of the click variable can be explained by the location variable. All four metrics agreed that the location is relatively important.