

Answers (Problem set 2)

Online S520

1. *Trosset exercise 3.7.7.*

(a) Venn diagram (see the picture below):

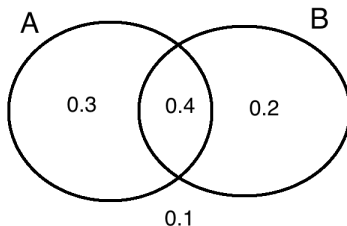
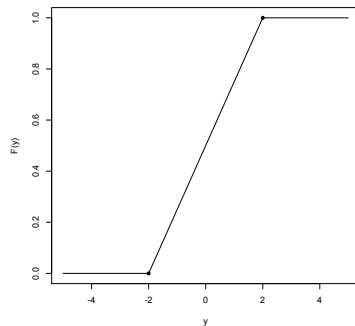


Figure 1: Venn diagram for Exercise 3.7.7.

- (b) No: $P(A \cap B) = 0.4$, not zero.
- (c) From the diagram, 0.8.
- (d) No: $P(A) \cdot P(B) = 0.7 \times 0.6 = 0.42$, which isn't quite the same as the value of $P(A \cap B)$ we found in part (b).
- (e) $P(A|B) = P(A \cap B)/P(B) = 2/3$.
2. (a) There are 26 males and 7 male undergraduates, so this is $7/26$.
- (b) There would be 34 students left including 11 undergrads, so $11/34$.
3. (e) Dependent. There is no structural reason to think that the proportion of Westerns is the same for color movies as for black-and-white movies. If you know some movie history, you might know that Westerns were more common in the black-and-white era than in the color era. If you don't, check IMDb: $P(\text{Western}|\text{Color}) \approx 2.3\%$, $P(\text{Western}|\text{Non-color}) \approx 4.3\%$.
- (f) Dependent. There's no structural reason to think that students are equally likely to attend any particular college regardless of state. In particular, William & Mary is in Virginia, so its students are much more likely to have gone to high school in Virginia (their website says 68% of their students are from Virginia, and there's no way that $P(\text{from Virginia}) = 68\%$ for all U.S. college freshmen.)
- (g) Dependent. There's no structural reason to think that the percentage of PhDs who were women was the same before and after 1950. Before 1950, only a tiny minority of PhDs were women, but the percentage has steadily increased to the point that now the majority of new PhDs are women. Pretty clearly $P(B|A) < P(B|A^c)$.

4. Graph of CDF:



(a)

$$P(X \leq 1) = F(1) = \frac{1+2}{4} = \frac{3}{4}$$

(b)

$$P(X > 1) = 1 - P(X \leq 1) = 1 - \frac{3}{4} = \frac{1}{4}$$

(c)

$$P(X \geq 1) = P(X > 1) + P(X = 1) = \frac{1}{4} + 0 = \frac{1}{4}$$

(d)

$$\begin{aligned} P(-1.5 < X < 0.5) &= P(X < 0.5) - P(X \leq -1.5) \\ &= P(X \leq 0.5) - P(X \leq -1.5) \\ &= F(0.5) - F(-1.5) \\ &= \frac{0.5+2}{4} - \frac{-1.5+2}{4} \\ &= \frac{2}{4} = 0.5 \end{aligned}$$

(e)

$$\begin{aligned} P(|X| > 1) &= P(X < -1) + P(X > 1) \\ &= P(X < -1) + 1 - P(X \leq 1) \\ &= P(X \leq -1) + 1 - P(X \leq 1) \\ &= F(-1) + 1 - F(1) \\ &= \frac{-1+2}{4} + 1 - \frac{1+2}{4} \\ &= \frac{1}{4} + 1 - \frac{3}{4} = 0.5 \end{aligned}$$

5. R tutorial and exercise: calculator