Supplemental Examples (Chapter 9)

Online S520

1. During the winter of 2012-2013, the average utility bill for Delaware residents was \$186 per month. A random sample of 40 customers was selected during the winter of 2013-2014, and the average bill was found to be \$178.10 with a sample standard deviation of \$22.40. What is the significance probability? Using $\alpha = 0.05$, does this sample provide enough evidence to conclude that the average utility bill in Delaware was lower in the winter of 2013-2014 than it was in the winter of 2012-2013?

   Solutions: Since compelling evidence is needed to conclude "the average utility bill in Delaware was lower in the winter of 2013-2014 than it was in the winter of 2012-2013", the alternative hypothesis should be "the average utility bill in 2013-2014 was lower than \$186". Accordingly, we will test the null hypothesis $H_0 : \mu \geq \mu_0 = 186$ against the alternative hypothesis $H_1 : \mu < \mu_0 = 186$.

   The test statistic is

   $$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = (178.10 - 186)/(22.4/\sqrt{40}) = -2.230535.$$

   Using a standard normal distribution, we can compute the P-value (or significance probability) (refer to the formula on Trosset p.217 Test (c))

   $p = P_{\mu_0}(T_n \leq -2.230535) \approx \Phi(-2.230535) = P(Z \leq -2.230535)$ (Refer to Trosset p. 215)

   ```
   > pnorm(-2.230535)
   [1] 0.01285597
   ```

   This is below the significance level of 0.05, so we reject the null hypothesis; there is strong evidence to conclude that the average utility bill in Delaware was lower in the winter of 2013-2014 than it was in the winter of 2012-2013.

   (You could also argue for a t-test here, in which case the P-value would be

   ```
   > pt(-2.230535, 100-1)
   [1] 0.01576807
   ```

   — not substantively different.)

2. Zingo's Grocery store claims that customers spend an average of 5 minutes waiting for service at the store's deli counter. A random sample of 45 customers was timed at the deli counter, and the average service time was found to be 5.5 minutes. Assume the standard deviation is 1.7 minutes per customer. Using $\alpha = 0.02$, does this sample provide enough evidence to reject the claim made by Zingo's management?

Solutions: Let X_i denote the time customer i spends waiting for service at the store's deli counter. The average service time is $\mu = E(X_i)$. Accordingly, it will test the null hypothesis $H_0 : \mu = \mu_0 = 5$ minutes against the alternative hypothesis $H_1 : \mu \neq \mu_0 = 5$ minutes.

The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = (5.5 - 5)/(1.7/\sqrt{45}) = 1.973001.$$

Using a standard normal distribution, we can compute the P-value (or significance probability) as below (refer to the formula on Trosset p.217 Test (a))

$$\begin{aligned} p &= P_{\mu_0}(|T_n| \geq |1.973|) \\ &= 1 - P_{\mu_0}(|T_n| < 1.973) \\ &= 1 - P_{\mu_0}(-1.973 < T_n < 1.973) \\ &= 1 - P_{\mu_0}(T_n < 1.973) + P_{\mu_0}(T_n < -1.973) \\ &\approx 1 - \Phi(1.973) + \Phi(-1.973) \quad \text{(Refer to Trosset p. 215)} \\ &= 1 - P(Z < 1.973) + P(Z < -1.973) \end{aligned}$$

```
> 1 - pnorm(1.973) + pnorm(-1.973)
[1] 0.04849556
```

Or use the results on Lecture Notes pages 6 and 7:
The two-tailed P-value is:

```
> t = (5.5 - 5)/(1.7/sqrt(45))
> 2 * (1 - pnorm(abs(t)))
[1] 0.04849543
```

(If use a t-distribution,

```
> t = (5.5 - 5)/(1.7/sqrt(45))
> 2 * (1 - pt(abs(t), 44))
[1] 0.05479811
```

This probability is bigger than $\alpha = 0.02$, so we do not reject H_0. The data is compatible with Zingo's claim that customers spend an average of 5 minutes waiting for service at the store's deli counter.

3. (Trosset exercise 9.6.4.) The U.S. Food and Drug Administration requires evaporated milk to contain "not less than 23 percent by weight of total milk solids." A company that sells evapaorated milk is sued by a group of consumers who are concerned that the company's product does not meet FDA standards. The two parties agree to binding arbitration. If the consumers win, the company will pay damages and enhance its product; if the company wins, then the consumers will issue a public apology. To resolve the dispute, the arbiter commissions a neutral study in which the percent by weight of total milk solids will be measured in a random sample of $n = 225$ packages produced by the company. Both parties agree to a standard of proof ($\alpha = 0.05$), but they disagree on which party should bear the burden of proof.

   (a) State appropriate null and alternative hypotheses from the perspective of the consumers.
   Solution: The consumers believe that the company should bear the burden of proving that its product meets FDA standards. From this perspective, the null hypothesis is $H_0 : \mu \leq 23$ and the alternative hypothesis is $H_1 : \mu > 23$.

   (b) State appropriate null and alternative hypotheses from the perspective of the company.
   Solution: The company believes that the plaintiff (the consumers) should bear the burden of proving their case. From this perspective, the null hypothesis is $H_0 : \mu \geq 23$ and the alternative hypothesis is $H_1 : \mu < 23$.

   (c) Suppose that the random sample reveals a sample mean of $\bar{x} = 22.8$ percent with a sample standard deviation of $s = 3$ percent. Compute t, the value of the test statistic.
   Solution: $t = (22.8 - 23)/(3/15) = -1$

   (d) From the consumers' perspective, what action should be taken? Why?
   Solution: The observed sample mean is $\bar{x} = 22.8 < 23 = \mu_0$. These data will not persuade the consumers that $\mu > 23$. The consumers will decline to reject $H_0 : \mu \leq 23$ and ask the arbiter to require the company to pay damages and upgrade its product.

   (e) From the company's perspective, what action should be taken? Why?
   Solution: The signficiance probability for testing $H_0 : \mu \geq 23$ versus $H_1 : \mu < 23$ is

$$\mathbf{p} \approx P(Z < -1) \doteq 0.1586553 > 0.05,$$

   so the company remains unconvinced that $\mu < 23$. It will ask the arbiter to dismiss the complaint and require the consumers to issue a public apology.

4. Use the binomial distribution to find the P-value for each of the following situations.

   (a) I roll a six-sided die repeatedly. Let p be the probability of getting a six. I wish to test $H_0 : p \geq 1/6$ vs. $H_1 : p < 1/6$. I end up getting 15 sixes in 100 rolls.

   Solutions: This is a left-tailed test with P-value `pbinom(15, 100, 1/6)` $= 0.39$.

   (b) I want to know if there's bias in coin tosses before Test cricket matches. My null hypothesis is that the home team and the away team both have a 50-50 chance of winning the tosses. I decide to perform a two-tailed test. I find that the home team has won 1150 times and the away team has won 1065 times.

   Solutions: This is a two-tailed test of the null $H_0 : p = 0.5$. The P-value is `2 * pbinom(1065, 1065 + 1150, 0.5)` $= 0.074$. You could also do `2 * (1 - pbinom(1149, 1065 + 1150, 0.5))`, giving the same result.

   (c) I wonder if my intro stats students are just guessing on their multichoice tests, or if they really know anything. I ask a total of 720 questions, each with four options. My null hypothesis is that all the students are randomly guessing on all questions; I want a small P-value to be evidence that they're doing better than that. I find they end up getting 237 out of 720 right.

   Solutions: This is a right tailed test of the alternative $H_1 : p > 0.25$. The P-value is `1 - pbinom(236, 720, 0.25)` $= 1.2 \times 10^{-6}$.

5. The number of long-term unemployed, defined as those out of work for more than 27 weeks, has been a detriment to the recent economic recovery. In 2013, 40% of the unemployed had been out of work for more than 27 weeks. Government policy makers feel that this percentage has declined recently as the job market has improved. To test this theory, a random sample of 300 unemployed people was selected, and it was found that 112 were unemployed for longer than 27 weeks. What is the significance probability? Based on this sample, can the federal government conclude that the percentage of unemployed who have been out of work for more than 27 weeks has recently decreased? Use $\alpha = 0.10$.

   Solutions: This is a left-tailed test with alternative $H_1 : p < 0.4$. The P-value is `pbinom(112, 300, 0.4)` $= 0.189 > \alpha$. So there is not sufficient evidence for the federal government to conclude that the percentage of unemployed who have been out of work for more than 27 weeks has recently decreased.

6. A 2010 survey by CarMD.com found that 64% of vehicle owners avoided automotive maintenance and repairs. Suppose Sears Automotive would like to perform a hypothesis test to challenge this finding. From a random sample of 170 vehicle owners that was recently taken, it was found that 103 avoid maintenance and repairs. What is the significance probability?

Based on this sample, can we conclude that the proportion of vehicle owners who avoid maintenance has changed since 2010? Use $\alpha = 0.02$.

Solutions: This is a two-tailed test of the null $H_0 : p = 0.64$.

[Use Binomial probability] The P-value is `2 * pbinom(103, 170, 0.64)` $= 0.40 > \alpha = 0.02$. So there is not enough evidence to conclude that the proportion of vehicle owner who avoid maintenance has changed.

[Use Normal approximation] (Please read Trosset page 215) The observed proportion of success is $103/170 = 0.606$, so the value of the test statistic is

$$z = \frac{0.606 - 0.64}{\sqrt{0.64 \times (1 - 0.64)/170}} = \frac{-0.034}{0.0368} = -0.924$$

and the significance probability (or P-value) is `2*pnorm(-0.924) = 0.355.`