

Problem set 8

Online S520

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1. (5 points) Trosset exercise 9.6.6.

Hint:

- In some other textbooks/statistical courses, you might find the following test:
 $H_0 : \mu = \mu_0$ vs $H_1 : \mu > \mu_0$. However, in this course, we only consider the three types of tests as listed in Trosset 9.4.1 (page 217), i.e. the hypotheses are collectively exhaustive.
 - When you formulate hypotheses for this problem, start with the alternative hypothesis. Think about what the costumers' complaints are. Are they dissatisfied because they believe the average number of hours of the light bulbs is less than 800, not equal to 800, or greater than 800?
 - After determining the alternative hypothesis, the complement statement will be the null hypothesis.
2. (5 points) Trosset exercise 9.6.7.
 3. A researcher wanted to see if live reggae music improved students' math test scores.¹ He selected a sample of 61 students from his university and gave them a math test. The students then studied for two and a half hours while an acoustic reggae band played quietly. The students then took another math test of the same difficulty.

The researcher's variable was the *change* in test score for each student — a positive change meant the student did better, while a negative change meant a student did worse. He found that the changes had an approximately normal distribution with mean 6.5 and standard deviation 12.
 - (a) (5 points) Assuming the sample is random, test the hypothesis that the population average is positive.
 - (b) (2 points) Does the study provide convincing evidence that live reggae music improves students' math test scores? Explain why or why not.
 4. Every semester, a professor gives his undergraduate statistics classes a test for psychic powers. The test consists of guessing which side of a screen a picture will appear on: left or right. In one trial of the test, a student has to guess "Left" or "Right"; then R's random number generator will randomly choose one side of the screen to display a picture of a star. The student repeats the process for a total of 20 trials.
 - (a) (1 point) In words, the null hypothesis for a particular student is that they don't have psychic powers and they're just randomly guessing. The alternative hypothesis is that

¹I am not making this up.

they do have psychic powers and in the long run, they can do better than randomly guessing.

Let p be the probability the student guesses correctly on any particular trial. Write down mathematical null and alternative hypotheses in terms of p .

- (b) (1 point) Suppose the null hypothesis is true for a particular student. Let Y be the number of times the student guesses correctly. Then if the null is true, Y has a Binomial(20, 0.5) distribution.

Which of the following is true?

- i. If the student guesses 13 right out of 20, the significance probability (P -value) is the probability under the null of guessing 13 or more out of 20.
- ii. If the student guesses 13 right out of 20, the significance probability (P -value) is the probability under the null of guessing 13 or fewer out of 20.

Hint: Remember that the smaller the P -value, the stronger the evidence for the *alternative* hypothesis.

- (c) (3 points) Suppose a student guesses 13 right out of 20. What is the P -value? Do you think 13 out of 20 is intriguing evidence that the student has psychic powers? (**Use Binomial probability**)
- (d) (3 points) Now suppose a student guesses 19 right out of 20. What is the P -value? Do you think 19 out of 20 is intriguing evidence that the student has psychic powers? (**Use Binomial probability**)
5. (5 points) Recently, the number of airline companies that offer in-flight Wi-Fi service to passengers has increased. However, it is estimated that only 10% of the passengers who have Wi-Fi available to them are willing to pay for it. Suppose Gogo, the largest provider of airline Wi-Fi service, would like to test this hypothesis by randomly sampling 125 passengers and asking them if they would be willing to pay \$4.95 for 90 minutes of onboard Internet access. Suppose that 20 passengers indicated they would use this service. What is the significance probability? Based on this sample, can we conclude that the proportion of airline passengers willing to pay for onboard Wi-Fi service is different than 10%? Use $\alpha = 5\%$. (**Use Binomial probability**)

Hint:

- Significance probability (or p -value) is a probability, so it should be a number between 0 and 1.
- In a two-sided hypothesis test, the significance probability is calculated by multiplying a probability of a tail region by 2. Note that this tail region should have an area less than 0.5.