Problem set 4
Online S520

Upload your answers through the Assignments tab on Canvas

1. (5 points) Suppose that X is a continuous random variable with probability density function (pdf) f defined as follows:
$$f(x) = \begin{cases} 0 & x < 20 \\ \frac{1}{20} & 20 \leq x \leq 40 \\ 0 & x > 40 \end{cases}$$

    (a) Graph f.

    (b) Verify that f is a pdf.

    (c) Compute $P(0 < X < 35)$.

    (d) Find the CDF of X, $F(y)$, for all y.

2. (3 points) Trosset chapter 5.6 exercise 2.

3. (3 points) Let X be a random variable with PDF
$$f(x) = \begin{cases} \frac{1}{30} & 0 \leq x < 20 \\ \frac{1}{60} & 20 \leq x < 40 \\ 0 & \text{otherwise.} \end{cases}$$

    Find the expected value of X.

4. (5 points) Trosset chapter 5.6 exercise 7. (Use R and give code.)

5. (5 points) Trosset chapter 5.6 exercise 8.
   Hint:

    - Note that a normal random variable is usually denoted as $N(\mu, \sigma^2)$.

    - It is helpful to review the following theorems:
      Theorem 5.2 (page 132)
      Theorem 4.2 (page 101), Theorem 4.4 (page 101)
      Theorem 4.7 (page 104), Theorem 4.8 (page 105)
      (Note: these chapter 4 theorems can be applied to continuous random variables as well).

6. (4 points) A local gas station waits 4 days to receive a delivery of regular gasoline to replenish its inventory. The wait period to receive inventory is known as the lead time. The demand during the lead-time period for regular gasoline, as measured in gallons, follows the normal distribution with a mean of 930 gallons and standard deviation of 140 gallons. The station manager places the next order for regular gas when the inventory is 1200 gals (known as the reorder point). Whats the probability that the station will run out of gas before the order arrives? (Use R and give code.)

7. (5 points) Let X be a standard normal random variable. Let $Y = X^2$. (Use R and give code.)

   (a) Find $P(-1.5 < X < 2.5)$.
   (b) Find $P(Y > 1)$.

   Notes:

   - You are not expected to and don't need to figure out the distribution of Y. Just convert the probability for Y to a probability involving X.
   - Algebra review:
     $$x^2 < 4 \Longrightarrow |x| < 2 \Longrightarrow -2 < x < 2$$
     $$x^2 > 4 \Longrightarrow |x| > 2 \Longrightarrow x > 2 \text{ or } x < -2$$
     $$1 < x^2 < 4 \Longrightarrow 1 < |x| < 2 \Longrightarrow 1 < x < 2 \text{ or } -2 < x < -1$$
   - Use R to find the final results