Online S520 Midterm 1

Instructions

- Type your answers in a Word document or Latex and submit through Canvas/Assignment/Midterm 1.
- This exam is due on Monday 11:59pm, Oct 2nd (Pacific Time). **Late submission will not be accepted!**
- **You must NOT discuss this exam with anyone other than the instructor and the TAs until the due date has passed.**
- Write explanations for all your answers. **Answers alone will not get credit.** For questions where you use R, you must give R code, but the code alone is not a sufficient explanation.
- You may use any R functions.
- Round answers sensibly, e.g. to 3 significant figures. Unrounded or inaccurately rounded answers may receive point deductions.
- **Example.** Let Z be a standard normal random variable. What is $P(2 < Z < 3)$?
  - Answer: 0.02140023. — **NO POINTS**
  - Answer: We can calculate this as the difference between the CDF of the standard normal at $y = 3$ and the CDF at $y = 2$. I used the R code `pnorm(3) - pnorm(2)` to get this difference. The answer is 0.0214, or about 2%. — **FULL POINTS**

What can I ask the instructor by email?

- If you think there is an error in the exam, notify the instructor immediately.
- General questions about course material or help handling the data. (However, it's easier to talk about these issues during office hours.)

What can I ask at the instructor's or TA's office hours?

- General questions about course material.
- Help handling the data.

What can I ask other students?

Nothing.

1. I toss six fair coins.

   (a) (4 points.) What is the probability I get exactly five heads?

   (b) (4 points.) If I get at least five heads, what is the conditional probability I get exactly five heads?

   (c) (4 points.) Are the events "I get exactly five heads" and "I get at least five heads" independent? Explain why or why not.

2. 48% percent of U.S. adults have visited a library in the past year. I select two U.S. adults at random. Let X be a discrete random variable representing the number of the adults I selected who have visited a library in the last year.

   (a) (3 points.) Find $P(X = 0)$, $P(X = 1)$, and $P(X = 2)$.

   (b) (5 points.) Write down an expression for $F(y)$, the cumulative distribution function (CDF) of X, for all y-values from $-\infty$ to ∞.

   (c) (4 points.) Find the expected value and variance of X.

3. Let X be a continuous random variable with the probability density function (PDF)

$$f(x) = \begin{cases} 0.1 & 0 \leq x < 8 \\ 0.05 & 8 \leq x < 12 \\ 0 & \text{otherwise.} \end{cases}$$

   To answer the following questions, it may help to draw a graph of this PDF.

   (a) (4 points.) Find $F(y)$, the cumulative distribution function (CDF) of X, for all y-values from $-\infty$ to ∞.

   (b) (4 points.) Find the median of X.

   (c) (4 points.) Find the expected value of X.

4. Let X be a random variable with the cumulative distribution function (CDF)

$$F(y) = \begin{cases} 0 & y < 0 \\ 0.25y^2 & 0 \leq y < 1 \\ 0.25y & 1 \leq y < 4 \\ 1 & y \geq 4. \end{cases}$$

   Calculus shows that $E(X) = 49/24$ and $E(X^2) = 43/8$. You may use these results below.

   (a) (4 points.) Is X discrete, continuous, or neither? Explain.

   (b) (4 points.) Find the standard deviation of X.

   (c) (4 points.) Let A and B be independent random variables with the same distribution as X. Let $Y = A - B$. What are the expected value and standard deviation of Y?

5. Let X be a continuous random variable with cumulative distribution function

$$F(y) = \begin{cases} 0 & y < 1 \\ 1 - \frac{1}{y^2} & y \geq 1. \end{cases}$$

and probability density function

$$f(x) = \begin{cases} \frac{2}{x^3} & x \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) (4 points.) Find the median of X.

   (b) (6 points.) Find the interquartile range of X.

   (c) (4 points.) Is the expected value of X less than, equal to, or greater than its median? Explain. (Hint: You don't have to use calculus. You can use your intuition and/or the shape of the distribution to explain.)

6. Let X be standard normal random variable. Let $Y = |X|$.

   (a) (4 points.) What is the median of Y?

   (b) (4 points.) What is $P(1 < Y < 2)$?

   (c) (4 points.) What is the 0.95-quantile of Y?

7. The heights of adult men in the U.S. approximately follow a normal distribution with mean 175.9 cm and standard deviation 7.5 cm. The heights of adult women in the U.S. approximately follow a normal distribution with mean 162.1 cm and standard deviation 7.3 cm.

   (a) (4 points.) I select an adult U.S. woman at random. What is the probability she is under five feet (152.4 cm) tall?

   (b) (4 point.) I select an adult U.S. man at random. What is the probability he is over six feet (182.88 cm) tall?

   (c) (4 point.) I select four adult U.S. men at random. What is the probability at least one of them is over six feet tall?

   (d) (6 point) I select at random an adult U.S. man and, independently, an adult U.S. woman. What is the probability the woman is taller? (Hint: What distribution does the difference of two independent normal random variables have)

8. One semester I gave a 20-question test for psychic powers to a class of 80 students.

   - Each student (independently) guessed "left" or "right," then (using the random number generator in R) a star appeared on either the left or the right side on the screen. (This is one question in the test.)
   - This process (question described above) was repeated 20 times, i.e. 20 questions total in the test.

   At the end, each student counted up the number of times they correctly guessed the side of the screen the star would appear on. The highest score was 15.

   (a) (4 points.) Suppose this test is taken by a student who does not have psychic powers. What is the probability she gets 15 or more correct?

   (b) (4 points.) Suppose this test is taken by eighty students who do **NOT** have psychic powers. What is the probability *at least one student* gets 15 or more correct?