Answers (Problem set 3)

Online S520

1. *Trosset exercise 4.5.1.*

   (a)

$$f(x) = \begin{cases} 0.1 & x = 1 \\ 0.4 & x = 3 \\ 0.4 & x = 4 \\ 0.1 & x = 6 \\ 0 & \text{otherwise} \end{cases}$$

   (b)

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.1 & 1 \le x < 3 \\ 0.5 & 3 \le x < 4 \\ 0.9 & 4 \le x < 6 \\ 1 & x \ge 6 \end{cases}$$

   (c) $EX = 1 \cdot 0.1 + 3 \cdot 0.4 + 4 \cdot 0.4 + 6 \cdot 0.1 = 3.5$

   (d)

$$\begin{aligned} E\left[X^2\right] &= 1^2 \cdot 0.1 + 3^2 \cdot 0.4 + 4^2 \cdot 0.4 + 6^2 \cdot 0.1 \\ &= 13.7 \\ VarX &= E\left[X^2\right] - (EX)^2 \\ &= 13.7 - 3.5^2 = 1.45 \end{aligned}$$

   (e) $\sigma = \sqrt{VarX} \approx 1.2$

2. *Trosset exercise 4.5.3.*

   (a)

$$f(x) = \begin{cases} 0.4 & x = 1 \\ 0.1 & x = 2 \\ 0.2 & x = 5 \\ 0.3 & x = 10 \\ 0 & \text{otherwise} \end{cases}$$

   (b)

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.4 & 1 \le x < 2 \\ 0.5 & 2 \le x < 5 \\ 0.7 & 5 \le x < 10 \\ 1 & x \ge 10 \end{cases}$$

(c) $EX = 1 \cdot 0.4 + 2 \cdot 0.1 + 5 \cdot 0.2 + 10 \cdot 0.3 = 4.6$

(d)

$$\begin{aligned} E\left[X^2\right] &= 1^2 \cdot 0.4 + 2^2 \cdot 0.1 + 5^2 \cdot 0.2 + 10^2 \cdot 0.3 \\ &= 35.8 \\ VarX &= E\left[X^2\right] - (EX)^2 \\ &= 35.8 - 4.6^2 = 14.64 \end{aligned}$$

(e) $\sigma = \sqrt{VarX} \approx 3.8$

3. *Trosset exercise 4.5.10.*

Each of the $n = 12$ attendees represents a Bernoulli trial. The possible outcomes are attendance and nonattendance. If we designate attendance as success and nonattendance as failure, then the probability of success is $p = 0.5 \cdot 0.8 = 0.4$. Let Y denote the observed number of successes, so that $Y \sim \text{Binomial}(12, 0.4)$. Then

$$\begin{aligned} P(Y > 7) &= 1 - P(Y \leq 7) \\ &= \texttt{1 - pbinom(7, 12, 0.4)} \\ &= 0.057 = 5.7\%. \end{aligned}$$

4. *Trosset exercise 4.5.14.*

(a) Each attempt to send/receive a symbol is a Bernoulli trial. There are 5 symbols, so the probability of success is $p = 0.2$. There are $n = 25$ trials, so the expected number of success is $np = 5$.

(b) This is a Binomial experiment. There are $n = 25$ Bernoulli trials, "success" is "guess correctly", the probability of success is $p = 0.2$. Let Y be the number of successes or score. Then $Y \sim \text{Binomial}(25, 0.2)$. A score indicative of ESP has to be greater than 7 $(Y > 7)$.

$$\begin{aligned} P(Y > 7) &= 1 - P(Y \leq 7) \\ &= \texttt{1 - pbinom(7, 25, 0.2)} \\ &= 0.1091 = 10.9\% \end{aligned}$$

(c) There are two Binomial experiments in this problem. One is the same as in part (b); the other one is "whether each receiver attains a score indicative of ESP". There are $n = 20$ trials, "success" is "score > 7", the probability of success is the final answer you got from part (b).

Let X be the number of successes or number of people out of 20 receiving a score indicative of ESP. We want to find $P(X \geq 1) = 1 - P(X = 0)$.

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= \texttt{1 - dbinom(0, 20, 0.1091)} \\ &= 1 - (1 - 0.1091)^{20} \\ &= 0.901 = 90.1\% \end{aligned}$$