## HUMBOLDT UNIVERSITY OF BERLIN

## EINFÜHRUNG IN DAS WISSENSCHAFTLICHE RECHNEN

# XXX

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#### 1 Worked Example 1

7											
1         5         7         4         9         2         8         3         0         6         (3) swap 7 and 5           1         5         4         7         9         2         8         3         0         6         (4) swap 7 and 4           1         5         4         2         9         7         8         9         0         6         (5) swap 7 and 2           1         5         4         2         3         0         8         9         7         6         (7) swap 7 and 0           1         5         4         2         3         0         6         9         7         8         (8) swap 8 and the pivot           1         5         4         2         3         0         6         9         7         8         (9) 6 is in the correct place           partition the sequence into (1, 5, 4, 2, 3, 0) and (9, 7, 8)           1         5         4         2         3         0         6         9         7         8         find the pivot           0         5         4         2         3         1         6         9         7         8         find	7	1		4	9	2	8	3	0	6	
1         5         4         7         9         2         8         3         0         6         (4) swap 7 and 4           1         5         4         2         9         7         8         3         0         6         (5) swap 7 and 2           1         5         4         2         3         0         8         9         7         6         (7) swap 7 and 0           1         5         4         2         3         0         6         9         7         8         (8) swap 8 and the pivot           1         5         4         2         3         0         6         9         7         8         (9) 6 is in the correct place           partition the sequence into (1, 5, 4, 2, 3, 0) and (9, 7, 8)           1         5         4         2         3         0         6         9         7         8         find the pivot           0         5         4         2         3         1         6         9         7         8         swap 1 and the pivot           0         5         4         2         3         1         6         9         7         8	1	7	5	4	9	2	8	3	0	6	(2) swap <b>7</b> and <b>1</b>
1 5 4 2 9 7 8 3 0 6 (5) swap 7 and 2 1 5 4 2 3 7 8 9 0 6 (6) swap 9 and 3 1 5 4 2 3 0 8 9 7 6 (7) swap 7 and 0 1 5 4 2 3 0 6 9 7 8 (8) swap 8 and the pivot 1 5 4 2 3 0 6 9 7 8 (9) 6 is in the correct place  partition the sequence into (1, 5, 4, 2, 3, 0) and (9, 7, 8)  1 5 4 2 3 0 6 9 7 8 sort left side 1 5 4 2 3 0 6 9 7 8 find the pivot 0 5 4 2 3 1 6 9 7 8 swap 1 and the pivot 0 5 4 2 3 1 6 9 7 8 or in the correct place  partition the sequence into () and (5, 4, 2, 3, 1)  0 5 4 2 3 1 6 9 7 8 sort right side 0 5 4 2 3 1 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 2 4 3 5 6 9 7 8 swap 4 and 2 0 1 2 3 4 5 6 9 7 8 swap 4 and the pivot	1	5	7	4	9	2	8	3	0	6	(3) swap <b>7</b> and <b>5</b>
1 5 4 2 3 7 8 9 0 6 (6) swap 9 and 3 1 5 4 2 3 0 8 9 7 6 (7) swap 7 and 0 1 5 4 2 3 0 6 9 7 8 (8) swap 8 and the pivot 1 5 4 2 3 0 6 9 7 8 (9) 6 is in the correct place  partition the sequence into (1,5,4,2,3,0) and (9,7,8)  1 5 4 2 3 0 6 9 7 8 sort left side 1 5 4 2 3 0 6 9 7 8 find the pivot 0 5 4 2 3 1 6 9 7 8 swap 1 and the pivot 0 5 4 2 3 1 6 9 7 8 ois in the correct place  partition the sequence into () and (5,4,2,3,1)  0 5 4 2 3 1 6 9 7 8 sort right side 0 5 4 2 3 1 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 swap 5 and the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 find the pivot 0 1 4 2 3 5 6 9 7 8 swap 4 and 2 0 1 2 4 3 5 6 9 7 8 swap 4 and 2 0 1 2 3 4 5 6 9 7 8 swap 4 and the pivot	1	5	4	7	9	2	8	3	0	6	(4) swap <b>7</b> and <b>4</b>
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1         5         4         2         3         0         6         9         7         8         sort left side           1         5         4         2         3         0         6         9         7         8         find the pivot           0         5         4         2         3         1         6         9         7         8         swap 1 and the pivot           0         5         4         2         3         1         6         9         7         8         sort right side           0         5         4         2         3         1         6         9         7         8         find the pivot           0         5         4         2         3         1         6         9         7         8         find the pivot           0         1         4         2         3         5         6         9         7         8         find the pivot           0         1         4         2         3         5         6         9         7         8         find the pivot           0         1         4         2         3	1	5	4	2	3	0	6	9	7	8	(9) 6 is in the correct place
1       5       4       2       3       0       6       9       7       8       find the pivot         0       5       4       2       3       1       6       9       7       8       swap 1 and the pivot         0       5       4       2       3       1       6       9       7       8       find the correct place         0       5       4       2       3       1       6       9       7       8       find the pivot         0       5       4       2       3       5       6       9       7       8       find the pivot         0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       4       2       3       5       6       9       7       8       5 is in the correct place         0       1       4       2       3       5       6       9       7       8       find the pivot         0       1<		J	part	itioı	n th	e se	que	nce	into	(1,	5,4,2,3,0) and $(9,7,8)$
0         5         4         2         3         1         6         9         7         8         swap 1 and the pivot           0         5         4         2         3         1         6         9         7         8         0 is in the correct place           partition the sequence into () and (5,4,2,3,1)           0         5         4         2         3         1         6         9         7         8         sort right side           0         5         4         2         3         1         6         9         7         8         find the pivot           0         1         4         2         3         5         6         9         7         8         swap 5 and the pivot           0         1         4         2         3         5         6         9         7         8         find the pivot           0         1         4         2         3         5         6         9         7         8         5 is in the correct place           0         1         4         2         3         5         6         9         7         8         find the pivot </td <td>1</td> <td>5</td> <td>4</td> <td>2</td> <td>3</td> <td>0</td> <td>6</td> <td>9</td> <td>7</td> <td>8</td> <td>sort left side</td>	1	5	4	2	3	0	6	9	7	8	sort left side
0 5 4 2 3 1 6 9 7 8 0 is in the correct place  partition the sequence into () and (5,4,2,3,1)  0 5 4 2 3 1 6 9 7 8 sort right side  0 5 4 2 3 1 6 9 7 8 find the pivot  0 1 4 2 3 5 6 9 7 8 swap 5 and the pivot  0 1 4 2 3 5 6 9 7 8 lis in the correct place  0 1 4 2 3 5 6 9 7 8 find the pivot  0 1 4 2 3 5 6 9 7 8 find the pivot  0 1 4 2 3 5 6 9 7 8 find the pivot  0 1 4 2 3 5 6 9 7 8 find the pivot  0 1 4 2 3 5 6 9 7 8 find the pivot  0 1 4 2 3 5 6 9 7 8 swap 4 and 2  0 1 2 3 4 5 6 9 7 8 swap 4 and the pivot	1	5	4	2	3	0	6	9	7	8	find the pivot
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0         5         4         2         3         1         6         9         7         8         find the pivot           0         5         4         2         3         1         6         9         7         8         find the pivot           0         1         4         2         3         5         6         9         7         8         1 is in the correct place           0         1         4         2         3         5         6         9         7         8         find the pivot           0         1         4         2         3         5         6         9         7         8         find the pivot           0         1         4         2         3         5         6         9         7         8         find the pivot           0         1         2         4         3         5         6         9         7         8         swap 4 and 2           0         1         2         3         4         5         6         9         7         8         swap 4 and the pivot	0	5	4	2	3	1	6	9	7	8	0 is in the correct place
0         5         4         2         3         1         6         9         7         8         find the pivot           0         1         4         2         3         5         6         9         7         8         swap 5 and the pivot           0         1         4         2         3         5         6         9         7         8         find the pivot           0         1         4         2         3         5         6         9         7         8         5 is in the correct place           0         1         4         2         3         5         6         9         7         8         find the pivot           0         1         2         4         3         5         6         9         7         8         swap 4 and 2           0         1         2         3         4         5         6         9         7         8         swap 4 and the pivot			]	part	itio	n th	e se	que	nce	into	() and $(5,4,2,3,1)$
0       1       4       2       3       5       6       9       7       8       swap 5 and the pivot         0       1       4       2       3       5       6       9       7       8       1 is in the correct place         0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       2       4       3       5       6       9       7       8       swap 4 and 2         0       1       2       3       4       5       6       9       7       8       swap 4 and the pivot	0	5	4	2	3	1	6	9	7	8	sort right side
0       1       4       2       3       5       6       9       7       8       1 is in the correct place         0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       2       4       3       5       6       9       7       8       swap 4 and 2         0       1       2       3       4       5       6       9       7       8       swap 4 and the pivot	0	5	4	2	3	1	6	9	7	8	find the pivot
0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       4       2       3       5       6       9       7       8       5 is in the correct place         0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       2       4       3       5       6       9       7       8       swap 4 and 2         0       1       2       3       4       5       6       9       7       8       swap 4 and the pivot	0	1	4	2	3	5	6	9	7	8	swap 5 and the pivot
0       1       4       2       3       5       6       9       7       8       5 is in the correct place         0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       2       4       3       5       6       9       7       8       swap 4 and 2         0       1       2       3       4       5       6       9       7       8       swap 4 and the pivot	0	1	4	2	3	5	6	9	7	8	1 is in the correct place
0       1       4       2       3       5       6       9       7       8       find the pivot         0       1       2       4       3       5       6       9       7       8       swap 4 and 2         0       1       2       3       4       5       6       9       7       8       swap 4 and the pivot	0	1	4	2	3	5	6	9	7	8	find the pivot
0 1 2 4 3 5 6 9 7 8 swap 4 and 2 0 1 2 3 4 5 6 9 7 8 swap 4 and the pivot	0	1	4	2	3	5	6	9	7	8	5 is in the correct place
0 1 2 3 4 5 6 9 7 8 swap 4 and the pivot	0	1	4	2	3	5	6	9	7	8	find the pivot
1	0	1	2	4	3	5	6	9	7	8	swap 4 and 2
0 1 2 3 4 5 6 9 7 8 3 is in the correct place	0	1	2	3	4	5	6	9	7	8	swap 4 and the pivot
	0	1	2	3	4	5	6	9	7	8	3 is in the correct place

### 2 Worked Example 2

7	1	5	4	9	2	8	3	0	6	find the pivot
1	7	5	4	9	2	8	3	0	6	swap 7 and 1
1	5	7	4	9	2	8	3	0	6	swap $7$ and $5$
1	5	4	7	9	2	8	3	0	6	swap $7$ and $4$
1	5	4	2	9	7	8	3	0	6	swap 7 and 2
1	5	4	2	3	7	8	9	0	6	swap 9 and 3
1	5	4	2	3	0	8	9	7	6	swap $7$ and $0$
1	5	4	2	3	0	6	9	7	8	swap 8 and the pivot

Now, the pivot 6 is on the right place and every element on the left side is smaller and every element on the right side is larger than the pivot.

1	5	4	2	3	0	6	9	7	8	1
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We partition the sequence into two smaller ones and apply the algorithm on each.

1	5	4	2	3	0	find the pivot
0	5	4	2	3	1	swap 1 and the pivot

The pivot 0 is correctly placed.

Since there is no left side of the pivot, we proceed with the right side.

	5	4	2	3	1	find the pivot
ĺ	1	4	2	3	5	swap 5 and the pivot

Again, 1 is placed correctly in the far left. The following sequence is left.

Now we have

since the pivot 5 is already correctly placed, there is no swapping to do. We continue with

4	2	3	find the pivot
2	4	3	swap $4$ and $2$
2	3	4	swap 4 and the pivot

After this, the left side of the inital partition is correctly sorted.

						_				
0	1	2	3	4	5		6	9	7	8

We continue with the right side.

9	7	8	find the pivot
7	9	8	swap $9$ and $7$
7	8	9	swap 9 and the pivot

At the end of the algorithm we have the correctly sorted list.

0   1   2   3   4   5   6   7   8   9
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