# Curve Tracing

Let

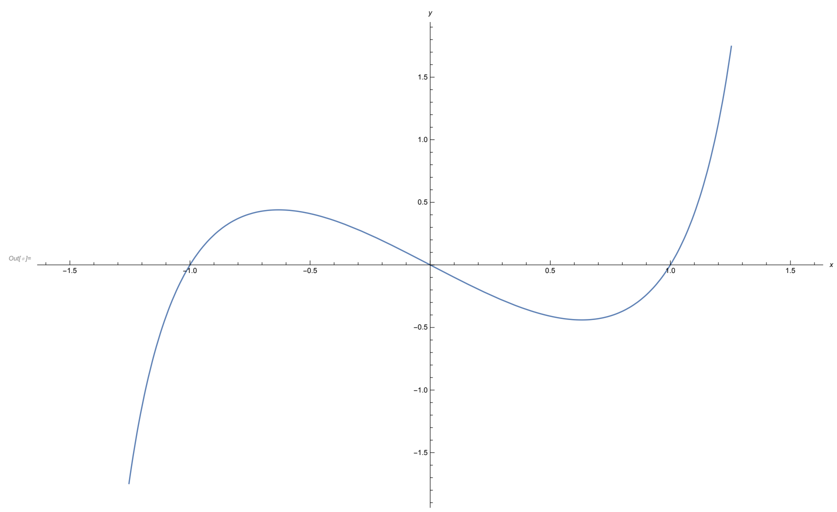
$\boldsymbol{f[\mathrm{x\_}] := \left(x^2 - 1\right) * \mathrm{Tan}[x]}$

be a function defined on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. We want to find its roots, extreme values i.e. minima, maxima, and saddle points, inflection points, and if applicable, singularities and limits.

Plot

The plot for the function f is the following.

$\mathbf{Plot}\left[\left(x^2 - 1\right) * \mathbf{Tan}[x], \left\{x, -\frac{\pi}{2}, \frac{\pi}{2}\right\}, \mathbf{ImageSize} \rightarrow \mathbf{1000}, \mathbf{AxesLabel} \rightarrow \{x, y\}\right]$




Roots

Mathematica will find the roots for us numerically.

$\mathbf{FindRoot}[f[x], \{x, -1\}]$

$\{x \rightarrow -1.\}$

$\mathbf{FindRoot}[f[x], \{x, 0\}]$

$\{x \to 0.\}$

$\mathbf{FindRoot}[f[x], \{x, 1\}]$

$\{x \to 1.\}$

Therefore, the function f has three roots (intersections with the x axis). These are (-1, 0); (0, 0); and (1, 0).

$\mathbf{Simplify}[f[x] = 0]$

0

Setting the function f to zero, we get the intersection with the y axis which is at (0, 0).

Extreme Values

In this section, we want to find all maxima, minima and saddle points of the function f.

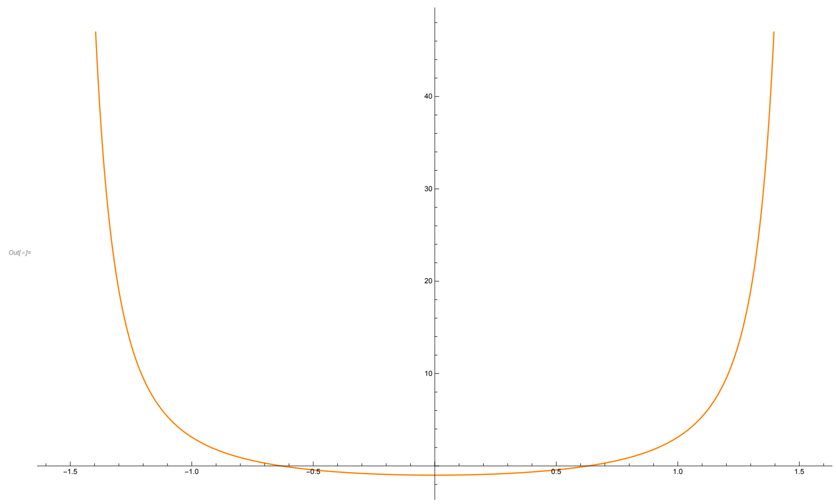
## Derivatives

Before we consider the extreme values for the function f, we want to find its derivatives. We have

$f'[x]$

$(-1 + x^2)\,\mathbf{Sec}[x]^2 + 2x\mathbf{Tan}[x]$

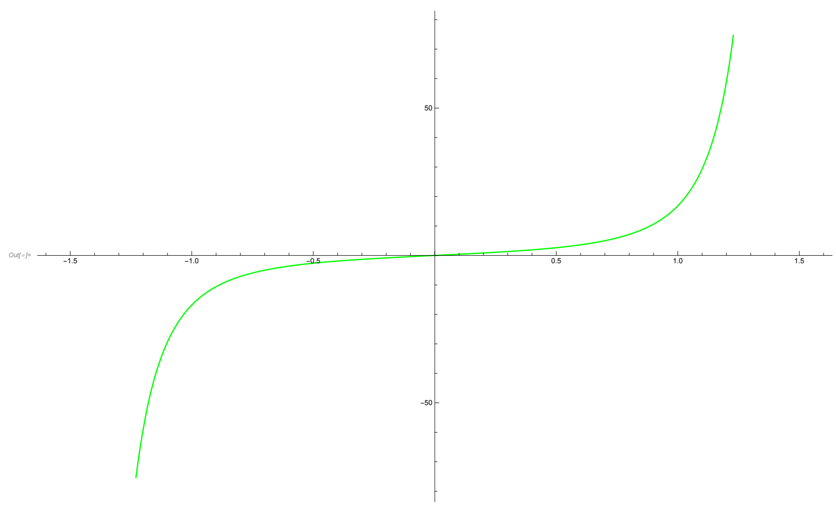
$\mathbf{Plot}\left[f'[x], \left\{x, \frac{-\pi}{2}, \frac{\pi}{2}\right\}, \mathbf{PlotStyle} \to \mathbf{Orange}, \mathbf{ImageSize} \to \mathbf{1000AxesLabel} \to \{x, y\}\right]$




**f”[x]**

$4x\text{Sec}[x]^2 + 2\text{Tan}[x] + 2\left(-1 + x^2\right)\text{Sec}[x]^2\text{Tan}[x]$

**Plot** $\left[f\text{''}[x], \left\{x, \frac{-\pi}{2}, \frac{\pi}{2}\right\}, \text{ImageSize} \to 1000, \text{PlotStyle} \to \text{Green}\right]$




Putting the original function and its two derivatives in a plot, we get

**Plot** $\left[\left\{\left(x^2 - 1\right)\text{Tan}[x], f'[x], f\text{''}[x]\right\}, \left\{x, \frac{-\pi}{2}, \frac{\pi}{2}\right\}, \text{ImageSize} \to 1000, \text{PlotStyle} \to \{\text{Dashed}, \text{Orange}, \text{Green}\}, \text{Plo}\right.$

## Extreme Values

For the extreme values we have

**FindMinimum** $\left[\left(x^2-1\right)\text{Tan}[x],\{x,0.5\}\right]$

$\{-0.439732,\{x\to 0.631264\}\}$

**FindMaximum** $\left[\left(x^2-1\right)\text{Tan}[x],\{x,-0.5\}\right]$

$\{0.439732,\{x\to -0.631264\}\}$

To find saddle points we have

**FindRoot**$[f'[x],\{x,-1\}]$

$\{x\to -0.631264\}$

**FindRoot**$[f'[x],\{x,1\}]$

$\{x\to 0.631264\}$

These extreme values are already a minimum or a maximum; therefore, the function f does not have any saddle points.

Inflection Points

To find inflection points, we compute

**FindRoot**$[f"[x],\{x,0\}]$

$\{x\to 0.\}$

$f"[0]$

0

Therefore, the function f does not have any inflection points.

Singularity

The function f does not have a singularity in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ since tan(x) $=\frac{\sin(x)}{\cos(x)}$, but cos(x) is never 0 for the given domain. In other words, the function f is defined everywhere on the given domain.

Limits

Since we do not have any singularity and the domain is a closed set, there is no limit to sensibly evaluate.