

Chapter 1

Interpolation

Chapter 2

Finite Element

Chapter 3

Newton and other algorithms

Definition 1 (Newton's Method). Let $D \subset \mathbb{K}^n$ an nonempty open domain and $x_0 \in D$. For $j \in \mathbb{N}$ Newton's Method is recursively defined as

$$x_{j+1} = x_j - \frac{f(x_j)}{f'(x_j)} \quad (3.1)$$

and stops if $x_{j+1} \notin D$ or $f'(x_j) = 0$.

Theorem 1.1. Let $x^* \in D$ be a root of $f \in C^1(D, \mathbb{K}^n)$ with $f'(x^*)$ regular (if f is one-dimensional, then $f'(x^*) \neq 0$), then the following hold.

1. There exists a $\epsilon > 0$ such that the Newton's Method does not break for all $x_0 \in \overline{B(x^*, \epsilon)}$. Moreover, we have the convergence

$$\lim_{j \rightarrow \infty} x_j = x^* \quad (3.2)$$

2. Superlinear convergence. For all $j \in \mathbb{N}$ and a null sequence q_j