# Chapter 1

# Rings

**Example 1.0.1.**

1. $(\mathbb{Z}, +, \cdot)$
2. All fields, such as $(\mathbb{Q}, +, \cdot)$, $(\mathbb{R}, +, \cdot)$, and $(\mathbb{C}, +, \cdot)$, are rings.
3. Let $R$ be a commutative ring, then $R[X]$, the set of polynomials with coefficients in $R$, is again a ring, e.g. $\mathbb{Z}[X]$, $\mathbb{Q}[X]$, and $\mathbb{R}[X]$.
4. For any ring $R$ and for any $n \in \mathbb{N}$, the set of all square $n$-by-$n$ matricies with entries from $R$, forms a ring with matrix addition and matrix multiplication as operations. If $n = 1$, this matrix ring is isomorphic to $R$ itself. For $n > 1$ (and $R$ not a zero ring), this matrix is noncommutative. More concretely, $\mathrm{Mat}_{3\times 3}(\mathbb{R})$ is a noncommutative ring.

## 1.1 No idea yet

**Definition 1.1** (Fractional Ideal)**.** Let $A$ be an integral domain.

1. A fractional ideal of $A$ is an $A$-submodule $I \subset \mathrm{Quot}(A)$ such that $dI \subset A$ for some denominator $d \in A \setminus \{0\}$.
2. A principal fractional ideal is a fractional ideal of the form $(r) = rA = \{ar \mid a \in A\}$

**Example 1.1.1.**

- The subset
$$\frac{3}{25}\mathbb{Z} = \left\{ \frac{3n}{25} \in \mathbb{Q} \,\middle|\, n \in \mathbb{Z} \right\} \subset \mathbb{Q} \tag{1.1}$$
is a principal fractional ideal of $\mathbb{Z}$