0.1 Exact Sequence

Definition 1.

Theorem 2.

$$0 \longrightarrow M' \longrightarrow M \longrightarrow M'' \longrightarrow 0$$

$M/M' \cong M''$

Example 2.1. 1.

$$0 \longrightarrow N \longrightarrow M \longrightarrow M/N \longrightarrow 0$$

$$0 \longrightarrow M' \longrightarrow M' \oplus M'' \longrightarrow M'' \longrightarrow 0$$

Definition 3. Split

1. $g \circ s = \mathrm{id}_{M''}$
2. s is injective
3. this does not imply that M and M'' are isomorphic, because for that we also need $s \circ g = \mathrm{id}_M$
4. g admits section is a very good naming, because basically it means that M'' lies in M
5. M'' may be viewed as a submodule of M
6. since f is injective, we may see M' as a submodule of M
7. $M' \cap M'' = 0 \iff f(M) \cap s(M'') = 0 \iff \ker(g) \cap s(M'') = 0$
8. And the last equation is true because if $x \in \ker(g) \cap s(M'')$, then $g(x) = 0$ and $x = s(m'')$, so $g(s(m'')) = m'' = 0$, putting it back together $x = s(m'') = s(0) = 0$.
9. $M = M' + M'' \iff M = f(M) + s(M'') \iff M = \ker(g) + s(M'')$
10. I'm not sure about this, but w/E
11. thus: $M = M' \oplus M''$

Definition 4.

$$\mathrm{coker}(f) = M/\mathrm{im}(f)$$

Example 4.1.

$$0 \longrightarrow \ker f \longrightarrow M' \longrightarrow M \longrightarrow \mathrm{coker} f \longrightarrow 0$$

0.2 Snake Lemma