**1.** Compute the values of the following sums.

1. $\sum_{k=0}^{\infty} x^k$, for $|x| < 1$,
2. $\sum_{k=0}^{\infty} \frac{x^k}{k!}$,
3. $\sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

**Solution:**

1. The sum submits to the ratio test and converges to some number. Denote this number with $S$. We have

$$S = \tag{1}$$

**2.** For $\lambda > 0$ let $X \sim \mathrm{Exp}(\lambda)$ and let

$$Y := \lceil X \rceil := \min\{\, n \in \mathbb{N} \mid n \geq X \,\} \tag{2}$$

Show that for the parameter $p = 1 - e^{-\lambda}$ holds $Y \sim \mathrm{Geo}(p)$.

**Solution:**
For the random variable $X$ we have the distribution $f^X(x) = \lambda e^{-\lambda x} \mathbb{1}_{[0,\infty)}(x)$ that generates the probability measure via

$$\mathbb{P}(X \in (a, b]) = \int_a^b \lambda e^{-\lambda x} \mathbb{1}_{[0,\infty)}(x)\, \mathrm{d}x, \qquad \text{for all } a, b \in \mathbb{R}. \tag{3}$$

If $Y = \lceil X \rceil$, then we have

$$\mathbb{P}(Y \in (a, b]) = \mathbb{P}(\lceil X \rceil \in (a, b]) \tag{4}$$