Problem 1

Let $\varphi : X_0 \subset X \longrightarrow Y$ be a continuous map. Recall the construction of glueing X to Y via φ from class.

1. Show that Y can be considered to be a subspace of $Y \cup_\varphi X$. Show that the same is true for X if φ is a homeomorphism onto its image.

   Proof.

$$
\begin{aligned}
Y \cup_\varphi X &= X \cup Y/ \sim \\
\mathcal{O}_{X \cup Y/\sim} &= \left\{ U \subset X \cup Y/ \sim \mid \pi^{-1}(U) \in \mathcal{O}_{X \cup Y} \right\} \\
\mathcal{O}_{X \cup Y} &= \{ U \times V \mid U \in \mathcal{O}_X, V \in \mathcal{O}_Y \}
\end{aligned}
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$\square$