Structure Theorem

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1 Category

Definition 1. A category $\mathcal{C}$ consists of

1. a class of objects, denoted by $\mathrm{Ob}(\mathcal{C})$, and
2. a collection of of morphisms, denoted by $\mathrm{Hom}(\mathcal{C})$

Example 1.1.
1. The category of sets
2. The category of groups
3. the category of rings
4. the category of fields
5. For a ring A, the category of left modules, right modules
6. the single element category

Definition 2 (Subcategory)**.**

Definition 3 (Opposite Category)**.**

2 Functors

Definition 4. Let $\mathcal{C}$ and $\mathcal{D}$ be two categories. A functor F from $\mathcal{C}$ to $\mathcal{D}$ is a mapping that

1. associates each object X in $\mathcal{C}$ to an object $F(X)$ in $\mathcal{D}$, and
2. associates each morphism $f : X \to Y$ in $\mathcal{C}$ to a morphism $F(f) : F(X) \to F(Y)$ in $\mathcal{D}$ such that
   a) $F(\mathrm{id}_X) = \mathrm{id}_{F(X)}$ for every object X in $\mathcal{C}$, and
   b) $F(g \circ f) = F(g) \circ F(f)$ for all morphisms $f : X \to Y$ and $g : Y \to Z$ in $\mathcal{C}$.

Definition 5 (Covariant and Contravariant)**.**

Definition 6 (Faithful and Full)**.**

Intuition. Faithful is basically injective, full is basically surjective.