Chapter 1

Rings

Concepts

1. zero-Divisors
2. nilpotent
3. ideals

Properties of Ring

1. local ring
2. noetherian ring
3. artinian ring
4. principal ideal domain
5. unique factorization domain
6. integral domain

Part I

Ideals

Chapter 2

Ideal Operation

Definition 2.1 — .
Let R be a ring and $\{\mathfrak{a}_i\}_{i \in I}$ a collection of ideals for an arbitary index set I.

1. The sum of ideals is the smallest ideal in R containing each $\mathfrak{a}_i$, i.e.

$$\sum_{i \in I} \mathfrak{a}_i := \left\{ \sum_{i \in I} a_i \mid a_i \in \mathfrak{a}_i \text{ for all } i \in I, \text{ and } a_i = 0 \text{ for almost all } i \in I \right\}. \tag{2.1}$$

   If $\mathfrak{a}$ and $\mathfrak{b}$ are ideals, then

$$\mathfrak{a} + \mathfrak{b} = \{ a + b \mid a \in \mathfrak{a} \text{ and } b \in \mathfrak{b} \}. \tag{2.2}$$

2. The product of ideals is the smallest ideal in R