Exercise Sheet 2

Exercise 1
Solution

1. $\mathbb{Z} \times \mathbb{Z}$ is not a Dedekind domain as it is not even an integral domain. Take $(1,0) \in \mathbb{Z} \times \mathbb{Z}$ and $(0,1) \in \mathbb{Z} \times \mathbb{Z}$ for example. $(1,0) \cdot (0,1) = (0,0)$ even though we chose nonzero elements.

2. We have $\mathbb{Z}[X]/(X^2+3) \cong \mathbb{Z}[\sqrt{-3}]$; therefore, $\mathbb{Z}[X]/(X^2+3)$ is an integral domain.

   $\mathbb{Z}[X]/(X^2+3)$ is also noetherian as $\mathbb{Z}$ is noetherian and therefore it's polynomial ring and the quotient ring.