Exercise 0.1
Let $\Omega = \{a, b, c, d, e\}$ and $M = \{\{a\}, \{a, b\}\}$.

1. Give $\sigma(M)$.
2. Give a σ-algebra $\mathcal{A}$ over Ω with $\mathcal{A} \supset \sigma(M)$ and $\mathcal{A} \neq \sigma(M)$.
3. Give two measures which are not identical, but have the same value on $\sigma(M)$.

Solution 0.1

1. In all σ-algebras Ω and $\varnothing$ are included, so we immediately have $\Omega, \varnothing \in \sigma(M)$. Further, all complements are included, hence we have $\{b, c, d, e\} \in \sigma(M)$ and $\{c, d, e\} \in \sigma(M)$. We also have $\{a\} \cup \{c, d, e\} = \{a, c, d, e\} \in M$ and its complement $\{b\} \in M$. Another union included is $\{b\} \cup \{c, d, e\} = \{b, c, d, e\}$. To sum up, we have

$$\{\Omega, \varnothing, \{a\}, \{b\}, \{a, b\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\} = M \tag{1}$$

   These are indeed all the subsets of M as there is no more new complements or unions to be made.

2. $\mathcal{P}(\Omega) =: \mathcal{A}$ is a σ-algebra and fullfills the required properties.
3.