Definition 1. A K-algebra $\mathcal{A}$ is a ring that is also a K-vector space, such that for $\lambda \in K$ and $a, b \in \mathcal{A}$ such that

$$\lambda(ab) = (\lambda a)b = a(\lambda b) \tag{1}$$

Example 1.1.

1. ring containing K such as field extensions
2. $K[X, Y]$
3. $K[[X]]$
4. $\mathrm{Mat}_{n \times n}(K)$

Definition 2. A R-algebra $\mathcal{A}$ is a ring that is also an R-module.

Theorem 3. Let K be a field that is finitely generated as a $\mathbb{Z}$-module then its finite.

Proof.

1. Let $x \in K$.
2. $x = z_1 x_1 + \cdots + z_n x_n$

□