

Exercise Sheet 1

Exercise 2

Let $k \in \mathbb{Z}_{>0}$.

1. Show that $k = a^2 + b^2$ for some $a, b \in \mathbb{Z}$ if and only if for every prime $p \equiv 3 \pmod{4}$, the exponent of p in the prime decomposition of k (in \mathbb{Z}) is even.
2. In this case, describe how to obtain all solutions $(a, b) \in \mathbb{Z}^2$.

Solution

1.

Let $k = a^2 + b^2$. According to Theorem 1.0.1. this is equivalent to $k = 2$ or $k \equiv 1 \pmod{4}$. If $k = 2$, then it is clear immediately. So consider the case $k \neq 2$.