Contents

[1]

Definition 0.0.1. For a subset T of the polynomial ring $A = k[X_1, \ldots, X_n]$, we define the zero set of T to be the common zeros of all the elements of T, i.e.

$$Z(T) = \{ P \in \mathbb{A}^n \mid f(P) = 0 \text{ for all } f \in T \} .$$

Definition 0.0.2. A subset Y of $\mathbb{A}^n$ is an algebraic set if there exists a subset $T \subset A = k[X_1, \ldots, X_n]$ such that $Y = Z(T)$.

Definition 0.0.3. If $Y \subset \mathbb{A}^n$ is an affine algebraic set, we define the affine coordinate ring $A(Y)$ of Y, to be $A/I(Y)$.

Exercise 0.0.1. *Let Y be the plane curve $y = x^2$ (i.e., Y is the zero set of the polynomial $f = y - x^2$). Show that $A(Y)$ is isomorphic to a polynomial ring in one variable over k.*

Solution. By definition 0.0.3, we simply have $A(Y) = k[X,Y]/(Y - X^2)$. The isomorphism follows from the isomorphism theorem and the map $f : k[X,Y] \to k[X]$ where we set $f(Y) = X^2$.

Exercise 0.0.2. *Let Z be the plane curve $xy = 1$. Show that $A(Z)$ is not isomorphic to a polynomial ring in one variable over k.*

Solution.

Bibliography

[Har77] Robin Hartshorne. *Algebraic Geometry*. New York: Springer, 1977.