

Category Theory

February 22, 2023

1 Category

Definition 1. A category \mathcal{C} consists of

1. a class of objects, denoted by $\text{Ob}(\mathcal{C})$, and
2. a collection of morphisms, denoted by $\text{Hom}(\mathcal{C})$

Example 1.1. 1. The category of sets

2. The category of groups
3. the category of rings
4. the category of fields
5. For a ring A , the category of left modules, right modules
6. the single element category

Definition 2 (Subcategory).

Definition 3 (Opposite Category).

2 Functors

Definition 4. Let \mathcal{C} and \mathcal{D} be two categories. A functor F from \mathcal{C} to \mathcal{D} is a mapping that

1. associates each object X in \mathcal{C} to an object $F(X)$ in \mathcal{D} , and
2. associates each morphism $f : X \rightarrow Y$ in \mathcal{C} to a morphism $F(f) : F(X) \rightarrow F(Y)$ in \mathcal{D} such that
 - a) $F(\text{id}_X) = \text{id}_{F(X)}$ for every object X in \mathcal{C} , and
 - b) $F(g \circ f) = F(g) \circ F(f)$ for all morphisms $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ in \mathcal{C} .

Definition 5 (Covariant and Contravariant).

Definition 6 (Faithful and Full).

Intuition. Faithful is basically injective, full is basically surjective.