

Exercise Sheet 8

Exercise 1

Solution to 1.

We want to show that I_α is a nonzero ideal. First we reformulate the statement to be proven.

$$I_\alpha = \{a \in A \mid aB \subseteq A[\alpha]\} \neq 0 \quad (1)$$

$$\iff \exists a \in A \setminus \{0\} : aB \subseteq A[\alpha] \quad (2)$$

$$\iff \exists a \in A \setminus \{0\}, \exists b \in B, \exists \lambda_1, \lambda_2 \in A : ab = \lambda_1 + \lambda_2 \alpha. \quad (3)$$

Consider the last equation $ab = \lambda_1 a + \lambda_2 \alpha$. Since both sides of the equation are elements of a field and a is nonzero, we can cancel out a and get

$$b = \lambda_1 + \lambda_2 a^{-1} \alpha. \quad (4)$$