Topology

K

September 20, 2022

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Chapter 1

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"⇒": Let $f : X_1 \longrightarrow X_2$ be a homeomorphism and fix a subset (not necessarily open) $U \in \mathcal{T}_1$.

1. Assume U is open in X_2. Because f is continuous, the image of open subsets are again open, thus $f(U)$ lies in $\mathcal{T}_2$.

2. On the other hand, if $f(U)$ is open in X_2, then since f is bijective we have

$$f^{-1}\left(f\left(U\right)\right) = U.$$

   Because f is continuous, the preimage of open subsets under f is open. We may therefore conclude U is open in X_1.

We have shown that if f is a homeomorphism, then $f(\mathcal{T}_1) = \mathcal{T}_2$.
"⇐": Let $f : X_1 \longrightarrow X_2$ be a bijective map such that $f(\mathcal{T}_1) = \mathcal{T}_2$. Fix a open subset $V \in \mathcal{T}_2$.