Chapter 1

Interpolation

Example 0.1. Consider the domain $[2, 10]$ partitioned into 5 points, i.e. $\{2, 4, 6, 8, 10\}$ and a function $f : [0, 10] \to \mathbb{R}$, $x \mapsto f(x) = \ln(x)$. The y-values then are

$$\ln(2) \approx 0.6931 \quad \ln(4) \approx 1.3862 \quad \ln(6) \approx 1.7917 \quad \ln(8) \approx 2.0794 \quad \ln(10) \approx 2.3025. \tag{1.1}$$

Computing the Lagrange polynomials gives

$$\begin{align}
L_1(x) &= \ln(2) \cdot \frac{x-4}{2-4} \cdot \frac{x-6}{2-6} \cdot \frac{x-8}{2-8} \cdot \frac{x-10}{2-10} \tag{1.2}\\
&= 5\ln(2) - \frac{77}{24}x\ln(2) + \frac{71}{96}x^2\ln(2) - \frac{7}{96}x^3\ln(2) + \frac{1}{384}x^4\ln(2) \tag{1.3}\\
L_2(x) &= \ln(2) \cdot \frac{x-2}{4-2} \cdot \frac{x-6}{4-6} \cdot \frac{x-8}{4-8} \cdot \frac{x-10}{4-10} \tag{1.4}\\
&= -10\ln(2) + \frac{107}{12}x\ln(2) - \frac{59}{24}x^2\ln(2) + \frac{13}{48}x^3\ln(2) - \frac{1}{96}x^4\ln(2) \tag{1.5}
\end{align}$$