1 Rigidity Conjecture

Definition 1. Let $f(X) \in \mathbb{C}[[X]]$ be a power series. We call a power series $f^{-1}(X) \in \mathbb{C}[[X]]$ the compositional inverse of f, if it satisfies $f(f^{-1}(X)) = f^{-1}(f(X)) = X$.

Proposition 2. A power series $f(X) = a_0 + a_1X + \cdots \in \mathbb{C}[[X]]$ has a compositional inverse if and only if $a_0 = 0$ and $a_1 \neq 0$. Moreover, if the compositional inverse exists, then it is unique.

Proof. Assume f has a compositional inverse and denote the compositional inverse by $f^{-1}(X) = b_0 + b_1X + b_2X^2 + \cdots$. Writing out $f(f^{-1}(X)) = X$ gives

$$a_0 + a_1(b_0 + b_1X + b_2X^2 + \cdots)$$

□