My notes on "The Strong Factorial Conjecture" by Eric Edo and Arno van den Essen. See: https://arxiv.org/abs/1304.3956

Theorem 1 (Conjecture 2.13)**.** Let $a(X) \in \mathbb{C}[X]$ be a polynomial of degree less or equal to $m+1 \in \mathbb{N}_+$ such that $a(X) \equiv X \mod X^2$. If the first m consecutive coefficient of the compositional inverse $a^{-1}(X)$ vanish, then $a(X) = X$.

Theorem 2 (Conjecture 2.14)**.** Let $a(X) \in \mathbb{C}[X]$ be a polynomial of degree less or equal to $m+1 \in \mathbb{N}_+$ such that $a(X) \equiv X \mod X^2$. If the coefficients of $X^{n+1}, \ldots, X^{n+m}$ of the compositional inverse vanish, then $a(X) = X$.

Remark. $R(m)$ if and only if $R(m)_n$ for all $n \in \mathbb{N}_+$.

Proof. Let $R(m)$ be true for a $m \in \mathbb{N}_0$.

Then $R(m)_1$ is true, i.e. if $\deg(a) \leq m+1$ and if the □

Remark. If we denote the polynomial $a(X)$ by $\sum_{k \in \mathbb{N}_0} a_k X^k$ for some $a_k \in \mathbb{C}$ for all $k \in \mathbb{N}_0$, then the condition $a(X) \equiv X \mod X^2$ amounts to $a_0 = 0$ and $a_1 = 1$.

Moreover, we have this:

A power series has a compositional inverse if and only if $a_1 \neq 0$. In that case, the inverse is unique.

See

https://www.amazon.com/dp/B00HMUGS4S

https://math.stackexchange.com/questions/2520744/finding-compositional-inverses-for-formal-power-series

My questions:

1. What if $a_0 \neq 0$? Pick $a_0 = 3$.

   Let $f \in \mathbb{C}[X]$ be a polynomial with $a_0 \neq 0$. Then we may write $f(X) = g(X) + a_0$ where g has a compositional inverse. Thus it it

$$\begin{aligned} g^{-1}(g(X) + a_0) &= g^{-1}(g(X)) + g^{-1}(a_0) \\ &= X + g^{-1}(a_0) \end{aligned}$$

$$\begin{aligned} h(X) &= g^{-1}(X) + g^{-1}(a_0) \\ h(f(X)) &= h(g(X) + a_0) \\ &= g^{-1}(g(X) + a_0) + g^{-1}(a_0) \\ &= X \end{aligned}$$

   Let $f \in \mathbb{C}[X]$ be a polynomial with $a_1 \neq 1$ and $a_1 \neq 0$. Then we may write $f(X) =$

https://www.math.uwaterloo.ca/ dgwagner/co430I.pdf

$^{p}roof$

Proposition 3. 1. The polynomial $a(X)$ is invertible for the composition.

2. For all $i \in \{1, \ldots, \deg(a-1)\}$, the coefficient a_i is nilpotent element in A.