

Number Theory

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Theorem 0.0.1 (Fundamental Theorem of Arithmetic).

Question: To what extent does this hold for other kind of numbers such as $\mathbb{Z}[i]$ or $\mathbb{Z}[\sqrt{5}]$.

Definition 0.1 (Number Field). A **number field** is an **algebraic extension** of \mathbb{Q} of **finite degree**.

Let k be a **number field**, then the **ring of integers** \mathcal{O}_k is the **subset** formed by the elements

Example 0.1.1. 1. Set $k = \mathbb{Q}[\sqrt{5}]$.

Question: Is $\mathcal{O}_k = \mathbb{Z}[\sqrt{5}]$?

Answer: No. Consider $a + b\sqrt{5} \in \mathbb{Q}[\sqrt{5}]$, then it satisfies $(x - a)^2 = b^2 5$.