

Integration and Integration

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Part I

Solving Integrals

Chapter 1

Trigonometric Functions

Chapter 2

Hyperbolic Functions

Chapter 3

Solving Integrals

Theorem 3.0.1 (Important Identities).

$$\int x^\alpha dx = \frac{1}{\alpha+1} x^{\alpha+1} + c \quad \text{For all } \alpha \in \mathbb{N} \quad (3.1)$$

$$\int \frac{1}{x} dx = \ln |x| + c \quad \text{If } x \neq 0. \quad (3.2)$$

$$\int e^x dx = e^x + c \quad (3.3)$$

$$\int \cos x dx = \sin x + c \quad (3.4)$$

$$\int \sin x dx = -\cos x + c \quad (3.5)$$

$$\int \frac{1}{1+x^2} dx = \arctan(x) + c \quad (3.6)$$

$$\int \quad (3.7)$$

Exercise 3.1

$$\int (1-t)^9 dx \quad (3.8)$$

Solution 3.1

Substitute $1-t = u \Rightarrow -1 dt = du$, then we have

$$\int (1-t)^9 dx = - \int u^9 du \quad (\text{Substitution.}) \quad (3.9)$$

$$= -\frac{u^{10}}{10} + c \quad (\text{Important identities: 3.1}) \quad (3.10)$$

$$= -\frac{(1-t)^{10}}{10} + c. \quad (u = 1-t.) \quad (3.11)$$

Exercise 3.2

$$\int (x^2+1)^2 dx \quad (3.12)$$

Solution 3.2

Substitute $x^2 + 1 = u \Rightarrow 2x \, dx = du$.

$$\int (x^2 + 1)^2 \, dx = \int x^4 + 2x^2 + 1 \, dx \quad (3.13)$$

$$= \int x^4 \, dx + 2 \int x^2 \, dx + \int 1 \, dx \quad (3.14)$$

$$= \frac{1}{5}x^5 + \frac{2}{3}x^3 + x \quad (3.15)$$

Exercise 3.3

$$\int \frac{1}{x^2 - x + 1} \, dx \quad (3.16)$$

Solution 3.3

Substitute $x - \frac{1}{2} = u \Rightarrow 1 \, dx = du$.

$$\int \frac{1}{x^2 - x + 1} \, dx = \int \frac{1}{(x - \frac{1}{2})^2 + \frac{3}{4}} \, dx \quad (3.17)$$

$$= \int \frac{1}{u^2 + \frac{3}{4}} \, dx \quad (3.18)$$

$$= \frac{3}{4} \int \frac{1}{\frac{4}{3}u^2 + 1} \, du \quad (3.19)$$

The last step was done in order to force the integrand to be in a form similar to

$$\frac{d}{dx} \arctan x = \frac{1}{x^2 + 1} \quad (3.20)$$

Lastly, substitute in the equation above $\frac{2}{\sqrt{3}}u = v \Rightarrow \frac{2}{\sqrt{3}} \, du = dv$. It is $u = \frac{\sqrt{3}}{2}v$ and $du = \frac{\sqrt{3}}{2} \, dv$, therefore, we have

$$\frac{3}{4} \int \frac{1}{\frac{4}{3}u^2 + 1} \, du = \frac{3}{4} \int \frac{1}{\frac{4}{3}(\frac{\sqrt{3}}{2}v)^2 + 1} \frac{\sqrt{3}}{2} \, dv \quad (3.21)$$

$$= \frac{3}{4} \frac{\sqrt{3}}{2} \int \frac{1}{v^2 + 1} \, dv \quad (3.22)$$

$$= \frac{3\sqrt{3}}{8} \arctan v \quad (3.23)$$

$$= \frac{3\sqrt{3}}{8} \arctan \left(\frac{\sqrt{3}}{2}u \right) \quad (3.24)$$

$$= \frac{3\sqrt{3}}{8} \arctan \left(\frac{\sqrt{3}}{2}(x - \frac{1}{2}) \right) \quad (3.25)$$