Problem 01.2

Let $\Omega, \mathcal{A}, \mathbb{P}$ a probability space and $M, \mathcal{F}$ a measurable space. Moreover, let $X : \Omega \to M$ a $(\mathcal{A}, \mathcal{F})$-measurable random variable. Show that

$$\mathbb{P}^X(B) := \mathbb{P}(X \in B) = \mathbb{P}(X^{-1}(B)), \qquad B \in \mathcal{F} \tag{1}$$

defines a probability measure on $(M, \mathcal{F})$.

Solution