

Chapter 1

Rings

Example 1.0.1. 1. $(\mathbb{Z}, +, \cdot)$

2. All fields, such as $(\mathbb{Q}, +, \cdot)$, $(\mathbb{R}, +, \cdot)$, and $(\mathbb{C}, +, \cdot)$, are rings.
3. Let R be a commutative ring, then $R[X]$, the set of polynomials with coefficients in R , is again a ring, e.g. $\mathbb{Z}[X]$, $\mathbb{Q}[X]$, and $\mathbb{R}[X]$.
4. For any ring R and for any $n \in \mathbb{N}$, the set of all square n -by- n matrices with entries from R , forms a ring with matrix addition and matrix multiplication as operations. If $n = 1$, this matrix ring is isomorphic to R itself. For $n > 1$ (and R not a zero ring), this matrix is noncommutative. More concretely, $\text{Mat}_{3 \times 3}(\mathbb{R})$ is a noncommutative ring.