1 Probability Space

Definition 1 (Sample Space)**.** An outcome is a possible result of an experiment or trial. The sample space (also called sample discription space or possibility space) of an experiment or random trial is the set of all possible outcomes or results of that experiment. A subset of a sample space is called an event. Moreover, we say the following.

- The empty event $A = \varnothing$ is called the impossible event.
- The sample space itself as an event $A = \Omega$ is called the certain event.
- The complementary event of any event A is the set of all outcomes not in A.

Definition 2 (Probability Measure)**.** Let Ω be a set and $\mathcal{A}$ a σ-algebra over Ω. A function $\mathbb{P} : \mathcal{A} \to [0, 1]$ is a probability measure if it is a measure and if $\mathbb{P}(\Omega) = 1$.

Definition 3 (Probability Space)**.** A probability space is the structrue $(\Omega, \mathcal{A}, \mathbb{P})$ consisting out of a sample space Ω, a σ-algebra over Ω and a probability measure $\mathbb{P} : \mathcal{A} \to [0, 1]$.

Definition 4 (Random Variable)**.** Let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space and $(S, \mathcal{S})$ a measureable space. A random variable is a measure function $X : \Omega \to S$. The distribution of a random variable X is given by the probability measure

$$\mathbb{P}^X(B) := \mathbb{P}(X \in B) = \mathbb{P}(X^{-1}(B)), \qquad B \in \mathcal{S}. \tag{1}$$

2 Discrete Distribution

Definition 5. The geometric distribution with success chance $p \in (0, 1]$ on $\Omega = \mathbb{N}$ is given by the discrete probability mass function

$$p_{\mathrm{Geo}(p)}(k) = (1 - p)^{k-1} p, \qquad k \in \mathbb{N}. \tag{2}$$

If $p^X = p_{\mathrm{Geo}(p)}$ for a random variable X, we say X is geometrically distributed and denote $X \sim \mathrm{Geo}(p)$.

3 Continuous Distribution

Definition 6. The exponential distribution with parameter $\lambda > 0$ is given by the continuous probability mass function

$$f_{\mathrm{Exp}(\lambda)}(x) = \lambda e^{-\lambda x} \mathbb{1}_{[0,\infty)}(x), \qquad x \in \mathbb{R}. \tag{3}$$

If $f^X = f_{\mathrm{Exp}(\lambda)}$ for a real-valued random variable X we write $X \sim \mathrm{Exp}(\lambda)$.