

1. Compute the values of the following sums.

1. $\sum_{k=0}^{\infty} x^k$, for $|x| < 1$,

2. $\sum_{k=0}^{\infty} \frac{x^k}{k!}$,

3. $\sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$

Solution:

1. The sum submits to the ratio test and converges to some number. Denote this number with S . We have

$$S = \tag{1}$$

2. For $\lambda > 0$ let $X \sim \text{Exp}(\lambda)$ and let

$$Y := \lceil X \rceil := \min\{ n \in \mathbb{N} \mid n \geq X \} \tag{2}$$

Show that for the parameter $p = 1 - e^{-\lambda}$ holds $Y \sim \text{Geo}(p)$.

Solution:

For the distribution of Y we have

$$f^Y(x) = \mathbb{P}(Y = x) = \mathbb{P}(\lceil X \rceil = x) = \mathbb{P}(x \leq X < x + 1) = F^X(x + 1) - F^X(x). \tag{3}$$