

1. true
2.  $A \uparrow B$

## 1 Sheffer Stroke

**Definition 1.1.**  $A \uparrow B$  if  $x$  is not in the intersection of  $A$  and  $B$ .

$$A \uparrow B = X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B) \quad (1)$$

**Proposition 1.1.1.** 1. Sheffer stroke is associative.

2. Sheffer stroke is commutative.

3. For  $X$  the only identity element is  $\emptyset$ .

*Proof.* Let  $A, B, C \subset X$ . We have

$$(A \uparrow B) \uparrow C = (X \setminus (A \uparrow B)) \cup X \setminus C \quad (2)$$

$$= (X \setminus A) \cup (X \setminus B) \cup (X \setminus C) \quad (3)$$

$$= (X \setminus A) \cup ((X \setminus B) \cup (X \setminus C)) \quad (4)$$

$$= A \uparrow ((X \setminus B) \cup (X \setminus C)) \quad (5)$$

$$= A \uparrow (B \uparrow C) \quad (6)$$

$$A \uparrow B = (X \setminus A) \cup (X \setminus B) \quad (7)$$

$$= (X \setminus B) \cup (X \setminus A) \quad (8)$$

$$= B \uparrow A \quad (9)$$

$$X \uparrow \emptyset = X \setminus (X \cap \emptyset) \quad (10)$$

$$= X \setminus \emptyset \quad (11)$$

$$= X \quad (12)$$

□

**Corollary 1.** Sheffer stroke does not form a group on the power set of  $X$ .