

Chapter 1

Interpolation

Chapter 2

Finite Element

Example 0.1. Let $K = [0, 1]$, \mathcal{P} be the set of linear polynomials and $\mathcal{L} = \{L_1, L_2\}$ where $L_1(p) = p(0)$ and $L_2(p) = p(1)$ for all $p \in \mathcal{L}$. Then $(K, \mathcal{P}, \mathcal{L})$ is a finite element.

Proof.

$$\begin{aligned}
 L_1(p_1) = 1 &\iff a_1 \cdot 0 + b_1 = 1 \\
 &\iff b_1 = 1 \\
 L_2(p_1) = 0 &\iff a_1 \cdot 1 + b_1 = 0 \\
 &\iff a_1 = -1 \\
 L_1(p_2) = 0 &\iff a_2 \cdot 0 + b_2 = 0 \\
 &\iff b_2 = 0 \\
 L_2(p_2) &=
 \end{aligned}$$

□

Example 0.2 (Counter Example to Nonconform P_2 -FE). Denote

$$P_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \quad P_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \quad P_3 = \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \quad (2.1)$$

and write $p(x) = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy$. Then, for L_1 we have

$$L_1(p) = p(Q_1) \quad (2.2)$$

$$= p(\mu P_2 + (1 - \mu)P_3) \quad (2.3)$$

$$= \mu p(P_2) + p(P_3) - \mu p(P_3) \quad (2.4)$$

$$\begin{aligned}
 &= \mu (a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy) \\
 &+ (a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy) \quad (2.5)
 \end{aligned}$$

$$- \mu (a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy)$$

$$\mu \quad (2.6)$$

Example 0.3. Let K be any rectangle, $\mathcal{P} = \mathcal{Q}_k$ and \mathcal{N} denote point evaluations at $\{(t_i, t_j) \mid 0 \leq i, j \leq k\}$ where $0 = t_0 < t_1 < \dots < t_{k-1} < t_k = 1$. Then $(K, \mathcal{P}, \mathcal{N})$ is a finite element.