Table of Contents

Introduction (Chapter 1) 1
Problems 1
Solutions 2
Financial Statements (Chapters 2,3,4) 4
Balance Sheets 4
Income Statements 4
Cash Flow Statements 5
Financial Ratios 6
Problems 7
Solutions 9
Time Value of Money (Chapters 5,6,7,8) 13
Key Terms 13
Key formulae: 14
Bonds 14
Mortgages 14
Dividends 15
Problems 16
Solutions 20
Budgeting for Capital Investments (Chapters 9,10) 24
NPV 24
Investment Decisions 24
Problems 25
Solutions 26
Market Risk (Chapters 12,13) 30
Variance and Standard Deviation 30
Measuring Risk 30
Problems 31
Solutions 33
Other Exam Tips 34

Introduction (Chapter 1)

There is nothing quantitative, and there isn't a huge amount of material in this section, so there's nothing much that can be added that isn't already in your notes and slides.

The exam problems tend to be limited to two forms:

- Choose the true or false statement out of four possibilities
- Choose all of the true or false statements (i, ii, iii, iv)

The main challenge – other than knowing the material – is to pay attention to what the problem is asking of you.

Problems

1.1. Which of the following is (are) not (a) disadvantage(s) of the corporate form of organization?

i. The principal-agent problem
ii. Double taxation of dividends
iii. Tax credits on interest payments
iv. Unlimited liability

a) i, ii
b) i, ii, iii
c) iii, iv
d) ii, iv

1.2. Your old friend Art Vandelay is looking to get out of architecture, and into the soup business. He has already decided on a name and location for his shop, but he is rather clueless when it comes to finance, and has given you a list of his four main goals. Which one of these goals actually makes sense for a business?

a) Minimize profits
b) Slow and unsteady earnings growth
c) Poor marketing and customer service
d) Minimize costs

1.3. Kramerica, Inc. is an established company listed on the NYSE, and recently its share price has soared. With many potential avenues of expansion, its managers have decided to take

advantage of its high stock price and raise money by issuing shares. In which market will this take place?

a) Primary market
b) Secondary market
c) Over the counter market
d) Black market

1.4. Which of the following is not a disadvantage of the corporate form of organization?

a) Double Taxation
b) Agency Problems
c) Ability to raise capital
d) Expensive to form
e) Agency problem

1.5. Which of the following is an example of agency costs?

A. A rival firm lures a client away
B. The CEO uses a corporate jet to visit his mistress in Buenos Aires
C. The share price collapses after a bad earnings report
D. An earthquake damages a major capital investment project

Solutions

1.1. (c) is the correct answer. If you get stumped on this sort of question, try to find at least one statement that you know matches the criteria of the problem. For example, if you know that (iv) is not a disadvantage, you can rule out choices (a) and (b). Even if you are still completely confused about the other statements, at the very least you've improved your odds on a guess from 1:4 to 1:2.

Also, note that if statement (iv) was "limited liability", it would still not be a disadvantage of the corporate form.

1.2. (d). The problem itself is pretty easy, but just make sure you're able to sift through the "fluff" that's amusing only to the person writing the questions and irrelevant to you.

1.3. (a). Because Kramerica is already listed, you might be tempted to choose (b), but that would be incorrect. Even a secondary issue of new equity takes place in the primary market. In the

secondary market, participants exchange ownership of existing equity, and the firm does not obtain any proceeds from those trades.

1.4. (c). Relative ease of raising capital is one of the advantages of incorporation, along with limited liability, ease of ownership transfer, and legal immortality.

1.5. (b). a, c and d are negative events but are not examples of a conflict of interest between shareholders and management. Since it was not specified that the CEO had company business in Argentina, there was a conflict.

Financial Statements (Chapters 2,3,4)

Balance Sheets

The key is that

$$\text{Assets} = \text{Liabilities} + \text{Equity}$$

If after completing a problem, you find that the two sides don't add up, then something's wrong. Check your calculations and make sure that all the components are in the right place.

ASSETS

- Current assets
  - Cash, inventories, accounts receivable, liquid securities, etc.
- Fixed assets
  - Tangibles such as plant, property, equipment, etc.
  - Intangibles such as intellectual property

=

LIABILITIES

- Current liabilities
  - Accounts payable, short term loans (<1 year), etc.
- Long term debt (>1 year)

EQUITY

- Owner's or shareholders' depending on firm organization
- Common and preferred shares, capital contributions, treasury stock, retained earnings etc.

Remember:

$$\text{Net Working Capital (NWC)} = \text{Current Assets} - \text{Current Liabilities}$$

Income Statements

In the most reduced form, an income statement boils down to:

$$\text{Income} = \text{Revenues} - \text{All Expenses}$$

Here is a pretty typical income statement, and the steps taken to obtain the next line:

Net Sales		1,000,000,000
Cost of goods	-	350,000,000
Depreciation	-	50,000,000

EBIT	=	600,000,000
Interest	-	500,000,000
EBT	-	100,000,000
Taxes (30%)	-	30,000,000
Net income	=	70,000,000
Dividends (20%)	-	14,000,000
Addition to retained earnings	=	56,000,000

In this case, the company looks set to make a pretty hefty profit from its $1 billion sales, until it is hit with a very large interest payment on its heavy debt burden. After taxes and dividend payments, there is only a paltry $56 million to add to retained earnings

It is important to remember that retained earnings from this period are added to the existing retained earnings in the equity section of the balance sheet. Don't just replace the existing retained earnings with the current year's additions.

Cash Flow Statements

Neither the balance sheet nor income statement deal with the amount of cash gained or lost during a period. The balance sheet only lists the total amount of cash, and while it mi

Similar to the balance sheet identity,

$$\text{CF } \mathbf{from} \text{ assets} = \text{CF } \mathbf{to} \text{ creditors} + \text{CF } \mathbf{to} \text{ stockholders}$$

Cash flow from assets

- Operating cash flow (OCF)
  - EBIT+Depreciation-Taxes
- Change in (Δ) NWC
  - End NWC – Start NWC
- Net capital spending (NCS)
  - End NFA+Dep-Start NFA
- Cash flow from assets
  - OCF- ΔNWC-NCS

=

CF to creditors

- Interest paid – net new borrowings

CF to stockholders

- Dividend paid – net new equity

Financial Ratios

Current Ratio

$$= \frac{\text{Current assets}}{\text{Current liabilities}}$$

Cash Ratio

$$= \frac{\text{Cash}}{\text{Current Liabilities}}$$

Quick Ratio

$$= \frac{\text{Current assets} - \text{Inventory}}{\text{Current liabilities}}$$

Equity Multiplier

$$= \frac{\text{Total Assets}}{\text{Total Equity}}$$
$$= 1 + \text{Debt/Equity}$$

Total Debt Ratio

$$= \frac{\text{Total Assets} - \text{Total Equity}}{\text{Total Assets}}$$

Debt to Equity Ratio

$$= \frac{\text{Total Debt}}{\text{Total Equity}}$$

Long Term Debt Ratio

$$= \frac{\text{Long Term Debt}}{\text{Long Term Debt} + \text{Total Equity}}$$

Times Interest Earned

$$= \frac{\text{EBIT}}{\text{Interest}}$$

Cash Coverage Ratio

$$= \frac{\text{EBIT} + \text{Depreciation}}{\text{Interest}}$$

Inventory Turnover Ratio

$$= \frac{\text{Cost of Goods Sold}}{\text{Inventory}}$$

Cash Coverage Ratio

$$= \frac{\text{EBIT} + \text{Depreciation}}{\text{Interest}}$$

Total Asset Turnover

$$= \frac{\text{Sales}}{\text{Total Assets}}$$

Profit Margin

$$= \frac{\text{Net Income}}{\text{Sales}}$$

Return on Assets (ROA)

$$= \frac{\text{Net Income}}{\text{Total Assets}}$$

Return on Equity (ROE)

$$= \frac{\text{Net Income}}{\text{Total Equity}}$$

Price to Earnings (PE)

$$= \frac{\text{Price Per Share}}{\text{Earnings Per Share}}$$

DuPont Identity

$$ROE = \frac{\text{Net Income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Assets}} \times \frac{\text{Assets}}{\text{Equity}}$$
$$= Profit\ Margin \times Total\ Asset\ Turnover \times Equity\ Multiplier$$

Dividend Payout Ratio

$$= \frac{\text{Dividends}}{\text{Net Income}}$$

Internal Growth Rate (IGR)

$$= \frac{\text{ROA} \times \text{b}}{1 - \text{ROA} \times \text{b}}$$

Sustainable Growth Rate (SGR)

$$= \frac{\text{ROE} \times \text{b}}{1 - \text{ROE} \times \text{b}}$$

Many of these are obvious choices to include on your cheat sheet. Just make sure to keep it clean – you don't want any of them to get smeared away.

Problems

2.1. What is the UCC of a $30,000 truck (class 10) at the end of year 4?

a) $12,495
b) $7,203
c) $8745.5
d) $8746.5

2.2. Consider a company with a net income in 2014 of $1.235B. It paid 22% in corporate taxes and $0.12B in interest. It somehow managed to avoid depreciation, and its cost of goods sold was 31%. Assuming that no other factors affected its net income, what were the company's revenues in 2014? (rounded to the nearest 1/10 of a billion)

a) $2.6
b) $2.5
c) $2.4
d) $2.3

Questions 2.3-2.8 all involve the same company.

2.3. In 2015, a company has sales of $4,000,000, costs and depreciation total $3,000,000, interest expense of $250,000, and net income of $450,000. What is the company's tax rate?

a) 64%
b) 60%
c) 40%
d) 30%

2.4. For the company from 2.3, if the dividend payout ratio is 30% and the retained earnings in 2014 were $600,000, what are the retained earnings for 2015?

a) $915,000
b) $1,050,000
c) $315,000
d) $735,000

2.5. For 2015, in addition to the retained earnings, the company from the above two questions has other equity of $1,135,000. What is the company's sustainable growth rate?

a) 17%
b) 19%
c) 7%
d) 18%

2.6. In 2015, with the information you have from questions 2.3-2.5, determine the company's level of assets.

a) $4,000,000
b) $5,000,000
c) $10,000,000
d) Not enough information

2.7. If the equity multiplier for the above company in 2015 is 1.75, its long term debt is $695,000 and net working capital is $125,000, what is the current ratio?

a) 0.87
b) 1.15
c) 1.25
d) 1.05

2.8. The equity multiplier for the company in 2015 is still 1.75. What is the internal growth rate?

a) 9.6%
b) 10.4%
c) 3.9%
d) 25.6%

2.9 and 2.10 involve the same company with the following balance sheet:

2.9. A company had sales of $2,625,000 in 2015 at 75% capacity, and wishes to expand its production to 100% capacity in 2016. It has costs of $2,137,500 and depreciation and interest expenses total $250,000. The company has an obscure offshoring strategy that allows it to avoid paying any corporate taxes for the next 5 years. Sales, costs, and liabilities all expand at the same rate as capacity. If all goes according to plan, what is the expected percentage increase in the profit margin in 2016 compared to 2015?

a) 15%
b) 0%
c) 43%
d) 26%

2.10. With the 2016 plan in place, the company is looking forward even further to 2017. Assuming that 2016 will turn out exactly as planned, the company wishes to expand and increase its 2017 capacity to 120% of its 2016 level. All the conditions from 2.9 still apply, and in addition the owners feel that they will be able to afford to pay a 20% dividend. Interest and depreciation will increase by $30,000. The company expects to have assets of $5,000,000 and equity of $2,500,000 in 2016. How much external financing will be needed for 2017, if any?

a) $125,000
b) $150,000
c) $100,000
d) $75,000

Solutions

2.1. (d). Remember the 50% rule for the first year, or you get answer (b), which is incorrect. (a) is the UCC at the beginning of year 4, and (c) is just there to take advantage of your potential carelessness.

Year	UCC_{start}	CCA	UCC_{end}
1	15000	4500	25500
2	25500	7650	17850
3	17850	5355	12495
4	12495	3748.5	8746.5

2.2. You are given the net income, and have to work backwards to find the sales figure. The first step is to find the earnings before taxes.

$$Net\ earnings = EBT - EBT \times T_c$$

This can be simplified to

$$Net\ earnings = EBT(1 - T_c)$$

We then invert it to find the earnings before taxes, given net earnings

$$EBT = {Net\ earnings}/{(1 - T_c)}$$

Since we already know net earnings and the tax rate, we can calculate an EBT of $1.5833b.

2.3. (c). The income statement looks like this:

Sales	4,000,000
Costs+Dep	3,000,000
EBIT	1,000,000
Interest	250,000
EBT	750,000
Taxes	?????????
Net Income	450,000

The difference between EBT and net income is the amount of taxes paid, which is 300,000.

$$\frac{300000}{750000} = 0.4 = 40\%$$

2.4. (a). The addition to retained earnings are $450000(1 - 0.3) = 315000.\ 315000 + 600000 = 915000$. Remember that the retained earnings number is the amount remaining after the dividends have been paid out. On the balance sheet, the current year's retained earnings are an addition to the existing retained earnings.

2.5. (d). Recall that SGR depends on ROE and retention ratio *b*, which is the proportion of income not paid out as dividends.

$$ROE = \frac{Income}{Equity} = \frac{450000}{915000 + 1135000} = 0.2195$$

$$b = 1 - 0.3 = 0.7$$

$$SGR = \frac{0.2195 \times 0.7}{1 - 0.2195 \times 0.7} = 0.1815 = 18\%$$

2.6. (d). The DuPont identity comes in handy:

$$ROE = \frac{\text{Net Income}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Assets}} \times \frac{\text{Assets}}{\text{Equity}}$$

$$= Profit\ Margin \times Total\ Asset\ Turnover \times Equity\ Multiplier$$

Since the assets cancel each other out in DuPont, if net income, sales and equity are fixed, you can have literally any nonzero amount of assets and still arrive at the same ROE value. This makes sense, because it is a return on *equity*. Sales can also cancel each other out, and you're left with the basic net income/equity definition of ROE. The DuPont identity is powerful because you will often not have all the information.

You're probably not going to get this sort of "trick" question on an exam, but it's always nice to have reminders about the DuPont identity.

2.7. (b). From 2.5, we know that total equity is $2,050,000. Equity multiplier of 1.75 allows us to find the assets and liabilities:

$$Assets = 2050000 \times 1.75 = 3587500$$

$$Liabilities = 3587500 - 2050000 = 1537500$$

Note that liabilities can also be found using the debt equity ratio, which is 1.75-1=0.75. Current liabilities exclude long term debt:

$$Current\ Liabilities = 1537500 - 695000 = 842500$$

The net working capital value allows us to find the current assets:

$$\text{Current Assets} = \text{Current Liabilities} + \text{NWC}$$

$$= 842500 + 125000 = 967500$$

Finally we can calculate the current ratio:

$$\frac{967500}{842500} = 1.15$$

2.8.(a) Finally a more "normal" question. From above, we know that the assets are $3,587,500, income is $450,000, and the retention ratio is 0.7.

$$ROA = \frac{450000}{3587500} = 0.1254$$

$$IGR = \frac{0.1254 \times 0.7}{1 - 0.1254 \times 0.7} = 9.6\%$$

2.9. (d). The company is currently operating at 75% capacity, so sales and costs need to be adjusted to find the full capacity values.

	2015		**2016**
Sales	2625000	/0.75	3500000
Costs	2137500	/0.75	2850000
Int&Dep	250000	------	250000
Net Income	237500		400000

The old profit margin here is 9.05%, while the new one is 11.43%. This is an increase of 26.3%.

2.10. (c). Sales and costs are multiplied by 1.2 while, again, the interest and depreciation are fixed.

	2016		**2017**
Sales	3500000	x1.20	4200000
Costs	2850000	x1.20	3420000
Int&Dep	250000	------	280000
Net Income	400000		500000

The company is also paying a 20% dividend amounting to $100,000, bringing retained earnings to 400,000. Since we are trying to find EFN, we need to examine the effects on the balance sheet. Since we know the assets and equity values for 2016, we can determine liabilities as 5,000,000-2,500,000 = $2,500,000. Assets and liabilities are increased by the 20%, the same as sales and costs.

	2016		**2017**
Assets	5000000	x1.20	6000000
Liabilities	2500000	x1.20	3000000
Equity	2500000	+400000	2900000

We can see here that liabilities and equity add up to only $5,900,000, short of the $6,000,000 in assets. $100,000 is the EFN required to balance things out.

Time Value of Money (Chapters 5,6,7,8)

The concept of time value of money is based on the tendency of people to value $1 now more than $1 later. In financial markets, interest rates are a reflection of this idea.

Key Terms

Present Value (PV) – the current value of an asset. It could represent the principal of a loan,

Future Value (FV) – the value of the asset at some time. When the entire value of an asset is to be amortized – such as in a loan repayment or term annuity – the FV will be 0. In a bond problem, the FV will be the face value (usually $1,000 unless otherwise specified).

Discount Rate (r) – the rate at which the real value of an asset depreciates over time. This is often represented by an interest rate or inflation rate, but it can also be based on an individual's personal factors, or the internal criteria of a firm.

Period (t) – the time period under consideration for a time value problem. FIN 300 problems will measure time in daily, weekly, monthly, quarterly, semiannual, or yearly units. The time period may be finite or infinite, as in the case of perpetuities and perpetual dividends.

Interest periods per year (I/Y) – the number of compounding periods per year.

Payment periods per year (P/Y) – the number of payments per year on an asset. In standard bond problems

Periodic payment (C) – The amount of money to be paid or received for each P/Y.

Growth rate (g) – The rate at which the periodic payment C increases.

Annual Percentage Rate (APR) – The interest rate without taking sub-annual periodic compounding. It is the quoted interest rate that you usually see in advertisements, since it is lower than the…

Effective Annual Rate (EAR) – The interest rate taking compounding into account. It is the rate at which an annual compounding would be equal to the APR compounded semiannually, monthly, weekly, daily, or continuously.

Key formulae:

Future value of investment	$FV = PV(1+r)^t$
Present value of future amount	$PV = \frac{FV}{(1+r)^t}$
Present value of an annuity	$PV = C\left(\frac{1-\frac{1}{(1+r)^t}}{r}\right)$
Present value of an annuity due (payments at the beginning of a period)	$PV = C\left(\frac{1-\frac{1}{(1+r)^t}}{r}\right)(1+r)$
Future value of an annuity	$FV = C\left(\frac{(1+r)^t-1}{r}\right)$
Future value of an annuity due	$FV = C\left(\frac{(1+r)^t-1}{r}\right)(1+r)$
Present value of a growing annuity	$PV = C\left(\frac{1-\left(\frac{1+g}{1+r}\right)^t}{r-g}\right)$
Present value of a perpetuity	$PV = \frac{C}{r}$
Present value of a growing perpetuity	$PV = \frac{C}{r-g}$
Effective annual rate	$EAR = \left(1+\frac{APR}{m}\right)^m - 1$
Standard bond, with face value F at maturity. Note that this is just the PV of the payments, plus the PV of the face value.	$Bond\ Value = C\left(\frac{1-\frac{1}{(1+r)^t}}{r}\right)+\frac{F}{(1+r)^t}$

Bonds

A standard bond price has two components: the present value of all future coupon cash flows + the present value of the face value. A strip/zero coupon bond does not have additional cash flows, and as such contains only the face value portion.

Mortgages

Mortgages present an interesting issue, since they require monthly payments, but are compounded semiannually. On a financial calculator, this is fairly simple since you can set

P/Y=12 and C/Y=2 and it does the heavy lifting for you. Using regular calculations, you have to first find the effective monthly rate, based on the semiannual compounding, before plugging it in to the annuity formula.

We start with the formula for converting APR to EAR:

$$EAR = \left(1 + \frac{APR}{m}\right)^{m} - 1$$

For semiannual compounding, the effective annual rate is:

$$EAR = \left(1 + \frac{APR}{2}\right)^{2} - 1$$

To find the monthly rate (let's call it *r* for simplicity) from an annual rate, we take the bracketed term and raise it to the power of 1/12, so that the exponent becomes 2/12, or 1/6.

$$r = \left(1 + \frac{APR}{2}\right)^{1/6} - 1$$

There are multiple examples of this in the problems section.

Dividends

When tackling growing dividend problems, make sure to read the question carefully, since it will give you the value of either the previous dividend or the next one. The current price will always be based on the next payment value, in which case it is equal to the present value of a growing perpetuity:

$$P_0 = \frac{D_1}{R - g}$$

Where P_0 is the current price (at time 0)

D_1 is the next period's dividend (at time 1)

R is the required rate of return, and g is the dividend's growth rate

If the question gives you the previous dividend, then you need to multiply it by (1+g) to obtain the next dividend:

$$P_0 = \frac{D_0(1 + g)}{R - g}$$

If there is no dividend growth then, the price is the present value of a simple perpetuity:

$$P_0 = \frac{D_1}{R}$$

You may also encounter problems with irregular dividends that have nonlinear growth for the first few periods, followed by linear or no growth in perpetuity afterward. The current price will still be the PV of all future cash flows, but you'll have to discount the first few separately, and then discount the perpetual portion again.

Consider an asset that pays D_1, D_2, D_3 for the next three periods, and then pays D_4, growing at g in perpetuity. The required rate is R.

$$P_0 = \frac{D_1}{(1+R)^1} + \frac{D_2}{(1+R)^2} + \frac{D_3}{(1+R)^3} + \frac{\frac{D_4}{R-g}}{(1+R)^3}$$

Period 1 Period 2 Period 3 Periods 4,5,6,7…

It may be tempting to discount the perpetual portion by 4 periods, rather than 3. But remember that in our basic example, the price at period 0 is based on the cash flow starting at period 1. Similarly, the "present" value at period 3 is based on the cash flow starting in period 4. Discounting the perpetuity at period 3 gives us our proper PV at time 0.

If the problem involves one growth rate g_1 for the first t periods, and then grows at g_2 in perpetuity, you can use the annuity PV formula for the first part.

$$P_0 = D_1 \left(\frac{1 - \left(\frac{1+g_1}{1+R}\right)^3}{R - g_1} \right) + \frac{\frac{D_1(1+g_1)^3}{R-g_2}}{(1+R)^3}$$

Since that formula can be unwieldly, if the dividend grows at R_1 for a small, manageable period (usually 2 or 3), you can also just calculate them individually:

$$P_0 = \frac{D_1}{(1+R)^1} + \frac{D_1(1+g_1)^1}{(1+R)^2} + \frac{D_1(1+g_1)^2}{(1+R)^3} + \frac{\frac{D_1(1+g_1)^3}{R-g_2}}{(1+R)^3}$$

It might help to make a chart with the dividend price for each period, to avoid error. For example, if the dividend starts at $1.50 per share, increasing 5% for the first three years, and 3% in perpetuity afterwards:

Problems

3.1. You have $4000 available to invest now, and you want your investment to grow to $10,000 in 10 years. What is the annual rate you will need to earn to make that possible?

a) 9.4%
b) 9.5%
c) 9.6%
d) 9.7%

3.2. Singer/political activist John Lenin is saving for his newborn daughter's future university education in 18 years, which he thinks will cost $100,000. He expects to earn 9.6%, compounded monthly. How much will he need to invest now?

a) $17,886.49
b) $19,204.81
c) $5,068.94
d) $5,555.56

3.3. A payday lending company offers 2 week loans of $300 for $20. Assuming no other fees, what is the effective annual rate of this loan? (Hint: count in days, not weeks, and assume a non-leap year)

a) 173%
b) 435%
c) 174%
d) 438%

3.4. Brandon Marlons, a local businessman, is making an offer you can't refuse. In exchange for certain services that we won't get into here, he will pay you $100,000 at the start of every year for 20 years. If your discount rate is 4%, what are these payments worth to you now?

a) $1,359,033
b) $1,413,394
c) $2,000,000
d) $912,774

3.5. There's something in the water, and Martin Brody needs a bigger boat. But he doesn't have enough money on hand to buy one without a loan. He will need to borrow $25,000 at 5%, compounded monthly. If the loan is expected to last for 5 years with payments due at the end of the month, what is the monthly payment that Brody will need to make?

a) $472
b) $581
c) $417
d) $470

3.6. Fredo C. has just set up shop in Nevada, and he is seeking to loan out $100,000 to legitimate local businesses. Three have been shortlisted: X will pay out 5% per year over 4 years; Y will

pay 4% per year over 5 years; and Z offers 8% per year over 3 years. All rates are compounded yearly. If Fredo considered future value as the only factor in his decision, how much will his investment be worth at the end of the specified period?

a) $125,971.20
b) $121,665.29
c) $121,550.63
d) $128,630.98

3.7. Fredo decides, instead, to change his mind and rather than invest in X, Y or Z, he now wishes to invest in company W since he likes the cut of their jib. W pays 3% compounded yearly, and Fredo would like for his investment to be worth $120,000 after 5 years. How much would he need to loan to W?

a) $24,000.00
b) $103,513.05
c) $100,000.00
d) $104,347.83

3.8. Knowing of Fredo's general incompetence with money (and other matters), his brother Michael has sent you to Nevada to oversee the operations. You realize that W is not a viable investment option, and Z has secured funding from Mr Green, so it is no longer available to you. You still have a choice between X and Y. But you also expect an inflation rate of 2%. Taking these facts into account, what is the present value of the superior investment?

a) $110,196.00
b) $121,665.29
c) $112,293.41
d) $121,550.63

3.9-3.11 are related.

3.9. You are looking to buy a $800,000 house and required a mortgage. Pyramid Mortgage Co. is offering terms of 4% compounded semiannually for 25 years, and requires a 25% down payment. What will be your monthly payment?

a) $4208
b) $3201
c) $3256
d) $3156

3.10. In the above mortgage, you are fixed to a rate of 4% for a 5-year term. At the end of this period, how much principal remains unpaid? (round to the nearest 100)

a) $522,300
b) $521,900
c) $171,000
d) $523,400

3.11. After 5 years, the rate has fallen to 3%, and you have an opportunity to pay only this rate for the remaining 20 years. What is the monthly payment at this lower rate? (For simplicity, use the rounded term from 3.10)

a) $3300
b) $2892
c) $2829
d) $3155

3.12. The Lime Lantern Corporation's preferred shares just paid a $1.00 yearly dividend. The market discount rate is 5%. What is the market value of these preferred shares? What if the dividend grows at 2% every year?

a) $20.00; $33.33
b) $33.33; $20.00
c) $20.00; $34.00
d) $20.00; $50.00

3.13. A company is offering 5-year bonds with a face value of $1,000 and a coupon rate of 6%. The market discount rate is 5%. What is the current price of the bond? (Round to the nearest cent)

a) $1043.29
b) $957.88
c) $1043.76
d) $1044.00

3.14. Consider two bonds: a 5-year bond and a 10-year bond, both with coupon rates of 4%. The market discount rate has fallen to 6% from 7%. In percentage terms, what can you say about the value of these two bonds?

a) The 5-year bond has lost more value.
b) The 10-year bond has lost more value.

c) The 5-year bond has gained more value.
d) The 10-year bond has gained more value.

Solutions

3.1. (c). Your PV is 4000, FV is 10000 and time is 10 years. You need to find the rate r.

$$4000(1+r)^{10} = 10000$$

$$(1+r)^{10} = 2.5$$

$$1+r = 2.5^{1/10}$$

$$r = 0.09595 = 9.6\%$$

Just remember to round up to get 9.6% and not 9.5%.

3.2. (a). If you forget to compound monthly, you'll end it with (b).

$$\frac{100000}{\left(1+\frac{0.096}{12}\right)^{18\times 12}} = 17886.49$$

3.3. (d). Since this loan is for 14/365 days, you can find the solution by compounding the solution to a full year.

$$\left(1+\frac{20}{300}\right)^{365/14} - 1 = 4.3796 = 438\%$$

Note that 20/300 = 0.06667 is the 2-week rate for the loan.

(b) is the rate you'd get if you count time by weeks; however, 52 weeks is only 364 days, so using 365/14 is more accurate. (a) and (b) are APRs using per day and per week measures. For example,

$$\frac{20}{300}\times\frac{365}{14} = 1.7381 = 174\%$$

This is actually pretty low for one of these places, but the "no other fees" assumption in the question might be a stretch.

3.4. (b). This is an annuity with start of period payments, so we use the present value annuity due formula.

$$PV = 100000\left(\frac{1-\frac{1}{(1+0.04)^{20}}}{0.04}\right)(1+0.04) = 1413394$$

(a) neglects the beginning payment, (c) ignores all discount rates, and (d) finds the present value of $2,000,000 in 20 years. Don't make those mistakes!

3.5. (a). For this you are given the present value of all the payments Brody must make, which is $25,000. All relevant values are given except for the monthly payment, which you must find.

$$25000 = PMT\left(\frac{1-\frac{1}{(1+0.05/12)^{5\times12}}}{0.05/12}\right)$$

$$PMT = 471.78 = 472$$

3.6. (a). There is not much subtlety to this question, and the key elements are laid out pretty clearly.

For business X, we know that r = 0.05, and t = 4; for Y, r = 0.04, and t = 5; for Z, r = 0.08, t = 3. The principal – our PV in this case – is $100,000.

$$Value\ of\ X = 100000(1+0.05)^4 = 121550.63$$

Similarly, we obtain values for Y and Z.

$$Value\ of\ Y = 100000(1+0.04)^5 = 121665.29$$

$$Value\ of\ Z = 100000(1+0.08)^3 = 125971.20$$

Investment Z will result in the highest amount. In the shortest time, too.

3.7. (b). We know the future value, and need to find the present value.

$$PV = \frac{120000}{(1+0.03)^5} = 103513.05$$

Notice that he only has $100,000 available, so his goal is not actually possible.

3.8. (c). From 3.6, we know that X is worth $121,550.63 after 4 years and Y is worth $121,665.29 after 5 years. To account for inflation, which can be considered a discount rate, we discount both values.

$$Adjusted\ X = \frac{121550}{(1+0.02)^4} = 112293.41$$

$$Adjusted\ Y = \frac{121665.29}{(1+0.02)^5} = 110196.00$$

Notice how even if you didn't account for inflation, investment Y already looked questionable since it took an entire extra year to gain only a slight advantage over X. Adjusting for inflation, Y becomes noticeably worse.

3.9. (d) After a 25% down payment, you are borrowing $600,000 for 25x12=300 months. The effective monthly rate is:

$$r = (1+0.04/2)^{1/6} - 1 = 0.0033058903$$

$$C = \frac{600000}{\left(1 - \frac{1}{(1+0.0033058903)^{300}}\right)/0.0033058903} = \$3156$$

On a BA II+ calculator: N=300, I/Y=4, C/Y=2, P/Y=12, PV=-600000, FV=0, calculate PMT.

3.10. (a). From 3.9 we can see that the present value of 300 $3156 monthly payment amounts to the original principal of $600,000. To find the remaining principal after 5 years, we can calculate the present value of 240 (300 less 60 months, or 20 years in months) payments.

$$C = (3156)\left(1 - \frac{1}{(1+0.0033058903)^{300}}\right)/0.0033058903 = 522303$$

3.11. (b). Since this is a lower rate without changes in the amortization period, you can expect the monthly payment to be lower and rule out (a). Again, we calculate the effective monthly rate, then plug it in to the amortization formula for the remaining 240 months:

$$r = (1+0.03/2)^{1/6} - 1 = 0.00248451672$$

$$C = \frac{522300}{\left(1 - \frac{1}{(1+0.00248451672)^{240}}\right)/0.00248451672} = \$2892$$

3.12. (c). The key here is that for the second part of the problem with annual dividend growth, you have to use the next payment as a basis. Even for the the no growth scenario, you are actually using the next payment; it just hasn't changed from the current payment.

$$\text{No growth scenario: } \frac{1}{0.05} = 20$$

$$\text{Growth scenario: } \frac{1.02}{0.05 - 0.02} = 34$$

3.13. (c) With a 6% coupon rate, the semiannual payment is 1000x0.03=$30. The 6% coupon rate is higher than the market discount rate of 5%, so this is a premium bond and you can expect its price to be above $1,000.

$$Price = 30\left(1 - \frac{1}{(1 + 0.05/2)^{5\times2}}/(0.05/2)\right) + \frac{1000}{(1 + 0.05/2)^{5\times2}}$$

$$= 262.56 + 781.20 = 1043.76$$

3.14. (d). Since the market rate has fallen, both bonds have gained value, allowing us to rule out (a) and (b). Additionally, the price sensitivity of a longer-term bond is greater than that of a shorter-term bond, so the 10-year bond is the most affected by the change in the discount rate. This can be illustrated with $1,000 face value bonds:

Market rate	7%	6%	% Change
5-year	$875.25	$914.70	+4.51%
10-year	$786.81	$851.23	+8.19%

Budgeting for Capital Investments (Chapters 9,10)

NPV

In the most basic form,

$$NPV = PV\ of\ future\ cash\ flows - Investment\ Cost$$

If the NPV > 0, then the project should be accepted (assuming no alternatives), and if NPV < 0, then it should be rejected (except in special cases that we won't get into here).

Also of interest is the amount of time it takes for the initial investment to be recouped.

$$Payback\ period = number\ of\ years\ for\ cash\ flow\ to\ equal\ investment$$

$$Discounted\ Payback\ period = number\ of\ years\ for\ discounted\ cash\ flow\ to\ equal\ investment$$

Potential projects will often be compared to each in terms of not only NPV, but also the payback period.

The internal rate of return (IRR) is the discount rate at which the NPV is 0. Think of it as a break-even point: with a lower discount rate, the project will be profitable; at a higher discount rate, it won't be. It can't be calculated with a simple formula, so you can either:

- Use your financial calculator's built-in IRR function
- Since the exams are multiple choice, try them out until you find the rate that gives an NPV of 0

The profitability index is another way to measure the value of a project.

$$PI = \frac{PV\ of\ future\ cash\ flows}{Investment\ Cost}$$

If PV > Cost, then PI > 1.0, and the NPV > 0.

If PV < Cost, then PI < 1.0, and the NPV < 0.

If PV = Cost, then PI = 1.0, and the NPV = 0.

Investment Decisions

Since NPV calculations place an emphasis on the PV of operating cash flow (OCF), it is important to know how to find this value.

There are three equivalent approaches, depending on the information you are given.

Bottom-up approach:

$$OCF = Net\ income + Depreciation$$

Top-down approach:

$$OCF = Sales - Costs - Taxes$$

Tax-shield approach:

$$OCF = (Sales - Costs)(1 - Taxes) + Depreciation \times T$$

Depreciation represents the loss of value of your assets, and does not affect your cash flow.

Problems

(4.1-4.5 are related)

The giant conglomerate Weyland-Yutani is financing an 8-year mining operation on Io, one of Jupiter's moons. The initial outlay is $2.0 trillion and the expected cash flows from the project are as follows:

Year	**Cash Flow**
2036	$200 billion
2037	$250 billion
2038	$450 billion
2039	$500 billion
2040	$650 billion
2041	$500 billion
2042	$450 billion
2043	$300 billion

4.1. What is the simple payback period of this project?

a) 4.8 Years
b) 4.9 Years
c) 5.0 Years
d) 5.1 Years

4.2. Assuming that Weyland-Yutani has a discount rate of 10%, and no salvage value for the initial outlay, what is the discounted NPV of the Io project?

a) $124.74 billion
b) $1,300 billion
c) -460.53 billion
d) $1,000 billion

4.3. What is the IRR of the project? (Rounding to the nearest one)

a) 8%
b) 10%
c) 12%
d) 14%

4.4. Unhappy with the current projections, the board of directors is considering a proposal from the engineering division. They claim that by using experimental equipment that will increase the initial outlays by $500,000, the Io project can potentially last forever. They project that instead of requiring a shutdown, the mine can instead earn $250 billion in perpetuity beginning in 2044. What will the NPV be under these conditions?

a) $1,291.01 billion
b) $1,060.24 billion
c) $791.01 billion
d) $560.24 billion

4.5. In considering the value of the proposal in 4.4, the company has noted the improvement in the profitability index. What was the PI for the mine in 4.2, and what is it under the 4.4 proposal?

a) 1.06; 1.32
b) 1.32; 1.06
c) 1.06; 1.53
d) 1.32; 1.65

4.6. Madeup Records is preparing to launch Jordyn Quick's new album, 1991. It estimates that it will require an initial outlay of $40,500,000 in various items necessary for the launch, as well as an increase in net working capital of $5,500,000. The investment will be straight-line depreciated to 0 in 12 months. The album is expected to generate $6,650,000 in gross income per month for the first 6 months, then $3,250,000 for the 6 months after that. Afterwards, it will generate $400,000 in cash flow monthly in perpetuity. Madeup's monthly discount rate is 1%, the corporate tax rate is 30%, and the salvage value of the various items mentioned above is $4.5 million. The NWC will be recovered after 12 months since it won't be necessary to support digital sales. What is the NPV of this record?

a) $99,400,000
b) -$99,400,000
c) $47,968,706
d) $-47,968,706

Solutions

1.4. (b). Think of $2 trillion as $2,000 billion. By simple addition, we can see that the total cash flows will be $1,400 billion up to 2039, and $2,050 in 2040, so the unadjusted breakeven point occurs somewhere between years 4 and 5. We just need to calculate how much of the fourth year we need.

$$\frac{2000 - 1400}{650} = 0.9$$

So the payback period is 4.9 years.

4.2. (a). Each value needs to be adjusted according to the company's 10% rate.

Year	**Cash Flow**	**Factor**	**Adjusted**
2036	$200 billion	$1/1.1^1$	$181.8182
2037	$250 billion	$1/1.1^2$	$206.6116
2038	$450 billion	$1/1.1^3$	$338.0917
2039	$500 billion	$1/1.1^4$	$341.5067
2040	$650 billion	$1/1.1^5$	$403.5989
2041	$500 billion	$1/1.1^6$	$282.2370
2042	$450 billion	$1/1.1^7$	$230.9212
2043	$300 billion	$1/1.1^8$	$139.9522
Totals	$3,300 billion		$2,124.74 billion

Subtracting the adjusted amount by the initial cost, we have $124.74 billion remaining.

4.3. (c) This is very easy with a financial calculator and potentially a real pain without one. On a BA II+, press the CF button, and enter -2500 for CF0, then the appropriate values for CF(1-8). Then press IRR and CPT. You get the answer of about 11.52%, which rounds up to 12%.

If you only have a regular calculator, you will need to improvise a little. First of all, consider the definition of IRR, which is the discount rate at which the project breaks even. At the company's 10% rate, the NPV is positive, which implies that the IRR is above 10%, and the only possible solutions are 12% and 14%.

Discounting each cash flow using factors based on a 12% rate, you will arrive at discounted future cash flows of $1,962.80, only slightly below the $2,000 necessary to break even. If you're still uncomfortable about it, consider that at 14%, the discounted cash flows will be even lower, and significantly below break-even. In this case, then, 12% is the only logical solution.

4.4. (c) The initial investment is increased to $2,500 billion, and a perpetuity of $250 billion is added at the end, but all other conditions are still the same. For the perpetual cash flow, we use the standard perpetuity formula:

$$Perpetuity\ beginning\ 2044 = \frac{250}{0.1} = 2500$$

Year	Cash Flow	Factor	Adjusted
2036			\$181.8182
2037			\$206.6116
2038			\$338.0917
2039			\$341.5067
2040			\$403.5989
2041			\$282.2370
2042			\$230.9212
2043			\$139.9522
2044+	\$2,500 billion	$1/1.1^8$	\$1,166.2685
Totals			\$3,291.01 billion

The discount factor's exponent remains at 8 for reasons discussed back in the dividends section.

The NPV is $-2500 + 3291.01 = 791.01$.

4.5. (a). PI is the total cash flows divided by the initial investment.

$$Old\ PI = \frac{2124.34}{2000} = 1.06$$

$$New\ PI = \frac{3291.01}{2500} = 1.32$$

4.6. (c). Everything is measured in months, so you will need to consider that. Luckily, the problem gives a monthly discount rate to make things a bit simpler.

$$Monthly\ depreciation = \frac{40500000}{12} = 3375000$$

$$After - tax\ salvage = 4500000(1 - 0.3) = 3150000$$

$$0 - 6\ month\ OCF = 6650000(1 - 0.3) + 0.3(3375000) = 5667500$$

$$7 - 12\ month\ OCF = 3250000(1 - 0.3) + 0.3(3375000) = 3287500$$

After one year, there is no more depreciation to add back to the cash flow.

$$Perpetual\ OCF\ after\ 1\ year = \frac{400000}{0.01} = 40000000$$

Next we find the present values of all cash flows.

$$PV\ of\ first\ 6\ months = 5667500\left(\frac{1 - 1/1.01^6}{0.01}\right) = 32845862.92$$

$$PV\ of\ second\ 6\ months = 3287500\left(\frac{1 - 1/1.01^6}{0.01}\right)\left(\frac{1}{1.01^6}\right) = 17948438.28$$

$$PV\ of\ Perpetuity = \frac{40000000}{1.01^{12}} = 35497969.01$$

$$PV\ of\ After - tax\ Salvage = \frac{3150000}{1.01^{12}} = 2795465.06$$

$$PV\ of\ NWC\ reclamation = \frac{5500000}{1.01^{12}} = 4880970.74$$

We now have all the elements necessary. Remember that NPV = costs + PV of all future cash flows.

$$NPV = -40500000 - 5500000 + 32845862.92 + 17948438.28 + 35497969.01 + 2795465.06 + 4880970.74 = 47968706.01$$

Market Risk (Chapters 12,13)

Variance and Standard Deviation

Variance and standard deviation are both measures of the distance of a set of numbers from the mean value of that set. In finance, they are used to measure the volatility of assets, which in turn is related to risk. These measures will become much more prominent later on. But for now you just need to know how they are calculated.

The variance is the sum of the squares of deviations from the mean, divided by 1 less the number of values in the set. The standard deviation is the square root of the variance.

For example:

Year	Return	Average	Deviation	Squared deviation
2014	0.05	0.0275	0.0225	0.00050625
2015	0.06	0.0275	0.0325	0.00105625
2016	-0.02	0.0275	-0.0475	0.00256250
2017	0.02	0.0275	-0.0075	0.00005625
Totals	0.11		0	0.00418125

$$Variance = \frac{0.00418125}{4-1} = 0.00139375$$

$$Standard\ Deviation = \sqrt{Variance} = 0.037332961$$

The value of the standard deviation implies a volatility in returns of 3.73%. Note that the sum of the deviations from an arithmetic mean is always going to be 0. When calculating variance, make sure that is the case.

Measuring Risk

The risk premium is the additional rate an asset pays over the risk-free rate.

$$Risk\ premium = Expected\ return - risk\ free\ rate$$

The risk-free rate is a theoretical construct that doesn't really exist, since any investment carries with it some risk, even if it is trivial. The RF rate is usually represented by US year Treasuries.

The expected return on a portfolio is the average of expected individual returns, weighted by each asset's share of the entire portfolio.

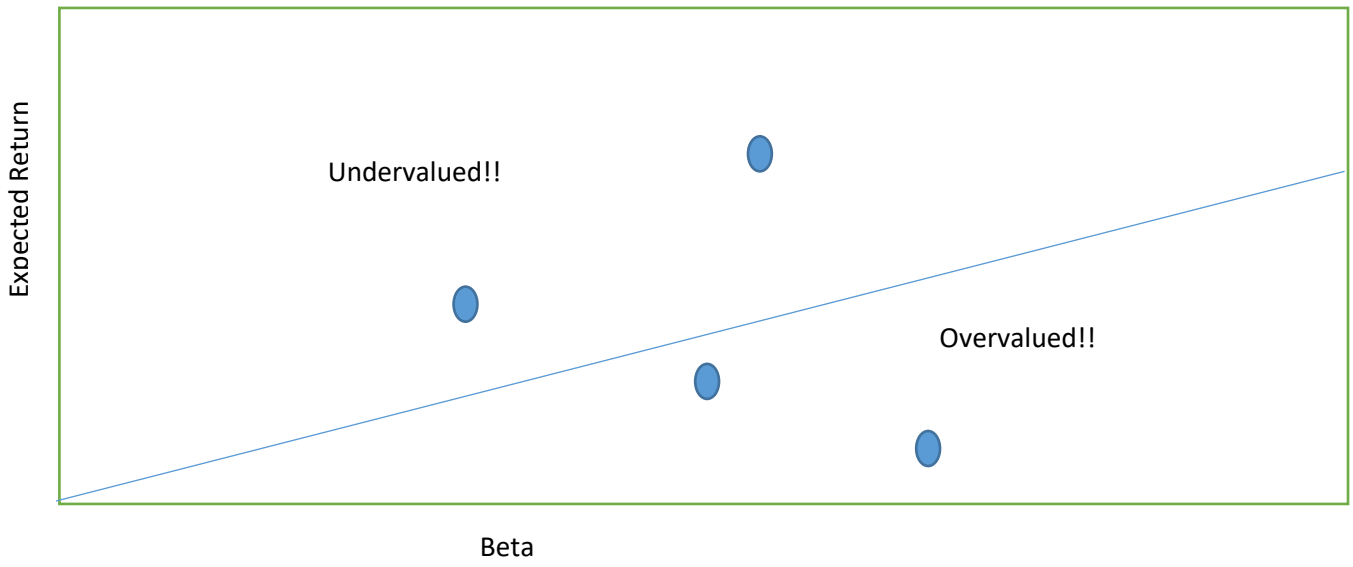
$$E(R_p) = x_1E(R_1) + x_2E(R_2) + \cdots + x_nE(R_n)$$

The standard model used to determine the expected return of an asset is called the capital asset pricing model (CAPM).

$$E(R_i) = R_f + \left[E(R_m) - R_f\right]\beta_i$$

In other words, the expected return is the risk free rate plus the risk premium modified by the asset's volatility or systematic risk, β.

The security market line (SML) is a visual representation of CAPM. The line represents the "acceptable" level of risk, as calculated by CAPM. Stocks above the line are undervalued, while those below it are overvalued.



This was going to be much fancier, but I think there's a sort of rustic charm to it.

Problems

(5.1-5.2 are related)

Your portfolio has experienced the following returns:

2008	-14%
2009	-4
2010	5%
2011	20%
2012	9%

5.1. What are the arithmetic and geometric average returns?

a) 3.2%; 2.5%
b) 2.5%; 3.2%
c) 10.4%; 10.2%
d) 3.2%; 10.2%

5.2. What is the standard deviation of the portfolio?

a) 1.67%
b) 12.91%
c) 14.95%
d) 3.53%

(5.3-5.4 are related)

You have a portfolio of two stocks and are considering their returns depending on the state of the economy:

State	**Probability**	**Return on X**	**Return on Y**
Recession	0.30	0.15	-0.10
Boom	0.70	-0.03	0.10

Additionally, the portfolio is 65% X and 35% Y.

5.3. What is the expected return of this portfolio?

a) 0.296%
b) 2.96%
c) 6%
d) 6.4%

5.4. What is the standard deviation of this portfolio?

a) 4.64%
b) 4.53%
c) 2.15%
d) 3.56%

5.5. Give the following average return information, find the risk premiums of the assets in the same order in which they are presented from left to right.

Large cap	**Small cap**	**LT Gov't bonds**	**Corporate Bonds**	**T-Bills**
15.5%	17.4%	5.5%	7.6%	3.1%

a) 12.4%; 14.3%; 2.4%; 4.5%; 0%
b) 14.3%; 12.4%; 4.5%; 2.4%; 0%

c) 0%; 4.5%; 2.4%; 14.3%; 12.4%
d) I don't know.

Solutions

5.1. (a). The sum of the returns is 16%, so the arithmetic average is 16/5=3.2%.

$$Geometric\ average = [(1-0.14)(1-0.04)(1+0.05)(1+0.20)(1+0.09)]^{1/5} - 1 = 0.025$$

5.2. (b). From above, we know that the mean is 3.2%, or 0.032.

$$Variance$$

$$= \frac{(-0.14-0.032)^2 + (-0.04-0.032)^2 + (0.05-0.032)^2 + (0.20-0.032)^2 + (0.09-0.032)^2}{5-1}$$

$$= 0.01667$$

$$Standard\ deviation = \sqrt{Variance} = 0.1291$$

5.3. (.) First we find the expected return for each economic state:

$$Recession: 0.65(0.15) + 0.35(-0.10) = 0.0625$$

$$Boom: 0.65(-0.03) + 0.35(0.10) = 0.0155$$

Now find the overall expected return based on the probabilities of each state:

$$Expected\ return = 0.30(0.0625) + 0.70(0.0155) = 0.0296$$

5.4. (c). We have the recession boom, and overall expected returns from the previous question. The overall expected return is the mean for this problem:

$$Standard\ deviation = \sqrt{\frac{0.3(0.0625-0.0296)^2 + 0.7(0.0155-0.0296)^2}{2-1}} = 0.0215$$

5.5. (a). Remember grade 1? It's back, in finance form.

Other Exam Tips

Before the Exam

Make sure to sleep well and eat breakfast. This is pretty simple advice, but you want as few distractions as possible, and few distractions are worse than exhaustion and hunger. Remember what happened to the Romans at Trebia.

Time Management

You have 3 hours to solve 50 problems, or around 3.6 minutes for each one. If you're well prepared, most of them should take less than a minute, but there are always several that will take longer.

They are all weighed the same, so at first, don't linger too long on questions that are giving you a hard time. You can keep note of questions you're temporarily skipping by, for example, writing it lightly on an unused section of your Scantron sheet, or making a light mark by the number. Just make sure to erase all extraneous markings afterwards.

Rechecking Solutions

If you finish before the time limit, you will be tempted to leave. But there is an almost 100% chance that you've made mistakes. For some of them, there is really nothing you can do, but there are small and simple errors that can be fixed.

Calculation mistakes are common. Even if you know how to solve a problem, there is always some chance that you've entered a wrong number into your calculator, or maybe you forgot a few brackets and did 0.05^{25} instead of 1.05^{25}, and it just so happened that your incorrect solution coincided with one of the multiple choice questions.

On a financial calculator, you might have set it up for semiannual compounding for a question that required only annual compounding. If you primarily use a financial calculator, it might be useful in these cases to have a second scientific calculator on hand to check your answers using the standard formulae.

In many cases, you can plug in the solution to see if it is consistent with the rest of the problem. For example, if you're asked to find an interest rate for a PV/FV problem, you can easily check to see if that rate will allow the PV to reach the FV in the given time period.