# Online Supplemental Information

Helping Friends or Influencing Foes: Electoral and Policy Effects of Campaign Finance Contributions

## Contents

# B Extensions and robustness

## B.1 Illustrative examples

Example 1 shows how the welfare effects of contributions, relative to banning contributions, depend on the parameters of the model. Example 2 shows that separating equilibria are robust to the inclusion of a large number of interest groups. In both examples, we focus on the implications of persuasion by switching sides, though similar analyses would apply to persuasion by restraint. Both examples are discussed briefly in the text and explained in greater detail below. In both cases, since the examples are simply for illustration, we do not show extensive calculations. Instead, a supplementary Mathematica file is included in the Supplemental Information to verify all work.

### B.1.1 Welfare effects of contributions

**Example 1.** Suppose that $\theta$ is distributed uniformly on $[0,1]$ and assume that $p(c^M, c^A)$ is a linear probability function

$$p(c^M, c^A) = \frac{1}{2} + \phi(c^M - c^A)$$

where $\phi \in \left(0, \frac{1}{2}\right)$ represents the marginal effect of contributions on electoral outcomes.

The conditional expectations of any signal or pair of signals is computed by performing the usual Beta-Binomial updating, noting that the uniform is equivalent to a Beta(1, 1) distribution. The expectation of $\theta$ following only $s_D$ is

$$\frac{1 + \mathbb{I}_{(s_D = B)}}{3}$$

and the expectation of $\theta$ following a pair of signals $(s_D, s_j)$ for $j \in \{M, A\}$ is

$$\frac{1 + \mathbb{I}_{(s_D = B)} + \mathbb{I}_{(s_j = B)}}{4},$$

where $\mathbb{I}_{(s_D = B)}$ is an indicator function that takes a value of one when $s_D = B$ and zero when $s_D = G$ and $\mathbb{I}_{(s_j = B)}$ is defined analogously for a generic player $j$'s signal. Thus, the expected policy choice from the Moderate for a Donor with signal $s_D$ who sends a contribution that induces the belief that $s_D = G$ is

$$\bar{x}(\hat{s}_D(c^M, c^A), s_D) = \frac{1+\mathbb{I}_{(s_D=B)}}{3}\frac{1}{2} + \left(1 - \frac{1+\mathbb{I}_{(s_D=B)}}{3}\right)\frac{1}{4}.$$

Similarly for a contribution inducing the belief that $s_D = B$:

$$\bar{x}(\hat{s}_D(c^M, c^A), s_D) = \frac{1+\mathbb{I}_{(s_D=B)}}{3}\frac{3}{4} + \left(1 - \frac{1+\mathbb{I}_{(s_D=B)}}{3}\right)\frac{1}{2}.$$

Therefore, we have:

$$\begin{aligned}
\bar{x}(G,G) &= \frac{1}{3} \\
\bar{x}(G,B) &= \frac{5}{12} \\
\bar{x}(B,G) &= \frac{7}{12} \\
\bar{x}(B,B) &= \frac{2}{3}.
\end{aligned}$$

Thus, our equilibrium tells us there exists a separating equilibrium if

$$\frac{8}{5} \leq \frac{(b-1)\left(\phi + \frac{1}{2}\right) + 1}{(b-1)\left(\frac{1}{2} - \phi\right) + 1}$$

which holds if $b > \frac{8}{5}$ and $\frac{3b+3}{26b-26} \leq \phi$. Under these conditions, the equilibrium contribution to the Moderate by the good type of Donor (found by setting persuasion costs equal to electoral costs for the bad type and solving for $c^M$) is

$$c^{m*} = \frac{b(3-16\phi)+16\phi+3}{10(b-1)\phi},$$

which results in the following probability of electing the Moderate given a good type of Donor:

$$p(c^{m*}, 0) = \frac{8b\phi - 4b - 8\phi + 1}{5-5b}.$$

We can now write the citizen's ex ante welfare as a function of the parameters:

$$\begin{aligned} W(\theta,b,\phi) &= \Bigg(-(1-\theta)\Bigg(\left(\frac{1}{45}(72\phi-39)+1\right)\left((1-\theta)\left(\frac{1}{40}-\theta\right)^2+\left(\frac{1}{20}-\theta\right)^2\theta\right) \\ &+ \frac{1}{45}(39-72\phi)\left((1-\theta)\left(\frac{1}{4}-\theta\right)^2+\left(\frac{1}{2}-\theta\right)^2\theta\right)\Bigg)-\theta\Bigg(\left(\phi+\frac{1}{2}\right)\Bigg((1-\theta)\left(\frac{1}{20}-\theta\right)^2 \\ &+ \left(\frac{3}{40}-\theta\right)^2\theta\Bigg)+\left(\frac{1}{2}-\phi\right)\left((1-\theta)\left(\frac{1}{2}-\theta\right)^2+\left(\frac{3}{4}-\theta\right)^2\theta\right)\Bigg)\Bigg) \end{aligned}$$

Since $\theta$ is unknown, we calculate the citizen's ex ante expected welfare (noting that the uniform density is a constant function $f(\theta)=1$ as

$$\int_0^1 W(\theta,b,\phi)d\theta = -\frac{b^2(158\phi+81)-4b(79\phi+35)+79(2\phi+1)}{480b^2}.$$

If contributions are banned, both candidates will win with probability $p(0,0)=\frac{1}{2}$ and update their beliefs based only on their own signals. Therefore, voter welfare is

$\int_0^1\left[\frac{1}{2}\left(-(1-\theta)\left(\frac{1}{3b}-\theta\right)^2-\theta\left(\frac{2}{3b}-\theta\right)^2\right)+\frac{1}{2}\left(-(1-\theta)\left(\frac{1}{3}-\theta\right)^2-\left(\frac{2}{3}-\theta\right)^2\theta\right)\right]d\theta=-\frac{7b^2-10b+5}{36b^2}$

Thus, there exists a separating equilibrium that is strictly preferred to banning contributions when the equilibrium conditions are met and also

$$-\frac{b^2(158\phi+81)-4b(79\phi+35)+79(2\phi+1)}{480b^2} > -\frac{7b^2-10b+5}{36b^2}.$$

This occurs when, in addition to the requirements that $b>\frac{8}{5}$ and $\frac{3b+3}{26b-26}\leq\phi$ we have $\phi<\phi^*(b)\equiv\frac{37b^2+20b-37}{474b^2-948b+474}$. The region of the parameter space for which there exists a separating equilibrium that dominates a ban on contribution is displayed in Figure 1. In this particular example, a ban seems to dominate the separating equilibrium over most of the parameter space. However, the example was chosen to be illustrative rather than realistic and other examples may support the opposite conclusion. Our main take-way is that full knowledge of the model parameters is often necessary in order to make clear statements about whether a ban on contributions would increase or decrease citizen welfare. Therefore, in most applications, the welfare prediction is ambiguous.

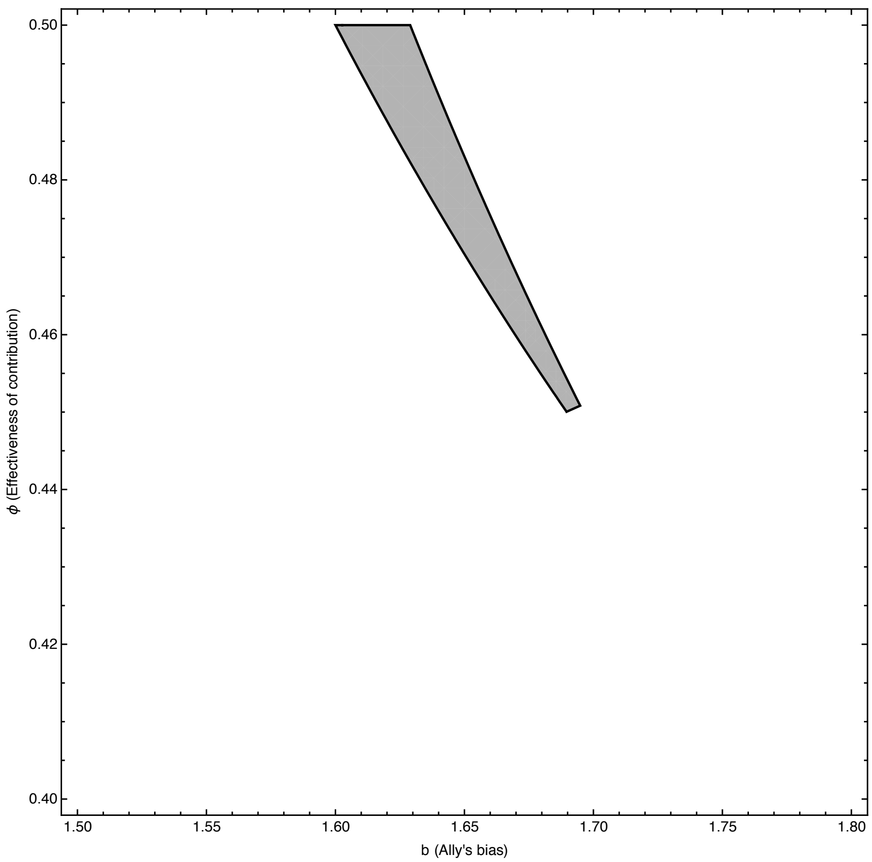


**Figure 1:** The grey region represents the set of parameters under which there exists a separating equilibrium that is preferred to banning contributions.

### B.1.2 Multiple donors

**Example 2.** Consider an $n$-donor example of the model. Suppose $\theta$ is uniform on $[0,1]$ and let $b = 10$. Now suppose there are $n$ identical donors that each receive conditionally independent signals and simultaneously choose contribution levels. Letting $T_M$ and $T_A$ denote the total amounts of contributions to the Moderate and the Ally respectively. The probability that the Moderate is elected is then,

$$p(T_M, T_A, n) = \log\left(\frac{1+e}{2} + \frac{e-1}{2n}(T_M - T_A)\right),$$

where the letter $e$ here represents the exponential constant and the log has base e. $p()$ is therefore log-linear and the intercept and slope are chosen to ensure that all probabilities fall between zero and one. We will show that there is a symmetric separating equilibrium even for large $n$. A separating equilibrium profile to this game involves each donor choosing $(c^M, c^A) = (0,1)$ when he is a bad type and $(c^M, c^A) = (c^{M*}, 0)$ for some $c^{M*} > 0$ when he is a good type. The winning candidate, in turn, updates her beliefs assuming all contributions with $c^M > c^{M*}$ and $c^A = 0$ indicate signals of $G$ and any contribution with $c^A \geq 0$ or $c^M < c^{M*}$ indicate signals of B. Since the uniform is a Beta(1, 1) distributon, this means that the posterior distribution of $\theta$ for candidate $j$ following a set of $n$ contributions and her own signal, letting $S$ denote the candidate's belief about the total number of bad signals, is distributed $Beta(1+S+\mathbb{I}_{(s_j=B)}, 1+n-S-\mathbb{I}_{(s_j=B)})$, where $\mathbb{I}_{(s_{j=B})}$ is an indicator that takes the value of one when $s_j = B$ and zero when $s_j = G$, and policy is chosen accordingly (equal to the expectation of $\theta$ for Moderate candidates and $1/b$ times that expectation for Allies). using this information, we can compute each type of donor's expected utility for choosing each signal on the path of play. The expected utility for type $s_D$ of contributing $(c^{M*}, 0)$ is given by,

$$U_{c^{M*}}(s_D, c^{M*}, n) = -\mathbb{E}\left[\frac{(S+1)(1-p(c^M(n-S)+c^M, S, n))}{b(n+2+1)} + \frac{(S+1)p(c^M(n-S)+c^M, S, n)}{n+2+1}\right],$$

where the expectation is taken with respect to $S$, which is distributed Beta-Binomial with parameters $\alpha = 1+\mathbb{I}_{(s_D=B)}$, $\beta = 2-\mathbb{I}_{(s_D=B)}$ and $n = n$ (the first two parameters are the updated beliefs about $\theta$ from the signal $s_D$ and the sample size $n$ reflects the number of signals – all other donors plus one candidate). Similarly, the expected utility from contributing $(0,1)$ is,

$$U_0(s_D, c^{M*}, n) - \mathbb{E}\left[\frac{(S+1+1)(1-p(c^M(n-S), S+1, n))}{b(n+2+1)} + \frac{(S+1+1)p(c^M(n-S), S+1, n)}{n+2+1}\right],$$

where $S$ is distributed in the same way as above. For this example we compute these expectations numerically and omit the analytical details of the calculations.

Notice that, unlike in the single donor model, the expected policy and the probability of electing the Moderate for a given contribution both depend on the players' types since they also affect expectations over other players' contributions. Nevertheless, our sorting condition still holds. The next plot verifies that the good type of donor has a stronger incentive to choose $(c^{M*}, 0)$ over $(0,1)$ than does the bad type of donor. This holds if the following expression, $\delta(c^{M*}, n)$ which is the difference in differences in expected utility for contributing $c^{M*}$ given $n$ donors, is negative for a given $n$ and all $c^{M*} \in (0,1)$:

$$\delta(c^{M*}, n) = (U_{c^{M*}}(1, c^{M*}, n) - U_0(1, c^{M*}, n)) - (U_{c^{M*}}(1, c^{M*}, n) - U_0(1, c^{M*}, n)).$$

Figure 2 verifies that this holds for several values of $n$.

The sorting condition implies that if there is a contribution to the moderate $c^{M*} > 0$ that would make the bad type of donor indifferent between sending $(c^{M*}, 0)$ and $(0,1)$ given the proposed strategy profile, the good type would strictly benefit from separating and sending $(c^{M*}, 0)$. We must now verify that some contribution would make the bad type of donor indifferent. As in the one donor model, we do so by showing that the bad type would not send the maximum amount to the Moderate. Then, by appealing to continuity, we know that some contribution makes the bad type indifferent. Figure 3 shows that this is true even for very large numbers of donors. In this figure, the number of donors is on the $x$-axis and the $y$-axis represents the bad type's utility gain from imitating by sending $(1,0)$. We note that this utility gain is negative for all of the examined values of $n$, which shows that the bad type can be deterred from imitating and therefore there is a symmetric separating

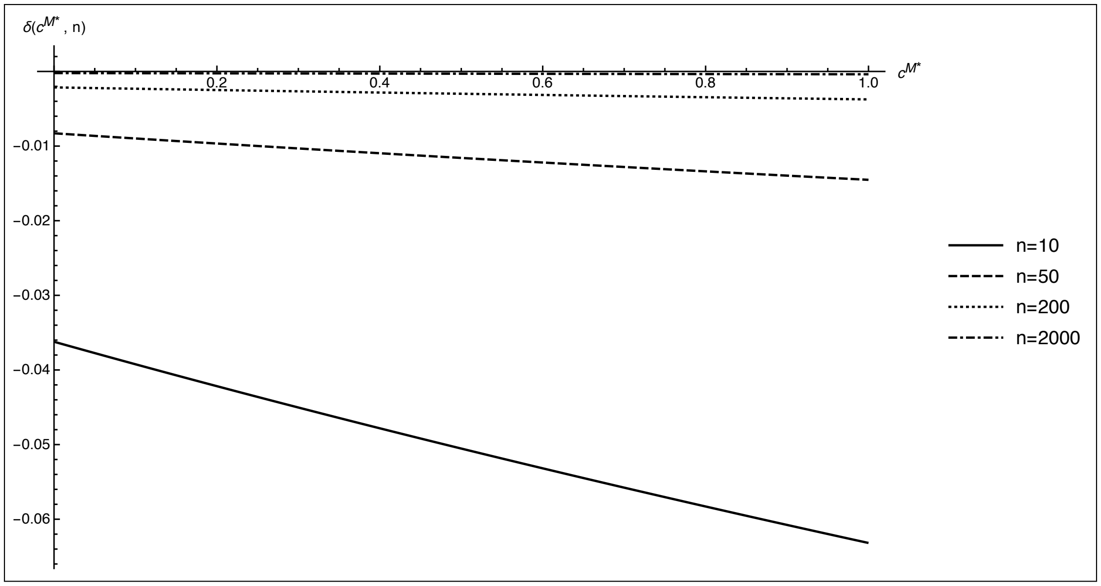


**Figure 2:** The lines show values of $\delta(c^{M*}, n)$ for each possible value of $c^{M*}$ and various values of $n$. Values below 0 verify our sorting condition: bad types of the donor have less of an incentive to contribute to the Moderate in order to be viewed as a good type of donor. Note also that the lines are decreasing in $c^{M*}$, since naturally this difference in incentives widens as we increase the required contribution and therefore increase the electoral disincentive to do so. However, the slope of the lines are lower for higher values of $n$ since the importance of any one contribution diminishes as $n$ increases.

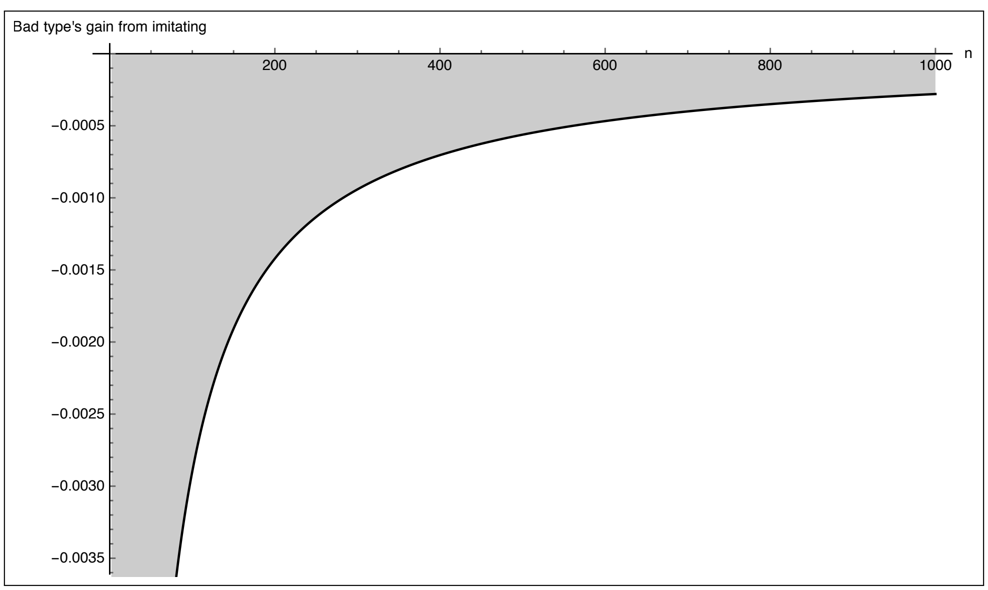


**Figure 3:** This plot shows the bad type of donor's net utility from deviating from the separating equilibrium by giving the maximum amount to the Moderate, for each value of $n$. Negative values indicate that the bad type would not deviate, which means that we can support a symmetric separating equilibrium for that value of $n$.

equilibrium. Note that this example does not show that separating equilibria are *always* robust to increasing the number of donors. In fact, for smaller values of $b$ the separating equilibrium is only sustainable for relatively small numbers of donors. As we have noted, the answer depends on how quickly the electoral and persuasion components of the effects of contributions go to zero as $n$ gets large: if the electoral effects converge to zero more slowly, as they do in this example, then the separating equilibrium is sustained for a larger number of donors (i.e., higher values of $n$).

## B.2 Separating conditions for more general Ally utility function

In this section we consider a more general candidate utility function in which the Ally might have as additive bias (as in Crawford-Sobel and other works) as well as the multiplicative bias that is in the baseline model. The purpose of this extension is to illustrate the impact of our assumption that candidate preferences diverge from each other more as their expectations about $\theta$ increase, which is the third assumption regarding candidate preferences discussed in the main text. The general message is that these two types of candidate preferences work much differently and the style of preferences used in the model are key to the mechanism in the paper. The key difference is that, with the multiplicative candidate biases in our model, the bad type of Donor perceives greater electoral stakes than does the good type and is therefore less willing to help the Moderate in order to influence the politicians. With Crawford-Sobel-style preferences, the two candidates' policy choices as a function of their expectations about $\theta$ are two parallel lines, so the perceived electoral stakes do not change as expectations about $\theta$ change. Below, however, we combine the two types of preferences. If Ally candidates have some additive bias *in addition to* the multiplicative bias, the conditions to support a separating equilibrium can be easier to satisfy, since the constant differences between the candidates serve as additional deterrent to the bad type. However, this only works because the sorting condition distinguishing between the types is bolstered by the mechanism from the baseline model. In this way, additive differences between candidates operate in much the same way as contribution costs.

Consider a variant of our original game with the following preferences for the ally:

$$u_A(\theta,x) = -(\theta b - x + \gamma)^2. \tag{1}$$

with $\gamma \geq 0$ and $b > 1$. The $\gamma$ parameter is the bias as in Crawford and Sobel and $b$ is the bias from our original model. Our purpose is to illustrate why the parameter $b$ is essential to the mechanism driving separating equilibria in this game while $\gamma$ is not.

Given a belief about $\theta$ the Ally's best reply then is to set $x = \mathbb{E}[\theta]/b - \gamma$. A player of type $s_D$ now prefers $(c^M, c^A)$ over $(\tilde{c}^M, \tilde{c}^A)$ if,

$$\begin{aligned} -p(c^M,c^A)\bar{x}(\hat{s}_D(c^M,c^A),s_D) - (1-p(c^M,c^A))\left(\frac{\bar{x}(\hat{s}_D(c^M,c^A),s_D)}{b} - \gamma\right) &\geq \\ -p(\tilde{c}^M,\tilde{c}^A)\bar{x}(\hat{s}_D(\tilde{c}^M,\tilde{c}^A),s_D) - (1-p(\tilde{c}^M,\tilde{c}^A))\left(\frac{(\bar{x}(\hat{s}_D(\tilde{c}^M,\tilde{c}^A),s_D)}{b} - \gamma\right) & \end{aligned}$$

Rearranging, type $s_D$ would prefer $(c^M, c^A)$ over $(\tilde{c}^M, \tilde{c}^A)$ if,

$$\begin{aligned} \bar{x}(\hat{s}_D(c^M,c^A),s_D)\left(\frac{p(c^M,c^A)-1}{b} - p(c^M,c^A)\right) - \bar{x}(\hat{s}_D(\tilde{c}^M,\tilde{c}^A),s_D)\left(\frac{p(\tilde{c}^M,\tilde{c}^A)-1}{b} - p(\tilde{c}^M,\tilde{c}^A)\right) \\ \geq \gamma(p(c^M,c^A) - p(\tilde{c}^M,\tilde{c}^A)). \end{aligned}$$

Consider a separating profile in which (a) the contribution $(\hat{c}^M, 0)$ induces the belief that $s_D = G$, (b) the contribution $(0,1)$ induces the belief that $s_D = B$. Type $s_D$ prefers $(\hat{c}^M, 0)$ over $(0,1)$ if

$$\bar{x}(G,s_D)\left(\frac{p(\hat{c}^M,0)-1}{b} - p(\hat{c}^M,0)\right) - \bar{x}(B,s_D)\left(\frac{p(0,1)-1}{b} - p(0,1)\right) \geq \gamma(p(\hat{c}^M,0) - p(0,1)). \tag{2}$$

To clarify these conditions, consider the limiting cases as $\gamma \to 0$ (the original case) and as $b \to 1$

(Crawford-Sobel-style preferences). When $\gamma = 0$ the condition in Equation 2 becomes

$$\overline{x}(G,s_D)\left(\frac{p(\hat{c}^M,0)-1}{b} - p(\hat{c}^M,0)\right) - \overline{x}(B,s_D)\left(\frac{p(0,1)-1}{b} - p(0,1)\right) \geq 0 \tag{3}$$

$$\overline{x}(G,s_D)\left(\frac{p(\hat{c}^M,0)-1}{b} - p(\hat{c}^M,0)\right) \geq \overline{x}(B,s_D)\left(\frac{p(0,1)-1}{b} - p(0,1)\right) \tag{4}$$

$$\frac{\overline{x}(G,s_D)}{\overline{x}(B,s_D)} \leq \frac{(b-1)p(0,1)+1}{(b-1)p(\hat{c}^M,0)+1} \tag{5}$$

which is exactly the condition from the baseline model. Note that the inequality changes direction in the last line since the parenthetical terms are negative. When $b \to 1$ our condition becomes $\overline{x}(B,s_D) - \overline{x}(G,s_D) \geq \gamma(p(\hat{c}^M,0) - p(0,1))$.

We now make a few observations about these conditions. First, while the term $\frac{\overline{x}(G,s_D)}{\overline{x}(B,s_D)}$ is larger when $s_D = B$ as shown in Lemma 5, the same cannot be said of the difference $\overline{x}(B,s_D) - \overline{x}(G,s_D)$. For instance, if we consider a $Beta(\alpha,\beta)$ prior, we would have

$$\overline{x}(B,G) - \overline{x}(G,G) = \overline{x}(B,B) - \overline{x}(G,B) = \frac{1}{\alpha+\beta+2}$$

so that the indifference curves of the good type and bad type would be identical in the model with $b = 1$. Second, when $\gamma > 0$ and $b > 1$ the single crossing condition is still satisfied: the new condition simply adds a type-independent term to the IC condition from the original model. Furthermore Equation 2 implies that increasing $\gamma$ makes separating equilibria easier to obtain as long as we have $b > 1$. The reasoning is as follows. Increasing $\gamma$ increases the RHS of Equation 2 meaning that an agent of any type is less inclined to send the contribution $(\hat{c}^M,0)$. This term increases the electoral stakes involved and makes either type of $D$ less inclined to make contributions that increase the likelihood of electing the Moderate. Though this effect is the same for good types as it is for bad types, the main issue for finding separating equilibria is whether or not the bad type can be deterred from imitating. By increasing the costs associated with contributing to the Moderate (or failing to contribute to the Ally), increases in $\gamma$ mean that separating equilibria can be supported more often and at lower overall contribution levels.