Problem Set 6: Support Vector Machines

This assignment requires a working IPython Notebook installation.

Only PDF files are accepted for submission. To print this notebook to a pdf file, you can go to "File" -> "Download as" -> "PDF via LaTex(.pdf)" or simply use "print" in browser.

Total: 100 points (+ 50 bonus points)

In this problem set you will implement an SVM and fit it using quadratic programming. We will use the CVXOPT module to solve the optimization problems.
You may want to start with solving Problem 1 and reading the textbook, this will help a lot in the programming assignment.

Problem 1 [30 pts]

1.1 Dual Representations [10 pts]

In class we saw that the SVM classifier with parameters $\mathbf{w}, b$ trained on n points $\{\mathbf{x}_i, y_i\}$ can be expressed in either the "primal" form

$$h(\mathbf{x}) = sign(\mathbf{w^T x} + b)$$

or the "dual" form

$$h(\mathbf{x}) = sign(\sum_{i=1}^{n} \alpha_i y_i \mathbf{x_i^T x} + b)$$

The dual form involves a "kernel function" which evaluates dot products $\mathbf{x_i^T x}$ between the input point and the training points. We can think of these values as similarities of the input $\mathbf{x}$ to the training points.

It turns out that many linear models we have seen before can be re-cast into an equivalent "dual representation" in which the predictions are also based on linear combinations of a kernel function evaluated at the training data points.

This is described in sections 6.0-6.1 in Bishop. Read it and work through all of the steps of the derivations in equations 6.2-6.9. You should understand how the derivation works in detail.

Write down in your own words: *how can the regularized least-squares regression be formulated in the dual form?*

We can solve for w by taking the derivative of the loss function and setting it equal to zero.

Then we use $a_n = -\frac{1}{\lambda} w^T \phi(x_n) - t_n$ and substitute that for w to form the dual representation.

In this representation, $\phi(x_n)^T \phi(x_m)$ is replaced by the gram matrix K_{nm}.

We can then take the derivative of the loss function again but with respect to a to get an equation. This allows us to rewrite the function so that we can work with kernel Knm instead of the feature vector $\phi(x)$ so we can impicitly use high dimensional feature spaces.

1.2 Kernels [10 pts]

Read Section 6.2 and verify the results (6.13) and (6.14) for constructing valid kernels, i.e. prove the kernels constructed by (6.13) and (6.14) are valid.

$k_1(x,x') = \phi(x)^T\phi(x')$, then $k(x,x') = ck_1(x,x') = c\phi(x)^T\phi(x')$
If we set new $\phi(x)$ as $\phi(x) = \sqrt{c}\phi(x)$ which is still valid (for c > 0).

For $k(x,x') = f(x)k_1(x,x')f(x') = f(x)\phi(x)^T\phi(x')f(x')$, we set $\phi(x) = f(x)\phi(x)$ then we get the valid kernel since $f(x)$ is polynomial.

1.3 Maximum Margin Classifiers [10 pts]

Read section 7.1 and show that, if the 1 on the right hand side of the constraint (7.5) is replaced by some arbitrary constant $\gamma > 0$, the solution for maximum margin hyperplane is unchanged.

$$t_n(w^T\phi(x_n) + b) \geq 1$$

$t_n(w^T\phi(x_n) + b) \geq 1$ classify data points as true if they are equal to or larger than 1. We can set any new constant $t_n(w^T\phi(x_n) + b) \geq C$ and still satisfy the constraints if we change the value for the point that is closest to the surface.

Implementing the SVM

Note, some of the code in cells below will take minutes to run, so feel free to test you code on smaller tasks while you go. Easiest way would be to remove both for-loops and run the code just once.

Quadratic Programming

The standard form of a Quadratic Program (QP) can be formulated as

$$\begin{aligned} \min_x \quad & \frac{1}{2}x^T Px + q^T x && (1) \\ \text{subject to} \quad & Gx \preceq h && (2) \\ & Ax = b && (3) \end{aligned}$$

where $\preceq$ is an element-wise $\leq$. The CVXOPT solver finds an optimal solution x^*, given a set of matrices P, q, G, h, A, b.

FYI, you can read about the methods for solving quadratic programming problems here (optional).

Problem 2 [10 points]

Design appropriate matrices to solve the following QP problem:

$$\begin{aligned} \min_x \quad & f(x) = x_1^2 + 4x_2^2 - 8x_1 - 16x_2 && (4) \\ \text{subject to} \quad & x_1 + x_2 \leq 5 && (5) \\ & x_1 \leq 3 && (6) \\ & x_2 \geq 0 && (7) \end{aligned}$$

Hint: first notice that if $x = [x_1, x_2]^T$ and P is a matrix

$$\begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix}$$

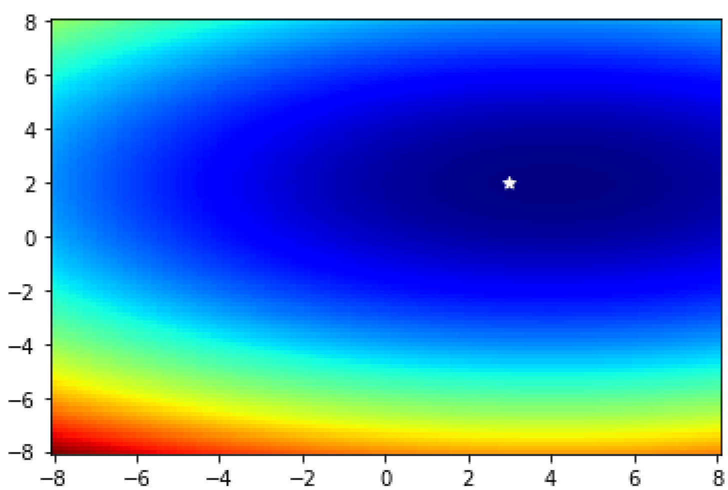
then $x^T P x = p_{11}x_1^2 + (p_{12} + p_{21})x_1 x_2 + p_{22}x_2^2$. We have filled in the correct P below.

In [6]:
```
!pip install cvxopt
from cvxopt import matrix, solvers
# Turns off the printing of CVXOPT solution for the rest of the notebook
solvers.options['show_progress'] = False

P = 2 * matrix([[1., 0.], [0., 4.]])
#------------------------------------------------------------------------------------------
# Define q, G, h
q = matrix([[-8.,-16.]])
G = matrix([[1.,1.,0.],[1.,0.,-1]])
h = matrix([[5.,3.,0.]])
#------------------------------------------------------------------------------------------

sol = solvers.qp(P, q, G, h)
x1, x2 = sol['x']
print('Optimal x: ({:.8f}, {:.8f})'.format(x1, x2))
```

```
Collecting cvxopt
  Downloading cvxopt-1.2.7-cp38-cp38-macosx_10_9_x86_64.whl (3.1 MB)
     |████████████████████████████████| 3.1 MB 1.9 MB/s eta 0:00:01
Installing collected packages: cvxopt
Successfully installed cvxopt-1.2.7
Optimal x: (2.99999993, 1.99927914)
```

Let's visualize the solution

In [7]:
```
%matplotlib inline
from matplotlib import pyplot as plt
import numpy as np

X1, X2 = np.meshgrid(np.linspace(-8, 8, 100), np.linspace(-8, 8, 100))
F = X1**2 + 4*X2**2 - 8*X1 - 16 * X2

plt.pcolor(X1, X2, F, cmap='jet', shading="auto")
plt.scatter([x1], [x2], marker='*', color='white')
plt.show()
```



Why is the solution not in the minimum?

Linear SVM

Now, let's implement linear SVM. We will do this for the general case that allows class distributions to overlap, i.e. the linearly non-separable case **(see Bishop 7.1.1)**.

As a linear model, linear SVM produces classification scores for a given sample x as

$$\hat{y}(x) = w^T \phi(x) + b$$

where $w \in \mathbb{R}^d$, $b \in \mathbb{R}$ are model weights and bias, respectively, and ϕ is a fixed feature-space transformation. Final label prediction is done by taking the sign of $\hat{y}(x)$.

Given a set of training samples $x_n \in \mathbb{R}^d$, $n \in 1, \dots, N$, with the corresponding labels $y_i \in \{-1, 1\}$ linear SVM is fit (*i.e.* parameters w and b are chosen) by solving the following constrained optimization task:

$$\begin{aligned} \min_{w,\xi,b} \quad & \frac{1}{2}||w||^2 + C\sum_{n=1}^{N} \xi_n & & \quad (8) \\ \text{subject to} \quad & y_n \hat{y}(x_n) \geq 1 - \xi_n, & n = 1, \dots, N & \quad (9) \\ & \xi_n \geq 0, & n = 1, \dots, N & \quad (10) \end{aligned}$$

Note that the above is a quadratic programming problem.

Problem 3.1 [50 points]

Your task is to implement the linear SVM above using a QP solver by designing appropriate matrices P, q, G, h. Complete the code below.

Hints

1. You need to optimize over w, ξ, b. You can simply concatenate them into $\chi = (w, \xi, b)$ to feed it into QP-solver. Now, how to define the objective function and the constraints in terms of χ? (For example, $b_1 + b_2$ can be obtained from vector $(a_1, b_1, b_2, c_1, c_2)$ by taking the inner product with $(0, 1, 1, 0, 0)$).
2. You can use `np.bmat` to construct matrices. Like this:

In [8]:

```
np.bmat([[np.identity(3), np.zeros((3, 1))],
         [np.zeros((2, 3)), -np.ones((2, 1))]])
```

Out[8]:

```
matrix([[ 1.,  0.,  0.,  0.],
        [ 0.,  1.,  0.,  0.],
        [ 0.,  0.,  1.,  0.],
        [ 0.,  0.,  0., -1.],
        [ 0.,  0.,  0., -1.]])
```

In [12]:

```
from sklearn.base import BaseEstimator

class LinearSVM(BaseEstimator):
    def __init__(self, C, transform=None):
        self.C = C
        self.transform = transform

    def fit(self, X, Y):
        """Fit Linear SVM using training dataset (X, Y).

        :param X: data samples of shape (N, d).
        :param Y: data target labels of size (N). Each label is either 1 or -1.
        """
        # Apply transformation (phi) to X
        if self.transform is not None:
            X = self.transform(X)
        d = len(X[0])
        N = len(X)

        #----------------------------------------------------------------------------
        # Construct appropriate matrices here to solve the optimization problem described
        # We want optimal solution for vector (w, xi, b).
```

```
        P = matrix(np.bmat([[np.identity(d),np.zeros((d,N+1))],[np.zeros((N+1,d+N+1))]]))
        q = matrix(np.bmat([[np.zeros((1,d)),self.C*np.ones((1,N)),np.zeros((1,1))]])).T

        xy = np.bmat([np.matrix(X[:,d1]*Y).T for d1 in range(d)])
        G = -matrix(np.bmat([[xy,np.identity(N),Y.reshape(N,1)],[np.zeros((N,d)),np.identi
        h = matrix(np.bmat([[-np.ones((N,1))],[(-1)*np.zeros((N,1))]]))
        #-------------------------------------------------------------------------------

        sol = solvers.qp(P, q, G, h)
        ans = np.array(sol['x']).flatten()
        self.weights_ = ans[:d]
        self.xi_ = ans[d:d+N]
        self.bias_ = ans[-1]

        #-------------------------------------------------------------------------------
        # Find support vectors. Must be a boolean array of length N having True for suppo
        # vectors and False for the rest.
        self.support_vectors = self.xi_ >= 0
        #-------------------------------------------------------------------------------

    def predict_proba(self, X):
        """
        Make real-valued prediction for some new data.
        :param X: data samples of shape (N, d).
        :return: an array of N predicted scores.
        """
        # return y_hat
        if self.transform is not None:
            X = self.transform(X)
        return np.dot(self.weights_,X.T) + self.bias_

    def predict(self, X):
        """
        Make binary prediction for some new data.
        :param X: data samples of shape (N, d).
        :return: an array of N binary predicted labels from {-1, 1}.
        """
        return np.sign(self.predict_proba(X))
```

Let's see how our LinearSVM performs on some data.

In [13]:

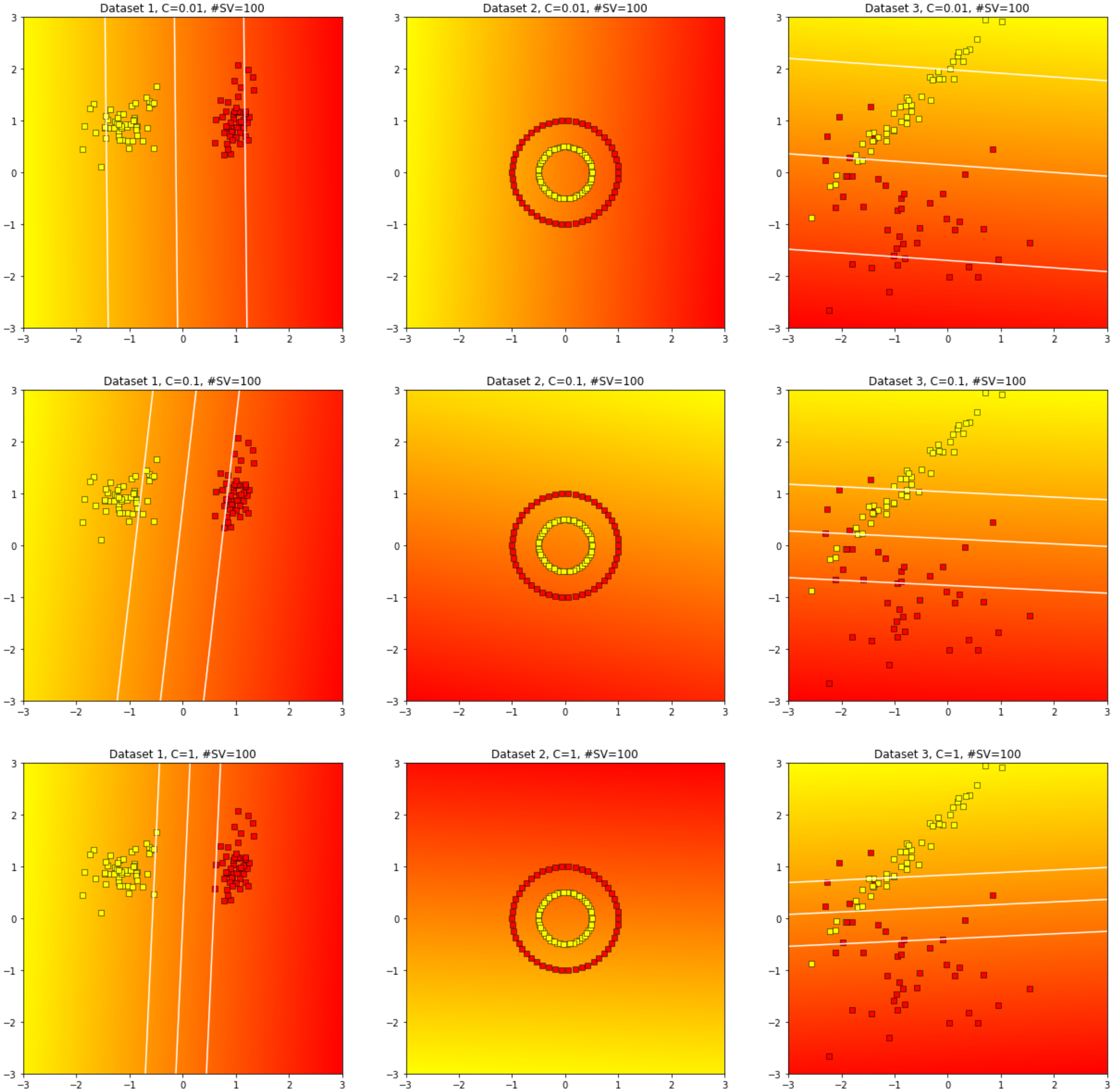
```
from sklearn.datasets import make_classification, make_circles
X = [None, None, None]
y = [None, None, None]
X[0], y[0] = make_classification(n_samples=100, n_features=2, n_redundant=0, n_clusters_pe
X[1], y[1] = make_circles(n_samples=100, factor=0.5)
X[2], y[2] = make_classification(n_samples=100, n_features=2, n_redundant=0, n_clusters_pe

# Go from {0, 1} to {-1, 1}
y = [2 * yy - 1 for yy in y]
```

In [15]:

```
C_values = [0.01, 0.1, 1]

plot_i = 0
plt.figure(figsize=(len(X) * 7, len(C_values) * 7))
for C in C_values:
    for i in range(len(X)):
        plot_i += 1
        plt.subplot(len(C_values), len(X), plot_i)
        #------------------------------------------------------------------------------
        model = LinearSVM(C=C)
        #------------------------------------------------------------------------------
        model.fit(X[i], y[i])
```

```
        sv = model.support_vectors
        n_sv = sv.sum()
        if n_sv > 0:
            plt.scatter(X[i][:, 0][sv], X[i][:, 1][sv], c=y[i][sv], cmap='autumn', marker=
                        linewidths=0.5, edgecolors=(0, 0, 0, 1))
        if n_sv < len(X[i]):
            plt.scatter(X[i][:, 0][~sv], X[i][:, 1][~sv], c=y[i][~sv], cmap='autumn',
                        linewidths=0.5, edgecolors=(0, 0, 0, 1))
        xvals = np.linspace(-3, 3, 200)
        yvals = np.linspace(-3, 3, 200)
        xx, yy = np.meshgrid(xvals, yvals)
        zz = np.reshape(model.predict_proba(np.c_[xx.ravel(), yy.ravel()]), xx.shape)
        plt.pcolormesh(xx, yy, zz, cmap='autumn', zorder=0, shading="auto")
        plt.contour(xx, yy, zz, levels=(-1, 0, 1,), colors='w', linewidths=1.5, zorder=1,

        plt.xlim([-3, 3])
        plt.ylim([-3, 3])
        plt.title('Dataset {}, C={}, #SV={}'.format(i + 1, C, n_sv))
plt.show()
```

```
/var/folders/q4/ptkzrlkn4jv7hpl3gcsd4vqc0000gn/T/ipykernel_24123/493890572.py:26: UserWarn
ing: No contour levels were found within the data range.
  plt.contour(xx, yy, zz, levels=(-1, 0, 1,), colors='w', linewidths=1.5, zorder=1, linest
yles='solid')
```



Why does the number of support vectors decrease as C increases?

For debug purposes: the very last model must have almost the same weights and bias as:

$$w = \begin{pmatrix} -0.0784521 \\ 1.62264867 \end{pmatrix}$$

$$b = -0.3528510092782581$$

In [16]:

```
model.weights_
```

Out[16]:

```
array([-0.0784521 ,  1.62264867])
```

In [17]:

```
model.bias_
```

Out[17]:

```
-0.35285100927825813
```

Problem 3.2 [10 points]

Even using a linear SVM, we are able to separate data that is linearly inseparable by using feature transformations.

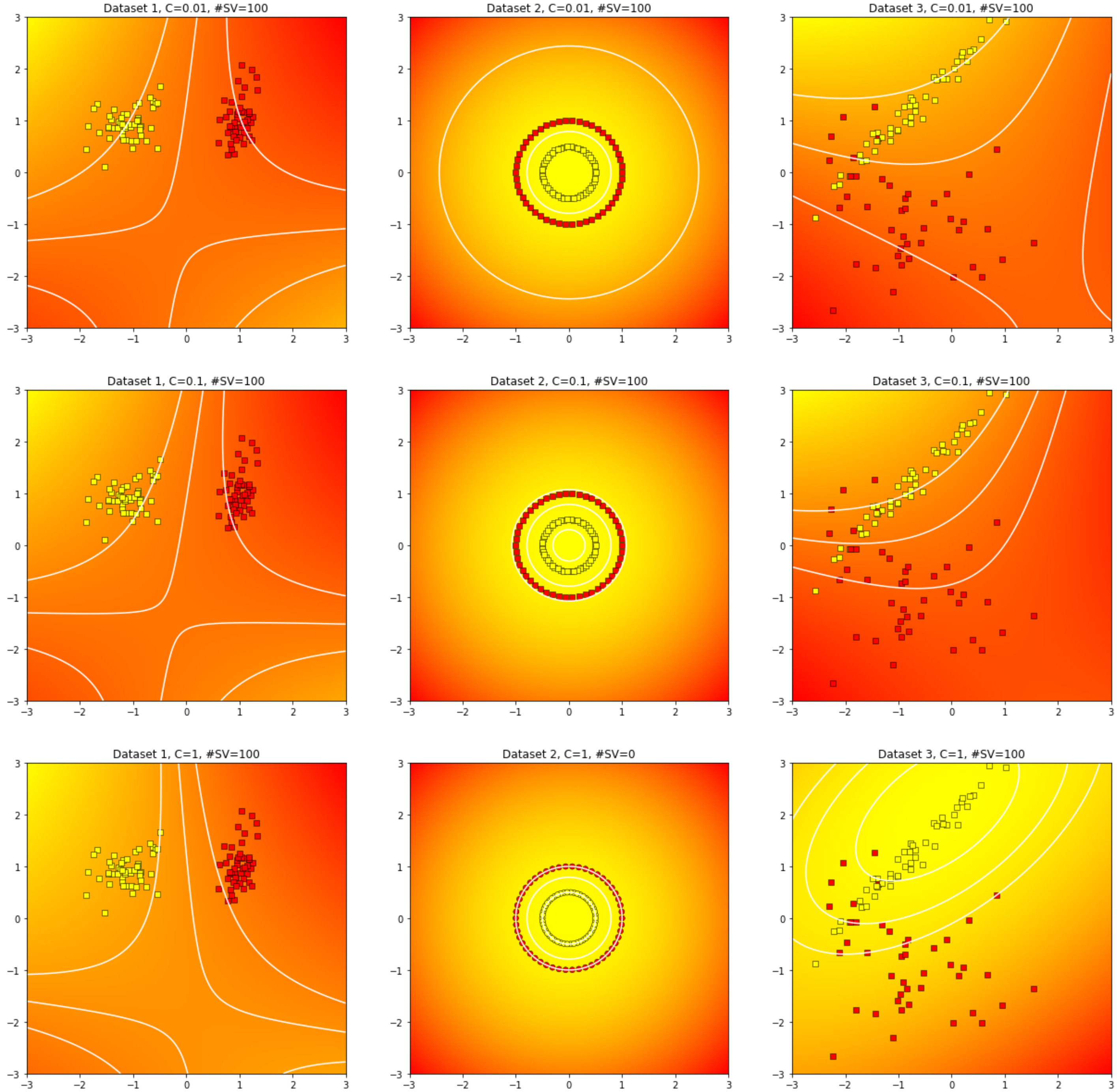
Implement the following feature transformation $\phi(x_1, x_2) = (x_1,\ x_2,\ x_1^2,\ x_2^2,\ x_1 x_2)$ and re-run your SVM.

In [18]:

```
def append_second_order(X):
    """Given array Nx[x1, x2] return Nx[x1, x2, x1^2, x2^2, x1x2]."""
    # return new_X
    X_0 = X[:,0].reshape(-1, 1)
    X_1 = X[:,1].reshape(-1, 1)
    new_X = np.concatenate([X_0, X_1, X_0*X_0, X_1*X_1, X_0*X_1], axis=1)

    return new_X

assert np.all(append_second_order(np.array([[1, 2]])) == np.array([[1, 2, 1, 4, 2]])), 'Tr
```

In [19]:

```
plot_i = 0
C_values = [0.01, 0.1, 1]
plt.figure(figsize=(len(X) * 7, len(C_values) * 7))
for C in C_values:
    for i in range(len(X)):
        plot_i += 1
        plt.subplot(len(C_values), len(X), plot_i)
        #----------------------------------------------------------------------------
        model = LinearSVM(C=C, transform=append_second_order)
        #----------------------------------------------------------------------------
        model.fit(X[i], y[i])
        sv = model.support_vectors
        n_sv = sv.sum()
        if n_sv > 0:
            plt.scatter(X[i][:, 0][sv], X[i][:, 1][sv], c=y[i][sv], cmap='autumn', marker=
                        linewidths=0.5, edgecolors=(0, 0, 0, 1))
        if n_sv < len(X[i]):
            plt.scatter(X[i][:, 0][~sv], X[i][:, 1][~sv], c=y[i][~sv], cmap='autumn',
                        linewidths=0.5, edgecolors=(0, 0, 0, 1))
        xvals = np.linspace(-3, 3, 200)
        yvals = np.linspace(-3, 3, 200)
        xx, yy = np.meshgrid(xvals, yvals)
        zz = np.reshape(model.predict_proba(np.c_[xx.ravel(), yy.ravel()]), xx.shape)
        plt.pcolormesh(xx, yy, zz, cmap='autumn', zorder=0, shading="auto")
        plt.contour(xx, yy, zz, levels=(-1, 0, 1,), colors='w', linewidths=1.5, zorder=1,

        plt.xlim([-3, 3])
        plt.ylim([-3, 3])
        plt.title('Dataset {}, C={}, #SV={}'.format(i + 1, C, n_sv))
plt.show()
```



Bonus part (Optional)

Dual representation. Kernel SVM

The dual representation of the maximum margin problem is given by

$$\max_{\alpha} \quad \sum_{n=1}^{N} \alpha_n - \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} \alpha_n \alpha_m y_n y_m k(x_n, x_m) \tag{11}$$

$$\text{subject to} \quad 0 \leq \alpha_n \leq C, \quad n = 1, \dots, N \tag{12}$$

$$\sum_{n=1}^{N} \alpha_n y_n = 0 \tag{13}$$

In this case bias b can be computed as $b = \frac{1}{|\mathcal{S}|}\sum_{n\in\mathcal{S}}\left(y_n - \sum_{m\in\mathcal{S}}\alpha_m y_m k(x_n, x_m)\right)$, and the prediction turns into $\hat{y}(x) = \sum_{n\in\mathcal{S}}\alpha_n y_n k(x_n, x) + b$.

Everywhere above k is a kernel function: $k(x_1, x_2) = \phi(x_1)^T \phi(x_2)$ (and the trick is that we don't have to specify ϕ, just k).

Note, that now

1. We want to maximize the objective function, not minimize it.
2. We have equality constraints. (That means we should use A and b in qp-solver)
3. We need access to the support vectors (but not all the training samples) in order to make a prediction.

Problem 4.1 [40 points]

Implement KernelSVM

Hints

1. What is the variable we are optimizing over?
2. How can we maximize a function given a tool for minimization?
3. What is the definition of a support vector in the dual representation?

In [21]:

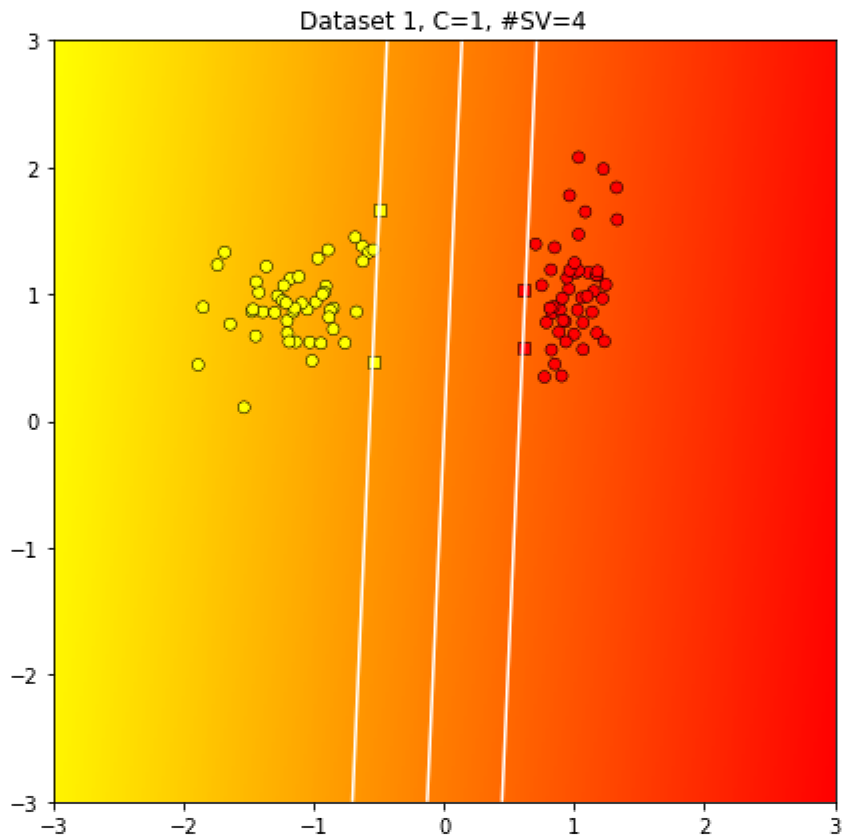
```
class KernelSVM(BaseEstimator):
    def __init__(self, C, kernel=np.dot):
        self.C = C
        self.kernel = kernel

    def fit(self, X, Y):
        """Fit Kernel SVM using training dataset (X, Y).

        :param X: data samples of shape (N, d).
        :param Y: data target labels of size (N). Each label is either 1 or -1. Denoted as
        """
        N = len(Y)

        #--------------------------------------------------------------------------------
        # Construct appropriate matrices here to solve the optimization problem described
        P = matrix([Y[i] * Y[j] * self.kernel(X[i], X[j]) for i in range(N) for j in range
        q = matrix(-np.ones(N))
        G = matrix(np.bmat([[-1. * np.identity(N)], [1. * np.identity(N)]]))
        h = matrix([0.] * N + [self.C] * N)
        A = matrix(1. * Y, (1, N))
        b = matrix(0.)
        #--------------------------------------------------------------------------------

        sol = solvers.qp(P, q, G, h, A, b)
        self.alpha_ = np.array(sol['x']).flatten()

        #--------------------------------------------------------------------------------
        # Find support vectors. Must be a boolean array of length N having True for suppor
        # vectors and False for the rest.
        self.support_vectors = self.alpha_ > 1e-5
        #--------------------------------------------------------------------------------

        sv_ind = self.support_vectors.nonzero()[0]
        self.X_sup = X[sv_ind]
        self.Y_sup = Y[sv_ind]
        self.alpha_sup = self.alpha_[sv_ind]
        self.n_sv = len(sv_ind)

        #--------------------------------------------------------------------------------
        # Compute bias
        self.bias_ = np.mean([self.Y_sup[i] - np.sum([self.alpha_sup[j] * self.Y_sup[j] *
                              self.kernel(self.X_sup[i], self.X_su
```

```
#-----------------------------------------------------------------------------
    def predict_proba(self, X):
        """
        Make real-valued prediction for some new data.
        :param X: data samples of shape (N, d).
        :return: an array of N predicted scores.
        """
        # return y_hat
        return [np.sum([self.alpha_sup[i] * self.Y_sup[i] * self.kernel(self.X_sup[i], X[j

    def predict(self, X):
        """
        Make binary prediction for some new data.
        :param X: data samples of shape (N, d).
        :return: an array of N binary predicted labels from {-1, 1}.
        """
        return np.sign(self.predict_proba(X))
```

We can first test our implementation by using the dot product as a kernel function. What should we expect in this case?

In [22]:

```
C = 1
i = 0

plt.figure(figsize=(7, 7))
#---------------------------------------------------------------------------
model = KernelSVM(C=C, kernel=np.dot)
#---------------------------------------------------------------------------
model.fit(X[i], y[i])
sv = model.support_vectors
n_sv = sv.sum()
if n_sv > 0:
    plt.scatter(X[i][:, 0][sv], X[i][:, 1][sv], c=y[i][sv], cmap='autumn', marker='s',
                linewidths=0.5, edgecolors=(0, 0, 0, 1))
if n_sv < len(X[i]):
    plt.scatter(X[i][:, 0][~sv], X[i][:, 1][~sv], c=y[i][~sv], cmap='autumn',
                linewidths=0.5, edgecolors=(0, 0, 0, 1))
xvals = np.linspace(-3, 3, 200)
yvals = np.linspace(-3, 3, 200)
xx, yy = np.meshgrid(xvals, yvals)
zz = np.reshape(model.predict_proba(np.c_[xx.ravel(), yy.ravel()]), xx.shape)
plt.pcolormesh(xx, yy, zz, cmap='autumn', zorder=0, shading="auto")
plt.contour(xx, yy, zz, levels=(-1, 0, 1,), colors='w', linewidths=1.5, zorder=1, linestyl

plt.xlim([-3, 3])
plt.ylim([-3, 3])
plt.title('Dataset {}, C={}, #SV={}'.format(i + 1, C, n_sv))
plt.show()
```



Problem 4.2 [5 points]

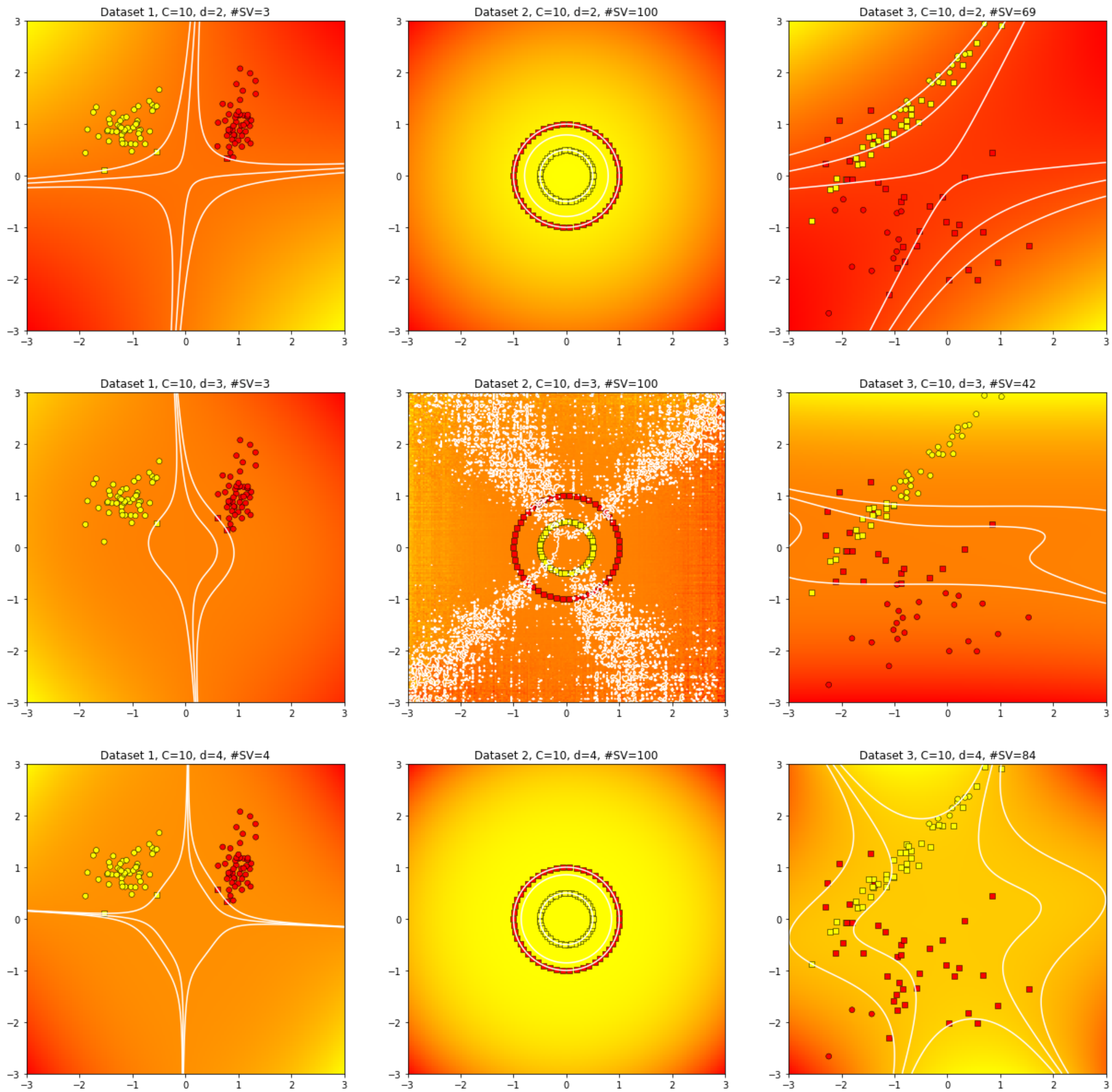
Implement a polynomial kernel function (wiki).

In [23]:
```
def polynomial_kernel(d, c=0):
    """Returns a polynomial kernel FUNCTION."""
    def kernel(x, y):
        """
        :param x: vector of size L
        :param y: vector of size L
        :return: [polynomial kernel of degree d with bias parameter c] of x and y. A scala
        """
        return (x.dot(y) + c) ** d
    return kernel

assert polynomial_kernel(d=2, c=1)(np.array([1, 2]), np.array([3, 4])) == 144, 'Polynomial
```

Let's see how it performs. This might take some time to run.

In [24]:
```
plot_i = 0
C = 10
d_values = [2, 3, 4]
plt.figure(figsize=(len(X) * 7, len(d_values) * 7))
for d in d_values:
    for i in range(len(X)):
        plot_i += 1
        plt.subplot(len(d_values), len(X), plot_i)
        #------------------------------------------------------------------------------
        model = KernelSVM(C=C, kernel=polynomial_kernel(d))
        #------------------------------------------------------------------------------
        model.fit(X[i], y[i])
        sv = model.support_vectors
        n_sv = sv.sum()
        if n_sv > 0:
```

```
            plt.scatter(X[i][:, 0][sv], X[i][:, 1][sv], c=y[i][sv], cmap='autumn', marker=
                        linewidths=0.5, edgecolors=(0, 0, 0, 1))
        if n_sv < len(X[i]):
            plt.scatter(X[i][:, 0][~sv], X[i][:, 1][~sv], c=y[i][~sv], cmap='autumn',
                        linewidths=0.5, edgecolors=(0, 0, 0, 1))
        xvals = np.linspace(-3, 3, 200)
        yvals = np.linspace(-3, 3, 200)
        xx, yy = np.meshgrid(xvals, yvals)
        zz = np.reshape(model.predict_proba(np.c_[xx.ravel(), yy.ravel()]), xx.shape)
        plt.pcolormesh(xx, yy, zz, cmap='autumn', zorder=0, shading="auto")
        plt.contour(xx, yy, zz, levels=(-1, 0, 1,), colors='w', linewidths=1.5, zorder=1,

        plt.xlim([-3, 3])
        plt.ylim([-3, 3])
        plt.title('Dataset {}, C={}, d={}, #SV={}'.format(i + 1, C, d, n_sv))
```



Problem 4.3 [5 points]

Finally, you need to implement a **radial basis function** kernel (wiki).

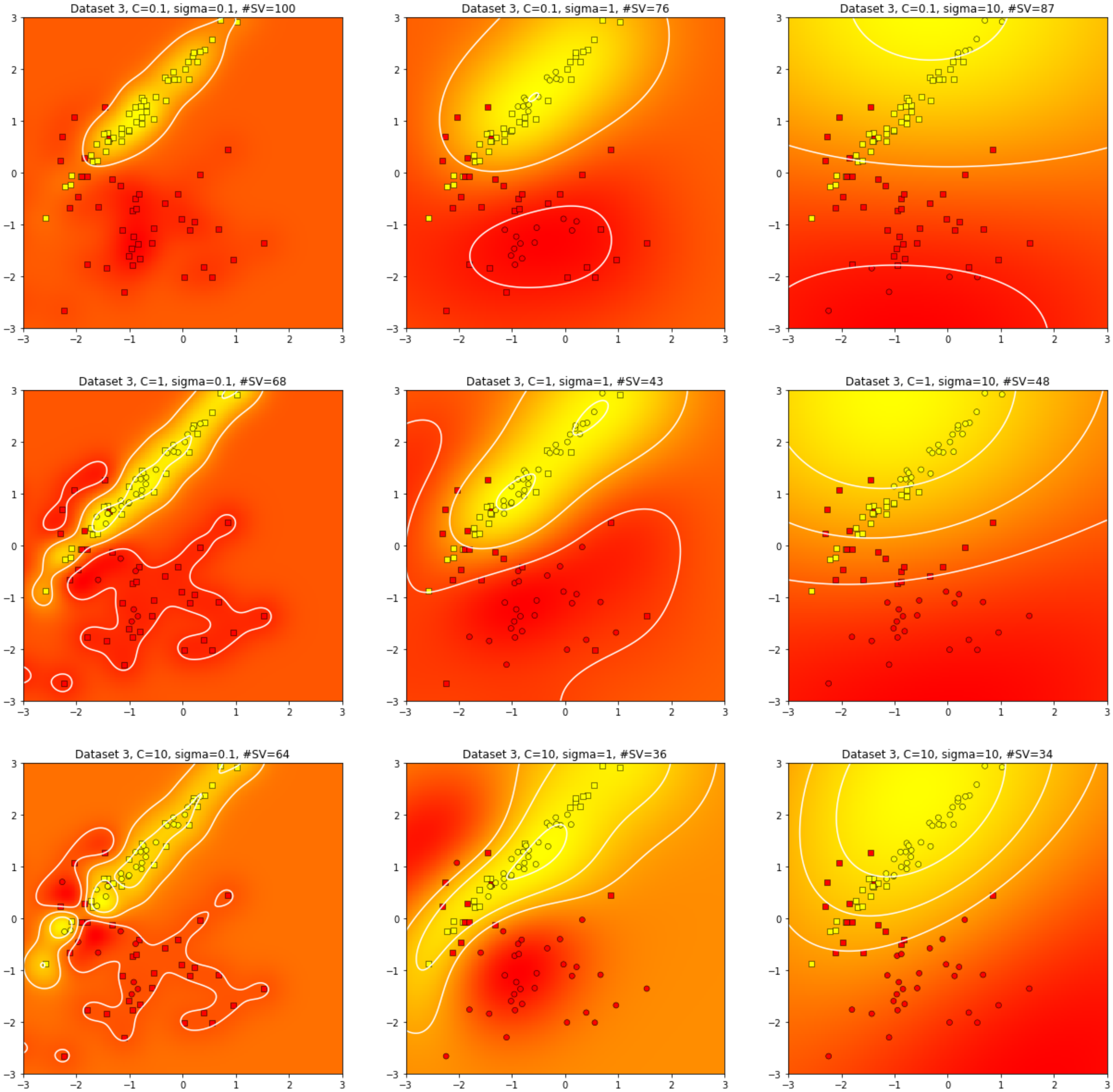
In [25]:

```
def RBF_kernel(sigma):
    """Returns an RBF kernel FUNCTION."""
    def kernel(x, y):
        """
        :param x: vector of size L
        :param y: vector of size L
        :return: [rbf kernel with parameter sigma] of x and y. A scalar.
        """
        return np.exp(-np.sum((x - y) ** 2) / (2. * sigma))
    return kernel
```

Let's see how it performs. This might take some time to run.

In [26]:

```
plot_i = 0
C_values = [0.1, 1, 10]
sigma_values = [0.1, 1, 10]
plt.figure(figsize=(len(sigma_values) * 7, len(C_values) * 7))
i = 2
for C in C_values:
    for sigma in sigma_values:
        plot_i += 1
        plt.subplot(len(C_values), len(X), plot_i)
        model = KernelSVM(C=C, kernel=RBF_kernel(sigma))
        model.fit(X[i], y[i])
        sv = model.support_vectors
        n_sv = sv.sum()
        if n_sv > 0:
            plt.scatter(X[i][:, 0][sv], X[i][:, 1][sv], c=y[i][sv], cmap='autumn', marker=
                        linewidths=0.5, edgecolors=(0, 0, 0, 1))
        if n_sv < len(X[i]):
            plt.scatter(X[i][:, 0][~sv], X[i][:, 1][~sv], c=y[i][~sv], cmap='autumn',
                        linewidths=0.5, edgecolors=(0, 0, 0, 1))
        xvals = np.linspace(-3, 3, 200)
        yvals = np.linspace(-3, 3, 200)
        xx, yy = np.meshgrid(xvals, yvals)
        zz = np.reshape(model.predict_proba(np.c_[xx.ravel(), yy.ravel()]), xx.shape)
        plt.pcolormesh(xx, yy, zz, cmap='autumn', zorder=0, shading="auto")
        plt.contour(xx, yy, zz, levels=(-1, 0, 1,), colors='w', linewidths=1.5, zorder=1,

        plt.xlim([-3, 3])
        plt.ylim([-3, 3])
        plt.title('Dataset {}, C={}, sigma={}, #SV={}'.format(i + 1, C, sigma, n_sv))
```



Well done!

Awesome! Now you understand all of the important parameters in SVMs. Have a look at SVM from scikit-learn module and how it is used (very similar to ours).

In [27]:

```
from sklearn.svm import SVC
SVC?
```

In [28]:

```
plot_i = 0
C = 10
d_values = [2, 3, 4]
plt.figure(figsize=(len(X) * 7, len(d_values) * 7))
for d in d_values:
    for i in range(len(X)):
        plot_i += 1
        plt.subplot(len(d_values), len(X), plot_i)
        #----------------------------------------------------------------------
        model = SVC(kernel='poly', degree=d, gamma='auto', probability=True)
```

```
        #----------------------------------------------------------------------------
        model.fit(X[i], y[i])
        plt.scatter(X[i][:, 0], X[i][:, 1], c=y[i], cmap='autumn', linewidths=0.5, edgecol
        xvals = np.linspace(-3, 3, 200)
        yvals = np.linspace(-3, 3, 200)
        xx, yy = np.meshgrid(xvals, yvals)
        zz = np.reshape(model.predict_proba(np.c_[xx.ravel(), yy.ravel()])[:, 1] * 2 - 1,
        plt.pcolormesh(xx, yy, zz, cmap='autumn', zorder=0, shading="auto")
        plt.contour(xx, yy, zz, levels=(-1., 0., 1.), colors='w', linewidths=1.5, zorder=1

        plt.xlim([-3, 3])
        plt.ylim([-3, 3])
        plt.title('Dataset {}, C={}, d={}, #SV={}'.format(i + 1, C, d, len(model.support_v
```

```
/var/folders/q4/ptkzrlkn4jv7hpl3gcsd4vqc0000gn/T/ipykernel_24123/654972113.py:19: UserWarn
ing: No contour levels were found within the data range.
  plt.contour(xx, yy, zz, levels=(-1., 0., 1.), colors='w', linewidths=1.5, zorder=1, line
styles='solid')
```