

CISC304: Homework 6

Due on December 2, 11:00 am

Keith Yong

Problem 1

Use the Hilbert Deductive System for First Order Logic (aka Predicate Calc.) to prove the following. a.

Problem 3

Use *Resolution*, utilizing the **Davis-Putnam procedure**, to show that the following set of propositional clauses is unsatisfiable. Show every step of the procedure and clearly indicate which clauses and literals are being resolved, eliminated and/or added in each step.

$$S = \{\{p, \neg r\}_1; \{q, \neg r\}_2; \{q, \neg s\}_3; \{\neg p, o\}_4; \{\neg q, \neg o\}_5; \{\neg q, r, o\}_6; \{p, s, \neg o\}_7; \{\neg p, q, r\}_8; \{q, r, s, o\}_9\}$$

Solution

1. No trivial or literal clauses found.

2. Resolve for p by resolving clauses $\langle 1, 4 \rangle, \langle 1, 8 \rangle, \langle 7, 4 \rangle, \langle 7, 8 \rangle$:

$$\{\{\neg r, o\}_{14}; \{\neg r, q, r\}_{18}; \{s, \neg o, o\}_{74}; \{s, \neg o, q, r\}_{78}; \{q, \neg r\}_2; \{q, \neg s\}_3; \{\neg q, \neg o\}_5; \{\neg q, r, o\}_6; \{q, r, s, o\}_9\}$$

3. Delete trivial clauses 18, 74

$$\{\{\neg r, o\}_{14}; \{s, \neg o, q, r\}_{78}; \{q, \neg r\}_2; \{q, \neg s\}_3; \{\neg q, \neg o\}_5; \{\neg q, r, o\}_6; \{q, r, s, o\}_9\}$$

4. Resolve for r by resolving clauses $\langle 78, 14 \rangle, \langle 78, 2 \rangle, \langle 6, 14 \rangle, \langle 6, 2 \rangle, \langle 9, 14 \rangle, \langle 9, 2 \rangle$:

$$\{\{o, \neg o, s, q\}_{7814}; \{s, \neg o, q\}_{782}; \{\neg q, o\}_{614}; \{\neg q, o, q\}_{62}; \{q, s, o\}_{914}; \{q, s, o, \neg s\}_{92}; \{q, \neg s\}_3; \{\neg q, \neg o\}_5\}$$

5. Delete trivial clauses 7814, 62, 92:

$$\{\{s, \neg o, q\}_1; \{\neg q, o\}_2; \{q, s, o\}_3; \{q, \neg s\}_4; \{\neg q, \neg o\}_5\}$$

6. Resolve for o by resolving clauses $\langle 2, 1 \rangle, \langle 2, 5 \rangle, \langle 3, 1 \rangle, \langle 3, 5 \rangle$:

$$\{\{s, q, \neg q\}_{21}; \{\neg q\}_{25}; \{q, s\}_{31}; \{q, s, \neg q\}_{35}; \{q, \neg s\}_4\}$$

7. Delete trivial clauses 21 and 35:

$$\{\{\neg q\}_1; \{q, s\}_2; \{q, \neg s\}_3\}$$

8. Resolve for q by resolving clauses $\langle 2, 1 \rangle, \langle 3, 1 \rangle$

$$\{\{s\}_{21}; \{\neg s\}_{31}\}$$

9. Resolve for s by resolving clauses $\langle 21, 31 \rangle$

$$\{\square\}$$

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