Question 5:

Finite Difference Method

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Problem Statement

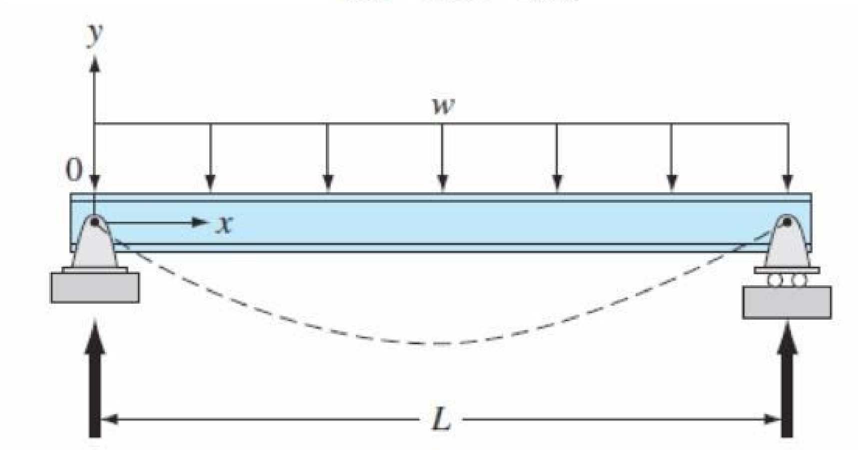
Solve the deflection of the beam using the FINITE DIFFERENCE approach. Compare numerical results to the analytical solutions, also using a graph.

2ME51P.0. The basic differential equation of the elastic curve for a uniformly loaded beam (as shown in figure) is given as

$$EI\frac{d^2y}{dx^2}=\frac{wLx}{2}-\frac{wx^2}{2}$$

where E = the modulus of elasticity and I = the moment of inertia. Solve for the deflection of the beam using (a) the finite-difference approach ($\Delta x = 600$ mm). The following parameter values apply: E = 210 GPa, $I = 3.3\times10^8$ mm^4, w = 15 kN/m, L = 3000 mm. Compare your numerical results to the analytical solution,

$$y=\frac{wLx^3}{12EI}-\frac{wx^4}{24EI}-\frac{wL^3x}{24EI}$$



Compare the analytical solution with graphical solution

Numerical Solution

→ Given

$L = 3000mm = 3m$

$\omega = 15\ kN/m = 15 \times 10^3\ N/m$

$\Delta x = 600mm = 0.6m$

$E = 210\ GPa = 210 \times 10^9\ Pa$

$I = 3.3 \times 10^3\ mm^4$

→ Converting to finite difference equation:-

$$EI \frac{d^2y}{dx^2} = \frac{\omega L x}{2} - \frac{\omega x^2}{2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2EI}(\omega L x - \omega x^2)$$

$$\frac{y_{i-1} - 2y_i + y_{i+1}}{(\Delta x)^2} = \frac{15 \times 10^3 \times (3x - x^2)}{(210 \times 10^3)(3.3 \times 10^8) \times 2}$$

$$y_{i-1} - 2y_i + y_{i+1} = (3.896 \times 10^{-17})(3x - x^2)$$

SCILAB Code

```
clc;
// Initialization - declaring variables
L=3;                    // in meters
h=0.6;                  // in meters
N=(L/h) - 1;            // N = 4;
CN = [0 0];             // +
A=zeros(N);             // +
w=15*10^3;              //converted kN/m to N/m
E=210*10^9;             // Converted GPa to Pa
I=3.3*10^8;             // in mm^4

// NUMERICAL
// Constant from: y'' = constant*(Lx - x^2) by rearranging given formula
constant = w*(h^2)/(2*E*I);
//constant = 3.896D-17

// For loop to create matrices
for i=1:N
    x=i*h;
    // for Matrix B
    B(i) = constant*(3*x-x^2);
    // for Matrix A
    for j=1:N
       if i==j
           A(i,j)=-2;
       else
           if j==i+1
               A(i,j)=1;
           elseif j==i-1
               A(i,j) = 1;
           end
       end
    end
end

M=[A, B]; // Augmented Matrix
printf("\n \tAugmented Matrix from finite-diff Method which is to be
solved");
disp(M);
Mx=rref(M); //converts matrix M into RREF(reduced row ec. form)
printf("\n*****************************************************************
***", 's');
printf("\n\n \tReduced Row-Echelon Form of Matrix");
disp(Mx);
Mx(:,$); // ($) selects last element - Hence this is last element of every
row(:)
YN = [CN(1)  Mx(:,$)' CN(2) ]; // NUMERICAL SOLUTIONS
```

```
printf("\n*****************************************************************
***", 's');
printf("\n\n",'s');
printf("\t OUR SOLUTIONS are as follows\n",'s');
for i=1:length(YN)
    printf("\tThe %d solution is \t %e\n",i,YN(i));
end

// Plotting for Numerical Solutions
xdata = 0:0.6:3; // X-axis points for Numerical
ydata = YN*10^16; // Storing Numerical Solutions (magnified)
xlabel("time (t)","fontsize", 3);
ylabel("y * 10^-16","fontsize", 3);
title("MiniProject #5",'color','red','edgecolor','blue','fontsize',5,
'fontname', 'times bold italic');
xgrid(4);
// NUMERICAL ENDS

// ANALYTICAL
function y_anal=g(x)
    y_anal = (w*L*(x^3) - (w*(x^4)/2) - (w*(L^3)*x/2))/(E*I*12);
endfunction
funcprot(0); // to avoid warning shown by scilab: redefining g(x)
for i=0:0.1:3
    y_analytical(10*i + 1)=g(i); // Analytical Solutions
end
y_analytical = y_analytical*10^16; // Storing Analytical Solutions,
(magnified)

printf("\n*****************************************************************
***", 's');
printf("\n\n\t   Numerical      Analytical       Error\n",'s');
for i=0:0.6:3
    n = i*(10/6)+1;
    //err(1) = 0;
    //err(6) = 0;
    err(n) = abs((YN(n) - g(i))/g(i))*100; // ERROR Calculation
    if n == 1 || n == 6      // at first and last node, error = 0
        err(n) = 0;          // NUM = ANA = 0 at {0, 3}
    end
    printf("\t %e \t %e \t %f%%\n",YN(n),g(i),err(n));
end

// Plotting of Analytical Graph
x_analytical=[0:0.1:3] // X-axis for Analytical
plot(xdata',ydata',"ro"); // Points for Numerical Solution
plot(xdata',ydata','diamonded:'); // Dot-Dashed line joining Numerical
Solution
yi=smooth([xdata;ydata],0.1); // obtaining smooth curve joining Numerical
Solutions
plot2d(yi(1,:)',yi(2,:)',10); // Plotting entire Numerical portion
plot2d(x_analytical',y_analytical', 1); // Plotting entire Analytical Portion
a = gca(); // get current axes
```

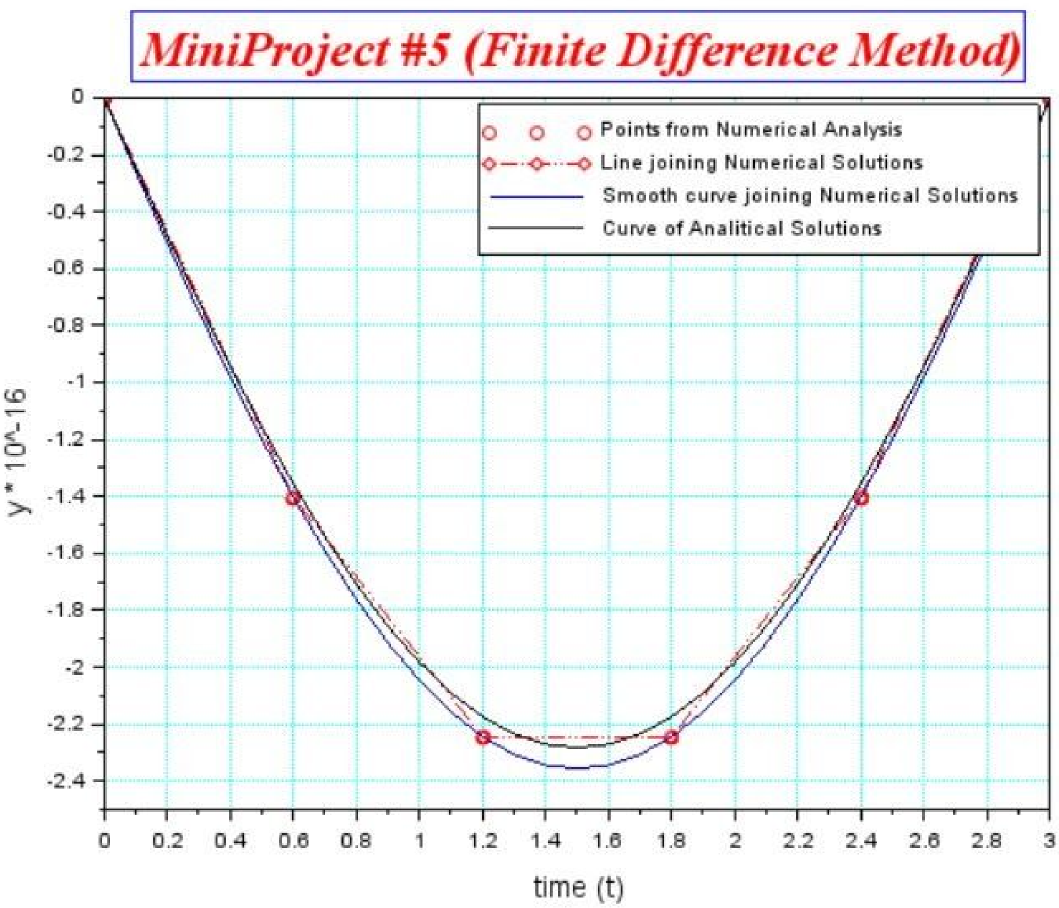
```
a.data_bounds = [0 -2.5; 3 0]; // Bounding X-axis and Y-axis to magnify our
graph
// ANALYTICAL ENDS

legend("Points from Numerical Analysis", "Line joining Numerical Solutions",
"Smooth curve joining Numerical Solutions", "Curve of Analytical
Solutions");
```

Link: [GitHub Folder of my Project]

Link to SciLab Code: [Mini Project CODE]

Output Graph



Conclusion

1. *Finite difference method* is **fairly accurate** (only ~3% error in each value)
2. The graph is nearly **quadratic** in nature within the range [0, 3]