5

$$EI\frac{d^2y}{dx^2} = \frac{wLx}{2} - \frac{wx^2}{2}$$

And $$\left(\frac{d^2y}{dx^2}\right)_{(i,j)} = \frac{y_{(i+1,j)} - 2y_{(i,j)} + y_{(i-1,j)}}{(\Delta x)^2}$$

$E = 210\ GPa = 210 \times 10^9\ Pa$

$I = 3.3 \times 10^8\ (mm)^4$

$w = 15\ kN/m = 15 \times 10^3\ N/m$

$L = 3000\ mm = 3\ m$

$$EI\frac{d^2y}{dx^2} = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$\frac{d^2y}{dx^2} = \frac{wLx}{2EI} - \frac{wx^2}{2EI}$$

$$\frac{d^2y}{dx^2} = (3.2467x - 1.08225 \times 10^{-3}x^2) \times 10^{-10}$$

$x_0 = 0$ $x_1 = 600mm$

$x_2 = 1200mm$ $x_3 = 1800mm$

$x_4 = 2400mm$ $x_5 = 3000mm$

$\Delta x = 600mm$ and $y_0 = 0$ $y_5 = 0$

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{(\Delta x)^2} = (3.2467x - 1.08225 \times 10^{-3}x^2) \times 10^{-10}$$

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{(\Delta x)^2} = (3.25x_i - 1.08\times10^{-3}x_i^2)\times10^{-10}$$

$\Delta x = 600$ and $i = 1$

$$\rightarrow y_2 - 2y_1 + y_0 = (3.25x_1 - 1.08\times10^{-3}x_1^2)\times10^{-10}\times(600)^2$$

$$y_2 - 2y_1 = (3.25\times600 - 1.08\times10^{-3}(600)^2)\times10^{-10}\times(600)^2$$

$$= (1950 - 388.8)\times360{,}000\times10^{-10}$$

$$= 1561.2 \times 3.6\times10^{-5}$$

$$y_2 - 2y_1 = 5.62\times10^{-2} \quad \text{— (1)}$$

$$\rightarrow y_3 - 2y_2 + y_1 = (3.25\times x_2 - 1.08\times10^{-3}x_2^2)\times10^{-10}\times(600)^2$$

$$= (1950\times2 - 388.8\times4)\times360{,}000\times10^{-10}$$

$$= 8.44\times10^5\times10^3\times10^{-10}$$

$$= 8.44\times10^{-2} \quad \text{— (2)}$$

$$\rightarrow y_4 - 2y_3 + y_2 = (3.25\times x_3 - 1.08\times10^{-3}x_3^2)\times10^{-10}\times360{,}000$$

$$= 8.46\times10^{-2} \quad \text{— (3)}$$

$$\rightarrow y_5 - 2y_4 + y_3 = (3.25\times x_4 - 1.08\times10^{-3}x_4^2)\times360{,}000\times10^{-10}$$

$$y_3 - 2y_4 = 5.68\times10^{-2} \quad \text{— (4)}$$

~~from eqn ②~~

from eqn ①

$$y_1 = \frac{y_2 - 5.62 \times 10^{-2}}{2}$$

put the value of $y_1$ in eqn ②

$$y_3 - 2y_2 + \frac{y_2}{2} - \frac{5.62 \times 10^{-2}}{2} = 8.44 \times 10^{-2}$$

$$y_3 - \frac{3y_2}{2} = (8.44 + 2.81) \times 10^{-2}$$

$$-3y_2 + 2y_3 = 22.5 \times 10^{-2} \quad \text{---} ⑤$$

from eqn ④, ③, ⑤

Let

$$-3y_2 + 2y_3 + 0y_4 = 22.5 \times 10^{-2} = A$$

$$y_2 - 2y_3 + y_4 = 8.46 \times 10^{-2} = B$$

$$0y_2 + y_3 - 2y_4 = 5.68 \times 10^{-2} = C$$

⇒ finding $y_2$, $y_3$, and $y_4$ by Gauss-Jordan method

$$\sim \left[\begin{array}{ccc|c} -3 & 2 & 0 & A \\ 1 & -2 & 1 & B \\ 0 & 1 & -2 & C \end{array}\right]$$

$$\simeq \frac{-R_1}{3}\begin{bmatrix} 1 & 2/3 & 0 & A/3 \\ 1 & -2 & 1 & B \\ 0 & 1 & -2 & C \end{bmatrix}$$

$$\simeq R_2 - R_1\begin{bmatrix} 1 & 2/3 & 0 & A/3 \\ 0 & -8/3 & 1 & B-A/3 \\ 0 & 1 & -2 & C \end{bmatrix}$$

$$\simeq R_{23}\begin{bmatrix} 1 & 2/3 & 0 & A/3 \\ 0 & 1 & -2 & C \\ 0 & -8/3 & -2 & B-A/3 \end{bmatrix}$$

$$\simeq R_3 + 3/8 \cdot 8/3 R_2\begin{bmatrix} 1 & 2/3 & 0 & A/3 \\ 0 & 1 & -2 & C \\ 0 & 0 & -22/3 & B-A/3+8C/3 \end{bmatrix}$$

$$\simeq -3/22 R_3\begin{bmatrix} 1 & 2/3 & 0 & A/3 \\ 0 & 1 & -2 & C \\ 0 & 0 & 1 & -(B-A/3+8C/3)3/22 \end{bmatrix}$$

$$\simeq R_2 + 2R_3\begin{bmatrix} 1 & 2/3 & 0 & A/3 \\ 0 & 1 & 0 & -3/11(B-A/3+8C/3) \\ 0 & 0 & 1 & -(B-A/3+8C/3)3/22 \end{bmatrix}$$

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$$\sim \left[\begin{array}{ccc|c} 1 & 2/3 & 0 & A/3 \\ 0 & 1 & 0 & (-3/11)(B - A/3 + \delta/3) \\ 0 & 0 & 1 & -3/22\,(B - A/3 + \delta/3) \end{array}\right]$$

Let $B - A/3 + \delta/3 = D = 16.11 \times 10^{-10}$

$$\sim \left[\begin{array}{ccc|c} 1 & 2/3 & 0 & A/3 \\ 0 & 1 & 0 & (-3/11)D \\ 0 & 0 & 1 & -3/22\,D \end{array}\right]$$

$$\sim R_1 - 3/4\,2/3\,R_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & A/3 - 2/3\,C + 2/11\,D \\ 0 & 1 & 0 & (-3/11)D \\ 0 & 0 & 1 & -3/22\,D \end{array}\right]$$

$$y_2 = A/3 - \frac{2C}{3} + \frac{2D}{11}$$

$$= 7.5 \times 10^{-2} - 3.787 * 10^{-2} + 2.929 \times 10^{-2}$$

$$y_2 = 6.642 \times 10^{-2}$$

$$y_3 = (5.68 - 4.39) \times 10^{-2}$$

$$y_3 = 1.286 \times 10^{-2}$$

$$y_4 = 2.1956 \times 10^{-2}$$

$$y_1 = \frac{y_2 - 5.62 \times 10^{-2}}{2} = 0.511 \times 10^{-2}$$