Planning and Learning

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with some slides from Rob Platt, U Alberta, Mykel Kochenderfer, and Frans Oliehoek

Announcements

- Exam
- Project proposals due 10/28
  - More formal (i.e., less English)
- Ex5 (TD) due 10/24
- Ex6 (Planning and learning) due 11/1
- Exam 2 on 11/25
- Read SB 9.1--9.5, 9.8

RL research opportunities

- Undergrad co-op on security for RL
- MS Research apprenticeships
    - Nominations
    - Projects

Overview: Planning and Learning

- We will talk about
  - various planning methods that use a model or simulator to generate solutions (planning)
  - Model-based RL methods that learn models of the dynamics as part of RL (which use planning with the model)

Planning and Learning: AlphaGo

Planning

What do you think of when you think about "planning"?

– often, the word "planning" often means a specific class of algorithm

– here, we use "planning" to mean any computational process that uses a model to create or improve a policy

Planning

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– often, the word "planning" often means a specific class of algorithm

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Model: "Anything that an agent can use to predict how the environment will respond to its actions." $p(s', r \mid s, a)$

For example: an unusual way to do planning

model $\longrightarrow$ simulated experience $\xrightarrow{\text{backups}}$ values $\longrightarrow$ policy

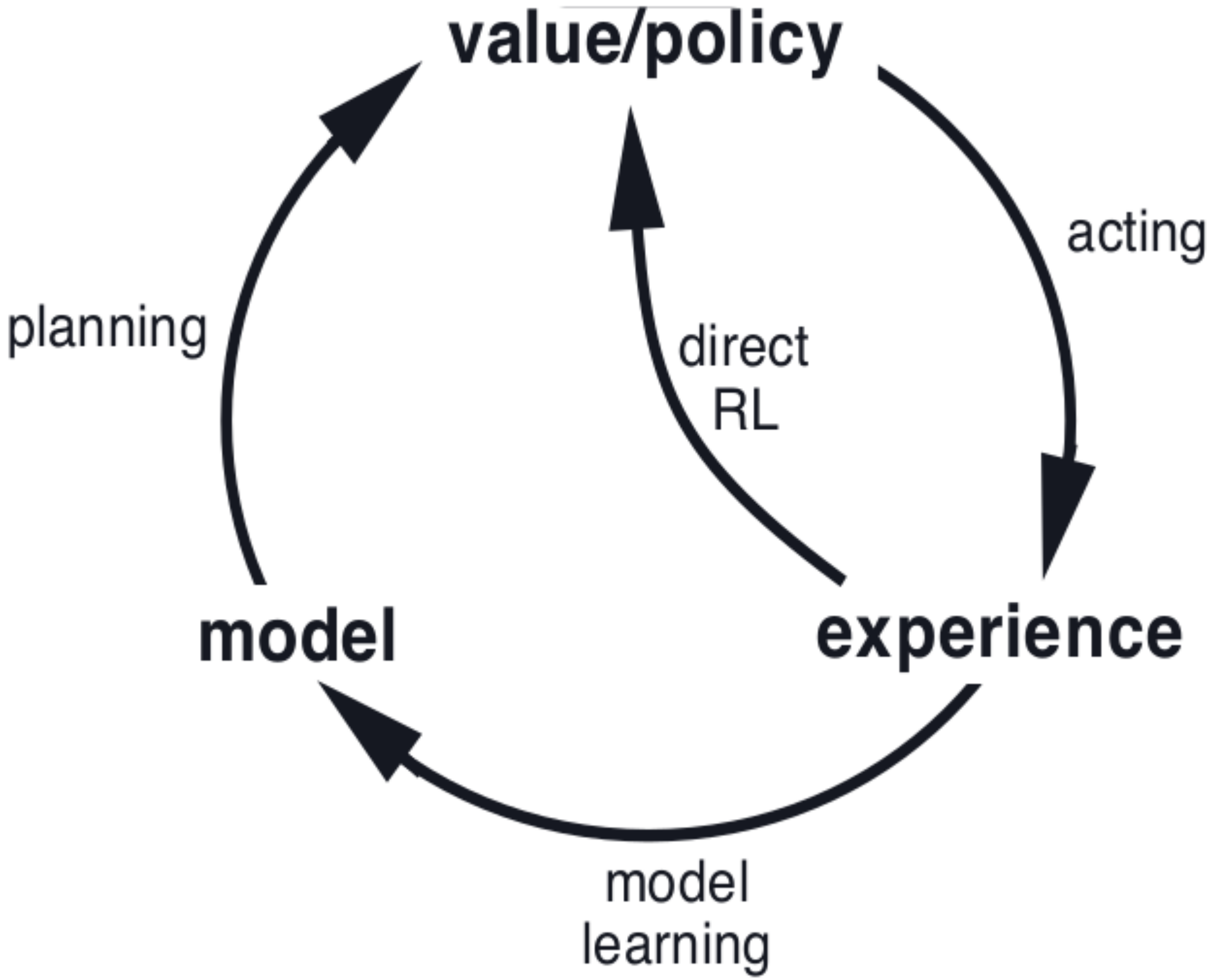
Random-sample one-step tabular Q-planning

Loop forever:

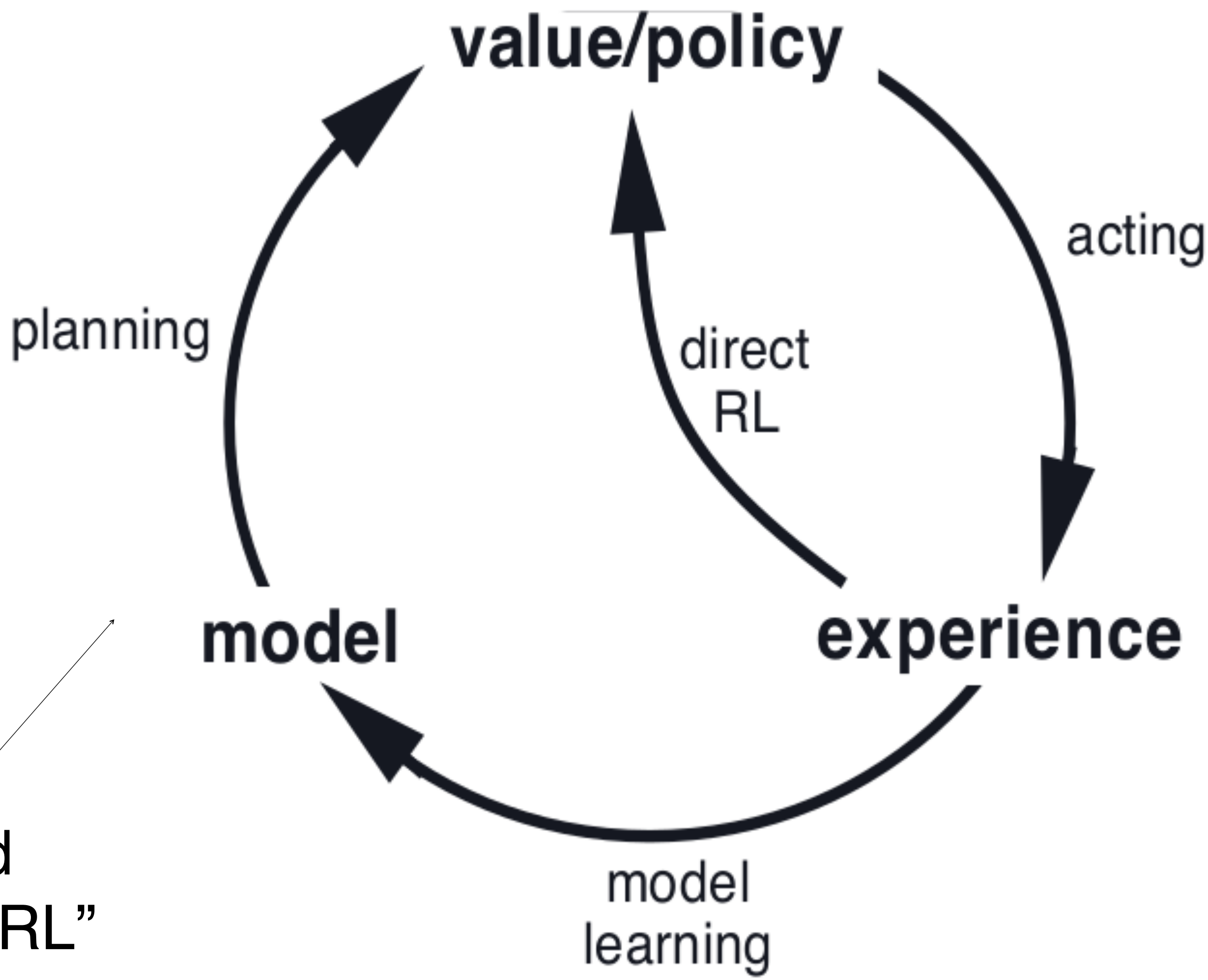
1. Select a state, $S \in \mathcal{S}$, and an action, $A \in \mathcal{A}(S)$, at random
2. Send S, A to a sample model, and obtain
   a sample next reward, R, and a sample next state, S'
3. Apply one-step tabular Q-learning to S, A, R, S':
   $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$

– why does this satisfy our expanded definition?

Planning vs Learning



Planning vs Learning



Often called
"model-based RL"

Models in RL

Model: anything the agent can use to predict how the environment will respond to its actions

Two types of models:

1. Distribution model: description of all transition possibilities and their probabilities

2. Sample model: a.k.a. a simulator
   – given a *s,a* pair, the sample model returns next state & reward
   – a sample model is often much easier to get than the distribution model

Models in RL

This is how we defined "model" at the beginning of this course

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Two types of models:

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   – a sample model is often much easier to get than the distribution model

In this section, we're going to use this type of model a lot

Planning vs learning

Planning typically assumes you use the model to look more than one step into the future (e.g., testing and evaluating possible policies)

Two types of models:

1. Distribution model: this is the typical planning case (use the distribution model to generate a full policy and then can execute that policy) --- this is what we did for dynamic programming (e.g., VI and PI)

2. Sample model: this is used for 'sample-based' or online planning where you only have access to a simulator and are not generating a full policy (e.g., a single action at a times) --- this is what we'll do for Monte-Carlo tree search

Here, we're using a sample model,
but we don't learn the model

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Planning methods don't learn the model by themselves

Planning

<u>An unusual way to do planning:</u>

$$\text{model} \longrightarrow \text{simulated experience} \xrightarrow{\text{backups}} \text{values} \longrightarrow \text{policy}$$

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Dyna-Q

Tabular Dyna-Q

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Direct RL: Do update directly using real experience

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Direct RL: Do update directly using real experience
Indirect RL: Learn model using real experience

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Direct RL: Do update directly using real experience
Indirect RL: Learn model using real experience,
then simulate experience using model, do 'planning' update using simulated experience

Question

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This version assumes deterministic transitions and rewards, how would you extend it to the stochastic case?

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Think of as many ways as you can to implement “model”

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Think of as many ways as you can to implement “model”
- just buffer all previously experienced transitions (really a model?)
- estimate tabular transition probabilities for s,a pairs
- learn a neural network that samples from next states

Question

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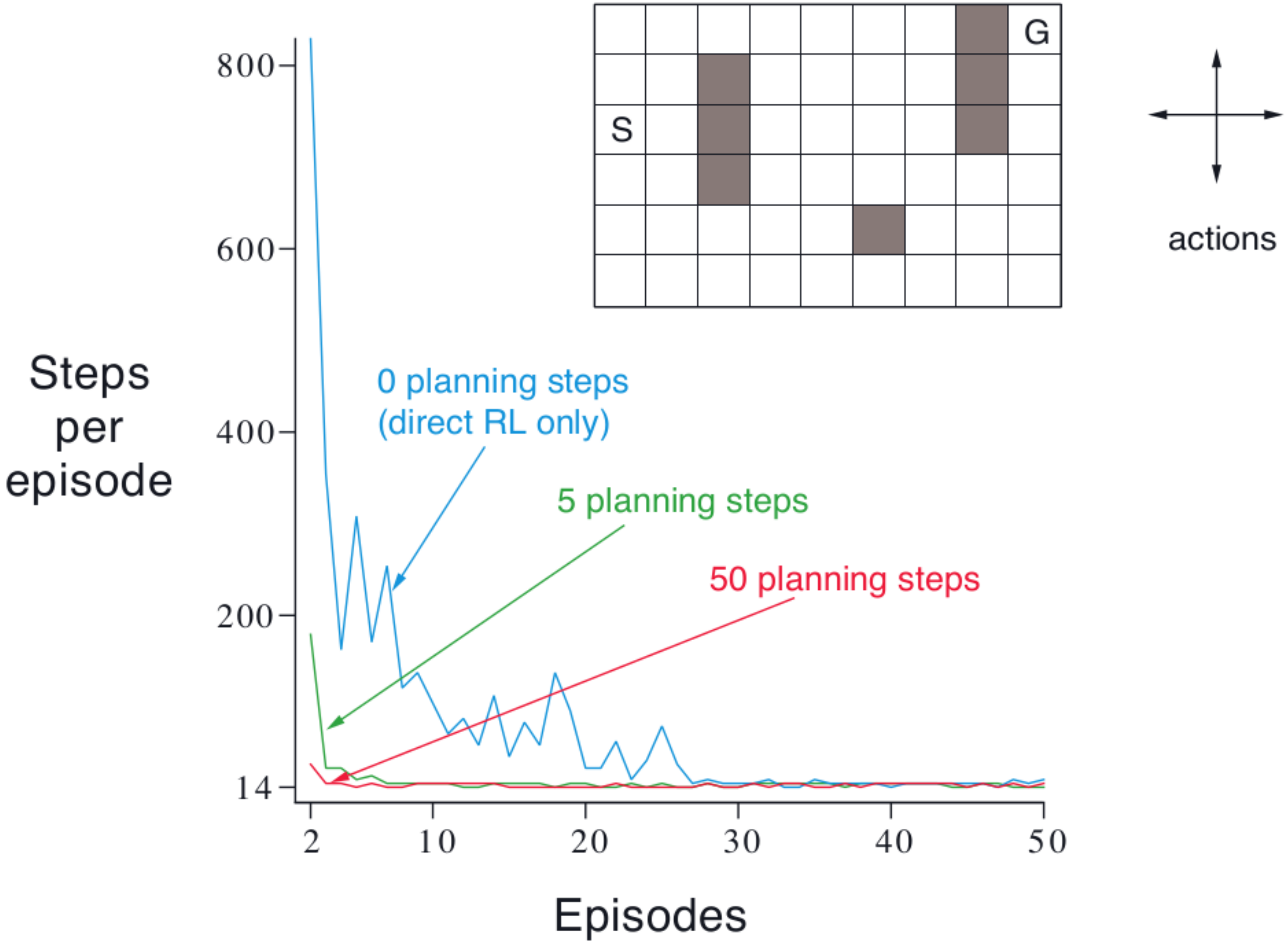
Modern versions learn a NN for a model

Dyna-Q on a Simple Maze



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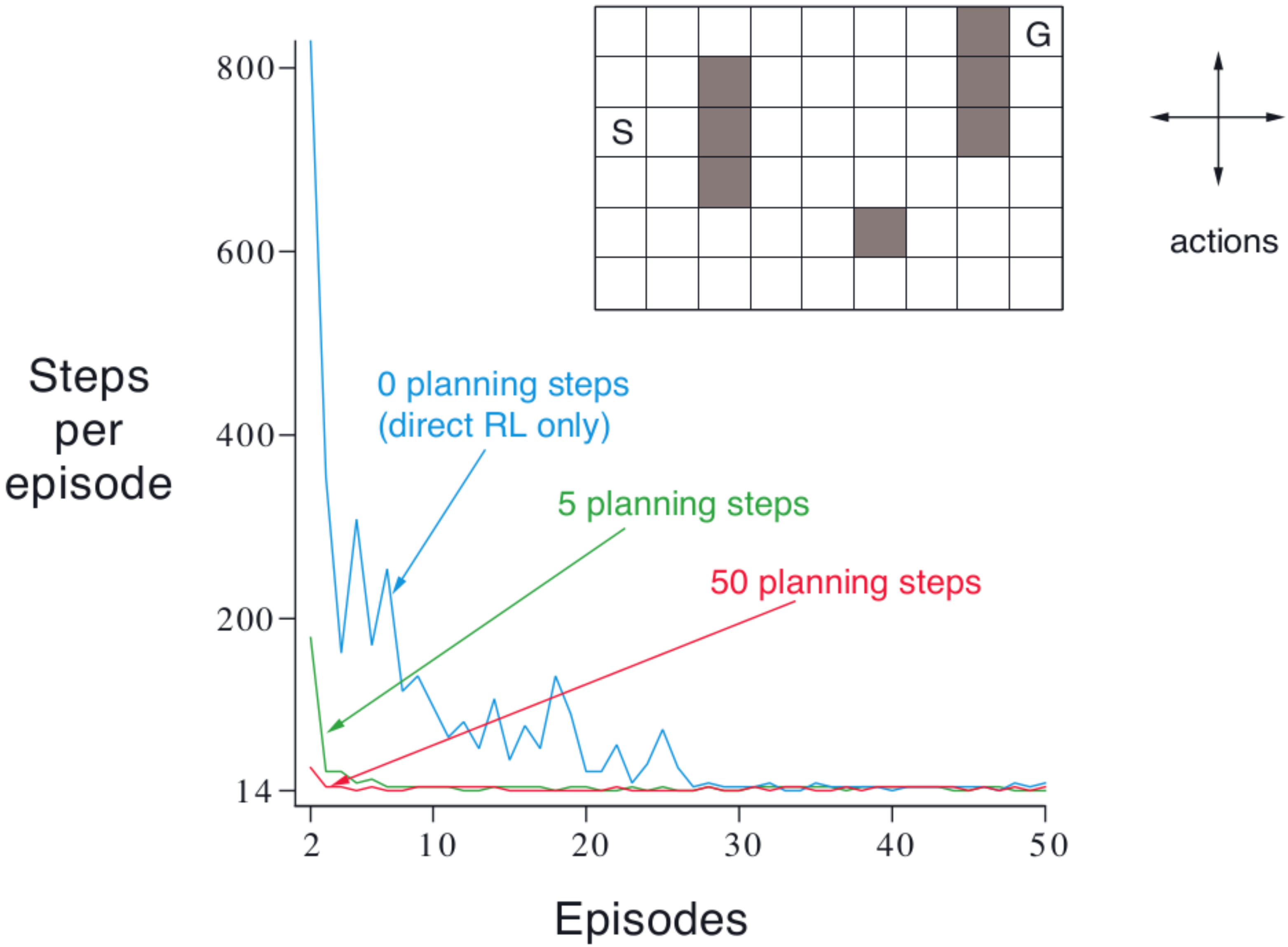
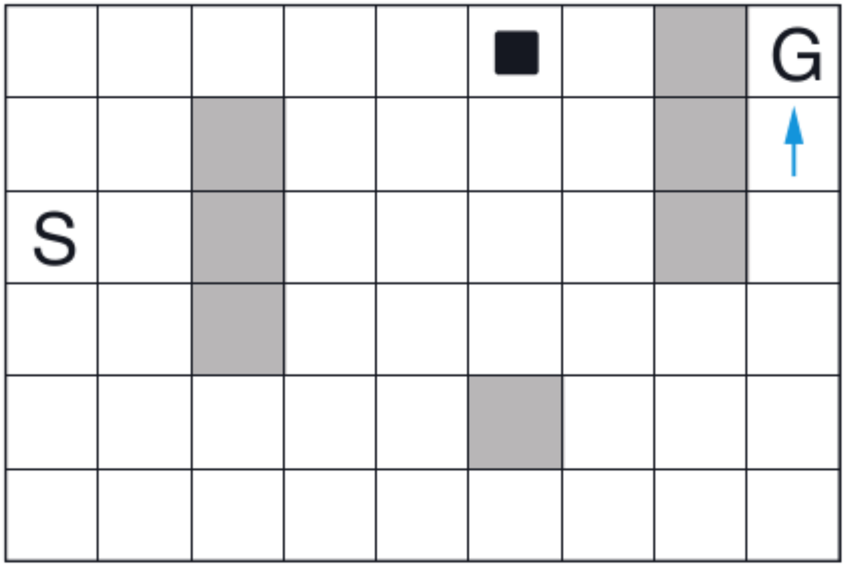


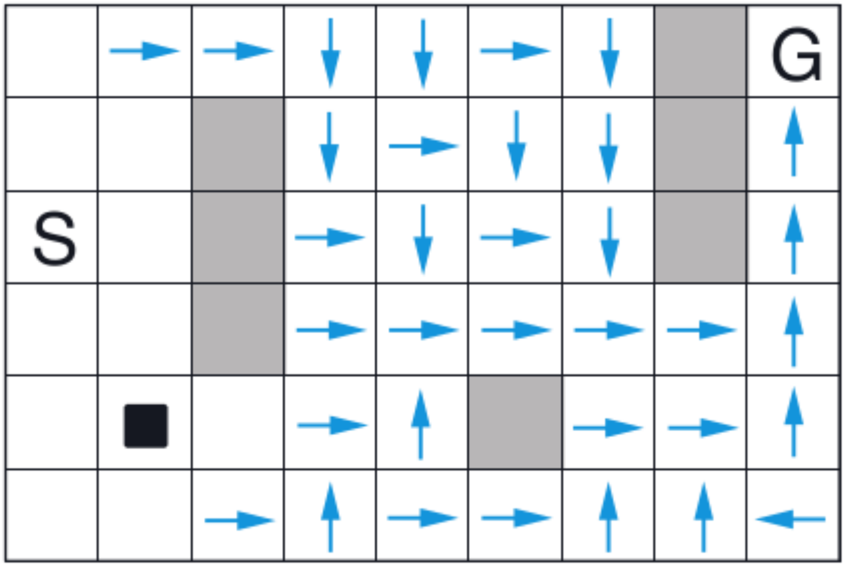
Would the results look as good if the domain was stochastic?

Why does Dyna-Q do so well?

WITHOUT PLANNING (n=0)



WITH PLANNING (n=50)



Policies found using Q-learning vs dyna-Q halfway through second episode

- dyna-Q w/ n=50
- optimal policy after three episodes!

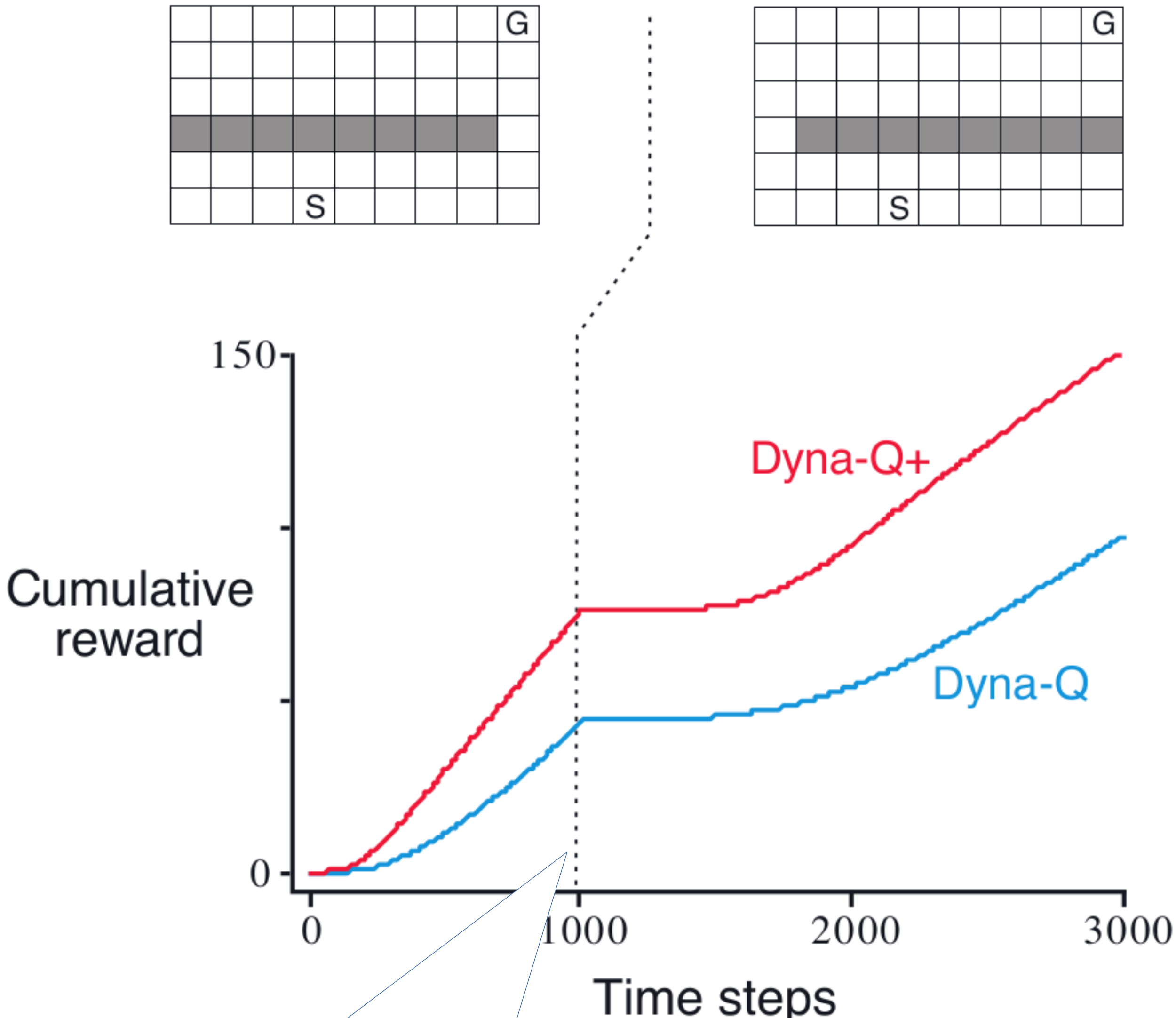
When the model is wrong

- So far, we have considered models that:
  - Start empty and are always updated with correct info
- The model can be wrong! Because:
  - environment might be stochastic and we have only seen a few samples
  - the environment has changed
- Planning is likely to compute a suboptimal policy in this case

- Imagine the world changed, and:
  - The suboptimal policy leads to discovery and correction of the modeling error

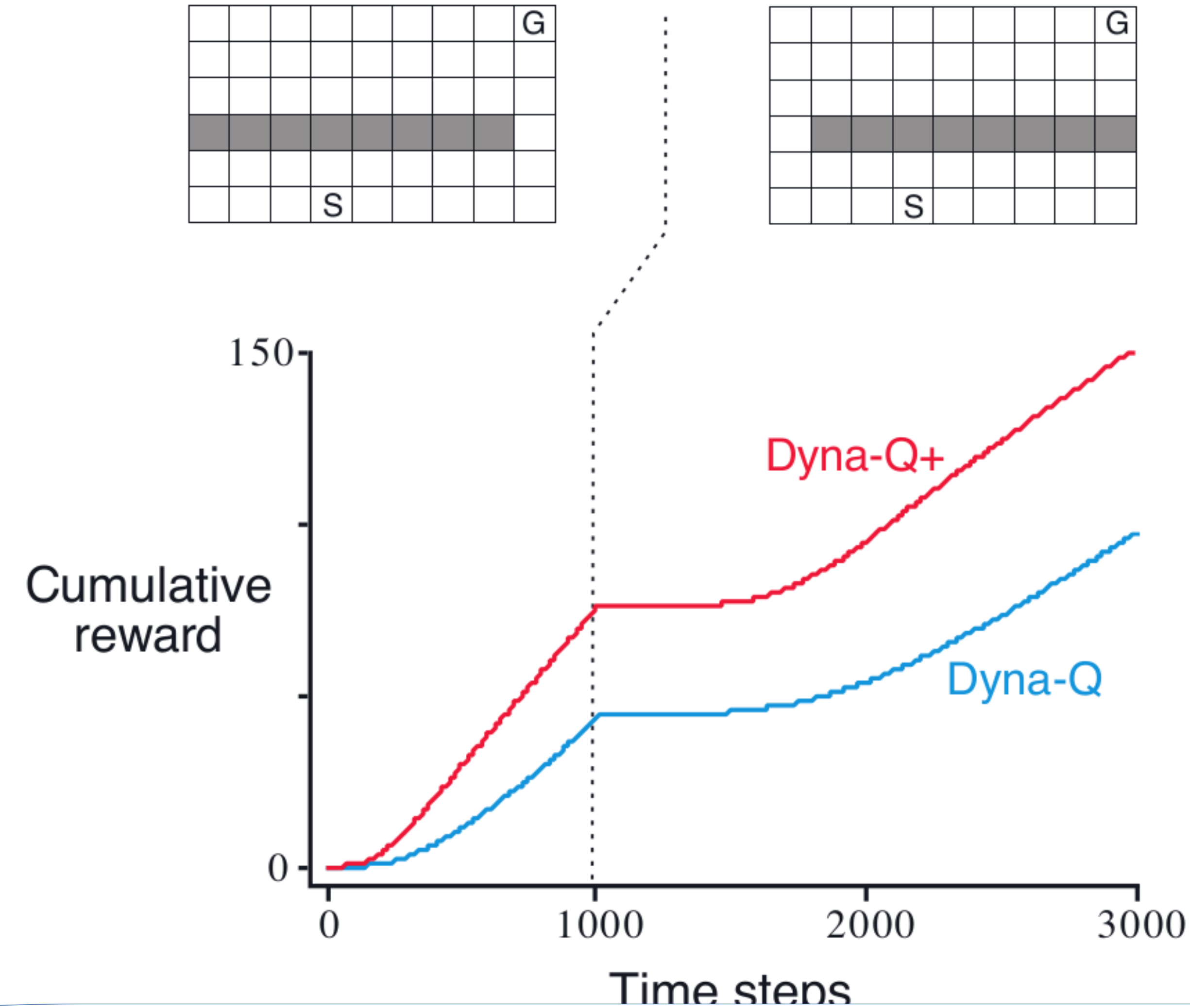
What happens with incorrect model?

(SB, Example 8.2)



Environment changes here

Think-pair-share



(SB, Example 8.2)

Questions:

- why does dyna-Q stop getting reward?
- why does it start again?
- what would happen if the change was the other way around (in either case)?

Think-pair-share

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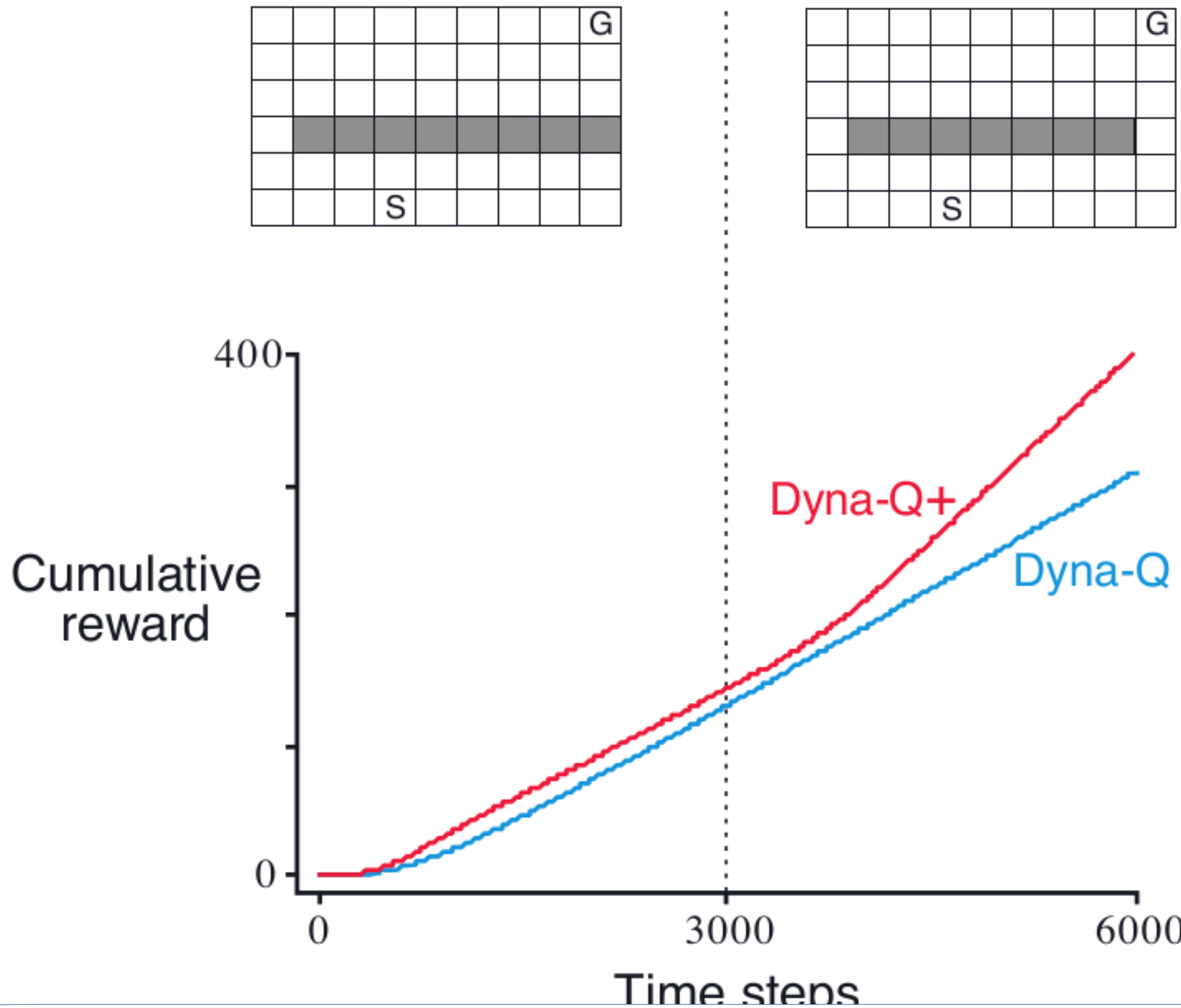

Questions:

– why does dyna-Q stop getting reward?
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What is Dyna-Q+

- Even with e-greedy policy Dyna-Q won't explore enough to find new path
- Dyna-Q+ uses an "exploration bonus" (like UCB):
  - Keeps track of time since each state-action pair was tried for real
  - An extra reward is added for transitions caused by state-action pairs related to how long ago they were tried: the longer unvisited, the more reward for visiting

$$R + \kappa\sqrt{\tau}$$

some small constant to balance exploration

time since last visiting the state-action pair

  - What impact does adding this exploration bonus have?

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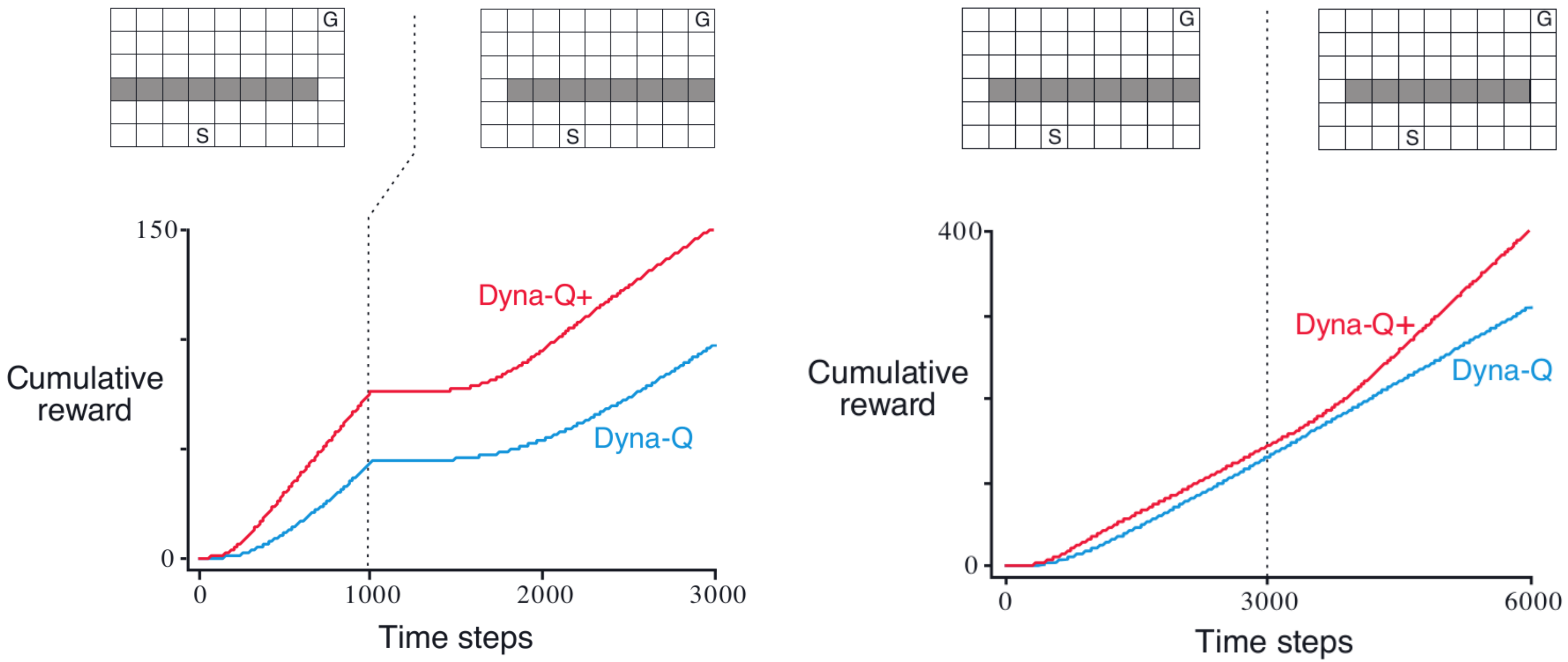
  - The agent actually “plans” how to visit long unvisited states

Note: Better exploration is a key benefit in model-based approaches

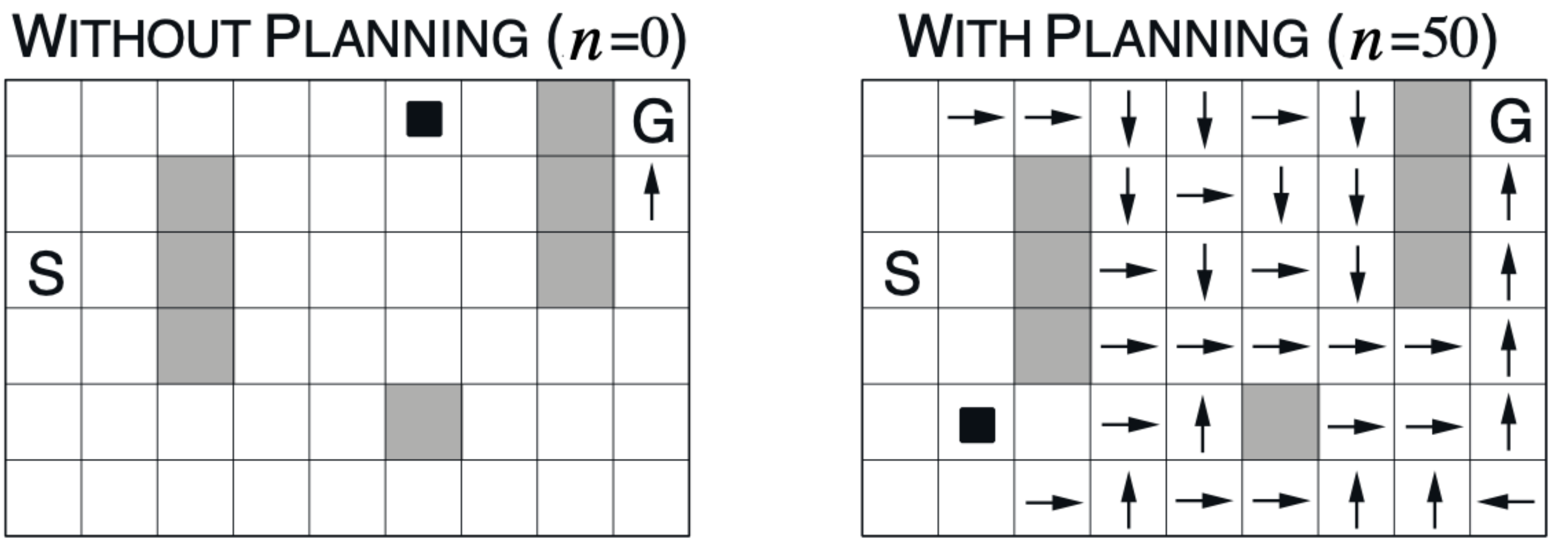
Think-pair-share



Exercise 8.2 Why did the Dyna agent with exploration bonus, Dyna-Q+, perform better in the first phase as well as in the second phase of the blocking and shortcut experiments? □

Prioritizing Search Control

- Consider the second episode in the Dyna maze
    - The agent has successfully reached the goal once…



- In larger problems, the number of states is so large that unfocused planning would be extremely inefficient

Prioritized Sweeping

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Unfocused replay from model

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Unfocused replay from model – can we do better?

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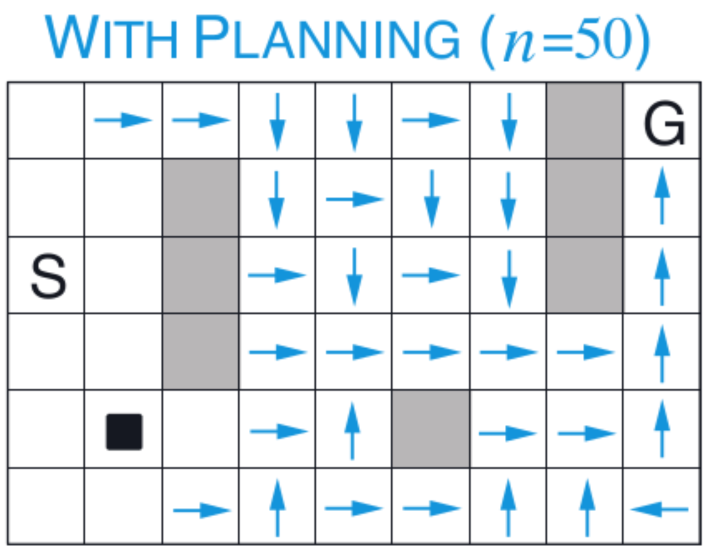
Unlike real experience, we can control which state-action pairs to replay

Prioritized Sweeping



Instead of replaying **all** of these transitions on each iteration, just replay the important ones…

– Which states or state-action pairs should be generated during planning?

– Work backward from states whose value has just changed

– Maintain a priority queue of state-action pairs whose values would change a lot if backed up, prioritized by the size of the change

– When a new backup occurs, insert predecessors according to their priorities

Prioritized Sweeping

Prioritized sweeping for a deterministic environment

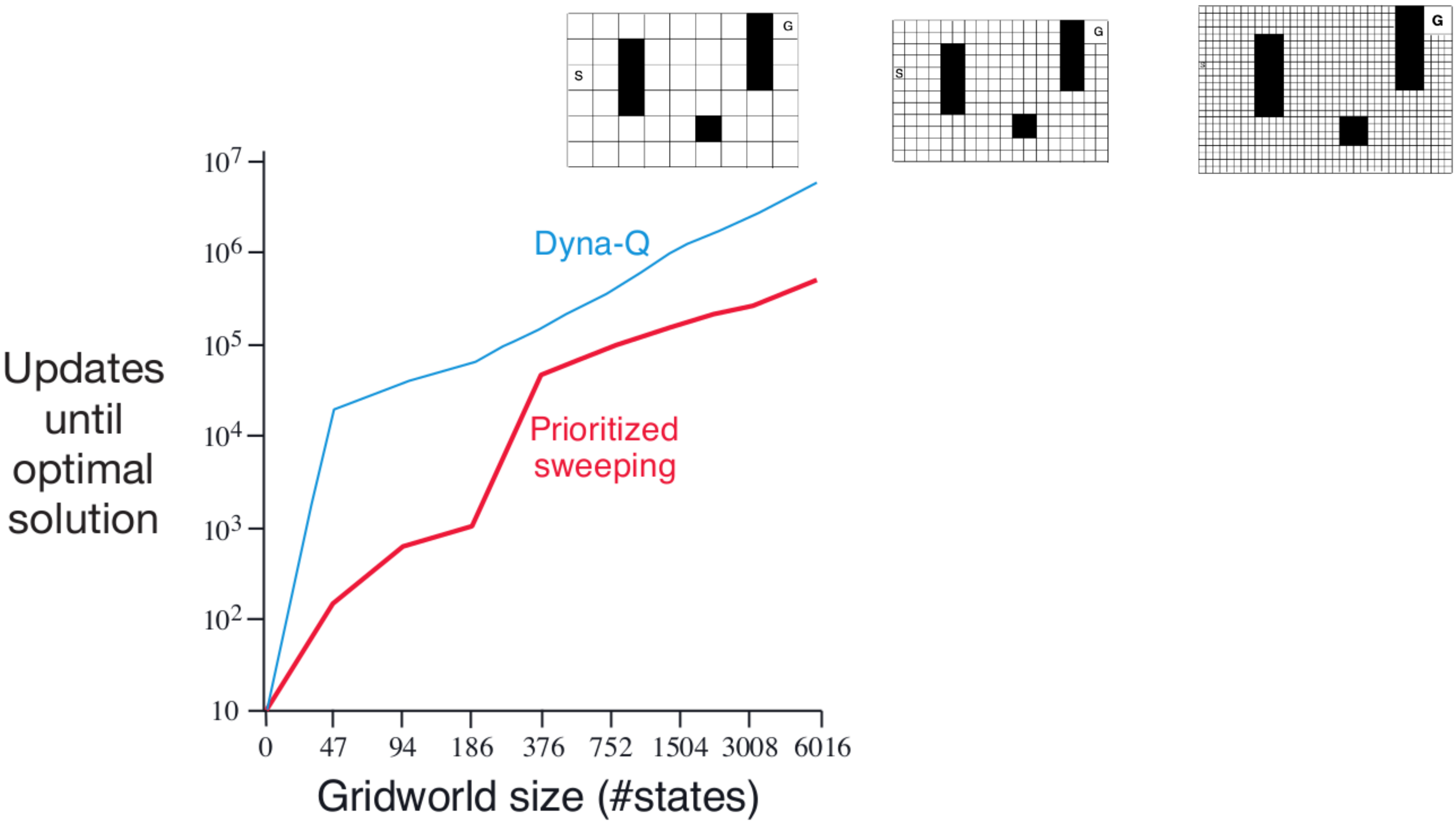
Initialize $Q(s,a)$, $Model(s,a)$, for all s,a, and $PQueue$ to empty
Loop forever:
  (a) $S \leftarrow$ current (nonterminal) state
  (b) $A \leftarrow policy(S,Q)$
  (c) Take action A; observe resultant reward, R, and state, S'
  (d) $Model(S,A) \leftarrow R, S'$
  (e) $P \leftarrow |R + \gamma \max_a Q(S',a) - Q(S,A)|$.
  (f) if $P > \theta$, then insert S,A into $PQueue$ with priority P
  (g) Loop repeat n times, while $PQueue$ is not empty:
    $S, A \leftarrow first(PQueue)$
    $R, S' \leftarrow Model(S,A)$
    $Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma \max_a Q(S',a) - Q(S,A)\right]$
    Loop for all $\bar{S}, \bar{A}$ predicted to lead to S:
      $\bar{R} \leftarrow$ predicted reward for $\bar{S}, \bar{A}, S$
      $P \leftarrow |\bar{R} + \gamma \max_a Q(S,a) - Q(\bar{S},\bar{A})|$.
      if $P > \theta$ then insert $\bar{S}, \bar{A}$ into $PQueue$ with priority P

TD error

what's this part doing?

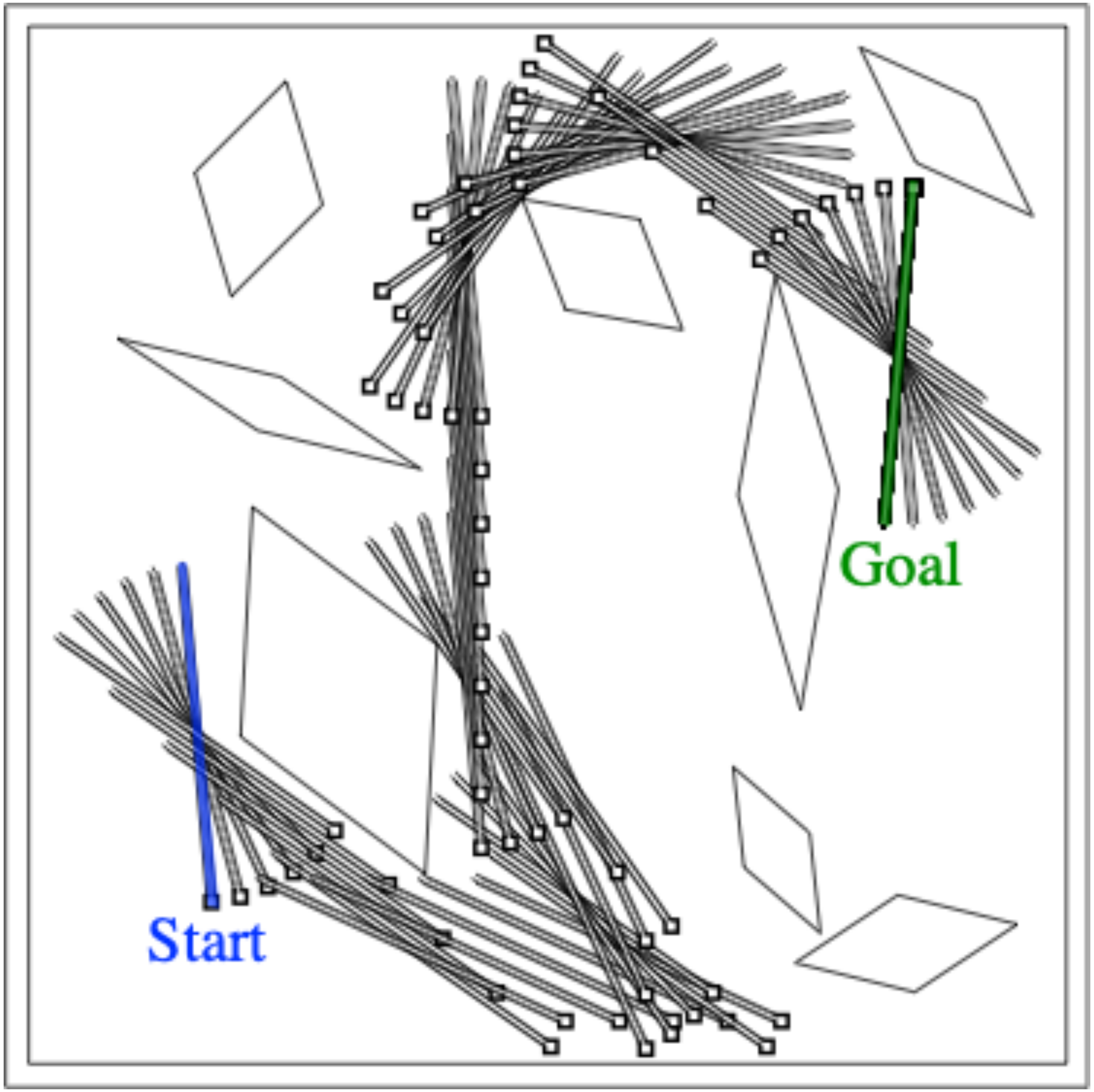
Prioritized sweeping isn't really a planning method so it can be used with lots of RL methods

Prioritized Sweeping: Performance



Both use n=5 backups per environmental interaction

Rod Maneuvering (Moore and Atkeson 1993)

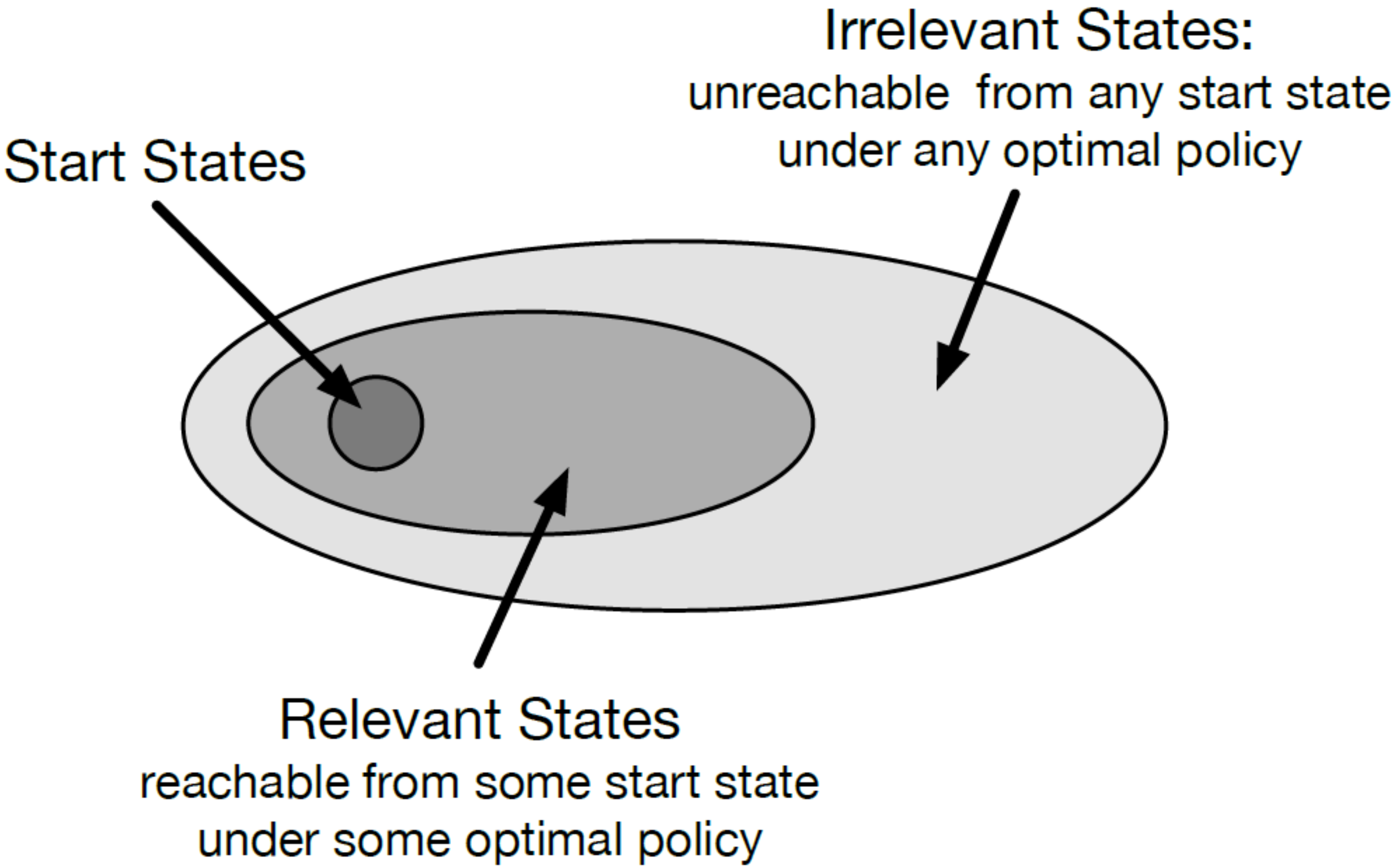


Solved by prioritized sweeping, but probably too large for unprioritized methods

Model-based RL and sample-based planning

- This chapter only touches the surface of both model-based RL and different planning methods
- Model-based RL
  - Typically, reinforcement learning methods that combine model-learning to improve exploration, transfer, etc.
  - Examples include Dyna, MuZero, World Models, Dreamer, etc.
- Sample-based planning
  - Typically, assumes the model is given in the form of a simulator (so can access any s,a and get s’, r)
  - Examples include, RTDP and MCTS (next)

Optimality without visiting all states



Online planning methods

- Dyna used planning to generate more data to improve the value function for all states (book calls this background planning)

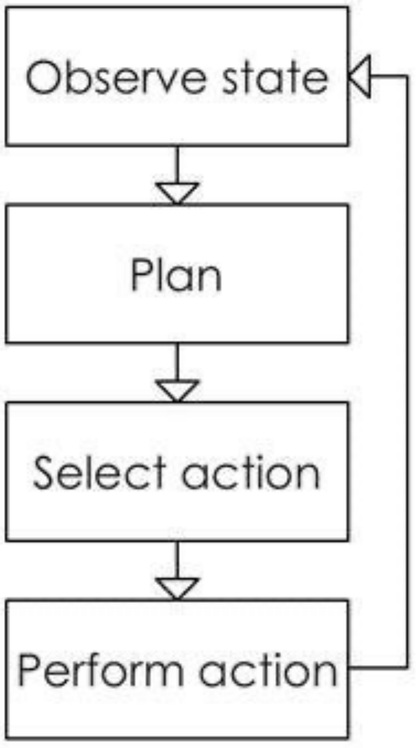
- Instead, online planning methods try to compute an optimal action from current state (book calls this decision-time planning)

  - Plan up to some horizon

  - States reachable from the current state is typically small compared to full state space

  - Heuristics and branch-and-bound techniques allow search space to be pruned

  - Monte Carlo methods provide approximate solutions



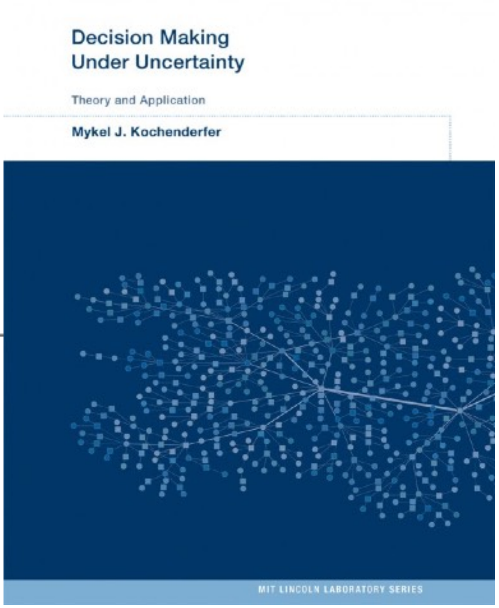
Forward Search

- Provides optimal action from current state s up to depth d
- Recall

Algorithm 4.6 Forward search

function SELECTACTION(s, d)
    if $d = 0$
        return $(\text{NIL}, 0)$
    $(a^*, v^*) \leftarrow (\text{NIL}, -\infty)$
    for $a \in A(s)$
        $v \leftarrow R(s, a)$
        for $s' \in S(s, a)$
            $(a', v') \leftarrow$ SELECTACTION$(s', d - 1)$
            $v \leftarrow v + \gamma T(s' \mid s, a) v'$
        if $v > v^*$
            $(a^*, v^*) \leftarrow (a, v)$
    return (a^*, v^*)

Pseudocode is from here:



- Time complexity is $O((|S| \times |A|)^d)$
- Tree search (expectimax) over and over again!

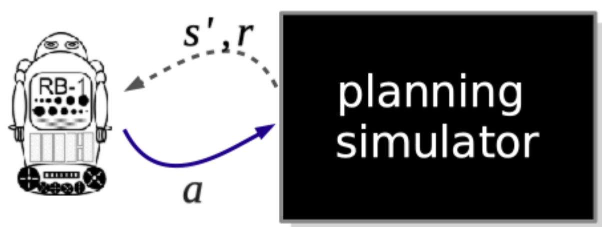
Online Planning

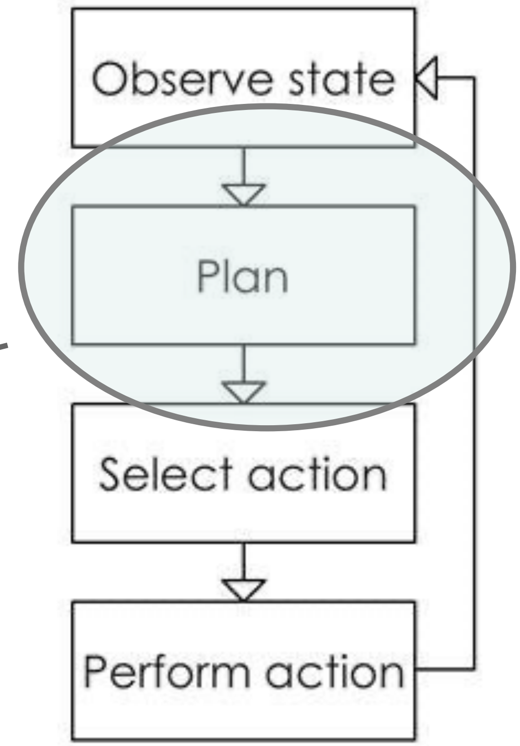
Most planning: use model to compute policy

- ►e.g., **dynamic programming**

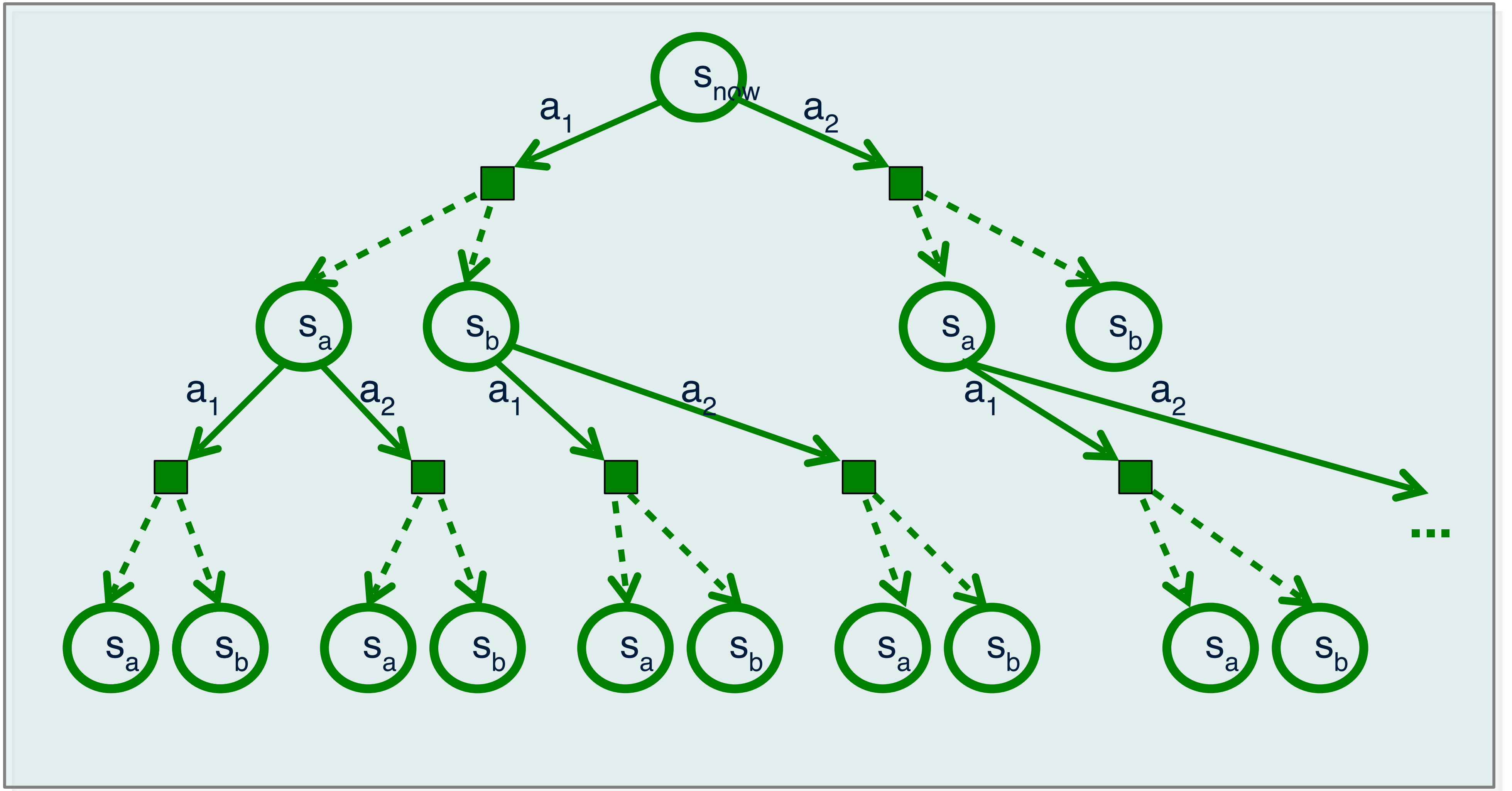
Simulation-based planning, e.g. **MCTS**:

- ►use sampled trajectories of simulated experience



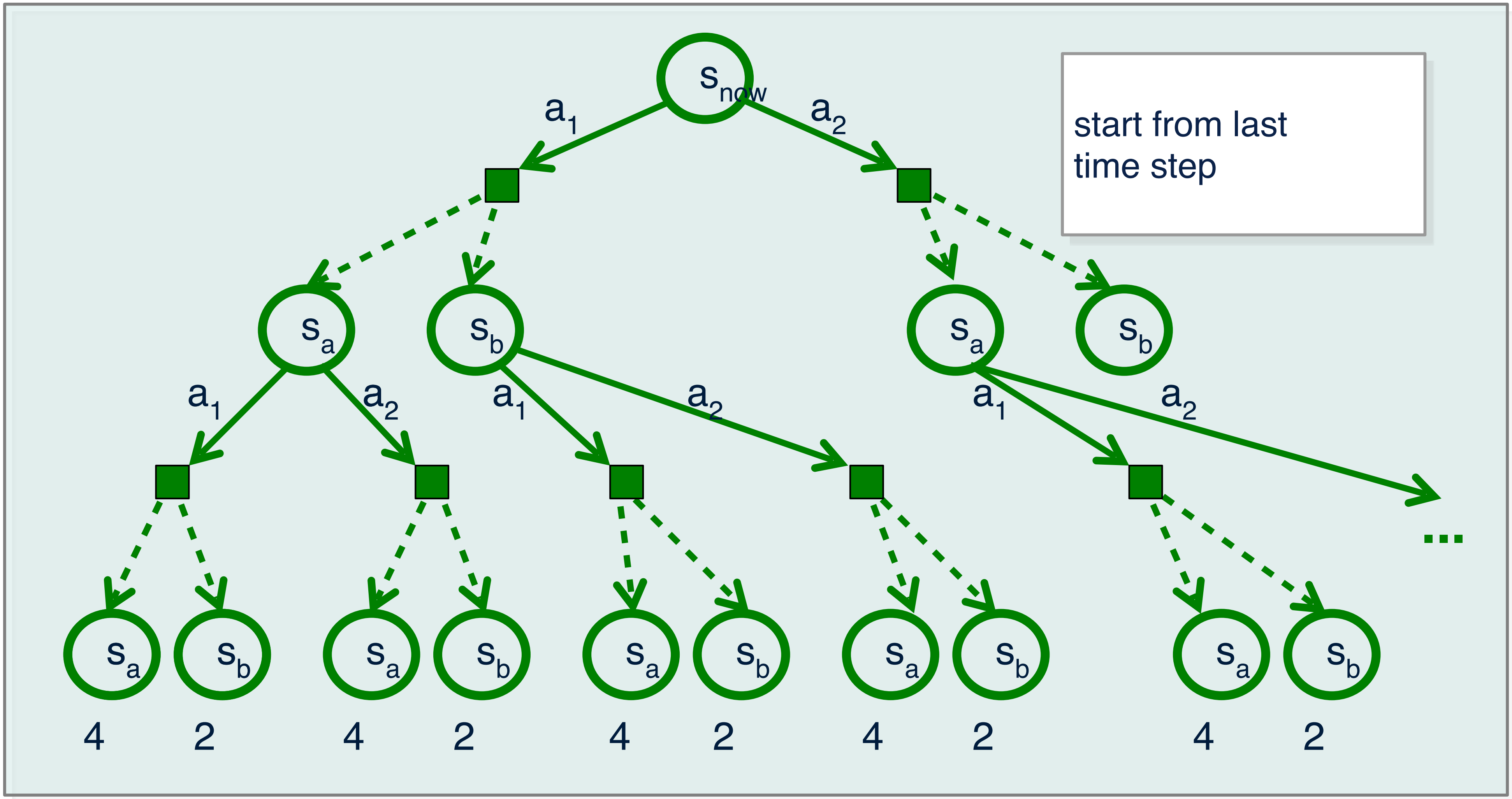


Forward Search

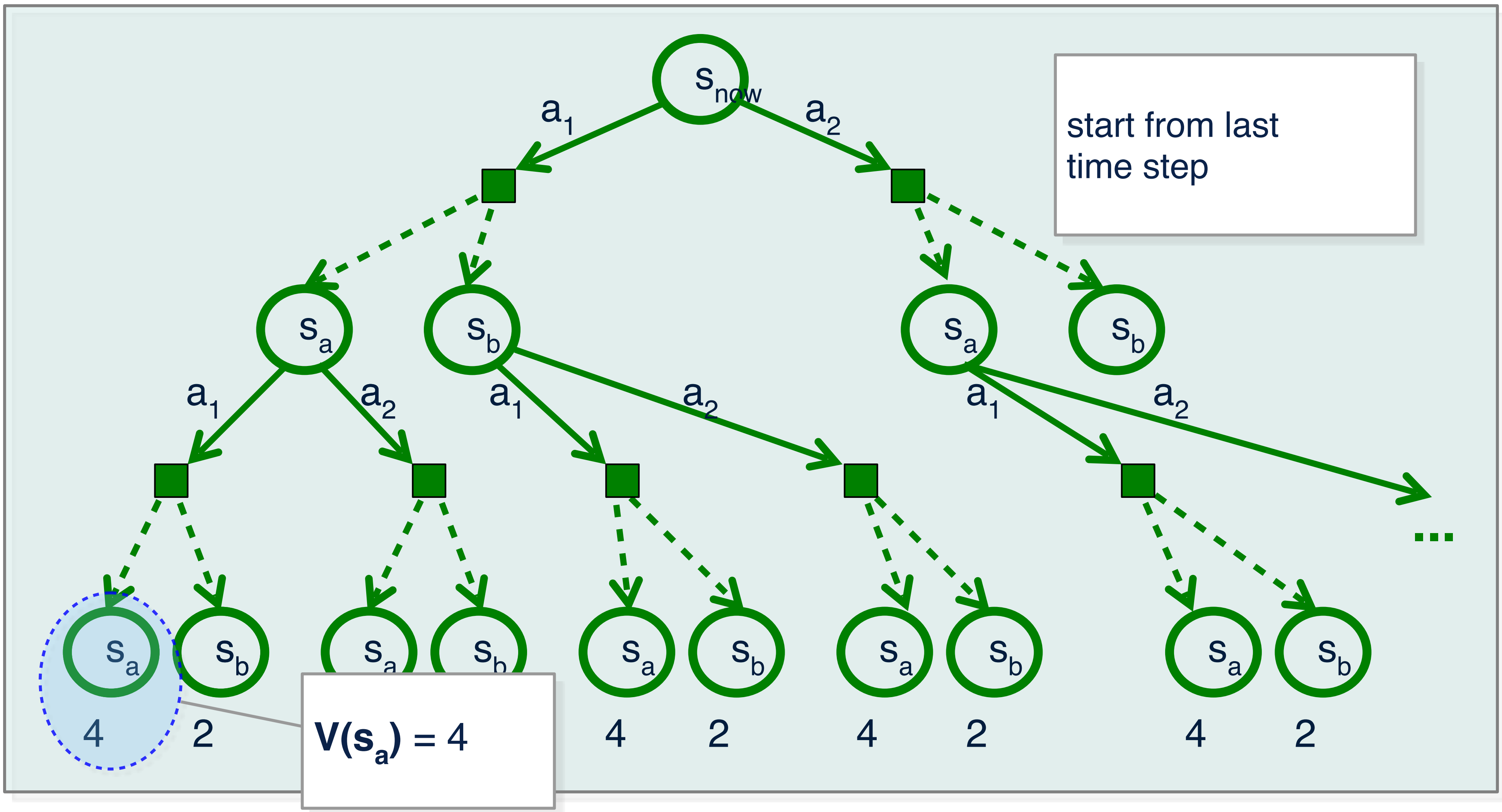


- Construct a plan for *T* time steps into the future

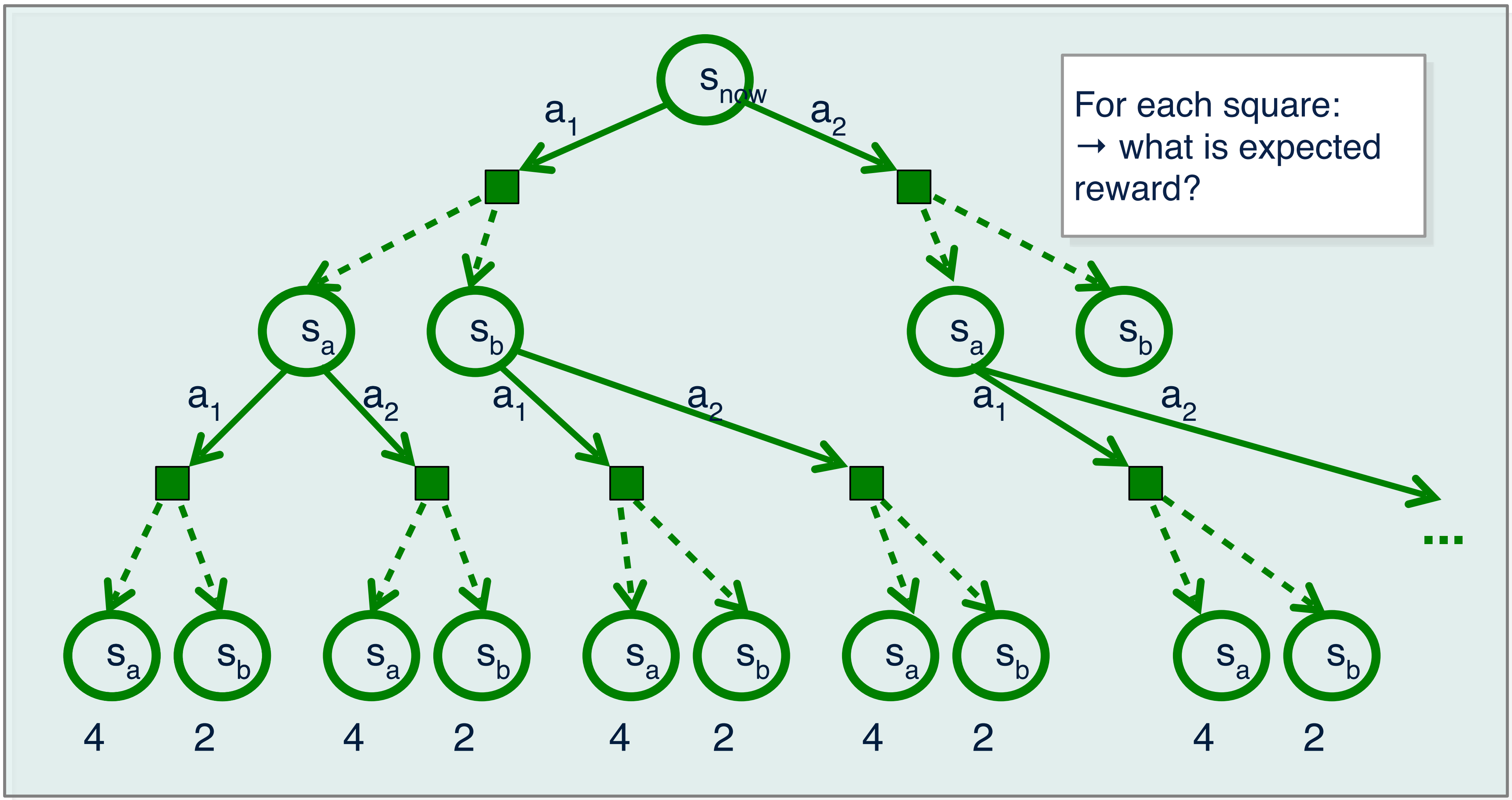
Forward Search



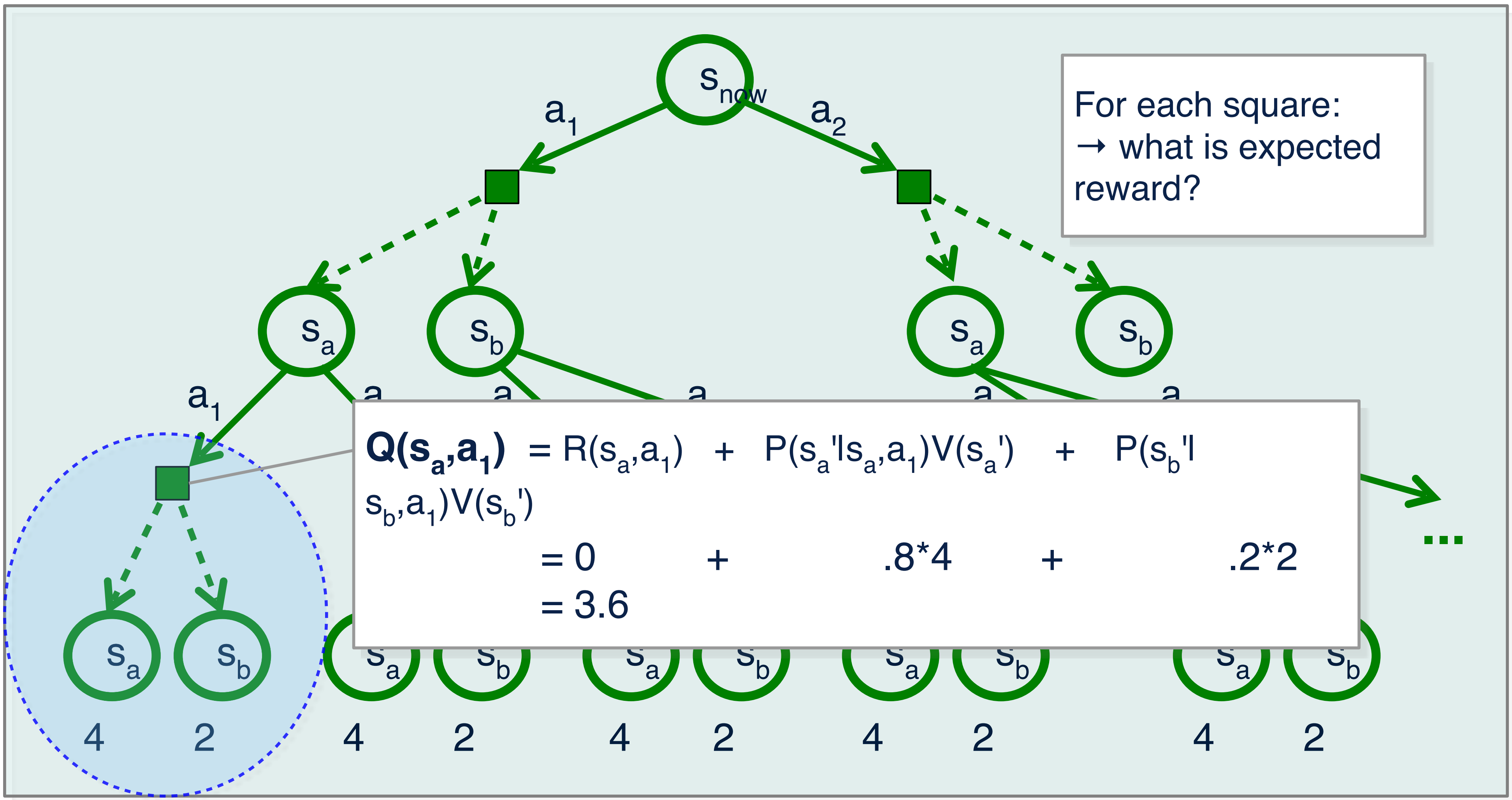
Forward Search



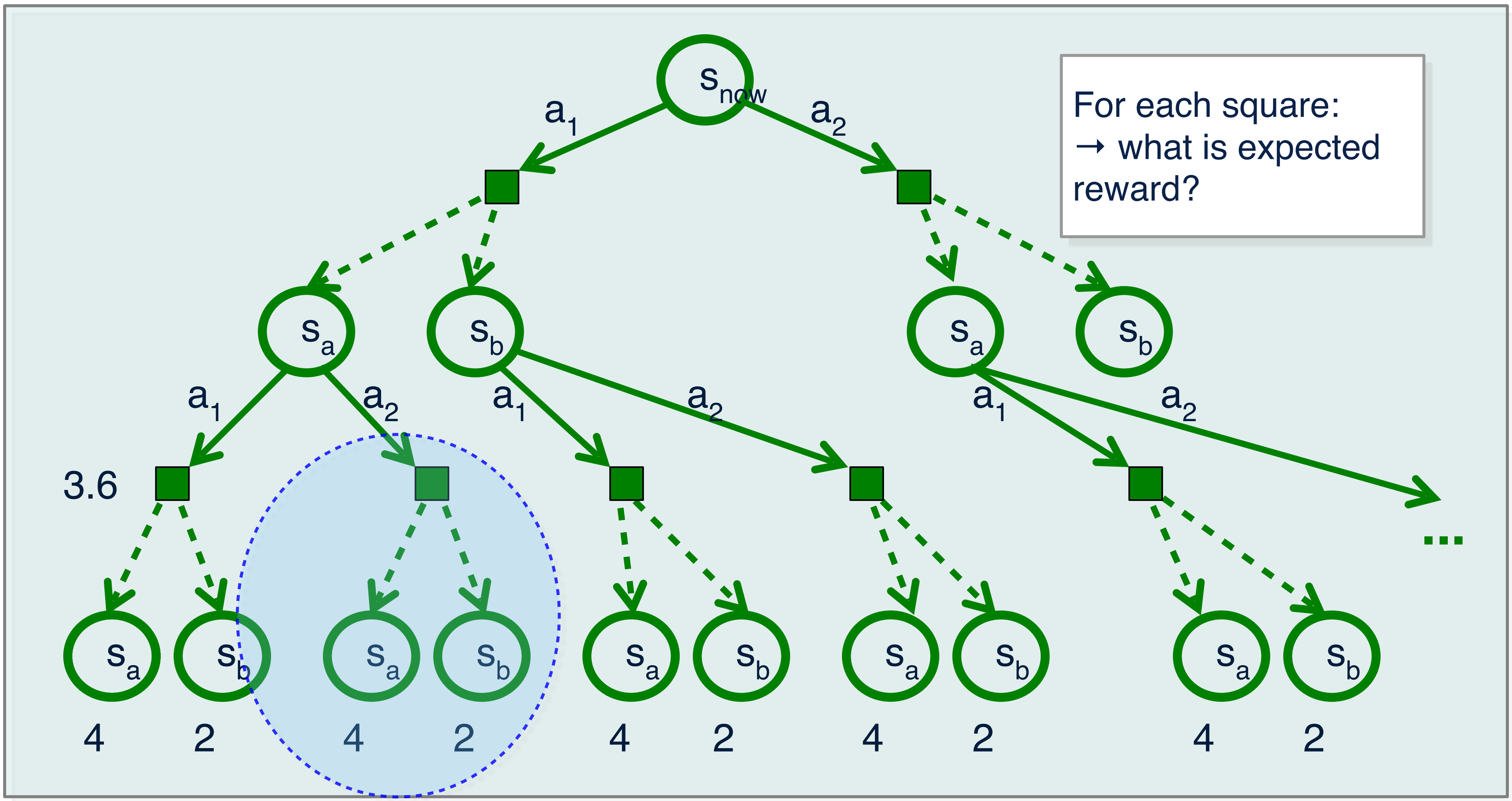
Forward Search



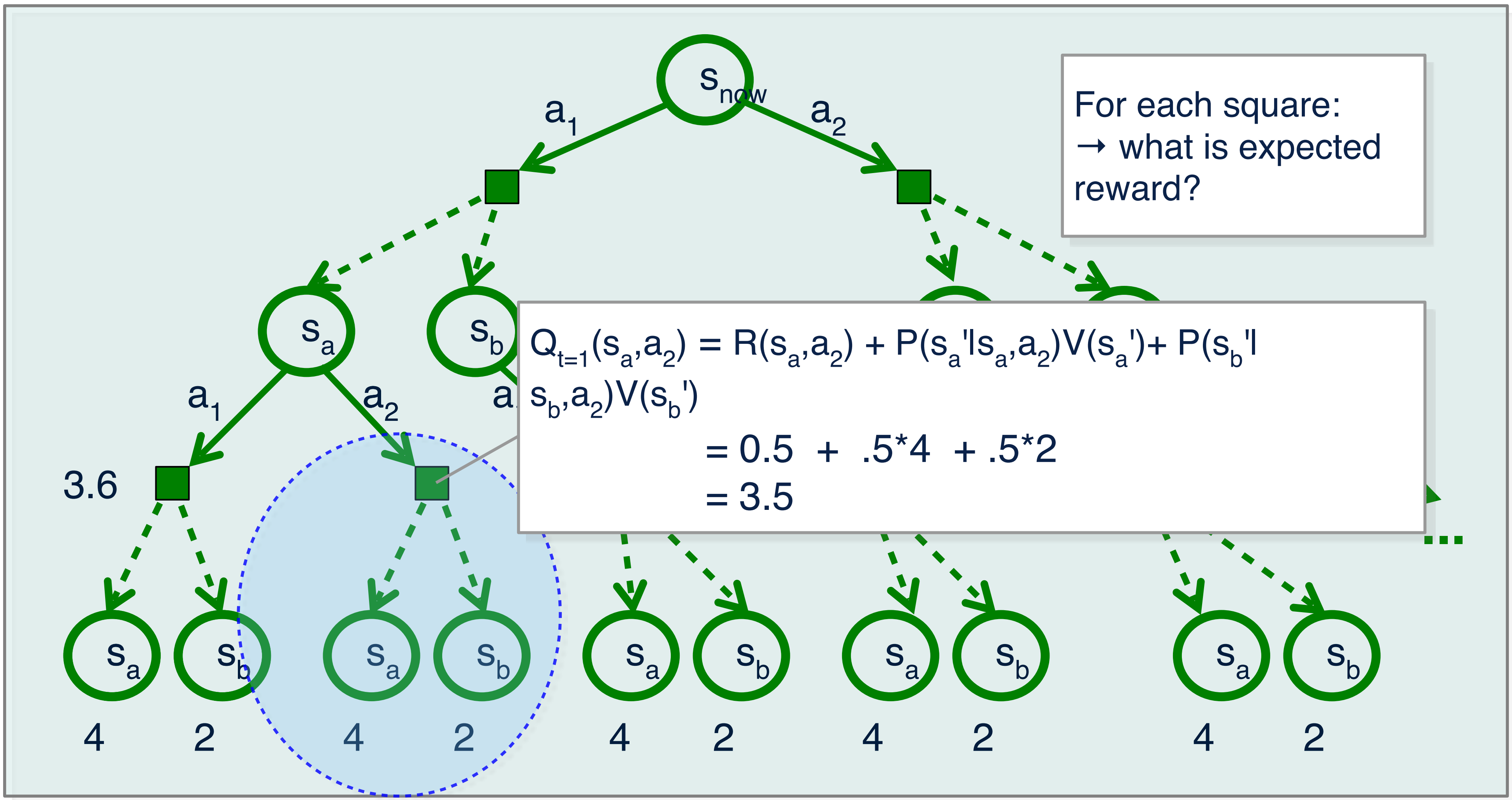
Forward Search



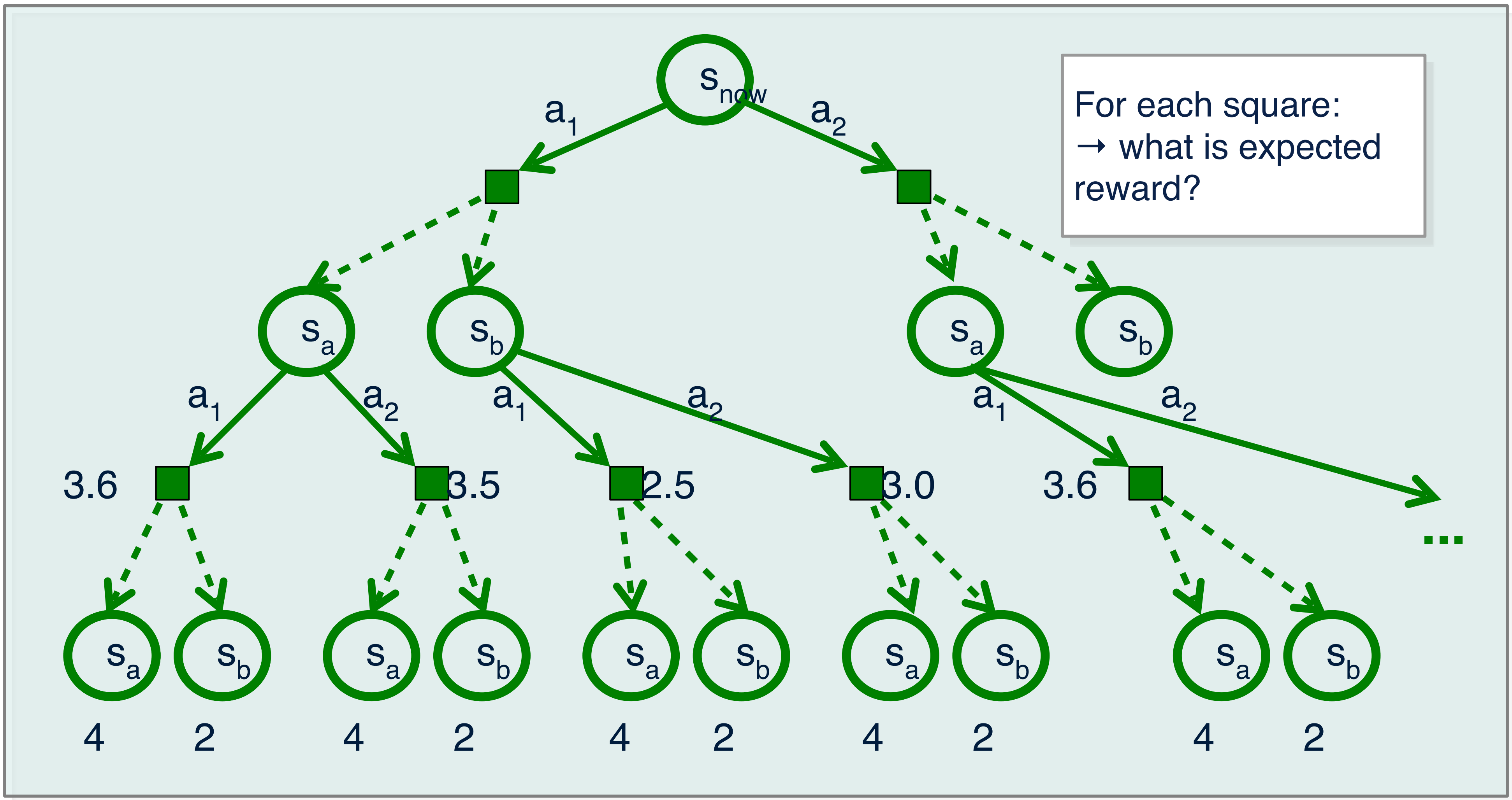
Forward Search



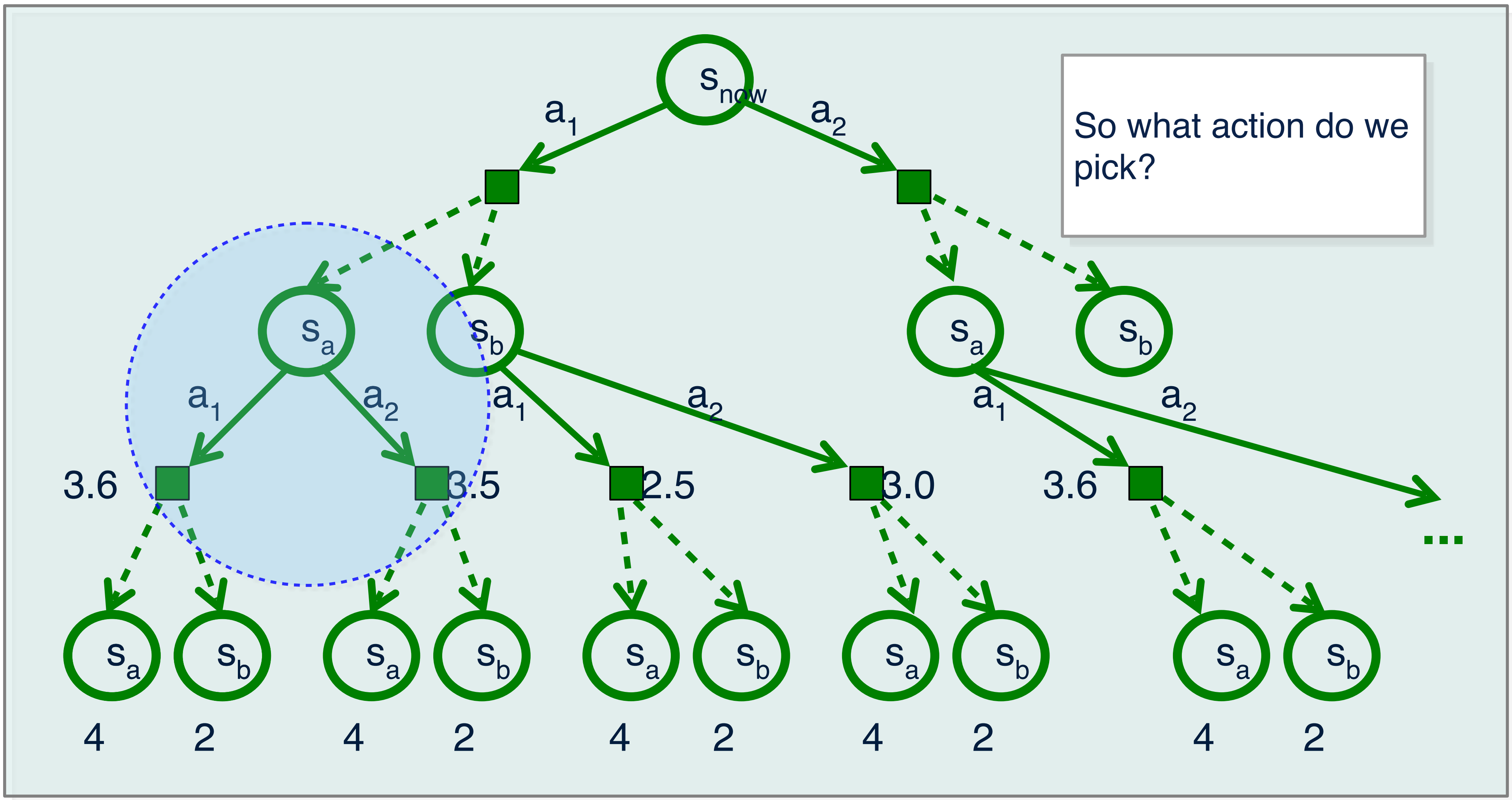
Forward Search



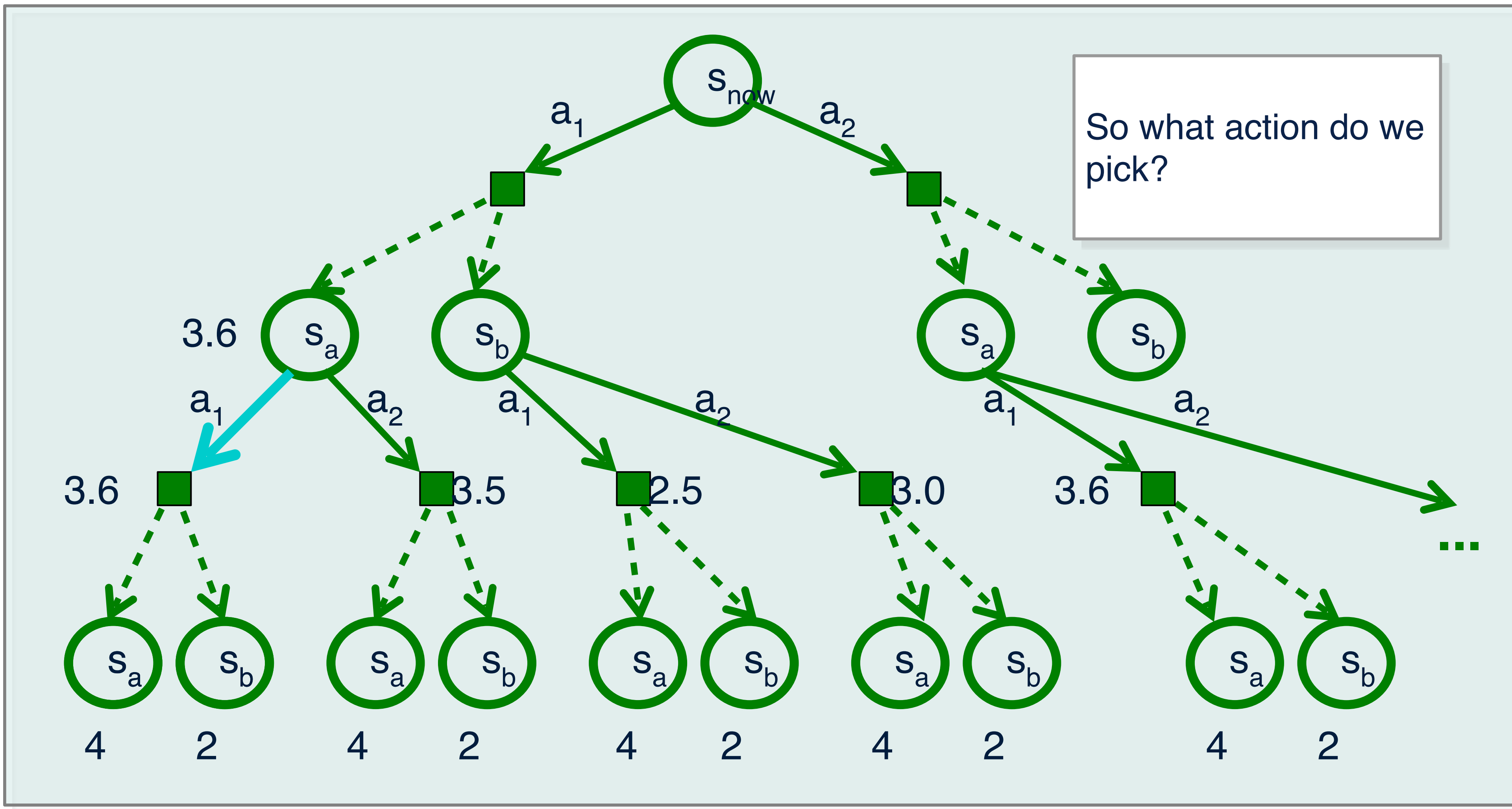
Forward Search



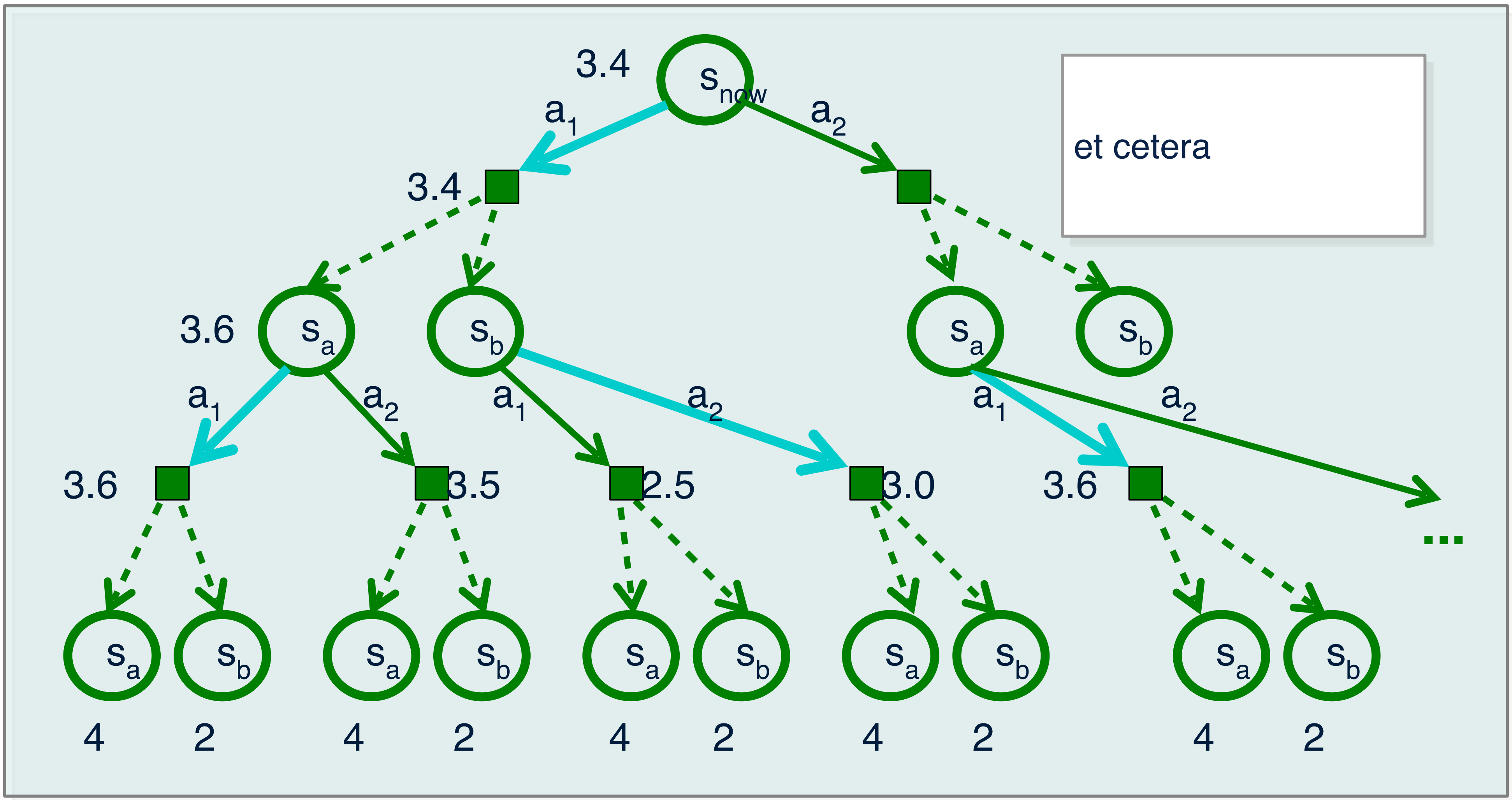
Forward Search



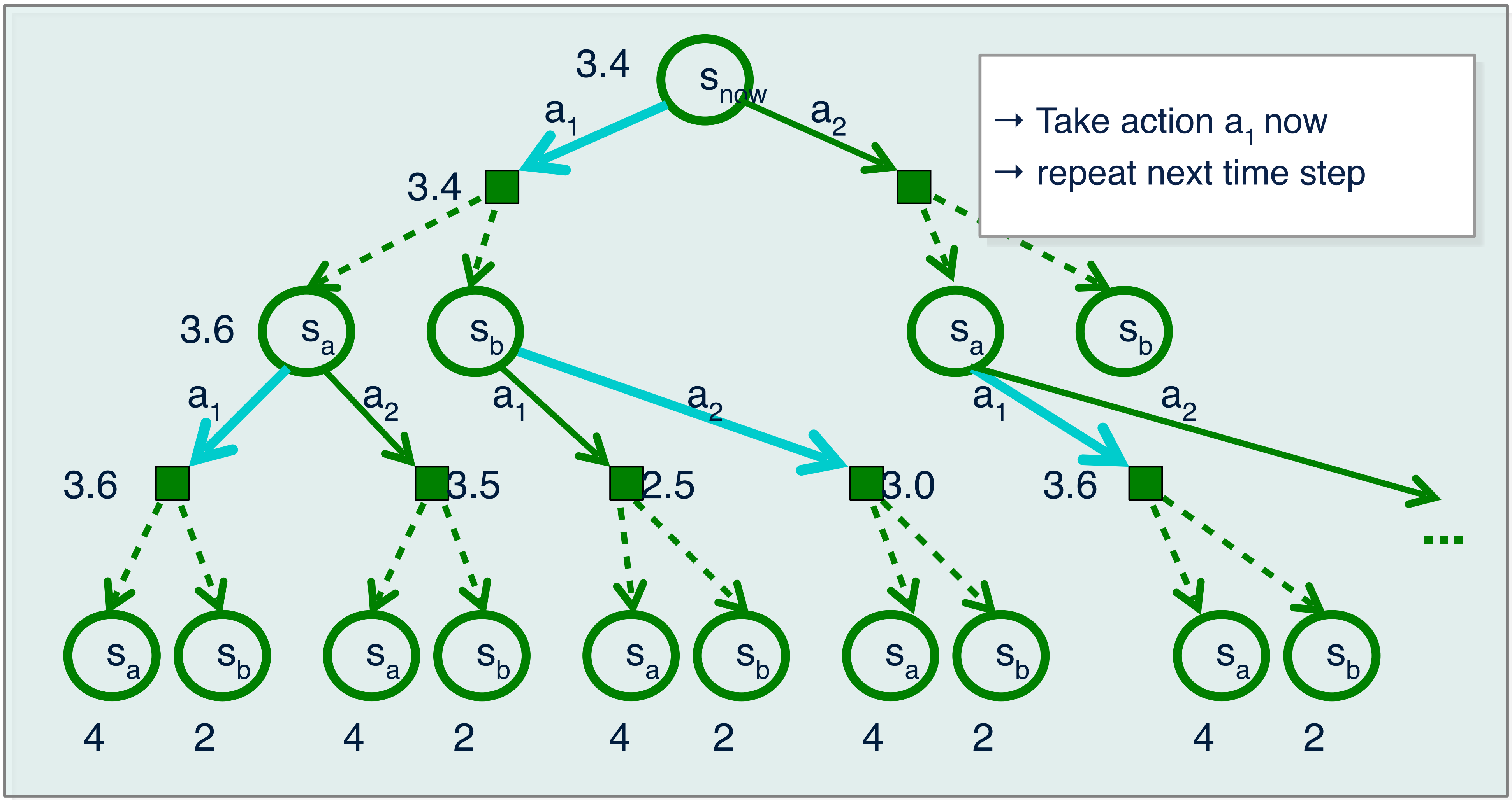
Forward Search



Forward Search



Forward Search



Forward Search: Limitations

- OK, so that seems nice enough...?
- Problem?

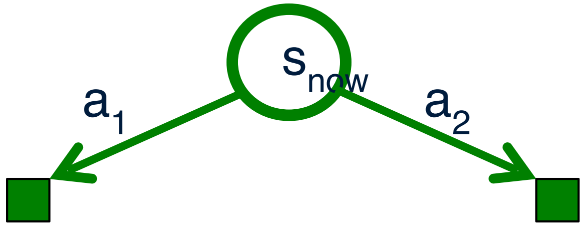
Forward Search: Limitations

- OK, so that seems nice enough...?
- Problem: trees get huge...!
  → not practical

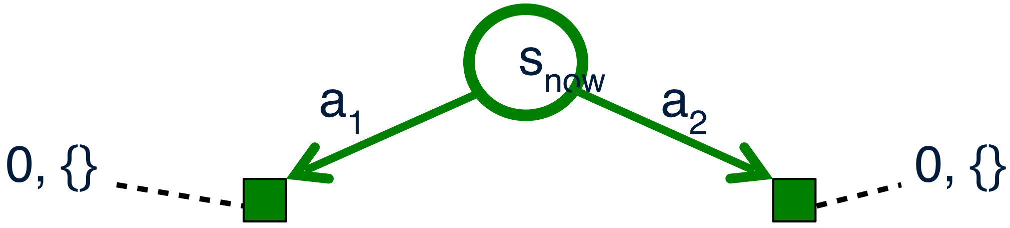
Monte Carlo Tree Search (MCTS)

- MCTS provides leverage by:
  - **incrementally** constructing a **sampled version** of the tree
  - focusing on **promising regions**

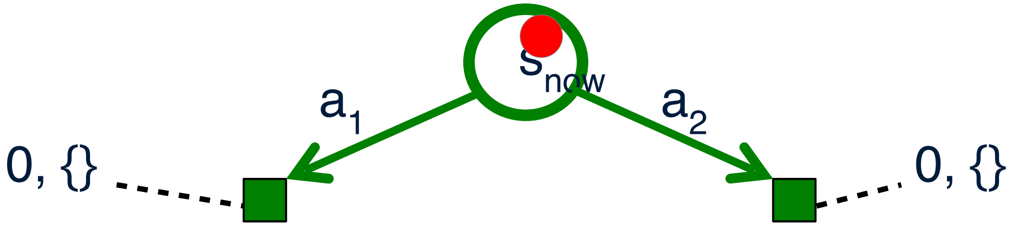
MCTS – Example



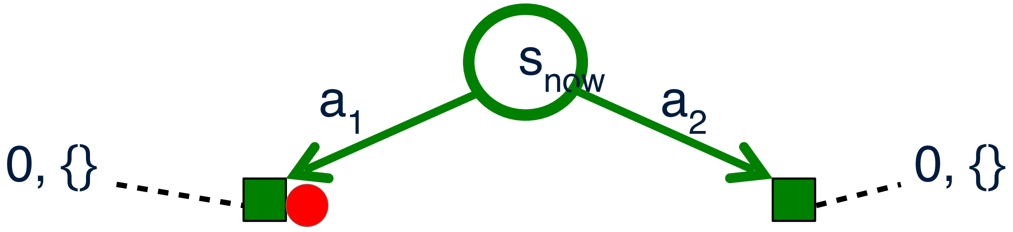
MCTS – Example



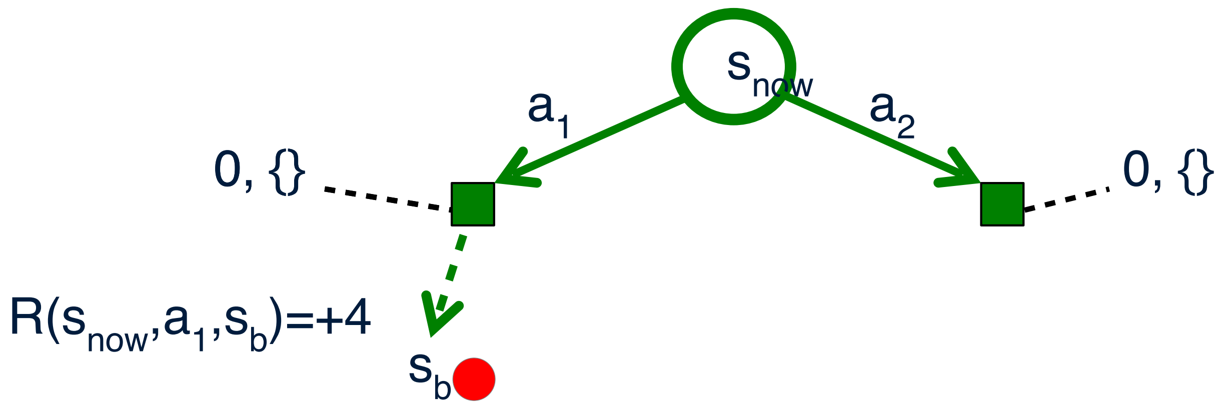
MCTS – Example



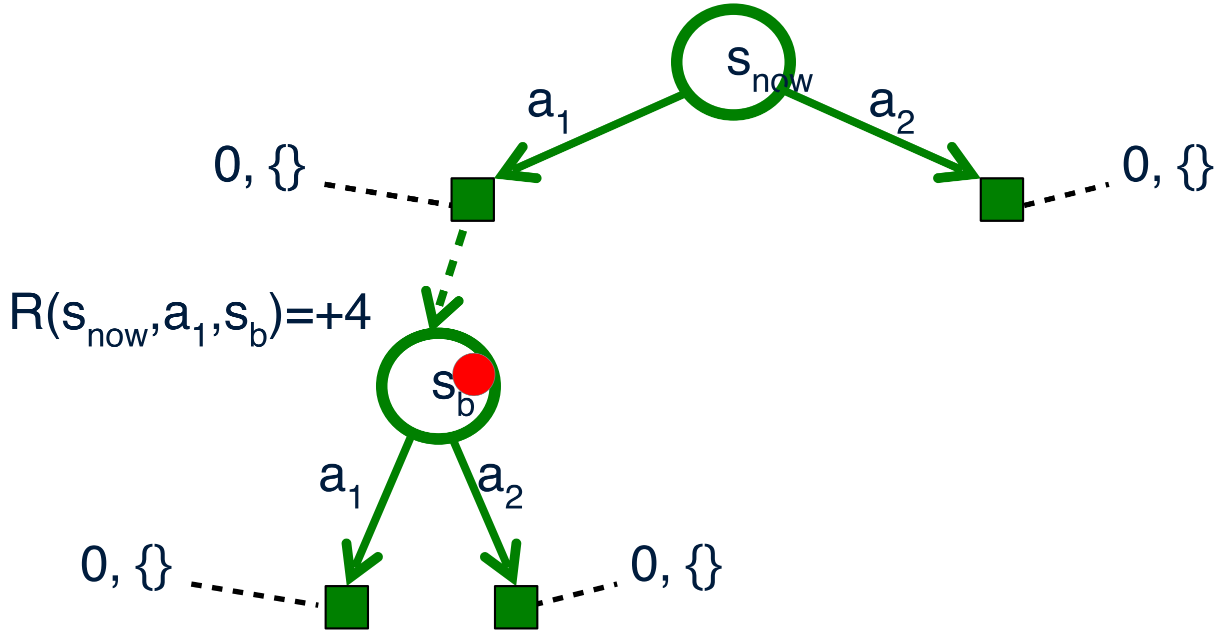
MCTS – Example



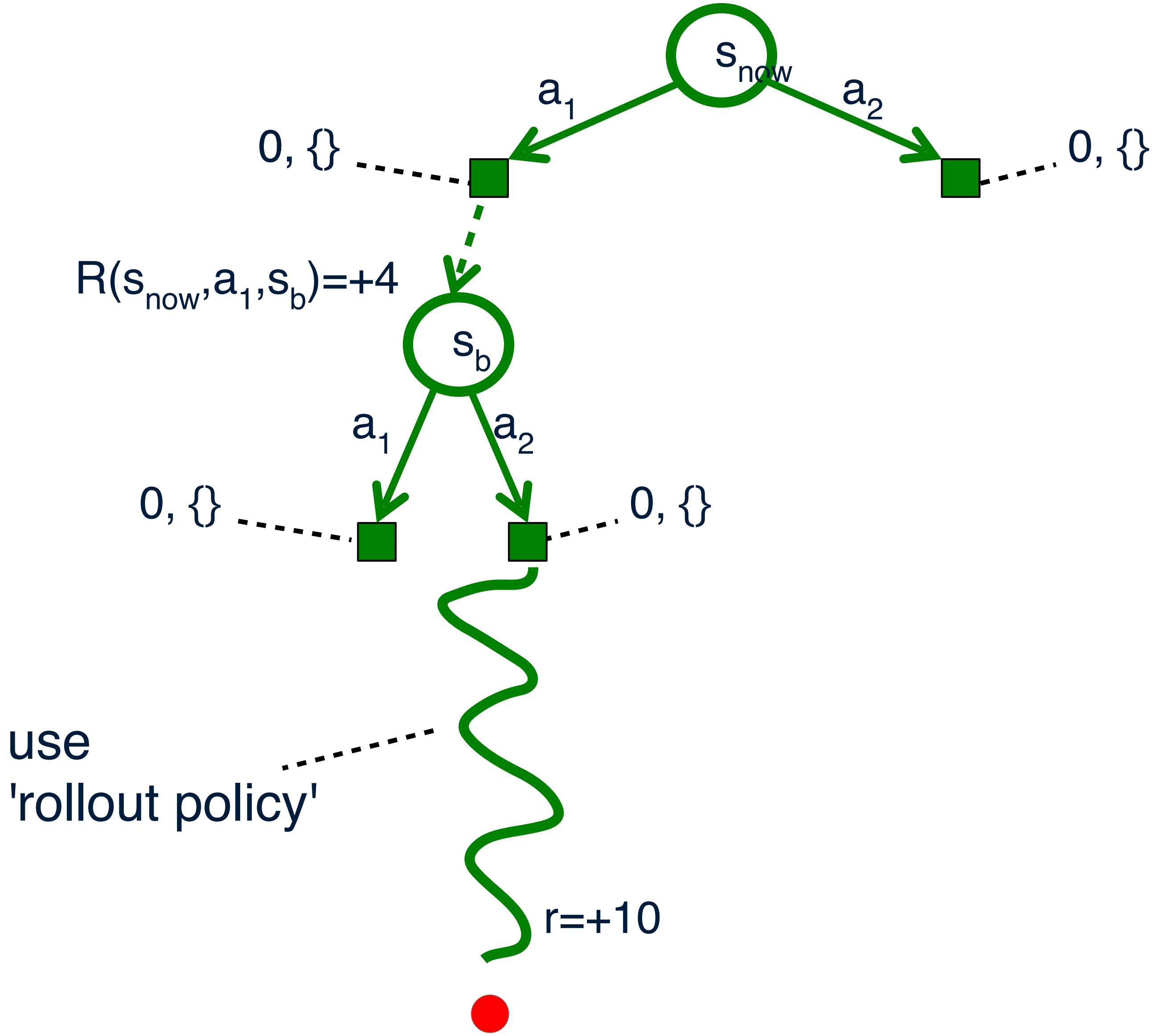
MCTS – Example



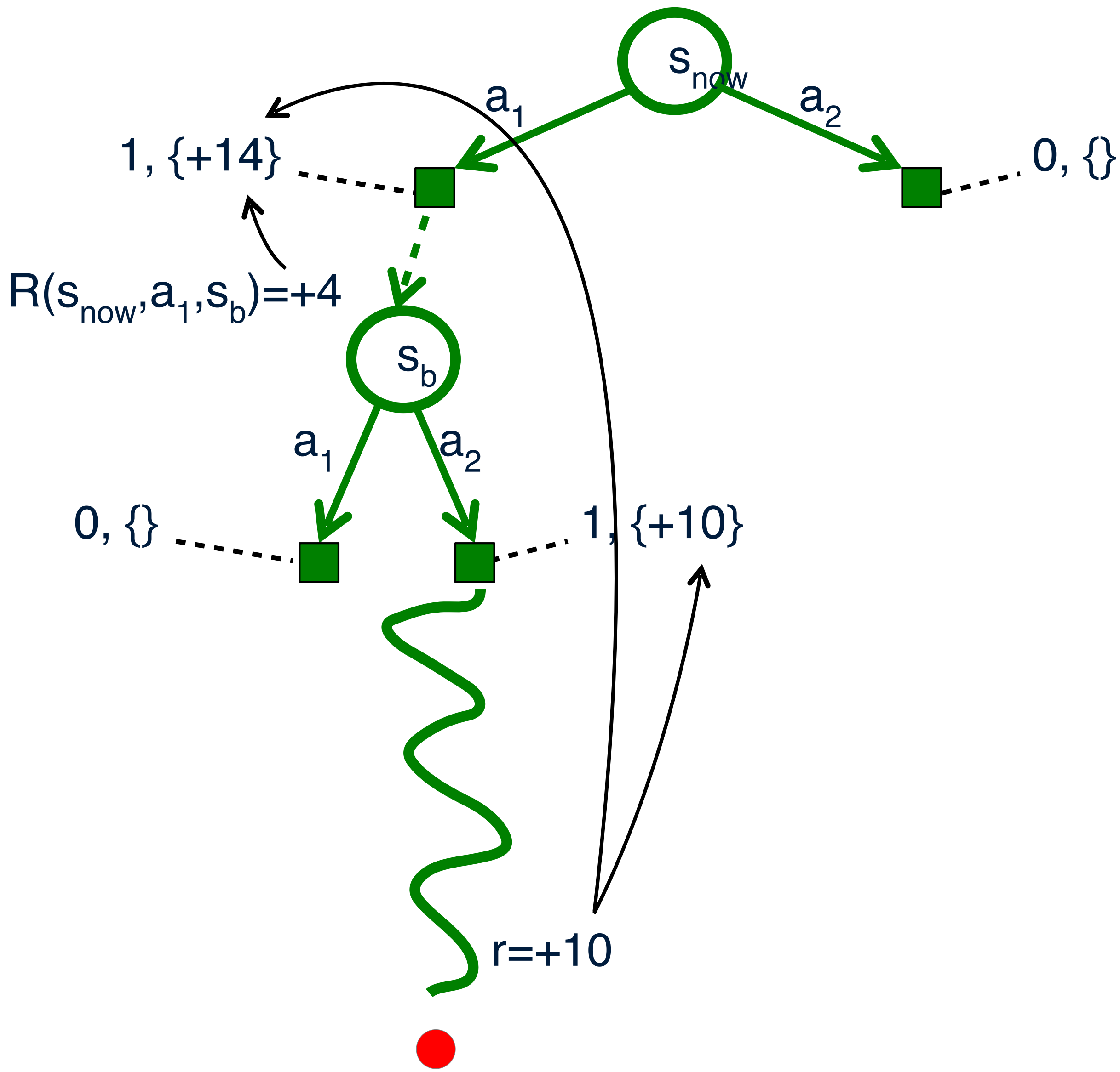
MCTS – Example



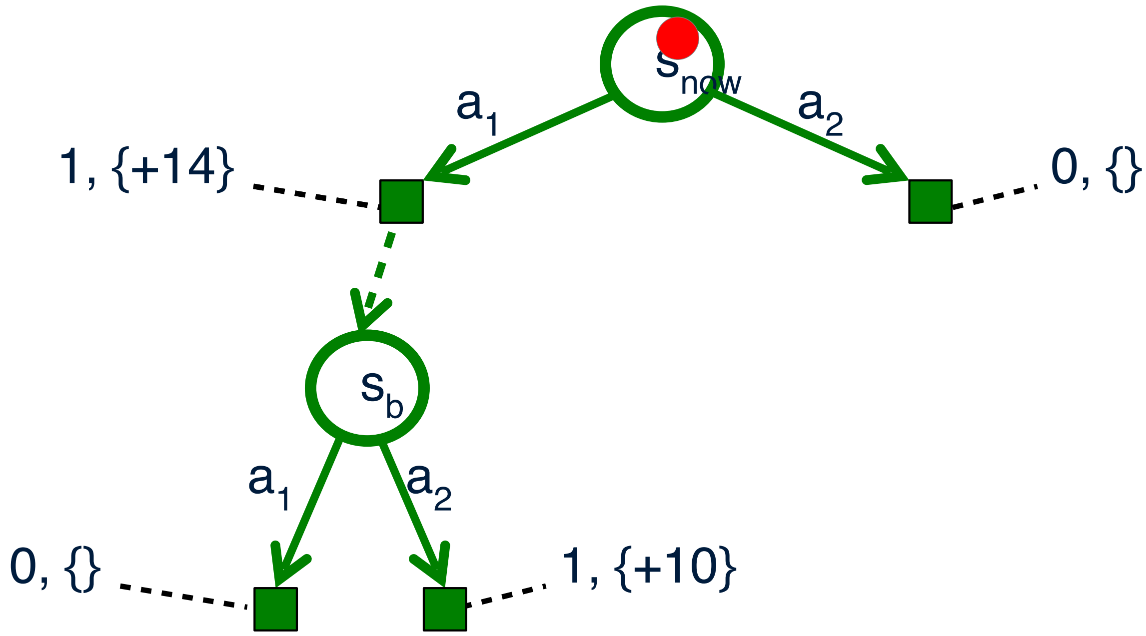
MCTS – Example



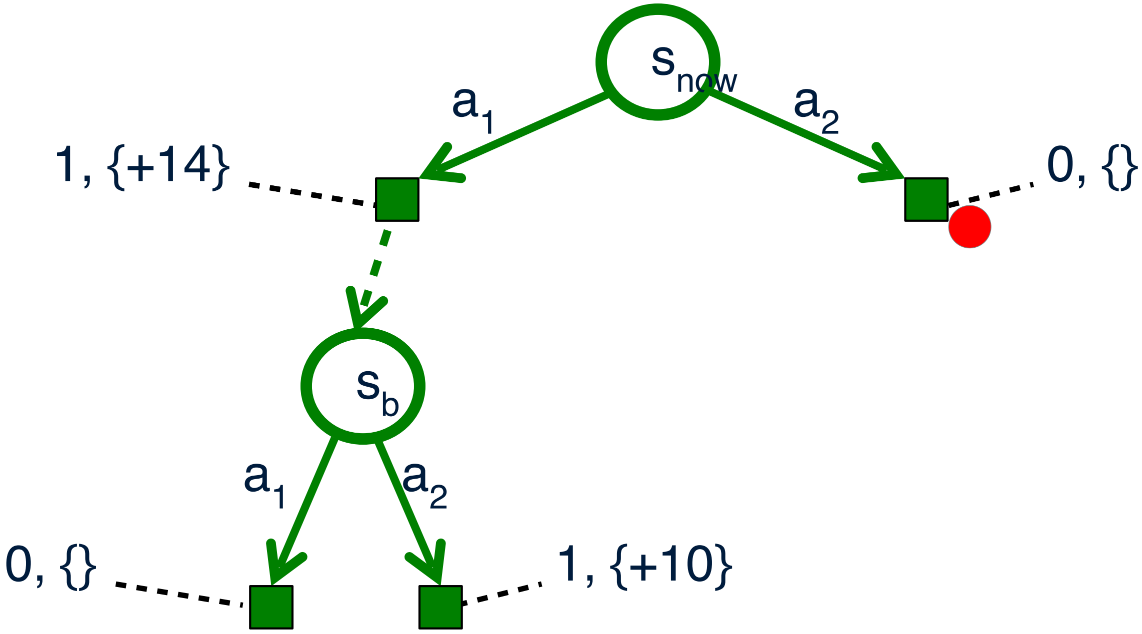
MCTS – Example



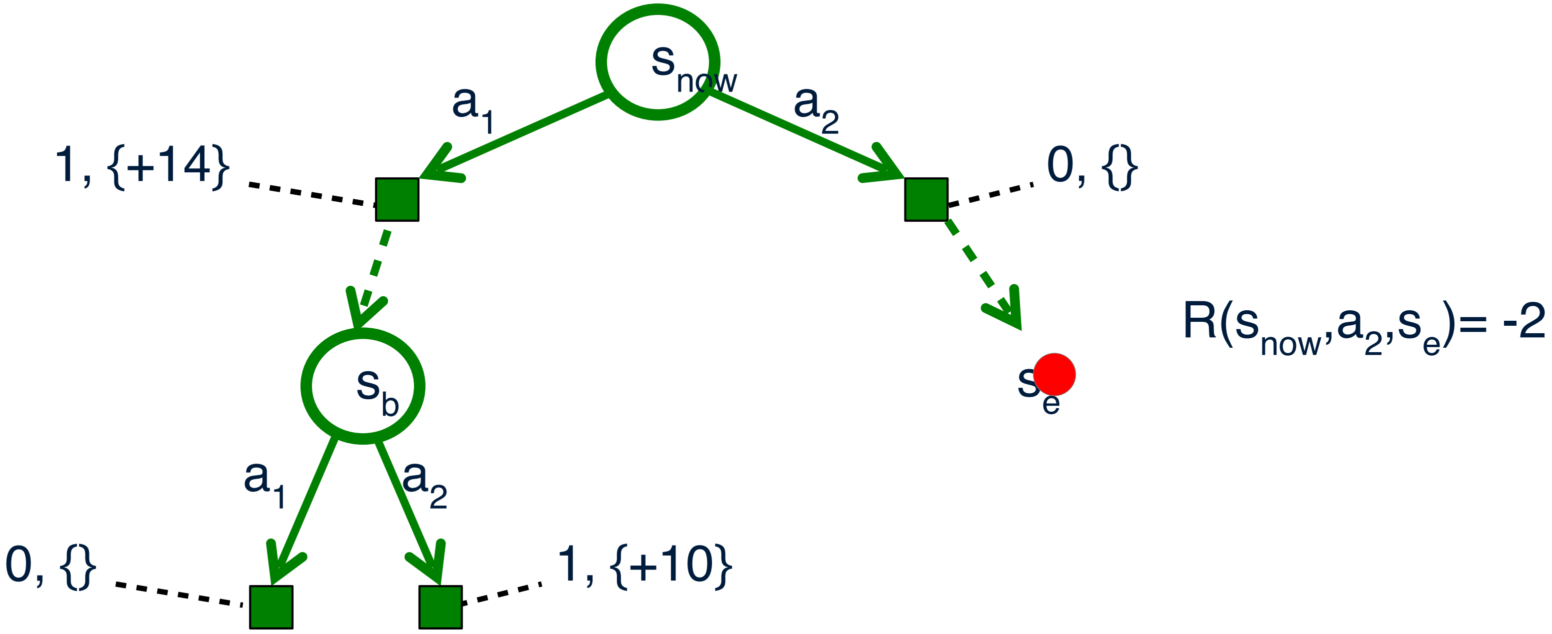
MCTS – Example



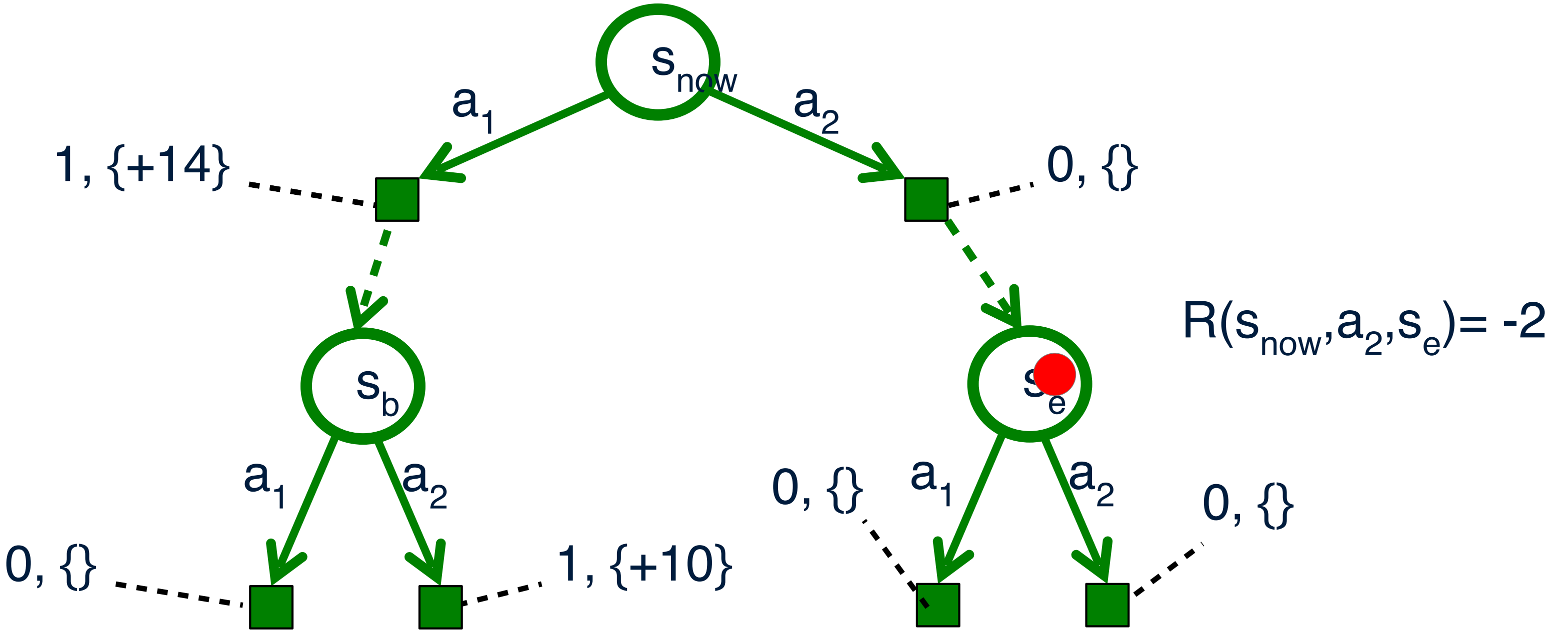
MCTS – Example



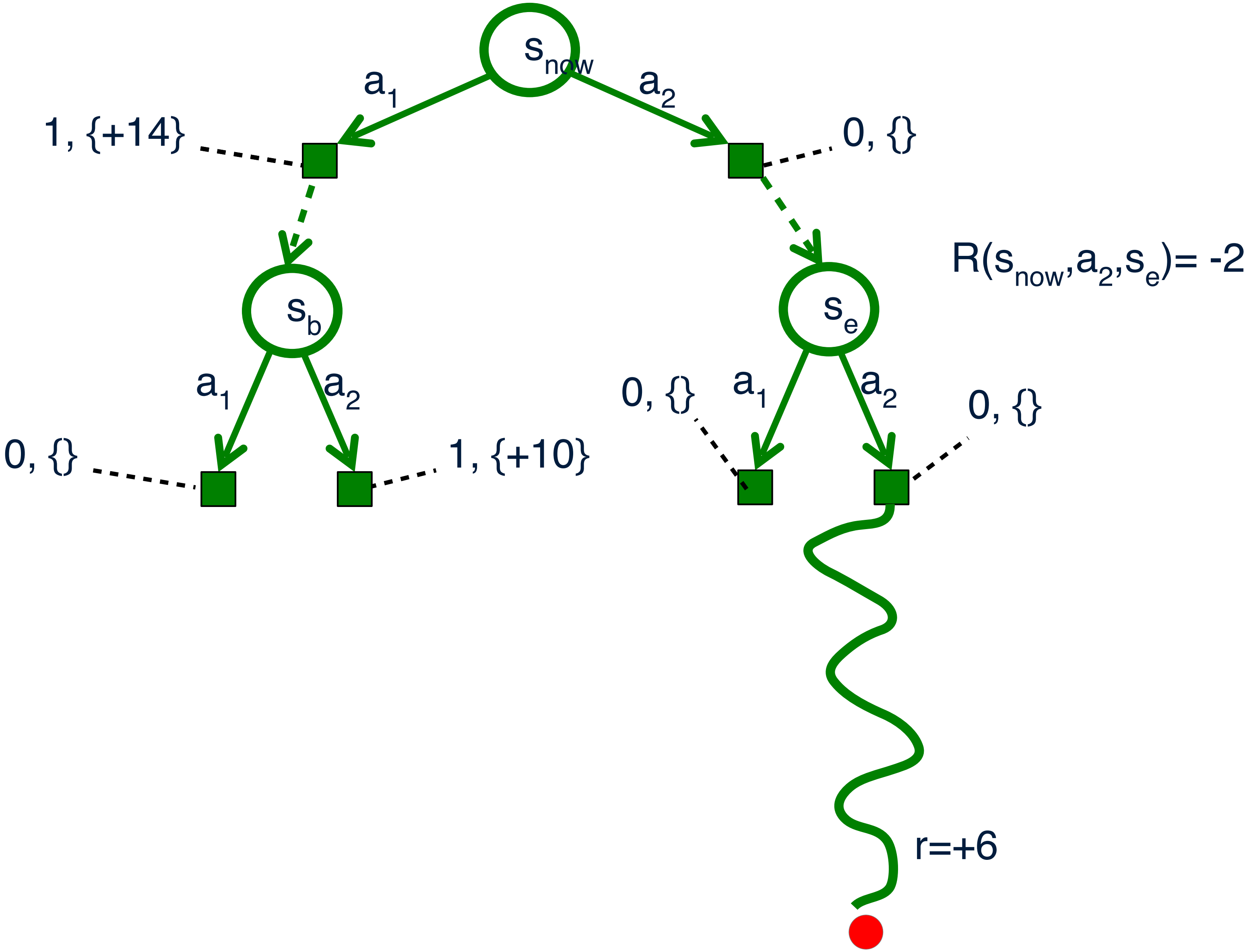
MCTS – Example



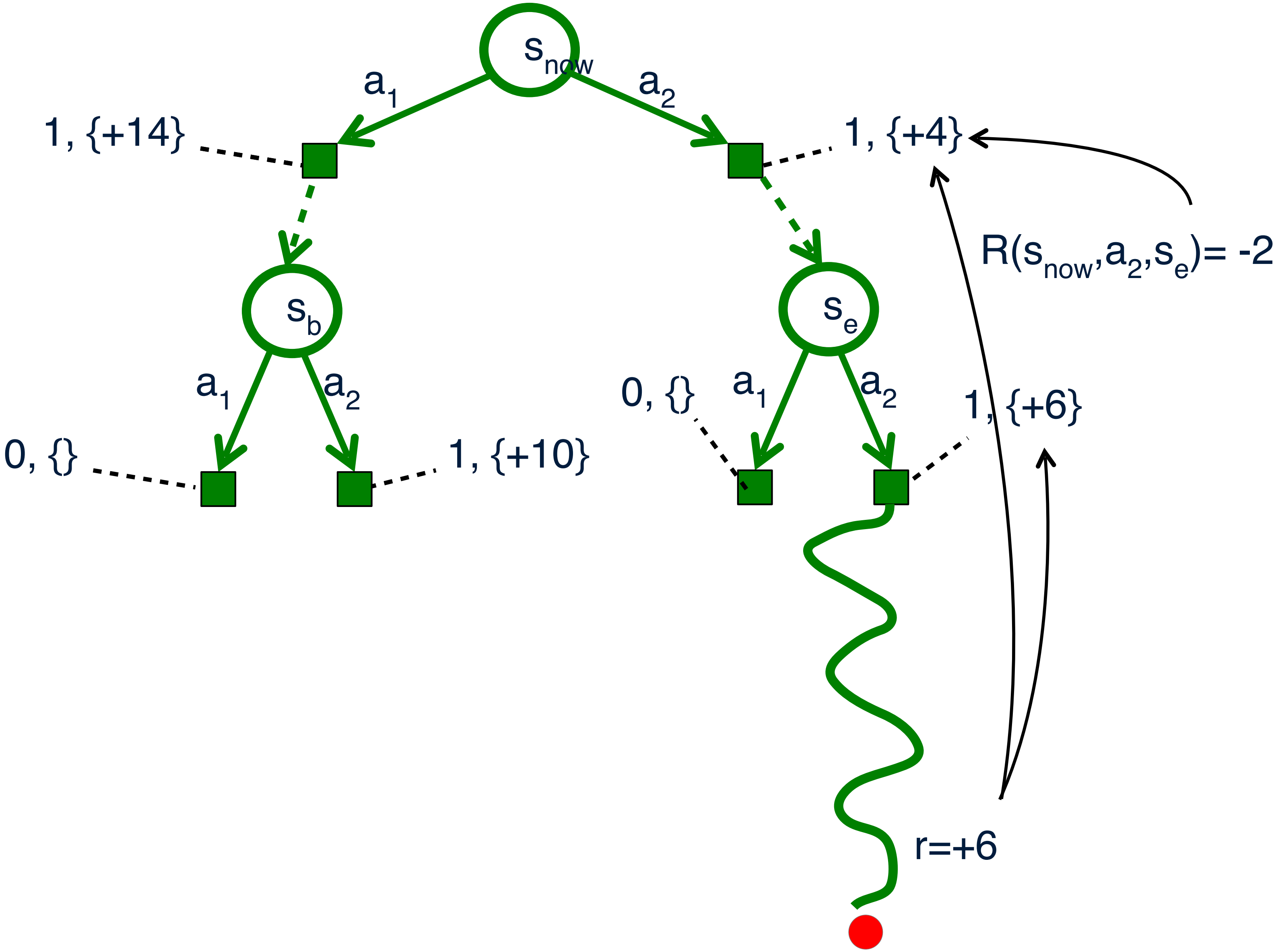
MCTS – Example



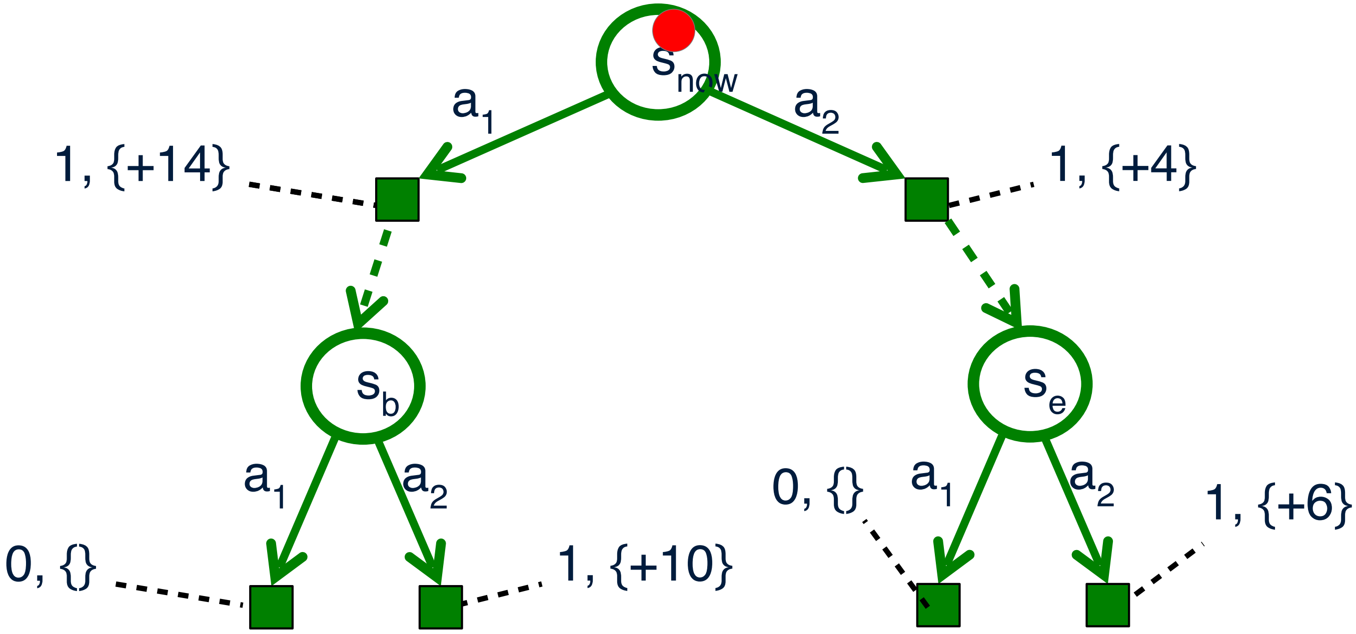
MCTS – Example



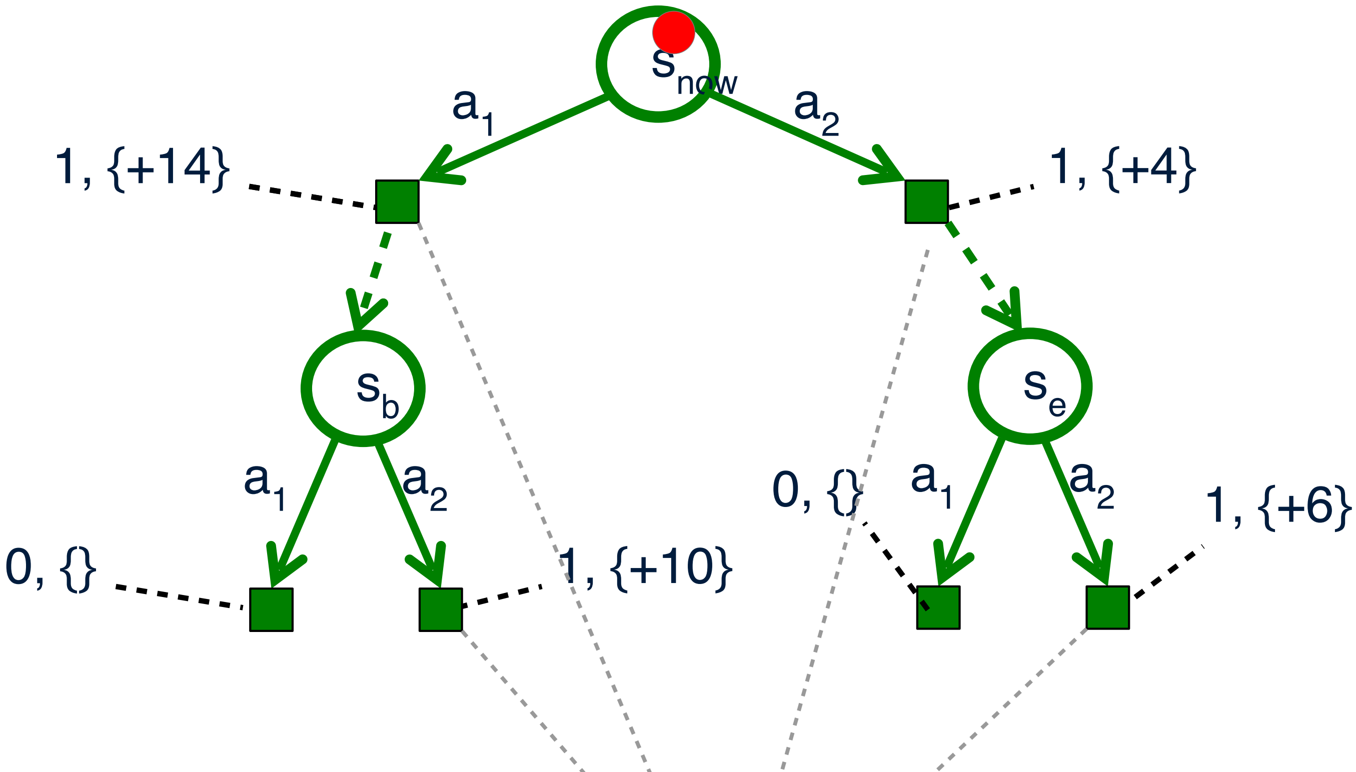
MCTS – Example



MCTS – Example

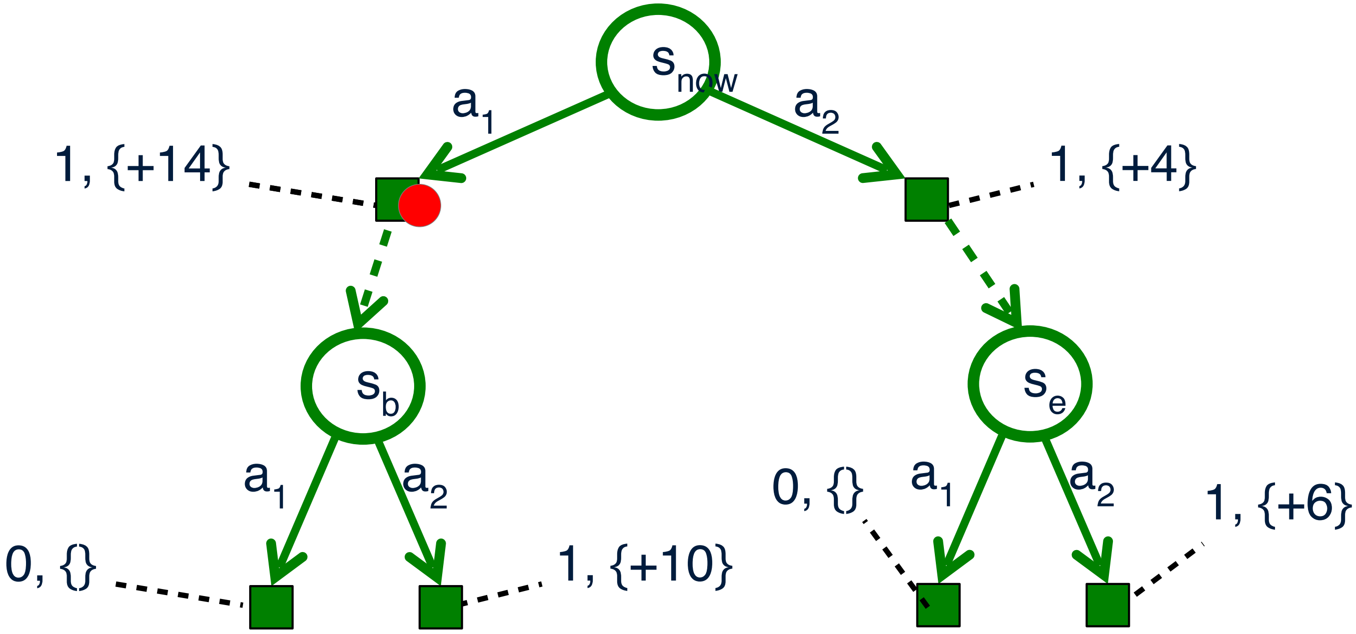


MCTS – Example

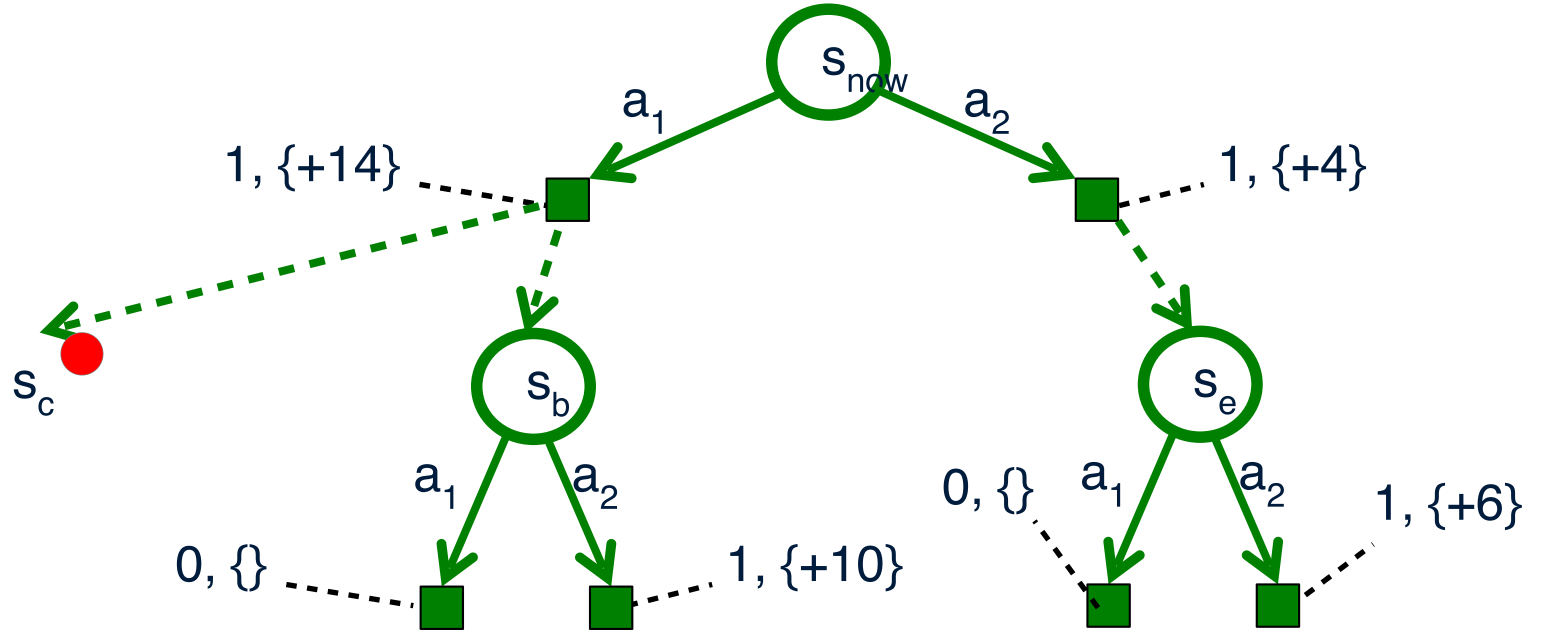


NOTE: the statistics maintained, represent an estimate of **Q(s,a)**

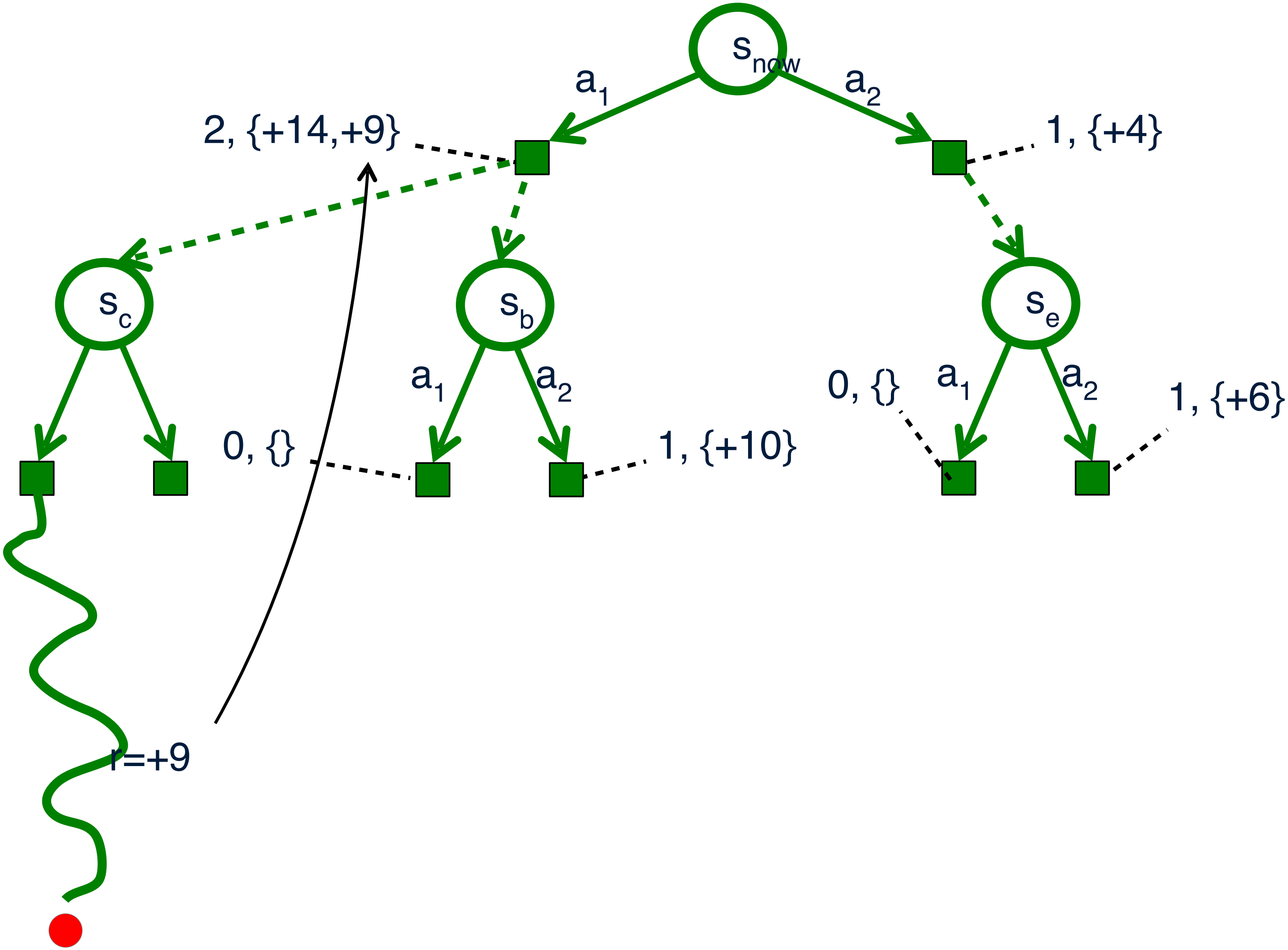
MCTS – Example



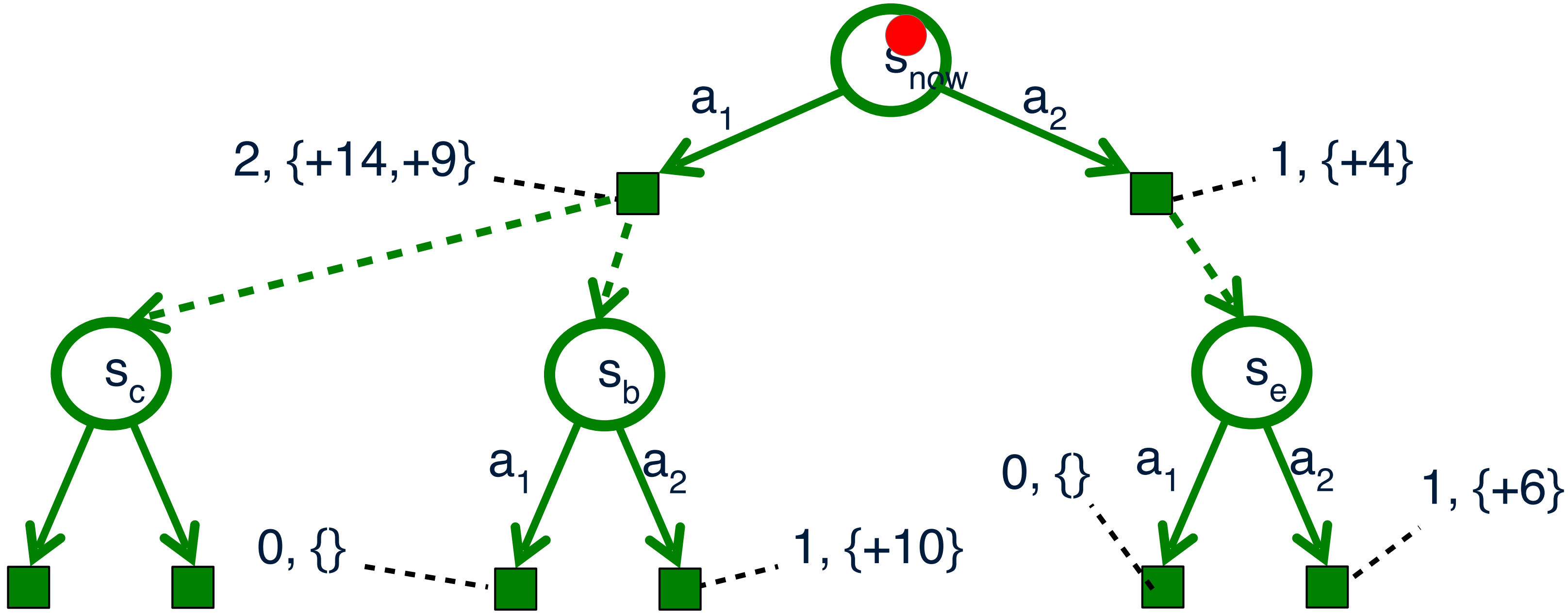
MCTS – Example



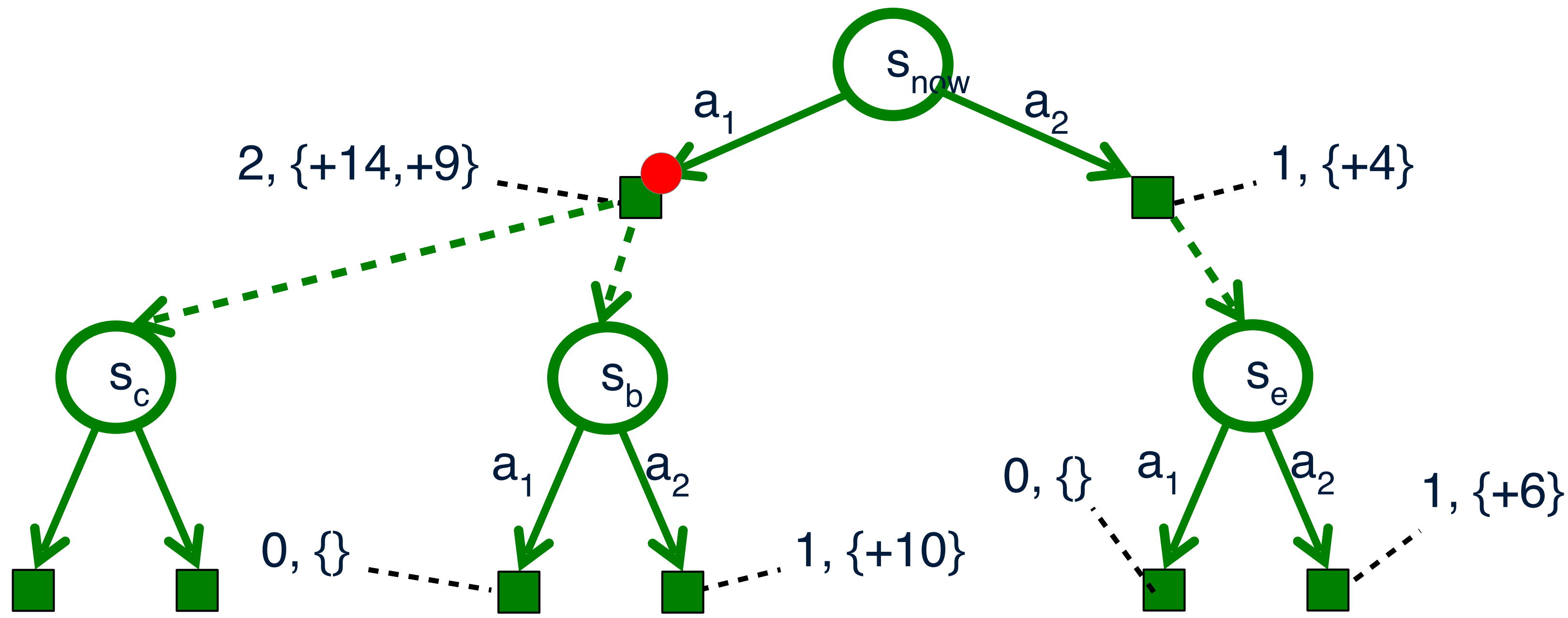
MCTS – Example



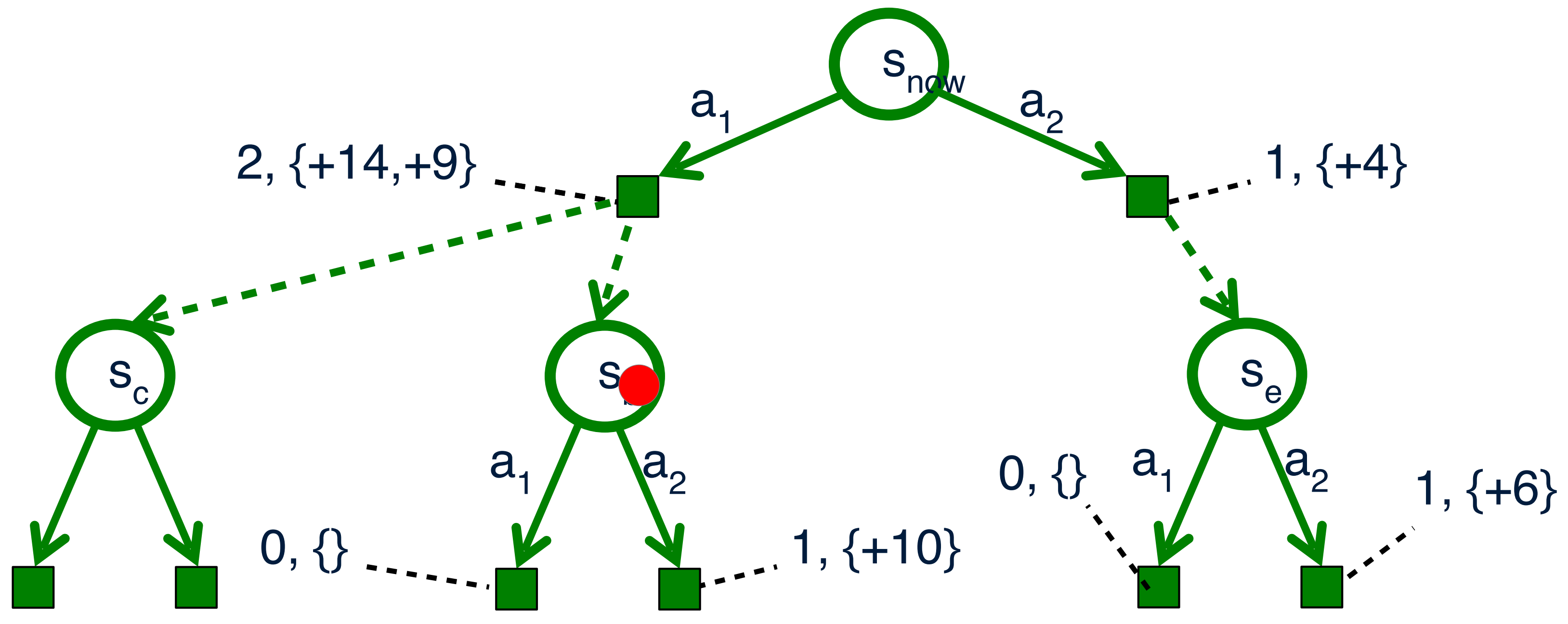
MCTS – Example



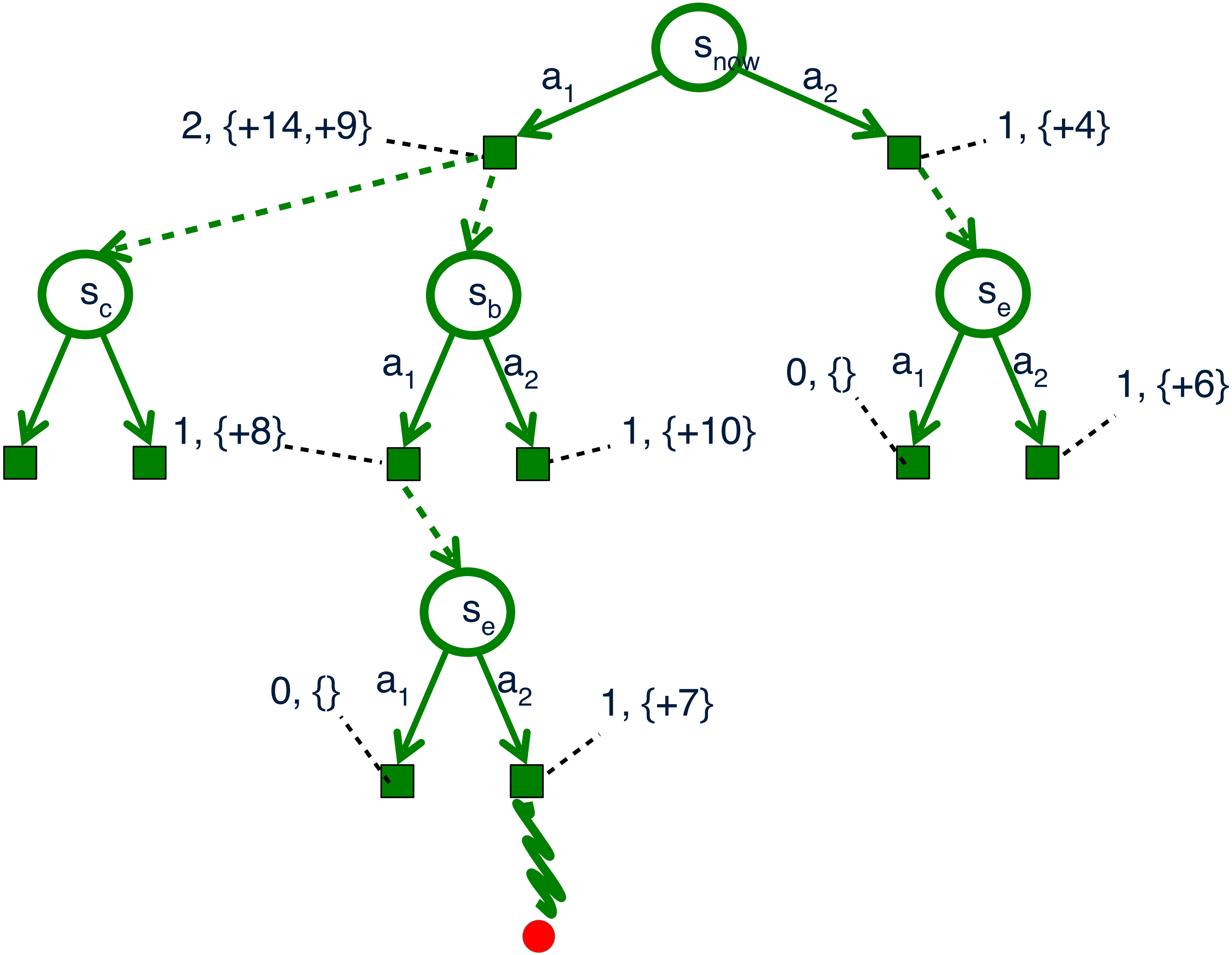
MCTS – Example



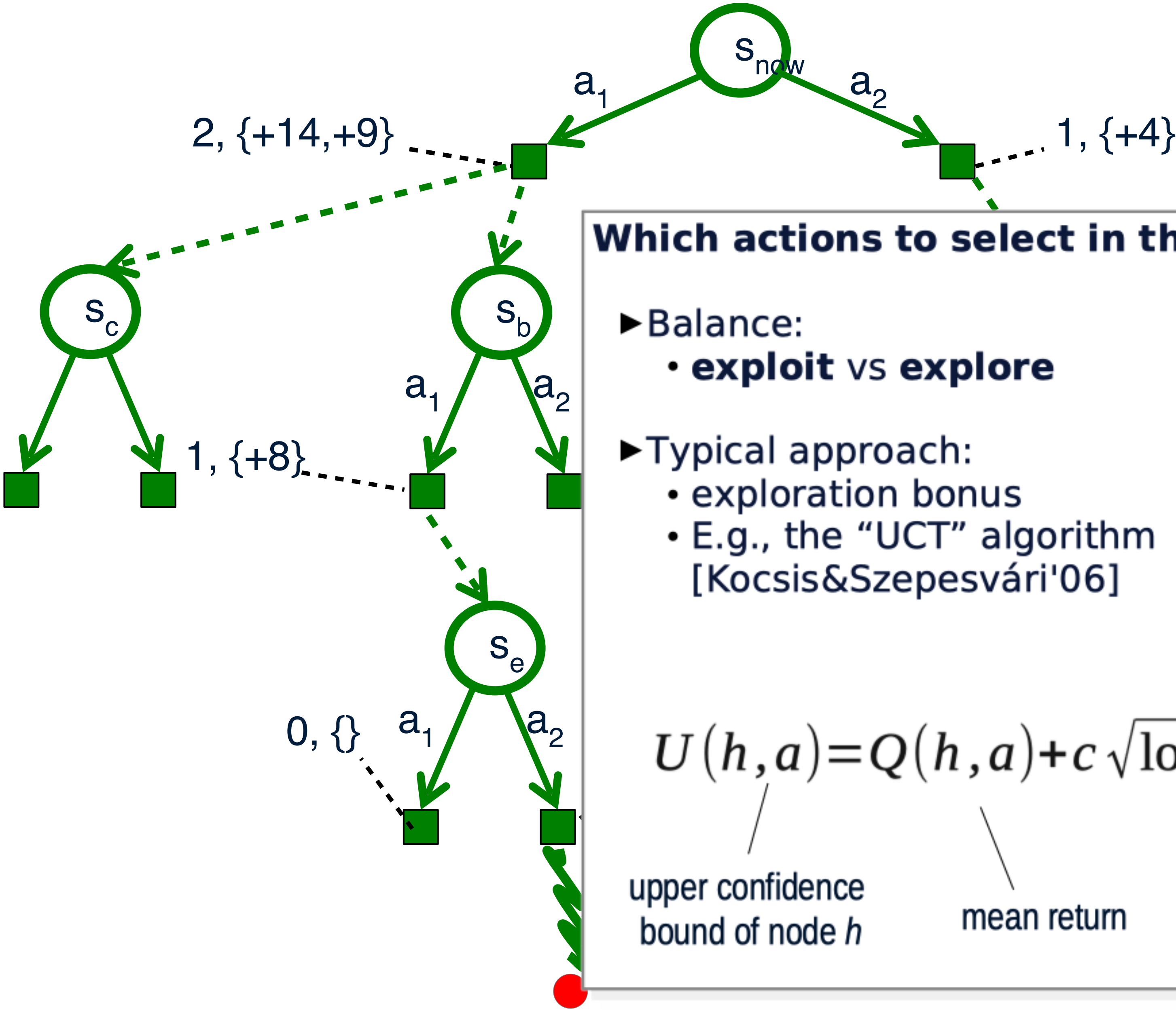
MCTS – Example



MCTS – Example



MCTS – Example

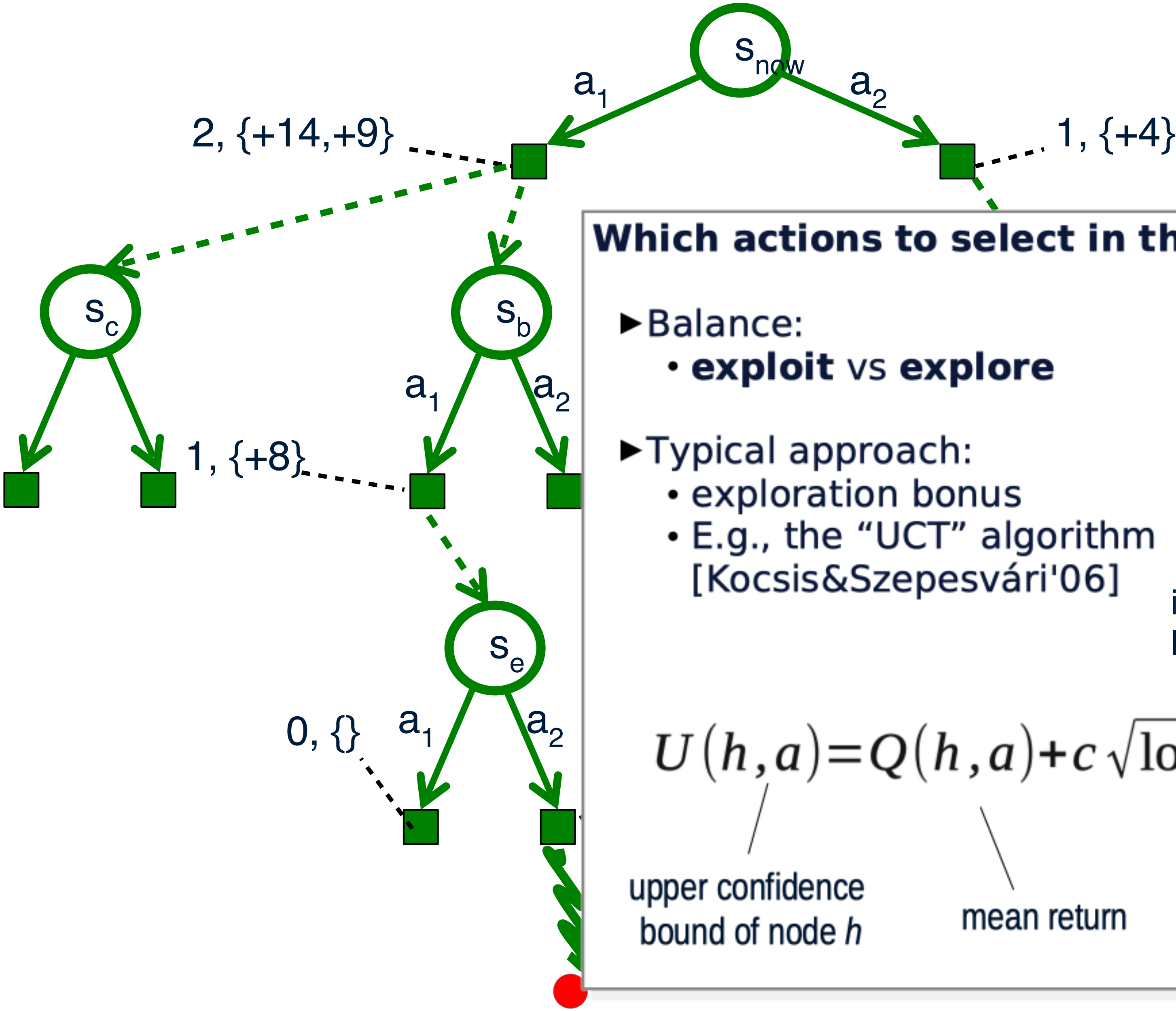


Which actions to select in the tree?

- Balance:
  - **exploit** vs **explore**
- Typical approach:
  - exploration bonus
  - E.g., the "UCT" algorithm [Kocsis&Szepesvári'06]

$$\underbrace{U(h,a)}_{\text{upper confidence bound of node } h}=\underbrace{Q(h,a)}_{\text{mean return}}+\underbrace{c\sqrt{\log(N_h+1)/N_a}}_{\text{exploration bonus}}$$

MCTS – Example



Which actions to select in the tree?

- Balance:
  - **exploit** vs **explore**
- Typical approach:
  - exploration bonus
  - E.g., the “UCT” algorithm [Kocsis&Szepesvári'06]

if *a* tried more often → less bonus

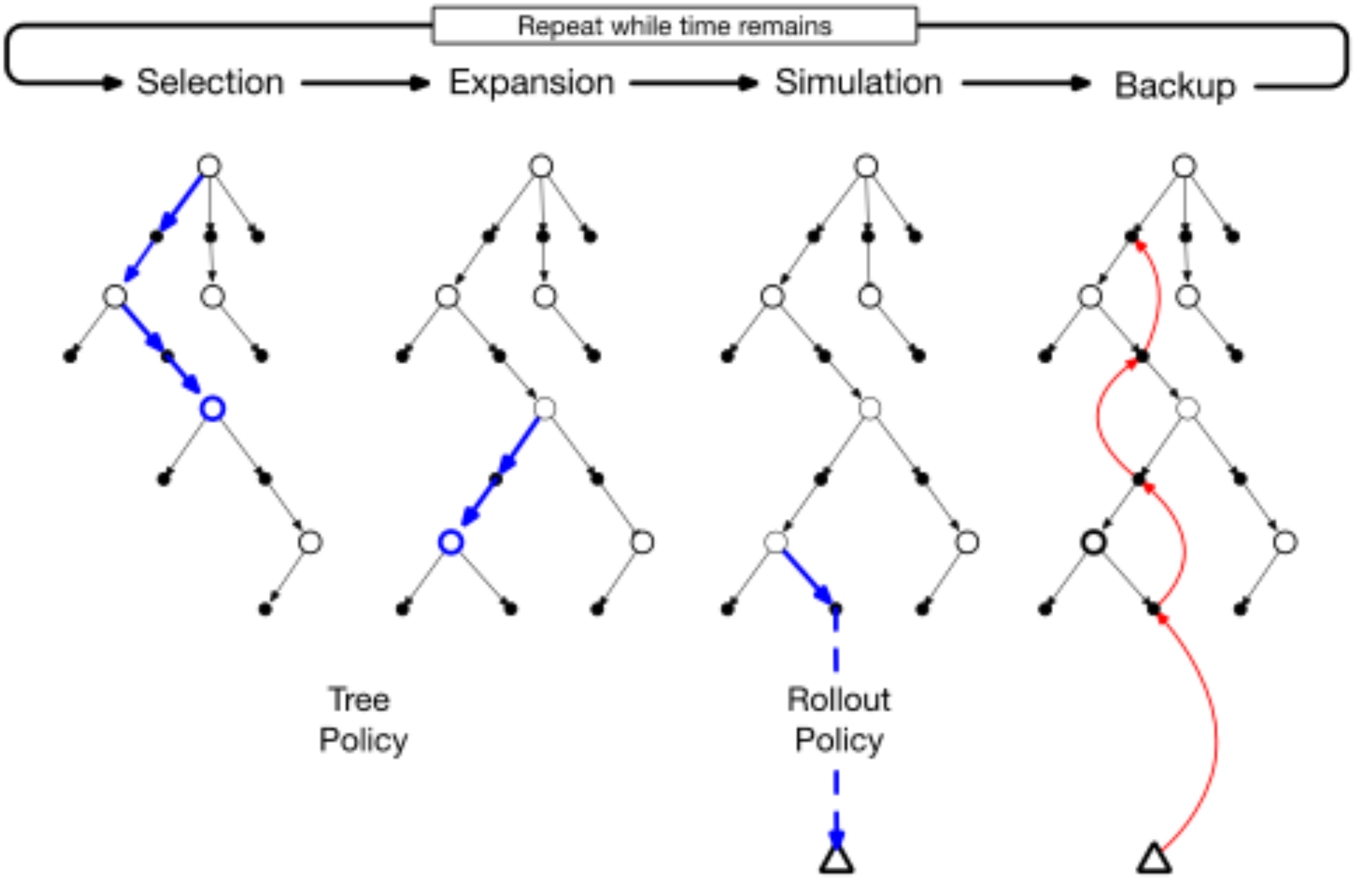
$$U(h,a) = Q(h,a) + c\sqrt{\log(N_h+1)/N_a}$$

upper confidence bound of node *h*

mean return

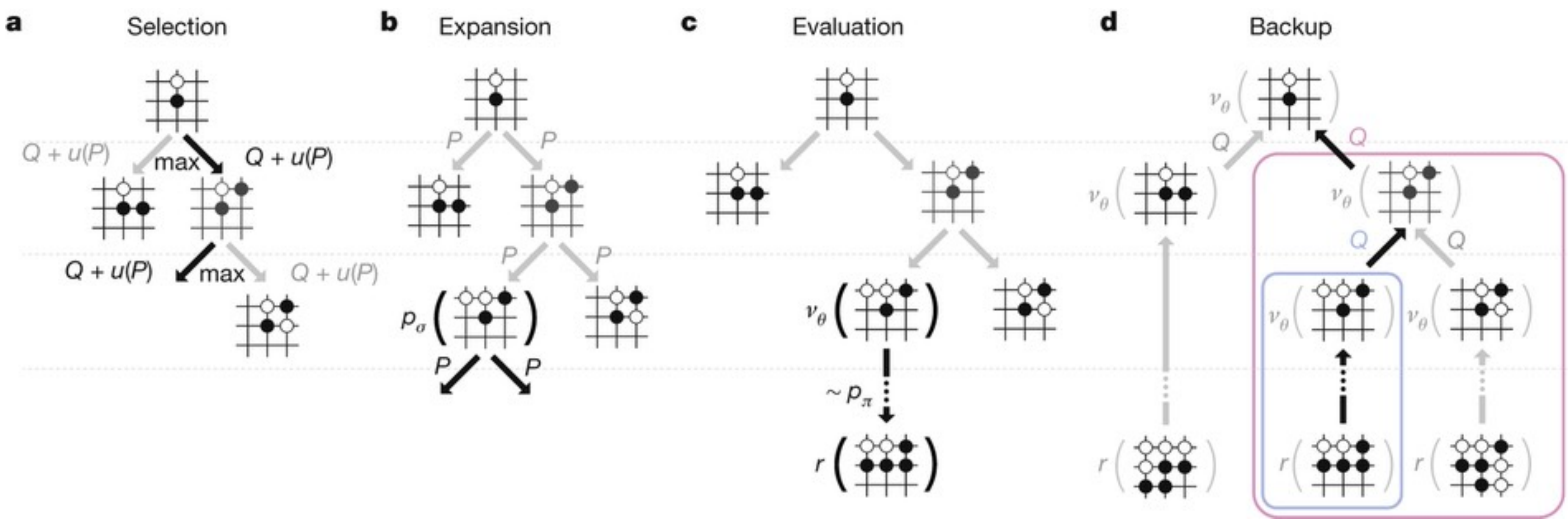
exploration bonus

Phases of MCTS



AlphaGo

- Uses UCT with neural net to approximate opponent choices and state values



Monte Carlo Evaluation

Algorithm 4.11 Monte Carlo policy evaluation

Depth

function MonteCarloPolicyEvaluation(λ, d)

  for $i \leftarrow 1$ **to** n

    $s \sim b$ Initial state distribution

    $u_i \leftarrow$ Rollout(s, d, π_λ) Rollout policy

  return $\frac{1}{n}\sum_{i=1}^{n} u_i$

Algorithm 4.10 Rollout evaluation

function Rollout(s, d, π_0)

  if $d = 0$

    return 0

  $a \sim \pi_0(s)$

  $(s', r) \sim G(s, a)$ Simulate

  return $r + \gamma$ Rollout($s', d - 1, \pi_0$)

- Estimate value of a policy by sampling from a simulator

Monte Carlo Tree Search

Algorithm 4.9 Monte Carlo tree search

function $\textsc{SelectAction}(s, d)$
  loop
    $\textsc{Simulate}(s, d, \pi_0)$
  return $\arg\max_a Q(s, a)$
function $\textsc{Simulate}(s, d, \pi_0)$
  if $d = 0$
    return 0
  if $s \notin T$
    for $a \in A(s)$
      $(N(s, a), Q(s, a)) \leftarrow (N_0(s, a), Q_0(s, a))$
    $T = T \cup \{s\}$
    return $\textsc{Rollout}(s, d, \pi_0)$
  $a \leftarrow \arg\max_a Q(s, a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$
  $(s', r) \sim G(s, a)$
  $q \leftarrow r + \gamma\,\textsc{Simulate}(s', d-1, \pi_0)$
  $N(s, a) \leftarrow N(s, a) + 1$
  $Q(s, a) \leftarrow Q(s, a) + \frac{q - Q(s,a)}{N(s,a)}$
  return q

UCT (Upper Confidence Bounds for Trees)

Just like our bandit case!

Monte Carlo Tree Search

Algorithm 4.9 Monte Carlo tree search

function $\textsc{SelectAction}(s, d)$
    loop
        $\textsc{Simulate}(s, d, \pi_0)$
    return $\arg\max_a Q(s, a)$

Don't use UCB to choose actions in the real world

function $\textsc{Simulate}(s, d, \pi_0)$
    if $d = 0$
        return 0
    if $s \notin T$

State isn't in tree (so initialize stats)

        for $a \in A(s)$
            $(N(s,a), Q(s,a)) \leftarrow (N_0(s,a), Q_0(s,a))$
        $T = T \cup \{s\}$
        return $\textsc{Rollout}(s, d, \pi_0)$

Add to tree and then do rollout

    $a \leftarrow \arg\max_a Q(s, a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$

UCT (Upper Confidence Bounds for Trees)

Just like our bandit case!

    $(s', r) \sim G(s, a)$
    $q \leftarrow r + \gamma \textsc{Simulate}(s', d - 1, \pi_0)$
    $N(s, a) \leftarrow N(s, a) + 1$
    $Q(s, a) \leftarrow Q(s, a) + \frac{q - Q(s,a)}{N(s,a)}$
    return q

UCT continued

- Search/Selection (within the tree, T)
  - Execute action that maximizes $$Q(s,a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$$
  - Update the value $Q(s,a)$ and counts $N(s)$ and $N(s,a)$
  - c is a exploration constant
- Expansion (outside of the tree, T)
  - Create a new node for the state
  - Initialize $Q(s,a)$ and $N(s,a)$ (usually to 0) for each action
- Rollout and Backup (outside of the tree, T)
  - Only expand once and then use a rollout policy to select actions (e.g., random policy)
  - Add the rewards gained during the rollout with those in the tree:

$$r + \gamma \textsc{Rollout}(s', d-1, \pi_0)$$

UCT continued

- Continue UCT until some termination condition
- Guarantees?

UCT continued

- Continue UCT until some termination condition (usually a fixed number of samples)
- MCTS (and UCT) will approach the true Q-values in the limit (for a fixed horizon)
  - You can see this by considering what happens when the full tree is generated
  - This doesn’t depend on rollout policy, but does require exploration of actions
  - In practice no one runs MCTS until optimality, so it does matter how rollout/evaluation and exploration is done

MCTS Pros/Cons

- Pros:
  - rapidly zooms in on promising regions
  - can be used to improve policies
  - basis of many successful application

- Limitations:
  - needle in the hay-stack problems
  - problems with high branching factor
  - both cause poor approximations in limited time

Summary

- Emphasized close relationship between planning and learning
- Important distinction between *distribution models* and *sample models*
- Looked at some ways to integrate planning and learning
    - synergy among planning, acting, model learning
- Distribution of backups: focus of the computation
    - prioritized sweeping
    - sample backups
    - trajectory sampling: backup along trajectories
    - heuristic search/MCTS
- Size of backups: full/sample; deep/shallow

Next time

- Linear function approximation (scaling beyond tabular methods)!
- Read SB 9.1--9.5, 9.8