EECE 5550

Mobile Robotics

Lecture 17: Robotic exploration

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Plan of the day

- The exploration problem
- Frontier-based exploration
- A brief introduction to information theory
- Information-based exploration

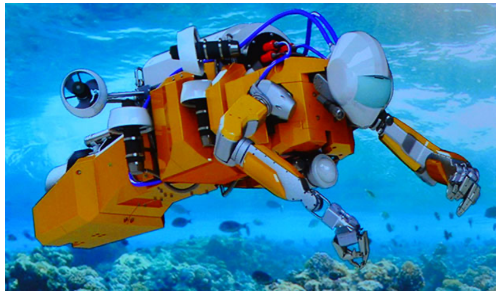
References

- B. Yamauchi: “A Frontier-Based Approach for Autonomous Exploration”
- F. Bourgault, A.A. Makarenko, S.B. Williams: “Information Based Adaptive Exploration”
- H. Carrillo, P. Dames, V. Kumar, J.A. Castellanos: “Autonomous Robotic Exploration Using Occupancy Grid Maps and Graph SLAM Based on Shannon and Renyi Entropy”
- C.E. Shannon: “A Mathematical Theory of Communication”

The exploration problem

Goal: Build a complete map of an initially unknown environment by selecting appropriate control actions which will lead the robot to visit previously unseen areas (or revisit highly uncertain areas)

Applications of autonomous exploration:



Stanford humanoid for deep-sea exploration

CMU/Oregon State U. robot for underground exploration

Drones for infrastructure exploration

NASA JPL VolcanoBot

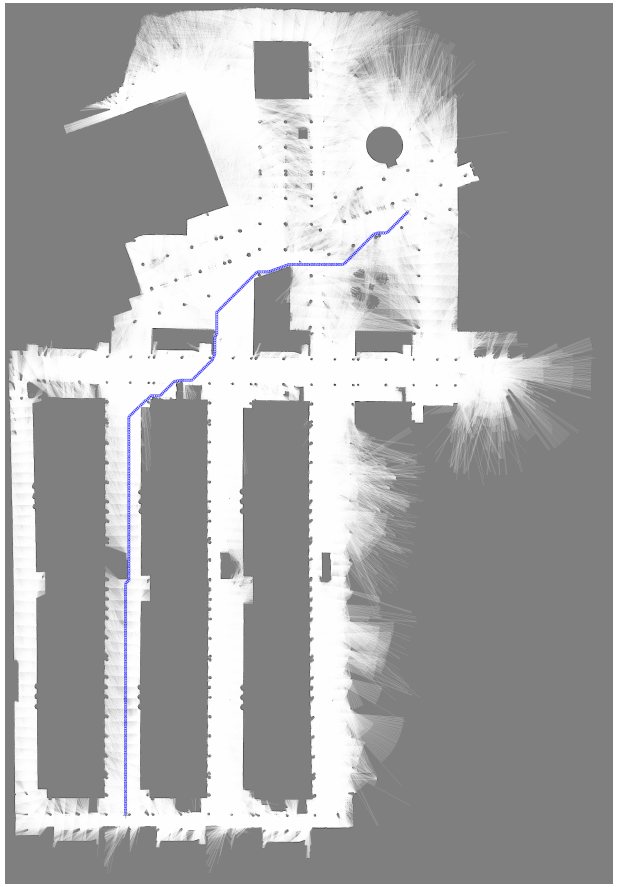
Space exploration

The exploration problem

Goal: Build a complete map of an initially unknown environment by selecting appropriate control actions which will lead the robot to visit previously unseen areas (or revisit highly uncertain areas)

Recall: SLAM algorithms

- enable us to build a map from sensor measurements and localize the robot in the built map
- do not address the trajectory planning of a robot (where to collect measurements).



Fundamental question: How should we decide what measurements to collect, given that we only have partial knowledge of the environment as we are building the map?

- *Decision-making problem*

Methods for robotic exploration

- **Human-directed:** No robot-autonomy; human is teleoperating the robot.

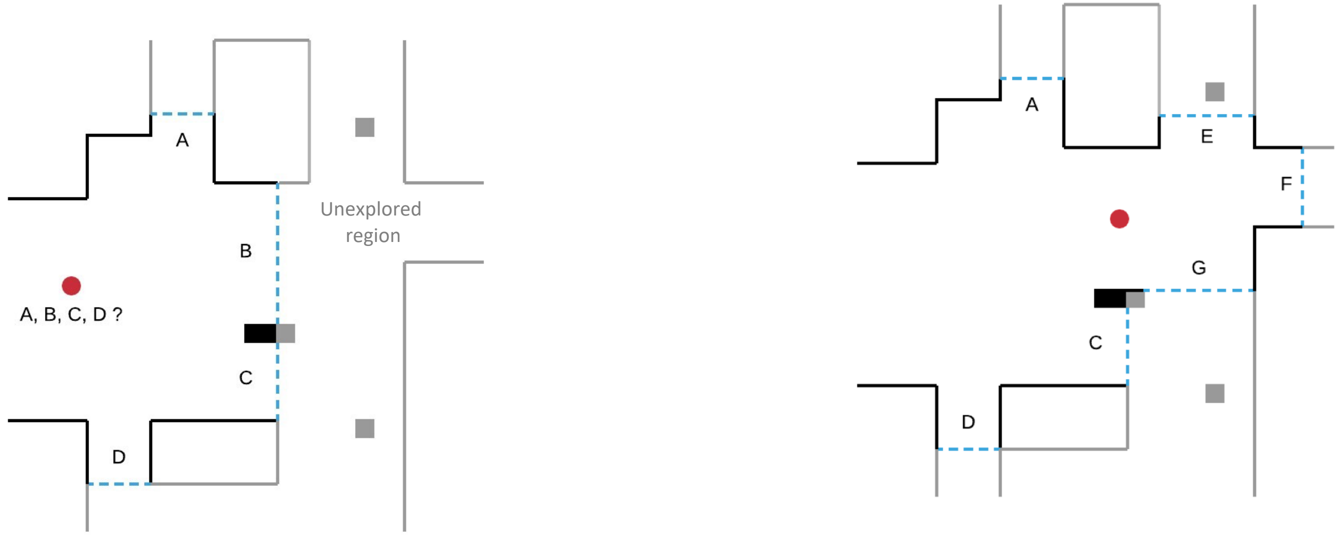
- **Random:** Select viewpoints based on a purely random approach.

- **Frontier-based:** Select viewpoints at the frontier of the explored space.

- **Information-based:** Select viewpoints to maximize the amount of new information obtained.

Frontier-based exploration

Main idea [Yamauchi]**:** To gain the most new information about the world, move to the boundary between the open space and unexplored territory.



Key question: How can we formalize the notion of "boundary" between free and unexplored space?

http://www.robotfrontier.com/papers/cira97.pdf

Frontier extraction in occupancy grids

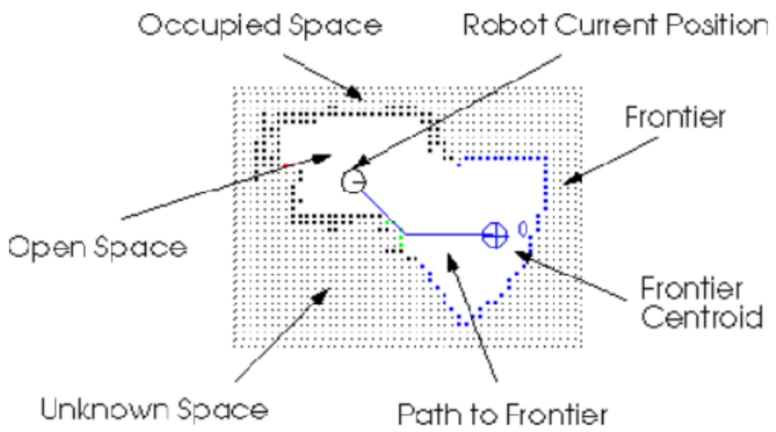
Occupancy grids are particularly nice for frontier-based exploration, because they admit a simple approach to classifying space as *free*, *occupied*, *and unknown*:

- **Free:** $p(occ) < prior$
- **Occupied:** $p(occ) > prior$
- **Unknown:** $p(occ) = prior$

We then define the set of *boundary cells* to be the set of all grid cells that are:

- Unoccupied
- Adjacent to an unknown cell

Finally, we define a *frontier* to be a maximal connected set of boundary cells.

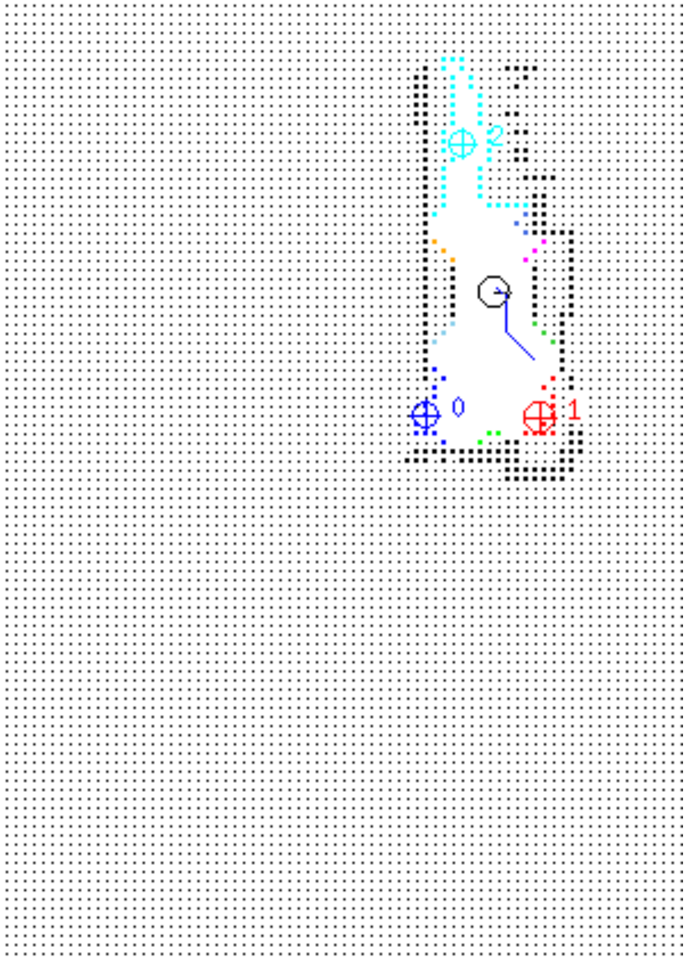


A Simple Frontier-Based Exploration Algorithm

while frontiers exist, repeat:

1. Plan a path to the nearest one
2. Follow the plan to collect measurements
3. Update map & frontiers

end while



Autonomous exploration with Turtlebot

https://youtu.be/op0L0LyGNwY

Frontier-Based Exploration in Occupancy Grids

Pro: Super simple ☺!

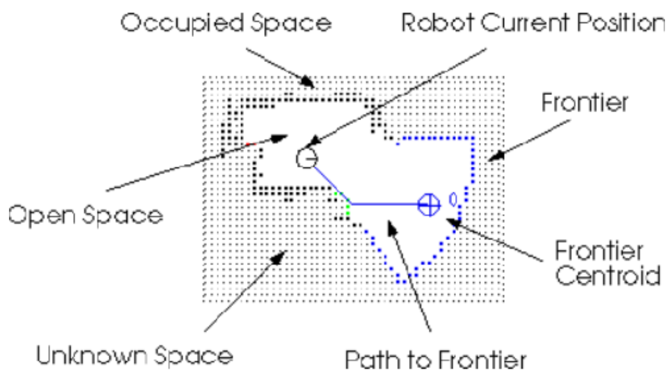
But: This approach does not quantify how valuable it would be to move the robot to a given frontier.

There exist other methods which take into account this!

- **Information based exploration:** Selecting next points that maximize the amount of new info
- How do we measure the information?

Frontier-Based Exploration

while frontiers exist, repeat:
1. Plan a path to the nearest one
2. Follow the plan to collect measurements
3. Update map & frontiers
end while

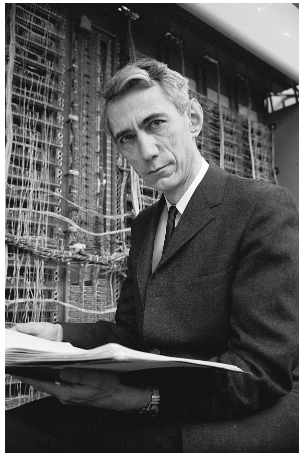


Entropy

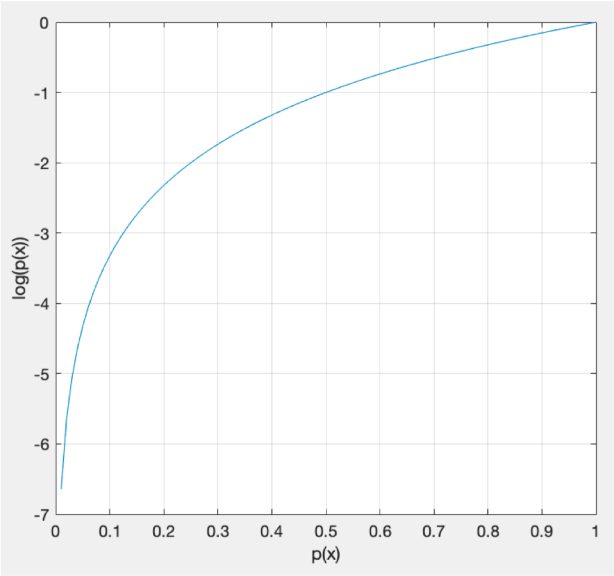
In information theory, the canonical measure of “uncertainty” associated with a random variable *X* is *entropy*.

If *X* is a discrete random variable taking values in the set $\{x_1, \dots, x_N\}$, its entropy is given by:

$$H(X) = -\sum_{i=1}^{N} P(x_i) \log P(x_i)$$



Claude E. Shannon



How does entropy measure "uncertainty"?

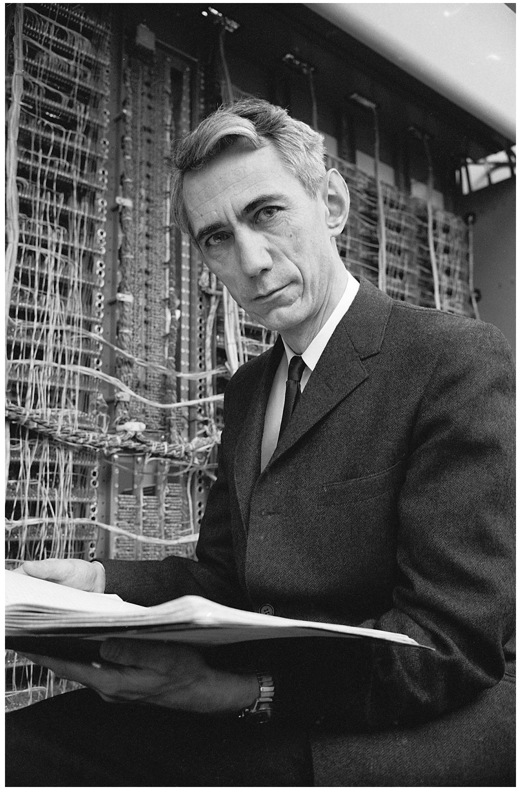
Shannon's key insight: The amount of "information" obtained by observing the outcome of a random process is directly related to how *unlikely* (or "surprising") that outcome is.

Ex: The entropy $H(X)$ is maximal for a uniform distribution, i.e., the uncertainty is highest when all outcomes are equally likely.

Let X has n possible outcomes, and $p(x_i) = \frac{1}{n}$

$$H(X) = -\sum_{i=1}^{n} \frac{1}{n} \log \frac{1}{n} = \log(n)$$

Key question: Can we come up with a good way of *measuring* the "information content" $I(X)$ of a random event X?)



Claude E. Shannon

How does entropy measure “uncertainty”?

Develop a measure $I(A)$ that assigns to a random event *A* telling us how “informative” observing that event is.

One approach: Let’s specify what properties we might like I to possess

Information axioms: Let *A* be a random event.

1. Nonnegativity: $I(A) \geq 0$
2. Monotonicity: $I(A)$ is *monotonically decreasing* in *p*(*A*).
   (Observing a *more likely* event is *less informative / surprising*).
3. If *p*(*A*) = 1 then $I(A) = 0$
   (Observing an almost-sure outcome provides *zero* information)
4. Additivity: If *B* is another event independent of *A*, then:

$$I(A \cap B) = I(A) + I(B)$$

Key point: Shannon showed that any function *I* satisfying 1-4 must be of the form:

$$I(A) = k \log p(A)$$

for k < 0.

How does entropy measure “uncertainty”?

Now let’s recall the definition of entropy:

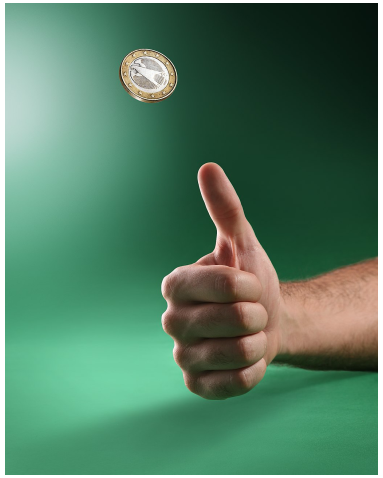
$$H(X) = -\sum_{i=1}^{N} P(x_i) \log P(x_i)$$

$$= \underbrace{\sum_{i=1}^{N} P(x_i)}_{\text{Expectation over } \textit{all} \text{ possible realizations of } X} \cdot \underbrace{(-\log P(x_i))}_{\text{Information / surprise of observing realization } X = x_i}$$

Therefore: The entropy $H(X)$ of a random variable X is measuring the *expected information / surprise* of observing a realization of X.

Example - Entropy

Consider tossing a coin.

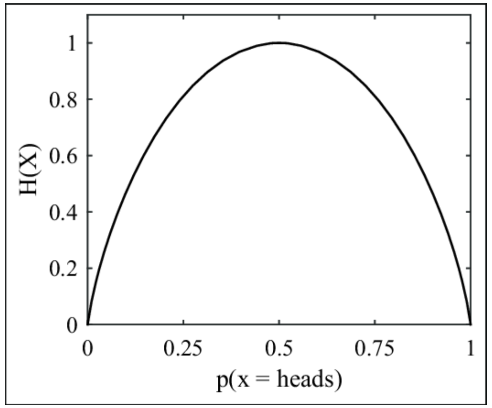


Scenario 1: The coin is fair (heads and tails have equal probability)

$$H(X) = -\sum_{i=1}^{N} P(x_i) \log P(x_i)$$

$$= -\sum_{i=1}^{2} \frac{1}{2} \log_2 \frac{1}{2} = -\sum_{i=1}^{2} \frac{1}{2}(-1) = 1$$

What about if $p(heads) = 1$?



Scenario 2: The coin is not fair ($p(heads) = 0.7$)

$$H(X) = -p(heads) \log_2 p(heads) - p(tails) \log_2 p(tails)$$

$$= -0.7 \log_2 0.7 - 0.3 \log_2 0.3 = 0.8816 < 1$$

Conditional entropy and mutual information

The notion of entropy can be used to measure how "informative" one random variable is wrt. another.

Suppose that *X* and *Y* are jointly distributed, with $P(X,Y) = P(Y|X)P(X)$, and we observe $Y = y$.
How can we quantify how much observing *Y* has reduced my uncertainty about *X*?

Before observing $\boldsymbol{Y}$: My belief over *X* was simply the *prior* $p(X)$.
After observing $\boldsymbol{Y}$: My belief over *X* is the *posterior* $p(X|Y=y)$.

Therefore: The reduction in my uncertainty over *X* due to measuring *Y* = *y* is:

$$H(X) - H(X|Y=y)$$

It follows that the *expected reduction* in *X*'s uncertainty after observing *Y* is:

$$H(X) - E_Y[H(X|Y=y)]$$

Conditional entropy and mutual information

Recall: The *expected reduction* in *X*'s uncertainty after observing *Y* is:

$$H(X) - E_Y[H(X|Y = y)]$$

The second term above is called the *conditional entropy*, denoted $H(X|Y)$:

$$H(X|Y) \triangleq E_Y[H(X|Y = y)]$$

Similarly, the expected reduction in *X*'s uncertainty after observing *Y* is called the information gain (or *mutual information* between *X* and *Y*) denoted $I(X;Y)$:

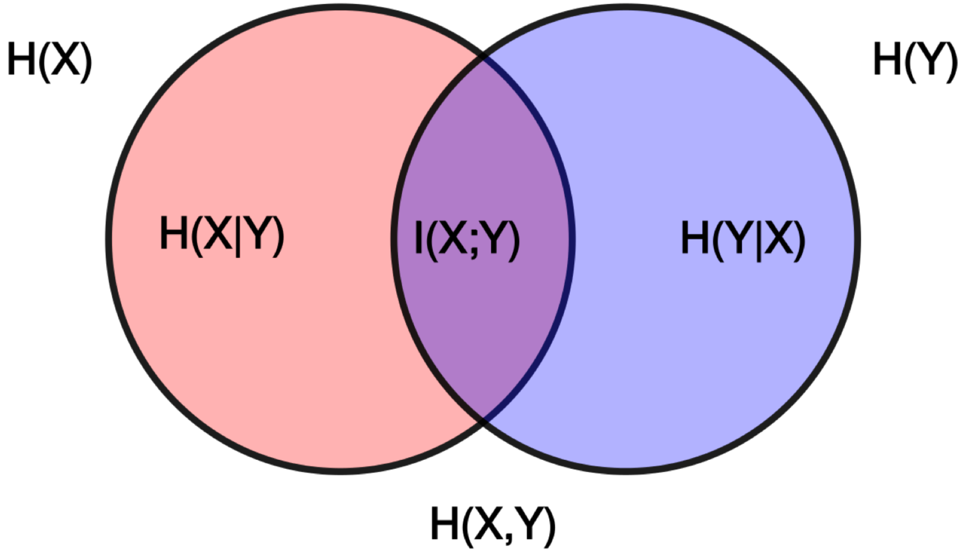
$$\boxed{I(X;Y) \triangleq H(X) - H(X|Y)}$$

⇒ The mutual information $I(X;Y)$ quantifies how much observing *Y* tells me about *X*.

Conditional entropy and mutual information

Graphical representation of the conditional entropy and the mutual information.

$$
\begin{aligned}
I(X;Y) &\triangleq H(X) - H(X|Y) \\
&= H(Y) - H(Y|X) \\
&= H(X) + H(Y) - H(X,Y)
\end{aligned}
$$



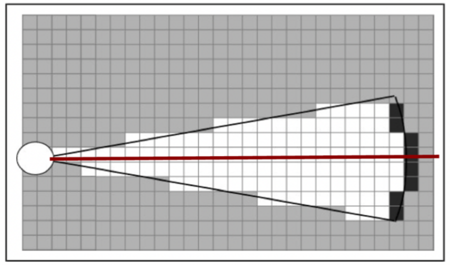
Measuring information in occupancy grid maps

Recall that

- Probabilistic occupancy grids encode a *belief* $p(M|z_{1:k})$ over the map M given measurements $z_{1:k}$
- The cells (the random variables) are **independent** of each other.

Initially: We (typically) have a uniform prior $p(m_i) = .5$ for all cells
⇒Intuitively: we are completely ignorant about the state of the world

After mapping: Ideally, we want either $p(m_i) \approx 1$ $p(m_i) \approx 0$ for all reachable m_i
⇒Intuitively: We are highly certain about the status of all cells



Inverse sensor model
$p(m_i|z_t, x_t)$

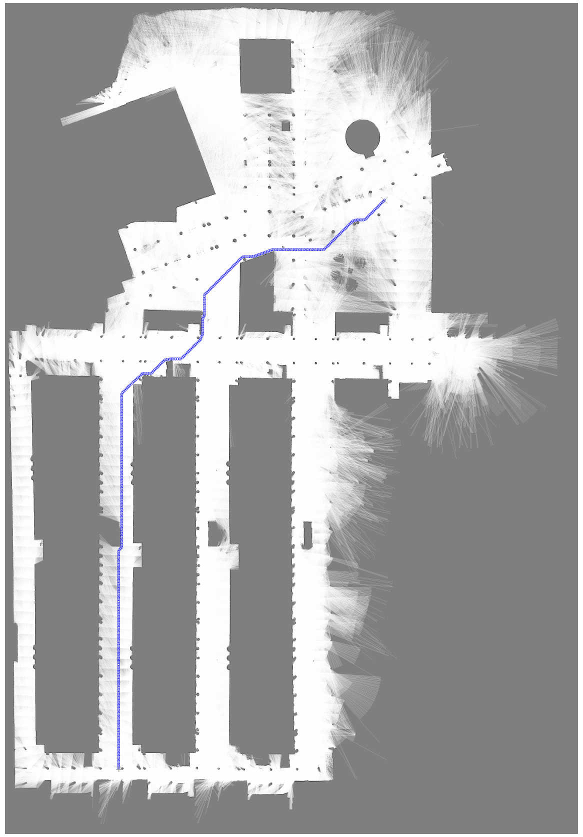
$$l(m_i|z_{1:t}, x_{1:t}) = l(m_i|z_t, x_t) + l(m_i|z_{1:t-1}, x_{1:t}) - l(m_i)$$

⇒The "quality" of information ↔ *posterior uncertainty* ↔ *entropy of the posterior*

Entropy in occupancy grid maps

$$H(M) = \sum_i H(m_i)$$

Intuitively: the uncertainty of the *entire map* is the *sum* of the uncertainties of each of its cells



Information-based exploration

Goal: Given a *current* belief $p(M)$ for the map, how do we determine where to scan next?

Simple greedy strategy: Select the scan that *maximizes the decrease in map entropy*:

$$x_k^* = argmax_{x \in SE(d)} \, H(M) - H(M|Z(x))$$

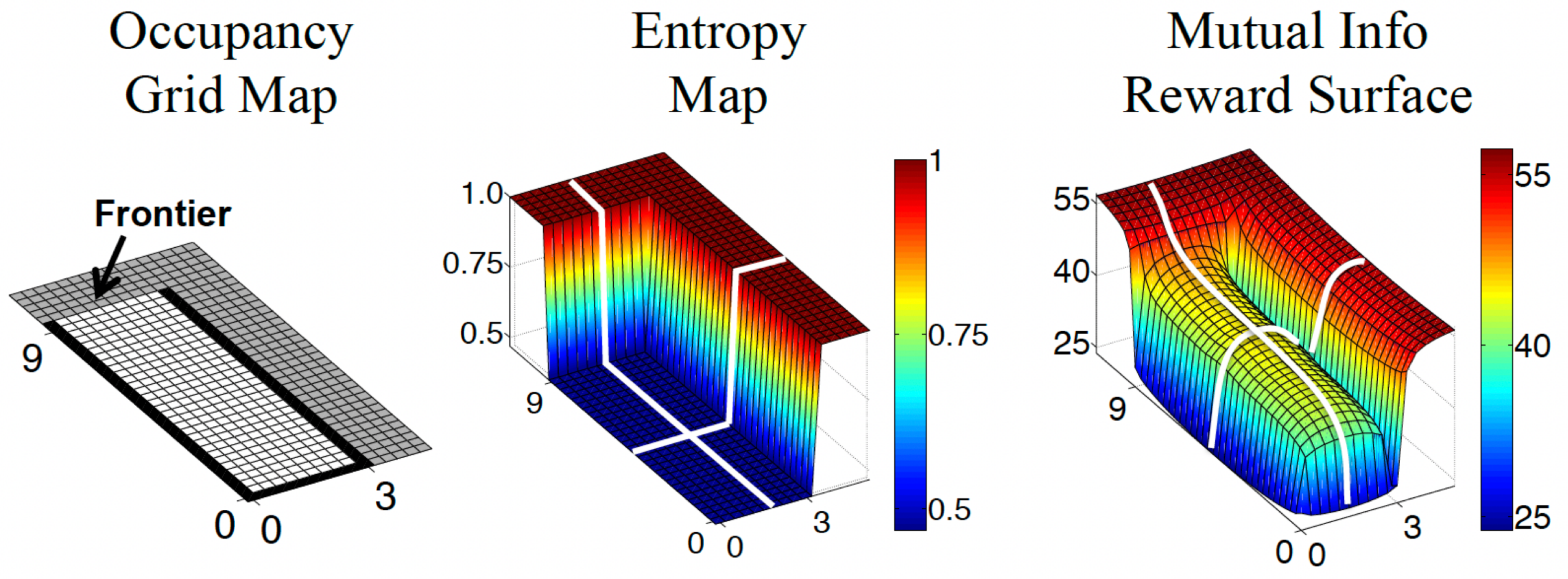
where here "$Z(x)$" means "a scan taken at pose *x*".

Equivalently: Choose the scan that *maximizes mutual information*:

$$x_k^* = argmax_{x \in SE(d)} \, I(M; Z(x))$$

Information-based exploration

$H(M)$ $\qquad$ $I(M; Z(x)) = H(M) - H(M|Z(x))$

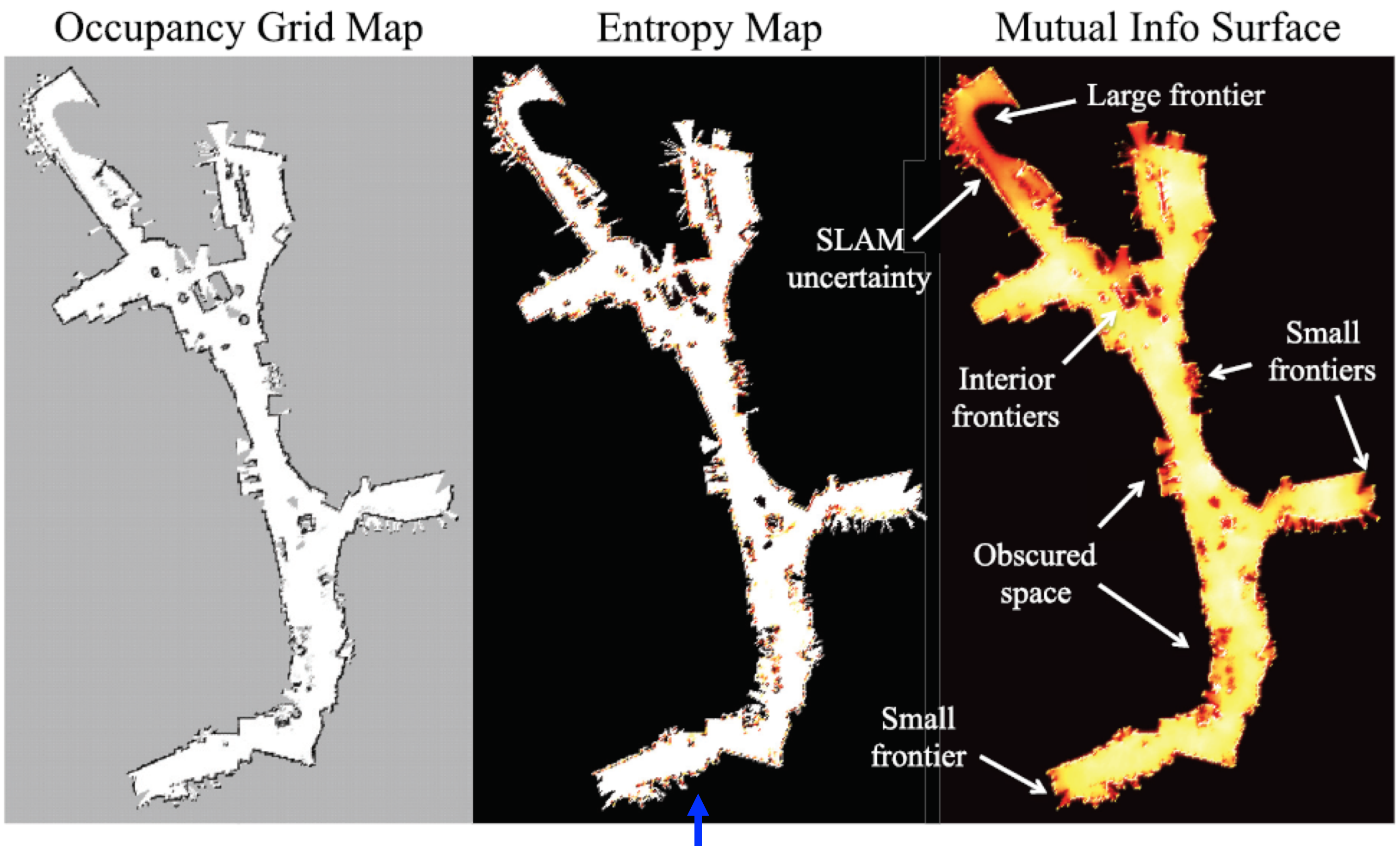


Mutual information

- predicts how much future measurements will decrease the robot's uncertainty assoc. with all cells.
- Is function of both the robot's position and the uncertainty of the surrounding cells.

B. Julian, S. Karaman, and D. Rus. "On mutual information-based control of range sensing robots for mapping applications." *The International Journal of Robotics Research* 33, no. 10 (2014): 1375-1392

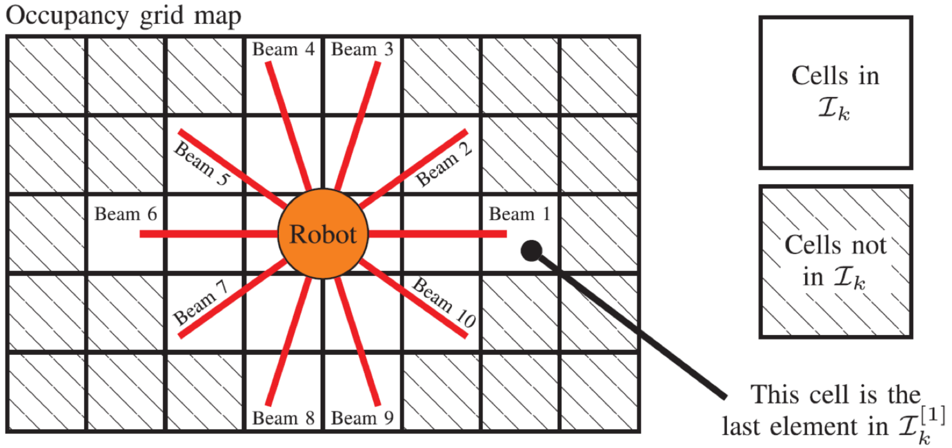
Information-based exploration



Showing cells with high uncertainty

Sensor footprints

Recall: We only update the occupancy probabilities of cells that lie in the sensor footprint



Therefore: Decrease in map entropy is equal to decrease in entropy of cells in the sensor footprint:

$$H(M) - H(M|Z(x_k)) = H\left(M_{I_k}\right) - H\left(M_{I_k}|Z(x_k)\right) = I\left(M_{I_k}; Z(x_k)\right)$$

Payoff: To calculate the value of a scan, we only need to consider the (relatively few) cells in the scan's sensor footprint

A simple information-based exploration algorithm

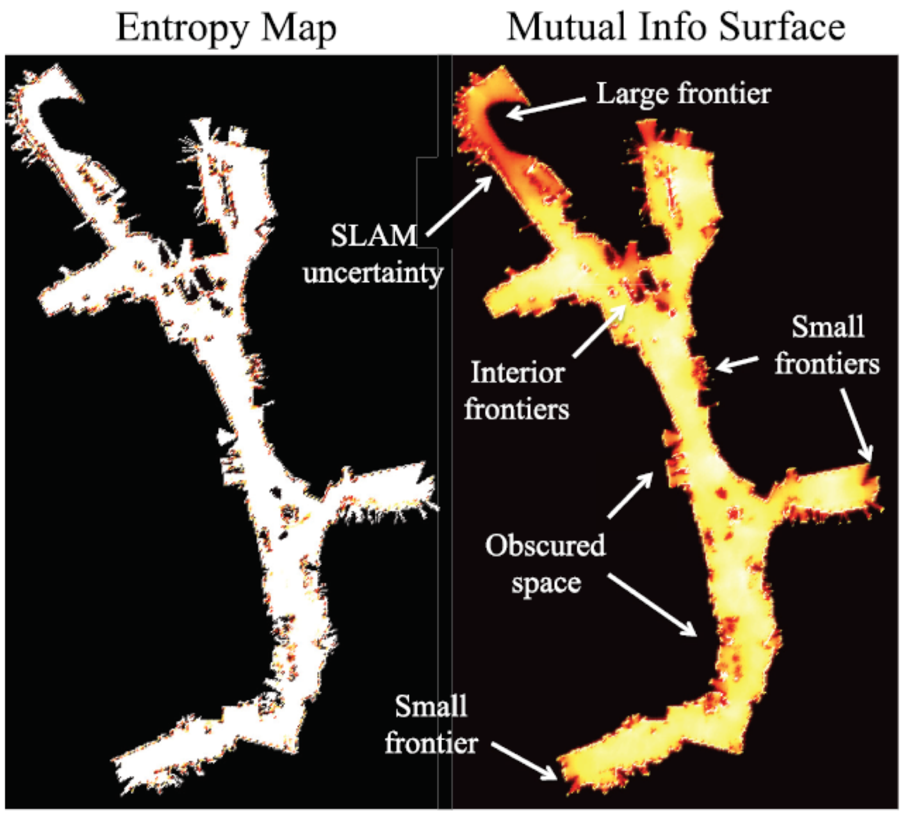
repeat

1. Calculate the next most-informative pose x_k^* at which to scan:

$$x_k^* = argmax_{x_k \in SE(d)} \; I\left(M_{I_k}; Z(x_k)\right)$$

2. Collect measurements
3. Update map

until (termination condition)



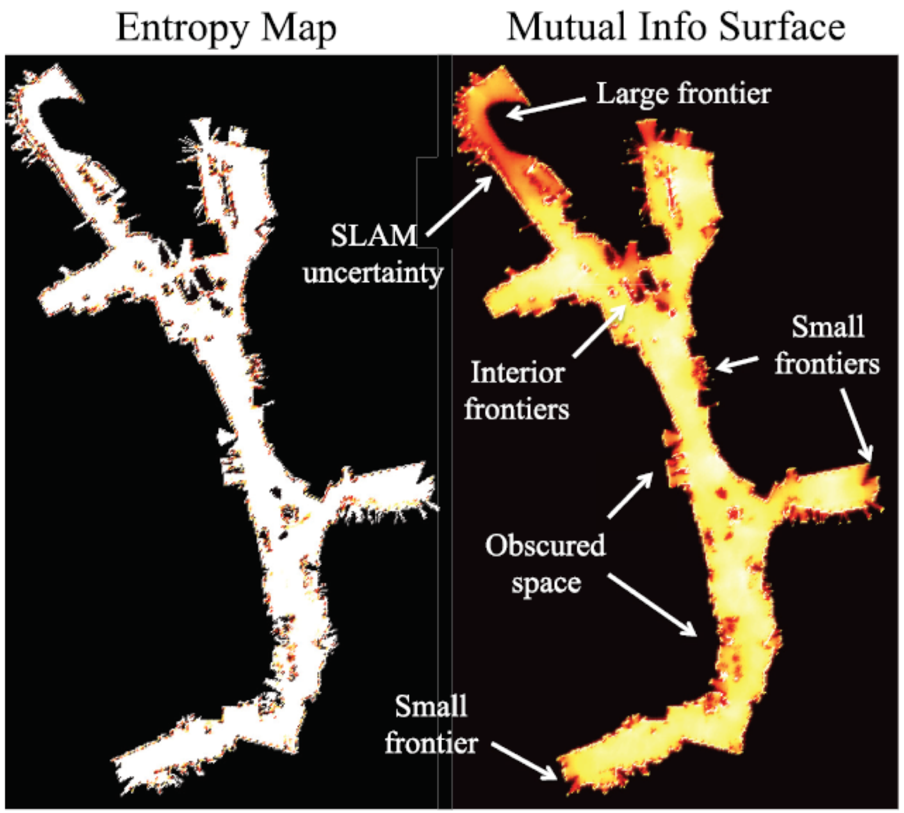
Practicalities

1. Computing the *exact* most-informative pose requires searching over *every possible* pose:

$$x_k^* = argmax_{x_k \in SE(d)} \; I\left(M_{I_k}; Z(x_k)\right)$$

This might get expensive …

2. Rather than planning for *single* measurements, we might like to design (approximately) optimal *trajectories*



MPC for information-based exploration

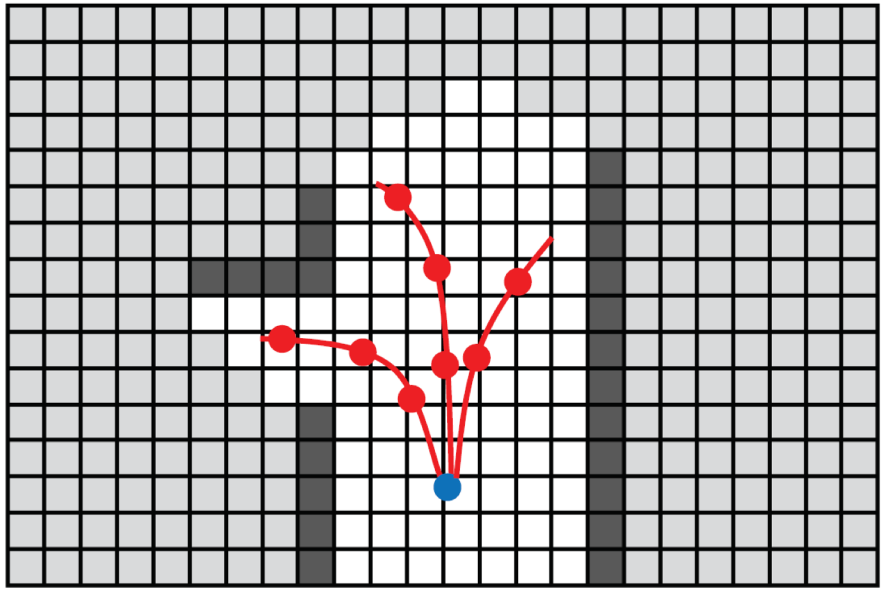
repeat

1. Calculate a (k-step, feasible) *trajectory* $x_{1:k}$ that (approximately) maximizes MI:

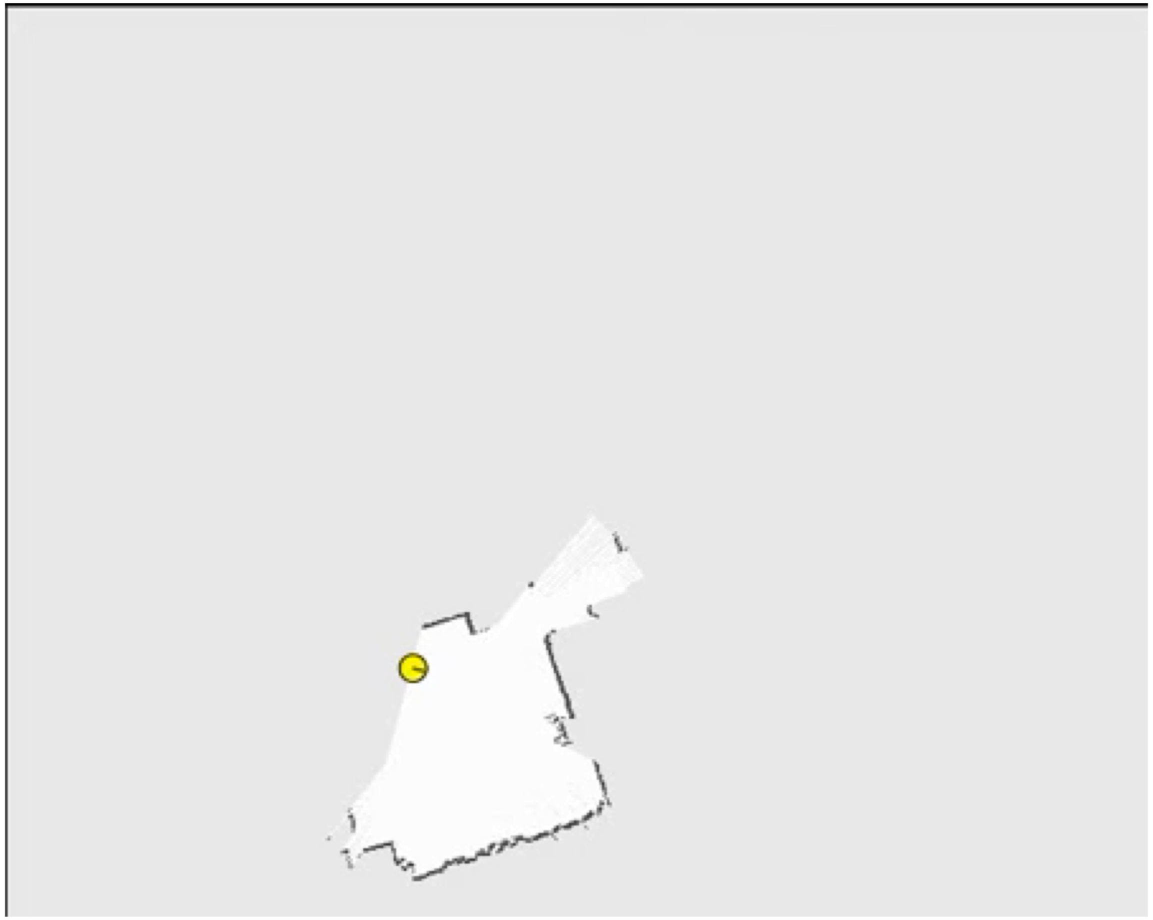
$$x_{1:k}^* = argmax_{x_i \in SE(d)} \; I(M; Z(x_{1:k}))$$

2. Execute first stage of plan
3. Update map

until (termination condition)



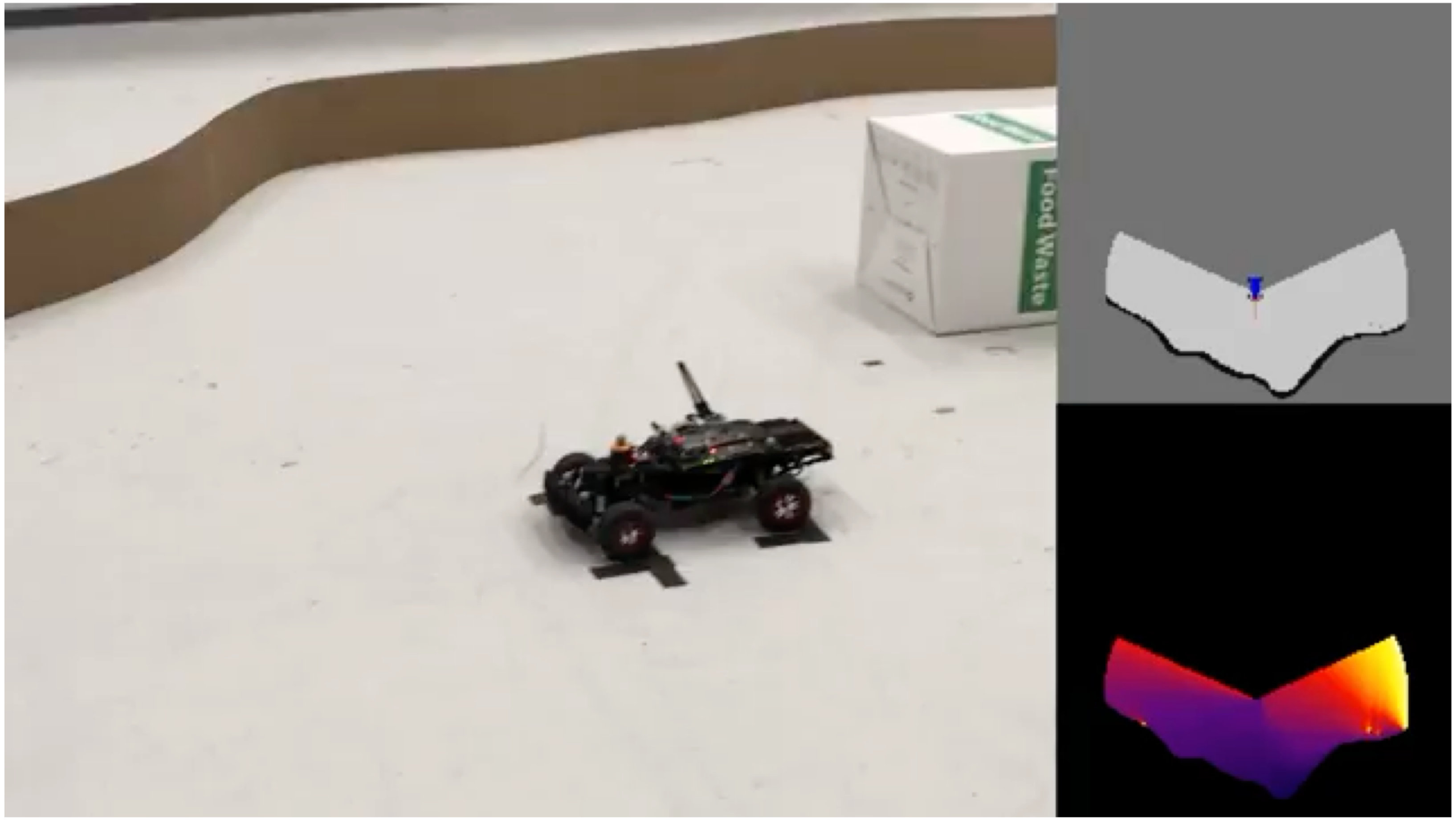
Information-based exploration



http://www.roboticsproceedings.org/rss01/p09.pdf

https://www.youtube.com/watch?v=1NuaUPnVFlk

Information-based exploration on MIT RACECAR



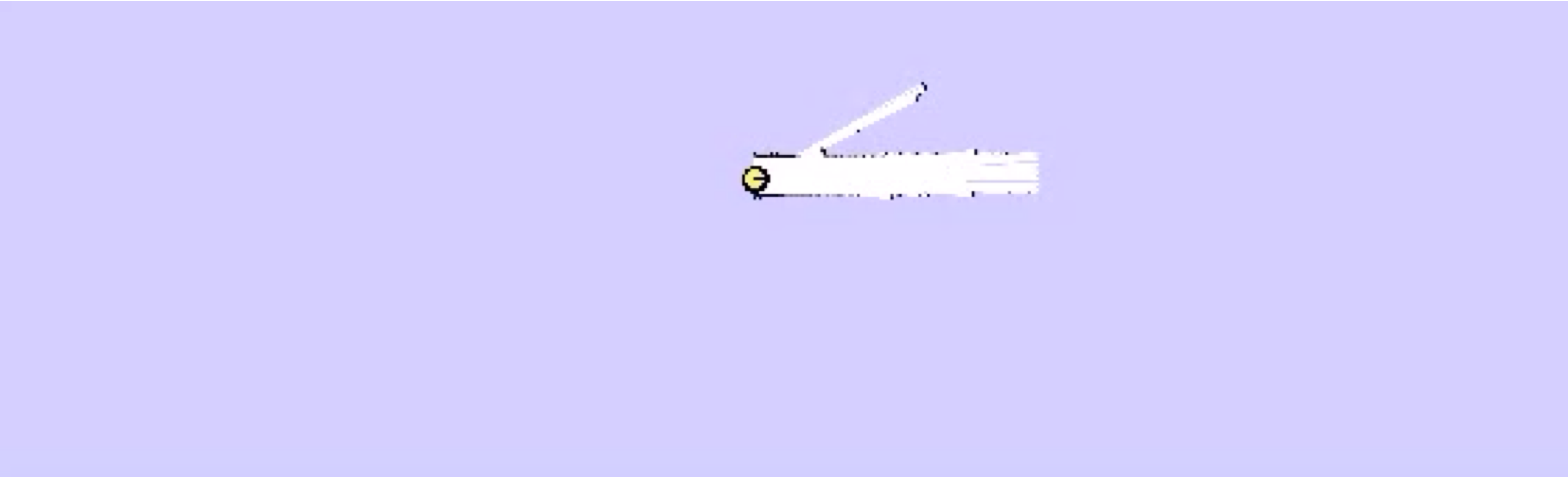
Red: a set of potential paths that the car is deciding between

Bright green: the highest mutual information reward

Mutual information surface: Bright spots have the highest mutual information.

https://youtu.be/6Ia0conjKMQ

Multi-robot exploration



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