EECE 5550
Mobile Robotics

Lecture 1: Overview

Derya Aksaray

Assistant Professor

Department of Electrical and Computer Engineering

Northeastern University

Today’s Agenda

Introductions

An overview of the course topics

Course logistics

Linear algebra review

Introductions

Instructor: Derya Aksaray (d.aksaray@northeastern.edu)

TAs:

Srinivas Peri
(peri.sr@northeastern.edu)

Rayyan Rafikh
(muhammadrafikh.r@northeastern.edu)

Students: You!

Robotics

From Britannica:

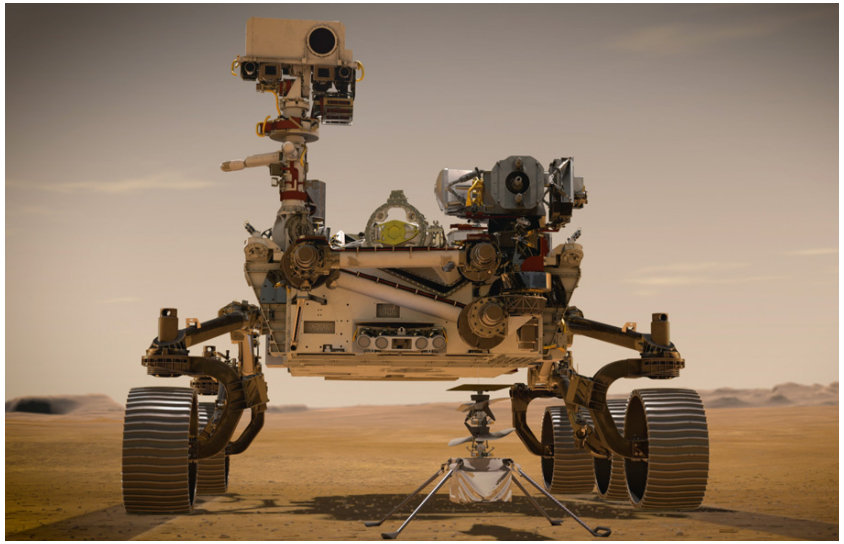
"design, construction, and use of machines (robots) to perform tasks done traditionally by human beings."

Self-Driving Vehicles

Drone Delivery

Many use areas: healthcare, food, postal deliveries…

Reconnaissance and Surveillance



Perseverance Rover and Ingenuity Mars Helicopter

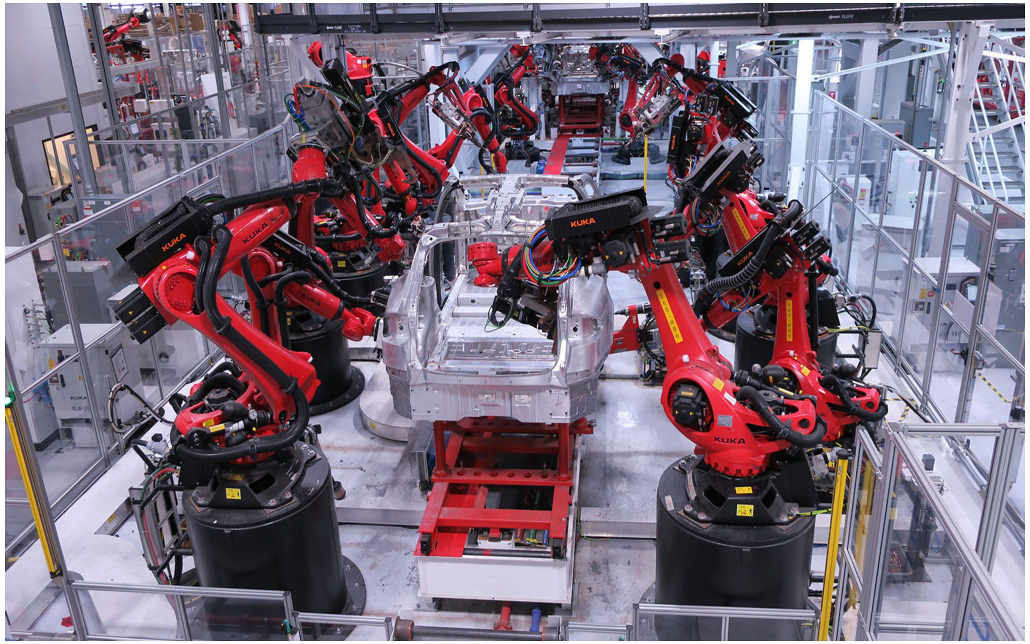
Pipe inspection robot

Infrastructure inspection drone

Factory/Warehouse Automation

Autonomous robots at Amazon

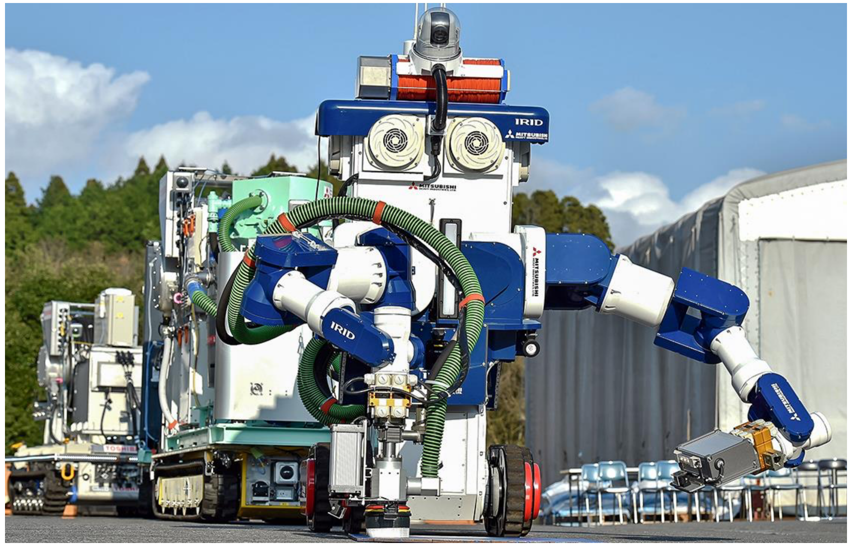
Autonomous arms at Tesla



Cleaning

Robot vacuum cleaners

Remote-controlled robots at Chernobyl, 1986



Remote-controlled robots at Fukushima Daiichi, 2011

Service Robots

Robot barista

Cooking robot

Robot lawn mower

Robots in Healthcare

Da Vinci Robot-Assisted Surgery

Robotic wheelchair

Robotics

From Britannica:

"design, construction, and use of machines (robots) to perform tasks done traditionally by human beings."

Robotics is an interdisciplinary branch of engineering and computer science.

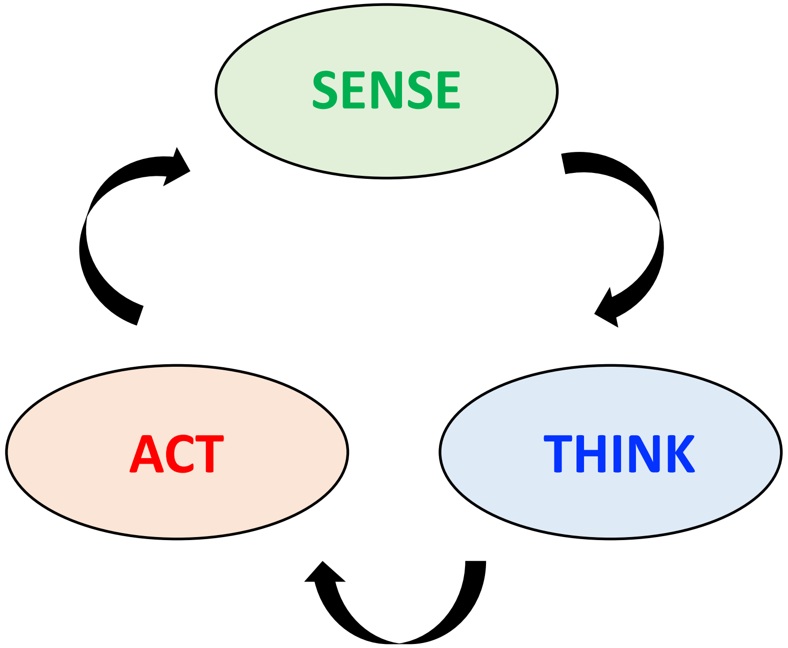
- Grounded in *real-world physics*: mechanics, electromagnetism, optics, etc. …
- Entails fundamental problems in *artificial intelligence*: representation (of the world), planning (how to achieve the goal), and control (to execute the plan).

The Central Dogma of Robotics

Sense: Process sensor data to construct a model of the world

Think: Construct a plan to move from the current state to the goal state.

Act: Control actuators to execute the plan



3 + 1 Pillars of (Robotics) Wisdom

Perception

Build a model of the world from sensor data

Planning

Use the world model to plan a course to goal

Control

Drive actuators to execute the plan

Mathematical Foundations

Geometry, probability & statistics, linear algebra, optimization

Objectives of this course

Prepare you for state-of-the-art work in robotics by teaching:

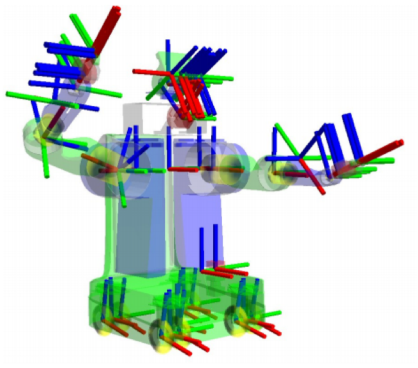
Theoretical foundations

- **Mathematics**: Geometry, probability & statistics, optimization, numerical linear algebra
- **Algorithms**: Foundations of perception, planning, and control

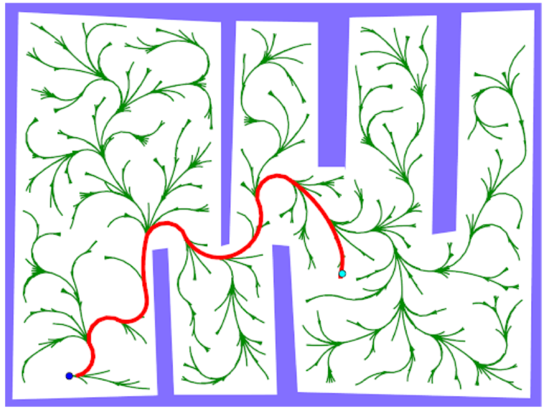
Practical skills

- **Tools of the trade:** Linux, Git, Robot Operating System (ROS)
- **Practical experience** building and deploying autonomous systems in the real world

Intended Learning Outcomes



1. Apply models of robot kinematics and sensing to mathematically model and simulate robotic systems.
2. Formalize robotic perception and estimation tasks using probabilistic graphical models, and devise corresponding inference algorithms to solve them.
3. Implement navigation and motion planning algorithms.
4. Develop new algorithms to address technical challenges in mobile robotics.



Expected Background

You should have a very strong foundation in:

- Linear algebra: Linear transformations, bases, eigenvalues, matrix factorizations
- Multivariable calculus: first- and second-order differentiation of multivariate functions, Taylor series
- Probability and statistics: probability density / mass functions, Bayes' Rule, independence and conditional independence, basic operations on distributions

Some prior programming experience (e.g. Python, MATLAB) would be helpful.

Course Outline

Lectures

- WF 11:45 am – 1:25 pm
- Snell Library, Room 031

Instructor office hours

- Friday 2:30-3:30 pm, EXP 710B

TA office hours (TBD)

Week	Date	Topic	Assignment
1	Sep 4	Course intro & linear algebra refresher Coordinate systems and geometry, Intro to Manifolds	
2	Sep 11	Robot modeling and sensing Fundamentals of computer vision	Assignment 1
3	Sep 18	Math foundations: Review of Probability Probabilistic Robotics	
4	Sep 25	Bayes filter implementations Mapping	Assignment 2
5	Oct 2	Localization	
6	Oct 9	Simultaneous localization and mapping (SLAM) Math foundations: Optimization	Assignment 3
7	Oct 16	Introduction to planning Robot motion planning	
8	Oct 23	Planning under uncertainty MDPs, POMDPs	Assignment 4
9	Oct 30	Robotic exploration Feedback control I	
10	Nov 6	Feedback control II Stability analysis	Assignment 5
11	Nov 13	Optimal control and model predictive control Advanced topics and recent trends	
12	Nov 20	Reading Week	
13	Nov 27	Thanksgiving	
14	Dec 4	Project presentations	

Course Outline

Assignments

- Roughly biweekly (5 total)
- Mix of theory and practical implementation (we will provide robotic hardware)
- Late policy

Reading Week

- Team-based (~4 students) paper presentation from the literature

Final project

- Team-based (~4 students)
- Focus on a challenging problem in mobile robotics, we can provide robotic hardware…
- **Deliverables**: Presentation, demo video, project report
- Stay tuned for details …

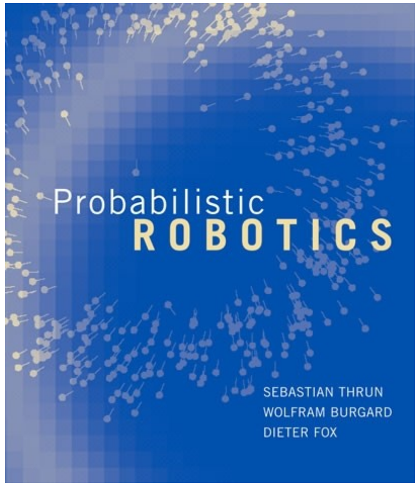
There will be no course exams

Grading

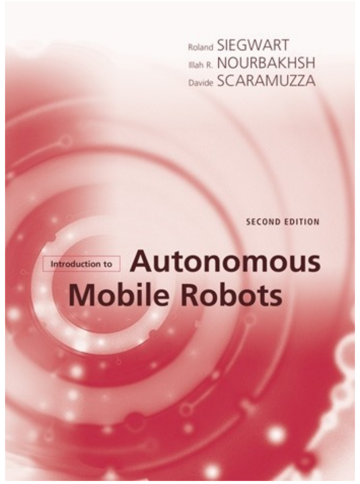
- 65% Assignments + 5% Reading week + 30% Final Project
- Letter assignments based on standard 90-80-70 thresholding

Reference Books

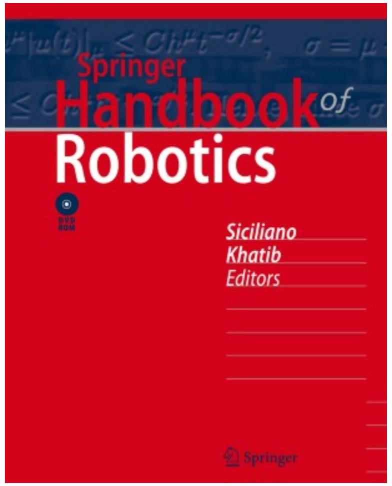
No required textbooks, some books are recommended and listed in the syllabus



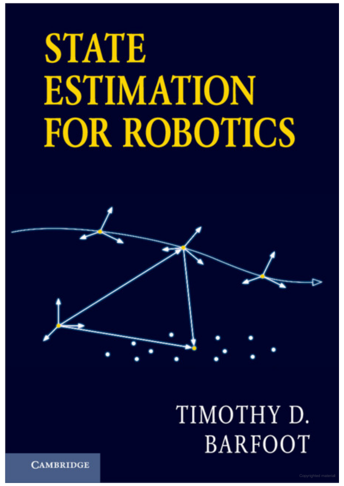
Probabilistic Robotics



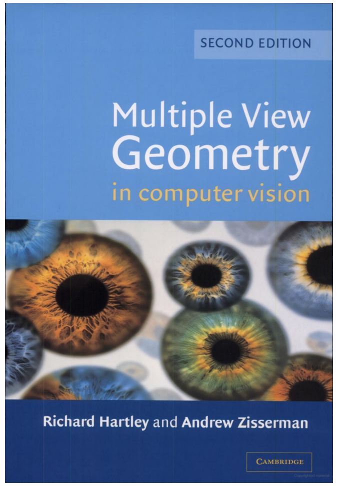
Autonomous Mobile Robots



Handbook of Robotics



State Estimation for Robotics



Multiple View Geometry in CV

Digital versions of all of these are all (legally) available gratis – see the links in the course syllabus.

Computing Resources

You will need access to a computer with:

- A scientific computing environment (e.g. Python + Jupyter notebook, MATLAB)
- The Robot Operating System (ROS)

Our "officially-supported" computing environment is:

- Ubuntu 20.04 LTS operating system
- Python3 w/ NumPy, SciPy libraries + Jupyter notebook
- ROS installed via Ubuntu's Advanced Packaging Tool (apt-get)

=> TA's will be able to help you set these up if you've not used them previously

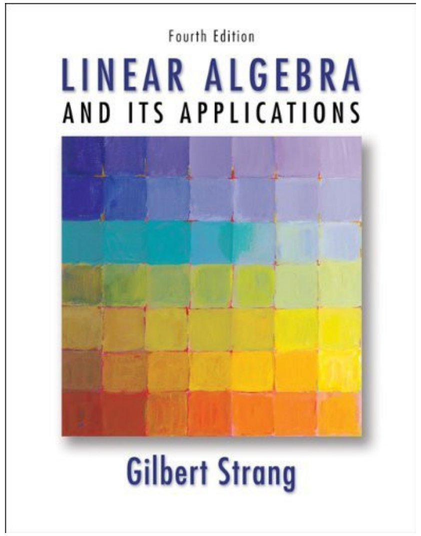
Academic Integrity

- I encourage you to discuss the course and assignments with your colleagues!
    - This is one of the best ways to both sharpen and expand your own thinking

- However, we have the following course rules:
    - If you collaborate on any part of the homework, **you must declare** who you worked with and what was discussed. This collaboration statement must appear at the very beginning of your submission.
    - Any work you submit must reflect your own understanding (i.e., you must be able to fully explain anything that you submit).

- Violating any of these rules will lead to penalties that could range from a zero on the assignment to an F in the course to a formal report with the Dean.

Linear Algebra Review



A series of recorded lectures from Gilbert Strang: https://ocw.mit.edu/courses/18-06-linear-algebra-spring-2010/

Basics

Linear algebra: Area of mathematics focusing on representing/solving linear equations/transformations.

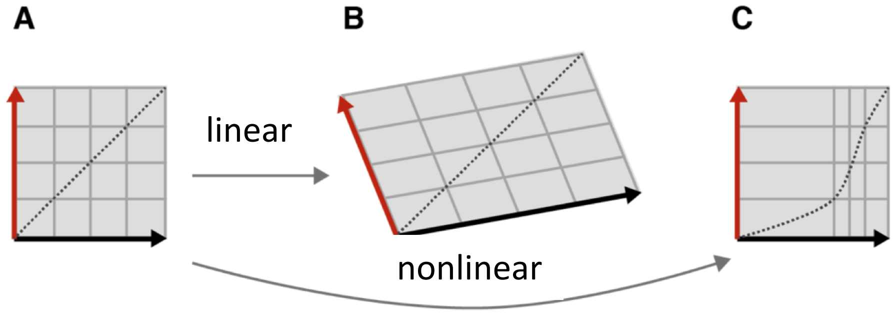
Linear equation: $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$

- **Variables:** $x_1, x_2, \dots, x_n \in \mathbb{R}$ (unknown),
- **Coefficients:** $a_1, a_2, \dots, a_n \in \mathbb{R}$ (known)
- **Constant:** $b \in \mathbb{R}$ (known)

A set of multiple equations is called *a system of linear equations.*

Linear transformation: $T: V \to W$

- **Vector spaces:** V, W
- **Linear map:** T



*preserves correlation between variables?

Vector (Linear) Spaces

A *vector space* over a field F is a set V together with two binary operations $+ : V \times V \to V$ ("vector addition") and $\cdot : F \times V \to V$ ("scalar multiplication") that satisfy the following **axioms** for all $u, v, w \in V$ and $a, b \in F$:

1. Associativity of addition: $u + (v + w) = (u + v) + w$
2. Commutativity of addition: $u + v = v + u$
3. Existence of additive identity: There exists an element of V, called "0", such that $0 + v = v$ for all $v \in V$.
4. Existence of additive inverses: For every $v \in V$, there is an *additive inverse* $-v \in V$ of v satisfying $v + (-v) = 0$.
5. Compatibility of scalar multiplication: $a(bv) = (ab)v$
6. Identity element of scalar multiplication: $1v = v$, where "1" is the multiplicative identity element in F.
7. Distributivity of scalar multiplication with respect to addition: $a(u + v) = au + av$
8. Distributivity of scalar multiplication of with respect to field addition: $(a + b)v = av + bv$.

Punchline: Vector (Linear) spaces "look like" generalizations of $\mathbb{R}^n$.

Some examples

- Euclidian space ($\mathbb{R}^n$)

E.g., $\mathbb{R}^2$ is the set of all pairs $(x, y) \in \mathbb{R}$

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$c(x, y) = (cx, cy)$$

- Polynomial vector spaces

E.g., $P_2(\mathbb{R})$ is the set of polynomials of degree 2 or less, i.e., $a_0 + a_1x + a_2x^2$, where $a_0, a_1, a_2 \in \mathbb{R}$.

$$(a_0 + a_1x + a_2x^2) + (b_0 + b_1x + b_2x^2) = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2$$
$$c(a_0 + a_1x + a_2x^2) = ca_0 + (ca_1)x + (ca_2)x^2$$

- Matrix vector space

E.g., $M_{2\times 2}(\mathbb{R})$ is the set of all 2×2 matrices with entries from $\mathbb{R}$ (e.g., A and B matrices)

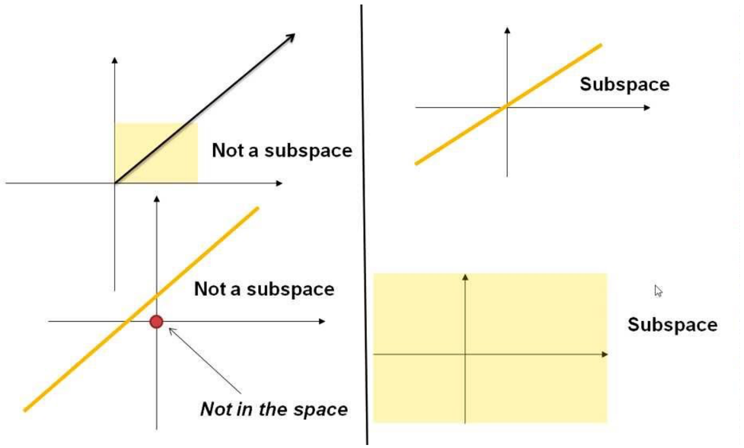
$$(A + B)_{ij} = A_{ij} + B_{ij}$$
$$(cA)_{ij} = cA_{ij}$$

Subspaces, Linear Combinations, Linear Spans

- A subset $W \subseteq V$ of a vector space V that is closed under vector addition and scalar multiplication is called a *(vector/linear) subspace*.
- Given a set of vectors $v_1, \ldots, v_n \in V$, a *linear combination* of these vectors is an element of the form:

$$c_1 v_1 + \cdots c_n v_n \in V$$

  for some choice of scalars $c_1, \ldots, c_n \in F$.



- Given a set of vectors $v_1, \ldots, v_n \in V$ their *linear span* is the set of *all* linear combinations they generate:

$$span(v_1, \ldots, v_n) \triangleq \{c_1 v_1 + \cdots + c_n v_n \mid c_i \in F\}$$

This is always a subspace of V (why?)

Linear Independence

A set of vectors $v_1, \ldots v_n \in V$ is called *linearly independent* if their linear combination is zero *only when* all of their coefficients are zero:

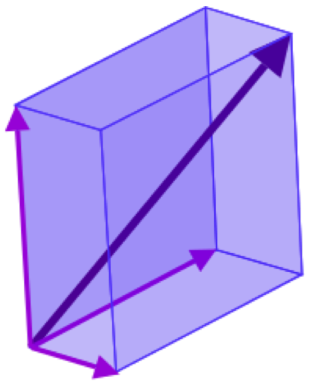
$$c_1 v_1 + \cdots + c_n v_n = 0 \quad \Leftrightarrow \quad c_1 = \cdots = c_n = 0.$$

Significance: If $v_1, \ldots, v_n \in V$ are linearly independent and $y \in span\{v_1, \ldots v_n\}$, then there is a *unique* set of scalars $c_1, \ldots, c_n \in F$ such that:

$$y = c_1 v_1 + \cdots + c_n v_n.$$

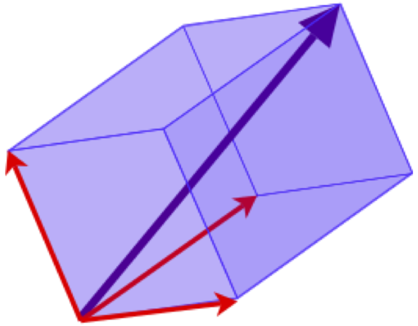
Bases

A linearly independent set $v_1, \dots, v_n \in V$ such that $V = span\{v_1, \dots, v_n\}$ is called a *basis* for *V*. The *dimension* of *V* is the cardinality of its bases.



The elements $(c_1, \dots c_n)$ are called the *coordinates* of *y* with respect to the basis $v_1, \dots, v_n$.

$$y = c_1 v_1 + \cdots + c_n v_n.$$



Ex: Consider the vector space $\mathbb{R}^n$ for n=2. What are its bases?

Linear Transformations

A *linear transformation* $T: V \to W$ between vector spaces V and W is a map that satisfies:

$$T(ax + by) = aT(x) + bT(y)$$

for all $x, y \in V$ and $a, b \in F$.

Let V and W be linear spaces and $T: V \to W$ a linear map, and fix bases $v_1, \dots, v_n$ for V and $w_1, \dots, w_m$ for W, respectively.

Let $x \in V$. Then there exist unique scalars $c_1, \dots, c_n$ such that $x = c_1 v_1 + \cdots + c_n v_n$.

$$T(x) = T(c_1 v_1 + \cdots + c_n v_n) = c_1 T(v_1) + \cdots + c_n T(v_n).$$

Notice: This shows that T is *completely determined* by where it sends the basis elements $v_1, \dots, v_n$.

Matrix representations of Linear Transformations

$$T(x) = T(c_1 v_1 + \cdots + c_n v_n) = c_1 T(v_1) + \cdots + c_n T(v_n)$$

$$T: V \to W$$

Since $T(v_j) \in W$ for each j, there are m unique corresponding scalars $a_{1j}, \ldots a_{mj}$ such that:

$$T(v_j) = a_{1j} w_1 + \cdots + a_{mj} w_m.$$

Substituting this into the previous equation for $T(x)$, we obtain:

$$T(x) = \sum_{j=1}^{n} c_j \left(\sum_{i=1}^{m} a_{ij} w_i \right) = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} a_{ij}\, c_j \right) w_i$$

This means that the ith coordinate of $T(x)$ with respect to the basis $w_1, \ldots, w_m$ is $y_i \triangleq \sum_{j=1}^{n} a_{ij}\, c_j$.

Matrix representations of Linear Transformations

Now suppose that we organize the scalars $\{a_{ij}\}$ in the above equation into a *matrix*:

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \in \mathbb{R}^{m \times n}$$

Then identifying *x* with its coordinate vector $c = (c_1, \dots, c_n)$, and *T*(*x*) with its coordinate vector $y = (y_1, \dots, y_m)$, we have that:

$$y = Ac$$

- The matrix *A* is a representation of the linear map $T: V \to W$ *in the coordinate systems determined by the bases* $v_1, \dots, v_n$ for *V* and $w_1, \dots, w_m$ for *W*.

A matrix is just a means of *representing* $T: V \to W$ given *specific choices* of bases for *V* and *W.*

Singular Value Decomposition

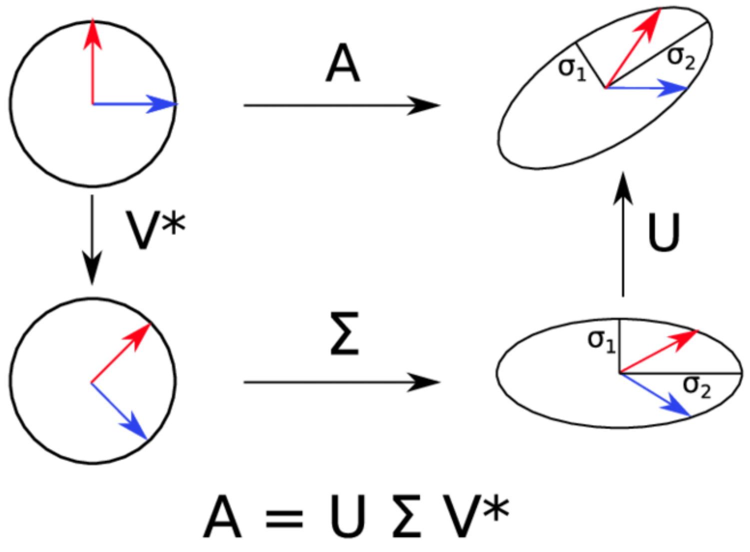
Let $A \in \mathbb{R}^{m \times n}$, and $k = \min(m, n)$. Then there exist orthogonal matrices $U \in O(m)$ and $V \in O(n)$, and scalars $\sigma_1, \ldots, \sigma_k \geq 0$ such that:

$$A = U\Sigma V^T,$$

where:

$$\Sigma = Diag(\sigma_1, \ldots \sigma_k) \in \mathbb{R}^{m \times n}$$

The scalars $\sigma_1, \ldots, \sigma_k$ are called *singular values*, and the factorization of A is called a *singular value decomposition*.



Geometric interpretation: The columns of U and V provide orthonormal bases for $\mathbb{R}^m$ and $\mathbb{R}^n$ (respectively) such that, when written with respect to these bases, the linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ represented by A takes an especially simple form:

$$T(v_i) = \sigma_i u_i \quad \forall 1 \leq i \leq k$$

Symmetric Eigendecomposition

Given a symmetric matrix $A \in S^{n \times n}$, there exists an orthogonal matrix U and scalars $\lambda_1, \dots, \lambda_n \in \mathbb{R}$ such that:

$$A = U \Lambda U^T,$$

where:

$$\Lambda = Diag(\lambda_1, \dots \lambda_n) \in \mathbb{R}^{n \times n}.$$

The scalars $\lambda_1, \dots, \lambda_n$ are the *eigenvalues* of *A*, the columns of *U* are the corresponding *eigenvectors*, and the factorization of *A* is called a *symmetric eigendecomposition*.

Geometric interpretation: Similarly to the SVD, the columns of *U* provide an orthonormal basis for $\mathbb{R}^n$ such that, when written with respect to this basis, the linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ represented by *A* takes an especially simple form:

$$T(u_i) = Au_i = \lambda_i u_i \quad \forall 1 \leq i \leq n$$

T simply acts by *scaling* each eigenvector u_i by its corresponding eigenvalue λ_i.

System of Linear Equations

Multiple equations: m equations, n variables

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ \underbrace{a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n}_{n \text{ variables}} &= b_m \end{aligned}\right\} m \text{ equations}$$

Solution: $x_1, x_2, \ldots, x_n$ that satisfy all the equations.

Number of Solutions

Does the system have a solution? Is there a unique solution?

For any system of linear equations, there are only three possibilities:

1) The system has infinitely many solutions.
2) The system has a unique solution.
3) The system has no solution.

Solving Linear Equations

We can multiply both sides of an equation with the same scalar $s \in \mathbb{R}$.

$$a_{11}x_1 + a_{12}x_2 = b_1 \qquad\qquad s(a_{11}x_1 + a_{12}x_2) = sb_1$$

We can add/subtract linear equations

$$a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \qquad\qquad a_{21}x_1 + \cdots + a_{2n}x_n = b_2$$

$$(a_{11} + a_{21})x_1 + \cdots + (a_{1n} + a_{2n})x_n = b_1 + b_2$$

If the system has a unique solution, we can find it via these operations (elimination).

Systems without a Unique Solution

No solution: The equations are inconsistent.

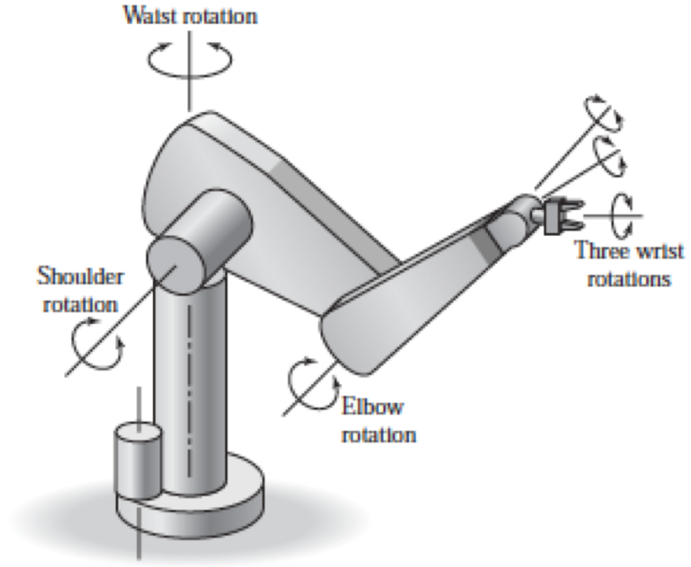
- Example: $x_1 + x_2 = 3, -x_1 - x_2 = -2$

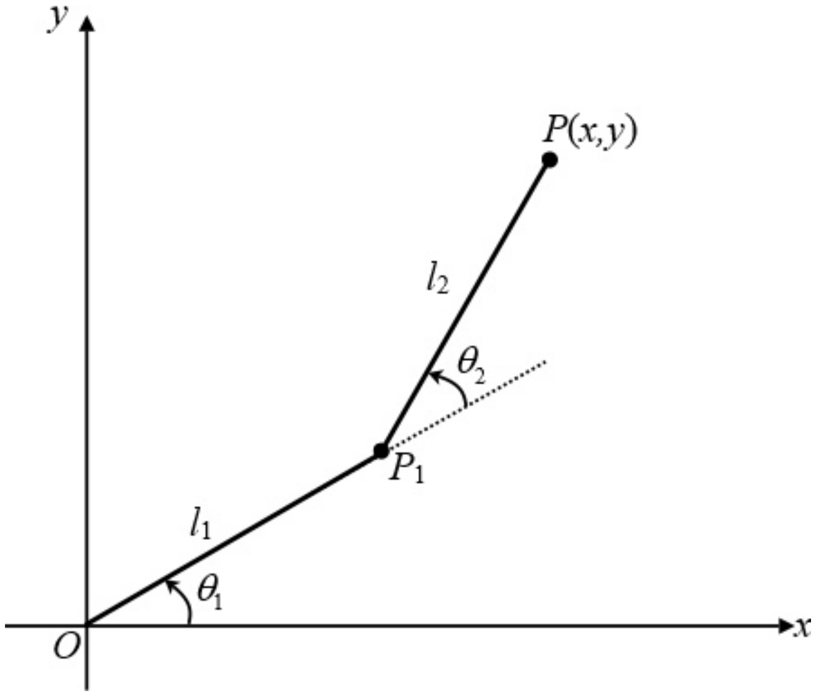
Infinitely many solutions: The equations are consistent, but they do not express a sufficient amount of independent information.

- Example 1: fewer equations (consistent) than unknowns: $x_1 + x_2 = 3$
- Example 2: $x_1 + x_2 = 3, 2x_1 + \; 2x_2 = 6$

Example 1

Given that $\theta_1 = \theta_2 = \frac{\pi}{6}\ rad$ results in $x = y = \frac{1+\sqrt{3}}{2}\ m$, find l_1 and l_2.





Matrix representation of linear equations

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

- **Solution:** $x_1, x_2, \ldots, x_n$ that satisfy all of the equations, i.e.,

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} \qquad \boldsymbol{Ax} = \boldsymbol{b}$$

Matrix representation of linear equations

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

- **Solution:** $x_1, x_2, \ldots, x_n$ that satisfy all of the equations, i.e.,

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

$$\boldsymbol{Ax} = \boldsymbol{b}$$

Number of solutions depends on $\boldsymbol{A}$ and $\boldsymbol{b}$.

Fundamental Subspaces

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} \boldsymbol{r_1^T} \\ \vdots \\ \boldsymbol{r_m^T} \end{bmatrix} = [\boldsymbol{c_1} \quad \cdots \quad \boldsymbol{c_n}]$$

Column space (Range/Image): $col(\boldsymbol{A}) = range(A) = span(\boldsymbol{c_1}, \dots, \boldsymbol{c_n})$

Row space: $row(\boldsymbol{A}) = span(\boldsymbol{r_1}, \dots, \boldsymbol{r_m})$

Null space (Kernel): $null(\boldsymbol{A}) = \ker(A) = \{\boldsymbol{v} \in \mathbb{R}^n | \boldsymbol{Av} = \boldsymbol{0}\}$

Existence and Uniqueness of Solution

$$\boldsymbol{Ax} = \boldsymbol{b} \qquad\qquad \boldsymbol{A} = [\boldsymbol{c_1} \quad \cdots \quad \boldsymbol{c_n}]$$

There exists a solution $\boldsymbol{x} \Leftrightarrow \boldsymbol{b}$ is in the column space of $\boldsymbol{A}$.

$$col(\boldsymbol{A}) = span(\boldsymbol{c_1}, \dots, \boldsymbol{c_n})$$

A unique solution $\boldsymbol{x}$ for any $\boldsymbol{b} \in col(\boldsymbol{A}) \Leftrightarrow \boldsymbol{c_1}, \dots, \boldsymbol{c_n}$ are lin. independent.

- For any square matrix $\boldsymbol{A} \in \mathbb{R}^{n \times n}$, this is equivalent to $\boldsymbol{A}$ being invertible (full rank).

Matrix Rank and Nullity

Dimension: Min. number of linearly independent vectors that span the space.
Rank: Dimension of the column space/row space.

$$\text{rank}(\boldsymbol{A}) = \dim(col(\boldsymbol{A})) = \dim(row(\boldsymbol{A}))$$

For any $\boldsymbol{A} \in \mathbb{R}^{m \times n}$

- $\text{rank}(\boldsymbol{A}) = \text{rank}(\boldsymbol{A}^T)$
- $\text{rank}(\boldsymbol{A}) \leq \min(m, n)$, $\boldsymbol{A}$ has full rank if $\text{rank}(\boldsymbol{A}) = \min(m, n)$

Nullity: Dimension of the null space/kernel.

$$nullity(\boldsymbol{A}) = \dim(null(\boldsymbol{A}))$$