

EECE 5550 Mobile Robotics

Lecture 4: Basic Models of Robot Motion and Sensing

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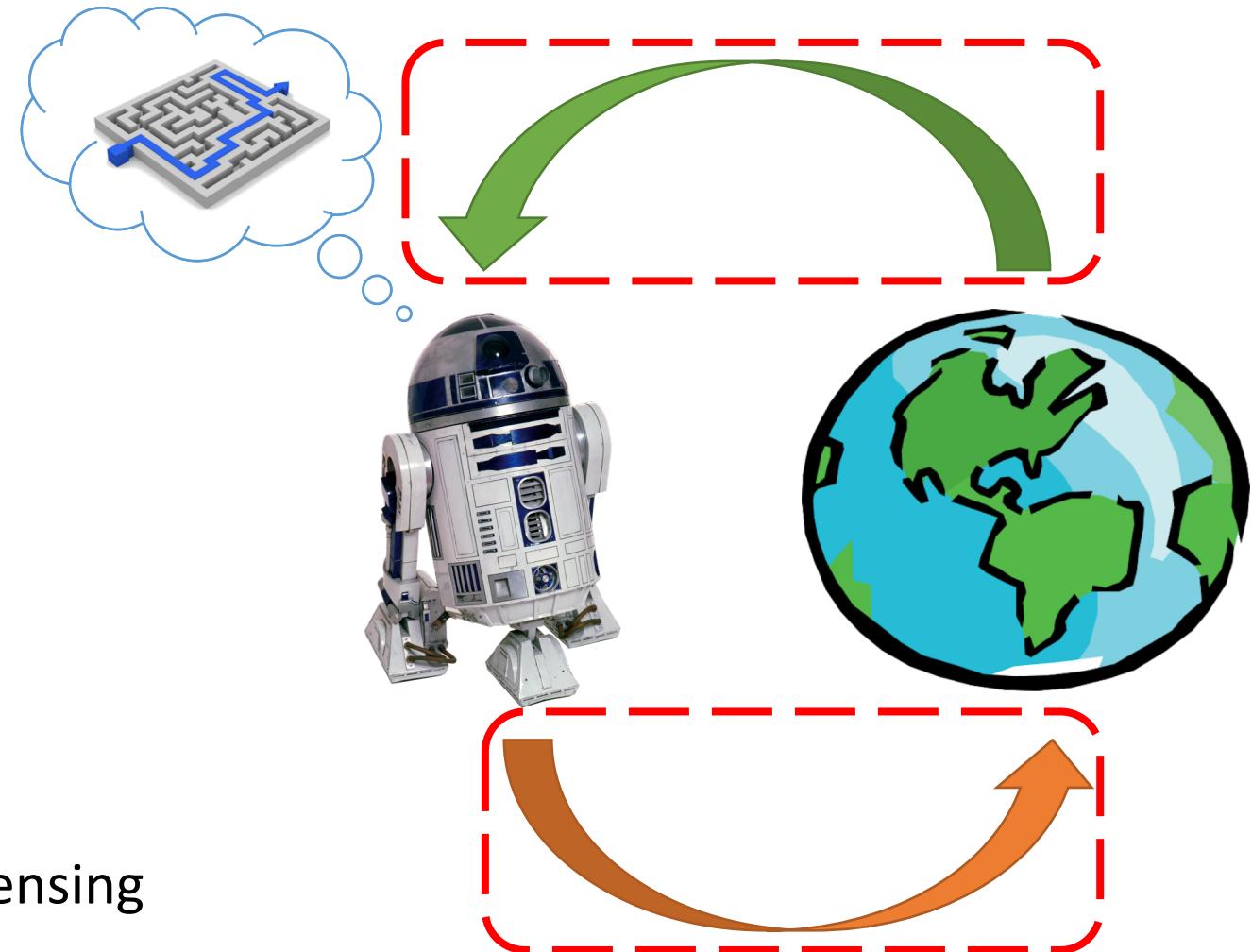
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University

Recap: The Central Dogma of Robotics

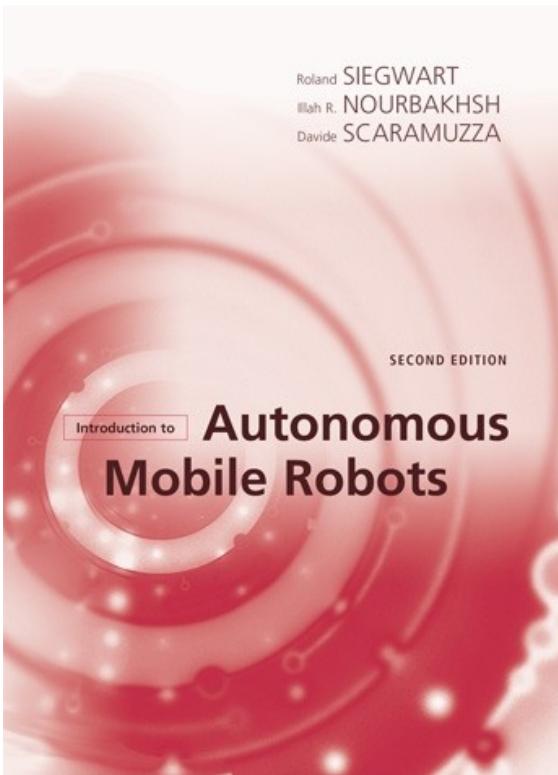
Sense → Think → Act

- Sense: Process **sensor** data to construct a model of the world
- Think: Construct a **plan** to move from the current state to the goal state
- Act: Control **actuators** to execute plan

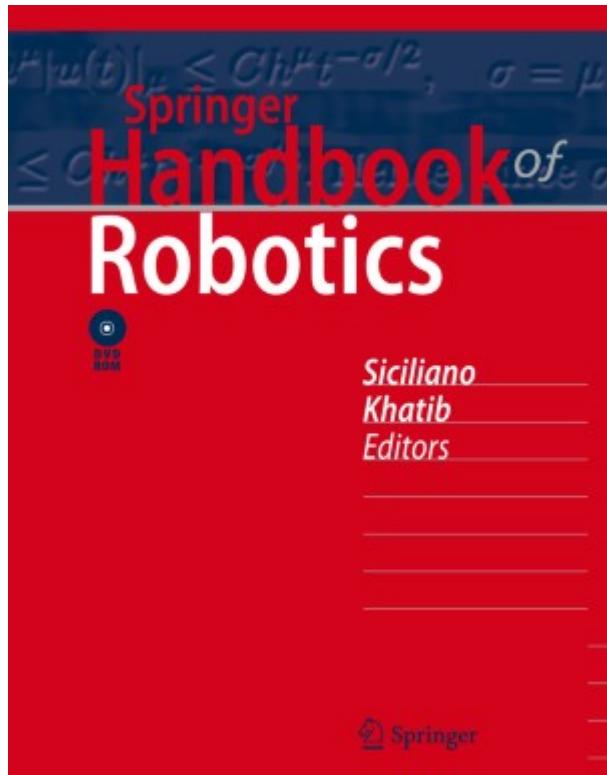
Today: Basic models of robot motion and sensing



References



Autonomous Mobile Robots



Handbook of Robotics

- Secs. 3.2 and 4.1 of *Autonomous Mobile Robots*
- Chps. 17 & 22 of *Handbook of Robotics*

Modeling

Mathematical: Describing important system characteristics via equations.

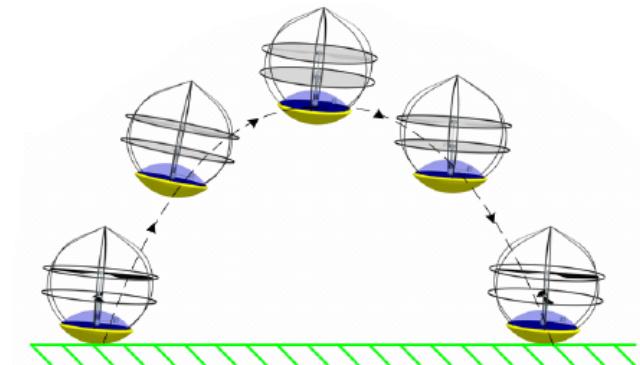
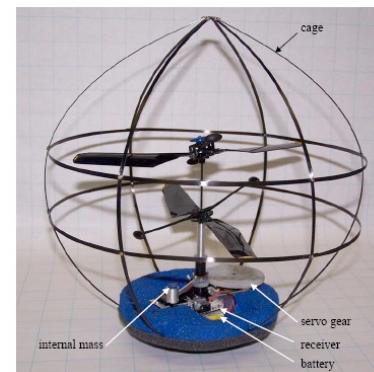
- e.g., applying laws of physics to model a physical system

Models may involve unknown parameters (estimated via experiments).

When well-established laws are not available, empirical data is used.

- Input-output relationships based on data

Hopping rotochute



Simplicity vs. Accuracy

Models with higher accuracy are usually more complicated to use/analyze.

- a compromise should be made between simplicity and accuracy.
- which aspects are negligible and which are essential for the task at hand?

All models are flawed, but some are very useful.



Robot motion

- Robots with rigid components under **rotational** and/or **translational** motion.
- Position of any point on each body in its attached coordinate frame is fixed.

Rigid

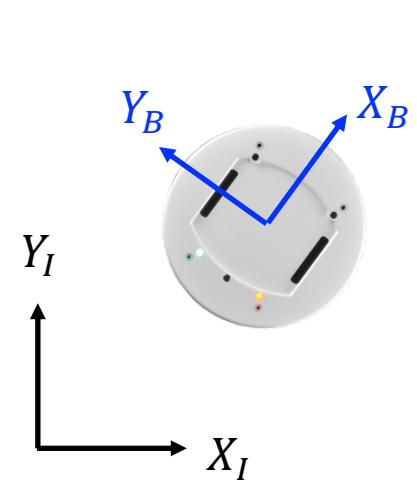
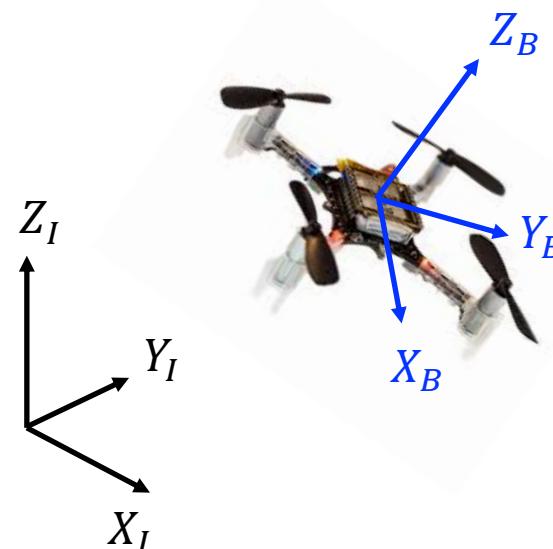
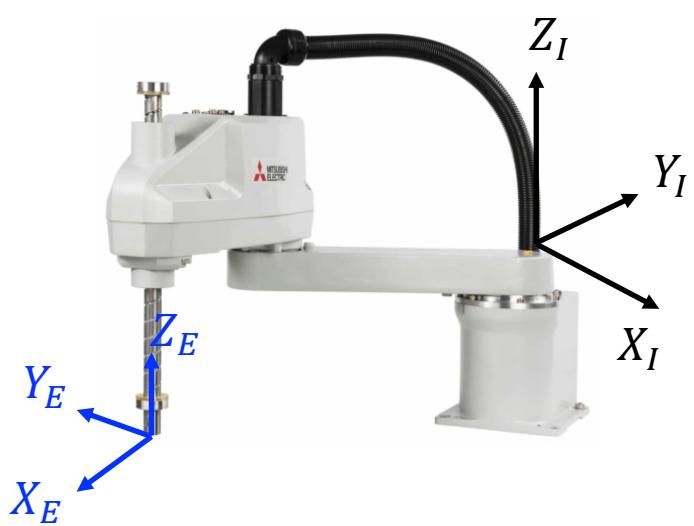


Deformable



Rigid motion

- Actuators (e.g., motors) apply force/torque to induce motion.
- Position and orientation of the body frame (**pose**) changes due to motion.



Modeling robot motion

How is the pose influenced by the actuators?

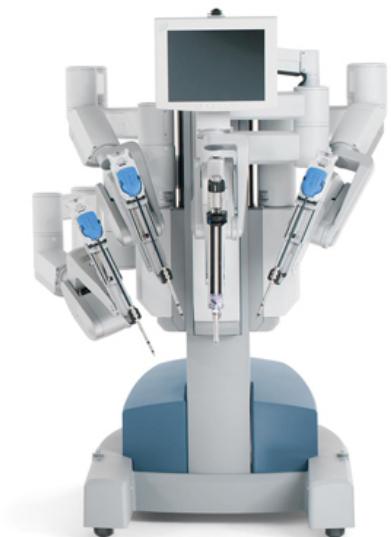
Kinematics: Geometry of motion (ignores forces/torques).

Dynamics: Physics of motion (relates motion to forces/torques).



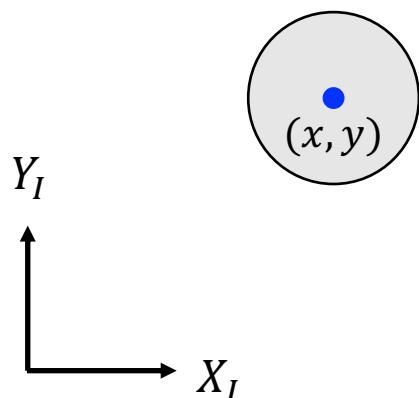
Choosing the model:

- **Kinematic model:** velocities (1st order ODEs).
- **Dynamic model:** accelerations (2nd order ODEs).
More accurate but also more complex.
- Decision depends on the application.
Amount of inertia, abrupt motions, precision,...



Single Integrator

- **Output:** position (x, y)
 - Orientation does not matter since the robot is assumed to move freely in any direction
- **Input:** velocity (v_x, v_y)



$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$



Omni wheel mobile robot

Single Integrator

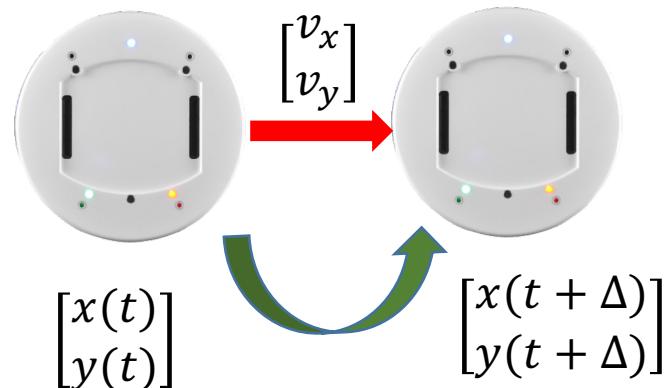
Too simplified for most robots, but very useful for high-level planning/control.

- where should the robot go?
- design the inputs for single integrators, then consider the additional complexity

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$

Discrete-time approximation

$$\begin{bmatrix} x(t + \Delta) \\ y(t + \Delta) \end{bmatrix} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \Delta \begin{bmatrix} v_x \\ v_y \end{bmatrix}$$



Sliding right is not possible, but there is a way to go there.

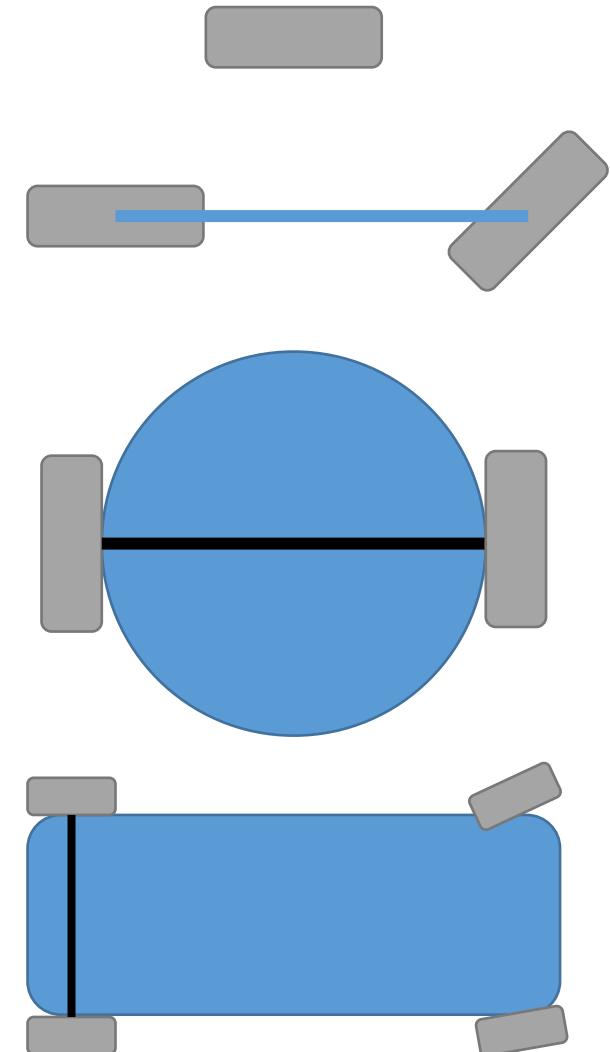
Kinematics of wheeled vehicles

LOTS of different possible designs for wheeled robots

Examples:

- Unicycle model
- Bicycle model
- Differential drive (2 or 3 wheels)
- Ackerman steering (car)

Main question: How does wheel geometry relate to robot motion (i.e. kinematics)?

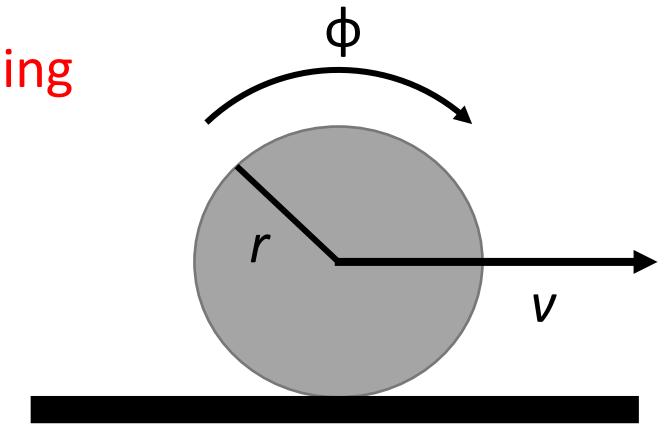


Basic modeling assumptions for wheels

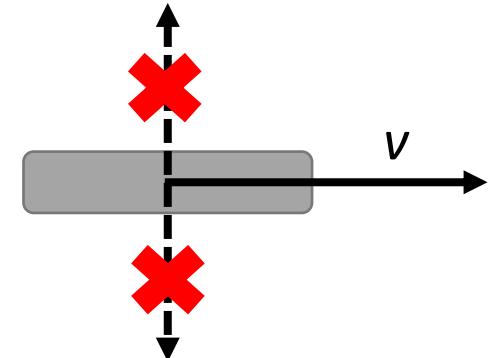
Main assumption: We assume that all wheels **rotate without slipping**

- *Forward* velocity v is determined by **wheel radius r** and **angular velocity $\dot{\phi}$** :

$$v = r\dot{\phi}$$

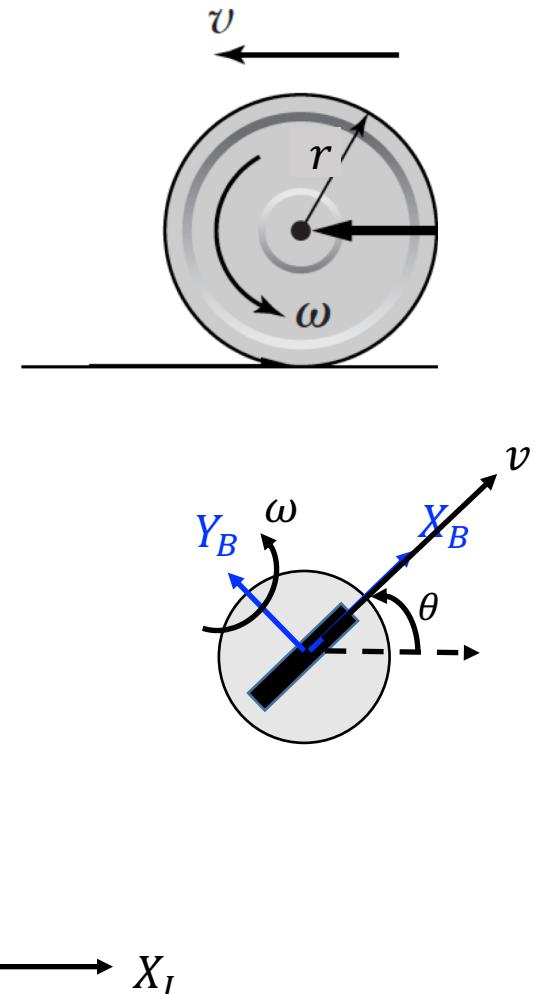


- *Lateral* motion is prohibited: **no side-slip!**
- **Key observation:** These conditions actually correspond to **constraints** on possible robot motion!



Unicycle

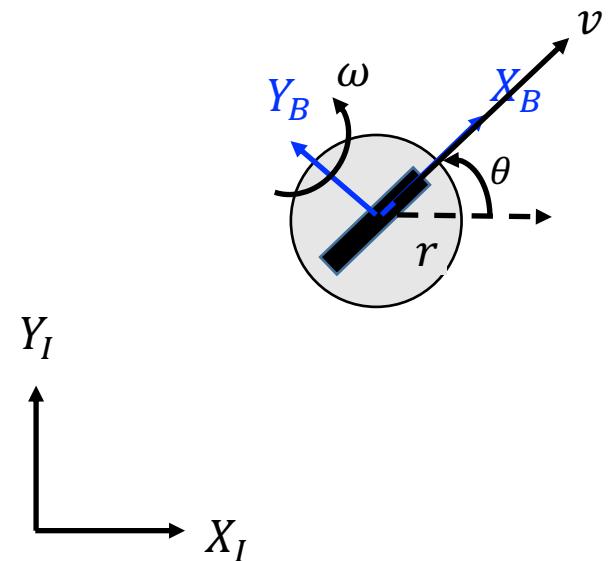
- A simple yet reasonable model for mobile robots.
 - Unlike the single integrator model, the heading matters.
- One **actuated, steered** wheel (pure rolling).
- **Output:** pose (x, y, θ)
- **Input:** angular velocities ω (wheel) and $\dot{\theta}$ (robot)



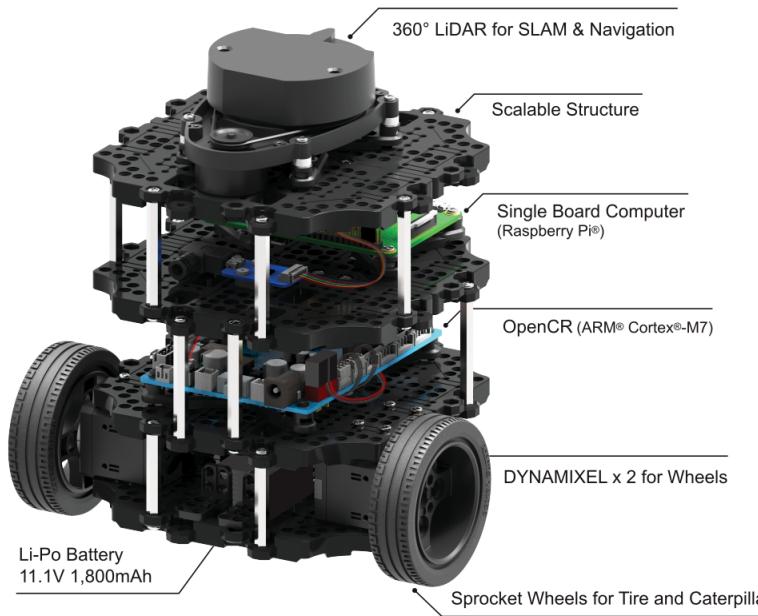
Unicycle – Kinematic Model

- **Output:** pose (x, y, θ)
- **Input:** angular velocity ω (wheel) and $\dot{\theta}$ (robot)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} r\cos(\theta) & 0 \\ r\sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega \\ \dot{\theta} \end{bmatrix}$$



Differential derive robots



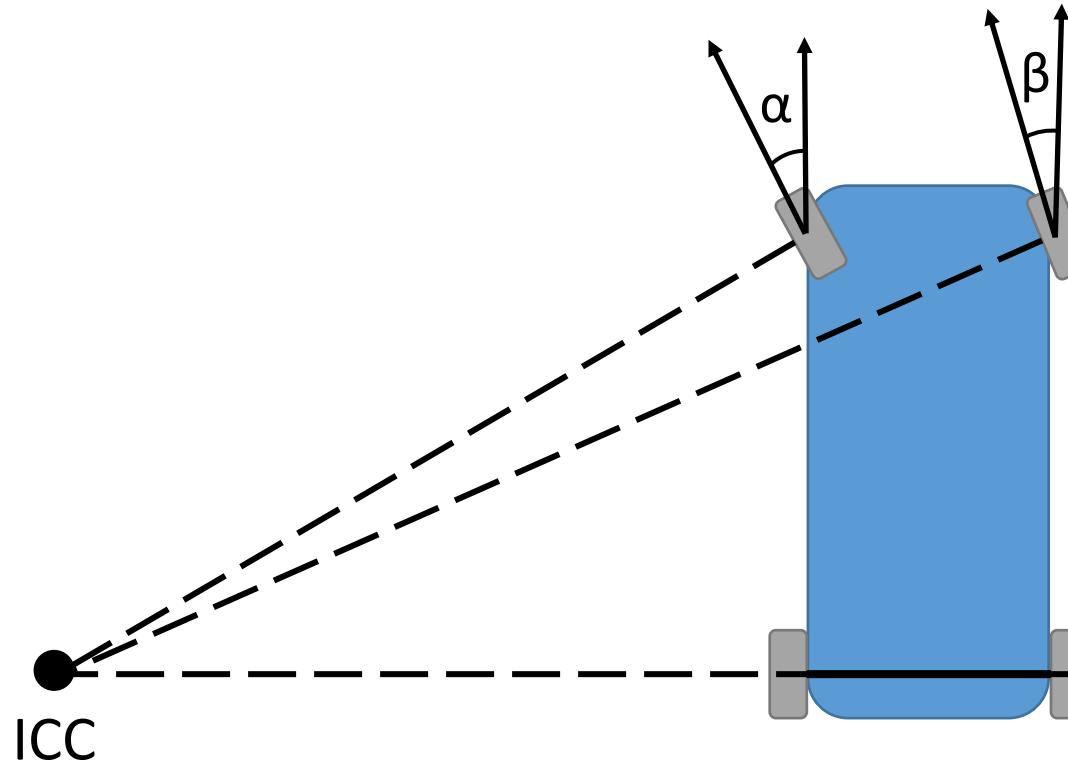
- Max translational velocity: 0.22 m/s
- Max rotational velocity: 2.84 rad/s
- Max payload: 15 kg
- Size (LxWxH): 138 mm x 178 mm x 192 mm
- Weight: 1 kg
- Expected operating time: 2.5 hours
- Expected charging time: 2.5 hours
- 360 Laser Distance sensor
- IMU
- Dynamixel: Actuator system connecting joints and mechanical structures



E-puck

- Diameter: 70 mm
- Height: 55 mm
- Weight: 150 gr
- Max operating time: 3 hours
- Max speed: 15 cm/s
- VGA color camera
- 3D accelerometer

Wheel constraints on robot motion



In order for each wheel to roll without slipping, they must all have a **common instantaneous center of curvature**

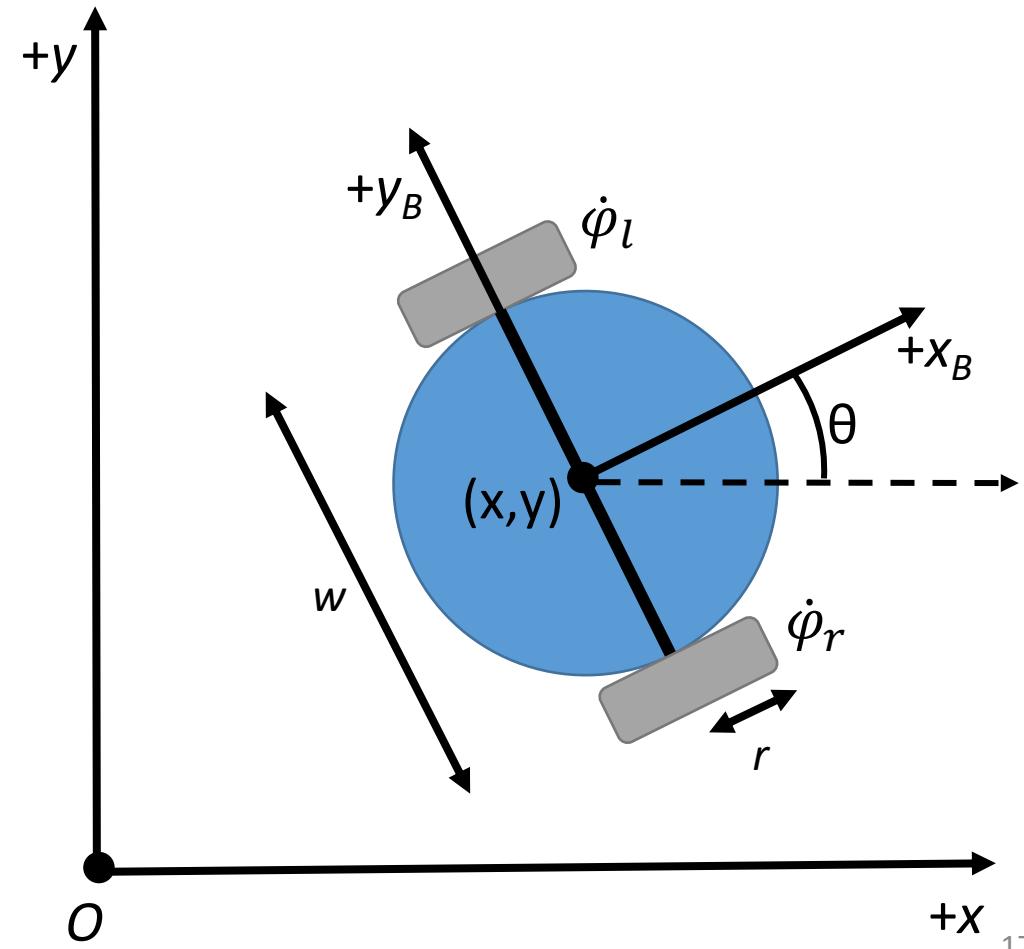
Kinematics of differential drive robots

Suppose we have a differential drive robot with:

- Wheels of radius r
- Wheel track of width w
- Pose (x, y, θ) with respect to the world frame O as shown in the diagram

Key question: What is the relation between the **angular velocities** $\dot{\phi}_l$ and $\dot{\phi}_r$ of the left and right wheels, and the **velocity** $v = (\dot{x}, \dot{y}, \dot{\theta})$ of the robot in the world frame O ?

(Assume that positive values of $\dot{\phi}_l$ and $\dot{\phi}_r$ correspond to forward motion of the robot)



Kinematics of differential drive robots

Let's consider the robot's body-centric coordinate system.

Recall that:

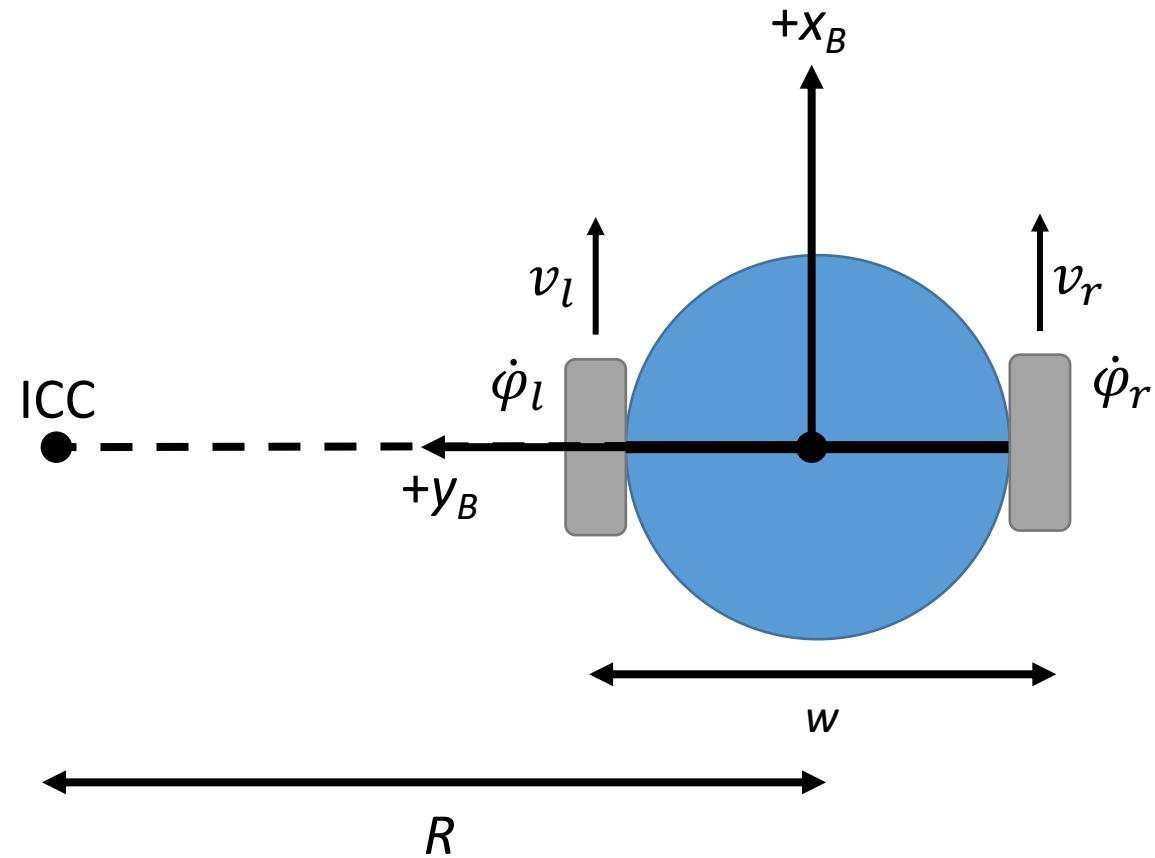
- Both wheels must have a **common ICC**
- There is **no lateral motion** ($\dot{y}_B = 0$)

⇒ The left and right wheels are following **circular arcs** of radii $R - \frac{w}{2}$ and $R + \frac{w}{2}$

⇒ The forward velocities of the left and right wheels satisfy:

$$v_l = \dot{\theta}_B \left(R - \frac{w}{2} \right)$$

$$v_r = \dot{\theta}_B \left(R + \frac{w}{2} \right)$$



Kinematics of differential drive robots

Let's consider the robot's body-centric coordinate system.

The forward velocities of the left and right wheels satisfy:

$$v_l = \dot{\theta}_B \left(R - \frac{w}{2} \right)$$

$$v_r = \dot{\theta}_B \left(R + \frac{w}{2} \right)$$

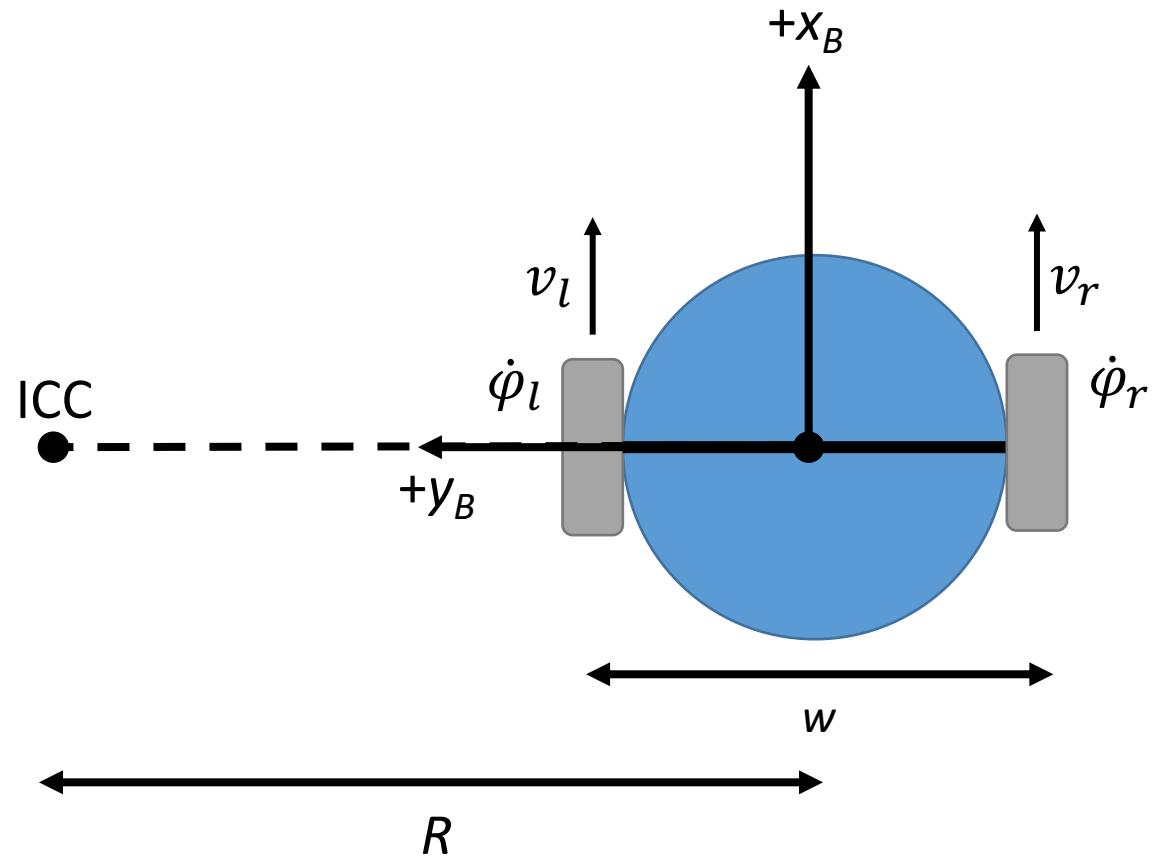
Summing these equations yields:

$$2R\dot{\theta}_B = v_l + v_r$$

But from the diagram, $R\dot{\theta}_B = \dot{x}_B$ is the forward velocity of the robot!

Therefore:

$$\dot{x}_B = \frac{v_l + v_r}{2}$$



Kinematics of differential drive robots

Let's consider the robot's body-centric coordinate system.

The forward velocities of the left and right wheels satisfy:

$$v_l = \dot{\theta}_B \left(R - \frac{w}{2} \right)$$

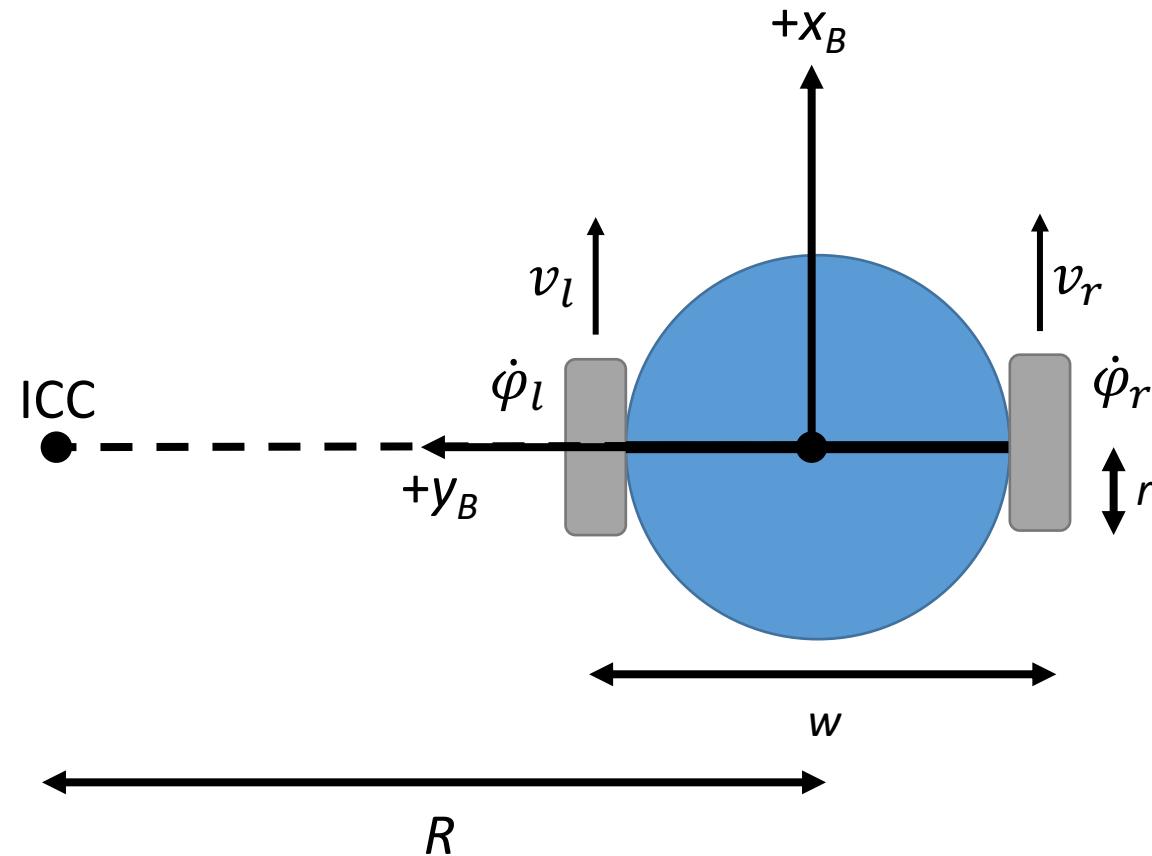
$$v_r = \dot{\theta}_B \left(R + \frac{w}{2} \right)$$

Similarly, subtracting v_l from v_r produces:

$$\dot{\theta}_B = \frac{v_r - v_l}{w}$$

Finally, recall the *no slip assumption* implies that the wheels' *translational* and *angular* velocities satisfy:

$$v_l = r\dot{\phi}_l \quad \text{and} \quad v_r = r\dot{\phi}_r$$



Kinematics of differential drive robots

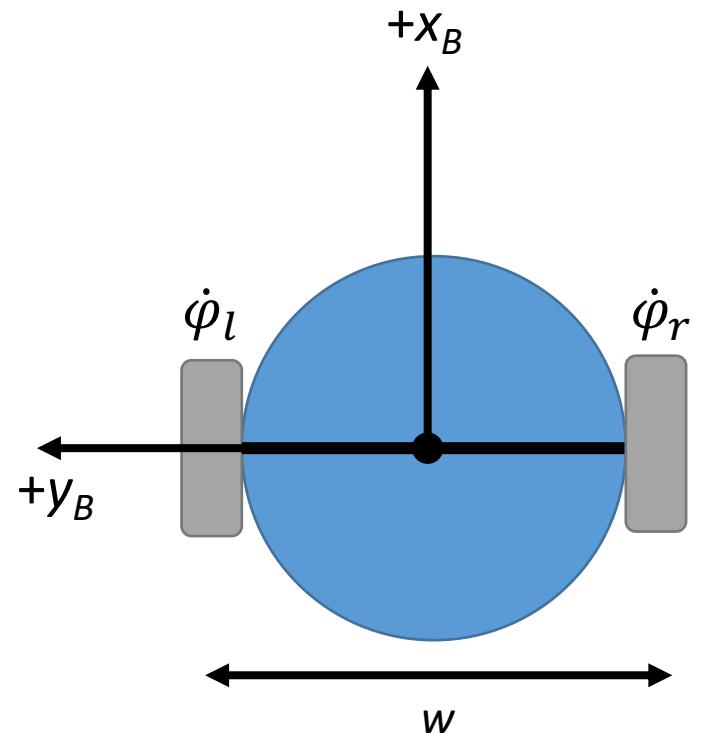
Let's consider the robot's body-centric coordinate system.

Combining these results, we have:

$$\begin{pmatrix} \dot{x}_B \\ \dot{y}_B \\ \dot{\theta}_B \end{pmatrix} = \begin{pmatrix} \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l) \\ 0 \\ \frac{r}{w}(\dot{\phi}_r - \dot{\phi}_l) \end{pmatrix}$$

But: These are in the robot's **body-centric frame**

Q: What about the **world reference frame**?



Kinematics of differential drive robots

Let's consider the robot's body-centric coordinate system.

Combining these results, we have:

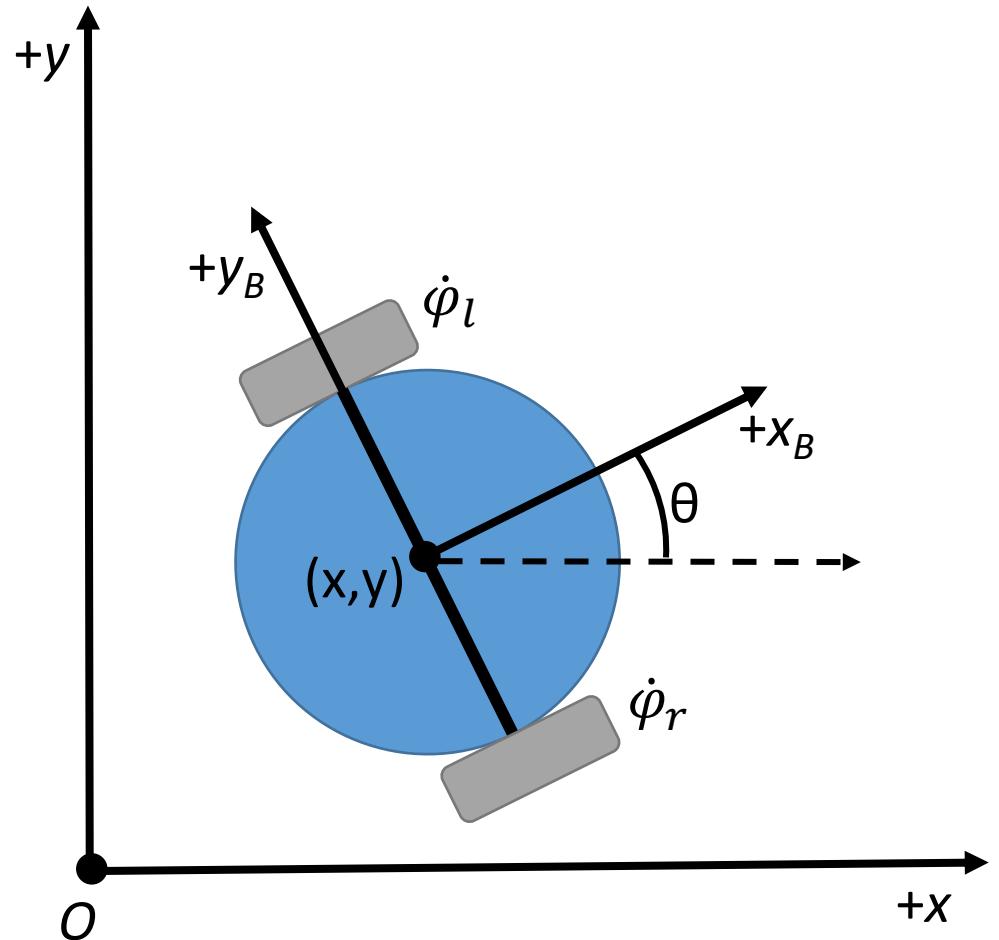
$$\begin{pmatrix} \dot{x}_B \\ \dot{y}_B \\ \dot{\theta}_B \end{pmatrix} = \begin{pmatrix} \frac{r}{2}(\dot{\phi}_r + \dot{\phi}_l) \\ 0 \\ \frac{r}{w}(\dot{\phi}_r - \dot{\phi}_l) \end{pmatrix}$$

Transforming these velocities into the *world frame* O:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} R(\theta) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x}_B \\ \dot{y}_B \\ \dot{\theta}_B \end{pmatrix}$$

where:

$$R(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$



Quadrotor

- A helicopter with four rotors
- Rotors are placed
 - in square formation



Quadrotor

- A helicopter with four rotors
- Rotors are placed
 - in square formation
 - equal distance from the center of mass



Quadrotor

- A helicopter with four rotors
- Rotors are placed
 - in square formation
 - equal distance from the center of mass
- Controlled by adjusting the angular velocity

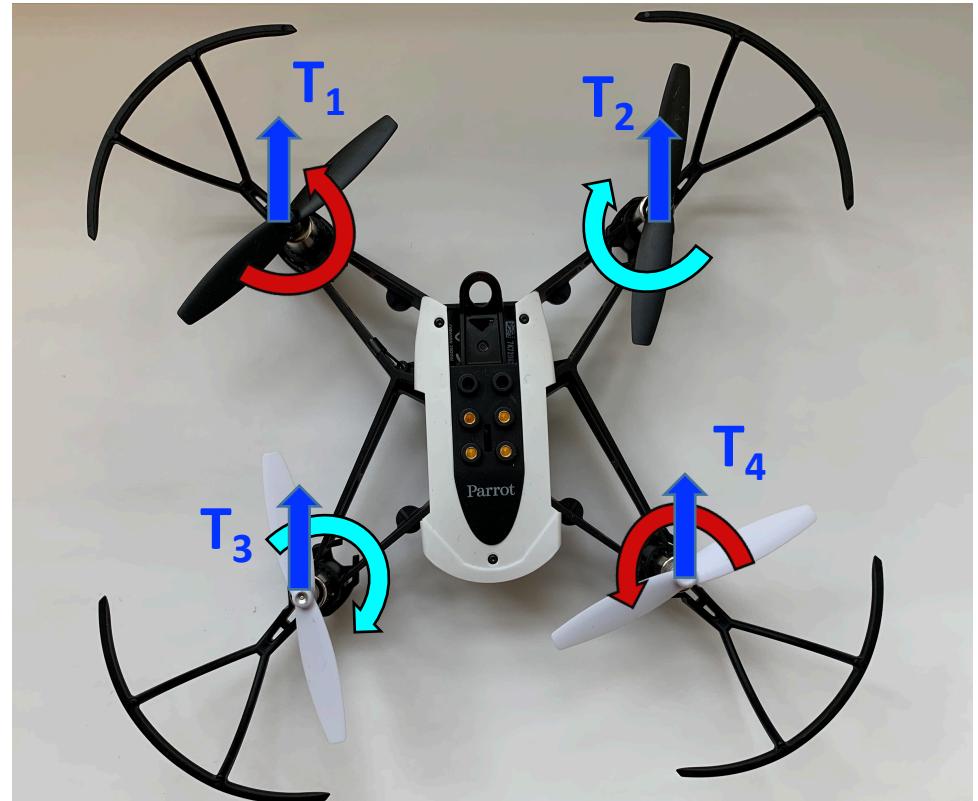


Quadrotor

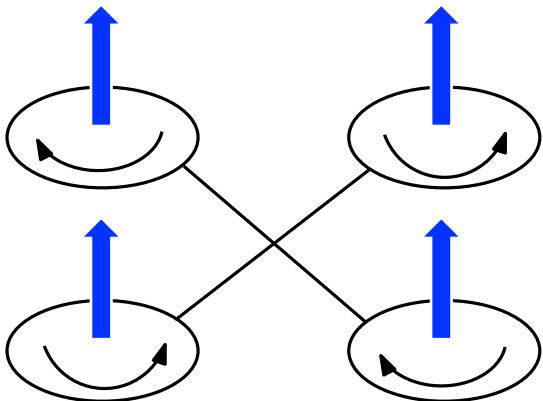
- A helicopter with four rotors
- Rotors are placed
 - in square formation
 - equal distance from the center of mass
- Controlled by adjusting the angular velocity

$$T_i = \frac{1}{2} \rho C_T \omega_i^2 = k_T \omega_i^2$$

Rotor design Rotor control



Quadrotor



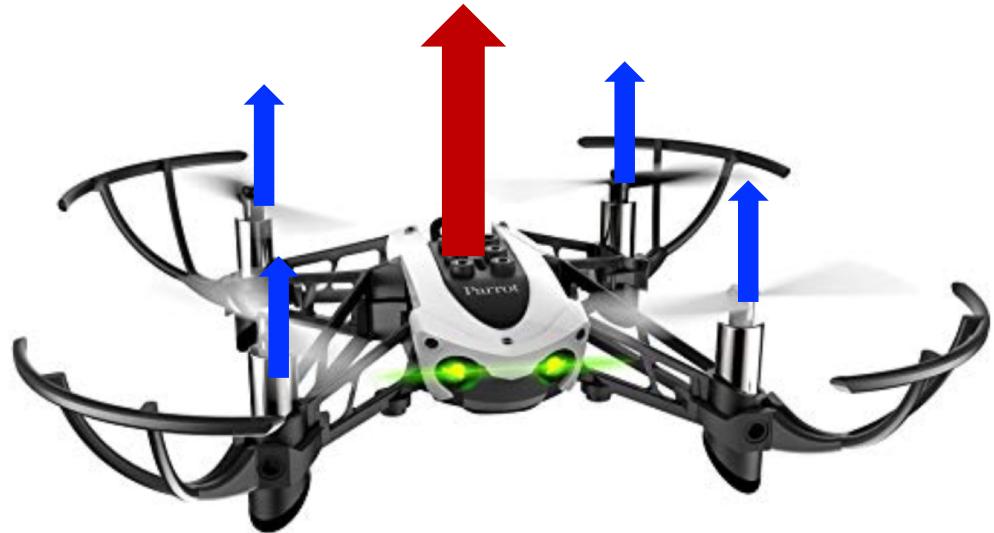
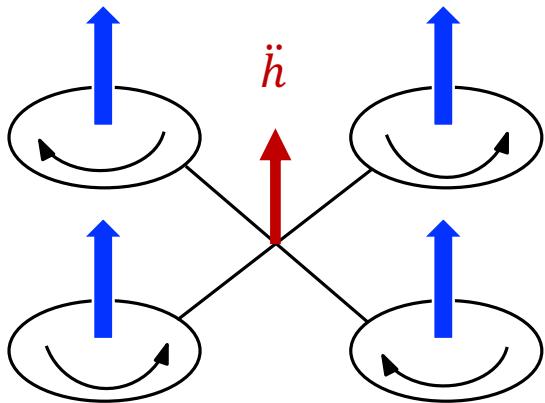
Increase the angular velocity



Thrust generation



Quadrotor

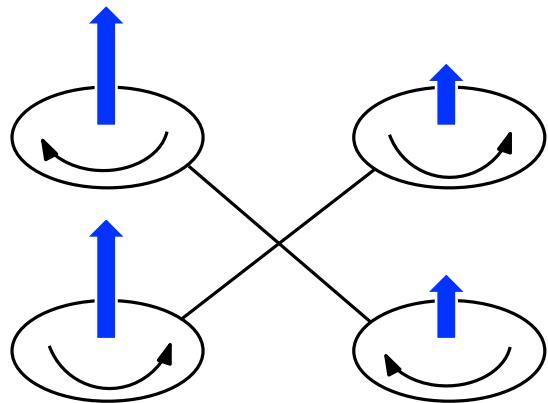


Increase the angular velocity



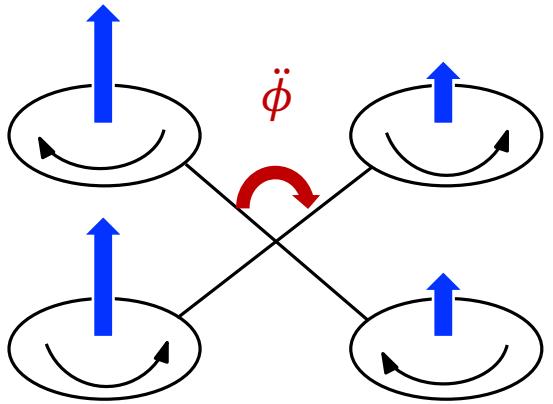
Thrust generation

Quadrotor



Imbalance in thrust generation → ?

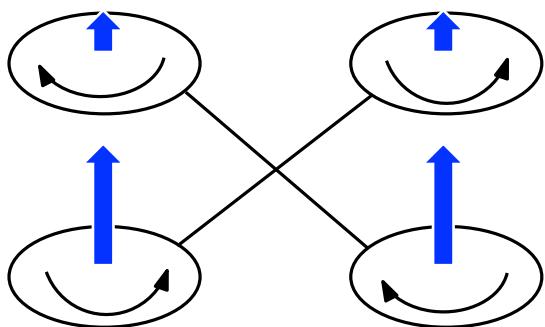
Quadrotor



Imbalance in thrust generation \rightarrow Roll

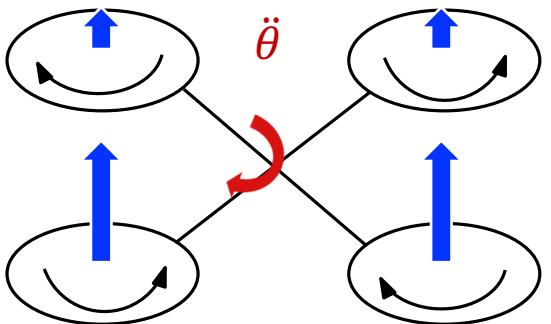


Quadrotor



Imbalance in thrust generation → ?

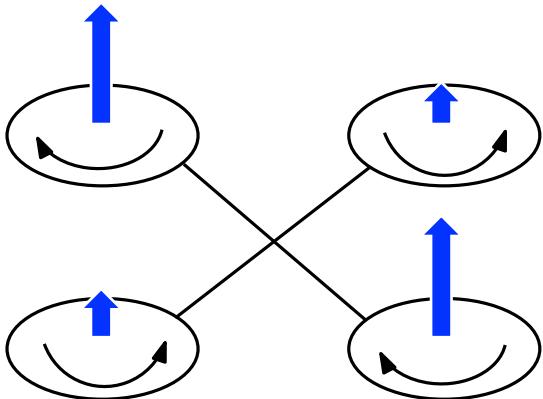
Quadrotor



Imbalance in thrust generation → Pitch

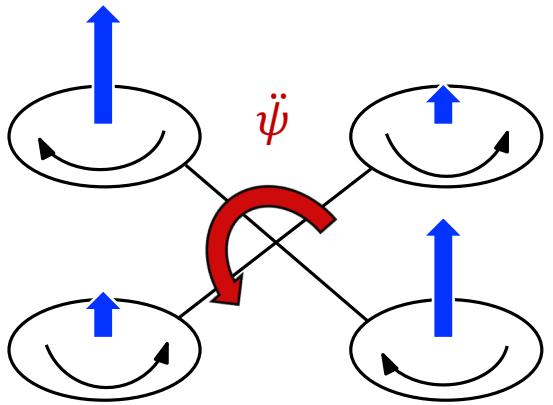


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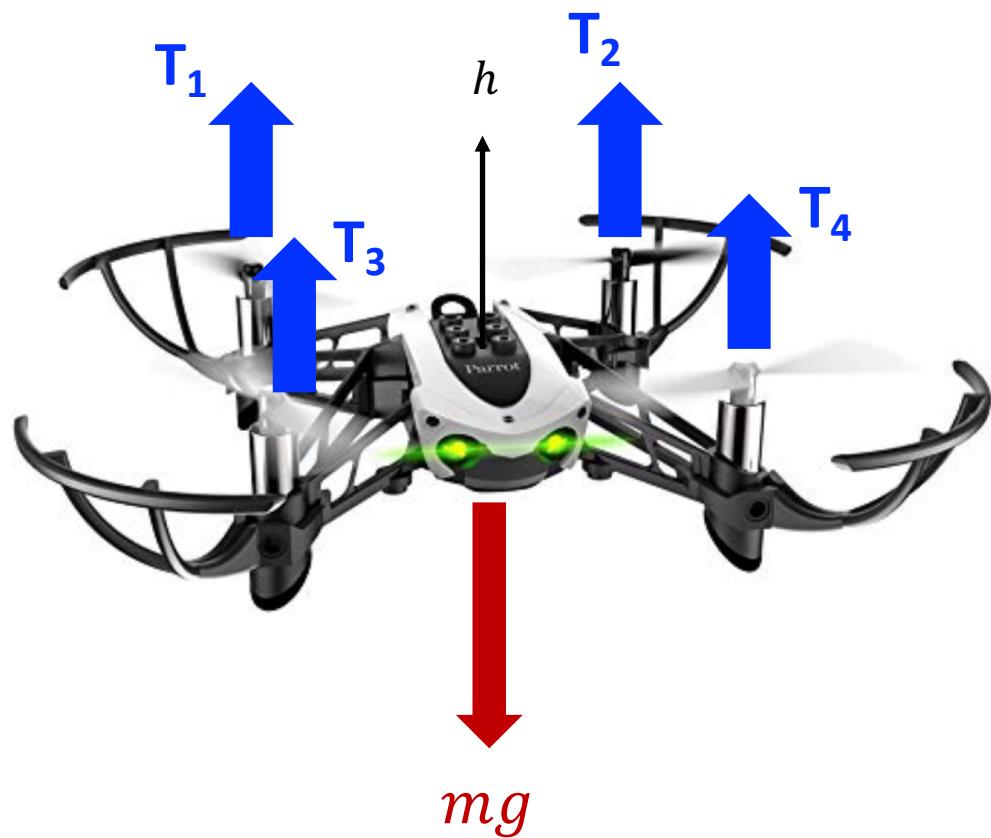
Imbalance in ??? generation  ?

Quadrotor



Imbalance in angular momentum generation → Yaw

A simple model for altitude dynamics



$$ma = \sum_{i=1}^4 T_i - mg$$

$$m\ddot{h} = k_T \sum_{i=1}^4 \omega_i^2 - mg$$

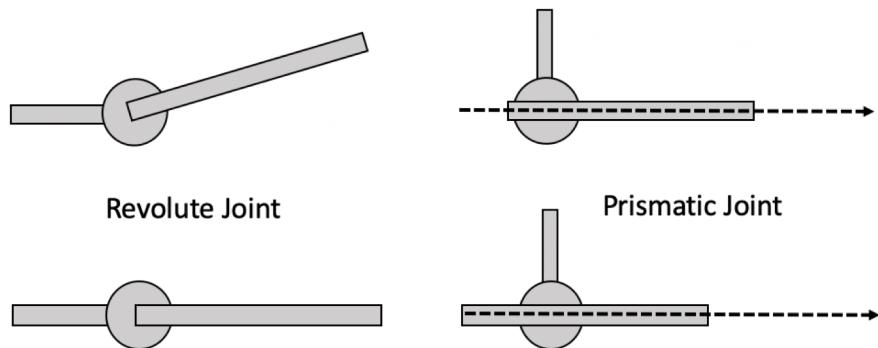
$$m\ddot{h} = 4k_T\omega^2 - mg$$

I.C.
$$\begin{aligned} h(0) &= h_0 \\ \dot{h}(0) &= \dot{h}_0 \end{aligned}$$

Manipulators

Rigid arms, industrial robots

- Rigid bodies (links) connected by joints
- Joints: Revolute or prismatic
- Drive: electric or hydraulic
- End effector mounted on a flange or plate secured to the wrist joint of robot

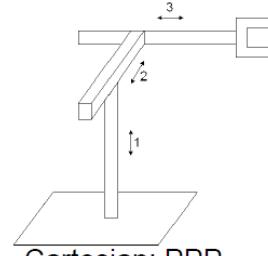


Manipulators

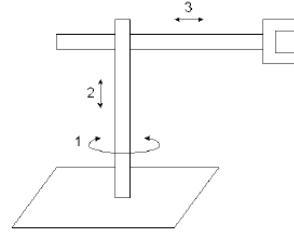
Motion control methods

- Point to point control
 - A sequence of discrete points (e.g., spot welding, pick-and-place, loading/unloading)
- Continuous path control
 - Follow a prescribed path (e.g., spray painting, arc welding)

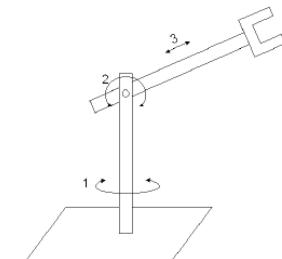
- Robot Configuration:



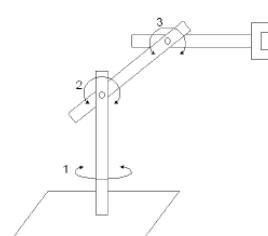
Cartesian: PPP



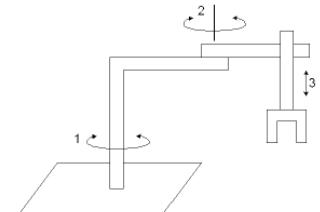
Cylindrical: RPP



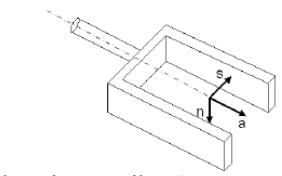
Spherical: RRP



Articulated: RRR



SCARA: RRP
(Selective Compliance Assembly Robot Arm)

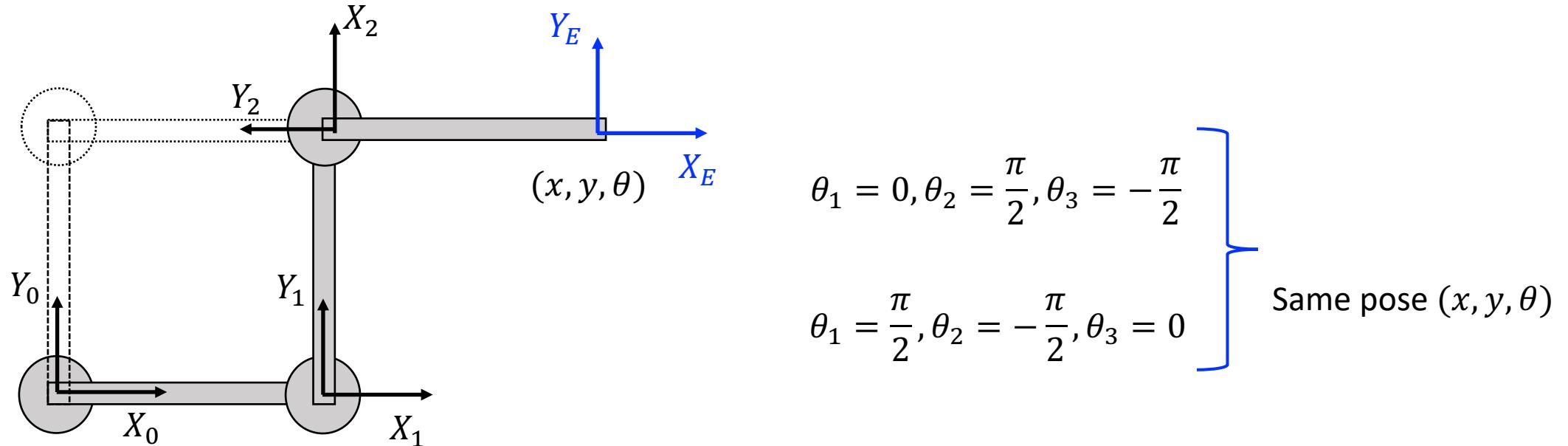


Hand coordinate:

n: normal vector; **s**: sliding vector;
a: approach vector, normal to the
tool mounting plate

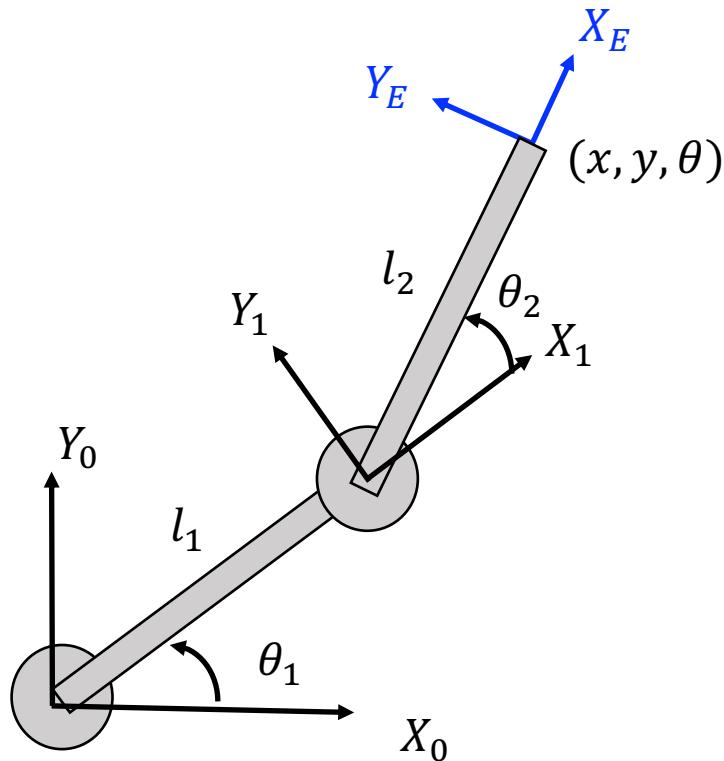
Kinematics of Robotic Arms

- **Forward:** Given the joint variables, find the end effector pose.
- **Inverse:** Given the end effector pose, find the joint variables.
 - May have multiple, unique, or no solution. Typically, harder than the forward problem.



Velocity Kinematics

- How does the pose change as the joint variables change?



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = J(\theta_1, \theta_2) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$J(\theta_1, \theta_2) = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \\ \frac{\partial \theta}{\partial \theta_1} & \frac{\partial \theta}{\partial \theta_2} \end{bmatrix}$$

Sensors

- A device that measures the state of the world or the state of the robot and converts it into data that can be interpreted by either a human or a machine.
- Passive vs. active sensors
 - Passive sensors use energy naturally present in the environment to obtain information.
 - Active sensors involve the emission of energy by a sensor apparatus into the environment, which is then reflected back in some manner to the robot.



Some sensors used in mobile robotics applications

Active sensors

Ability to obtain measurement anytime

High cost

Passive sensors

Low cost

Sensitive to environmental conditions

General classification (typical use)	Sensor Sensor System	A or P
Tactile sensors (detection of physical contact or closeness; security switches)	Contact switches, bumpers Optical barriers Noncontact proximity sensors	P A A
Wheel/motor sensors (wheel/motor speed and position)	Brush encoders Potentiometers Synchros, resolvers Optical encoders Magnetic encoders Inductive encoders Capacitive encoders	P P A A A A A
Heading sensors (orientation of the robot in relation to a fixed reference frame)	Compass Gyroscopes Inclinometers	P P A/P
Ground-based beacons (localization in a fixed reference frame)	GPS Active optical or RF beacons Active ultrasonic beacons Reflective beacons	A A A A
Active ranging (reflectivity, time-of-flight, and geometric triangulation)	Reflectivity sensors Ultrasonic sensor Laser rangefinder Optical triangulation (1D) Structured light (2D)	A A A A A
Motion/speed sensors (speed relative to fixed or moving objects)	Doppler radar Doppler sound	A A
Vision-based sensors (visual ranging, whole-image analysis, segmentation, object recognition)	CCD/CMOS camera(s) Visual ranging packages Object tracking packages	P

Active or passive

General characteristics of sensors

Sensor measurements are noisy, and it requires additional process to interpret the data.

Sensor performance varies with respect to its operational environment.

- Well-controlled laboratory environment vs. real-world

It is crucial to quantify the performance characteristics of a sensor.

General characteristics of sensors

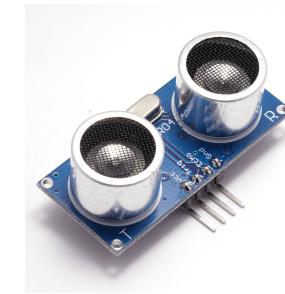
Range: The minimum and maximum value of physical variable that the sensor can sense or measure.



Resolution: The minimum difference between two values that can be detected by a sensor.



Resolution 1 cm →



Temperature sensor
-40°C to +125°C

Ultrasonic distance sensor
2 to 400 cm

General characteristics of sensors

Bandwidth or frequency: The speed with which a sensor can provide a stream of readings

- the number of measurements per second is defined as the sensor's frequency in *hertz*.
- mobile robots limited in maximum speed by the bandwidth of their obst. detection sensors.
- increasing the bandwidth of ranging and vision-based sensors is crucial.

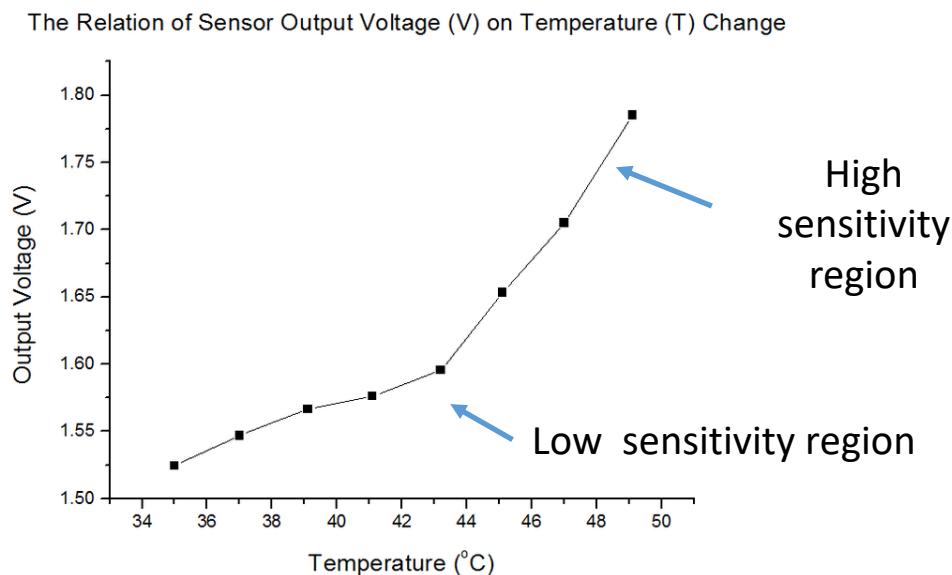
Linearity: A measure governing the behavior of the sensor's output signal as the input signal varies.

- A linear response indicates that if two inputs x and y result in the two outputs $f(x)$ and $f(y)$, then for any values a and b , $f(ax + by) = af(x) + bf(y)$.
- This means that a plot of the sensor's input/output response is simply a straight line.
- A more linear sensor output is easier to calibrate the sensor.

SNR - signal-to-noise ratio: a measure that compares the level of desired signal to the level of background noise

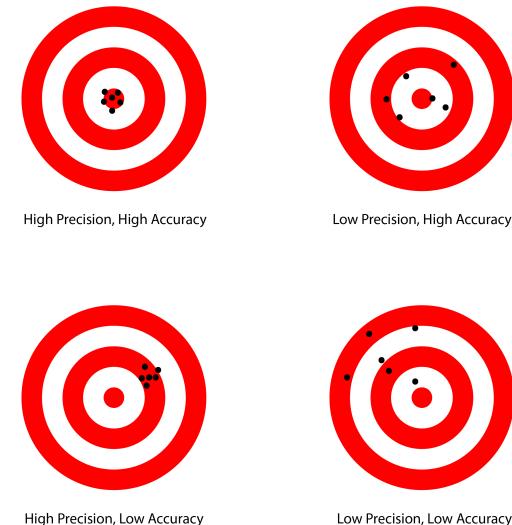
General characteristics of sensors

Sensitivity: The minimum input of physical parameter that will create a detectable output change. (slope of the output curve)



Accuracy: difference between measurement and actual.

Precision: the degree of reproducibility of a measurement



Example sensors on an autonomous car

Tactile or “bumper” sensors

- Detect physical contact with the environment
- Can be used for simple proximity sensing, e.g. early Roomba

Rotary (wheel) encoders

- Measure position / velocity of rotating shafts
- Useful for **dead reckoning** (w/ kinematic models)

Laser scanners

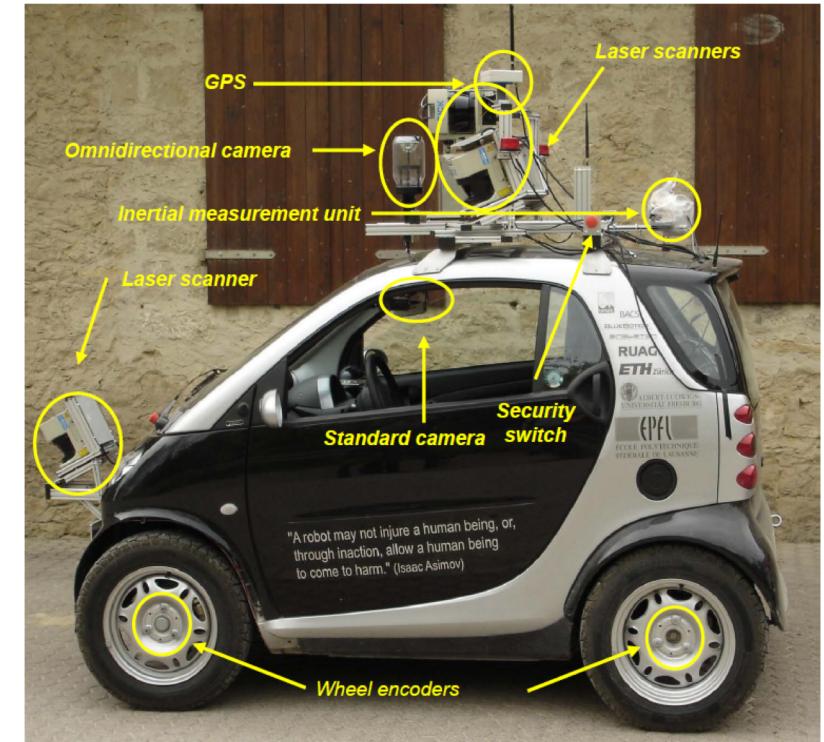
- Measures range and bearing to points in the environment
- Captures **high-accuracy geometry** (although often only in a *plane*)

Vision sensors

- Provides **dense field-of-view information** about the environment’s **visual appearance** (color, texture, etc.).
- A **very rich source** of information for scene understanding

Global position system (GPS)

- Direct measurement of **absolute position** on Earth’s surface
- Requires **clean line-of-sight** to GPS satellites



Other sensors: Gyroscope, magnetometer, accelerometer, etc.

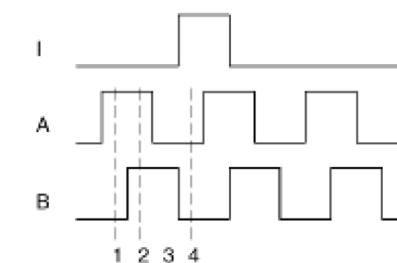
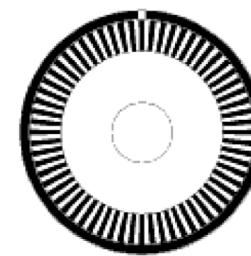
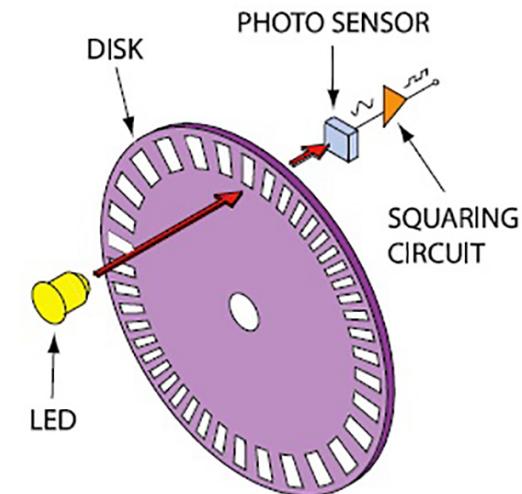
Optical rotary encoders

Principle of operation: Measure the rotation of a shaft by “counting” pulses of light that pass through a thin grating

Typical resolution (for robotics) is ~2000 cycles per revolution

Used for measuring:

- Joint angles (for manipulators)
- Wheel speed / odometry
⇒ Can be used for *dead reckoning* (w/ kinematic models)



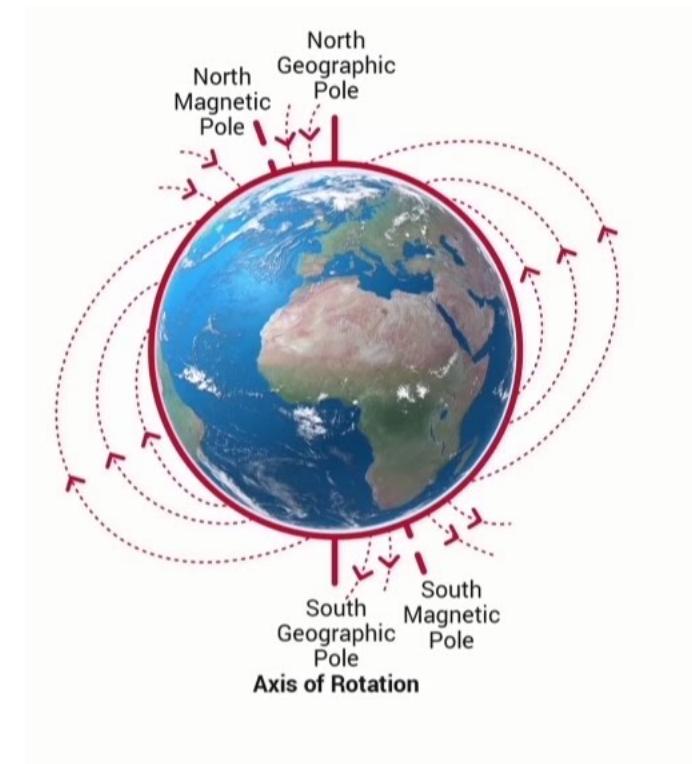
State	Ch A	Ch B
S_1	high	low
S_2	high	high
S_3	low	high
S_4	low	low

Magnetometer (Compass)

Measures the strength and direction of a magnetic field

Prone to inaccurate reading due to disruptive source of magnetic field (power lines, electrical devices, etc.)

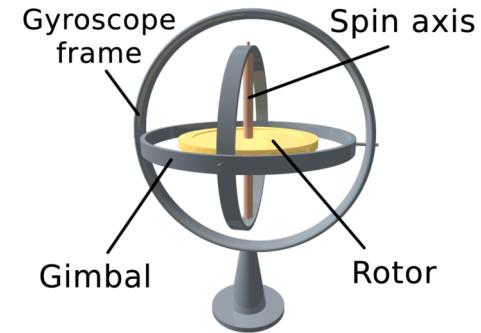
- Periodic calibration



Gyroscope & Accelerometer

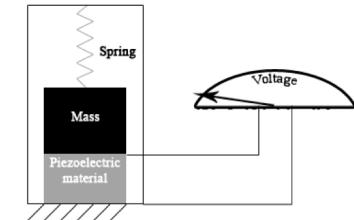
Gyroscope measures orientation

- Rate of rotation, degree of tilt, and angular velocity of a moving object
- Principle of conservation of angular momentum
- Cheap



Accelerometer measures linear movement along an axis.

- A piezoelectric accelerometer uses microscopic crystals that generate a current when they undergo stress. This stress can be brought about by accelerative forces, such as the movement of an object.

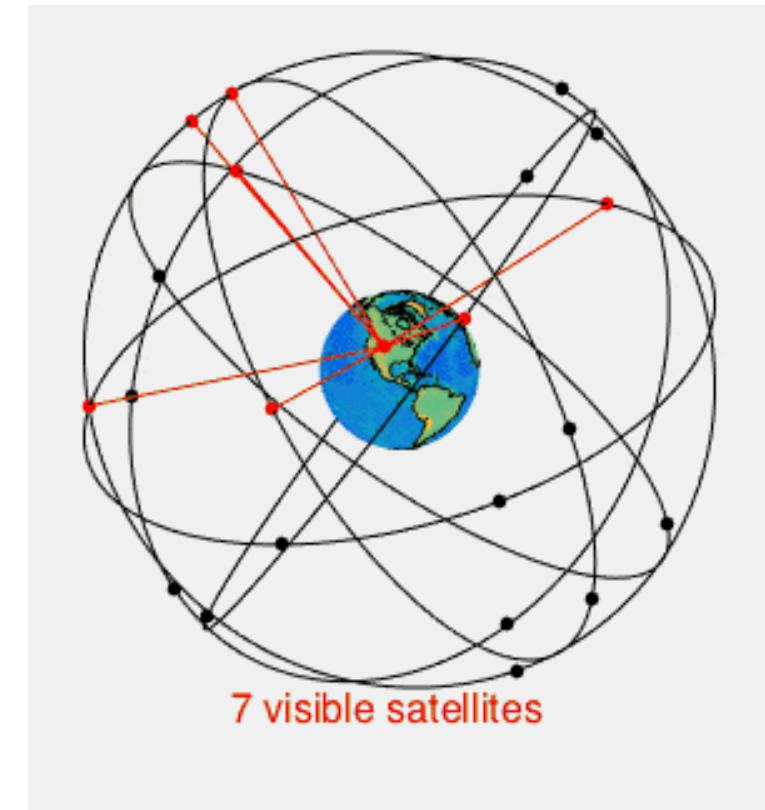
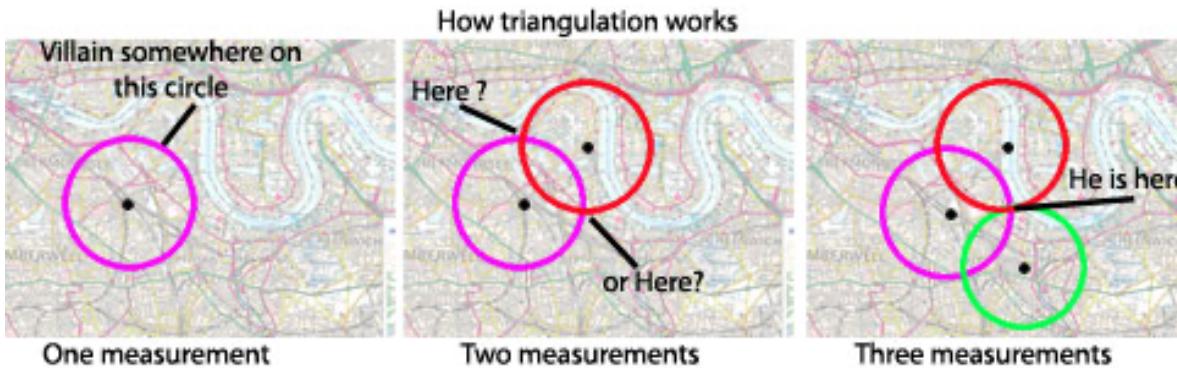


E.g., 3-axis accelerometer and 3-axis gyroscopes is commonly referred to 6-axis gyro stabilization. This setup allows a drone to maintain horizontal, vertical, and rotational stability while hovering.

Global positioning system (GPS)

Principle of operation: Measure the arrival times of radio signals from a constellation of satellites.

- Each satellite continuously transmits data that indicate its location and the current time.
- Time-of-flight for each signal \times speed of light = range to each satellite
- Receiver's position can then be determined via *multilateration*

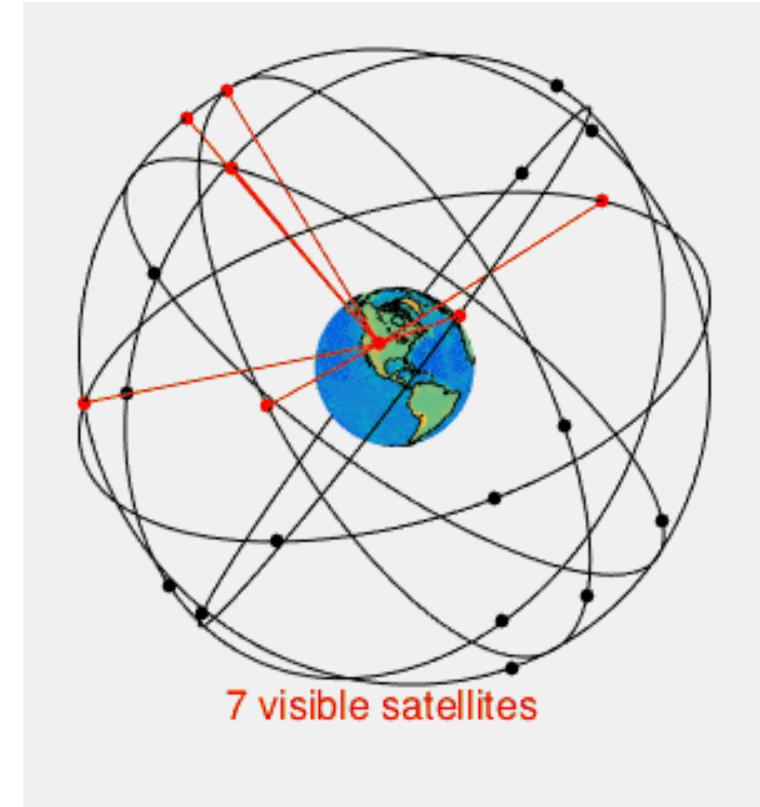


Global positioning system (GPS)

Pro: Enables direct measurement of *absolute position*

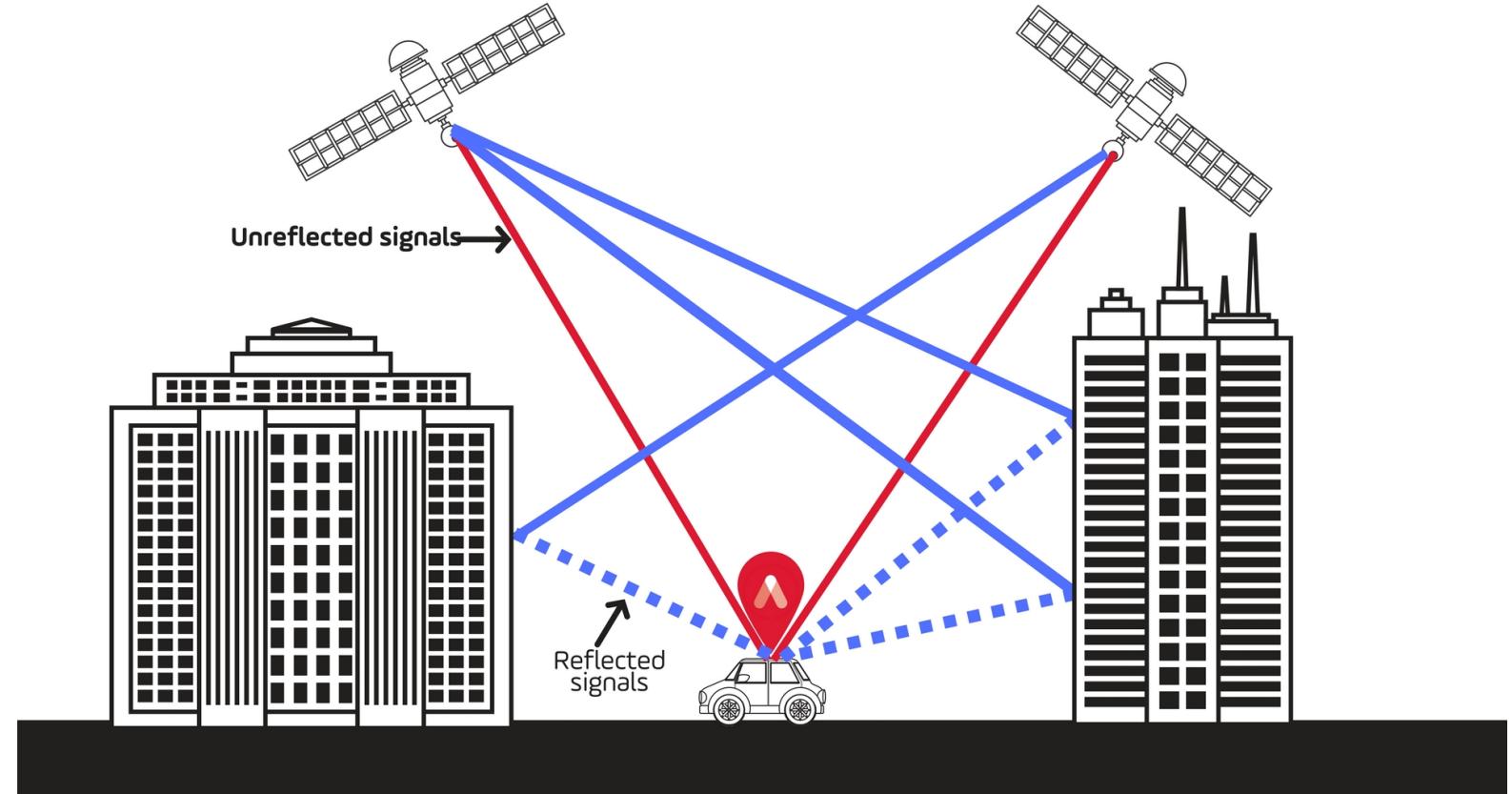
Con: Accuracy can be significantly affected by several external factors

- Constellation position (how many satellites are visible?)
- Atmospheric effects (affects EM propagation speed)
- Requires clear line-of-sight to satellites
 - Only works outdoors
- **Multipath effects** are a **significant** problem



GPS Multipath

*A signal arrives a receiver via two or more routes.



Inertial measurement unit (IMU)

Integration of various sensors

- Accelerometer
- Gyroscope
- Magnetometer

Can detect changes in location and rotational attributes

Integrate the acceleration of the drone to calculate velocity and position

Advantage: Works in GPS denied environments (indoors or GPS drops)

Disadvantage: Sensor errors accumulate due to integration

Active ranging sensors

For obstacle detection and avoidance, most mobile robots rely heavily on active ranging sensors.

Ultrasonic range finders

- Sounds waves
- Soft surfaces absorb most of the sound energy
- Underwater applications

Laser range finders

- Lasers
- More range
- More expensive

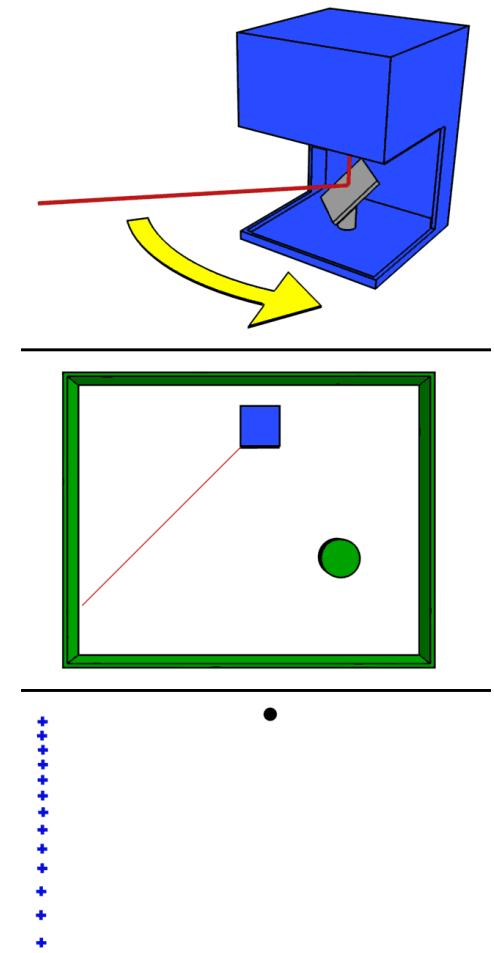
LIDAR

Principle of operation: Light **detection and ranging**. Measure range and bearing to points in the environment by measuring *time-of-flight* of a reflected laser beam

Typically these units will “sweep” a single laser through a planar section of the environment using a rotating mirror

Produces a scan of 2D points and intensities

- (x,y) in the laser’s frame of reference
- Intensity is related to the material of the object that reflects the light
- Certain surfaces are problematic for LIDAR: e.g. glass



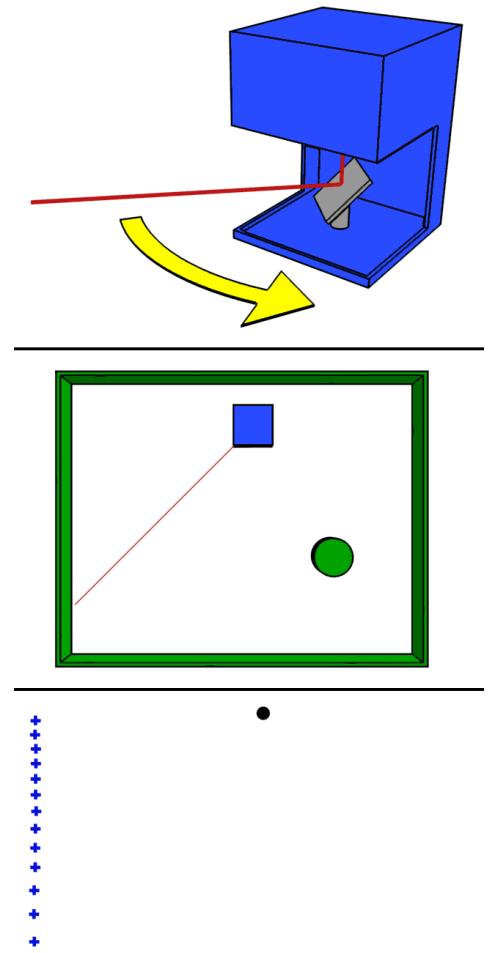
LIDAR

Pro:

- Captures **high-precision geometry at long range** (cm-level accuracy at distances of tens to hundreds of meters)
- **Active sensing**: Less sensitive to environmental properties (e.g. texture)

Con:

- Standard LIDARs only scan **a 2D-cross-section** of the environment – can't capture 3D information (e.g. variation w/ height)
- **Active sensing**: Requires power (can be an issue for resource-limited mobile platforms)



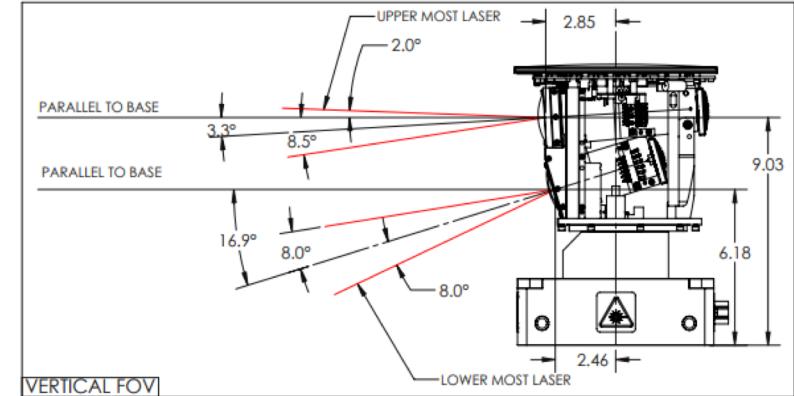
3D LIDAR

Some recently-developed lidars (e.g. Velodyne) can return **3-dimensional, 360-degree field-of-view** point clouds

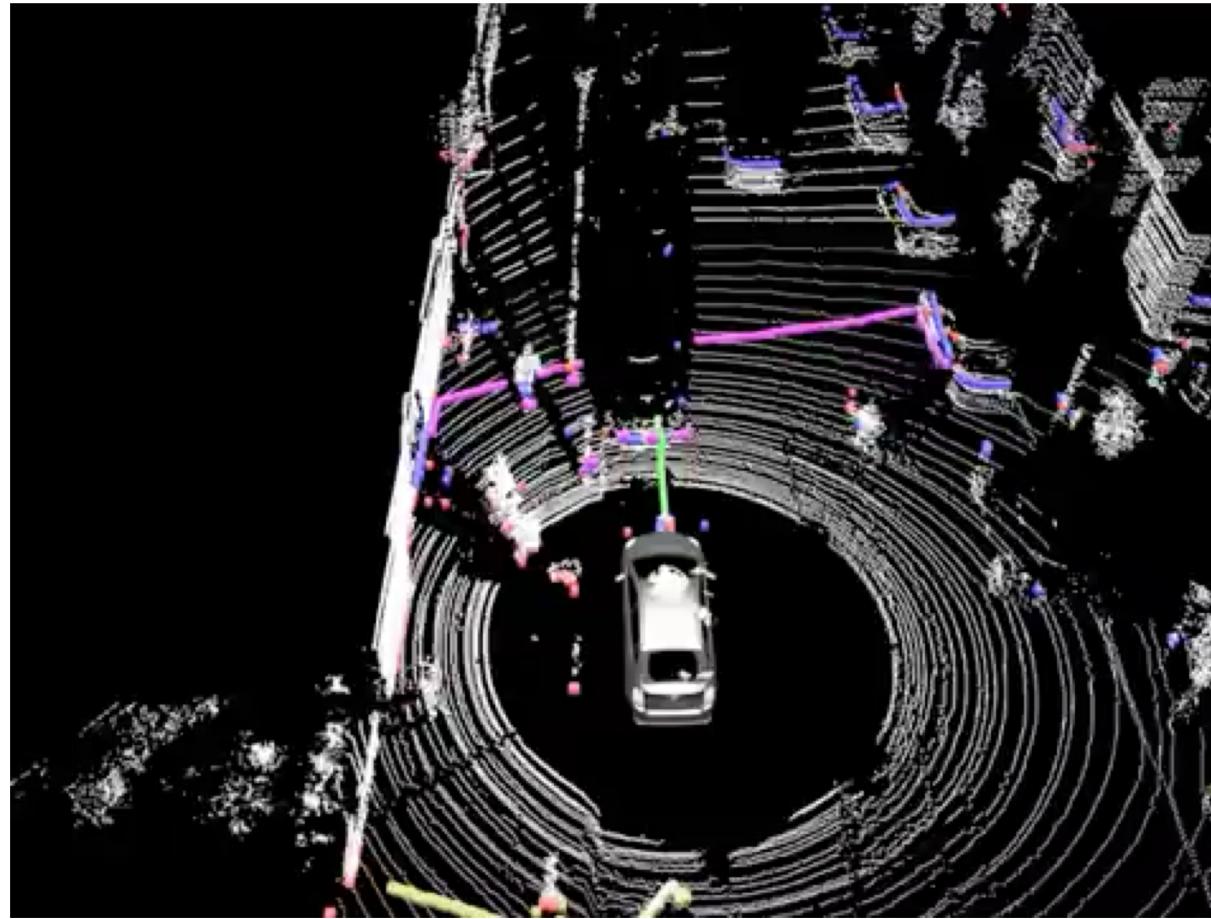
Main idea: Stack **multiple** (e.g. 64) lasers pitched at different elevations in a common housing, and rotate the **entire unit**

These are commonly used for **autonomous vehicles**
(Range < 300m)

Not very robust to adverse weather conditions: rain, snow, smoke, fog etc.



Example: 3D LIDAR on an autonomous vehicle



Cameras

Principle of operation: Focus ambient light rays onto a photo-sensitive surface (in robotics, a digital sensor)

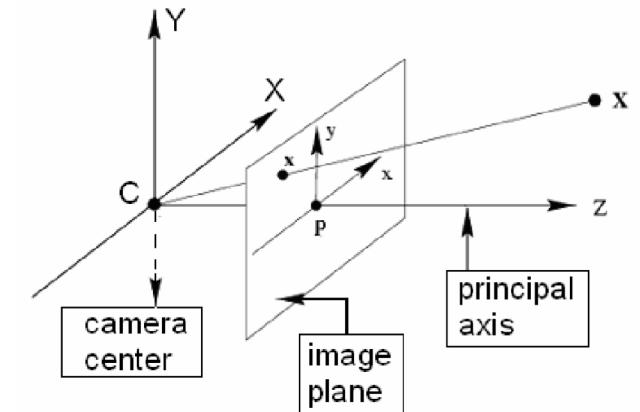
The location at which an incident beam is imaged depends upon its *direction of origin* – cameras are *orientation* sensors

Pro:

- Can capture extremely **rich, dense visual information** over a **large field of view** – very useful for scene classification, obj. recognition
- Passive sensing: Relatively low power (compared to lasers)

Con:

- Passive sensing: Efficacy depends upon **external environmental factors** (illumination, visual texturing)
- Cameras may not be as geometrically accurate as lasers, esp. at long ranges (more on this later)



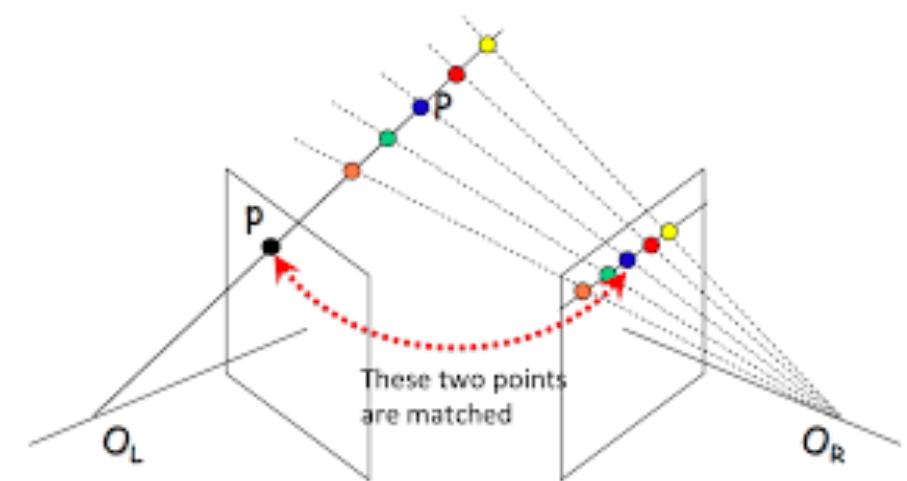
Stereo cameras

Recall: Cameras are *bearing* sensors -- can't determine depth (position) of a point from a single image



Idea: Use *two* cameras! If *relative pose* of the cameras is known (calibrated), the position of a point seen in both images can be *triangulated*

- Provides information about *geometry and visual appearance* over a field of view
- **But:** This depends upon *finding corresponding points* in the two images
⇒ *Requires visual texture!*



Example stereo depth map

Left



Right



Disparity map



Depth map

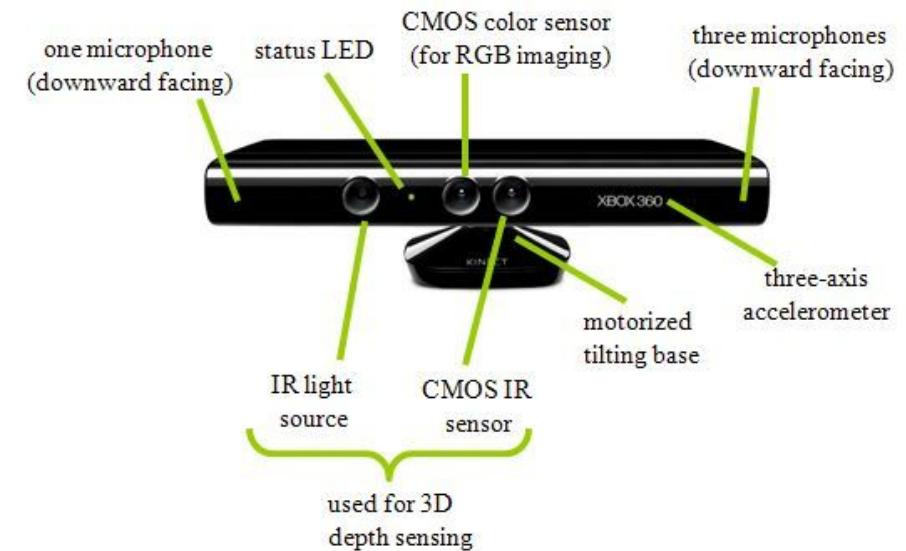


Structured light sensors

Principle of operation: Same as stereo, but one camera is replaced with an *emitter* that projects a *known pattern*

Solves two main challenges in (passive) stereo:

- Visual texture: The camera makes its own
- Feature matching: The projected pattern is designed to make *feature matching* easy



Kinect dot pattern



Sensor fusion

- Each sensor has its own advantages/disadvantages
- The process of **merging data from multiple sensors** such that to reduce the amount of uncertainty that may be involved in a robot navigation motion or task performing.

Examples:

- **Aircraft:** Fusing data from GPS and IMU for low-cost accurate attitude measurement
- **Autonomous driving:** Fusing data from LIDAR, camera, and ultrasonic sensors to interpret environmental conditions
- **Ground robots:** Fusing data from wheel encoders and IMU for better estimation of pose