the stochasticity of these problems, the stochastic optimization involves formulating a large-scale scenario program, which is in general computationally demanding.

In another effort, [24] incorporated the min-max behavior of RO problems to solve them by an oracle-based approximate robust optimization algorithm based on oracle-based subgradient descent and interior point methods. The proposed algorithms find an approximate robust solution using a number of calls to an oracle that solves the original (non-robust) problem. However, the solution would be approximated with some predetermined accuracy, η, with the number of iterations growing to $\mathcal{O}(\frac{1}{\eta})$, as the algorithm approximates the RC by invoking the Oracle a polynomial number of times. Cutting-plane approaches [25] and the column-and-constraint generation technique [26] handle $\mathcal{RO}$ as a semi-infinite programming problem. However, these approaches may have limitations in scenarios where pessimization (inner maximization) oracles are approximate, or when the number of scenarios becomes large, leading to scalability issues. The main contributions of this paper are as follows.

We introduce a continuous-time dynamical system that provably converges to the optimal robust solution for a broad class of robust optimization (RO) problems. Our approach builds on classical primal–dual dynamical systems [27], [28] but addresses the unique challenges posed by the min–max structure inherent in RO. To the best of our knowledge, this work presents the first continuous-time dynamical system specifically designed for solving robust optimization problems that are convex in the decision variables and concave in the uncertainty. Despite this convex–concave structure, the problem is not jointly concave in (λ, u), where λ denotes the dual variable and u the uncertainty. In contrast to classical primal–dual methods, the uncertainty variable here is treated as a dynamical state rather than as a fixed parameter. Moreover, due to the lack of joint concavity, the proposed dynamics cannot be derived as the gradient flow of a Lagrangian function, distinguishing our method from standard primal–dual gradient systems. We establish the saddle-point property of the equilibrium even in the absence of joint concavity in (λ, u). The non-classical structure of the dynamics necessitates the construction of a novel Lyapunov function to analyze stability. Using this Lyapunov function, we prove that the proposed dynamics are globally asymptotically stable and converge to the robust optimal solution.

A distinctive feature of our dynamical system is its amenability to model-free implementation when deployed in physical systems where agents can sense local gradients but do not possess global knowledge of objective or constraint functions. Each agent requires only the ability to measure local gradient information $\nabla_x f_i(x, u_i)$ and $\nabla_{u_i} f_i(x, u_i)$ through sensing or finite-difference approximations at their current state, along with information from neighboring agents through local communication. This enables implementation in distributed settings where the global problem formulation may not be explicitly known to individual agents, yet the collective dynamics converge to the robust optimal solution. This model-free characteristic distinguishes our approach from robust counterpart methods that require complete a priori knowledge of problem structure.

The proposed continuous-time optimization system can solve a general class of $\mathcal{RO}$ problems where the cost function and the constraints are convex (concave) with respect to the decision variables (uncertain variables) and the uncertainty sets are convex[1]. This class includes all the cases in [19, Table 1].

The remainder of this paper is organized as follows. In Section III, we present the problem statement of $\mathcal{RO}$ in a slightly generalized form. In Section IV, the characterization of saddle property and KKT optimality conditions along with the Lagrangian function for $\mathcal{RO}$ problem are provided. The main results on how to construct the $\mathcal{RO}$ dynamics is presented in Section V. This section also includes the Lyapunov-based global convergence result. We then present simulation results in Section VII followed by conclusions in Section VIII. Finally, detailed proofs are mentioned in the Appendix.

II. Notations

We define $i = 0, \cdots, N$ as $i \in [N]$, and $i = 1, \cdots, N$ as $i \in [N]^+$. In addition, $[\cdot]^+_\eta$ shows the positive projection defined as follows for the scalar-valued function P

$$[P]^+_\eta := \begin{cases} P & \text{if} \quad P > 0 \text{ or } \eta > 0 \\ 0 & \text{otherwise} \end{cases} . \tag{1}$$

For vector-valued P, the projection is defined element-wise.

III. Robust Optimization Problems

Given an objective $f_0(x)$ to optimize, subject to constraints $f_i(x, u_i) \le 0$ with uncertain parameters, $\{u_i\}$, a classical robust optimization problem has the following form

$$\begin{aligned} &\mu := \min_x f_0(x) \\ &\text{s.t. } \ f_i(x, u_i) \le 0, \ \forall u_i \in \mathcal{U}_i, \ i \in [N]^+. \end{aligned}$$

where $x \in \mathbb{R}^n$ is a vector of decision variables, f_0 and f_i are $\mathbb{R}^n \to \mathbb{R}$ functions, and the uncertainty parameters $u_i \in \mathbb{R}^{m_i}$ are assumed to take arbitrary values in certain convex compact uncertainty sets $\mathcal{U}_i \subseteq \mathbb{R}^{m_i}$. The above problem is typically a semi-infinite optimization due to the cardinality of the $\mathcal{U}_i$. However, each robust constraint can be rewritten as a maximization problem as[2]

$$\begin{aligned} &\mu := \min_x f_0(x) \\ &\text{s.t. } \ \max_{u_i \in \mathcal{U}_i} f_i(x, u_i) \le 0, \quad i \in [N]^+ \end{aligned} . \tag{2}$$

We consider a slight variation of (2), which takes the following under the assumptions stated below.

$$\mathcal{RO}_0 \begin{cases} \mu := \min_x \max_{u_0 \in \mathcal{U}_0} f_0(x, u_0) \\ \text{s.t. } \ \max_{u_i \in \mathcal{U}_i} f_i(x, u_i) \le 0 \ , \ i \in [N]^+, \\ \\ \mathcal{U}_i := \{u_i \in \mathbb{R}^{m_i} \ : \ h_{ij}(u_i) \le 0, \\ \qquad j \in [K_i]\} \ , \ i \in [N] \end{cases} \tag{3}$$

[1]Preliminary convergence results for a special formulation appeared without proofs in [29]. In this paper, we generalize the problem formulation and provide complete proofs, which are the core of our contributions.

[2]Since the uncertainty set is compact and the constraint functions are continuous, the supremum is attained within the set; therefore, we can replace “sup” with “max” in our formulation.