Response to Reviewer 6

***Summary:** The paper proposes a novel dynamical system-based approach, called RO dynamics, for solving robust optimization problems in a general convex-concave framework. Unlike traditional primal-dual gradient dynamics, this approach does not rely on the gradient of the Lagrangian function. Instead, it treats the uncertain variable as a dynamical state and shows that the globally asymptotically stable equilibrium point of RO dynamics can recover robust optimal solutions for a broad class of convex-concave robust optimization problems.*

The papers present interesting approaches to the addressed problems. However, some aspects require clarification to better understand the contribution of the work.

Major Comment 1: *My concern lies in the motivation behind the problem formulation (4). Does it offer any advantages compared to formulation (3)? The authors should emphasize the main reason for introducing (4), beyond merely presenting it as a more general version of (3).*
Response: We added a remark explaining the regularization terms:

"Our formulation (4) adds regularization terms c_i to the classical $\mathcal{RO}$ problem to prevent singularity when constraints are inactive ($\lambda_i = 0$), improve numerical stability, and allow recovery of the classical problem as $c_i \to 0$. We maintain separate c_i and λ_i rather than a combined $\gamma_i = c_i + \lambda_i$ to preserve the dual variable interpretation and enable our Lyapunov construction."

Section VI (Theorem 2) proves convergence as $\varepsilon \to 0$, demonstrating that solutions approach the classical problem's optimum.

Major Comment 2: *The paper also considers a slight variation of problem formulation (2), presented in (3). The authors should provide more details on this new formulation and explain how it differs from the classical one. For instance, what is the motivation behind presenting the sets U_i as nonlinear inequalities?*
Response: We added a remark:

"Formulation (3) generalizes the standard form (2) by allowing uncertainty in the objective function itself and by explicitly representing uncertainty sets through inequality constraints, which facilitates the min-max-max-min structure needed for our dynamical system approach."

Major Comment 3: *In equation (10), to derive the Lagrangian function, the authors introduce λ_i as multipliers. However, one could instead consider multipliers of the form $\gamma_i := c_i + \lambda_i$, which would reduce to the Lagrangian function of formulation (3). Therefore, I still do not see the novelty or specific role of the c_i terms.*
Response: Maintaining separate terms is crucial because: (i) λ_i preserves its interpretation as a dual variable with convergence properties, (ii) c_i provides independent regularization that can be systematically reduced to zero, (iii) the separation enables our specific Lyapunov function construction in Theorem 1, and (iv) asymptotic analysis in Section VI requires tracking how solutions behave as $c_i \to 0$ while λ_i converges to optimal duals.

Major Comment 4: *In formulation (2), the authors take the maximum over the constraint functions, which significantly increases the problem's complexity compared to the classical robust optimization problem (1). Specifically, if the constraint functions are smooth, taking the maximum introduces non-smoothness, making the problem harder to solve than formulation (1).*
Response: Our continuous-time dynamics naturally handle the non-smoothness introduced by taking the maximum over constraint functions. The dynamics decompose the problem through dual variables (λ_i and v_i) which effectively manage the non-smooth structure. The projection operators in the dynamics ensure proper handling of constraint boundaries, and the continuous-time formulation provides implicit regularization.

Major Comment 5: *Lemma 1 is a well-known result, or am I missing something? Definition 1 for convex problems seems to be stating the KKT conditions, which are then referred to as a saddle point condition, when certain constraint qualifications hold (e.g., Slater's condition).*
Response: Lemma 1 is NOT a standard result—it is a central contribution. We added a remark:

"Classical results such as Sion's minimax theorem or Rockafellar's saddle point theorem require joint concavity in the maximization variables. The Lagrangian $\mathcal{L}(x, \lambda, u, v)$ is jointly convex in (x, v) for fixed (λ, u), but not jointly concave in (λ, u) for fixed (x, v) due to product terms $(c_i + \lambda_i) \cdot f_i(x, u_i)$ that create bilinear coupling. This violation of joint concavity renders existing primal-dual methods inapplicable."

Minor Comment 6: *Note that even if U_0 is a singleton, formulation (2) is defined over general sets U_i, whereas in formulation (3), the variables u_i are specifically defined through the convex functions $h_{i,j}$. My point is that, in this case, formulation (2) remains more general than formulation (3).*
Response: We acknowledge that formulation (2) with general sets U_i is indeed more general than formulation (3) where uncertainty sets are defined through constraint functions h_{ij}. Our formulation (3) focuses on the practically relevant case where uncertainty sets have explicit functional representations, which is necessary for gradient-based dynamics.

Minor Comment 7: *Is the set U_i in formulation (3) still compact under the assumption that the functions $h_{i,j}$ are convex? I believe some continuity assumptions are also needed.*