Response to Reviewer 5

This paper proposes a novel continuous-time dynamical system, termed RO dynamics, for solving robust optimization (RO) problems in a convex-concave framework. While the approach is innovative and has potential, the manuscript requires significant revisions to improve clarity, technical rigor, and presentation. Below are specific comments and suggestions for improvement.

Comment 1: *The paper discusses contributions and motivation in multiple sections, but these points are not clearly articulated or cohesively presented. To improve readability and impact, I recommend reorganizing the introduction to explicitly highlight the key contributions.*
Response: We reorganized the Introduction to clearly articulate key contributions:

"We introduce a continuous-time dynamical system that provably converges to the optimal robust solution... this work presents the first continuous-time dynamical system specifically designed for solving robust optimization problems... The non-classical structure of the dynamics necessitates the construction of a novel Lyapunov function to analyze stability."

Comment 2: *Algorithm (23) is presented in isolation without sufficient explanation or context. The authors should provide a detailed discussion immediately after introducing the algorithm, including the intuition behind its design, a clear comparison with existing methods to highlight key differences, and specific improvements or advantages over traditional approaches.*
Response: After presenting the dynamics in equation (23), we added explanation of the state vector structure describing the role of each component: x dynamics (gradient descent on Lagrangian), λ_i dynamics (dual ascent with projection), u_i dynamics (worst-case uncertainty search), and v_i dynamics (dual variables for uncertainty constraints). We also provided two illustrative examples (min-max problem and one uncertain constraint) that highlight key differences from standard primal-dual systems.

Comment 3: *The paper lacks a detailed theoretical analysis of the proposed algorithm's convergence performance compared to existing methods.*
Response: Theorem 1 establishes global asymptotic stability with rigorous Lyapunov analysis. While explicit $O(\cdot)$ convergence rate bounds are not provided, the theorem guarantees global convergence to the optimal $\mathcal{RO}$ solutions.

Comment 4: *The structure of the manuscript needs improvement for better readability, as equations (5) and (6) are referenced in Assumption 2 before they are formally introduced in the text.*
Response: The strong duality remark has been moved to after the "Regularized Formulation" subsection where the referenced equations are formally defined, eliminating the forward reference issue.

Comment 5: *The conditions in Assumption 2 seem restrictive. Can they be relaxed, as in Ref. [34]? If not, please provide a detailed explanation.*
Response: Assumption 2 now includes: *"a regularity condition such as Slater condition"* which is the standard constraint qualification in robust optimization literature ensuring strong duality.

Comment 6: *The need for Assumption 1 should be justified. Can the framework be extended to more general cases?*
Response: Assumption 1 (convexity in decision variable, concavity in uncertainty) ensures computational tractability and is satisfied by most practical $\mathcal{RO}$ problems, particularly when uncertainty enters affinely. This assumption is fundamental to the robust optimization framework.

Comment 7: *The meaning of h_{ij} and K_i in equation (3) should be clearly understood, and the purpose of introducing them here should be explicitly justified.*
Response: We added clarification:

"Here, $h_{ij}(u_i)$ represents the j-th constraint function defining the i-th uncertainty set $\mathcal{U}_i$, and K_i denotes the total number of constraints that define the uncertainty set $\mathcal{U}_i$."

Comment 8: *The Lagrangian function (15) for the RO problem (4) appears to be straightforward, raising questions about the necessity of the lengthy and intricate analysis preceding it.*
Response: We added Remark stating:

"The Lagrangian (16) unifies the nested min-max structure and permits continuous-time dynamics over all variables simultaneously. The saddle property derived from this formulation is essential for Lyapunov stability analysis."

Comment 9: *The abbreviation "RC" in the line before equation (9) has been explained earlier and could be deleted here. The meaning of the abbreviation "RHS" after inequality (51) needs to be explained.*
Response: We reviewed the manuscript and ensured abbreviations are properly defined on first use without redundancy.

Comment 10: *In Section V-D, since the Appendix B only contains conclusions without detailed proofs, it is recommended to incorporate these conclusions directly into the main text.*
Response: The key results from Appendix B regarding solution existence and properties are incorporated into the main theoretical development.

Comment 11: *We note that the proof of Lemma 4 is not provided in Ref. [41], and this issue needs to be addressed.*
Response: We verified that the Lemma 4.4 in [40] is the correct reference which proves our Lemma 4.