

Robust Optimization via Continuous-Time Dynamics

Keivan Ebrahimi, Nicola Elia, and Umesh Vaidya

Abstract—We propose a continuous-time dynamical system for solving robust optimization problems in a general setting where the objective is convex in the decision variables and concave in the uncertainty. Unlike classical primal–dual gradient dynamics developed for standard optimization problems, the proposed dynamics do not rely on the gradient of a Lagrangian function to define the vector field. We establish that the globally asymptotically stable equilibrium of the proposed system recovers the robust optimal solution without requiring problem-specific reformulations. The continuous-time formulation is well suited for real-time operation in dynamic environments and naturally supports decentralized implementations. To demonstrate the effectiveness and generality of the approach, we present simulation studies including a nonlinear optimization problem with no tractable robust counterpart, as well as a robust localization and placement problem with time-varying anchor positions that is solved in a decentralized manner using the proposed dynamics.

I. INTRODUCTION

With emerging applications that require solving real-time optimization problems in a reactive manner, this paper describes how interacting dynamical systems can find a robust solution for a broad class of robust optimization problems. This is particularly relevant in situations where physical systems must be steered towards optimal operating conditions using gradient-based feedback mechanisms. Specifically, we propose a real-time approach to solving robust optimization problems, which has become increasingly important in recent years [1]. Optimization problems often involve various forms of uncertainty in the problem data. Two different approaches can be used: stochastic optimization, where uncertainty is treated as a random variable, or robust optimization (\mathcal{RO}), where uncertainty is assumed to be deterministic and bounded. Unlike stochastic optimization, \mathcal{RO} does not require any known probability distributions in the problem data. Instead, \mathcal{RO} assumes that the uncertain data reside within a predefined uncertainty set, for which constraint violation cannot be tolerated. For more information on robust and stochastic optimization-based approaches, see [2], [3], and the references therein. Early works on \mathcal{RO} include [4], which considered robust linear optimization with ellipsoidal uncertainty sets, and [5], which presented exact solutions of inexact linear

programs as a simple case of \mathcal{RO} . The problem of robust linear programming was studied in [6], while robust conic-quadratic optimization and robust semi-definite optimization were discussed in [7] and [8], respectively. Refer to [3] and [9] for a survey of different \mathcal{RO} problem solutions. [10] and [11] showed that some of the machine learning algorithms such as the norm-regularized support vector machine and the Lasso problem could be interpreted as \mathcal{RO} problems. In addition to that, several papers focused on making machine learning problems robust against outliers, parameter uncertainties, and data perturbations by techniques such as adversarial training [12]–[18].

One of the main standard approaches for solving \mathcal{RO} problems involves constructing a robust counterpart (RC) equivalent to the \mathcal{RO} problem [2]. This widely used method essentially tries to find a deterministic equivalent to \mathcal{RO} problem through a reformulation. In this sense, the practicality of robust programming depends on whether or not its RC is computationally tractable. An overview of different \mathcal{RO} problems with tractable conjugates can be found in [19] Table 1 for some of which RC cannot be found. The reformulation approach to solving the \mathcal{RO} problem, which is often a challenging, albeit usually convex, optimization problem, has the deficiency of suffering from case-by-case scenarios depending on the specific form of the uncertainty constraint and the specific form of the uncertainty set. In other words, depending on how simple the uncertainty set looks and based on the optimization problem type (whether it is linear programming, quadratic programming, second-order cone programming, semi-definite programming, etc.), an RC is being calculated and provided at hand. These reformulations may be computationally more expensive than other approaches [20]. Hence, the absence of a unified framework for solving a general class of \mathcal{RO} problems through continuous-time dynamics is strongly felt.

By calculating a concave conjugate of nonlinear constraint functions and supporting the function of uncertainty sets, [21] and [19] used the Fenchel duality to obtain a tractable RC for new classes of robust nonlinear optimization (RNO) problems. However, when no closed form is available for the convex conjugate function of an uncertainty set or convex/concave conjugates of a non-linear constraint, these approaches still cannot obtain the RC for many sets of nonlinear uncertainties and constraints.

An alternative randomized approach based on constraint sampling is an approximate probabilistic relaxation solution to \mathcal{RO} problems as seen in [22] and [23] where a finite set of high-dimensional deterministic optimization problems obtained from sampling are solved. This approach does not require concavity of the constraint in the uncertain variable but due to the large number of required scenarios to approximate

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