

Red Black Tree

Insertion

1. **Z** is Root
Color Z, black
2. **Z.Uncle** and **Z.Parent** are red
Recolor Z.Parent, Z.grandParent & Z.uncle
3. **Z.Uncle** is black (**Z**, **Z.Parent** & **Z.GrandParent** for a Triangle)
Rotate Z.Parent opposite direction of Z:
 - If **Z** is the left child, rotate right
 - If **Z** is the right child, rotate left
4. **Z.Uncle** is black (**Z**, **Z.Parent** & **Z.GrandParent** for a Line)
Rotate Z.GrandParent the opposite direction of Z and recolor Z.Parent & Z.GrandParent:
 - If **Z** is the left child, rotate right. Then recolor **Z.Parent** & **Z.GrandParent**
 - If **Z** is the right child, rotate left. Then recolor **Z.Parent** & **Z.GrandParent**

4 scenarios

1. **Z** = root → *color black*
2. **Z.uncle** = red → *recolor*
3. **Z.uncle** = black (triangle) → *rotate Z.parent*
4. **Z.uncle** = black (line) → *rotate z.grandparent & recolor*

Deletion

1. Initial Step - Choosing replacement
 - If the node we deleted has two **NIL** children, its replacement **X** is **NIL**
 - If the node we deleted has 1 **NIL** child and 1 **Non-NIL** child, its replacement is the **Non-NIL** child
 - If the node we deleted has 2 **Non-NIL** children, set the **X** to replacement's (successor's) right child before the replacement is spliced out.
2. Initial Step - Coloring
 - If the node we deleted is **Red** its replacement is **Red** or **NIL**, we are done
 - If the node we deleted is **Red** its replacement is **Black**, recolor the replacement and Proceed to appropriate case
 - If the node we deleted is **Black** its replacement is **Red**, recolor the replacement and Proceed to appropriate case
 - If the node we deleted is **Black** its replacement is **Black** Proceed to appropriate case
3. Cases
 - 1) Node **X** is **Red**
Color **X**, **Black**
 - 2) Node **X** is **Black** & Its sibling **W** is **Red**
 - I. Color **W**, **Black**
 - II. Color **X.Parent** **Red**
 - III. Rotate **X.Parent** in the same direction of **X**
 - If **X** is **left** child, rotate **left**
 - If **X** is **right** child, rotate **right**
 - IV.
 - If **X** is **left** child, set **W** = **X.Parent.Right**
 - If **X** is **Right** child, set **W** = **X.Parent.Left**
 - 3) Node **X** is **Black** & Its sibling **W** is **Black** and both **W**'s children are **Black**
 - I. Color **W**, **Red**
 - II. Set **X** = **X.Parent**
 - If **New X** is **Red**, color **X** **Black**
 - If **New X** is **Black**, decide on other cases
 - 4) Node **X** is **Black** & Its sibling **W** is **Black** and
 - a. If **X** is left child, **W**'s left child is **Red** & **W**'s right child is **Black**
 - I. Color **W.Left**, **Black**
 - II. Color **W**, **Red**
 - III. Rotate **W** to the right
 - IV. Set **W** = **X.Parent.Right**
 - V. Proceed to case 5
 - b. If **X** is right child, **W**'s right child is **Red** & **W**'s left child is **Black**
 - I. Color **W.Right**, **Black**

- II. Color **W**, Red*
- III. Rotate **W** to the left*
- IV. Set **W** = **X.Parent.Left***
- V. Proceed to case 5*

5) Node **X** is **Black** & Its sibling **W** is **Black** and

- a. If **X** is left child & **W**'s right child is **Red**
 - I. Color **W** the same color as **X.Parent***
 - II. Color **X.Parent**, **Black***
 - III. Color **W.Right**, **Black***
 - IV. Rotate **X.Parent** to the left*
- b. If **X** is right child & **W**'s left child is **Red**
 - I. Color **W** the same color as **X.Parent***
 - II. Color **X.Parent**, **Black***
 - III. Color **W.Left**, **Black***
 - IV. Rotate **X.Parent** to the right*