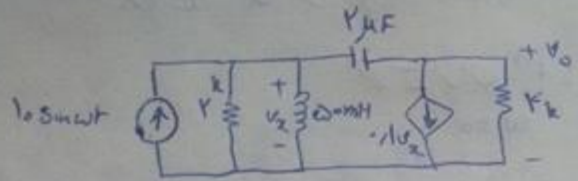
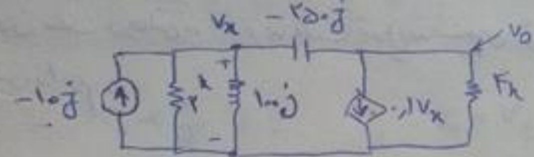
جواب سوالات Homwork 6

-۱

۱) حالت فرکانسی مدار مقابل به صورت زیر است:



$$C \rightarrow \frac{1}{j2\mu \times 2k} = -250j$$



$$L \rightarrow 2k \times 50^{m} j = 100j$$

$$10\sin\omega t \longrightarrow 10\cos(\omega t - 90)$$

فازوری ↓

$$10e^{-90}$$

$$10(\cos 90 - j\sin 90)$$

$$= -10j$$

$$\begin{cases} \dfrac{V_x}{2k} + \dfrac{V_x}{100j} + 10j + \dfrac{V_x - V_o}{-250j} = 0 \\[2ex] \dfrac{V_o}{4k} + \dfrac{V_o - V_x}{-250j} + 0.1 V_x = 0 \end{cases}$$

$$\begin{cases} (12+j)V_x + 8V_o = 20000 \\ (19+400j)V_x + (j-16)V_o = 0 \end{cases}$$

$$V_o = \frac{320000 + 8\times10^{4} j}{321 + 3202j}$$

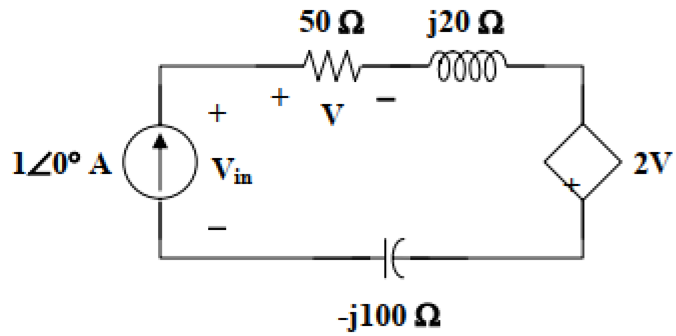
$$V_o = \frac{800.6397 \angle 87.7^\circ}{3220 \angle 84.27^\circ} \Rightarrow V_o = 2486 \angle 3.43^\circ$$

$$V_o(t) = 2486\cos(2000t + 3.43^\circ)$$

-٢

$$2\text{ mH} \longrightarrow j\omega L = j(10\times10^3)(2\times10^{-3}) = j20$$

$$1\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10\times10^3)(1\times10^{-6})} = -j100$$

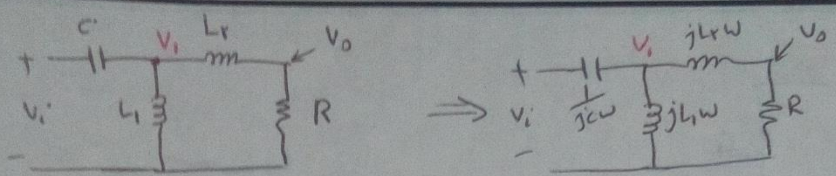


$$\mathbf{V} = (1\angle0°)(50) = 50$$

$$\mathbf{V}_{in} = (1\angle0°)(50 + j20 - j100) + (2)(50)$$
$$\mathbf{V}_{in} = 50 - j80 + 100 = 150 - j80$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{1\angle0°} = \underline{\mathbf{150 - j80\ \Omega}}$$

(۳



kcl در V_1 $\Rightarrow \dfrac{V_1-V_i}{\frac{1}{jC\omega}} + \dfrac{V_1}{jL_1\omega} + \dfrac{V_1-V_o}{jL_2\omega} = 0$ (1)

kcl در ۲ $\rightarrow \dfrac{V_o}{R} + \dfrac{V_o-V_1}{jL_2\omega} = 0 \rightarrow V_1 = \dfrac{R+jL_2\omega}{R} V_o$ (۲

از (۱) و (۲) $H = \dfrac{V_o}{V_i} = \dfrac{-\omega^2 R L_1 C}{R(1-\omega^2 L_1 C) + j\omega(L_1+L_2-\omega^2 L_1 L_2 C)}$

صورت و مخرج را در مزدوج مخرج ضرب می‌کنیم:

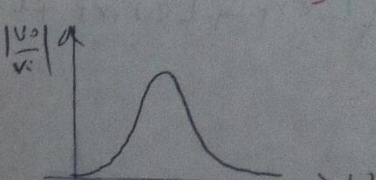
$$\frac{V_o}{V_i} = \frac{-\omega^2 R L_1 C\left[R(1-\omega^2 L_1 C) - j\omega(L_1+L_2-\omega^2 L_1 L_2 C)\right]}{R^2(1-\omega^2 L_1 C)^2 + \omega^2(L_1+L_2-\omega^2 L_1 L_2 C)^2}$$

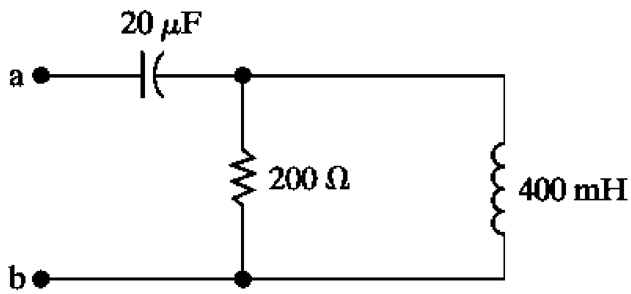
$$\frac{V_o}{V_i} = \frac{-\omega^2 R^2 L_1 C(1-\omega^2 L_1 C) + j\omega^3 R L_1 C(L_1+L_2-\omega^2 L_1 L_2 C)}{R^2(1-\omega^2 L_1 C)^2 + \omega^2(L_1+L_2-\omega^2 L_1 L_2 C)^2}$$

$$\left|\frac{V_o}{V_i}\right| = \frac{\sqrt{\left[-\omega^2 R^2 L_1 C(1-\omega^2 L_1 C)\right]^2 + \left[\omega^3 R L_1 C(L_1+L_2-\omega^2 L_1 L_2 C)\right]^2}}{\sqrt{\left[R^2(1-\omega^2 L_1 C)^2 + \omega^2(L_1+L_2-\omega^2 L_1 L_2 C)^2\right]^2}}$$

$\omega \rightarrow 0 \quad \left|\frac{V_o}{V_i}\right| = 0$

$\omega \rightarrow \infty \quad \left|\frac{V_o}{V_i}\right| = 0$





[a] $$\frac{1}{j\omega C} + R\|j\omega L = \frac{1}{j\omega C} + \frac{j\omega RL}{j\omega L + R}$$

$$= \frac{j\omega L + R - \omega^2 RLC}{j\omega C(j\omega L + R)}$$

$$= \frac{(R - \omega^2 RLC + j\omega L)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)}$$

The denominator in the expression above is purely real; set the imaginary part of the numerator in the above expression equal to zero and solve for ω:

$$-\omega^3 L^2 C - \omega R^2 C + \omega^3 R^2 C^2 L = 0$$

$$-\omega^2 L^2 - R^2 + \omega^2 R^2 LC = 0$$

$$\omega^2 = \frac{R^2}{R^2 LC - L^2} = \frac{200^2}{200^2(0.4)(20 \times 10^{-6}) - (0.4)^2} = 250{,}000$$

$$\therefore \qquad \omega = 500 \text{ rad/s}$$

[b] $$Z_{\mathrm{ab}}(500) = -j100 + \frac{(200)(j200)}{200 + j200} = 100\,\Omega$$