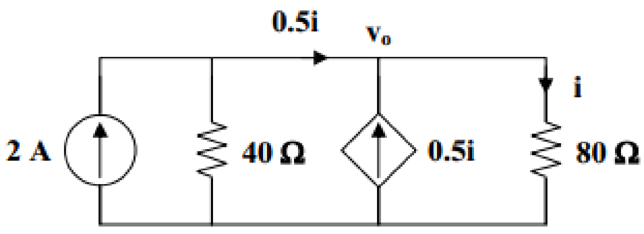
جواب تمرینات سری چهارم

۲-

Before $t = 0$, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.



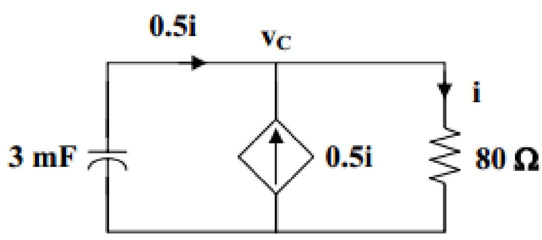
(a)

$$0.5i = 2 - \frac{v_o}{40}, \qquad i = \frac{v_o}{80}$$

Hence, $\frac{1}{2}\frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$

$$i = \frac{v_o}{80} = \underline{\mathbf{0.8\ A}}$$

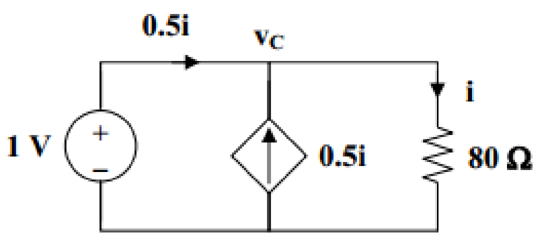
After $t = 0$, the circuit is as shown in Fig. (b).



(b)

$$v_C(t) = v_C(0)e^{-t/\tau}, \qquad \tau = R_{th}C$$

To find R_{th}, we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



(c)

$$i = \frac{v_C}{80} = \frac{1}{80}, \qquad i_o = 0.5i = \frac{0.5}{80}$$

$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160\,\Omega, \qquad \tau = R_{th}C = 480$$

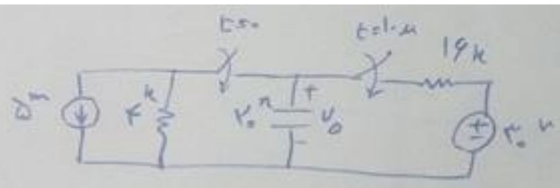
$$v_C(0) = 64 \text{ V}$$

$$v_C(t) = 64e^{-t/480}$$

$$0.5i = -i_C = -C\frac{dv_C}{dt} = -3\left(\frac{1}{480}\right)64e^{-t/480}$$

$$i(t) = \underline{0.8e^{-t/480}\ u(t)\text{A}}$$

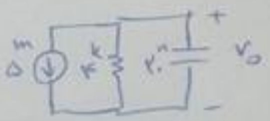
۳) برای حل این نوع سوالات که چند حالت کلیدزنی دارند باید مدار را به ازای زمان‌های مختلف حل کرد.



$V_o(0) = 0$

چون خازن دارای ولتاژ اولیه نیست و همچنین در t<0 منبع ولتاژ هنوز وصل نشده است.

$0 < t < 10^{ms}$



$V_o(\infty) = -4^k \times 5^m = -20^v$

پس معادله V_o در بازه $0<t<10^{m}$ به صورت زیر است:

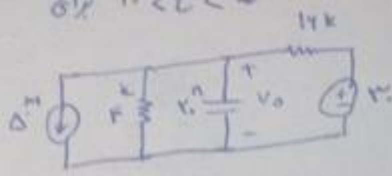
$\tau = R_{eq}C = 4^k \times 20^n = 80^{ms}$

$V_o = V_o(\infty) + (V_o(0) - V_o(\infty))e^{-t/\tau}$

$V_o = -20 + (0 - (-20))e^{-t/80^m}$

$V_o = -20(1 - e^{-12500t}) \quad 0<t<10^{ms}$

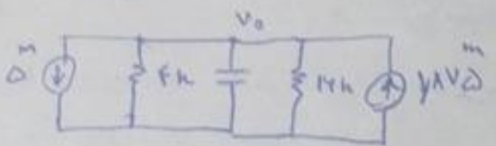
برای $10^{ms} < t < \infty$



چون ولتاژ خازن تغییر آنی ندارد پس ولتاژ خازن در $V_o(10^-)$ برابر است با ولتاژ خازن در $V_o(10^+)$ پس داریم: (از معادله ۲۰ در بازه $0<t<10^m$ بدست می‌آید)

$V_o(10) = -20(1 - e^{-12500 \times 10^m}) = -2.35$

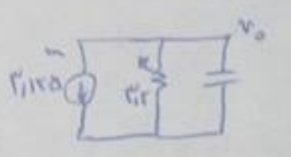
$V_o(\infty) = ?$



$R_{eq} = 14 \| 4 = 3.2^k$

$I_{eq} = 5 - 1.875 = 3.125^m$

|||

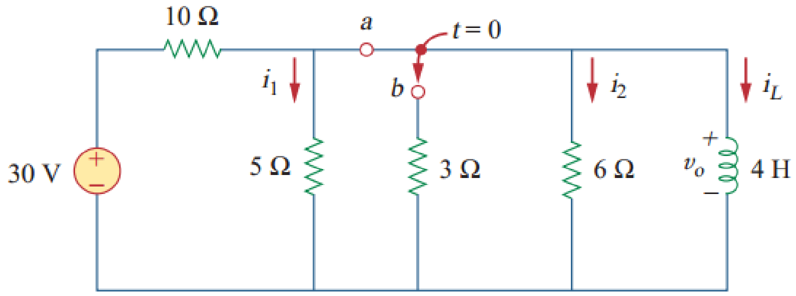


$V_o(\infty) = -3.2^k \times 3.125^m = -10$

$\tau = R_{eq}C = 3.2^k \times 20^n = 64^{\mu}$

$V_o(t) = -10 + (-2.35 + 10)e^{-\frac{(t-10^m)}{64^\mu}}$

$$V_o(t) = \begin{cases} 0 & t<0 \\ -20(1-e^{-12500t}) & 0<t<10^m \\ -10 + 7.65\,e^{-15625(t-10^m)} & 10^m<t<\infty \end{cases}$$



(a) When the switch is in position A, the 5-ohm and 6-ohm resistors are short-circuited so that

$$\underline{i_1(0) = \quad i_2(0) = v_o(0) = 0}$$

but the current through the 4-H inductor is $i_L(0) = 30/10 = 3A$.

(b) When the switch is in position B,

$$R_{Th} = 3//6 = 2\Omega, \qquad \tau = \frac{L}{R_{Th}} = 4/2 = 2\sec$$

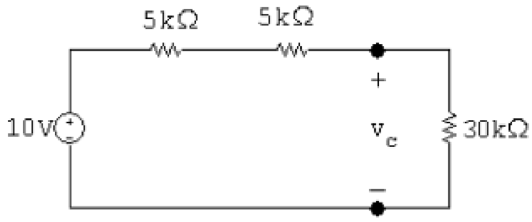
$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau} = 0 + 3e^{-t/2} = \underline{3e^{-t/2}\ A}$$

(c) $i_1(\infty) = \dfrac{30}{10+5} = \underline{2\ A}, \qquad i_2(\infty) = -\dfrac{3}{9} i_L(\infty) = \underline{0\ A}$

$$v_o(t) = L\frac{di_L}{dt} \longrightarrow \underline{v_o(\infty) = 0\ V}$$

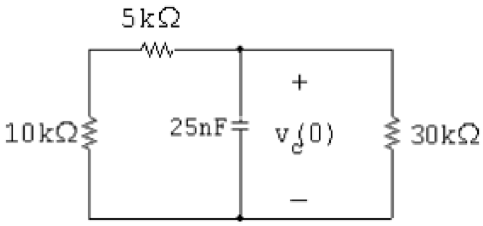
Note that for $t > 0$, $v_o = (10/15)v_c$, where v_c is the voltage across the 25 nF capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{30}{40}(10) = 7.5\,\text{V}$$

$0 \leq t \leq 0.2\,\text{ms}$:



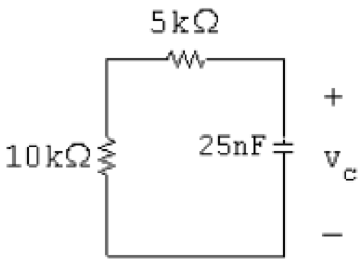
$$\tau = R_eC, \qquad R_e = 15{,}000\|30{,}000 = 10\,\text{k}\Omega$$

$$\tau = (10 \times 10^3)(25 \times 10^{-9}) = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$$

$$v_c = 7.5e^{-4000t}\,\text{V}, \qquad t \geq 0$$

$$v_c(0.2\,\text{ms}) = 7.5e^{-0.8} = 3.37\,\text{V}$$

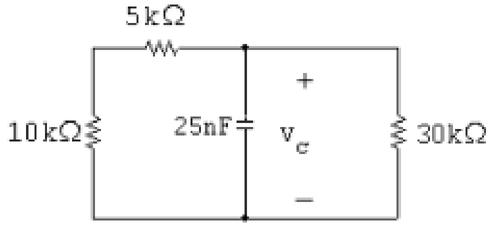
$0.2\,\text{ms} \leq t \leq 0.8\,\text{ms}$:



$$\tau = (15 \times 10^3)(2.5 \times 10^{-9}) = 375\,\mu\text{s}, \qquad \frac{1}{\tau} = 2666.67$$

$$v_c = 3.37e^{-2666.67(t-200\times10^{-6})}\ \text{V}$$

$0.8\,\text{ms} \le t <$:



$$\tau = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$$

$$v_c(0.8\,\text{ms}) = 3.37e^{-2666.67(800-200)10^{-6}} = 3.37e^{-1.6} = 0.68\,\text{V}$$

$$v_c = 0.68e^{-4000(t-0.8\times10^{-3})}\ \text{V}$$

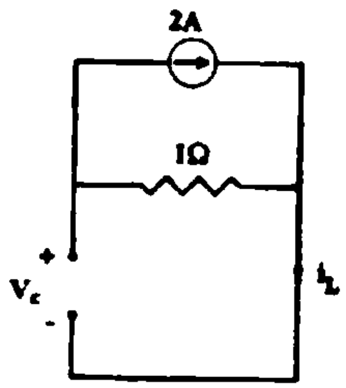
$$v_c(1\,\text{ms}) = 0.68e^{-4000(1-0.8)10^{-3}} = 0.68e^{-0.8} = 0.306\,\text{V}$$

$$v_o = (10/15)(0.306) = 0.204\,\text{V}$$

$$\frac{dv_c}{dt}(0^+) = \frac{1}{C} i_c(0^+)$$

$$\frac{di_L}{dt}(0^+) = \frac{1}{L} V_L(0^+)$$

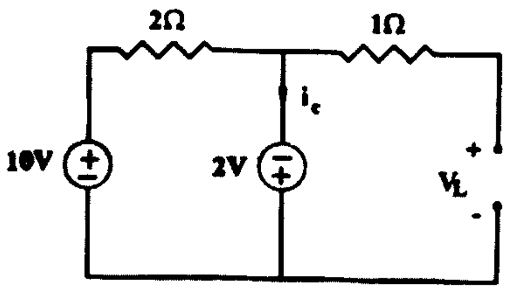
$in \qquad t = 0^-$



$i_L(0) = 0$

$V_c(0) = -2$

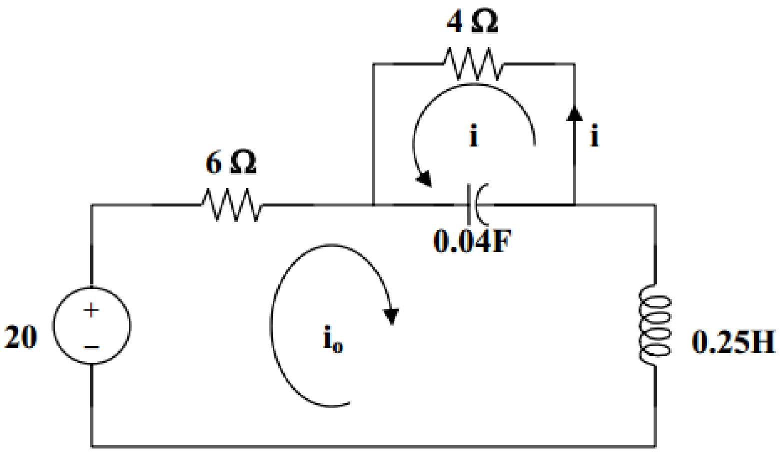
$in \qquad t = 0^+$



$$\begin{aligned} V_L(0^+) &= -2V \\ i_C(0^+) &= 6A \end{aligned} \Rightarrow \begin{cases} \dfrac{dV_C(0^+)}{dt} = 1 \times 6 = 6 \\ \dfrac{di_L(0^+)}{dt} = \dfrac{1}{1} \times -2 = -2 \end{cases}$$

For $t < 0$, $i(0) = 0$ and $v(0) = 0$.

For $t > 0$, the circuit is as shown below.



Applying KVL to the larger loop,

$$-20 + 6i_o + 0.25di_o/dt + 25\int (i_o + i)dt = 0 \qquad (1)$$

For the smaller loop, $\quad 4i + 25\int (i + i_o)dt = 0$ or $\int (i + i_o)dt = -0.16i \qquad (2)$

Taking the derivative, $\quad 4di/dt + 25(i + i_o) = 0$ or $i_o = -0.16di/dt - i \qquad (3)$

and $\quad di_o/dt = -0.16d^2i/dt^2 - di/dt \qquad (4)$

From (1), (2), (3), and (4), $\quad -20 - 0.96di/dt - 6i - 0.04d^2i/dt^2 - 0.25di/dt - 4i = 0$

Which becomes, $\quad d^2i/dt^2 + 30.25di/dt + 250i = -500$

This leads to, $\quad s^2 + 30.25s + 250 = 0$

$$\text{or } s_{1,2} = \frac{-30.25 \pm \sqrt{(30.25)^2 - 1000}}{2} = -15.125 \pm j4.608$$

This is clearly an underdamped response.

Thus, $i(t) = I_s + e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t))A$.

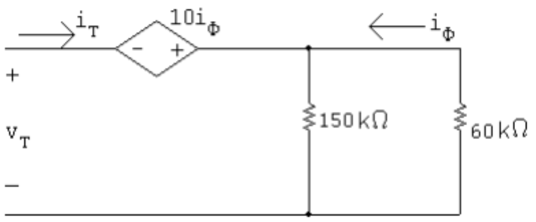
At $t = 0$, $i_o(0) = 0$ and $i(0) = 0 = I_s + A_1$ or $A_1 = -I_s$. As t approaches infinity, $i_o(\infty) = 20/10 = 2A = -i(\infty)$ or $i(\infty) = -2A = I_s$ and $A_1 = 2$.

In addition, from (3), we get $di(0)/dt = -6.25i_o(0) - 6.25i(0) = 0$.

$di/dt = 0 - 15.125\, e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t)) + e^{-15.125t}(-A_1 4.608\sin(4.608t) + A_2 4.608\cos(4.608t))$. At $t=0$, $di(0)/dt = 0 = -15.125A_1 + 4.608A_2 = -30.25 + 4.608A_2$ or $A_2 = 30.25/4.608 = 6.565$.

This leads to,

$$i(t) = \underline{\mathbf{(-2 + e^{-15.125t}(2\cos(4.608t) + 6.565\sin(4.608t))\ A}}$$



$$v_T = -10i_\phi + i_T\left(\frac{(150)(60)}{210}\right) = -10\frac{-i_T(150)}{210} + i_t\frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500 + 9000}{210} = 50\,\Omega$$

$$V_o = \frac{4000}{10{,}000}(50) = 20\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{20}{50} = -0.4\,\text{A}$$

$$\frac{i_C(0)}{C} = \frac{-0.4}{8 \times 10^{-6}} = -50{,}000$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(51.2 \times 10^{-3})(8 \times 10^{-6})}} = 1562.5 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50)(8 \times 10^{-6})} = 1250 \text{ rad/s}$$

$\alpha^2 < \omega_0^2$ so the response is underdamped

$$\omega_d = \sqrt{1562.5^2 - 1250^2} = 937.5$$

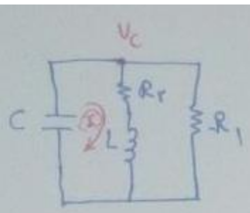
$$v_o = B_1 e^{-1250t}\cos 937.5t + B_2 e^{-1250t}\sin 937.5t$$

$$v_o(0) = B_1 20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{i_C(0)}{C}$$

$$\therefore \qquad -1250(20) + 937.5B_2 = -50{,}000 \qquad \text{so} \qquad B_2 = -26.67$$

$$v_o = 20e^{-1250t}\cos 937.5t - 26.67e^{-1250t}\sin 937.5t\,\text{V}, \qquad t \geq 0$$



(KCL) $\Rightarrow$ $i_C + i_L + \frac{v_C}{-R_1} = 0$

$$C\frac{dv_C}{dt} + i_L - \frac{v_C}{R_1} = 0 \qquad (1)$$

$(KVL)_I \Rightarrow v_C = R_2 i_L + L\frac{di_L}{dt} \qquad (2)$

$$(1,2) \Rightarrow R_2 C\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} + i_L - \frac{R_2}{R_1} i_L - \frac{L}{R_1}\frac{di_L}{dt} = 0$$

$$LC\frac{d^2 i_L}{dt^2} + \left(R_2 C - \frac{L}{R_1}\right)\frac{di_L}{dt} + \left(1 - \frac{R_2}{R_1}\right) i_L = 0$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{LC}\left(R_2 C - \frac{L}{R_1}\right)\frac{di_L}{dt} + \frac{1}{LC}\left(1 - \frac{R_2}{R_1}\right) i_L = 0$$

شرط نوسانی بودن: $\begin{cases} 2\alpha = 0 \\ \omega_0^2 > 0 \end{cases}$

$$2\alpha = \frac{1}{LC}\left(R_2 C - \frac{L}{R_1}\right) = 0 \qquad R_1 = \frac{L}{R_2 C}$$

$$\omega_0^2 = \frac{1}{LC}\left(1 - \frac{R_2}{R_1}\right) > 0 \qquad R_1 > R_2$$