جواب سوالات Homwork 8

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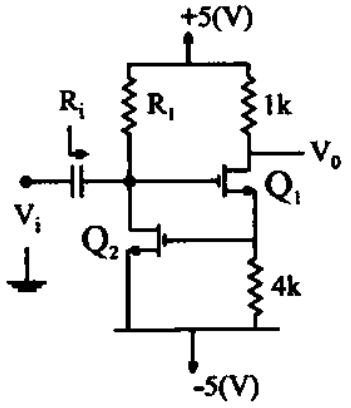


الف):

$$v_0 = 4v \quad \Rightarrow \quad I_{D_1} = \frac{5-4}{1k} = 1mA$$

$$V_{GS_2} = 4k(1mA) = 4V \quad \Rightarrow \quad I_D = k\left(V_{GS_2} - V_T\right)^2 = 1mA$$

$$R_1 = \frac{5-(-5)-V_{GS_1}-V_{GS_2}}{1mA} = 2k\Omega$$

ب)

$$R_i = \frac{V_i}{I_i}$$

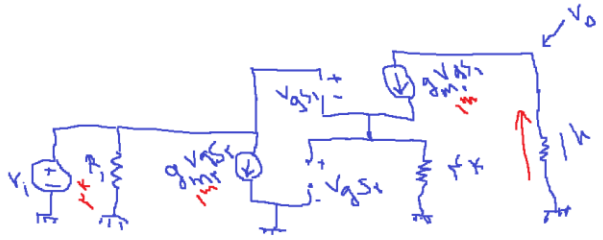
$$v_i = v_{gs_1} + g_m v_{gs_1}(4k) \quad , \quad g_m = k\left(V_{GS} - V_T\right) = 1\frac{mA}{V}$$

$$V_i = 5\left(V_{gs_1}\right) \quad , \quad V_{gs_2} = g_m v_{gs_1}(4k) = 4vgs_1$$

$$v_i = \frac{5}{4}\left(v_{gs_2}\right)$$

$$I_i = \frac{V_i}{R_1} + g_m v_{gs_2}$$

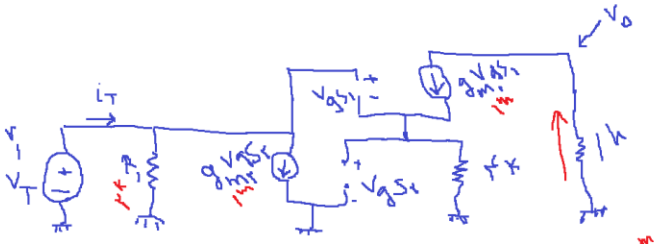
$$\frac{v_i}{I_i} = R_i = 0.77\ k\Omega$$



$$V_o = -1^k g_{m_1} V_{gs_1} = -V_{gs_1}$$

$$V_i = V_{gs_1} + r^k g_{m_1} V_{gs_1} = 6 V_{gs_1}$$

$$\frac{V_o}{V_i} = -\frac{1}{6}$$

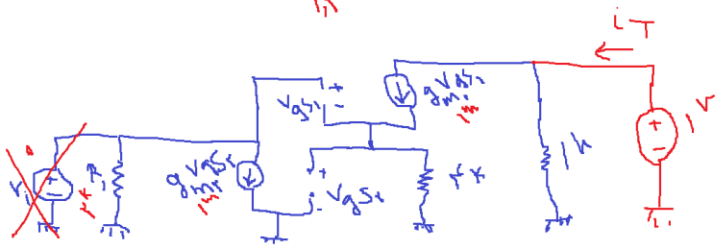


$$i_T = i_{D_2} + g_{m_2} V_{gs_2} = -D^m + i\lambda^m = 1_1 \mu^m$$

$$1 = V_{gs_1} + V_{gs_2} \quad , \quad V_{gs_2} = r^k g_{m_1} V_{gs_1} = r V_{gs_1}$$

$$1 = \frac{1}{r} V_{gs_2} + V_{gs_2} \rightarrow V_{gs_2} = \frac{r}{D}$$

$$R_i = \frac{1}{1 \mu^m} = V79\ \Omega$$



$$i_T = 1^m + g_{m_1} V_{gs_1} = 1^m$$

$$\left.\begin{aligned} V_{gs_1} + V_{gs_2} &= 0 \\ V_{gs_2} &= r^k g_{m_1} V_{gs_1} \end{aligned}\right\} \quad V_{gs_1} = V_{gs_2} = 0$$

$$R_o = \frac{1^V}{1^m} = 1^k$$

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الف)

$$I_{D_1} = I_{D_2} \Rightarrow \frac{K_1}{2}(V_{GS_1} - V_{T_1})^2 = \frac{K_2}{2}(V_{GS_2} - V_{T_2})^2$$

از طرفی: $\begin{cases} K_1 = K_2 \\ V_{T_1} = V_{T_2} \end{cases} \Rightarrow \frac{K_1}{2}(V_{GS_1} - V_{T_1})^2 = \frac{K_1}{2}(V_{GS_2} - V_{T_1})^2$

$$\Rightarrow (V_{GS_1} - V_{T_1})^2 = (V_{GS_2} - V_{T_1})^2 \Rightarrow V_{GS_1} = V_{GS_2}$$

$$I_{D_1} = I_{D_2} \Rightarrow \frac{K_1}{2}(V_{GS_1} - V_{T_1})^2 = \frac{K_2}{2}(V_{GS_2} - V_{T_2})^2$$

ب)

از طرفی: $\begin{cases} V_{G_1} = 10^V \\ V_{S_1} = V_O \end{cases}$, $\begin{cases} V_{G_2} = V_O \\ V_{S_2} = 0^V \end{cases} \Rightarrow (10 - V_O) = (V_O - 0) \Rightarrow V_O = 5^V$

$$I_{D_1} = I_{D_2} \Rightarrow \frac{k_1}{2}(V_{GS_1} - V_{T_1})^2 = \frac{k_2}{2}(V_{GS_2} - V_{T_2})^2$$

ج)

از طرفی: $\begin{cases} K_1 = \frac{K_2}{4} \\ V_{T_1} = V_{T_2} \end{cases} \Rightarrow \frac{K_2}{2\times 4}(V_{GS_1} - V_{T_1})^2 = \frac{K_2}{2}(V_{GS_2} - V_{T_1})^2$

$$\Rightarrow V_{GS_1} - V_{T_1} = 2(V_{GS_2} - V_{T_1}) \quad , \quad \begin{cases} V_{G_1} = 10 \\ V_{S_1} = V_O \end{cases} , \quad \begin{cases} V_{G_2} = V_O \\ V_{S_2} = 0V \end{cases}$$

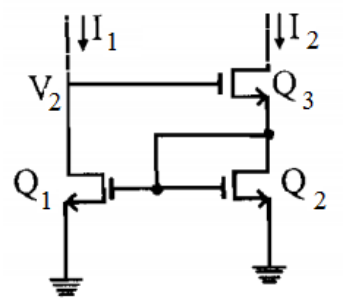
$$\Rightarrow (10 - V_O - V_{T_1}) = 2(V_O - 0 - V_{T_1}) \Rightarrow 3V_O = 10 + V_{T_1} \Rightarrow V_O = \frac{10 + V_{T_1}}{3}$$

$$I_{D_1} = I_{D_2} \Rightarrow \frac{K_1}{2}(V_{GS_1} - V_{T_1})^2 = \frac{K_2}{2}(V_{GS_2} - V_{T_2})^2 \quad , \quad \begin{cases} K_1 = K_2 \\ V_{T_1} = 4V_{T_2} \end{cases}$$

$$\Rightarrow (V_{GS_1} - ۴V_{T_2})^۲ = (V_{GS_2} - V_{T_2})^۲ \Rightarrow V_{GS_1} - ۴V_{T_2} = V_{GS_2} - V_{T_2}$$

$$\begin{cases} V_{G_1} = ۱۰ \\ V_{S_1} = V_O \end{cases}, \quad \begin{cases} V_{G_2} = V_O \\ V_{S_2} = ۰ \end{cases} \Rightarrow (۱۰ - V_O - ۴V_{T_2}) = (V_O - ۰ - V_{T_2}) \Rightarrow ۲V_O = ۱۰ - ۳V_{T_2}$$

$$\Rightarrow V_O = \frac{۱۰ - ۳V_{T_2}}{۲}$$



$$I_1=\frac{K_1}{2}(V_{GS_1}-V_{T_1})^2 \Rightarrow 1mA=\frac{2}{2}(V_{GS_1}-2)^2 \Rightarrow 1=(V_{G_1}-V_{S_1}-2)^2$$

$$V_{S_1}=0V \Rightarrow 1=(V_{G_1}-2)^2 \Rightarrow \begin{cases} V_{G_1}-2=1 \Rightarrow V_{G_1}=3V \\ V_{G_1}-2=-1 \Rightarrow V_{G_1}=1V \end{cases}$$

$V_{G_1}=1V$ قابل قبول نمی‌باشد چون $V_{GS}=1V>V_T$ نمی‌باشد.

$V_{G_1}=3V$ قابل قبول $\Rightarrow V_1=V_{G_1}=3V$

$$I_2=I_{D_2}=I_{D_1}=\frac{K_2}{2}(V_{GS_2}-V_{T_2})^2 \Rightarrow \begin{cases} V_{S_2}=0 \\ V_{G_1}=V_{G_2}=3V \end{cases}$$

$$\Rightarrow I_2=\frac{2}{2}(3-2)^2=1mA$$

$$I_{D_3}=I_{D_2}=1mA \Rightarrow 1mA=\frac{K_3}{2}(V_{GS_3}-V_{T_3})^2 \Rightarrow$$

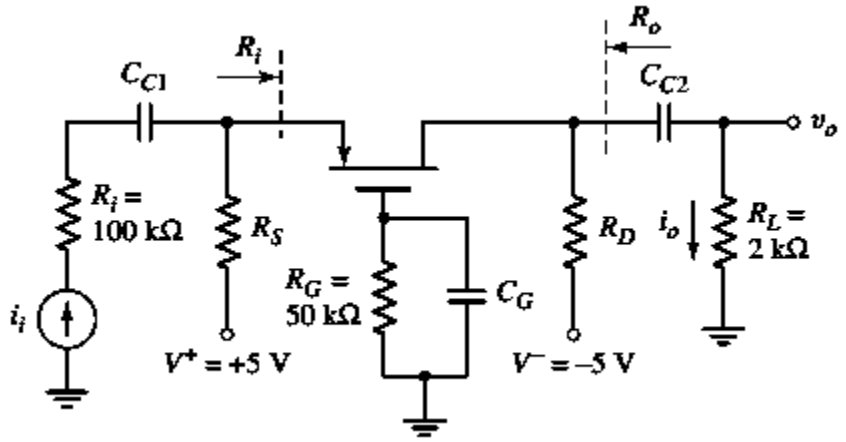
$$1mA=\frac{2}{2}(V_{G_3}-V_{S_3}-2)^2$$

از طرفی: $V_{S_3}=V_1=3V \Rightarrow 1=(V_{G_3}-3-2)^2 \Rightarrow$

$$V_{G_2} - 5 = \pm 1 \Rightarrow \begin{cases} V_{G_2} = 6\,V \\ V_{G_2} = 4\,V \Rightarrow \end{cases}$$ قابل قبول نمی‌باشد

$V_{G_2} = 4\,V$ باشد در این صورت $V_{GS_2} = 4 - 3 = 1V$ می‌شود که از V_T کمتر می‌باشد پس $V_{G_2} = 4\,V$ قابل قبول نمی‌باشد پس داریم:

$$V_{G_2} = 6\,V \Rightarrow V_2 = V_{G_2} = 6\,V$$



a.

$$I_{DQ} = K_p\left(V_{SG} + V_{TP}\right)^2$$

$$0.75 = (0.5)\left(V_{SG} - 1\right)^2 \Rightarrow V_{SG} = 2.225 \text{ V}$$

$$5 = I_{DQ}R_S + V_{SG} \Rightarrow R_S = \frac{5 - 2.225}{0.75} \Rightarrow \underline{R_S = 3.70 \text{ k}\Omega}$$

$$V_{SDQ} = 10 - I_{DQ}\left(R_S + R_D\right)$$

$$6 = 10 - (0.75)\left(3.70 + R_D\right) \Rightarrow \underline{R_D = 1.63 \text{ k}\Omega}$$

b.

$$R_i = \frac{1}{g_m}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.225 \; mA/V$$

$$R_i = \frac{1}{1.225} \Rightarrow \underline{R_i = 0.816 \text{ k}\Omega}$$

$$R_o = R_D \Rightarrow \underline{R_o = 1.63 \text{ k}\Omega}$$

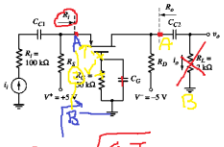
c.

$$i_0 = \left(\frac{R_D}{R_D + R_L}\right)\left(\frac{R_S}{R_S + [1/g_m]}\right) \cdot i_i$$

$$i_0 = \left(\frac{1.63}{1.63 + 2}\right)\left(\frac{3.70}{3.70 + 0.816}\right) i_i$$

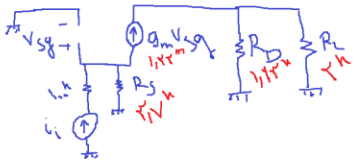
$$i_0 = 0.368 i_i = \underline{i_0 = 1.84 \sin \omega t \, (\mu A)}$$

$$v_0 = i_0 R_L = (1.84)(2) \sin \omega t \Rightarrow \underline{v_0 = 3.68 \sin \omega t \, (\text{mV})}$$



$g_m = \sqrt{K I_D}$

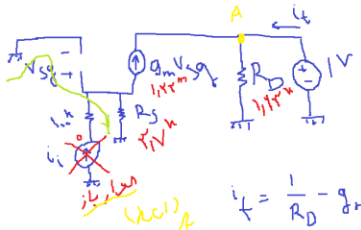
$R_i = \frac{V_t}{i_t}$



(KVL) $\rightarrow \quad V_t = V_{sg}$

$g_m V_{sg} = 1 \qquad V_{sg} = \frac{1}{g_m}$

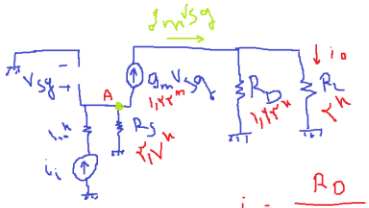
$R_i = \frac{V_t}{i_t} = \frac{1}{g_m} = \frac{1}{1{,}\text{؟؟}0^m} = \text{؟؟}\,\Omega$



(KCL) $\rightarrow \quad i_t = \frac{1}{R_D} - g_m V_{sg} = \frac{1}{R_D}$

(KVL) $\quad -V_{sg} - R_S g_m V_{sg} = 0 \Rightarrow V_{sg} = 0$

$R_o = \frac{1}{i_t} = R_D = 1{,}\text{؟؟}\,k\Omega$

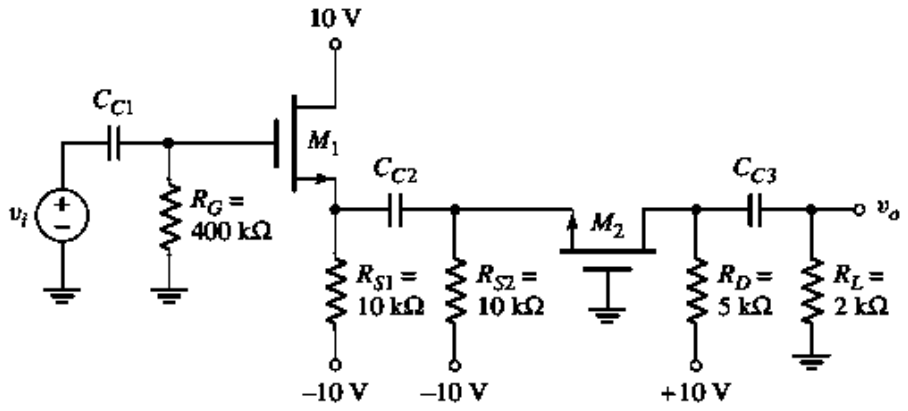


$i_o = \frac{R_D}{R_D + R_L} \times g_m V_{sg} \quad (1)$

(KCL)$_A \Rightarrow \quad \frac{V_{sg}}{R_S} + g_m V_{sg} - i_i = 0$

$V_{sg} = \frac{i_i}{\frac{1}{R_S} + g_m} \quad (2)$

$$i_o = \frac{R_D}{R_D + R_L} \times g_m \times \frac{1}{\frac{1}{R_S} + g_m} \, i_i$$



(a)

$$I_{DQ1} = \frac{10 - V_{GS1}}{R_{S2}} = K_{n1}\left(V_{GS1} - V_{TN1}\right)^2$$

$$10 - V_{GS1} = (4)(10)\left(V_{GS1}^2 - 4V_{GS1} + 4\right)$$

$$40V_{GS1}^2 - 159V_{GS1} + 150 = 0$$

$$V_{GS1} = \frac{159 \pm \sqrt{(159)^2 - 4(40)(150)}}{2(40)} \Rightarrow V_{GS1} = 2.435 \text{ V}$$

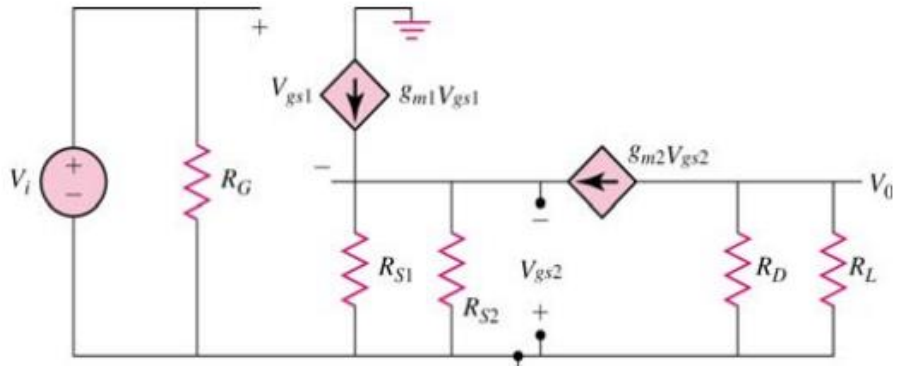
$$I_{DQ1} = (4)(2.435 - 2)^2 \Rightarrow \underline{I_{DQ1} = 0.757 \text{ mA}}$$

$$V_{DSQ1} = 20 - (0.757)(10) \Rightarrow \underline{V_{DSQ1} = 12.4 \text{ V}}$$

Also $\underline{I_{DQ2} = 0.757 \text{ mA}}$

$$V_{DSQ2} = 20 - (0.757)(10 + 5) \Rightarrow \underline{V_{DSQ2} = 8.65 \text{ V}}$$

(b) $$g_{m1} = g_{m2} = 2\sqrt{KI_{DQ}} = 2\sqrt{(4)(0.757)} \Rightarrow \underline{g_{m1} = g_{m2} = 3.48 \; mA/V}$$



$$V_0 = -\left(g_{m2}V_{gs2}\right)\left(R_D \| R_L\right)$$

$$V_{gs2} = \left(-g_{m1}V_{gs1} - g_{m2}V_{gs2}\right)\left(R_{S1} \| R_{S2}\right)$$

$$V_i = V_{gs1} - V_{gs2} \Rightarrow V_{gs1} = V_i + V_{gs2}$$

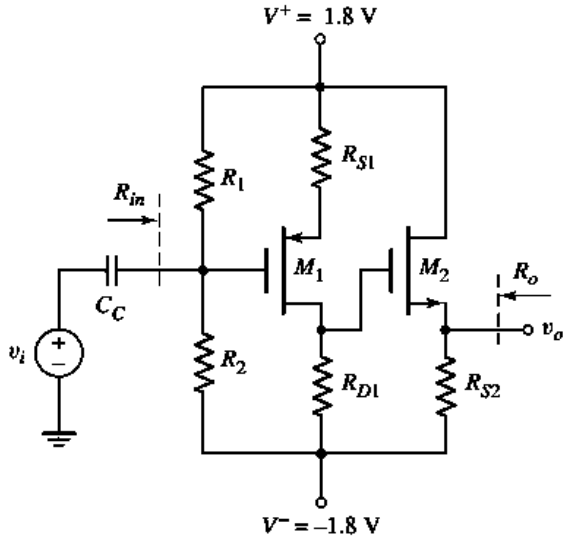
$$V_{gs2} + g_{m2}V_{gs2}\left(R_{S1} \| R_{S2}\right) = -g_{m1}\left(V_i + V_{gs2}\right)\left(R_{S1} \| R_{S2}\right)$$

$$V_{gs2} + g_{m2}V_{gs2}\left(R_{S1} \| R_{S2}\right) + g_{m1}V_{gs2}\left(R_{S1} \| R_{s2}\right) = -g_{m1}V_i\left(R_{S1} \| R_{S2}\right)$$

$$V_{gs2} = \frac{-g_{m1}V_i\left(R_{S1} \| R_{S2}\right)}{1 + g_{m2}\left(R_{S1} \| R_{S2}\right) + g_{m1}\left(R_{S1} \| R_{S2}\right)}$$

$$A_v = \frac{V_0}{V_i} = \frac{g_{m1}g_{m2}\left(R_{S1} \| R_{S2}\right)\left(R_D \| R_L\right)}{1 + \left(g_{m1} + g_{m2}\right)\left(R_{S1} \| R_{S2}\right)}$$

$$A_v = \frac{\left(3.48\right)^2\left(10 \| 10\right)\left(5 \| 2\right)}{1 + \left(3.48 + 3.48\right)\left(10 \| 10\right)} \Rightarrow \underline{A_v = 2.42}$$



(a) $R_{S1} = \dfrac{0.6}{0.1} = 6\,\text{k}\Omega$

$V_{D1} = 1.8 - 0.6 - 1 = 0.2\ \text{V}$

$R_{D1} = \dfrac{0.2 - (-1.8)}{0.1} = 20\,\text{k}\Omega$

$I_{DQ1} = K_{p1}\left(V_{SGQ1} + V_{TP}\right)^2$

$0.1 = 0.4\left(V_{SGQ1} - 0.4\right)^2 \Rightarrow V_{SGQ1} = 0.90\ \text{V}$

$I_{DQ2} = K_{n2}\left(V_{GSQ2} - V_{TN}\right)^2$

$0.3 = 4\left(V_{GSQ2} - 0.4\right)^2 \Rightarrow V_{GSQ2} = 0.6739\ \text{V}$

$V_{G1} = 1.8 - 0.6 - V_{SGQ1} = 1.8 - 0.6 - 0.9 = 0.3\ \text{V}$

$V_{G1} = \left(\dfrac{R_2}{R_1 + R_2}\right)(3.6) - 1.8$

$0.3 = \dfrac{1}{R_1}(200)(3.6) - 1.8 \Rightarrow R_1 = 343\,\text{k}\Omega$

$$R_1 \| R_2 = 200\,\text{k}\Omega \Rightarrow R_2 = 480\,\text{k}\Omega$$

$$V_{D1} = 1.8 - 0.6 - 1.0 = 0.2\ \text{V}$$

$$V_{S2} = V_{D1} - V_{GSQ2} = 0.2 - 0.6739 = -0.4739\ \text{V}$$

$$R_{S2} = \frac{-0.4739 - (-1.8)}{0.3} = 4.42\,\text{k}\Omega$$

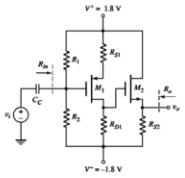
(b) $$A_v = \left(\frac{-g_{m1}R_{D1}}{1+g_{m1}R_{S1}}\right)\left\|\left(\frac{g_{m2}R_{S2}}{1+g_{m2}R_{S2}}\right)\right.$$

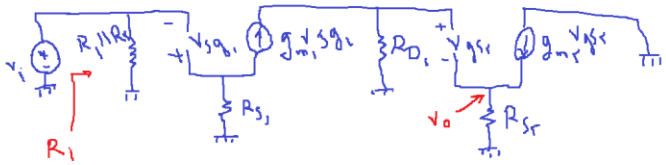
$$g_{m1} = 2\sqrt{K_{p1}I_{DQ1}} = 2\sqrt{(0.4)(0.1)} = 0.4\,\text{mA/V}$$

$$g_{m2} = 2\sqrt{K_{n2}I_{DQ2}} = 2\sqrt{(4)(0.3)} = 2.191\,\text{mA/V}$$

$$A_v = \frac{-(0.4)(20)}{1+(0.4)(6)} \cdot \frac{(2.191)(4.42)}{1+(2.191)(4.42)} = -2.13$$

(c) $$R_o = \frac{1}{g_{m2}} \| R_{S2} = \frac{1}{2.191} \| 4.42 = 0.4564 \| 4.42 \Rightarrow R_o = 414\,\Omega$$





R_1

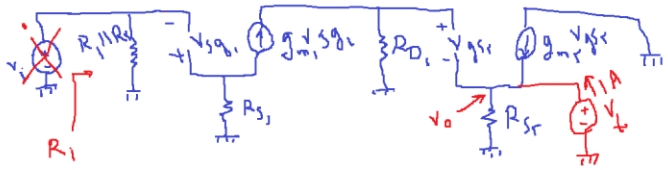
$$V_o = R_{S_2} g_{m_2} V_{gs_2}$$

$$g_{m_1} V_{sg_1} R_{D_1} = V_{gs_2}\left(1 + g_{m_2} R_{S_2}\right)$$

$$V_i = -V_{sg_1}\left(1 + g_{m_1} R_{S_1}\right)$$

$$V_i = -\underbrace{\left(1 + g_{m_1} R_{S_1}\right)\left(\frac{1 + g_{m_2} R_{S_2}}{g_{m_1} R_{D_1}}\right)}_{A} V_{gs_2}$$

$$\frac{V_o}{V_i} = \frac{R_{S_2} g_{m_2}}{A} = \frac{-R_{S_2} g_{m_2}\left(g_{m_1} R_{D_1}\right)}{\left(1 + g_{m_1} R_{S_1}\right)\left(1 + g_{m_2} R_{S_2}\right)}$$



R_1

$$V_t = R_{S_2}\left(1 + g_{m_2} V_{gs_2}\right) = R_{S_2} - R_{S_2} g_{m_2} V_t$$

$$V_{sg_1} = 0$$

$$V_{gs_2} = -V_t$$

$$V_t = R_o = \frac{R_{S_2}}{1 + R_{S_2} g_{m_2}}$$