Matrix Sum of Squares Relaxations for LPV System Analysis

Keivan Zavari, Goele Pipeleers, Jan Swevers
Department of Mechanical Engineering, PMA Division, KU Leuven, B-3001 Leuven, Belgium
Email: keivan.zavari@mech.kuleuven.be

1 Introduction

Linear parameter varying (LPV) systems have received a lot of attention during the last decade. The possibility to analyze and to design control strategies to cope with time-varying parameters, plays a big role in the recent developments.

This paper exploits a recent sum of squares (SOS) relaxation technique for analysis of LPV systems [1] and addresses the existing issues in this technique. The aim is to present how assumptions on time-varying parameters of an LPV system (and likewise bounded variation rate of parameters) can be subsumed under the mentioned SOS technique. Different approaches are proposed to model bounded variation rate in LPV $\mathscr{H}_\infty$ analysis using the SOS relaxations.

2 Matrix SOS relaxations

We consider the analysis of the following discrete-time LPV system $H(\lambda(k))$:

$$H(\lambda(k)) = \left[\begin{array}{c|c} A(\lambda(k)) & B(\lambda(k)) \\ \hline C(\lambda(k)) & D(\lambda(k)) \end{array}\right], \quad \lambda(k) \in \mathscr{L}, \qquad (1)$$

where $\mathscr{L}$ is a general set where parameters reside in. The time-varying parameter of the LPV system is denoted by $\lambda(k)$ and its variation rate is defined as $\Delta\lambda(k) \equiv \lambda(k+1) - \lambda(k)$. For reasons of conciseness, the dependency on k is omitted below.

The LPV $\mathscr{H}_\infty$ analysis problem can be concisely stated as the following [2]. There exists some symmetric positive definite polynomial matrix $P(\lambda, \Delta\lambda)$ such that:

$$F(\lambda, \Delta\lambda, P, \mu) > 0, \quad \forall(\lambda, \Delta\lambda) \in \mathscr{G}, \qquad (2)$$

then the system $H(\lambda)$ is exponentially stable and $\|H(\lambda)\|_\infty < \mu$. $\mathscr{G}$ is the parameter set while F is a matrix composed of the system matrices and dependent on the variables $\lambda, \Delta\lambda, P, \mu$.

It is well known that (2) is a semi-infinite problem and can be reformulated to a tractable, yet conservative convex problem. In [1], SOS sufficient conditions for problem (2) are presented where the parameter set $\mathscr{G}$ is described as

$$\mathscr{G} = \{(\lambda, \Delta\lambda) \mid G(\lambda, \Delta\lambda) < 0\}. \qquad (3)$$

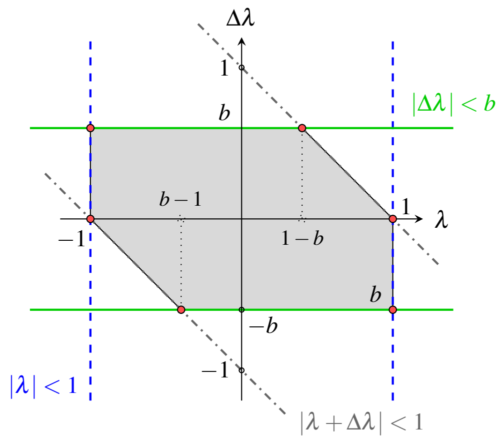

Figure 1: Parameter set for time varying parameters

3 Representation of parameter set

Representation of $\mathscr{G}$ has a large effect on the trade-off between computational effort and conservatism of the SOS relaxations technique.

Fig. 1 shows a set $\mathscr{G}$ for a scalar parameter λ described by $\{|\lambda| < 1, \ |\Delta\lambda| < b\}$ that commonly occurs in practice. In this research various representations of this set e.g. quadratic inequalities are considered. The effect of representation on the resulting SOS relaxations is analyzed as well as the merit of approximating it by inner and outer ellipsoids.

Acknowledgements This work benefits from KU Leuven-BOF PFV/10/002 Center-of-Excellence Optimization in Engineering (OPTEC), the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office (DYSCO), European research project EMBOCON FP7-ICT-2009-4 248940, KU Leuven's Concerted Research Action GOA/10/11, and G.0377.09 of the Research Foundation Flanders (FWO Vlaanderen). Goele Pipeleers is a Postdoctoral Fellow of the Research Foundation - Flanders.

References

[1] W. H. S.G. Dietz, C.W. Scherer, "Linear parameter-varying controller synthesis using matrix sum-of squares," in *Proceedings of the XVI Brazilian Automation Conference and XII Latin-American Congress on Automatic Control*, 2006.

[2] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia: SIAM, 1994.