Gaussian Processes Using A Sliding Window

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***Abstract*—This paper demonstrates the sliding window method as applied to gaussian processes–and examines the effects varying levels of noise have on the graph of the result. Source code to reproduce the experiments is given at the end of the paper.**

I. Introduction

We see from experiments with linear regression that the Bayes' rule effectively solves conditional probabilistic problems. If we know a past probability, we can improve a prediction about the future. The sliding window method applies these principles to non-stationary signals.

II. The Sliding Window

For this experiment we are given a signal, and we model our current output as a linear combination of our previous signals, plus the previous output. Our current output y_n is a simple prediction from previous samples, $y_{n,j}$ such that $i \in (-1, 1)$.

Our prediction $\hat{y_n} = f(\mathbf{y_{n-1}})$ is defined such that:

$$\mathbf{y}_{n-1} = [y_{n-1}, ..., y_{n-D}]^T \tag{1}$$

And it would follow that:

$$\hat{y_n} = \mathbf{y}_{n-1} = \mathbf{w}^T \mathbf{y}_{n-1} \tag{2}$$

Using these definitions, we construct a sliding window using $m = d = 3$ and apply the linear kernel:

$$k(x, x')_{linear} = \sigma_1^2 x^T x' + \sigma_0^2 + \sigma^2 \delta(x - x') \tag{3}$$

First, we run the program with $\sigma = 0.05$ and see the following:

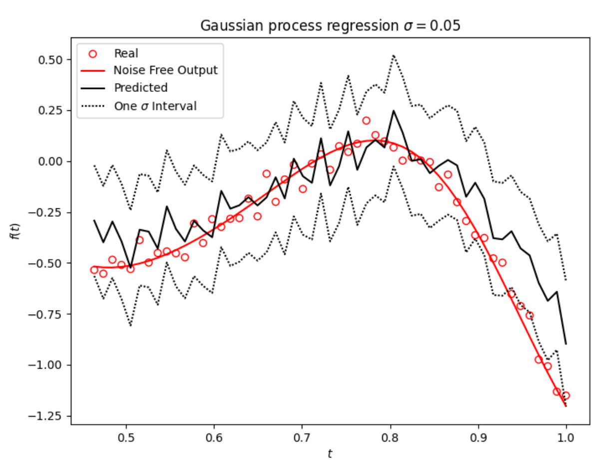

Fig. 1. Regression using the sliding window, $\sigma = 0.05$

Reducing the value of σ to 0.01 reveals the following:

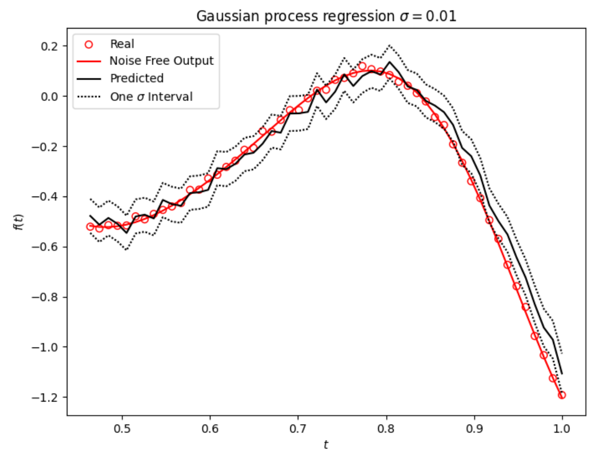

Fig. 2. Regression using the sliding window, $\sigma = 0.01$

III. Conclusion

We see that the value we choose for σ in our data generation function directly affects the margin on our graph. In this program, we used a perfectly discretized, synthetic signal as our input; however, the methods are sound and should apply to other classes of signals as well.

IV. Source Code

https://github.com/keithhbova/support_vector_machines/

References

[1] *Gaussian Processes for Machine Learning*. Rasmussen, Williams [Online]. Available: https://gaussianprocess.org/gpml/chapters/RW.pdf