# Linear Gaussian Processes

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***Abstract*—This paper examines one dimensional regression as applied to gaussian processes. Source code to reproduce the experiments is given at the end of the paper.**

## I. Introduction

GAUSSIAN processes are stoichastic processes; as such, they define a given probability of an event. When talking about gaussian processes, we can use the Bayes' rule that states the posterior is proportional to the prior multiplied by the likeleyhood. This relationship can be expressed mathematically as follows:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{1}$$

Using these two principles, we can construct a one-dimensional regression that allows us to define a distribution over a prediction.

## II. One Dimensional Regression

For this experiment, we describe a linear model $y_n$ as a linear combination of a uniform random variable $x_n$, and gaussian noise $\omega_n$:

$$y_n = 0.5x_n + 0.5 + \omega_n \tag{2}$$

Using this equation, we define a set of data points $x_n, y_n$ and generate a set of 50 using $\sigma = 0.05$ and initialize the linear gaussian process with $\sigma_n = 0.9$. We plot them in $\mathbb{R}^2$. Using a gaussian processes regressor, we define a kernel using the dot product and white nose, and plot the first two sigma bands predicted from the regressor. We see the results below.

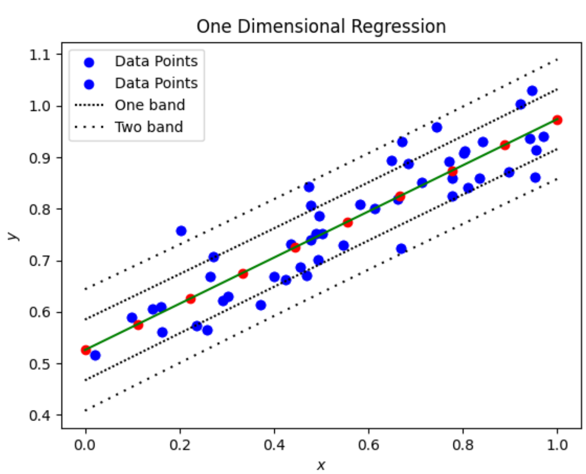


Fig. 1. linear gaussian process visualized in $\mathbb{R}^2$

## III. Conclusion

Using a gaussian regressor and the bayes rule, we can extract a linear relationship from a series of points–given the points follow a normal distribution. We see that most of the points cluster in the first $\sigma$ band, and continue to disperse up to the higher band. Now that we have properly defined this process for linear spaces, it would follow that we can repeat this method for non-linear functions.

## IV. Source Code

https://github.com/keithhbova/support_vector_machines/

## References

[1] *Gaussian Processes for Machine Learning*. Rasmussen, Williams [Online]. Available: https://gaussianprocess.org/gpml/chapters/RW.pdf