Gaussian Processes in a Feature Hilbert Space

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Abstract

For this assignment, we construct a gaussian process to compute a varying load forecast of different New England states. We apply the sliding window method to improve our accuracy over similar intervals. Source code to reproduce the experiments is given at the end of the paper.

I. Introduction

We have previously seen the sliding window method as applied to perfectly discretized signals; however, we want to evaluate this process using real-world data. In the following experiments, we construct a hilbert space and use the sliding window method to make a prediction–using two different types of kernels: the linear and the RBF. The kernels are defined as follows:

$$k(x,x')_{linear} = \sigma_1^2 x^T x' + \sigma_0^2 + \sigma^2 \delta(x - x') \tag{1}$$

$$k(x,x')_{RBF} = \sigma_1^2 e^{\gamma ||x - x'||^2} + \sigma_0^2 \tag{2}$$

We define our forecast hourly time horizon as m, such that:

$$m = [1, 6, 12] \tag{3}$$

These different values of m correspond to predicting 1 hour, six hours, and 12 hours ahead, respectively. We train a gaussian process using each of the two kernels for the first week of January, and the first week of July.

II. Data

The data can be found at https://www.iso-ne.com/, and also with our source code. We are given a time-series dataset that contains several text files–each with 8760 lines that correspond to an hour in the year. Given that there are 8760 hours in a year and 168 hours in a week, we define the indices:

$$January = [0 : 167] \tag{4}$$

$$July = [4367 : 4535] \tag{5}$$

We represent each text file as a vector and train six gaussian processes for each state: two kernels with three time horizions each. We see the following results below.

III. CT

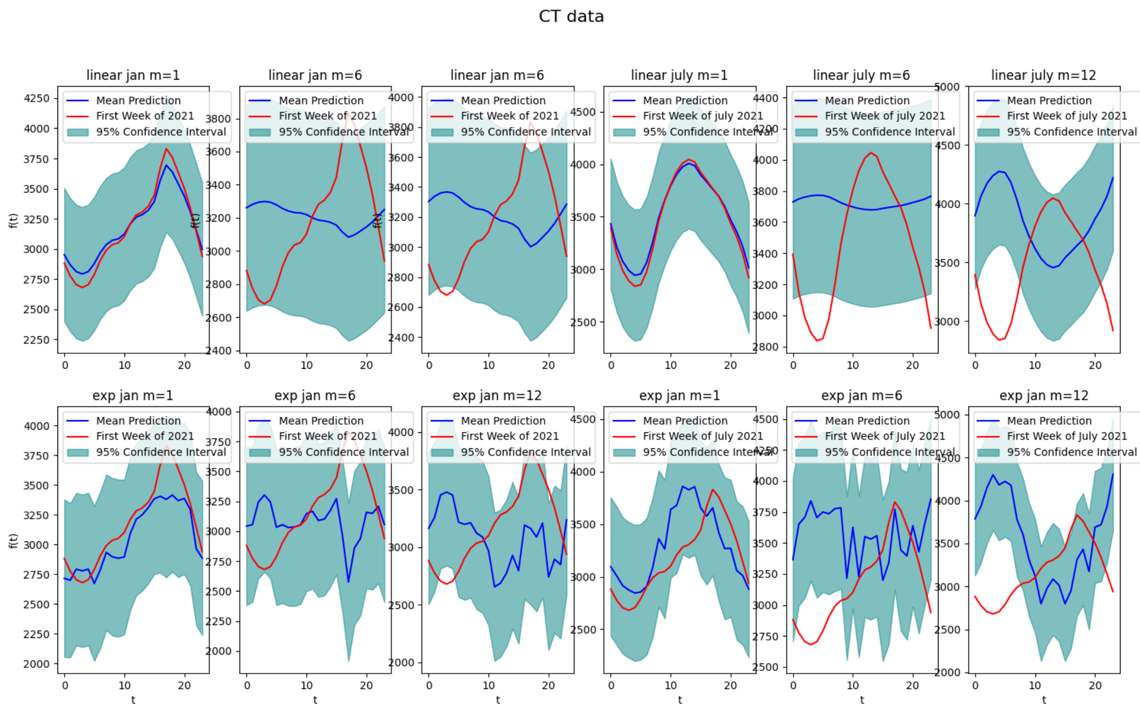

Fig. 1. Gaussian Process for CT Region

IV. ME

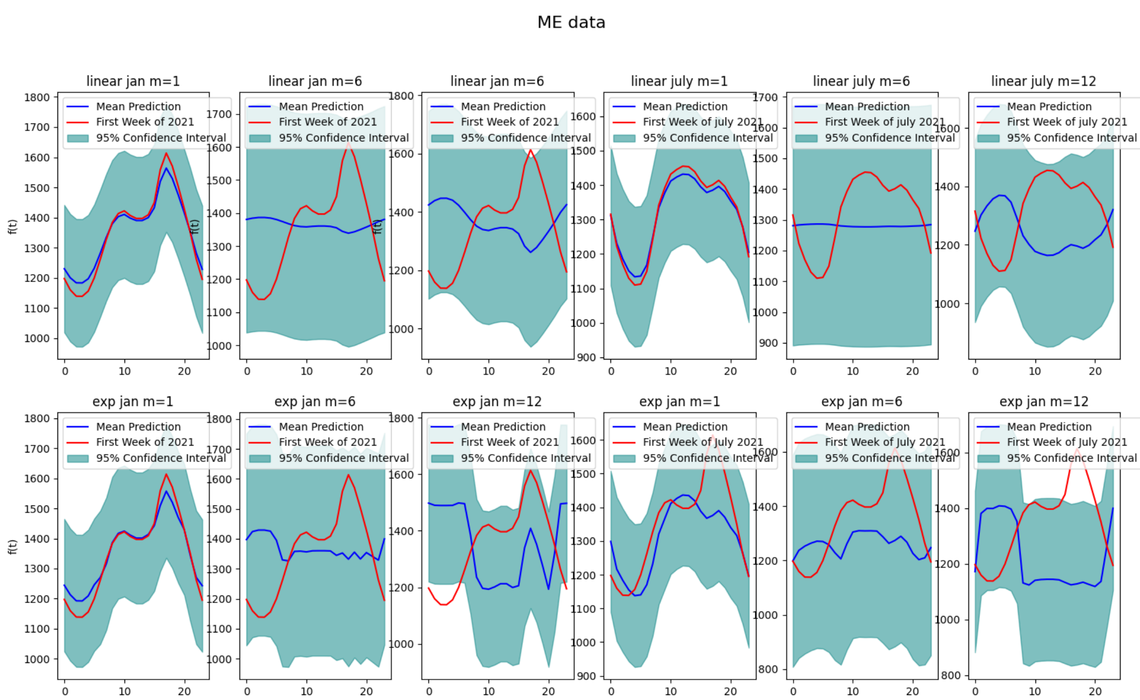

Fig. 2. Gaussian Process for ME Region

V. NEMA

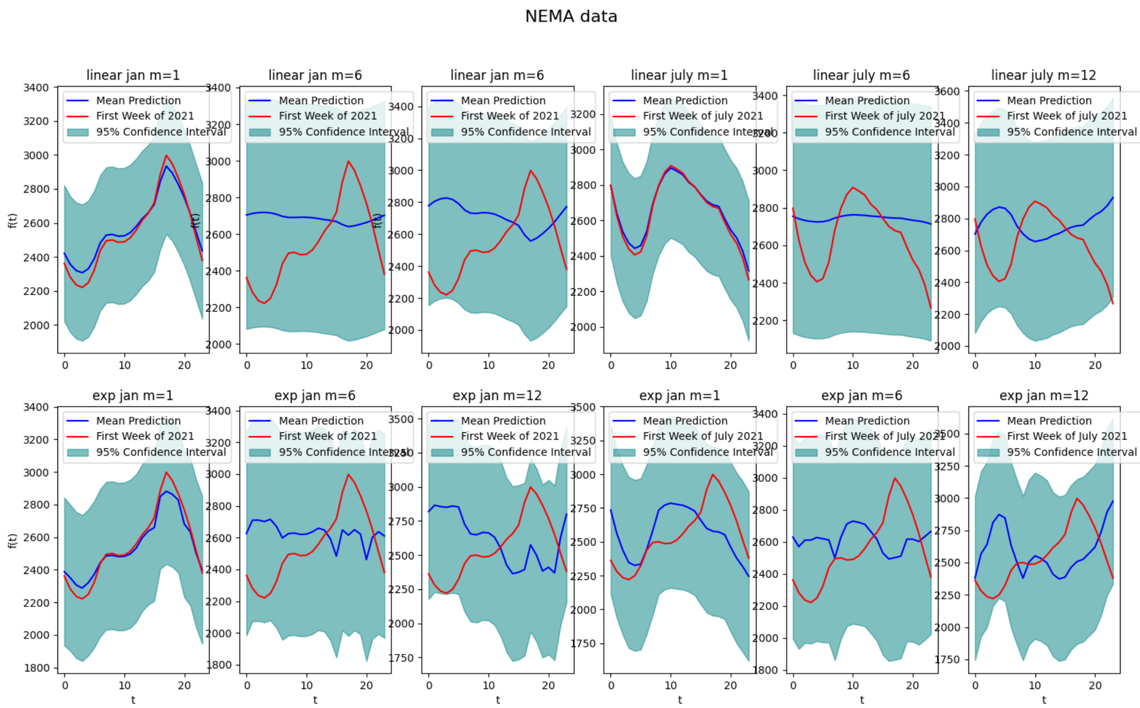

Fig. 3. Gaussian Process for NEMA Region

VI. NH

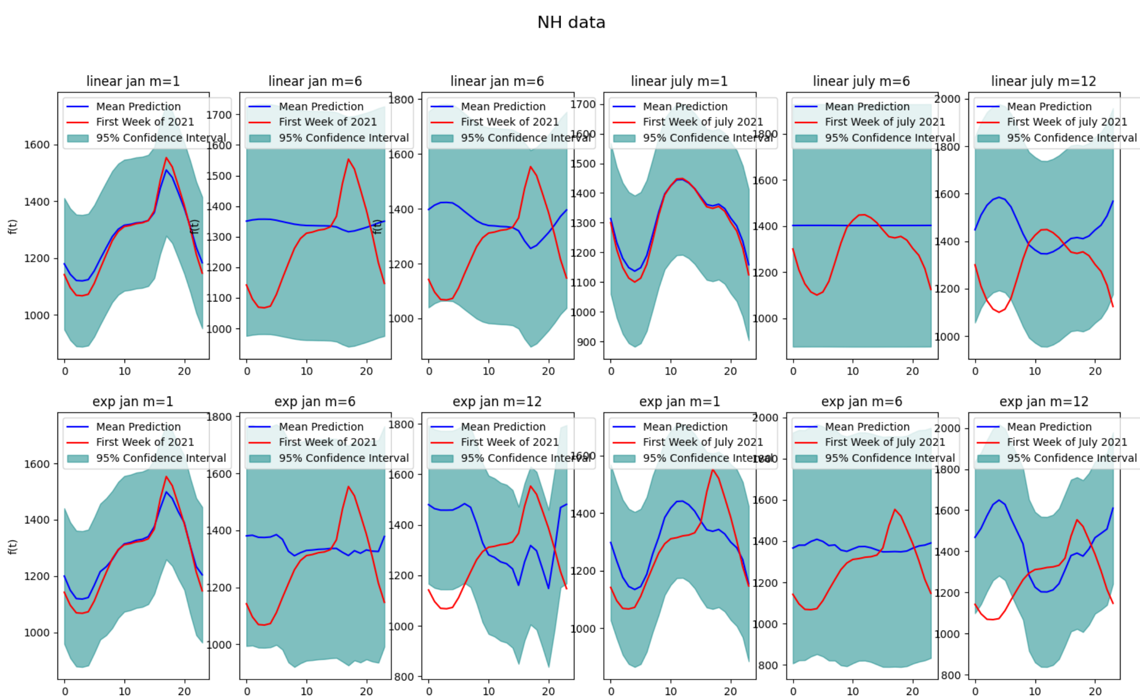

Fig. 4. Gaussian Process for NH Region

VII. RI

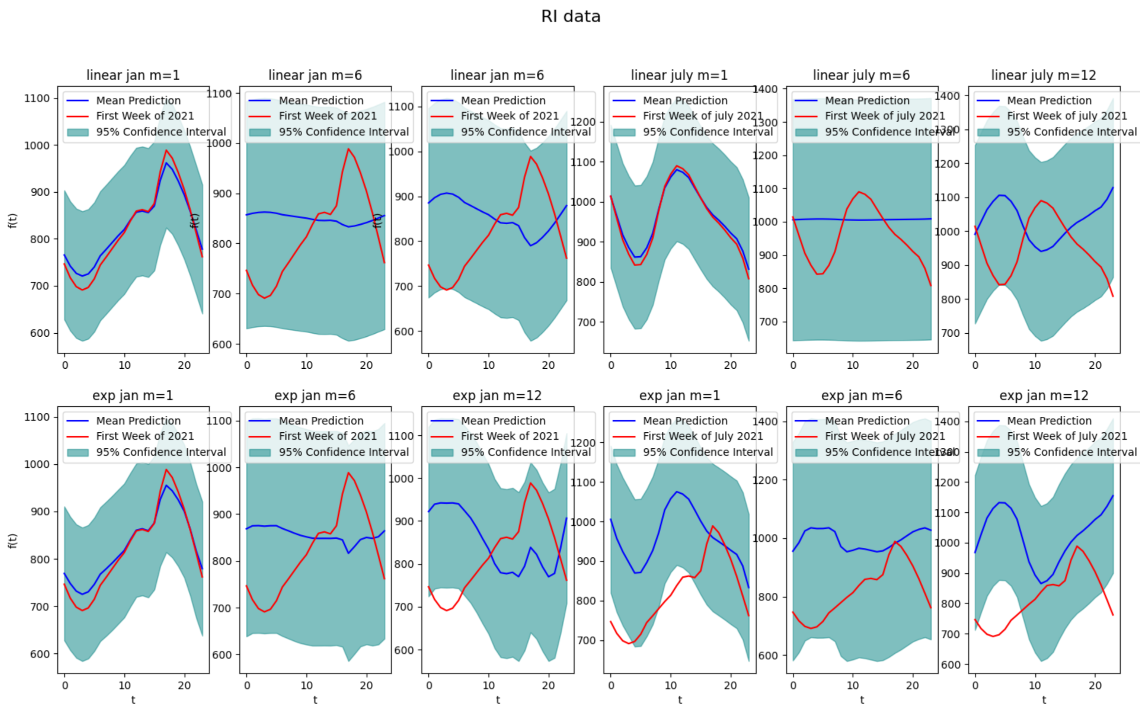

Fig. 5. Gaussian Process for RI Region

VIII. SEMA

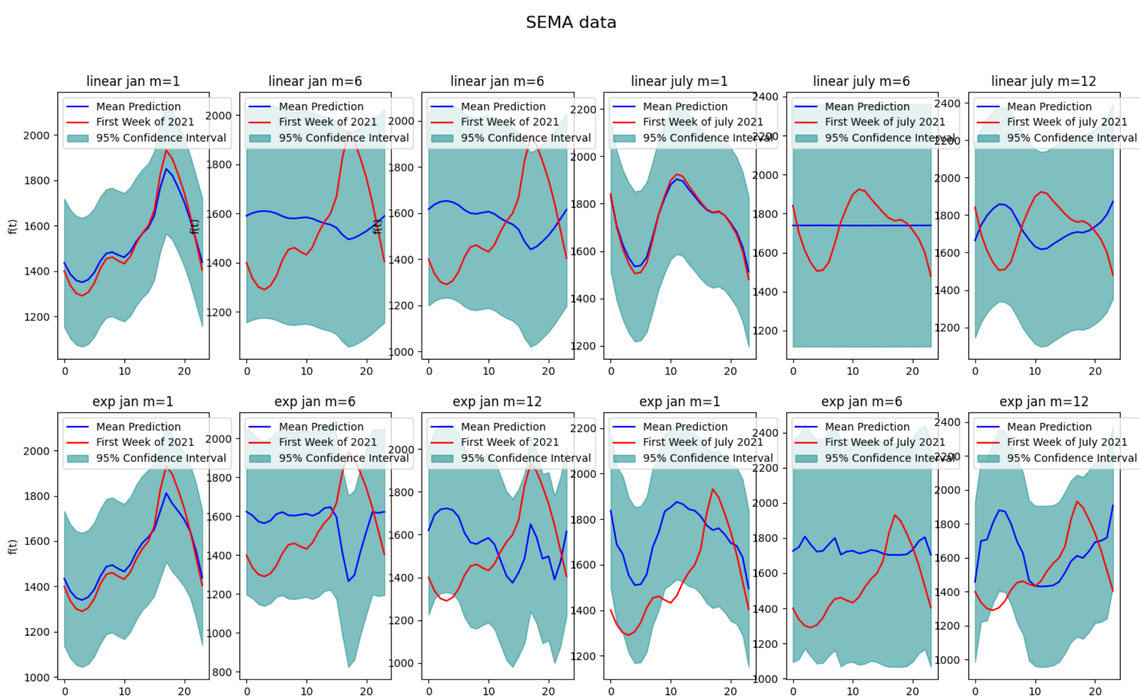

Fig. 6. Gaussian Process for SEMA Region

IX. VT

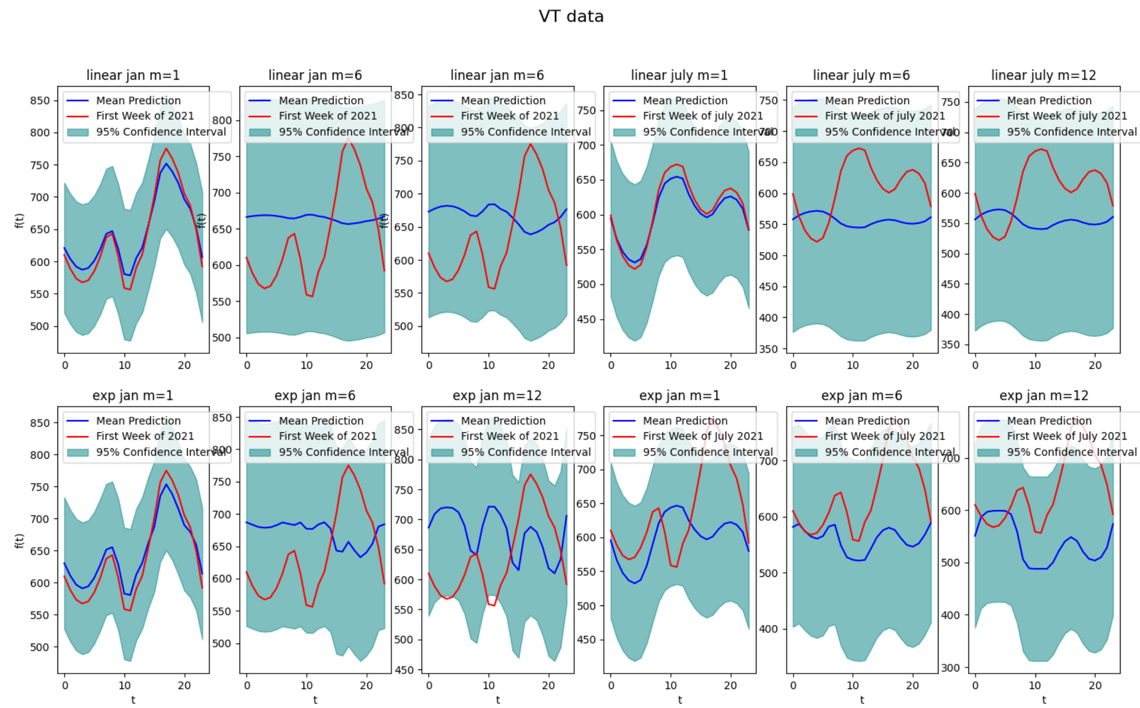

Fig. 7. Gaussian Process for VT Region

X. WCMA

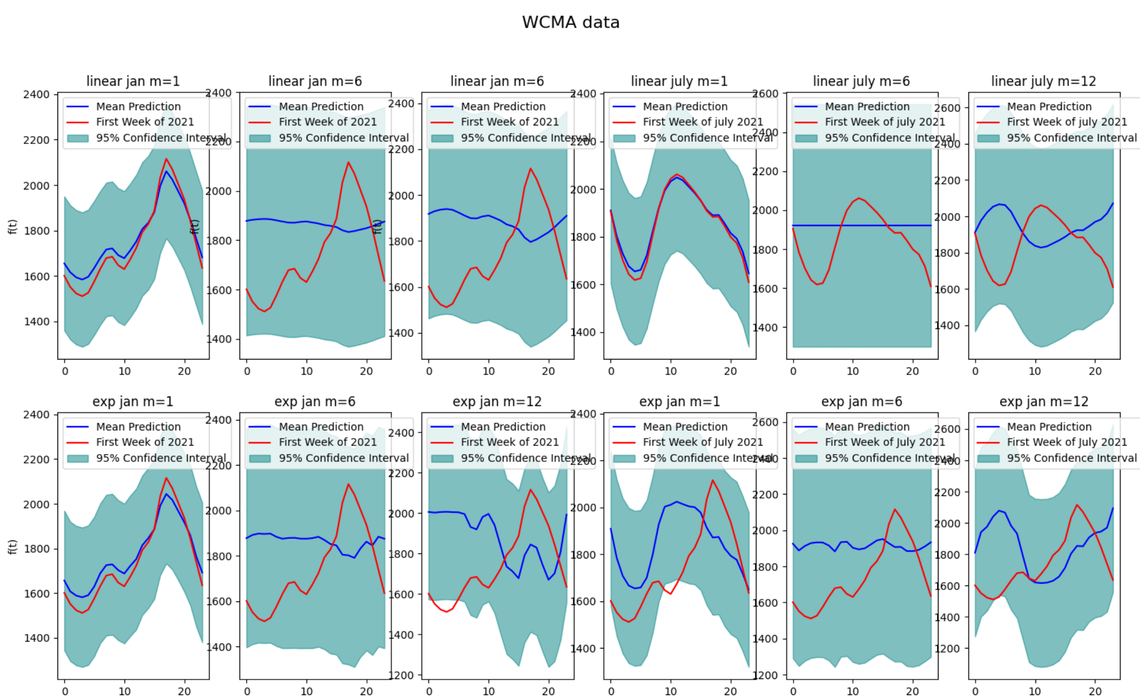

Fig. 8. Gaussian Process for WCMA Region

XI. Conclusion

We see from the above results that a lower value of m corresponds with a mean prediction that more closely follows the original signal; conceptually, this makes sense–we trade accuracy for generality in this case. When the width of our horizon increased beyond $m = 1$, our prediction starts deviating substantially.

XII. Source Code

https://github.com/keithhbova/support_vector_machines/

References

[1] *Gaussian Processes for Machine Learning*. Rasmussen, Williams [Online]. Available: https://gaussianprocess.org/gpml/chapters/RW.pdf