

Multiscale Depth Local Derivative Pattern for Sparse Representation Based 3D Face Recognition

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Abstract—3D face recognition is a popular research area due to its vast application in biometrics and security. Local feature-based methods gain importance in the recent years due to their robustness under degradation conditions. In this paper, a novel high-order local pattern descriptor in combination with sparse representation based classifier (SRC) is proposed for expression robust 3D face recognition. 3D point clouds are converted to depth maps after preprocessing. Multi-directional derivatives are applied in spatial space to encode the depth maps based on the local derivative pattern (LDP) scheme. Directional pattern features are calculated according to local derivative variations. Since LDP computes spatial relationship of neighbors in a local region, it extracts distinct information from the depth map. Multiscale depth-LDP is presented as a novel descriptor for 3D face recognition. The descriptor is employed along with the SRC to increase the range data distinctiveness. A histogram on the derivative pattern creates a spatial feature descriptor that represents the distinctive micro-patterns from 3D data. We evaluate the proposed algorithm on two famous 3D face databases, FRGC v2.0 and Bosphorus. The experimental results demonstrate that the proposed approach achieves acceptable performance under facial expression.

I. INTRODUCTION

For many years 2D face recognition has been studied among researchers as an important and popular biometrics. However, degradation conditions such as illumination and pose variations have influenced on 2D face recognition system performance. To overcome these limitations, 3D face data that contains spatial information has attracted researchers attention. Face recognition is categorized into holistic, feature-based, and hybrid approaches according to classification presented in [1]. The whole face or large region of the face is used to generate information from 3D data in holistic methods. The principle component analysis (PCA)-based method [2] and closest normal points (CNPs) [3] are examples of the holistic category. In feature-based methods, local features or regions are fed into a classifier for recognition such as methods presented in [4], [5]. Hybrid methods apply both the whole face region and local features or 2D and 3D face data like the approaches for 3D textured face recognition in [6], [7]. The feature-based methods are more robust under facial expression, pose variations, and occlusions [8] such as keypoint-based methods. Shape maps are applied in [7] in combination with scale invariant feature transform (SIFT)-like

matching framework [9] for 3D face recognition along with 2D face data. Li et al. [10] extended the SIFT-like matching scheme to mesh data and proposed a curvature-based 3D keypoint detectors using shape maps.

Applying efficient descriptors in 2D face recognition on depth images helps researchers to achieve high performance in 3D dimensionality like Gabor wavelet [11] and LBP [4]. Huang et al. [4] proposed feature-based method using shape index (SI) and local binary pattern (LBP) for 3D facial surface representation. LBP is considered as a simple and most efficient local 2D face descriptor that is first proposed by Ojala et al. [12]. Recently, LBP has been applied by researchers as an effective local descriptor in 3D face area. Multiscale extended LBP [13] that is a facial surface descriptor extracts local shape changes and applies SIFT-based matching for face recognition. In [14], depth and normal information of 3D data are extracted and encoded using LBP to create a face descriptor. The surface normal that determines a surface orientation at each point and includes local shape information also is applied by Li et al. [5] for feature-based 3D face recognition. Local normal pattern inspired by LBP is used to describe shape information and extended as a multiscale and multicomponent descriptor to improve the recognition system performance. To handle facial expression, they applied a weighted sparse representation-based classifier (W-SRC). The whole face is divided into local patches and local normal-based features are extracted and used in the training step to learn weights. The W-SRC is also employed in [15] along with region-based extended LBP descriptor for 3D face recognition.

Sparse representation [16] which is a subspace algorithm can be used as a feature representation method to extract more distinct feature and dimension reduction for depth images. The sparse regression model is proposed in [17] to embed the facial descriptors into the low dimensional matrix and handle occlusions and hair covering. In [18], authors apply sparse representation framework in combination with feature pooling and ranking scheme for low-level features. A local descriptor called local shape pattern (LSP) is presented by Huang et al. [19] to extract both differential structure and orientation information from 3D face data. They applied SRC to classify the local shape features.

In this paper, we propose multiscale depth local derivative

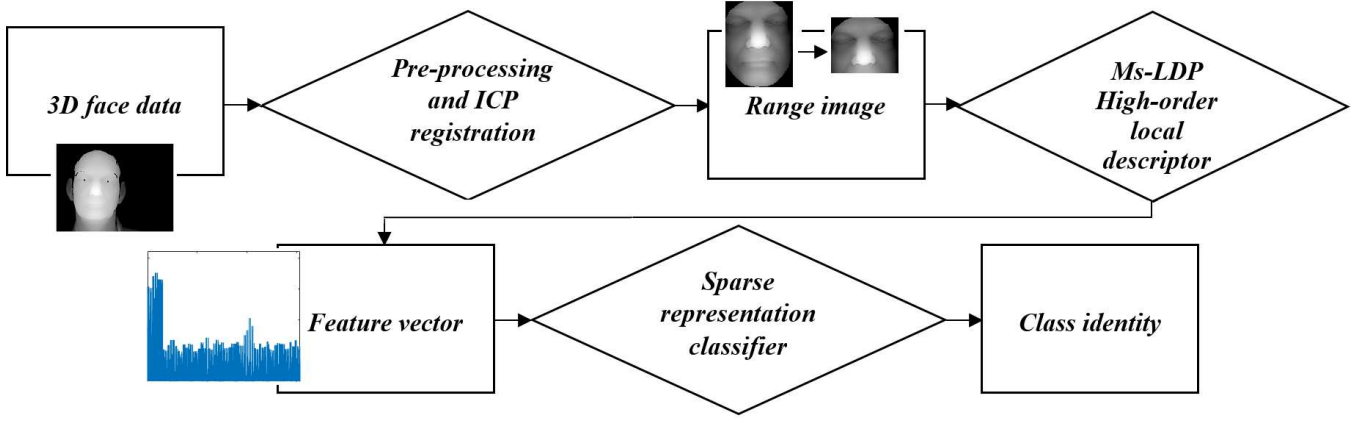


Fig. 1. The framework of the proposed method

pattern (MsDLDP) for 3D face recognition. Unlike LBP that is a non-directional first-order local pattern, LDP captures the change of derivative directions among local neighbors. High-order LDP performs better results for extracting more discriminative features compared to LBP. In our proposed algorithm, we apply learning-based approach using sparse representation (SR) to select prominent features and boost recognition rate. Similar to LBP, LDP is also modeled using a histogram of the extracted micro patterns. In the proposed approach, the histogram of MsDLDP is fed into SR classifier to do recognition task. The overview of the proposed approach has been presented in Fig. 1.

The remaining part of this work is presented as follows: in section II, preprocessing of 3D face data is explained. Section III provides the proposed method that consists of feature extraction, MsDLDP descriptor, and classification using SR. Experimental results including the proposed algorithm performance and comparison with state-of-the-art is presented in section IV, and section V describes conclusion and future work.

II. PREPROCESSING

The output of the 3D capturing device is noisy and needs to be smoothed. In the preprocessing part, the 3D face scans are processed to smooth noise, remove spikes, and fill holes [8]. The region of interest (ROI) extraction is next part that is performed using nose tip detection.

In this paper, we apply 3D face preprocessing tool developed by Szepticki et al. [20]. The median filter is used to remove spikes and noises. Hole filling is done using the square surface fitting. The curvature-based method is employed to detect nose tip and ROI extraction.

We apply iterative closest point (ICP) algorithm [21] to correct pose by considering five frontal scans with neutral expression from each database as models. We resize each preprocessed range image into 120×96 for next steps. To handle facial expression, we consider the rigid (nose) and semi-rigid (eye-forehead and cheek) areas and exclude the most impressed area by the expression, non-rigid area (mouth).

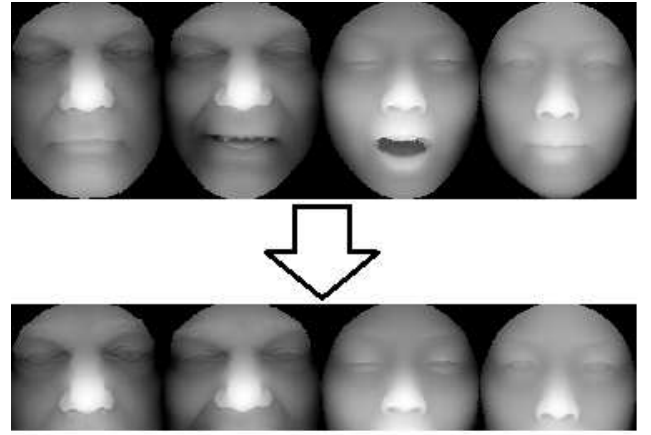


Fig. 2. Illustration of samples of the face preprocessing on the FRGC v2.0 database

The preprocessed faces from FRGC v2.0 database [22] have been shown in Fig. 2.

III. PROPOSED METHOD

A. Multiscale depth local derivative pattern descriptor

1) *depth-LDP descriptor*: Local derivative pattern on depth map that is a high-order multi-directional derivative texture pattern descriptor is proposed in this section for 3D face. LDP first is introduced by Zhang et al. as a texture pattern descriptor for 2D images [23]. It can be considered as a denoising function for its special binarization function.

To calculate LDP for a depth image $D(P)$, the first-order derivative $D'(P)$ for different directions including 0° , 45° , 90° , and 135° is calculated using the following equations

$$D'_{0^\circ}(P_0) = D(P_0) - D(P_4) \quad (1)$$

$$D'_{45^\circ}(P_0) = D(P_0) - D(P_3) \quad (2)$$

$$D'_{90^\circ}(P_0) = D(P_0) - D(P_2) \quad (3)$$

$$D'_{135^\circ}(P_0) = D(P_0) - D(P_1) \quad (4)$$

where, P_0 is a point in $D(P)$ and $P_i, i = 1, \dots, 8$ is the neighboring point around the P_0 as Fig. 3 shows.

To compute second-order directional LDP with direction α at P_0 , the following equation is applied.

$$DLDP_{\alpha}^2(P_0) = (f(D'_{\alpha}(P_0), D'_{\alpha}(P_1)), f(D'_{\alpha}(P_0), D'_{\alpha}(P_2)), \dots, f(D'_{\alpha}(P_0), D'_{\alpha}(P_8))) \quad (5)$$

A binary coding function f is used to determine local pattern transition types. The consistency of two neighboring derivatives is described using f defined as

$$f(D'_{\alpha}(P_0), D'_{\alpha}(P_i)) = \begin{cases} 0, & \text{if } f(D'_{\alpha}(P_0), D'_{\alpha}(P_i)) > 0 \\ 1, & \text{if } f(D'_{\alpha}(P_0), D'_{\alpha}(P_i)) \leq 0 \end{cases} \quad (6)$$

The encoding system is applied on the binary derivative calculations to make the integer value of the LDP descriptor. In our proposed method, according to above explanation, each pixel in depth map is assigned an integer value at specific direction α . Each depth map is divided into some local patches. The statistical distribution of the calculated features in the local regions is computed and presented using a histogram. The length of each histogram is 2^m which m is the number of the neighbors around the center point P_0 . The calculated histograms from each local patch are concatenated together to make the histogram in each direction. Final descriptor is created using histogram concatenation of different directions (See Fig. 4).

2) *Multiscale approach*: Like LBP, LDP can be extended with different local neighborhood sizes for different scales. A set of sampling points around the central point P_0 is considered as the local neighborhood. The arrangement of the sampling points is defined using a various number of the points and radius (P, R). Fig. 3 illustrates different LDP neighborhoods. MsLDP is defined by changing the value of radius R . This scheme for LBP, MsLBP, firstly is proposed by Ojala et al. for texture classification [12] and applied for 2D face recognition by Chan et al. [24]. Later, Huang et al. [4] applied it for 3D face recognition. In this work, we propose a multiscale strategy for depth-LDP calculation that is quite a different and new presentation of LDP for 3D face recognition. The local derivative pattern at different radius computes local shape variations and extracts highlight details.

3) *n^{th} -order MsDLDP descriptor*: The higher-order derivatives of the MsDLDP is computed by applying equations (1) through (4) on MsDLDP iteratively and using equation (6) for binarization. In this way, the n^{th} -order derivative for our proposed descriptor is calculated as follows:

$$MsDLDP_{\alpha}^n(P_0) = f(D_{\alpha,R}^{n-1}(P_0), D_{\alpha,R}^{n-1}(P_i)), i = 1, \dots, 8 \quad (7)$$

where R is the different values for the radius to generate the multiscale descriptor.

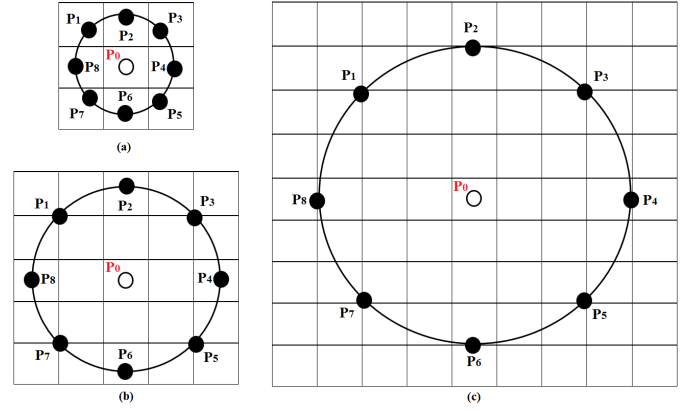


Fig. 3. Eight neighborhood around P_0 with different R , (a) $R=1$, (b) $R=2$, (c) $R=3$

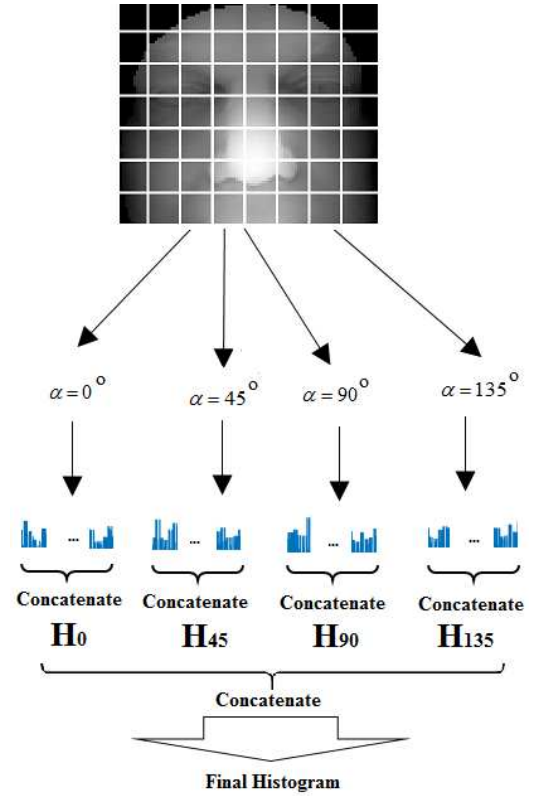


Fig. 4. Construction of depth-LDP descriptor

B. Sparse Representation-based classifier

The sparse representation classifier firstly introduced by Wright et al. [16] for 2D face recognition. They consider the problem of recognizing of the frontal faces under varying expression and lighting can be addressed using sparse signal representation. L1-minimization is used to compute a sparse representation as a general classification algorithm. This framework provides the insight that if the sparsity could be harnessed properly, the performance of the classification would be improved. Base on this representation, for a frontal test sample, the sparsity of the coefficient vector is high except

for the same class samples. These coefficients for the ones from other classes are zero or close to zero.

We apply the above framework for 3D face scans by considering the probe face as a sparse linear combination of gallery samples. Given n_i training 3D face samples of the class i , $A_i = [v_{i,1}, v_{i,2}, \dots, v_{i,n_i}] \in R^{m \times n_i}$, any probe sample from the same class is defined using the linear relation of the training samples in class i based on the following equation

$$y = \beta_{i,1}v_{i,1} + \beta_{i,2}v_{i,2} + \dots + \beta_{i,n_i}v_{i,n_i} \quad (8)$$

where $\beta_{i,j} \in R$, $j = 1, 2, \dots, n_i$. A new matrix A is defined for all training samples by concatenating of the n samples from i different 3D face classes. In this way, the linear representation of the probe sample can be defined as follows:

$$y = Ax_0 \in R^m, \\ x_0 = [0, \dots, 0, \beta_{i,1}, \beta_{i,2}, \dots, \beta_{i,n_i}, 0, \dots, 0]^T \in R^n \quad (9)$$

where x_0 is a coefficient vector with entries equal to zero or close to zero except those related to the subject of y . It is obvious that when $m > n$, the system based on the equation (9) is over-determined and the vector x_0 can easily be found as its unique solution. However, in 3D face recognition application, it is worth noting that for each subject there is only one sample in the gallery as training sample which is based on the most common setting in 3D face recognition systems. Consequently, in 3D face recognition applications the system $y = Ax_0$ is typically under-determined that its solution is not unique. For the frontally aligned faces, the expression variation problem is another challenge that we handle it in our proposed method by considering the rigid, and semi-rigid parts of the face as it is explained in section 2. To solve the sparse vector x the following solution using the minimum l^0 -norm according to [16] is applied as follows:

$$\hat{x}_0 = \operatorname{argmin} \|x\|_0 \quad \text{s.t.} \|Ax - y\|_2 \leq \varepsilon \quad (10)$$

where $\varepsilon \in R^n$ and represents a deviation vector. For sparse x_0 the above equation can be solved using solving the problem of L_1 -Norm [16] and reconstruction residuals $r_i(y)$ calculation as follows:

$$\hat{x}_2 = \operatorname{argmin} \|x\|_2 \quad \text{s.t.} \|Ax - y\|_2 \leq \varepsilon \quad (11)$$

$$r_i(y) = \|y - A\delta_i(\hat{x}_1)\|_2^2 \quad (12)$$

where δ_i represents a characteristic function that is used to select the coefficient related to the i^{th} sample of the gallery. Consequently, the identity of the probe y is determined with the index of minimal $r_i(y)$.

IV. EXPERIMENTAL RESULTS

In this section, we evaluate the proposed approach that consists of a new facial local descriptor along with sparse representation classifier. The following databases are used for the comprehensive evaluation of the proposed method under expression variations.

A. FRGC DB

The FRGC v2.0 [22] database consists of 4007 texture and 3D face scans under different facial expression from 466 persons. It is the largest set of the 3D face database that has been used in literature as the benchmark for 3D face recognition algorithms evaluation. The face acquisition system is the Minolta Vivid 900 scanner. The face scans are in controlled lighting and pose and they are under facial expressions such as happiness and surprise. In the experiments, the gallery consists of 3D face scans with neutral expression from each subject that are 466 samples to make the dictionary A of the SRC after applying MsDLDP descriptor of these samples. The remaining samples including 3541 3D face scans make up probe samples, y in equation (8).

B. Bosphorus DB

To further evaluation of the proposed algorithm under facial expression variations, in this section, we apply the Bosphorus database [25] that comprises of 4666 texture and 3D records from 105 persons. This database contains 34 facial expressions including action units and 6 emotions, 13 different pose variations that consist of the pitch, yaw, and cross rotations, and 4 occlusions including hair, eye glasses, eye, and mouth with the hand. The hardware device used to capture 3D face scans is the Inspeck Mega Capturor II 3D scanner. For the experiments using the Bosphorus database, we apply same approach for preprocessing used for the FRGC database. In this experiment, we consider nearly frontal faces with expression changes and partial occlusions. These scans consist of 3301 samples. A gallery set (105 scans) is made up using the first sample with the neutral expression and the remaining samples make up the probe set (3196 scans).

C. Experiments

To compare different orders of the proposed multiscale local descriptor, the recognition rate of different orders using SRC has been reported for neutral vs. all experiment based on the experimental protocols presented in [22]. Increasing order of the local pattern can improve the recognition results for second and third-order descriptors based on the results presented in Fig. 5. Since the accuracy of the recognition system using fourth-order has been decreased, it means that not only increasing the order to four does not add more information but also causes to convert the facial image to the noisy data and destroy the recognition rate.

In the next experiment, we evaluate the effectiveness of sparse representation-based classifier. The Chi-square distance is used in the literature as a popular similarity measurement for histogram-based descriptors like LBP, LDP, and etc. [26]. To

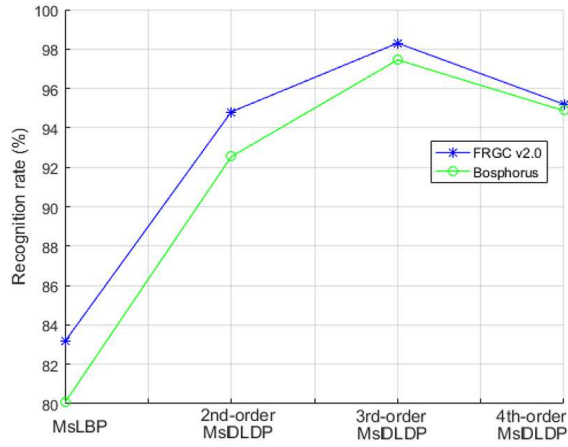


Fig. 5. Comparison of rank-one recognition rate of MsDLDP descriptor on FRGC v2.0 and Bosphorus databases

TABLE I
THE COMPARISON OF RR1 FOR TWO DIFFERENT CLASSIFIERS ON THE FRGC v2.0 DATABASE

Local descriptors	RR1 (Chi-square)	RR1 (SR)
MsLBP	75.34%	83.16%
MsDLDP(2nd-order)	82.7%	94.81%
MsDLDP(3rd-order)	88.46%	98.3%
MsDLDP(4th-order)	83.6%	95.2%

show the effectiveness of sparse-based classifier, we compare the recognition rate of SRC and Chi-square based classifier using the same facial descriptor with different orders. Tables 1 and 2 present the rank-one recognition rate (RR1) for neutral vs. all experiment on FRGC v2.0 and Bosphorus databases respectively. From the tables, it is obvious that the SRC outperforms the performance of Chi-square classifier and shows applying sparse-based classifier is effective along with local derivative pattern descriptor.

To evaluate the effectiveness of the proposed approach under facial expression, we test our algorithm on two sets from FRGC v2.0 database by dividing the probe samples into two different sets including neutral and non-neutral facial scans. In this experiment, we consider the third-order descriptor that is the most effective one based on the experimental results. Table 3 reports the performance of the system under facial expression and compares the obtained results with the state-of-the-art. As the results illustrate our proposed method is robust under expression by applying the local derivative pattern in the multiscale scheme that extracts discriminant enough information and as we exclude the non-rigid parts in

TABLE II
THE COMPARISON OF RR1 FOR TWO DIFFERENT CLASSIFIERS ON THE BOSPHORUS DATABASE

Local descriptors	RR1 (Chi-square)	RR1 (SR)
MsLBP	72.8%	80.06%
MsDLDP(2nd-order)	79.1%	92.54%
MsDLDP(3rd-order)	85.9%	97.45%
MsDLDP(4th-order)	82.41%	94.87%

TABLE III
THE PERFORMANCE OF THE PROPOSED METHOD UNDER FACIAL EXPRESSION ON THE FRGC v2.0 DATABASE

Methods	RR1 (Neutral)	RR1 (Non-neutral)
Huang et al. [4]	99.1%	92.5%
Li et al. [5]	98%	94.2%
MsDLDP(3rd-order)+SRC	99.3%	96.5%

the preprocessing step.

In the next experiment, the proposed approach is compared with the state-of-the-art 3D face recognition approaches that belong to LBP-based category method. We evaluate our algorithm on both databases and compare with other methods under the same experimental conditions. The performance comparison is reported in Table 4. From the results presented in the table, we can find that our proposed descriptor in combination with sparse representation-based classifier outperforms state-of-the-art. Among different local pattern-based descriptor in Table 4, the local derivative pattern can extract more distinct features since it works based on the spatial relationship of the neighbor points in the local region.

V. CONCLUSION

In this paper, a new facial descriptor called high-order multiscale depth local derivative pattern (MsDLDP) has been presented. The descriptor contains more spatial information compared to local binary pattern (LBP), since it encodes the various distinct spatial relationship in a local region. The proposed multiscale strategy provides more discriminative information and represents local shape features more comprehensively. In addition, to select most distinct features and improve recognition performance sparse representation-based classifier has been employed. The experimental results illustrate that the SRC is more efficient than distance-based

TABLE IV
THE PERFORMANCE COMPARISON WITH LBP-BASED METHODS ON THE
FRGC v2.0 AND BOSPHORUS DATABASES

Methods	RR1 (FRGC v2.0 DB)	RR1 (Bosphorus DB)
Huang et al. [13]	97.6%	97%
Tang et al. [14]	94.89%	-
Li et al. [5]	96.3%	95.4%
Lv et al. [15]	97.8%	-
This work	98.3%	97.45%

classifier. The algorithm can handle expression variations properly since we exclude the expression sensitive non-rigid areas in the pre-processing step. The presented algorithm is robust under facial expression for aligned nearly frontal faces. However, it is sensitive to pose variations.

In the future, we will extend our proposed descriptor on different shape maps and Gabor features. The weighted classifier to handle expression is another research direction that we will work on it to apply the whole face including non-rigid areas. Also, working on the recognition system robustness under pose variations is another plan to extend our work to face with this challenge.

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