# Gappy wavelet neural network for 3D occluded faces: detection and recognition

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Abstract The first handicap in 3D faces recognizing under unconstrained problem is the largest variability of the visual aspect when we use various sources. This great variability complicates the task of identifying persons from their 3D facial scans and it is the most reason that bring to face detection and recognition of the major problems in pattern recognition fields, biometrics and computer vision. We propose a new 3D face identification and recognition method based on Gappy Wavelet Neural Network (GWNN) that is able to provide better accuracy in the presence of facial occlusions. The proposed approach consists of three steps: the first step is face detection. The second step is to identify and remove occlusions. Occluded regions detection is done by considering that occlusions can be defined as local face deformations. These deformations are detected by a comparison between the input facial test wavelet coefficients and wavelet coefficients of generic face model formed by the mean data base faces. They are beneficial for neighborhood relationships between pixels rotation, dilation and translation invariant. Then, occluded regions are refined by removing wavelet coefficient above a certain threshold. Finally, the last stage of processing and retrieving is made based on wavelet neural network to recognize and to restore 3D occluded regions that gathers the most. The experimental results on this challenging database demonstrate that the proposed approach improves recognition rate performance from 93.57 to 99.45 % which represents a competitive result compared to the state of

**Keywords** 3D face recognition; Wavelets · Wavelet neural network · Gappy data · Occlusion detection

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#### 1 Introduction

3D facial processing has become an important biometric modality because of its aptitude in managing challenging problems. For example, for a single subject face image appearance is affected by lighting conditions [4], by head pose and orientation, by facial expression, in some cases image may be corrupted by the presence of occluding objects. The use of 3D facial shape modality, either alone or together with 2D texture channel, has shown to be helpful in recent large-scale face recognition benchmarks [21, 22]. Nevertheless, changes due to facial emotions and occlusions are still important factors that disgrace the accuracy of 3D face recognizers. Newly, many researches present algorithms which are especially thoughtless to emotion's detection. The regular analysis of the occlusion problem was absent due to the unavailability of such variations in 3D face databases. However, the accuracy of rigid registration methods decreases if some parts of the face enclose non-rigid surface deformations. As a solution to this problem, region-based systems and deformable registration techniques were suggested in [5, 13, 14, 17]. In region-based face recognition, a face is defined as a composition of facial components. Furthermore, deformable methods employ more complex registration schemes.

Another problem that affects performance in face recognition is due to the existence of partial facial occlusions. In 2D face recognition applications, local matching approaches have been used to compensate wrong occluded local regions classification by correct classification by the use of non-occluded parts [15, 16, 25]. In 3D case, there are very few studies about facial occlusions. This is mainly due to the scans requiring in 3D face databases with occlusion variations. In [6], a database composed of 208 3D faces from 22 different subjects. A training set of 132 images with various non-occluded facial and a test set of 76 faces. 52 of the test samples included a variety of occlusions such as glasses, scarves, caps, or by the subject's hand. Their approach consists of detecting the occluded area and then restoring the missing region using the information from the non-occluded parts [6, 8]. The occlusions are detected by calculating a distance between face with a generic facial model, and utilizing thresholding on facial surface distances.

We propose a new 3D face identification and recognition method based on Gappy Wavelet that is able to provide better accuracy in the presence of expression variations and facial occlusions. The proposed approach consists of three steps: the first is face detection by the mean of wavelet transformation in order to detect its presence in test image. The second step is to identify and remove occlusions. Occluded regions detection is done by considering that occlusions can be defined as local face deformations. These deformations are detected by a comparison between the input facial test wavelet coefficients and wavelet coefficients of generic face model formed by the mean data base gallery to benefit of wavelet neighborhood relationships between pixels rotation, dilation and translation invariant. Then, occluded regions are refined by removing pixels above a certain threshold. Finally, the last stage of processing and retrieving is made based on multi library wavelet neural network [2, 3, 19, 20], to recognize and to restore 3D occluded regions that gather the most.

In this work, we introduce a new 3D face identification and recognition method which is capable to achieve robustly under presence of significant occlusions amount motivated by Gappy PCA 'Principal Gappy Componement Analysis' by Colombo et al. in [10], Fuzzy PCA 'Fuzzy Principal Componement Analysis' by Zhi-ming et al. in [26] and T-PCA method 'Principal Componement Analysis on the tangent



space' by Drira et al. in [12]. We suggest applying discreet wavelet transform (DCT) for occlusion detection. The distance between the gallery and test faces obtained are then used to determine occluded region. Finally a multi library wavelet neural network is used for face recognition.

This paper is divided into three parts. In the first part, we present Gappy PCA approach (GPCA). Then, we detail our approach called Gappy Wavelet Neural Network (GWNN). Finally, we present experimental results in UMD-DB database [9] and BOSPHORUS database [23] for 3D occluded face identification and recognition.

# 2 Proposed approach

The proposed approach for face detection and recognition using Gappy wavelet is resumed in Fig. 1. Face detection used in the proposed approach is based on an improved version of Colombo algorithm [11]. Regions occluded detection is based on the idea that occlusions are considered as local face deformations. These deformations are initially calculated by the difference between wavelet coefficients of input depth image at level l and their equivalent one of a generic face model for each pixel (i, j). At this point, each transformed level depth image  $I_{wl}$  is compared with the transformed level depth mean face  $M_{wl}$  to detect occluding objects. A Multi Library Wavelet Neural Network [19] (MLWNN) classifier is used to restore occlusion.

#### 2.1 Face detection

In this study 3D training set and candidate input faces are normalized (angle face orientation is equal to zero) Fig. 2 represents a detailed diagram showing the main steps of face identification.

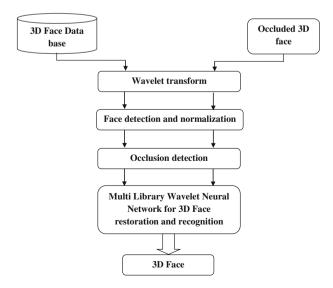


Fig. 1 The proposed algorithm diagram



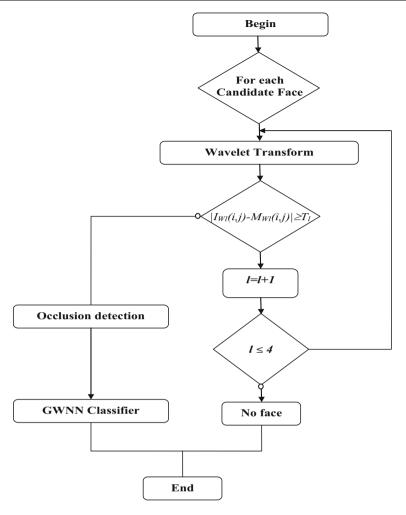


Fig. 2 face detection algorithm: detailed diagram

Wavelet transformation is applied on each candidate depth image and then we calculate the distance between the transformed depth image and the one of the mean depth image in data base according to Eq. (1):

$$|I_{wl}(I,j) - M_{wl}| \le T_l \forall 1 < i < N, 1 < j < M \tag{1}$$

Where

 $I_{wl}$  is the wavelet transform of candidate face depth image at level l

 $M_{wl}$  is the wavelet transform of mean face depth image at level l

 $T_l$  is a threshold at level l fixed in advance according to data base, it may be considered as tuning parameter.

In one case, if condition (1) fails at all levels (level is a tuning parameter fixed to 4), the considered depth image is considered as non face. In the other case, face is present and we proceed to detect occlusion in the next step.



## 2.1.1 Occlusion detection

Let  $I: D \subset \mathcal{H} \times \mathcal{H}^+ \to \mathcal{H}^2$ ;  $(x, y) \mapsto I(x, y)$  be a depth gray scale image on a domain D. The relation between occluded and non occluded depth image is given by:

$$I(x) = \begin{cases} I((w(x, y) + n(x, y); (x, y) \in D/\Omega(p)) \\ \phi(x, y); (x, y) \in \Omega(p) \end{cases}$$
 (2)

Where  $w: D \times \mathfrak{R}^+ \to \mathfrak{R}, (x,y) \mapsto w(x+dx,y+dy) \equiv x+v(x),y+v(y)$  is the domain deformation mapping I(x,y) onto I(x+dx,y+dy).

The occluded region  $\Omega$  can change and it can take any form from independently to the test image I(x, y). In the limit  $p \rightarrow 0$ ,  $\Omega(p) = \varphi$ ; ie: there is no occlusion.

The term n(x,y) in Eq (2) compound the effects of many parameters like illumination changes, Lambertian reflection [24], rotation, scale, etc....

Let's take the simplified hypothesis: face are normalized, the illumination changes and reflection are neglected compared to occlusion effects. In this case we assume that  $n(x,y) \rightarrow 0$ .

The residual error e(x, y) can be written as:  $e:D \rightarrow \Re;(x,y) \mapsto e(x,y)$ 

$$e(x, y) \equiv I(x, y) - I(w(x, y)) \begin{cases} n(x, y), (x, y) \in D/\Omega \\ \phi(x, y) - I(w(x, y), (x, y) \in \Omega \end{cases}$$
(3)

Considering the developed hypothesis and the mean depth image obtained from the data base, the deformation v(x,y) can be calculated by using Eq (4):

$$v(x, y) = |I_M(x, y) - I(w(x, y))| + e(x, y)$$
(4)

To well localize occlusion region we build a binary masque  $M_l(I,j)$  at level l according to the following equation:

$$M_l(i,j) = \begin{cases} 1 & \text{if } |I_{Ml}(i,j) - I_l(w(x,y))| < T_l \\ 0 & \text{else} \end{cases}$$
 (5)

Once binary masque  $M_l$  is constructed, at this point, we made a morphological operator (logical and) between candidate face and  $M_l$  in order to localize occlude object noted  $O_l$  as presented in Eq. (6):

$$O_l = I_l(w(x, y)) \wedge M_l(i, j) + E_l \tag{6}$$

Where,  $E_l$  is region occlusion error at level l.

At limit in the case where there is no occlusion (the masque M is a null matrix) we will have:



Appling in Eq. (6), we obtain:

We obtain:

# 2.2 Gappy multi library wavelet neural network classifier

MLWNN as presented in [2] and shown in Fig. 3, consists of three layers: a linear input layer composed of  $N_i$  neurons which receive the difference between each neutral depth image in data base and the calculated occluded objet  $O_i$ . A hidden layer constituted by  $N_w$  wavelets and a linear output neuron receiving pondered output of multi-dimensional wavelets and a refinement parts.

2D MLWNN input noted  $\Gamma(x,y)$  is given by the following equation:

$$\Gamma_l(x, y) = I_k - O(x, y) \tag{7}$$

Where,  $I_k$  is the neutral depth image in data base.

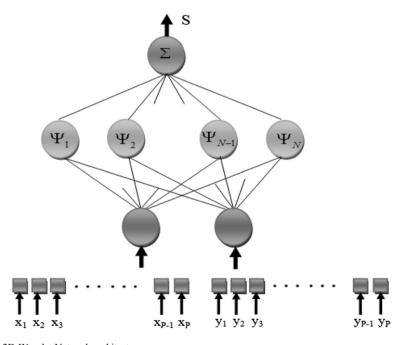


Fig. 3 2D Wavelet Network architecture





Fig. 4 Sample images illustrating occluded object with correspondence to their 3D acquisition

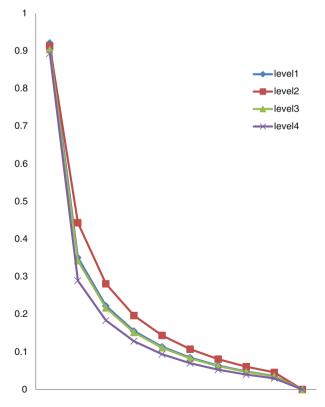


Fig. 5 Occlusion detection curves in term of threshold variation and wavelet level decomposition



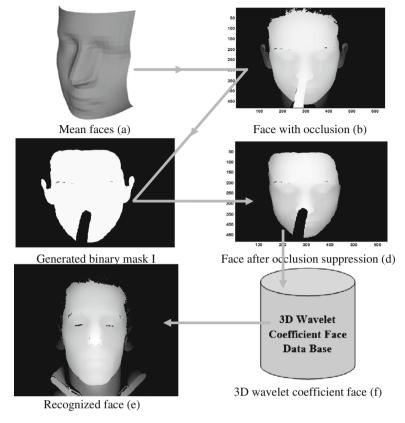


Fig. 6 Example of occlusion detection and face recognition

2D MLWNN output formalism,  $F: \mathcal{R} \rightarrow \mathcal{R}, (x,y) \mapsto \hat{F}(x,y)$  as

$$\hat{F}(x,y) = \sum \alpha_{i,j} \Psi_i \left( \frac{x - t_i}{d_i} \right) \Psi_j \left( \frac{y - t_j}{d_j} \right) + a_1 x + a_2 y + b$$
 (8)

From Eqs. (7) and (8), we get:

$$\hat{\Gamma}_{k}(x,y) = I_{k}^{\wedge}(x,y) - \hat{O}_{k}(x,y)$$
(9)

$$\hat{\Gamma}_{k}(x,y) = \sum \alpha_{i,j} \Psi_{i} \left( \frac{x_{I_{k}} - t_{i}}{d_{i}} \right) \Psi_{j} \left( \frac{y_{I_{k}} - t_{j}}{d_{j}} \right) - \sum a_{i,j}^{'} \Psi_{i} \left( \frac{x_{O_{k}} - t_{i}}{d_{i}} \right) \Psi_{j} \left( \frac{y_{O_{k}} - t_{j}}{d_{j}} \right) + a_{1}^{'} x + a_{1}^{'} y + b^{'}$$

$$\tag{10}$$

Table 1 Total rate of restitution

Database	Classifier	Total face	Restituted face
UMD-BD (non-occluded)	GWNN	951	100 %
UMD-BD (occluded)		951	92,44 %



Object type	Total face number	Restitution Percentage Level=1	Restitution Percentage Level=2	Restitution Percentage Level=3	Restitution Percentage Level=4
Hand with cap	91	95.60 %	95.60 %	96.70 %	94.51 %
eyeglasses	184	98.91 %	99.46 %	99.46 %	99.46 %
Free hands	181	85.64 %	88.95 %	88.95 %	88.40 %
Newspaper	77	87.01 %	85.71 %	83.12 %	79.22 %
Hand with phone	97	95.88 %	96.91 %	96.91 %	96.91 %
Scarf	123	86.99 %	88.62 %	89.43 %	86.99 %
Scissors	109	93.58 %	93.58 %	93.58 %	93.58 %
Hat	89	94.38 %	95.51 %	94.38 %	94.38 %

Table 2 Rate of restitution for each occlusion

We define the Distance from Wavelet Network Face Space (DWNFS) as:

$$DWNFS_k = \Gamma_k(x, y) - \hat{\Gamma}_k(x, y) \tag{11}$$

DWNFS error needs some kind of normalization because occlusion region is not fixed. In this work we simply normalize by dividing by the number of non-zero masque  $n_O$ ; but other normalization approaches could be tested with as well. The Normalized distance DWNFS (NDWNFS) for the depth image k becomes:

$$NDWNFS_k = \frac{\Gamma_k(x, y) - \hat{\Gamma}_k(x, y)}{n_0} \tag{12}$$

The overall problem can be written as the minimization of the cost function NDWNFS. In other words we should adjust wavelet neural network structural parameters using MLWNN architecture and training algorithm which provides  $O(1/k^2)$  convergence in k iterations [3], whereas for standard gradient descent is O(1/k) [18].

At this stage we obtain K distances. There are many algorithms for decision such as C-means, KNN, fuzzy logics, etc.... In this work, we adopt KNN algorithm for decision. To

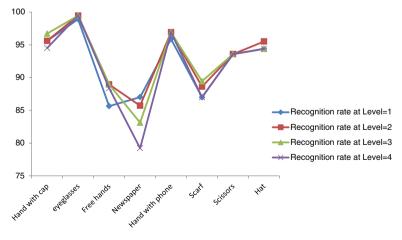


Fig. 7 Recognition rate for each occlusion from level 1 to level 4



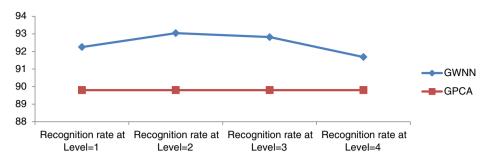


Fig. 8 UMB-DB global recognition rate for GWNN and GPCA

reconstruct the occluded region from the Gappy depth image, we used two morphological operators (AND then OR) on candidate image.

The reconstructed image (RI) is obtained by applying Eq. (13)

$$RI = (I(w(x,y)) \land M(x,y)) \lor \stackrel{\wedge}{\Gamma}(x,y)$$
(13)

## 3 Implementation and results

To test the proposed method we used two data bases: UMB-DB and BOSPHORUS.

## 3.1 UMB-DB

UMB-DB (University of Milano-Bicocca Data Base, [11]) dataset which is composed of 132 images without occlusions and 76 test set with artificially different kinds of occlusion. The test patterns are occluded by Cap, by glasses, by free hands, by newspaper, by hand with phone, by scarf, by scissors and by hat. Different occlusion objects are represented in Fig. 4. All the faces have been manually normalized in a standard position and scale. (see [7] for a description of an automatic normalization). A face of UMB database is represented like a 3D object surface as

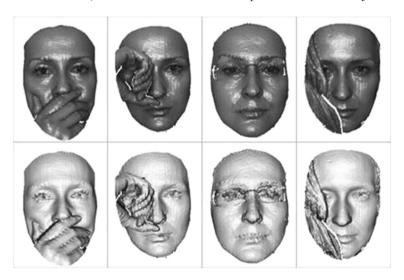


Fig. 9 Sample images illustrating occlusions in the Bosphorus 3D face database



Table 3	Comparison	of the	two
approach	es		

Database	Classification	Restitution Percentage
UMB-DB	GPCA [10] GWNN LEVEL=1	89.8 % 92.24 %
	GWNN LEVEL=2 GWNN LEVEL=3 GWNN LEVEL=4	93.04 % 92.81 % 91.67 %

the pixels matrix X, Y and Z. Each component or matrix X(I, j), Y(I, j) or Z(I, j) has 480x640 pixels. The component that carries the information is the depth component Z. Figure 4, shows 3D acquisition and some occlusions.

Figure 5 shows occlusion detection curves in term of threshold variation and wavelet level decomposition for the GWNN classifier. The curves have been computed considering only the wavelet transform candidate faces from level 1 to level 4. As we can see, performance deteriorates when threshold decreases. This is an obvious consequence due to information deficiencies. Threshold should be chosen according to application requirements.

Figure 6 shows different intermediate results of the proposed approach. Occlusions are detected by comparing the coefficients candidate face (Fig. 6b) with their omologues of mean face (Fig. 6a) previously calculated. Coefficients that their distance is greater than 2 from mean face coefficients will be considered occluded and so deleted. This treatment is done by applying a binary mask, shown in Fig. 6c. For deleting occluding objects (using morphological operator defined in Eq. (13)). The result is indicated in Fig. 6c after inverse wavelet transformation. At this stage, the binary masks are used to remove occlusions. After that GWNN is used for face recognition and occlusion restoration. The face that has the minimum difference from wavelet network parameters space (weights connection) (Fig. 6) are considered the most similar.

Table 1 reports the results obtained using different artificially occlusions (at least half of the face image must be non-occluded) for a classifier threshold  $T_i$  manually fixed in order to reduce false positives. A fraction of 90.43 % has been successfully detected. Results are acceptable considering problem hardiness and the fact that 3D acquisitions are contaminated with some errors. The proposed detector performs very well on non-occluded faces, success 100 % of the detected faces of a total number of 951 faces.

According to Table 2, we observed that recognition rate change according to the type of objects occluding. Recognition rate depends on occlusion size and position. For example, false positive is equal to 2 for eyeglasses, equal to 11 for Newspaper and 5 for hat.

Table 4 Rate of restitution for each occlusion

	ARM [1]	Proposed approach			
Object type	Restitution Percentage	Restitution Percentage Level=1	Restitution Percentage Level=2	Restitution Percentage Level=3	Restitution Percentage Level=4
Mouth with hand	91.49 %	89.68 %	91.43 %	92.67 %	92.47 %
Eye with hand	87.23 %	86.65 %	86.96 %	89.46 %	89.14 %
Eye glasses	95.74 %	95.32 %	96.19 %	97.12 %	97.02 %
Hair	89.66 %	89.32 %	90.71 %	93.91 %	89.22 %



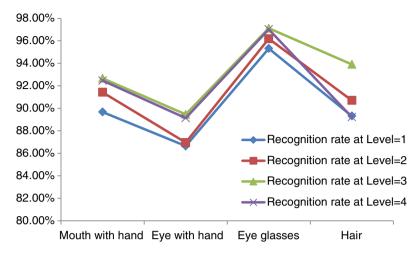


Fig. 10 Recognition rate for each occlusion from level 1 to level 4

Observing Figs. 7, 8 and 9 we notice that when more than half face is not occluded recognition rate is high (99.46 % for eyeglasses at level 2, 3 and 4). The lowest rate is obtained by newspaper occlusion (79.22 % at level 4). This result is obvious because the newspaper recovers the biggest surface in face. We can also remark that when occlusion is located on eyes or nose, such as scarf, recognition rate is low (86.99 %, 88.62 %, 89.43 % and 86.99 %, respectively at level 1, level 2, level 3 and level 4).

Table 3, shows that in all cases our approach is more efficient than the one obtained by GPCA algorithm. It can be explicated by the fact that wavelet transform reduce information redundancy and the use of wavelet neural network as classifier performs classification procedure. The performance is due to orthogonal projection of each 3D wavelet coefficient face on wavelet network regressor space.

$$Rate of restution = \frac{number of returned faces}{total number} * 100$$

## 3.2 Bosphorus DB

To evaluate the performance of our approach, we test the described algorithm in the previous section on the BOSPHORUS basis. BOSPHORUS data base is suitable for this assessment. Indeed, it contains 3D scans of 60 men and 45 women, 105 subjects with different expressions,

**Table 5** Comparison of the two approaches

Database	Classification	Restitution Percentage
BOSPHORUS DB	ARM [1] GWNN LEVEL=1 GWNN LEVEL=2 GWNN LEVEL=3 GWNN LEVEL=4	91,03 % 90,24 % 91,32 % 93,29 % 91,96 %



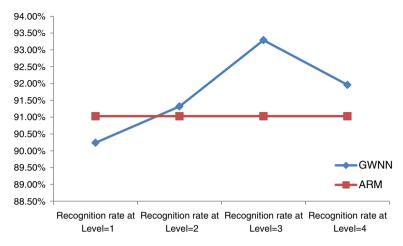


Fig. 11 Bosphorus-DB global recognition rate for GWNN and ARM

poses, and in the presence of external occlusions (glasses, hand, hair). The majority of subjects were aged between 25 and 35 years. The total number of scans is 4,652 with 54 scans per person. However, each subject has at least three scans with external occlusions (mouth with hand, eyes with glasses, eye with hand). Some topics have more than one occlusion with the hair, giving a total of 381 sessions with occlusions.

In Table 4, both accuracies obtained for ARM-based registration [1] and GWNN approaches are given. As the results exhibit, in ARM approach, little improvement is achieved compared to GWNN at the first level of decomposition. In GWNN approach, when the level decomposition increases to three level of decomposition, the proposed approach introduces better results. Figure 10 shows that when the decomposition level increases the recognition rate for each occlusion is better.

Table 5, shows that in all cases our approach is more efficient than the one obtained by ARM algorithm [1]. Fig. 11 illustrates the global recognition rate for the two approaches.

## 4 Conclusion and Perspectives

As a result, we proposed an approach exploiting the coefficients of decomposition of different wavelet for face restitution. This clearly shows the advantage in adopting wavelet to solve the occluding objects in 3D face recognition.

We developed a powerful system which is able to restitute 3D faces with different objects obscuring.

We noticed by this technique that the recognition rate is higher than 3.6 % then Gappy PCA approach using the same basic UMB-DB. We obtained also interesting results in comparison with ARM techniques proposed by Nese et al., in [1] when we used Bosphorus database.

The prospects of our work and future research are multiple. We are planning to improve both face and occlusion detection phase. Our aim is also to improve this approach by testing it with other databases faces and by using separate types of wavelets with changing levels of decomposition (wavelet Beta). At the end, by analogy Fuzzy PCA approach we aim to create a new approach called Fuzzy Gappy Wavelet Neural Network which consists in integrating the coefficients of wavelet decomposition.



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