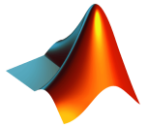
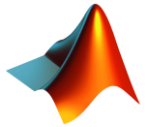
MATLAB 프로그래밍 및 실습

13강. 기초 수치해석 2

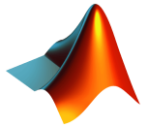


오늘 배울 내용

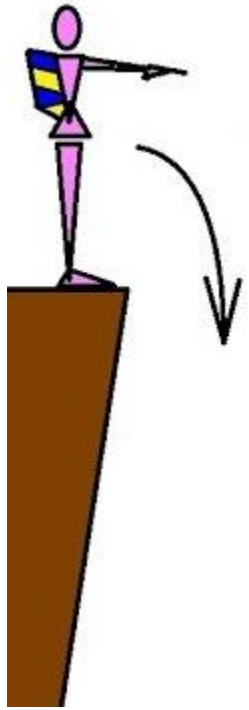
- 방정식의 해
- 함수 최소값 및 최적화
- 수치미분, 수치적분
- 미분방정식
- 몬테카를로 시뮬레이션



solution of equations



낙하산병 문제



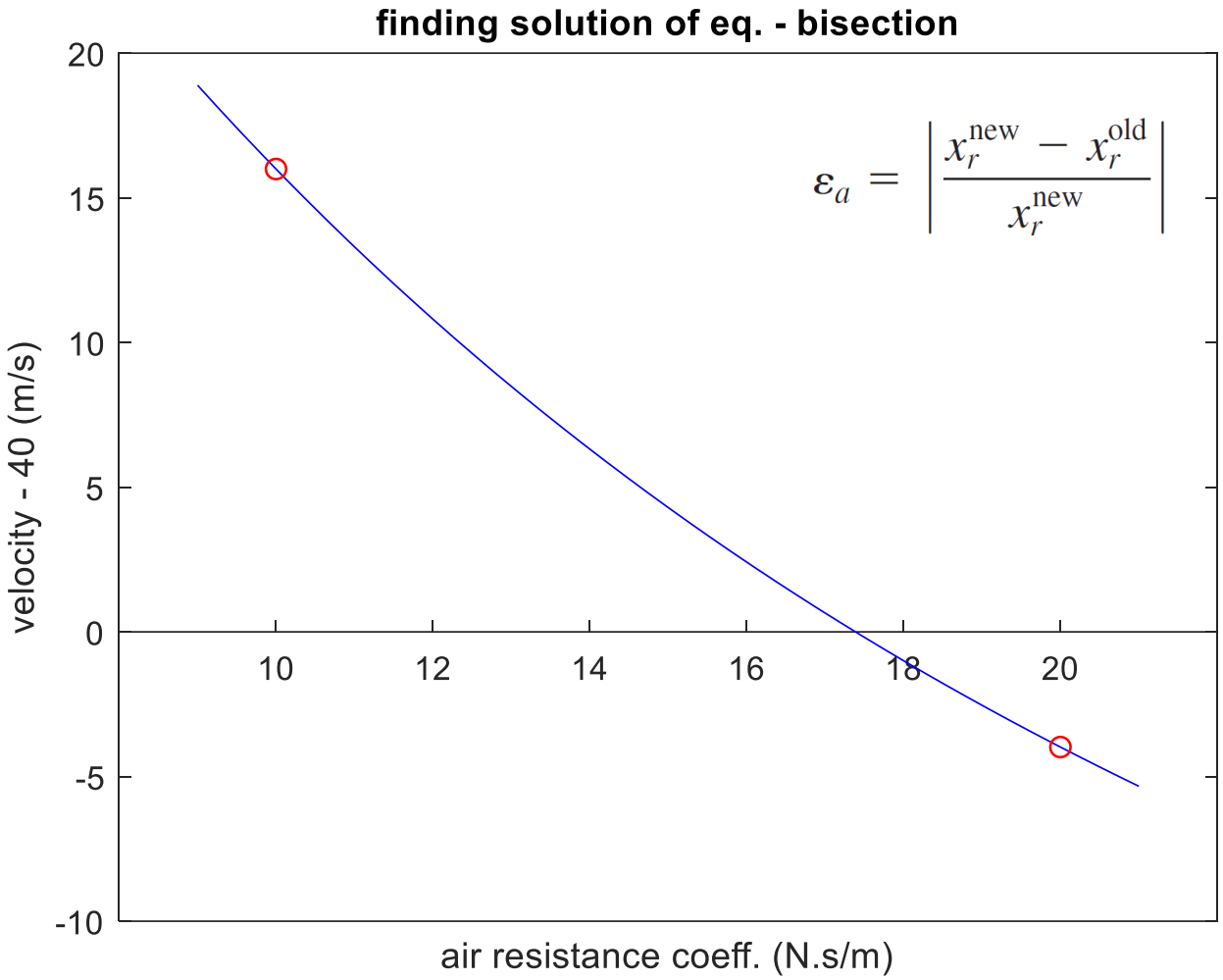
$$v(t) = \frac{mg}{k}\left(1 - e^{-(k/m)t}\right)$$

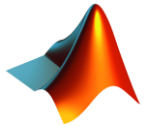
- 80 kg 낙하산병이 낙하 10초 후 40 m/s에 도달하기 위한 공기저항계수는?

  ➔ Find a value k such that

$$f(k) = \frac{mg}{k}\left(1 - e^{-(k/m)t}\right) - 40 = 0$$

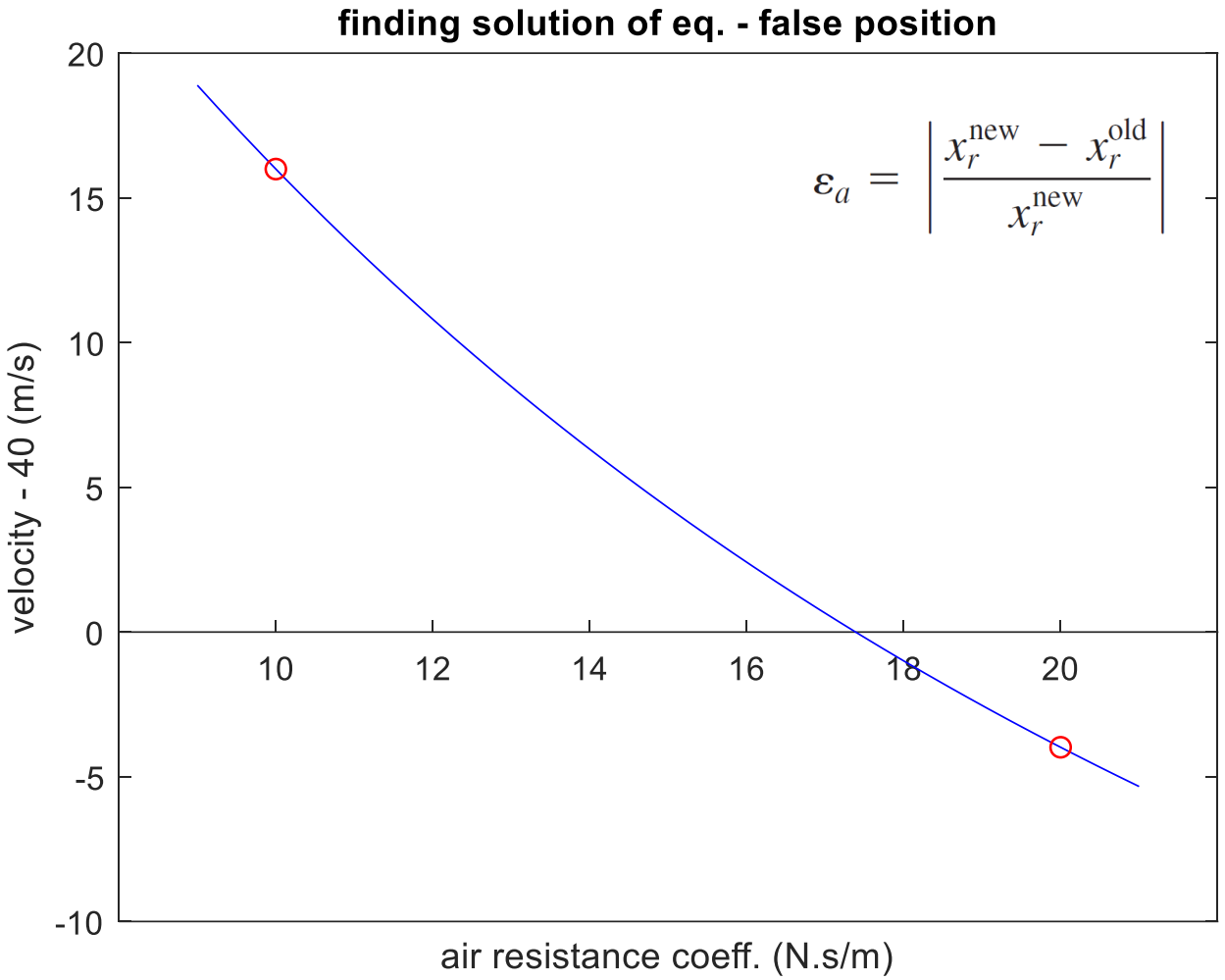
bisection

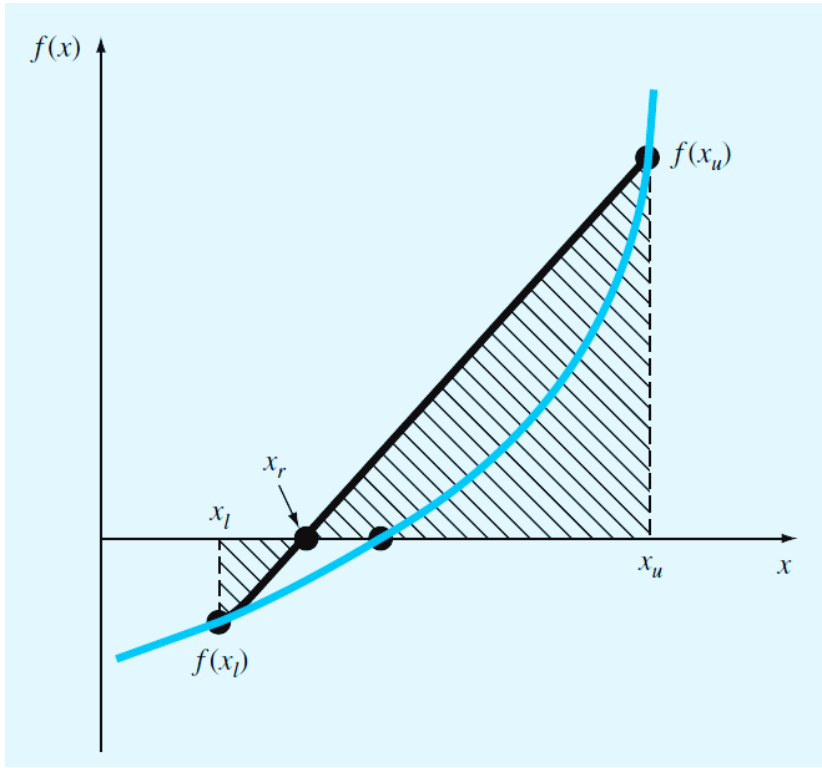




https://www.youtube.com/watch?v=OzFuihxtbtA

false position method

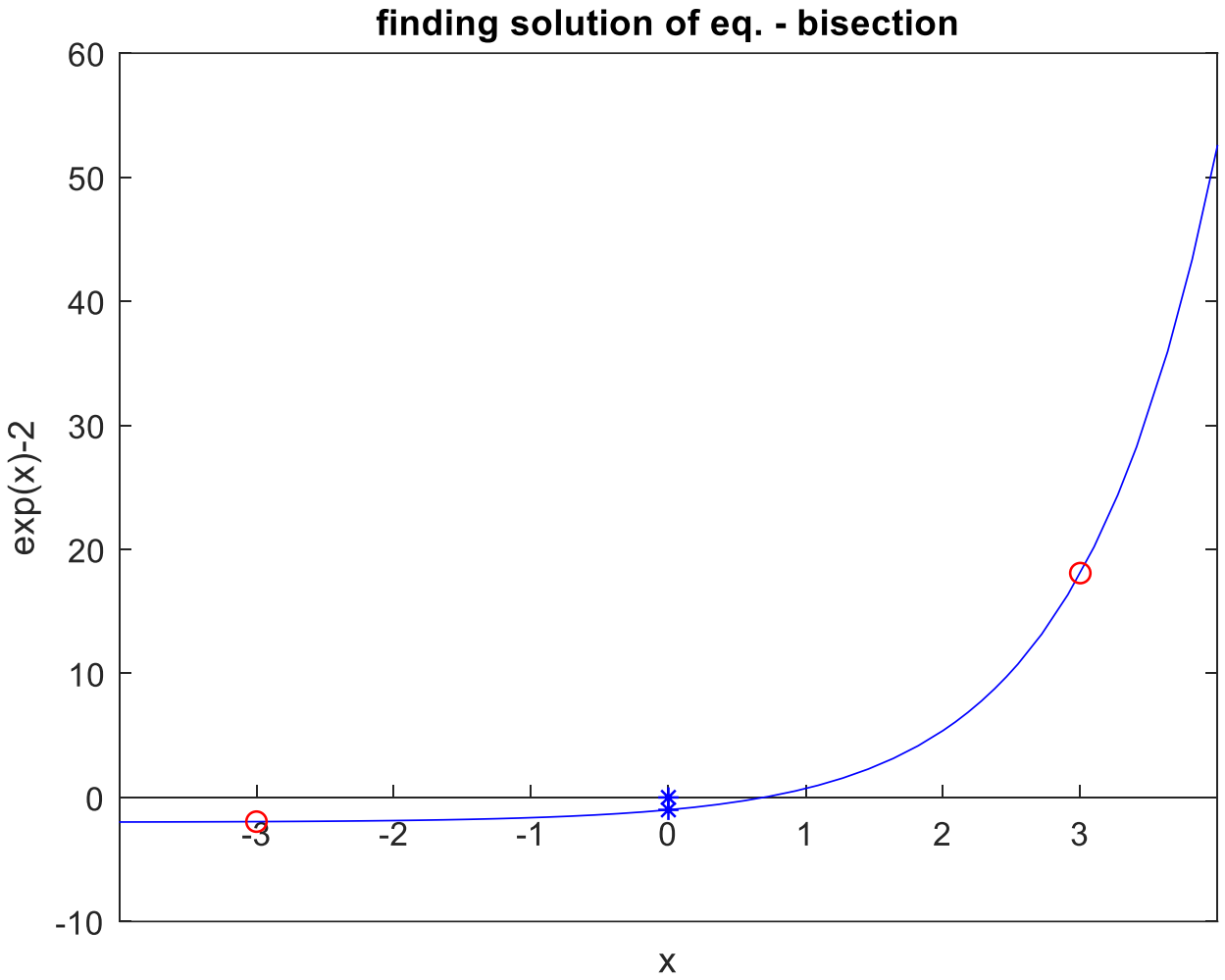


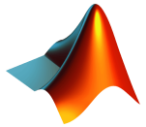


$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$$

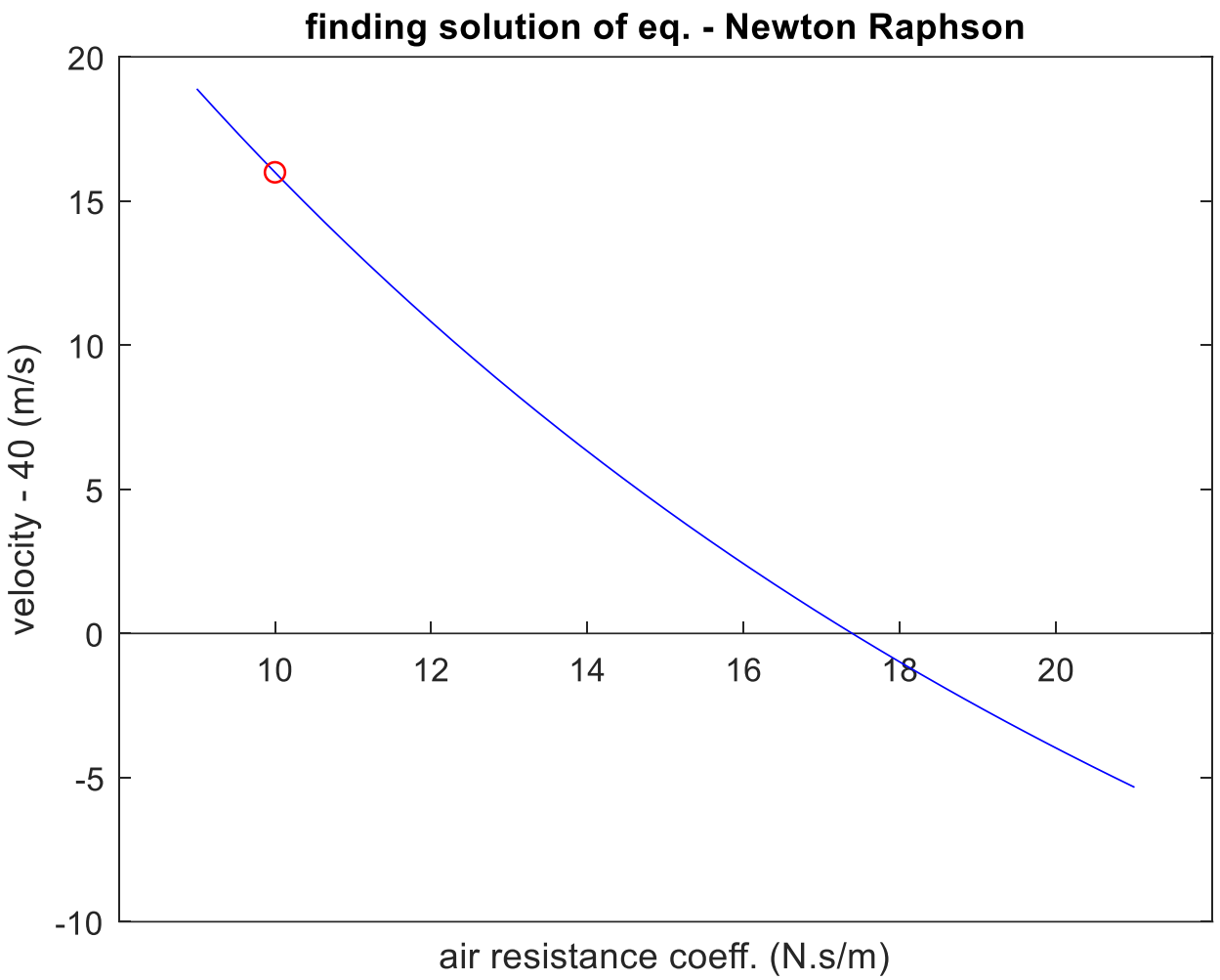
https://www.youtube.com/watch?v=g_bBf0i3NEI

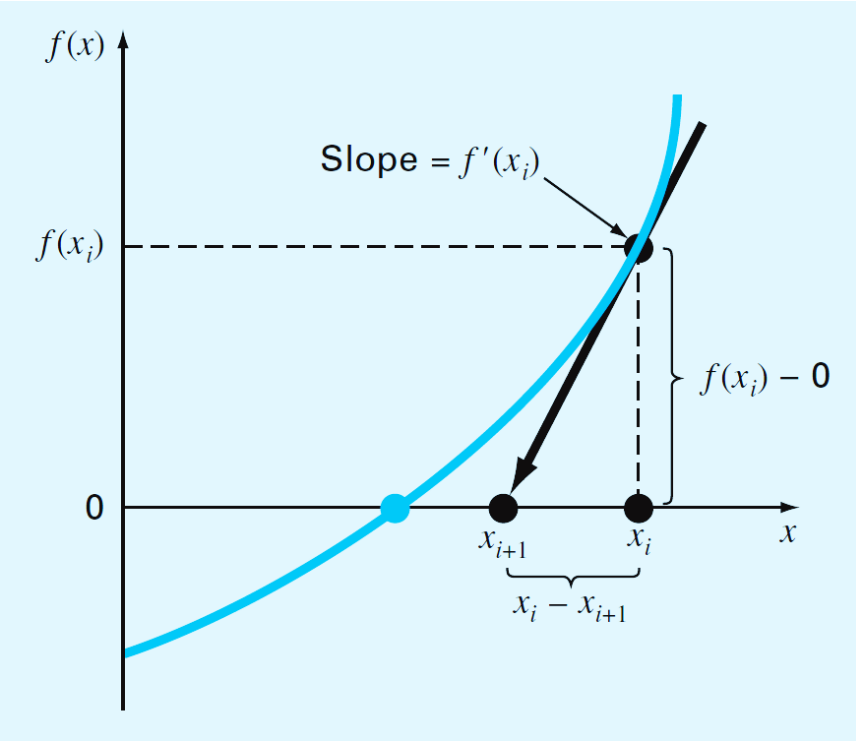
bisection < false position?





Newton-Raphson method

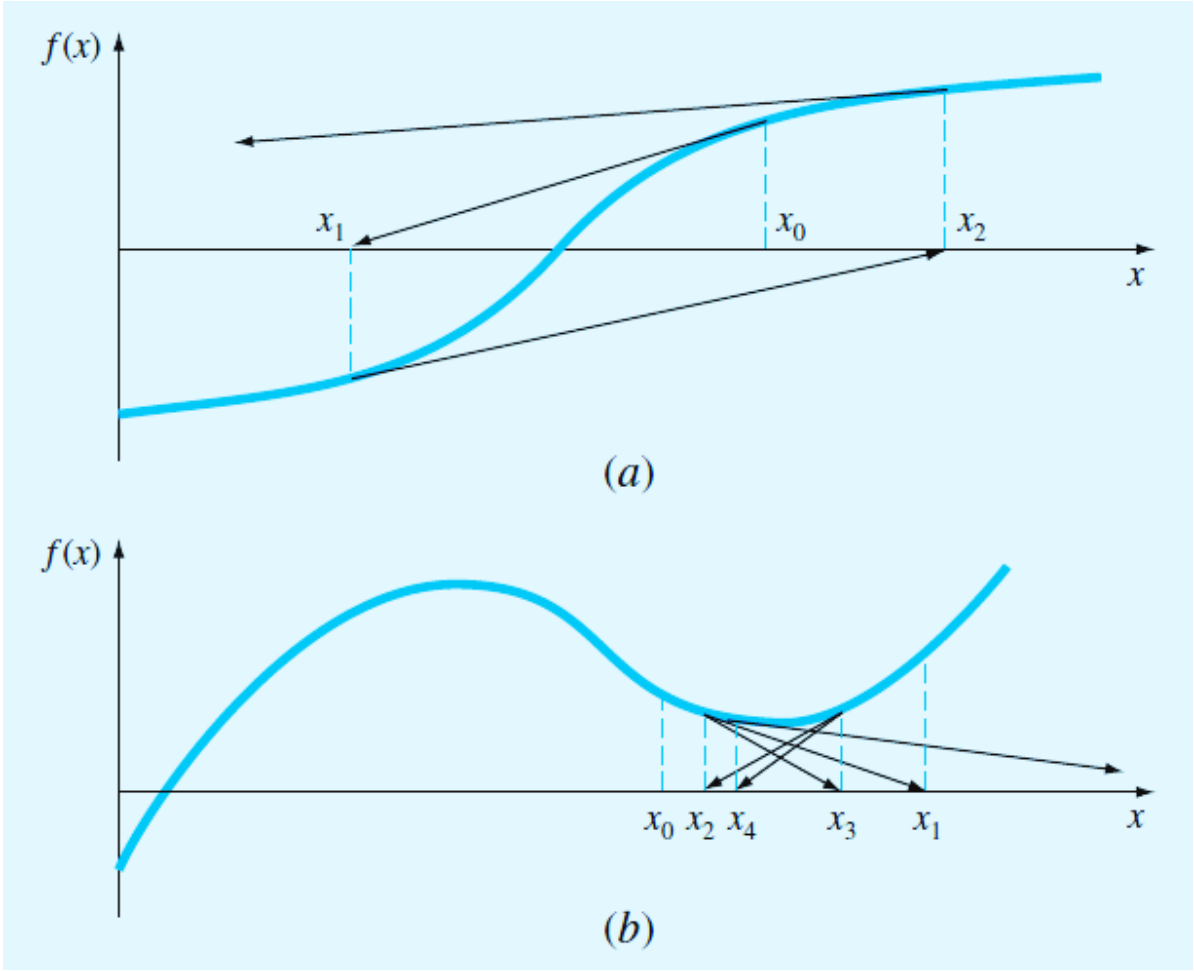


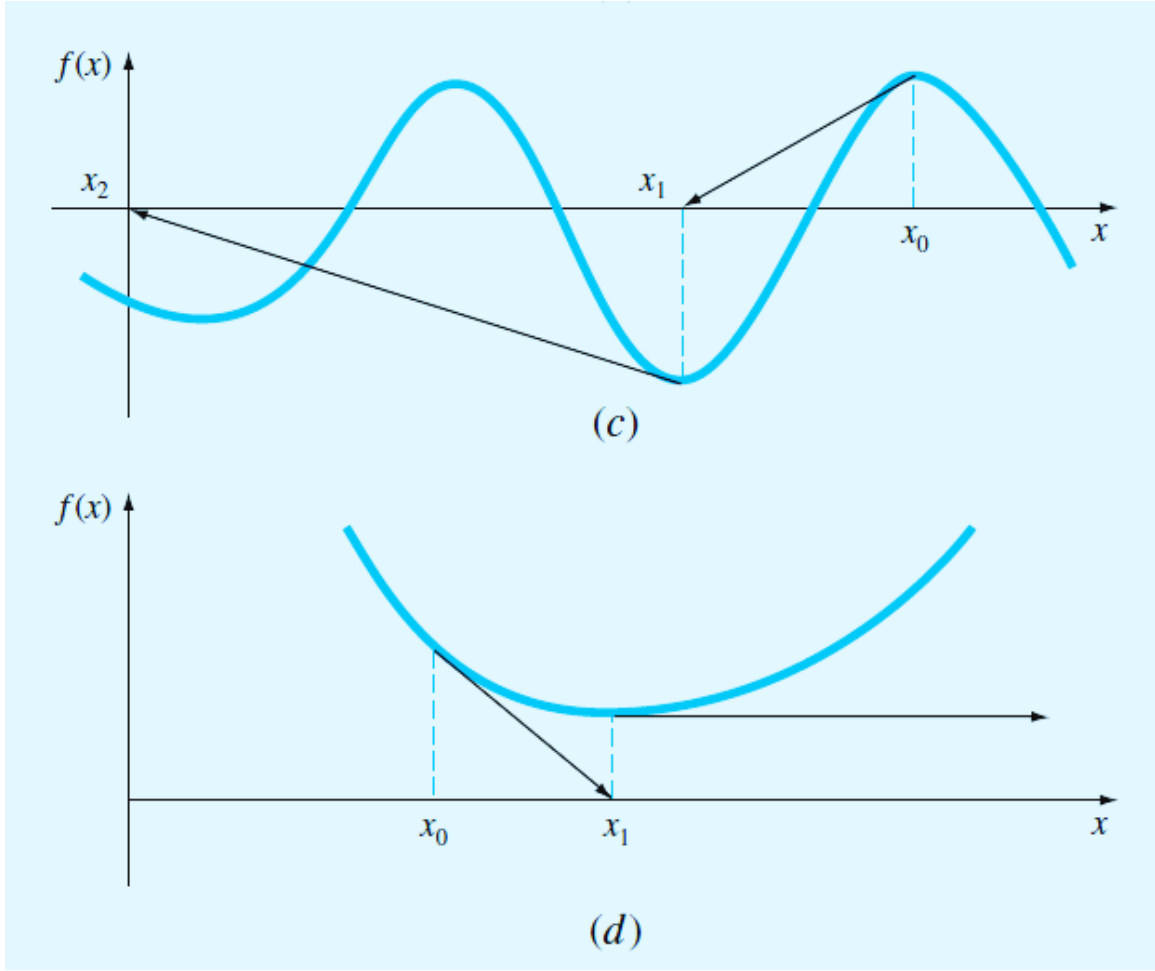


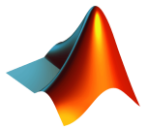
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

https://www.youtube.com/watch?v=cOmAk82cr9M

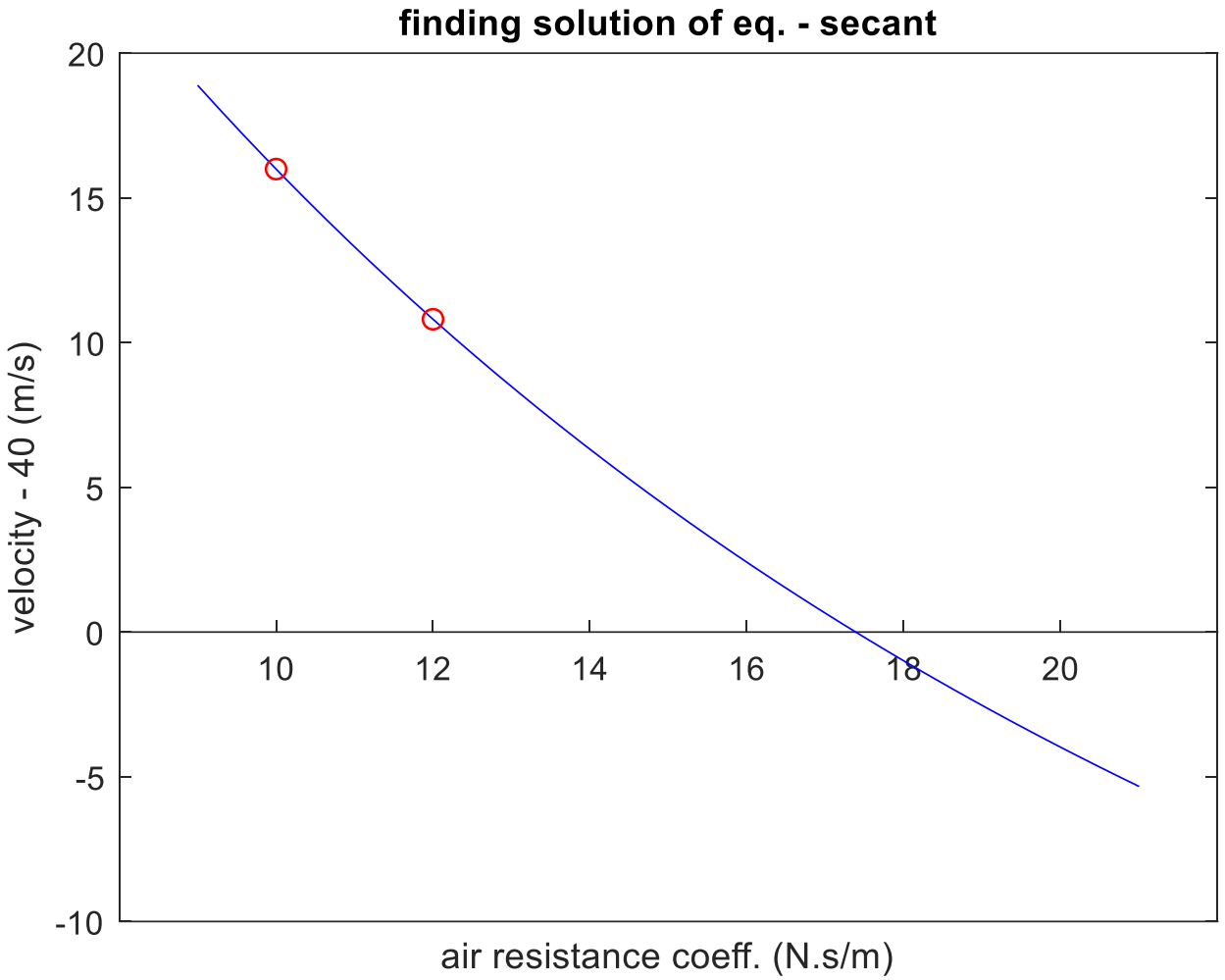
Newton-Raphson (sometimes) fails.

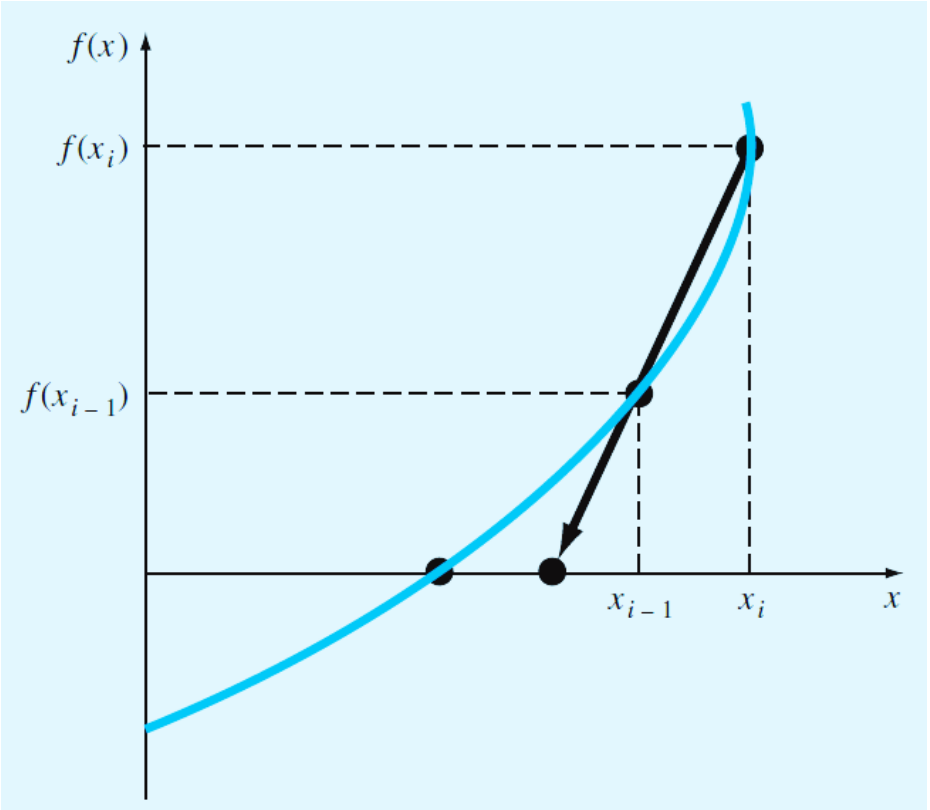




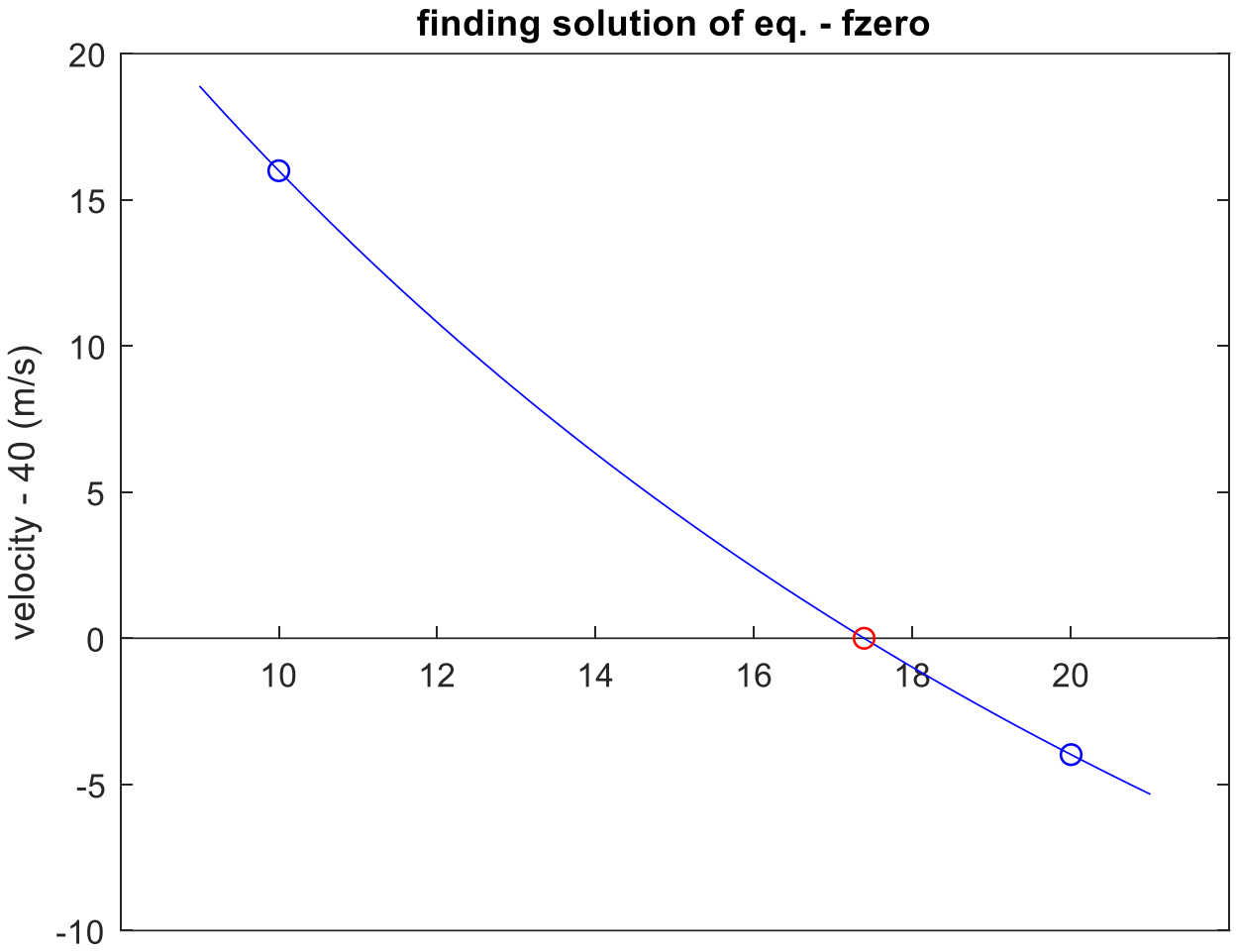


secant method





matlab function - fzero

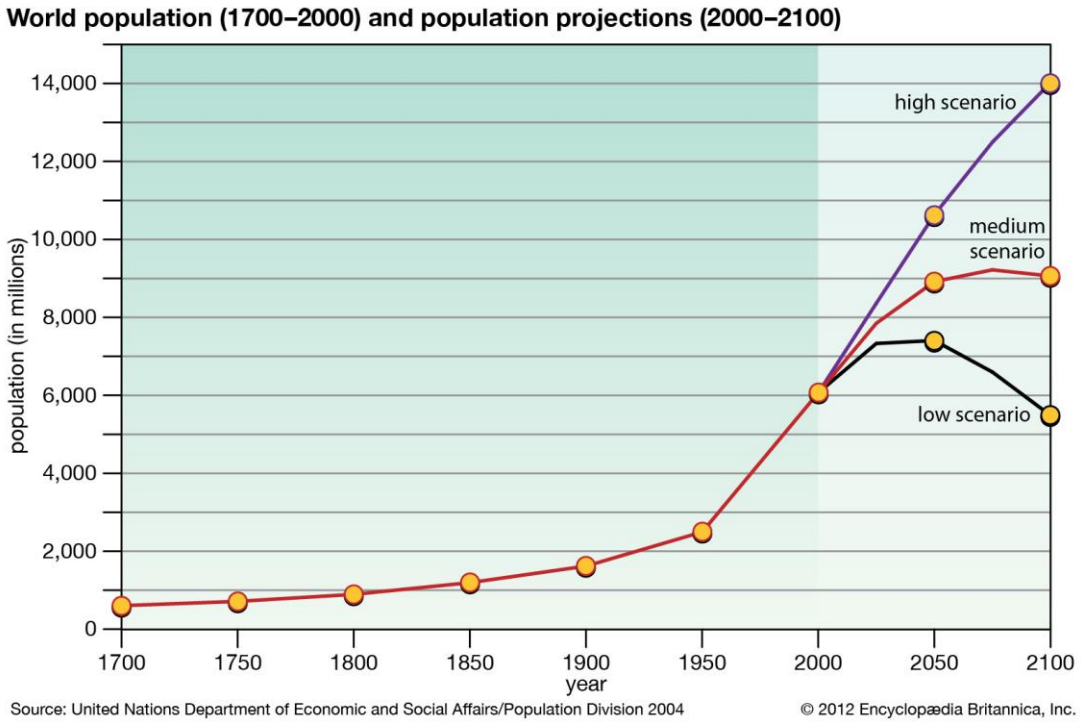


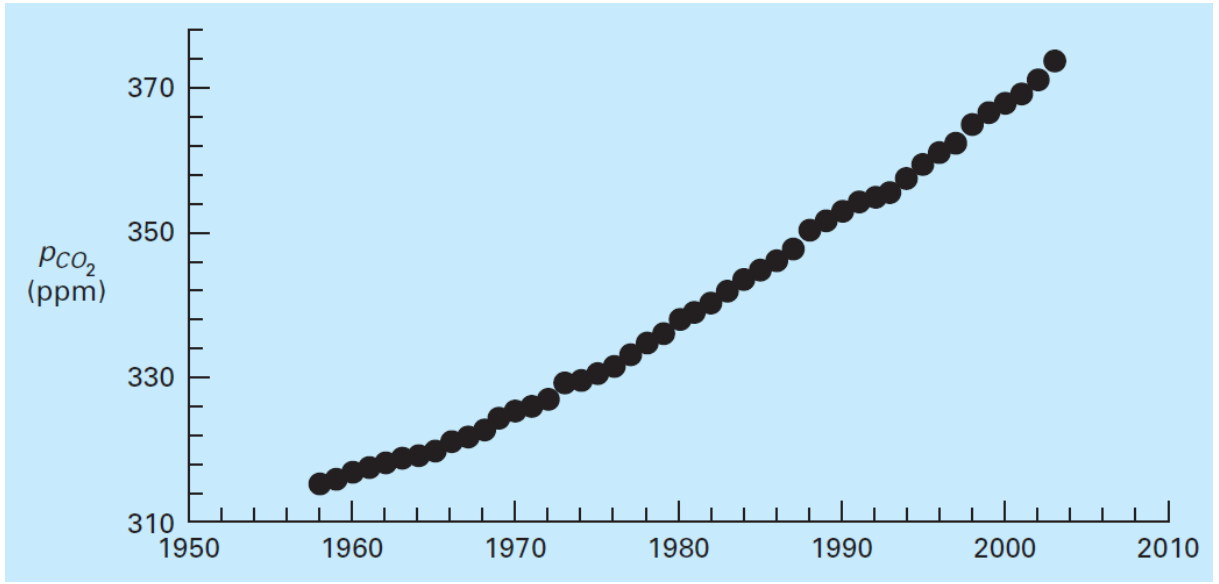

```
xl = 10;
xu = 20;
x = fzero(v, [xl, xu]);
```

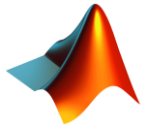
※ fzero는 부호가 바뀌는 지점을 찾는 알고리즘으로,
f(x) = x^2과 같은 경우 해를 찾지 못함

※ bisection method, secant method를 혼합하여 사용함

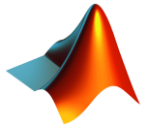
applications



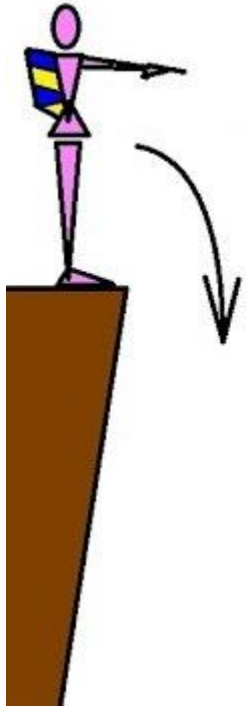




minimum of a function



영원히 고통받는 낙하산병



- 낙하 2.8초 후, 최대한 많이 내려가되 속도는 낮았으면 좋겠다.

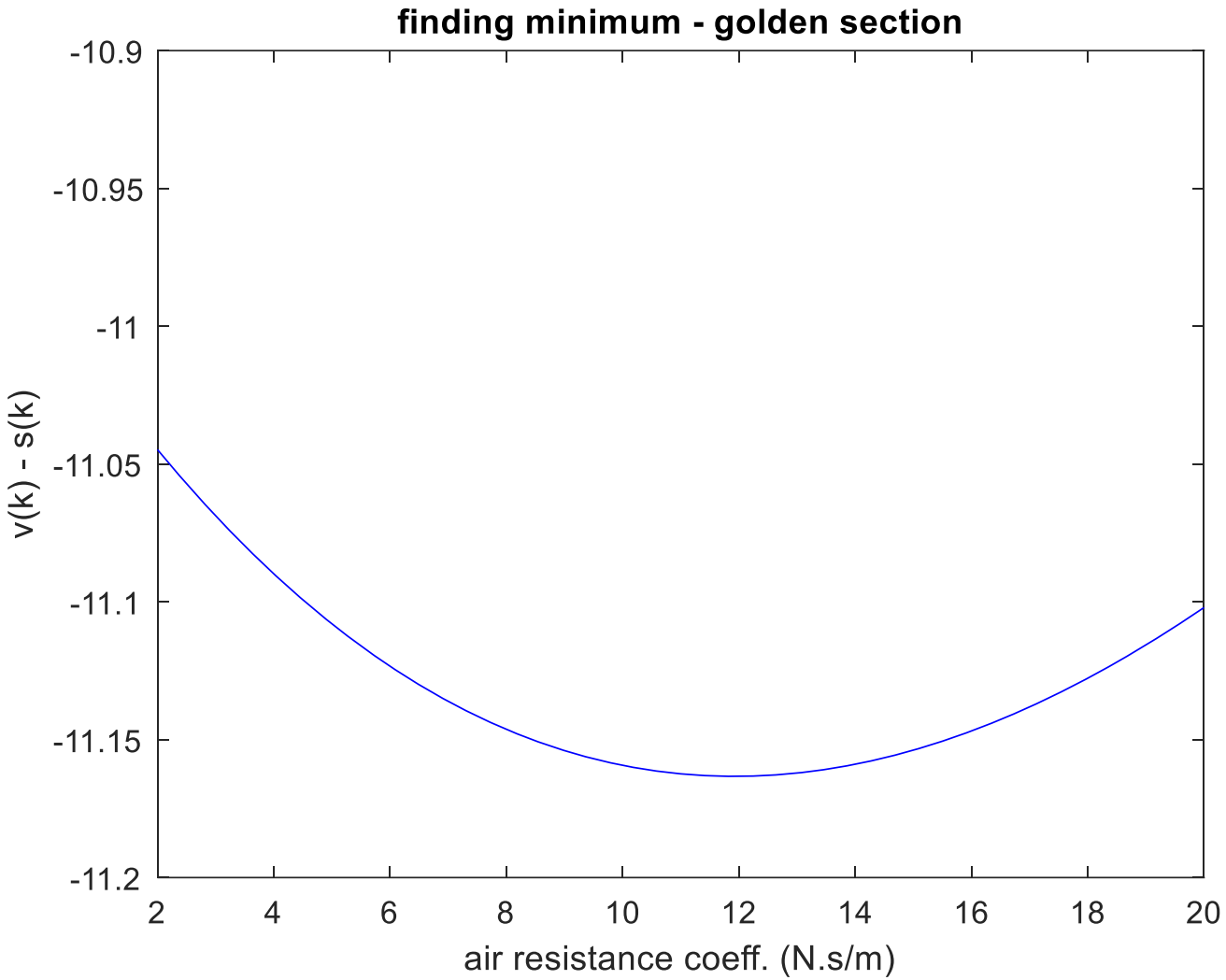
➔ Find a value k to minimize

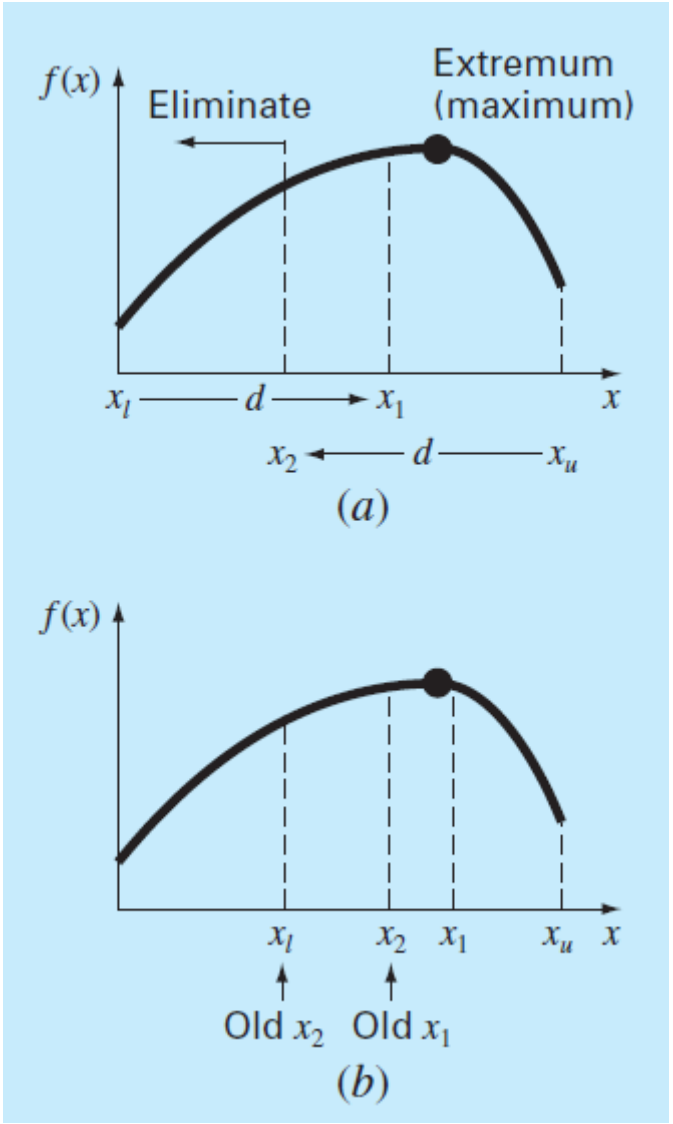
$$v(t) = \frac{mg}{k}\left(1 - e^{-(k/m)t}\right)$$

$$s(t) = \frac{mg}{k}\left(t + \frac{m}{k}\left(e^{-(k/m)t} - 1\right)\right)$$

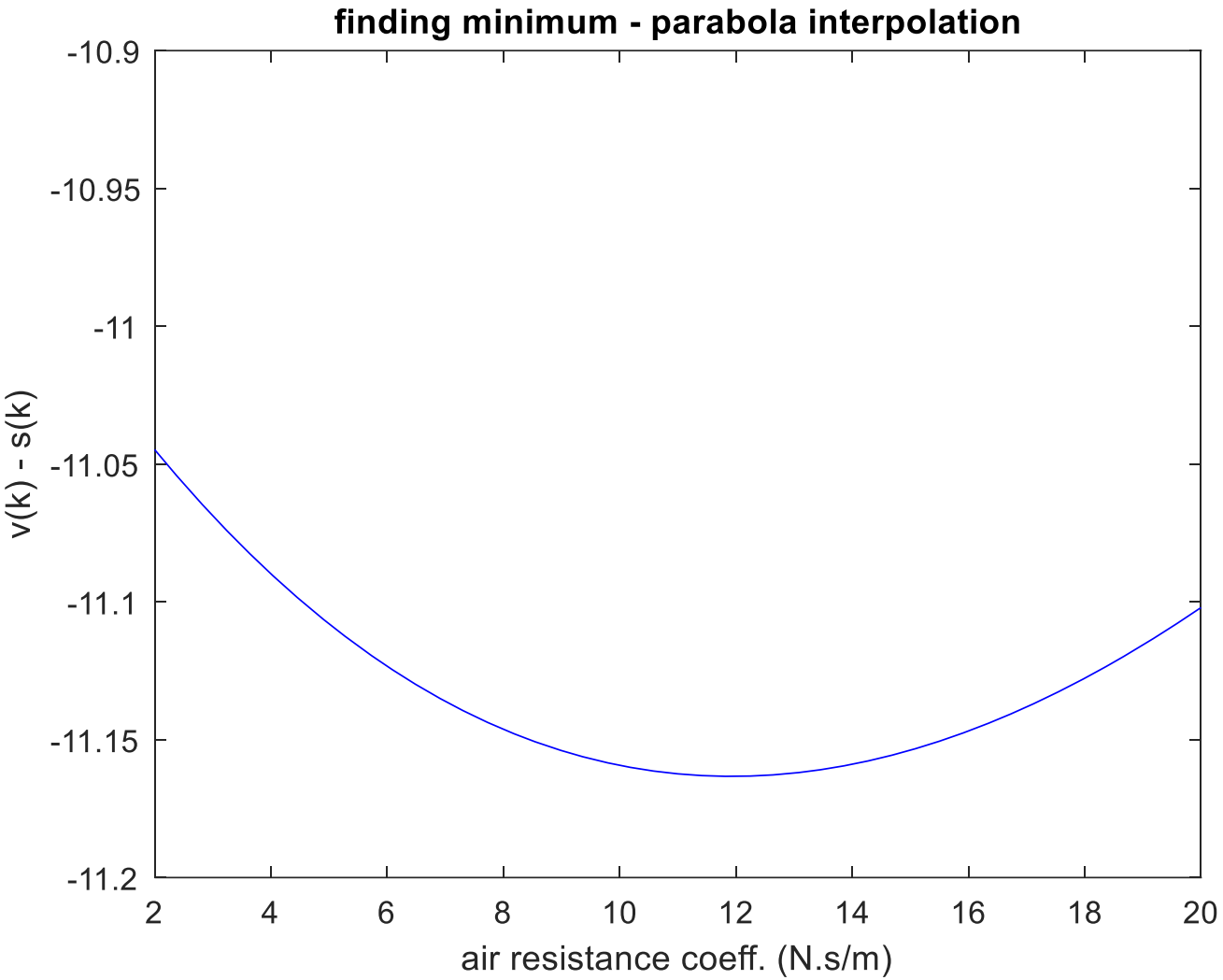
$$f(k) = v(k) - s(k)$$

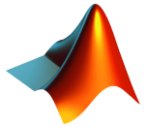
golden section search



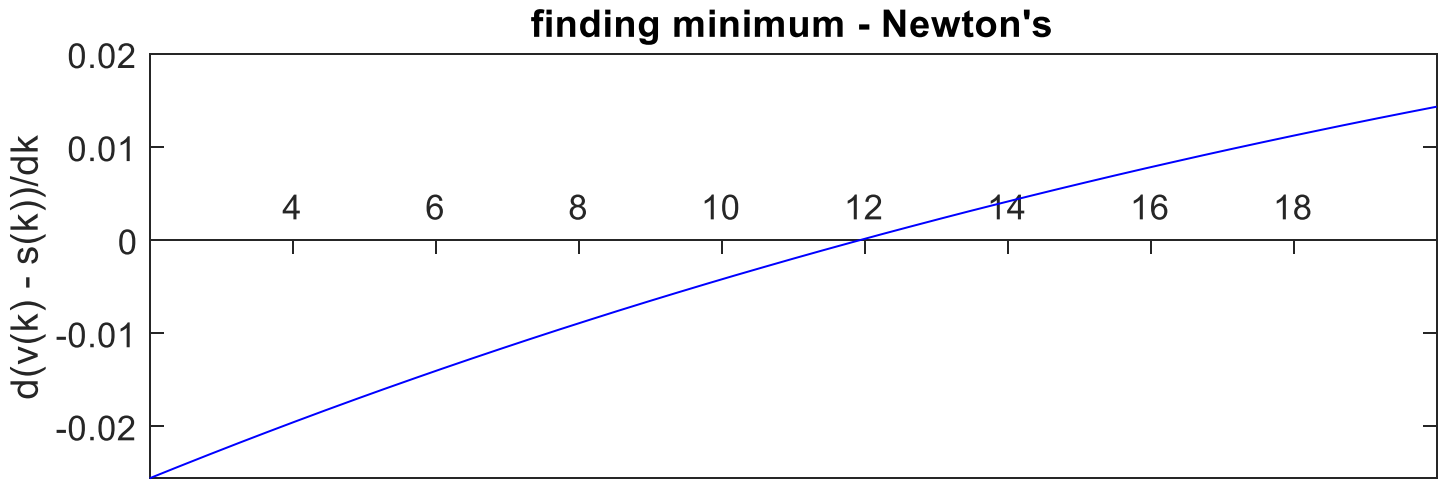


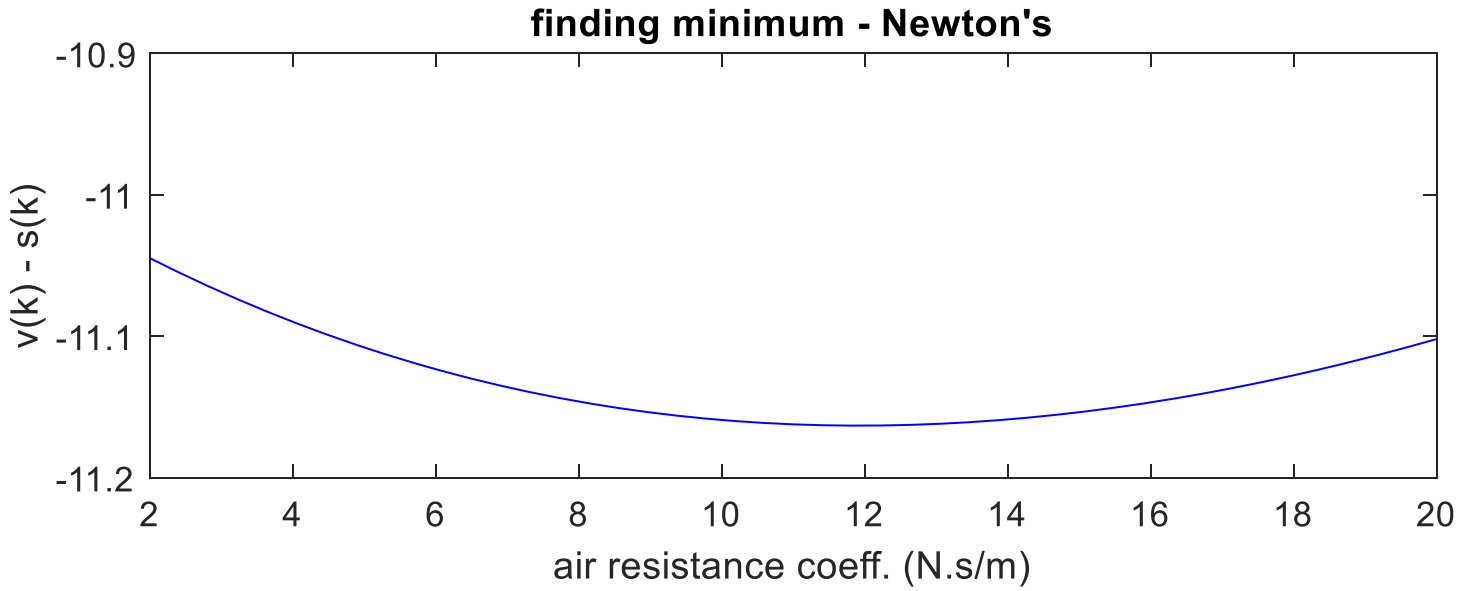
parabola interpolation



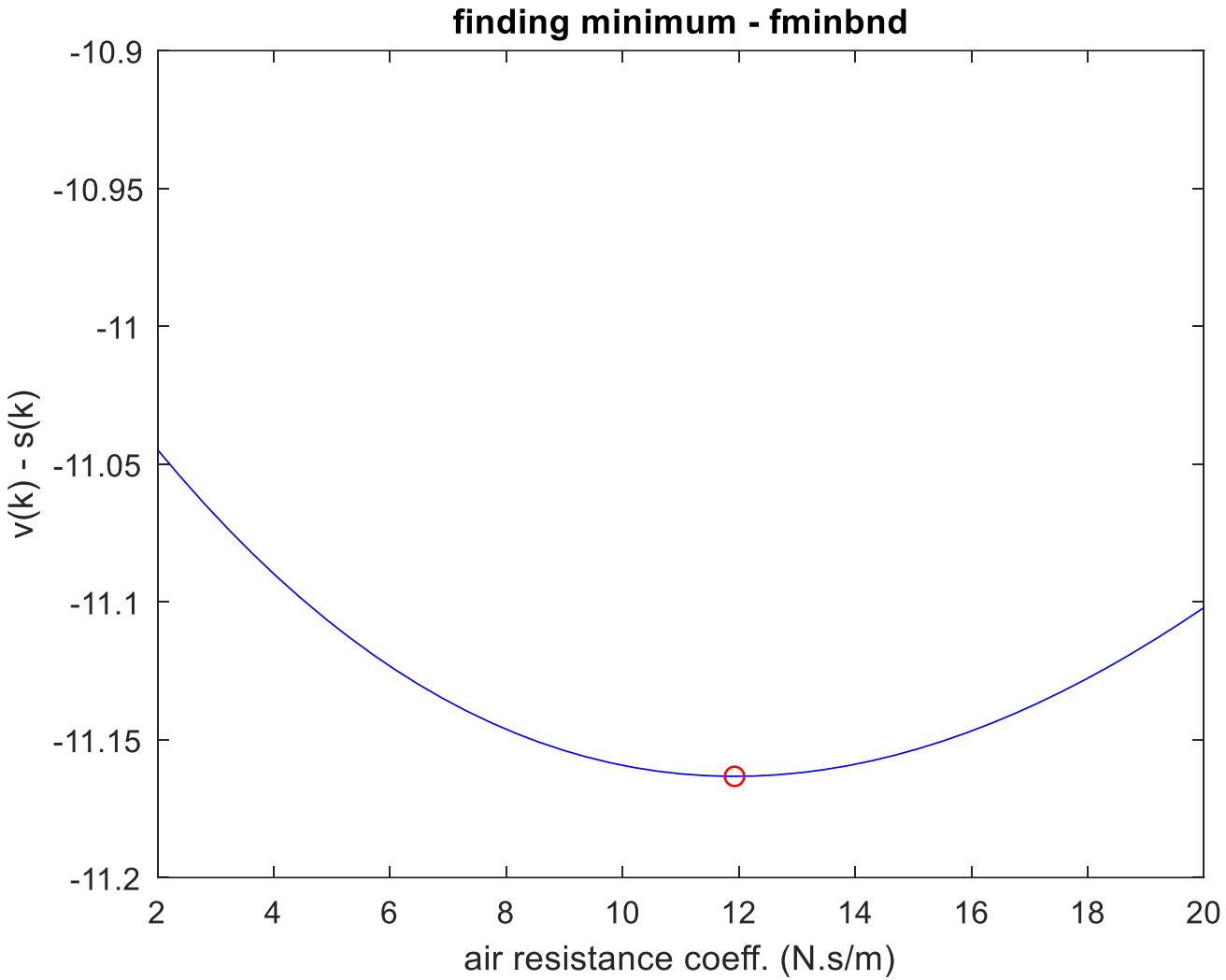


Newton's method





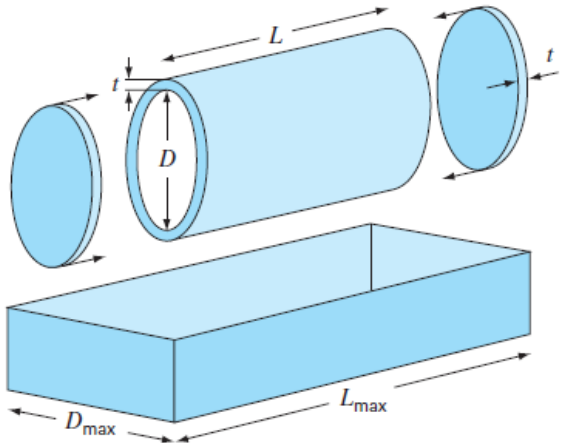
matlab function – fminbnd

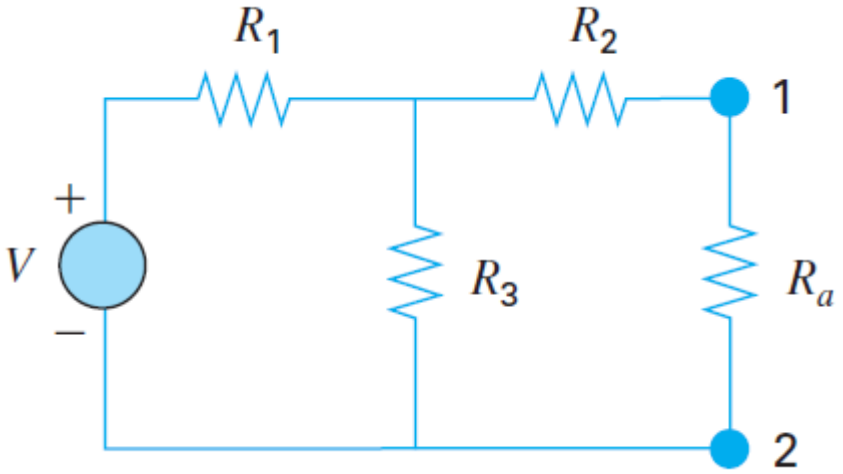


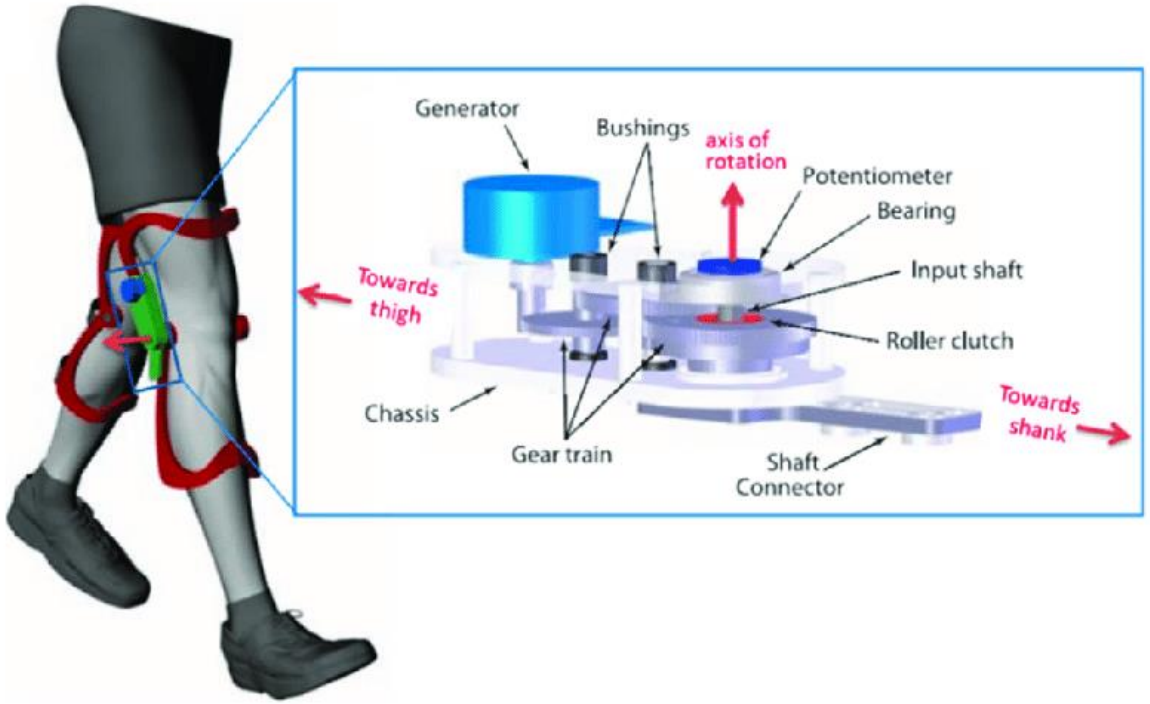

```
xl = 2;
xu = 20;
[x, fval] = fminbnd(f, xl, xu);
```

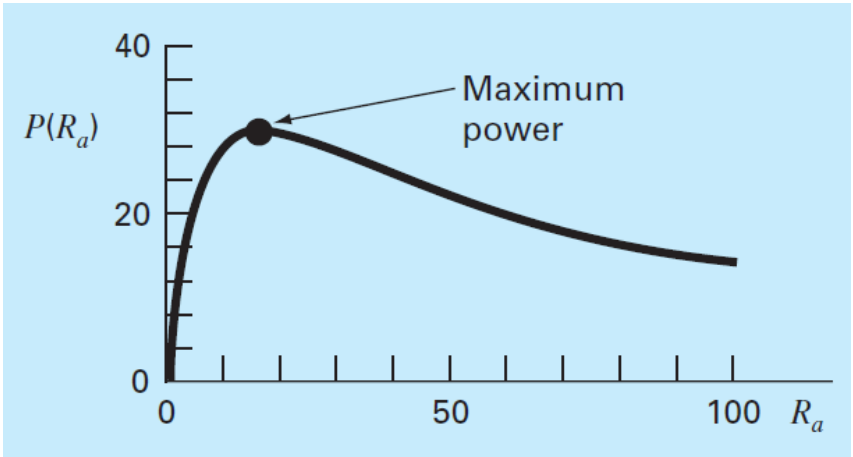
※ golden section과 parabola interpolation을 사용

applications

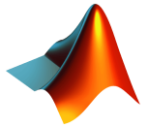








numerical differentiation and integration



high-order numerical differentiation

First Derivative

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$$

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h}$$

Second Derivative

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$$

$$f''(x_i) = \frac{-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)}{h^2}$$

Third Derivative

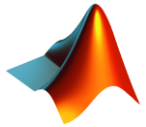
$$f'''(x_i) = \frac{f(x_{i+3}) - 3f(x_{i+2}) + 3f(x_{i+1}) - f(x_i)}{h^3}$$

$$f'''(x_i) = \frac{-3f(x_{i+4}) + 14f(x_{i+3}) - 24f(x_{i+2}) + 18f(x_{i+1}) - 5f(x_i)}{2h^3}$$

Fourth Derivative

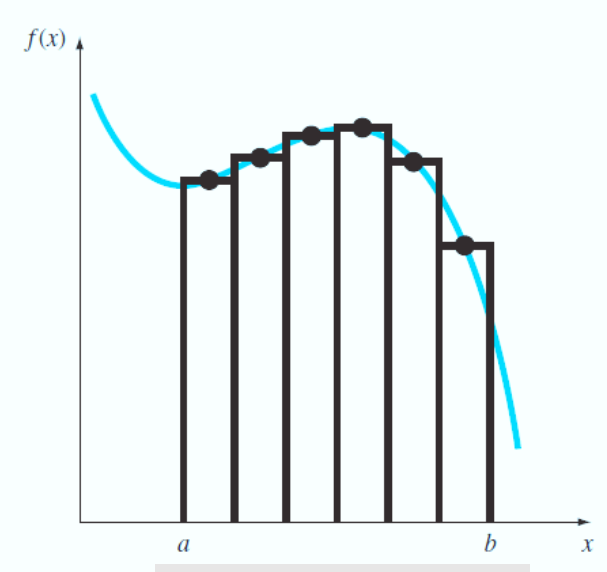
$$f''''(x_i) = \frac{f(x_{i+4}) - 4f(x_{i+3}) + 6f(x_{i+2}) - 4f(x_{i+1}) + f(x_i)}{h^4}$$

$$f''''(x_i) = \frac{-2f(x_{i+5}) + 11f(x_{i+4}) - 24f(x_{i+3}) + 26f(x_{i+2}) - 14f(x_{i+1}) + 3f(x_i)}{h^4}$$

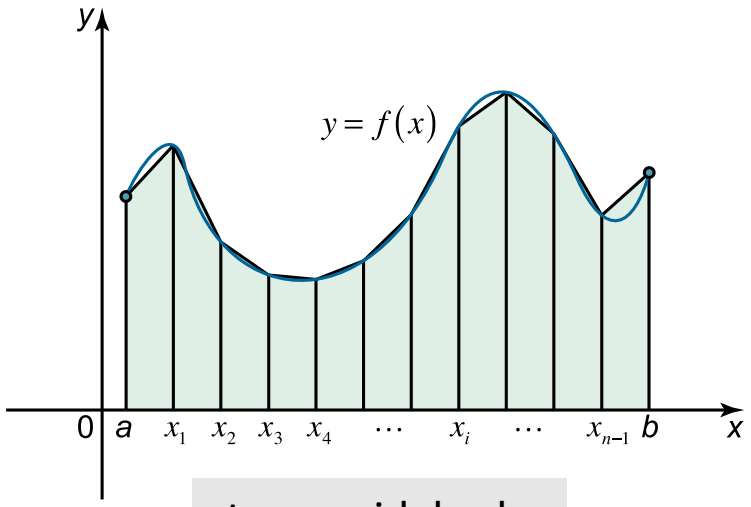


https://en.wikipedia.org/wiki/Numerical_differentiation

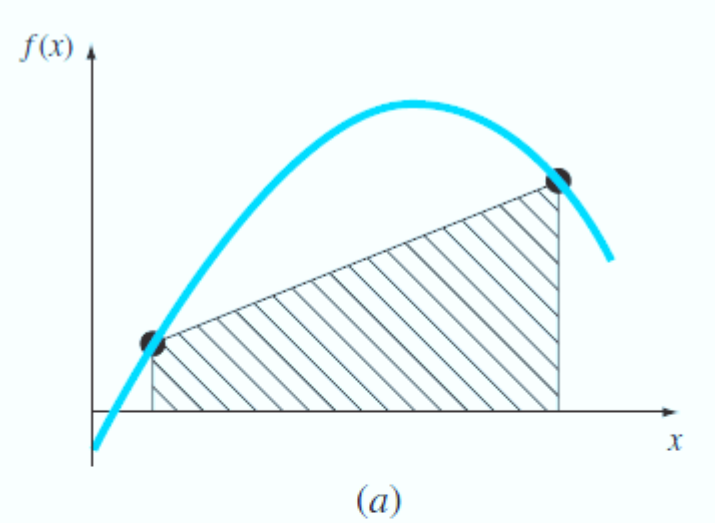
numerical integration

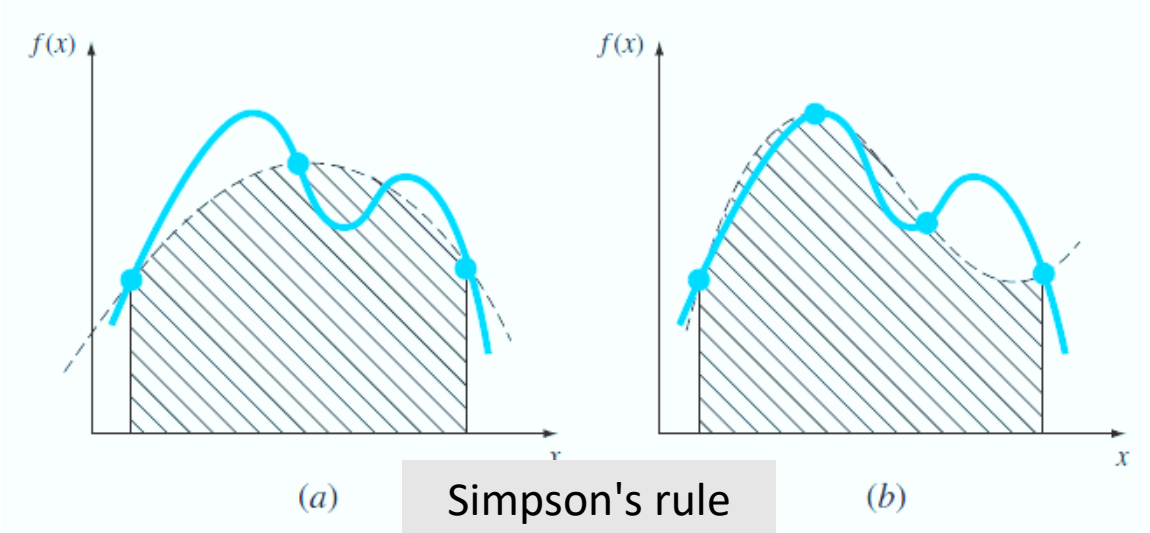


rectangular rule

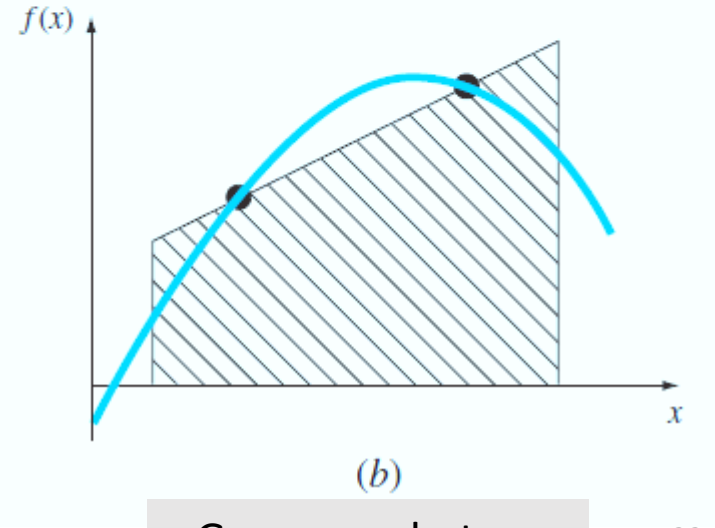


trapezoidal rule



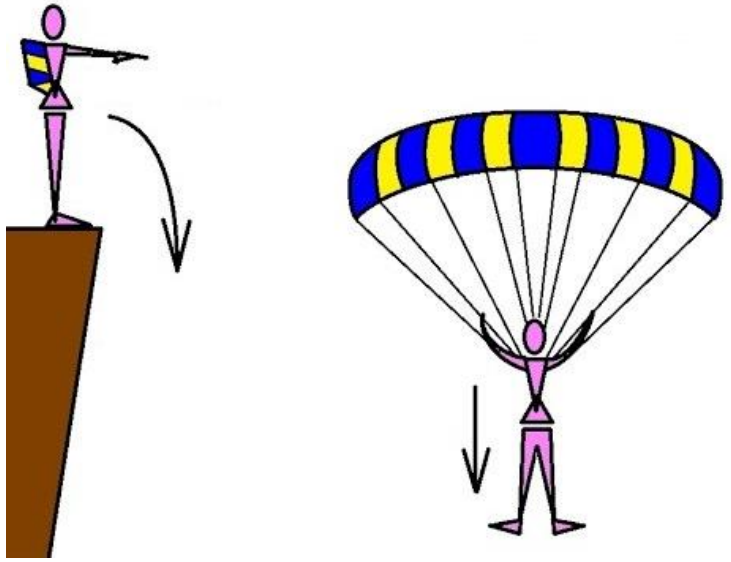


Simpson's rule



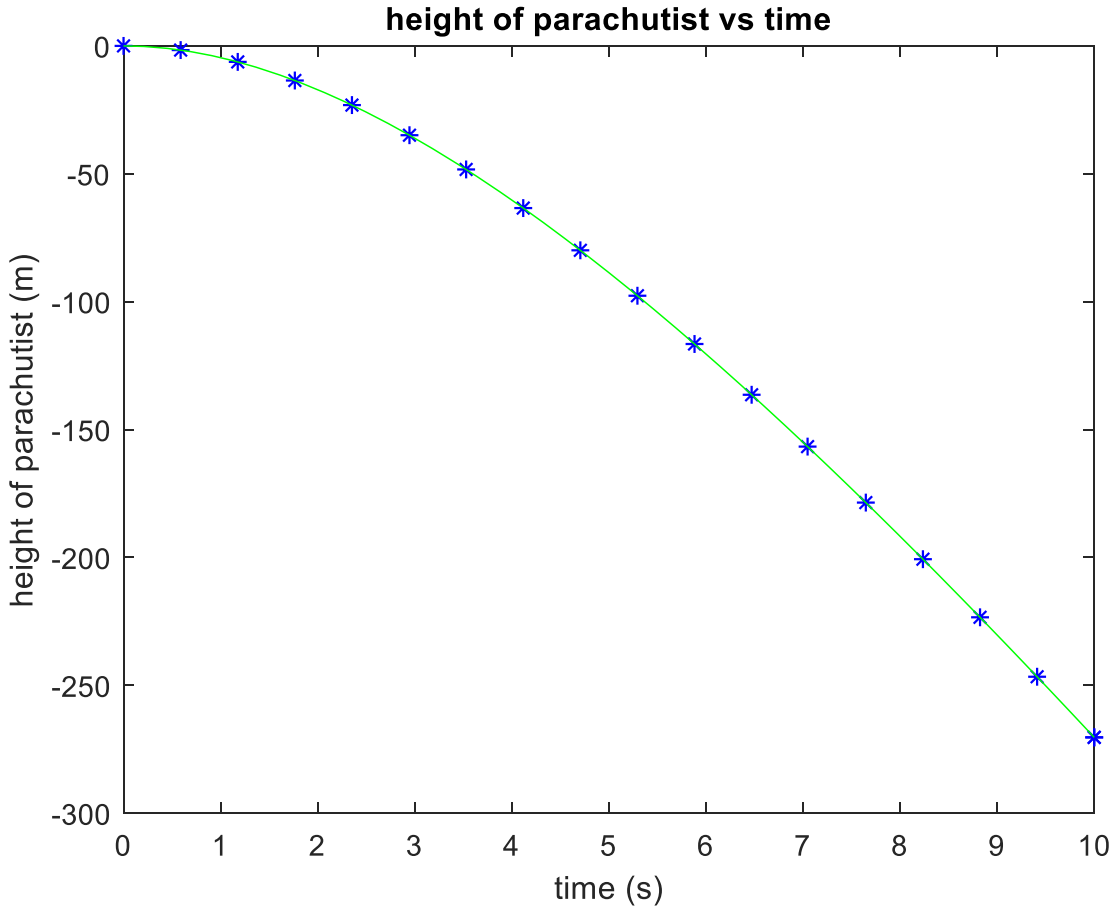
Gauss quadrature

matlab function – integral



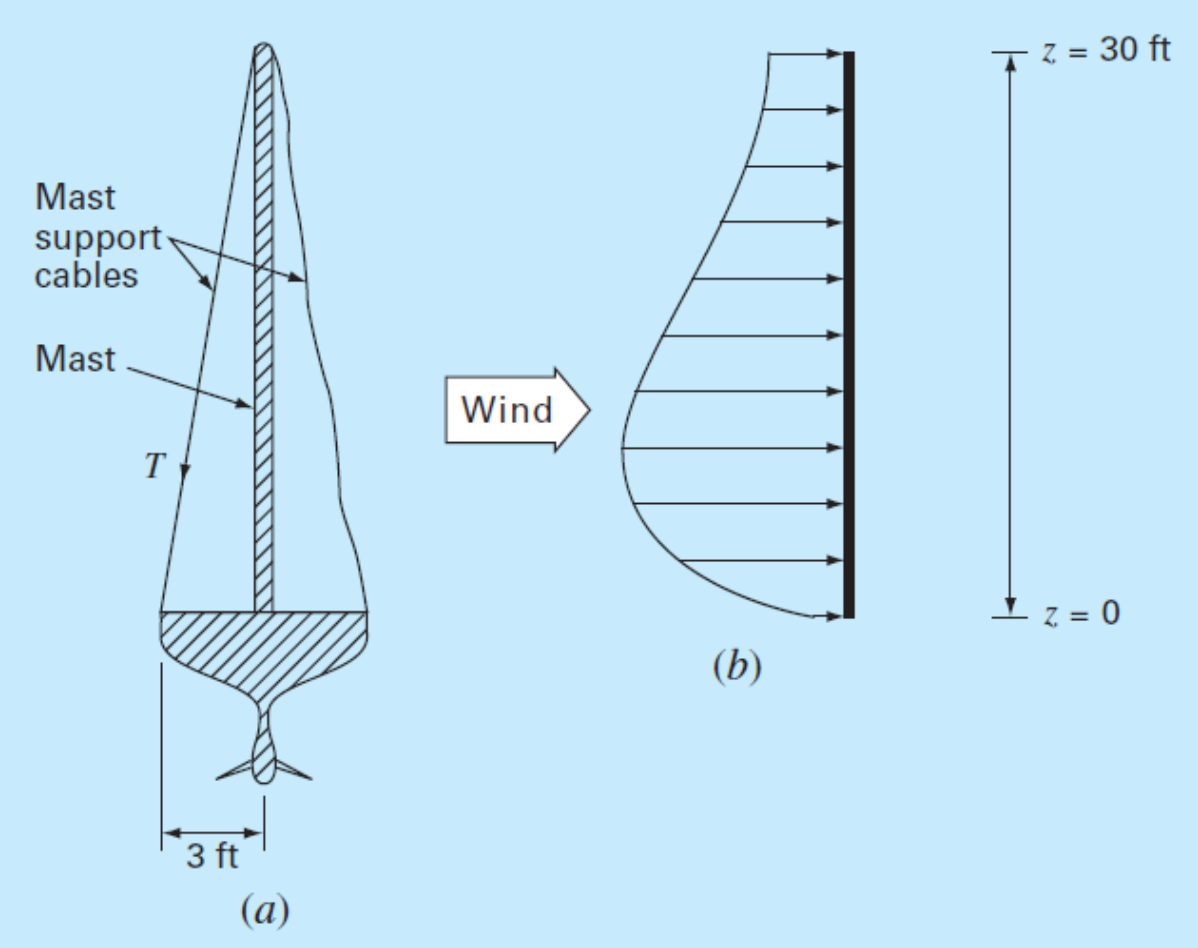
$$v(t) = \frac{mg}{k}\left(1 - e^{-(k/m)t}\right)$$

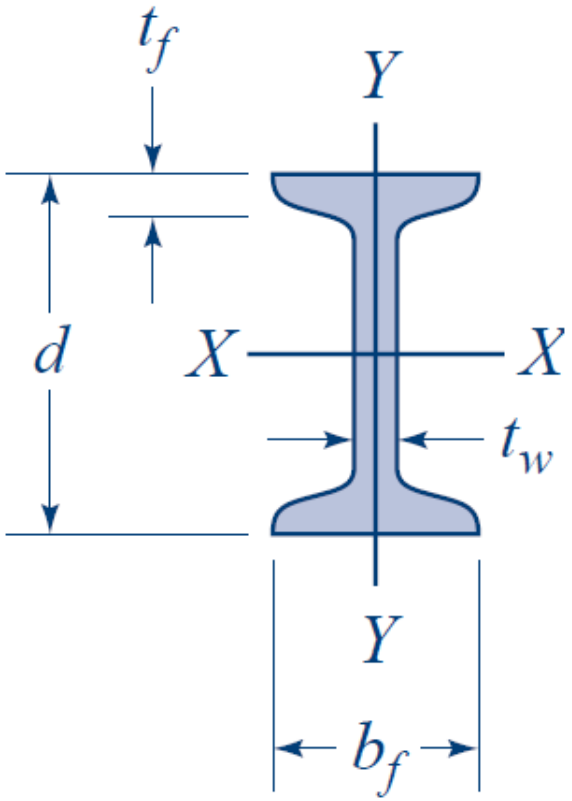
$$s(t) = \frac{mg}{k}\left(t + \frac{m}{k}\left(e^{-(k/m)t} - 1\right)\right)$$

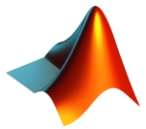



```
v = @(t) -m*g/k*(1-exp(-k/m*t));
s_exact = @(t) -m*g/k*(t+m/k*(exp(-k/m*t)-1));
s_num = @(t) -integral(v, 0, t);
```

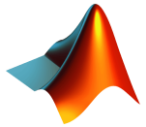
applications





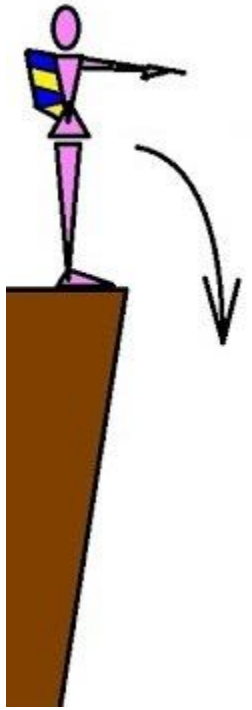


differential equation



이제 그만...

$$F = ma$$

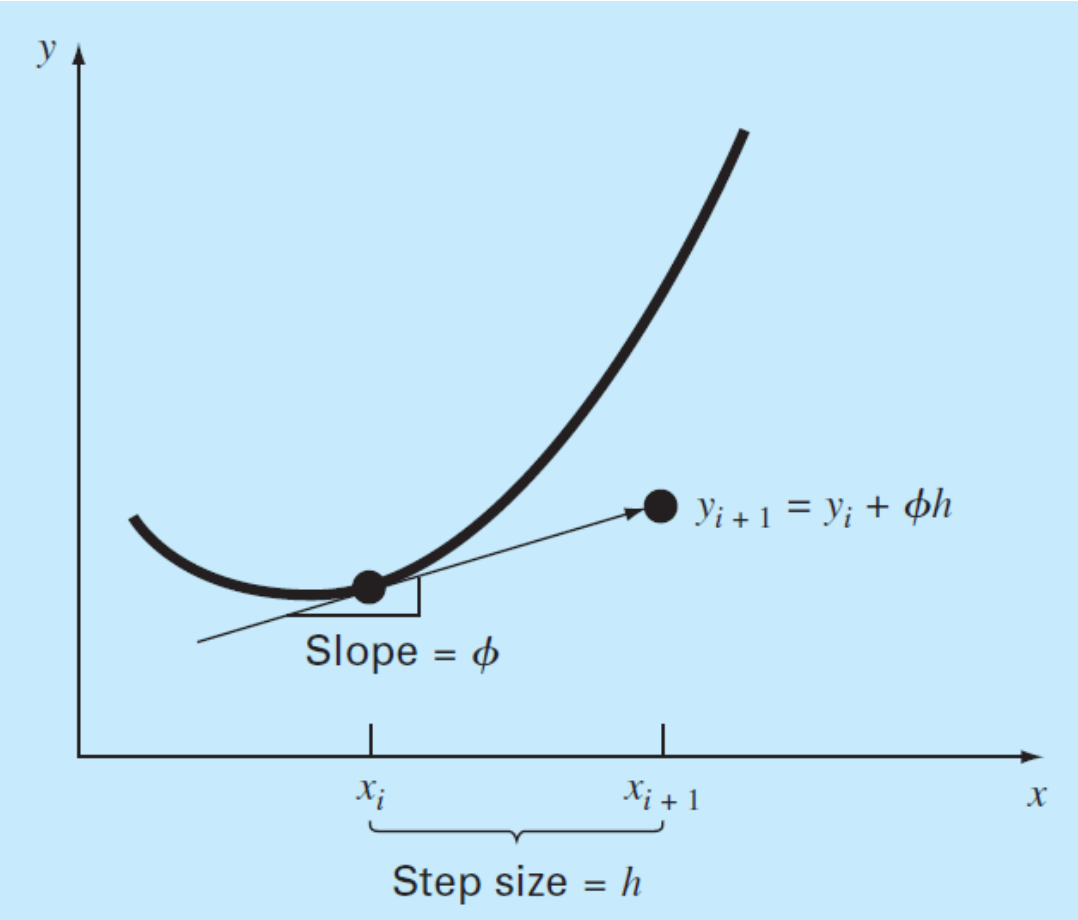


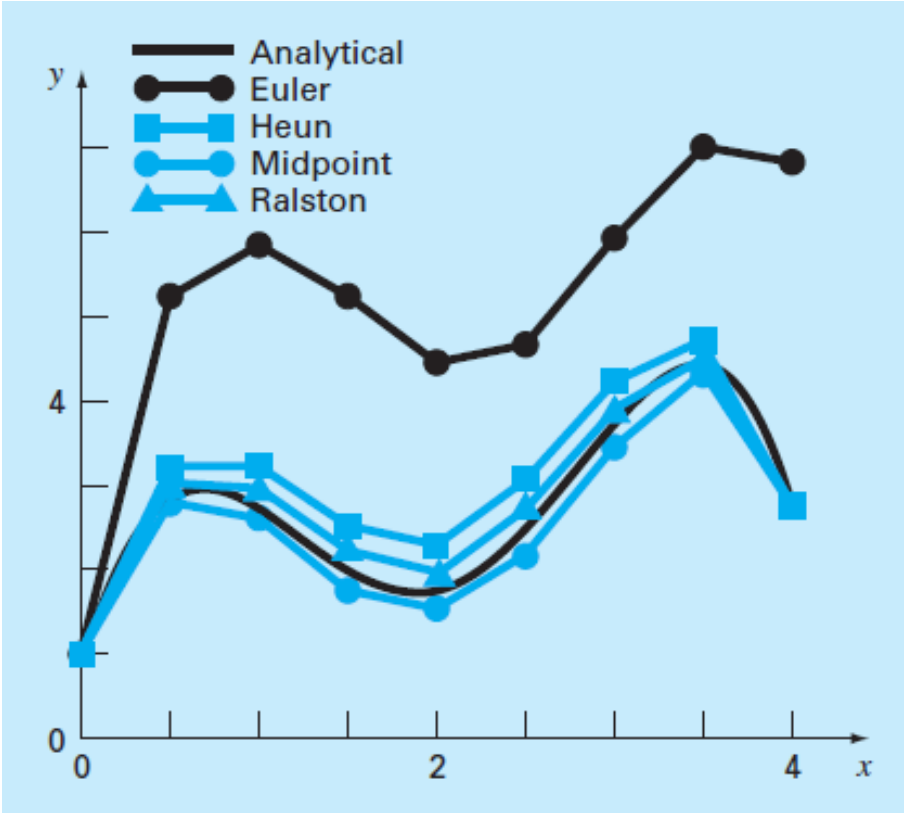
idea

$$\frac{dy}{dx} = f(x, y)$$

New value = old value + slope × step size

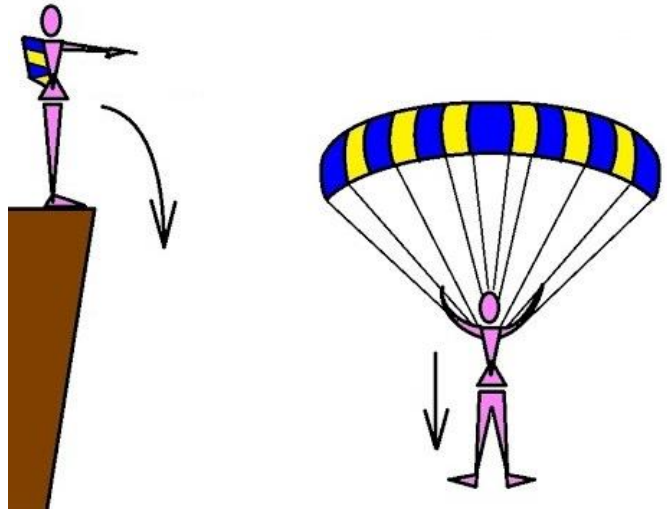
$$y_{i+1} = y_i + \phi h$$





ode45

$$y' = f(t, y)$$



```
[t, y] = ode45(odefun, tspan, y0);
```

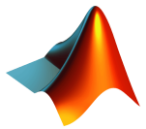
- 풀이 결과 $y(t)$
- $f(t, y)$에 대한 함수 핸들
- t의 범위
- y의 초기값

$$v(t)' = g - \frac{k}{m}v(t)$$

```
g = 9.81;    % m/s2
m = 80;      % kg
k = 17;      % N.s/m
v0 = 0;      % m/s
tspan = [0 20];

f = @(t, v) g - (k/m)*v;

[t, v] = ode45(f, tspan, v0);
```

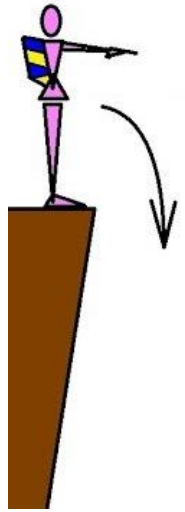
* analytic solution:

$$v(t) = \frac{mg}{k}\left(1 - e^{-(k/m)t}\right)$$



공기저항이 속도²에 비례한다면?

$$y' = f(t, y)$$



```
[t, y] = ode45(odefun, tspan, y0);
```

- 풀이 결과 $y(t)$
- $f(t, y)$에 대한 함수 핸들
- t의 범위
- y의 초기값

$$v(t)' = g - \frac{k}{m} v(t)^2$$

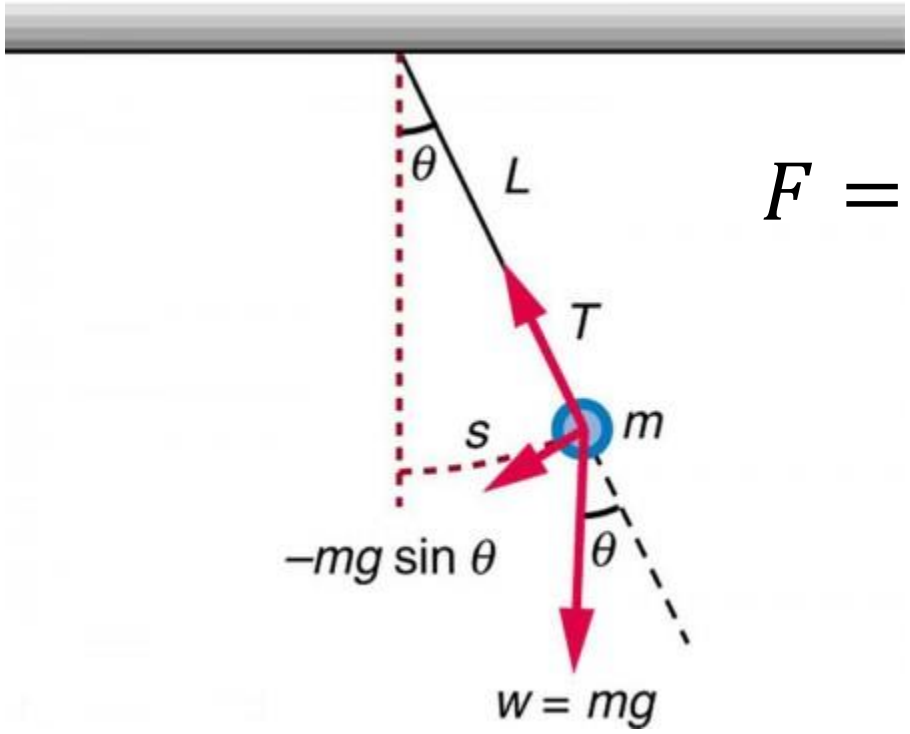
```
g = 9.81;    % m/s2
m = 80;      % kg
k = 3;       % N.s/m
v0 = 0;      % m/s
tspan = [0 10];

f = @(t, v) g - (k/m)*v^2;

[t, v] = ode45(f, tspan, v0);
```

* analytic solution:

$$v(t) = \frac{\sqrt{\frac{mg}{k}}\left(1-\exp\left(2t\sqrt{\frac{kg}{m}}\right)\right)}{1+\exp\left(2t\sqrt{\frac{kg}{m}}\right)}$$

https://xaktly.com/DifferentialEquations.html

단진자 운동

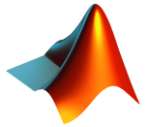


$$F = ma$$

$$y' = f(t, y)$$

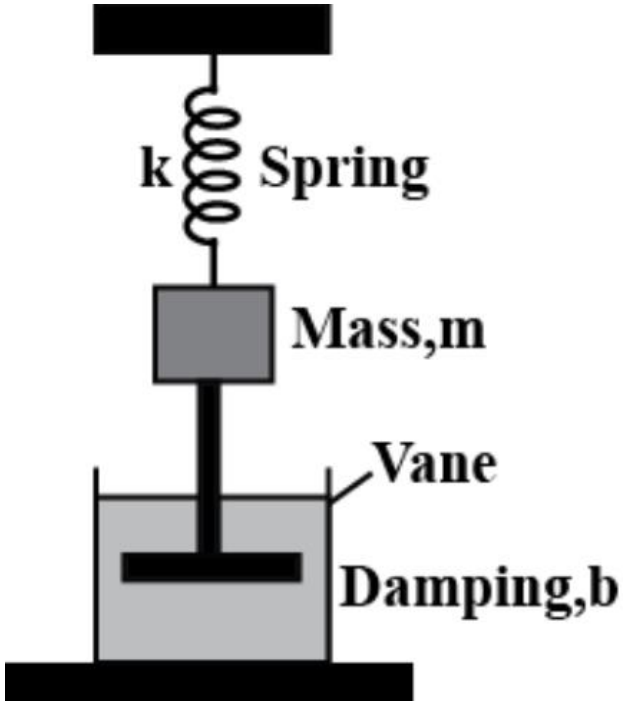
$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} f_1(t, y_1, y_2) \\ f_2(t, y_1, y_2) \end{bmatrix}$$

```
g = 9.81;    % m/s2
L = 1;       % m
x0 = 0.05;   % m
v0 = 0;      % m/s
tspan = [0, 10];

dxdt = @(t, x) [x(2); -g*x(1)/L];

[t, x] = ode45(dxdt, tspan, [x0, v0]);
```



https://courses.lumenlearning.com/physics/chapter/16-4-the-simple-pendulum/

damped vibration system



$$F = ma$$

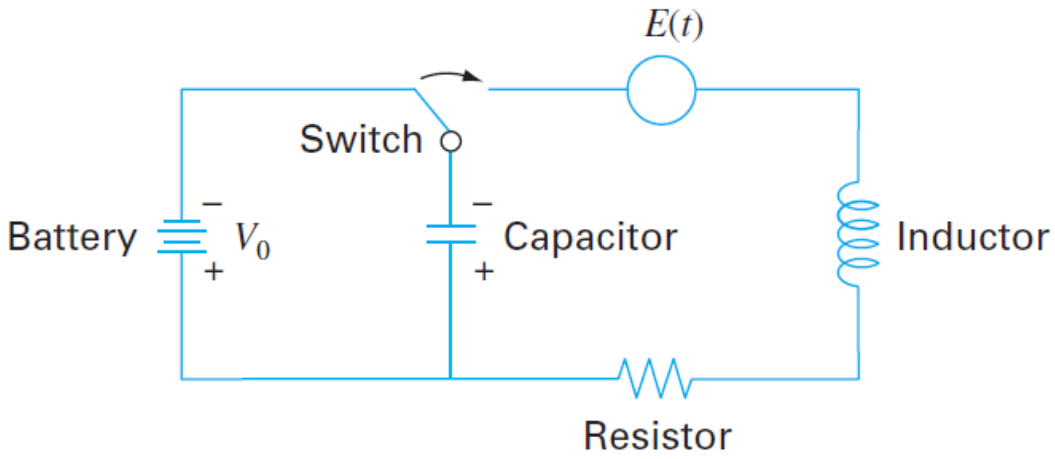
$$y' = f(t, y)$$

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} f_1(t, y_1, y_2) \\ f_2(t, y_1, y_2) \end{bmatrix}$$

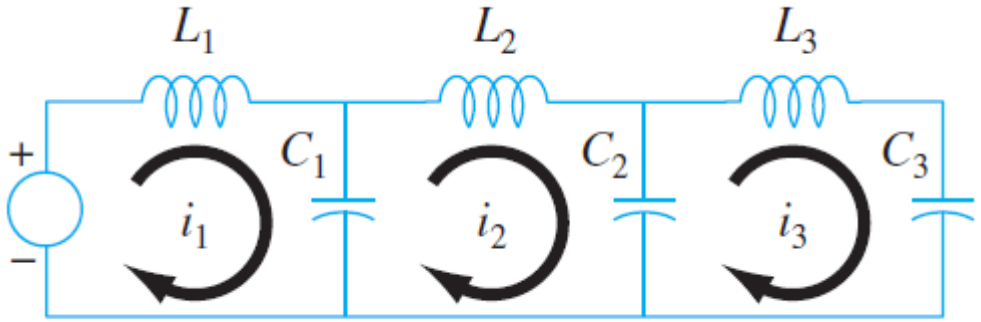
```
m = 10;        % kg
k = 100;       % N/m
b = 5;         % N.s/m
x0 = 1;
v0 = -b/2/m;
tspan = [0, 10];

dxdt = @(t, x) [x(2); -b/m*x(2)-k/m*x(1)];

[t, x] = ode45(dxdt, tspan, [x0, v0]);
```

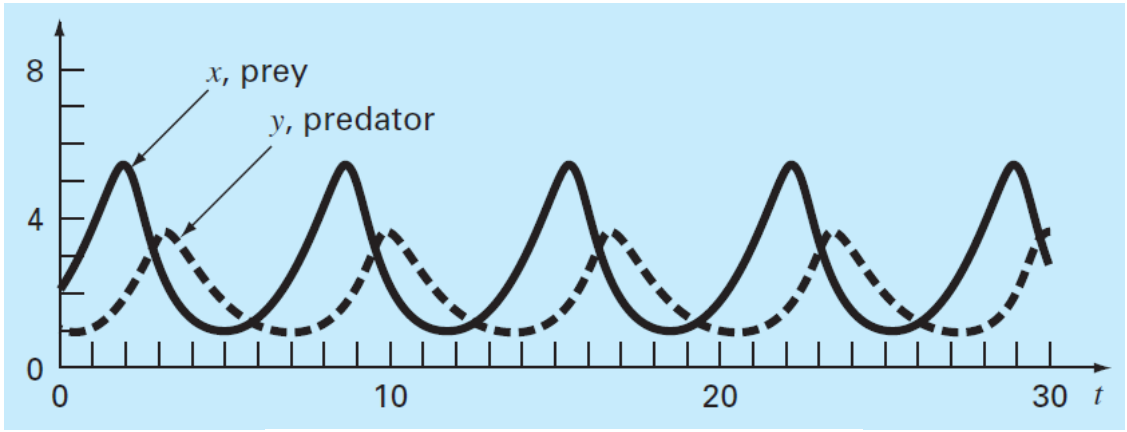
https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Map%3A_University_Physics_I_-_Mechanics_Sound_Oscillations_and_Waves_(OpenStax)/15%3A_Oscillations/15.06%3A_Damped_Oscillations

applications



$$L\frac{di}{dt} + Ri + \frac{q}{C} - E(t) = 0$$

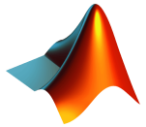




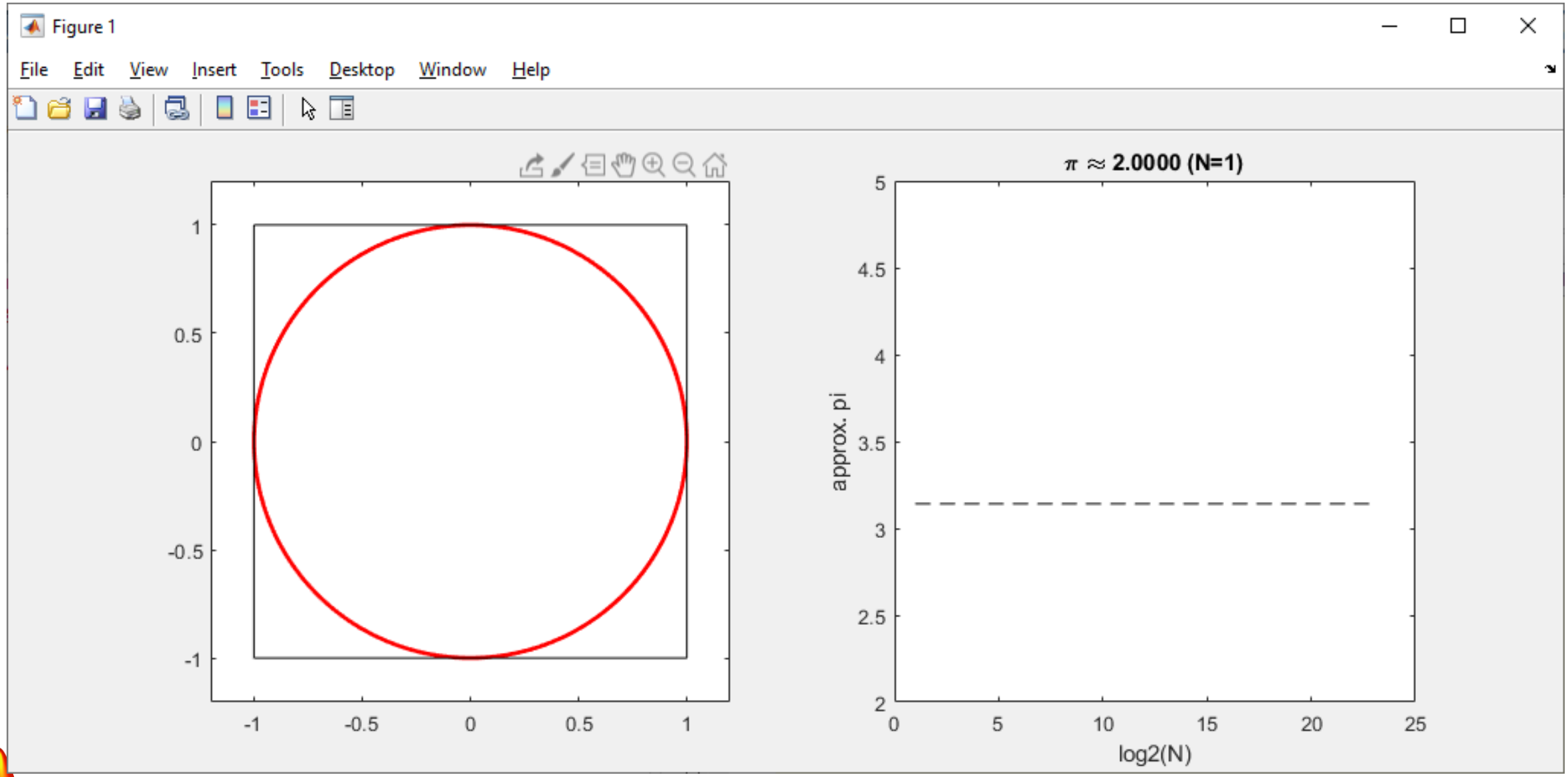
$$\frac{dx}{dt} = ax - bxy$$

$$\frac{dy}{dt} = -cy + dxy$$

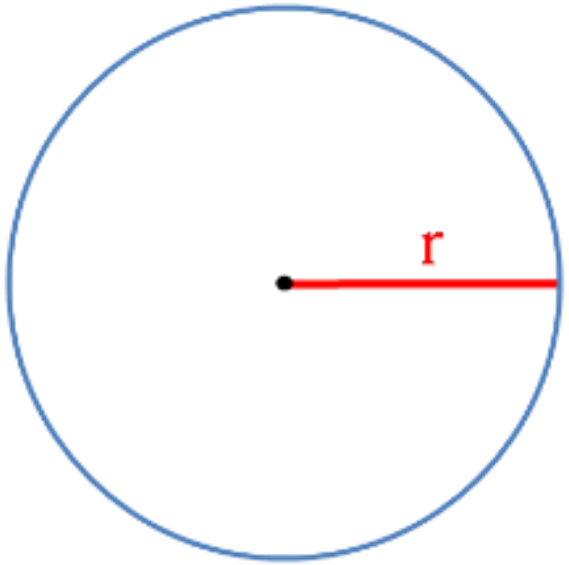
Monte Carlo simulation



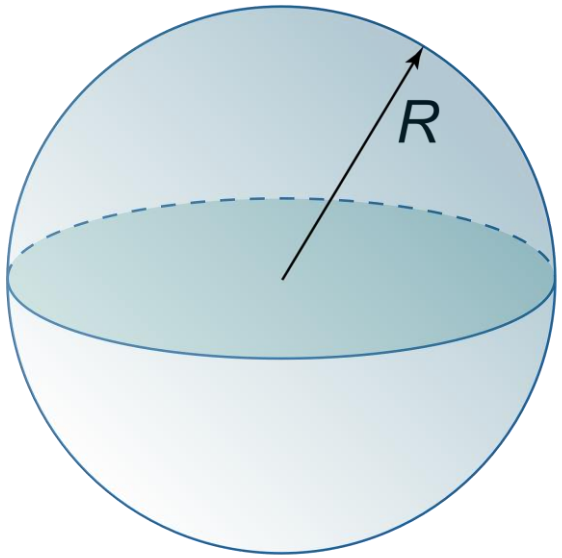
원주율을 구하는 새로운 방법



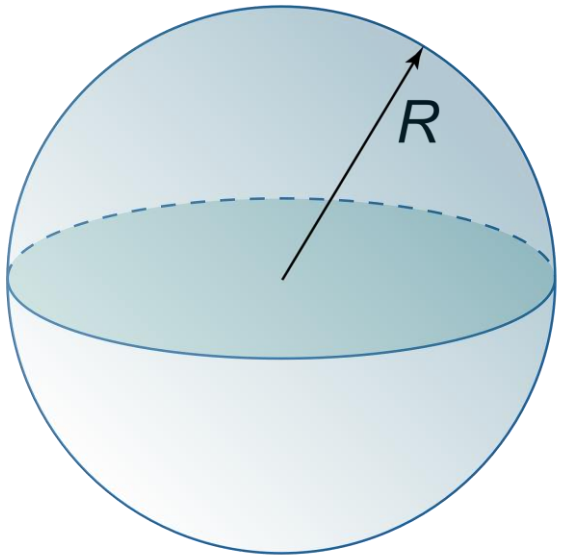
n차원 구의 부피는?



2차원 sphere



3차원 sphere



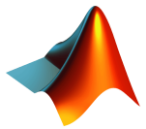
n차원 sphere?

$$V_2 = \pi R^2$$

$$V_3 = \frac{4}{3}\pi R^3$$

$$V_n = \underline{C}R^n$$

?



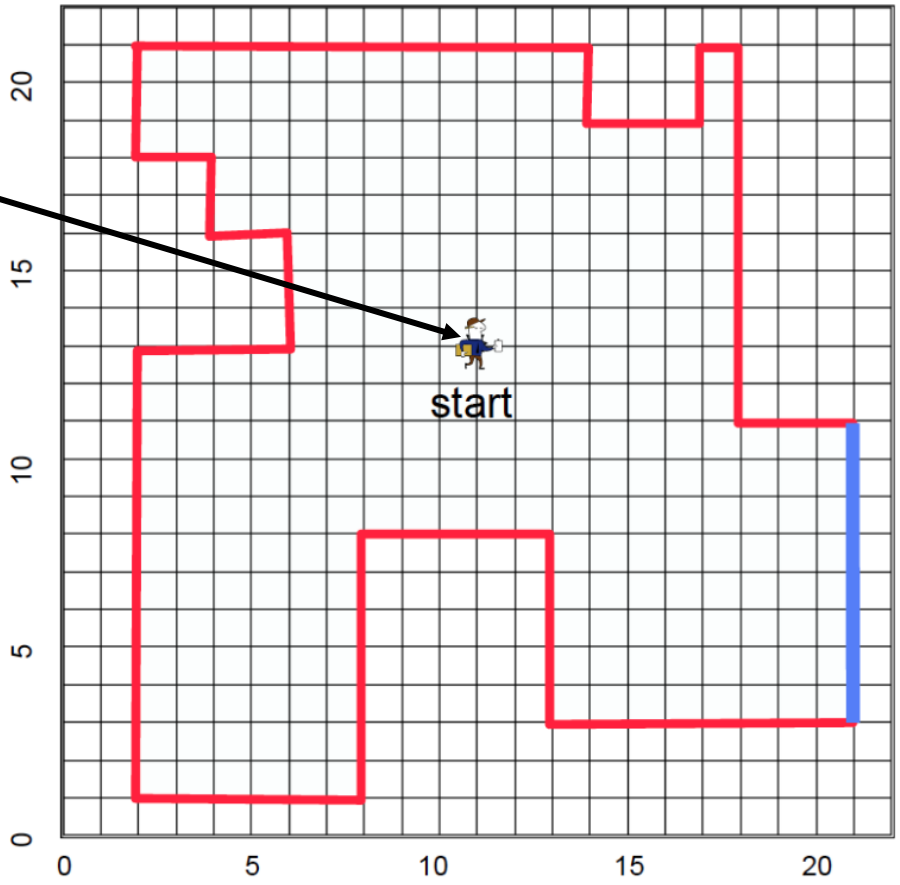
https://en.wikipedia.org/wiki/Volume_of_an_n-ball

※ 구가 아니라면?

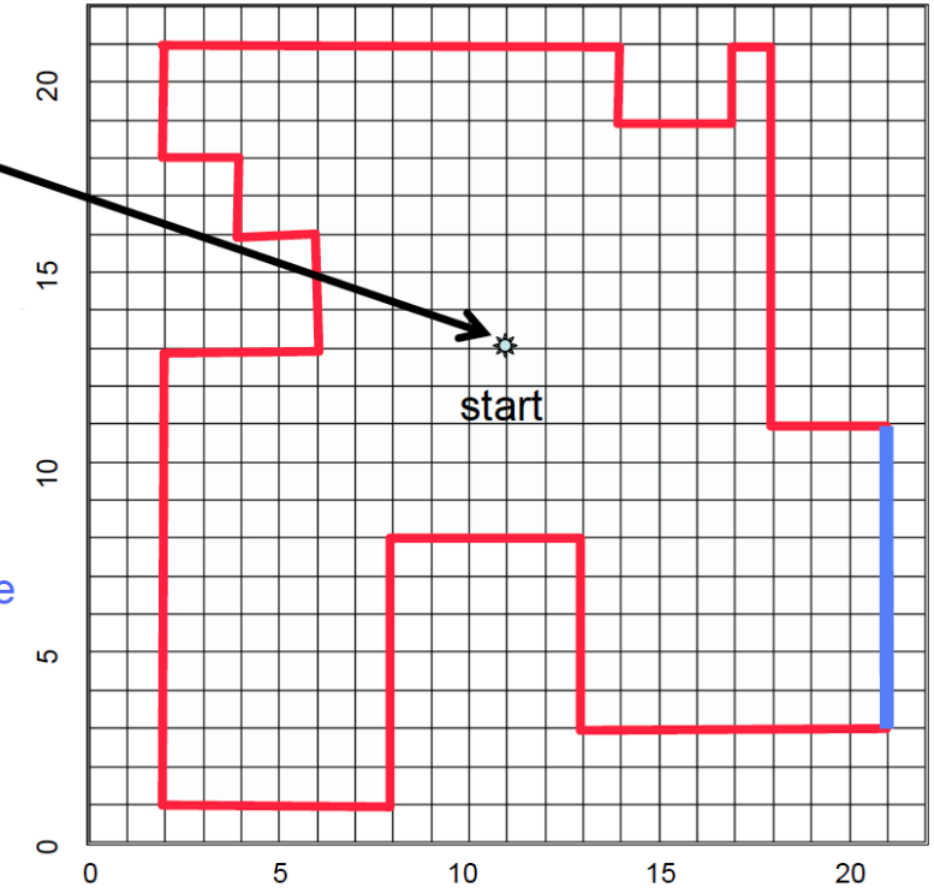
몬티홀 문제

random walk

파란색으로
나갈 확률은?



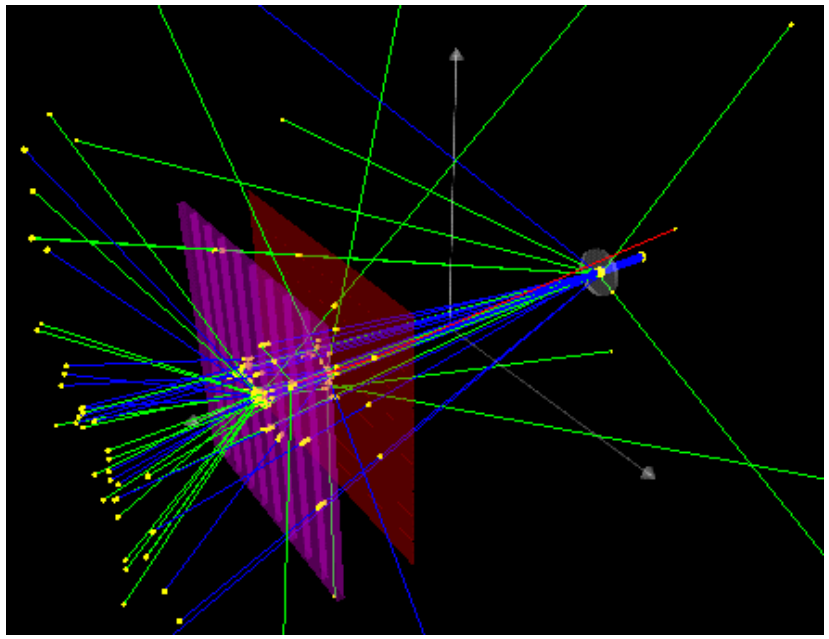
이 지점의
온도는?

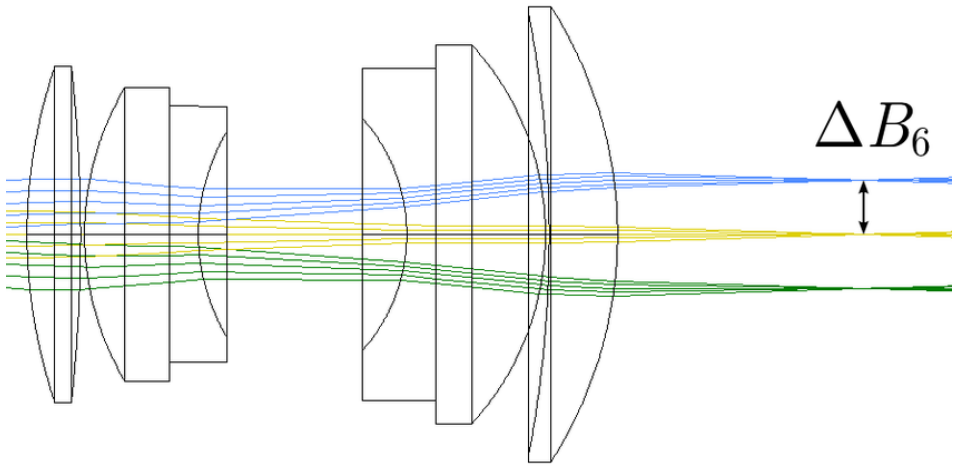


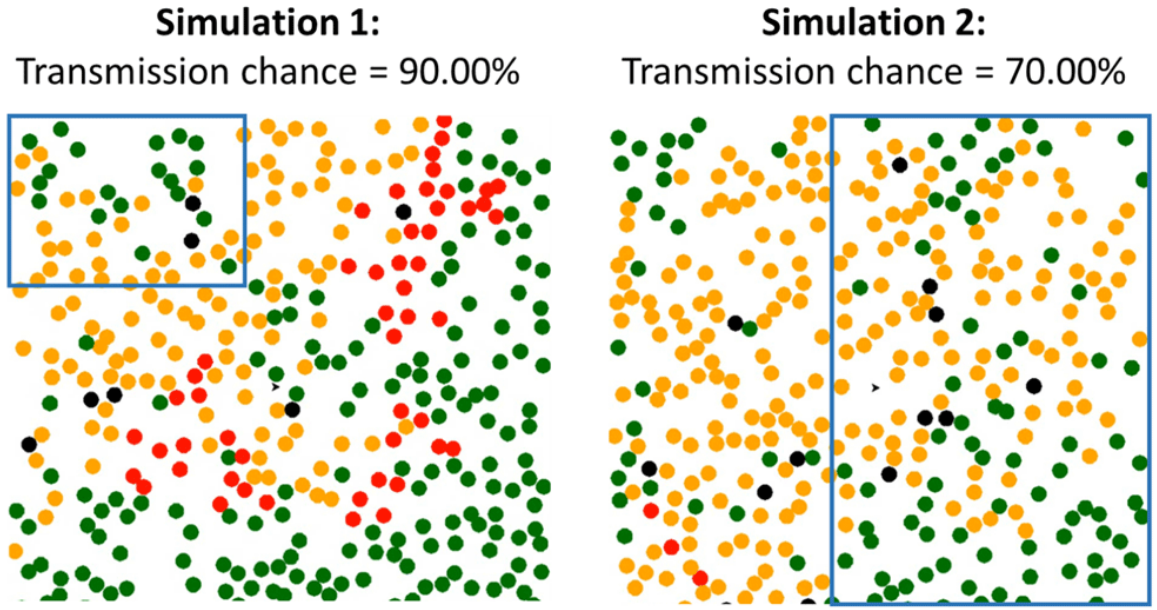
T=1 on blue
T=0 on red

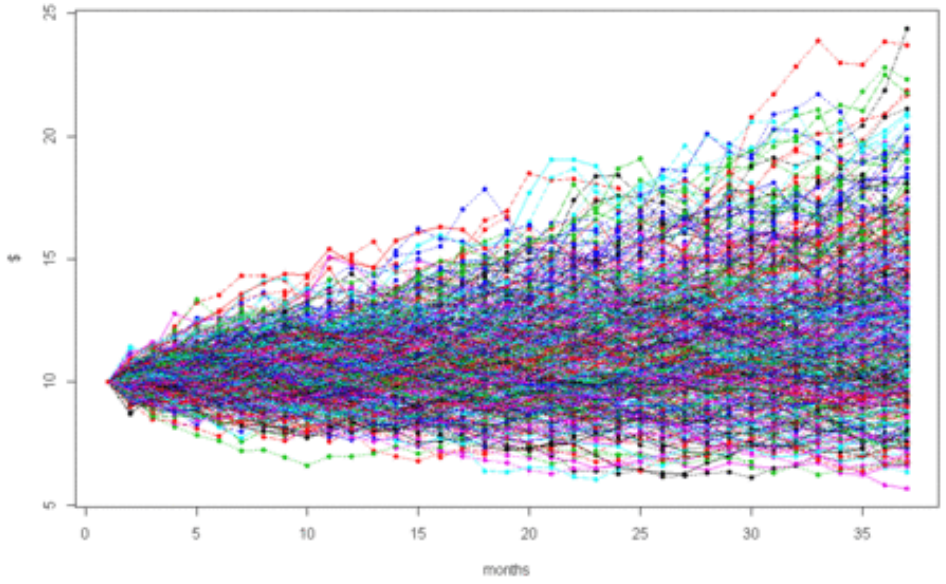
https://www.researchgate.net/publication/279532247_Monte_Carlo_High_School_Examples_Draft_2

applications









Q&A