MATLAB 프로그래밍 및 실습

12강. 기초 수치해석 1

오늘 배울 내용

- Polynomial
- Curve fitting
- Interpolation

- 다음주
  - 방정식의 해
  - 함수 최소값 및 최적화
  - 수치미분, 수치적분
  - 미분방정식
  - 몬테카를로 시뮬레이션

polynomial

polynomial

- Definition (n-th order polynomial)

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ \dots + a_1 x + a_0$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are constants and n is nonnegative integer.

- 대수학의 기본정리

$p(x) = \sum_{i=0}^{n} a_i x^i = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 (a_n \neq 0, n \geq 1)$에 대해 복소수 α가 존재하여 $p(\alpha) = 0$이다.

- 따름 정리

$$\begin{aligned} p(x) &= a_n x^n + a_{n-1} x^{n-1} + \ \dots + a_1 x + a_0 \\ &= a_n (x - r_1)(x - r_2) \dots (x - r_n) \end{aligned}$$

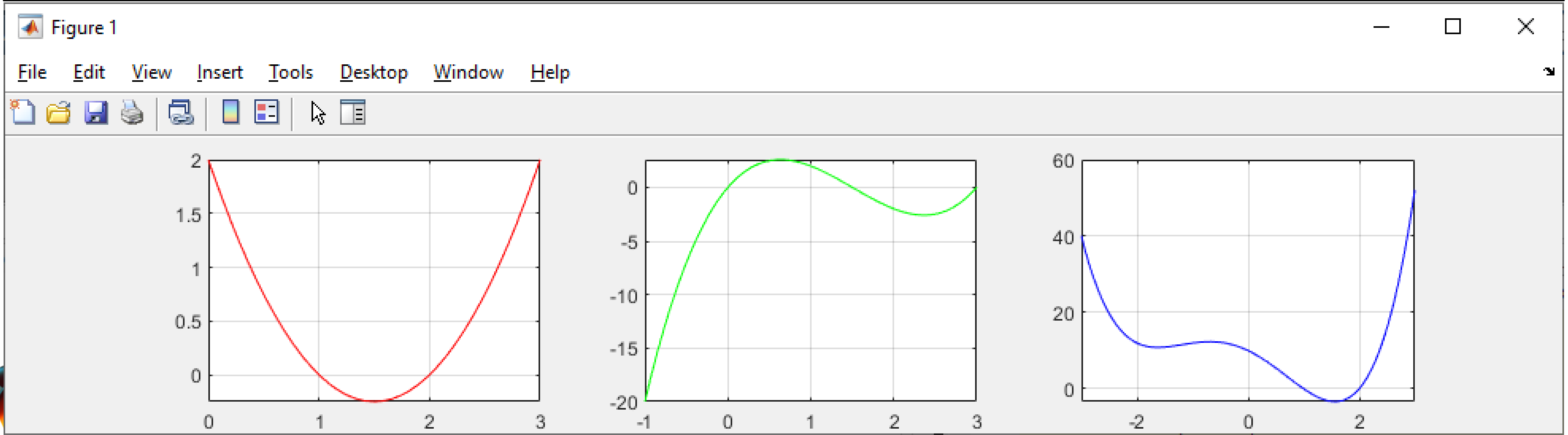
$$r_i \in \boldsymbol{C}$$

매트랩에서 polynomial 표현

```
x = linspace(-4, 4);

p1 = [1, -3, 2];
fx1 = polyval(p1, x);

p2 = [2, -9, 9, 0];
fx2 = polyval(p2, x);

p3 = [1, 1, -5, -7, 10];
fx3 = polyval(p3, x);

figure,
subplot(1,3,1), plot(x,fx1,'r'), grid on, xlim([0 3])
subplot(1,3,2), plot(x,fx2,'g'), grid on, xlim([-1 3])
subplot(1,3,3), plot(x,fx3,'b'), grid on, xlim([-3 3])
```

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \ \ldots + a_1 x + a_0$$

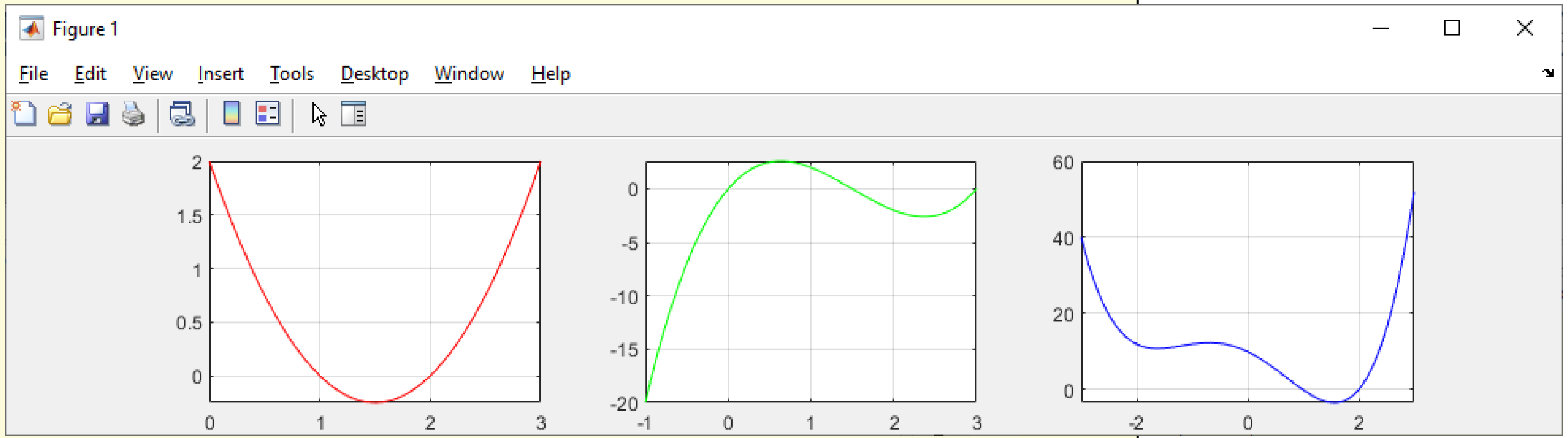
※ 계수가 0인 경우에도 0으로 자리를 채움



polyval vs anonymous function vs fplot

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \ \dots + a_1 x + a_0$$

```
x = linspace(-4, 4);

p1 = [1, -3, 2];
fx1 = polyval(p1,x);

p2 = [2, -9, 9, 0];
fx2 = polyval(p2,x);

p3 = [1, 1, -5, -7, 10];
fx3 = polyval(p3, x);

f1 = @(x) x.^2 - 3*x + 2;
f2 = @(x) 2*x.^3 - 9*x.^2 + 9*x;
f3 = @(x) x.^4 + x.^3 - 5*x.^2 - 7*x + 10;

f1m = @(x) p1(:)'*[x(:).^2 x(:) ones(numel(x),1)]';
f2m = @(x) p2(:)'*[x(:).^3 x(:).^2 x(:) ones(numel(x),1)]';
f3m = @(x) p3(:)'*[x(:).^4 x(:).^3 x(:).^2 x(:) ones(numel(x),1)]';

figure,
subplot(1,3,1), fplot(@(x) polyval(p1, x), [ 0, 3], 'r'), grid on
subplot(1,3,2), fplot(@(x) polyval(p2, x), [-1, 3], 'g'), grid on
subplot(1,3,3), fplot(@(x) polyval(p3, x), [-3  3], 'b'), grid on
```

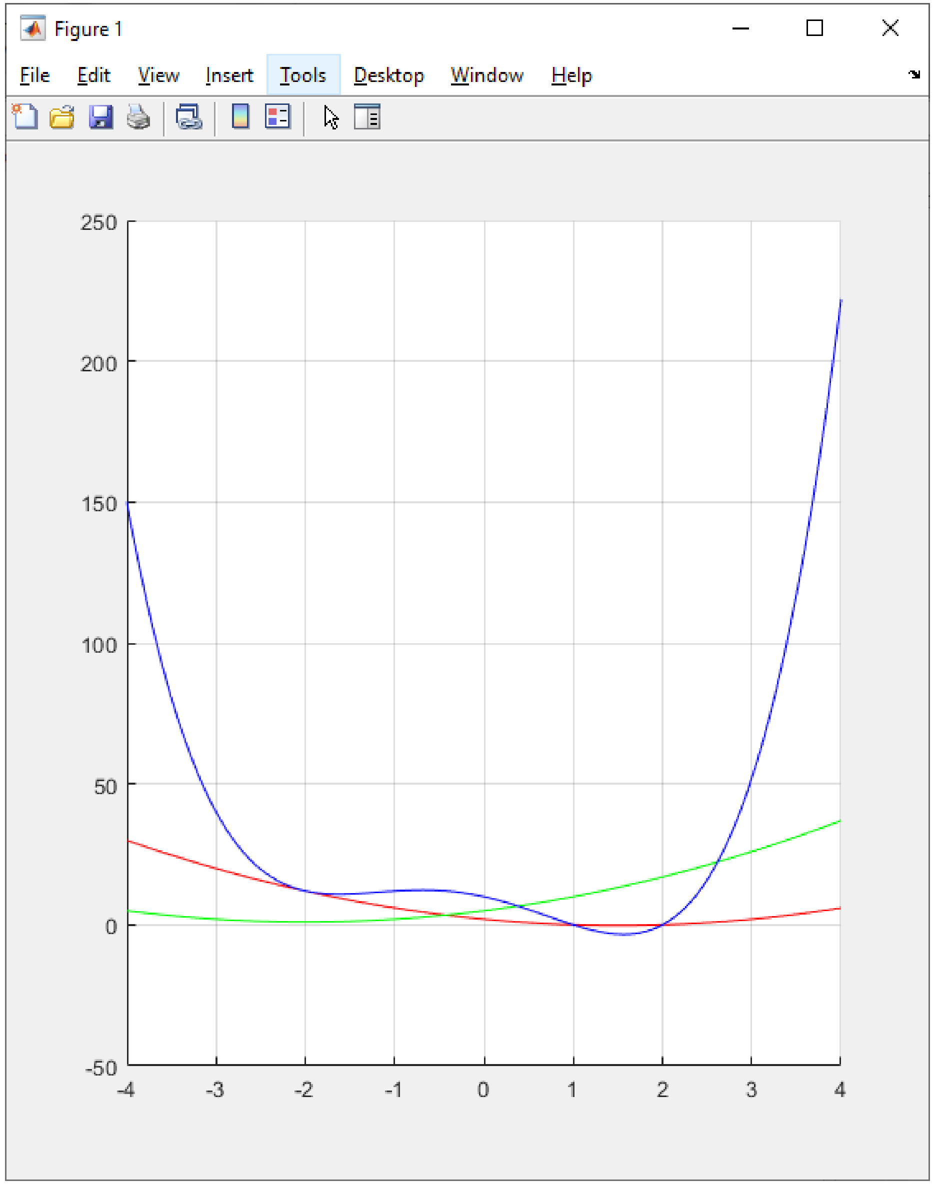


다항식 곱하기, 나누기: conv, deconv

```
x = linspace(-4, 4);

p1 = [1, -3, 2];
fx1 = polyval(p1,x);

p2 = [1, 4, 5];
fx2 = polyval(p2,x);

p3 = conv(p1, p2);
fx3 = polyval(p3,x);
```

```
u = [2, 9, 7, -6];
v = [1, 3];
[q, r] = deconv(u, v);

u = [2, -13, 0, 75, 2, 0, -60];
v = [1, 0, -5];
[q, r] = deconv(u, v);
```

* 더하기: p1 + p2



다항식의 해 -> 다항식의 계수: poly

$$p_1(x) = x^2 - 3x + 2 = (x-1)(x-2)$$

$$p_2(x) = 2x^3 - 9x^2 + 9x = 2x(x-3)(2x-3)$$

$$p_3(x) = x^4 + x^3 - 5x^2 - 7x + 10 = (x-1)(x-2)\big(x-(-2+i)\big)\big(x-(-2-i)\big)$$

```
x = linspace(-4, 4);

p1 = poly([1, 2]); % p1 = [1, -3, 2];
p2 = poly([0, 3, 3/2]); % p2 = [2, -9, 9, 0]/2;
p3 = poly([1, 2, -2+1i, -2-1i]); % p3 = [1, 1, -5, -7, 10];

fx1 = polyval(p1, x);
fx2 = polyval(p2, x);
fx3 = polyval(p3, x);
```

다항식의 계수 -> 다항식의 해: roots

* roots와 poly는 역함수

$$p_1(x) = x^2 - 3x + 2 = (x-1)(x-2)$$

$$p_2(x) = 2x^3 - 9x^2 + 9x = 2x(x-3)(2x-3)$$

$$p_3(x) = x^4 + x^3 - 5x^2 - 7x + 10 = (x-1)(x-2)\big(x-(-2+i)\big)\big(x-(-2-i)\big)$$

```
x = linspace(-4, 4);

p1 = [1, -3, 2];
fx1 = polyval(p1, x);
r1 = roots(p1);

p2 = [2, -9, 9, 0];
fx2 = polyval(p2, x);
r2 = roots(p2);

p3 = [1, 1, -5, -7, 10];
fx3 = polyval(p3, x);
r3 = roots(p3);
```

```
figure,
subplot(1,3,1), hold on, grid on, xlim([0 3])
plot(x,fx1,'r'), plot(r1, 0, 'ko'),

subplot(1,3,2), hold on, grid on, xlim([-1 3])
plot(x,fx2,'g'), plot(r2, 0, 'ko'),

subplot(1,3,3), hold on, grid on,
plot(x,fx3,'b'), xlim([-3 3])
for i=1:length(r3)
    if ~imag(r3(i))
        plot(r3(i), 0, 'ko')
    end
end
```

※ polynomial만 가능
※ nonlinear -> fzero

다항식의 미분과 적분

```
k = polyder(p);
```

$$k(x) = \frac{d}{dx} p(x)$$

```
k = polyder(a, b);
```

$$k(x) = \frac{d}{dx} [a(x) b(x)]$$

```
[q, d] = polyder(a, b);
```

$$\frac{q(x)}{d(x)} = \frac{d}{dx} \left[\frac{a(x)}{b(x)} \right]$$

```
q = polyint(p, C); % C 기본값은 0
```

$$q(x) = \int p(x) dx$$

```
p = [4, -9, 2, -3];
q = polyint(p);
d1 = polyval(q, 3) - polyval(p, 1);
d2 = diff(polyval(q, [1, 3]));
```

$$p(x) = 4x^3 - 9x^2 + 2x - 3$$

$$q(x) = \int p(x) dx = x^4 - 3x^3 + x^2 - 3x + C$$

$$\int_1^3 p(x) dx = q(3) - q(1) = 4$$

curve fitting

zero가 아닌 점을 지나는 polynomial은?

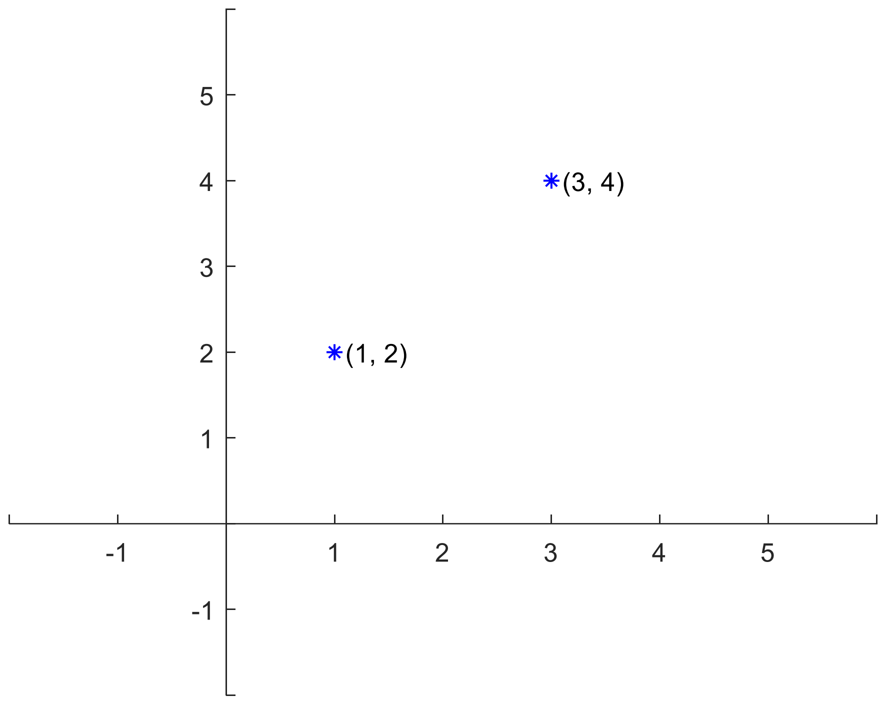
Q1) (1,2), (3,4)를 지나는 직선의 방정식을 구하시오.

Q2) (-1, 6), (3, 2), (5, 12)를 지나는 2차 함수를 구하시오.

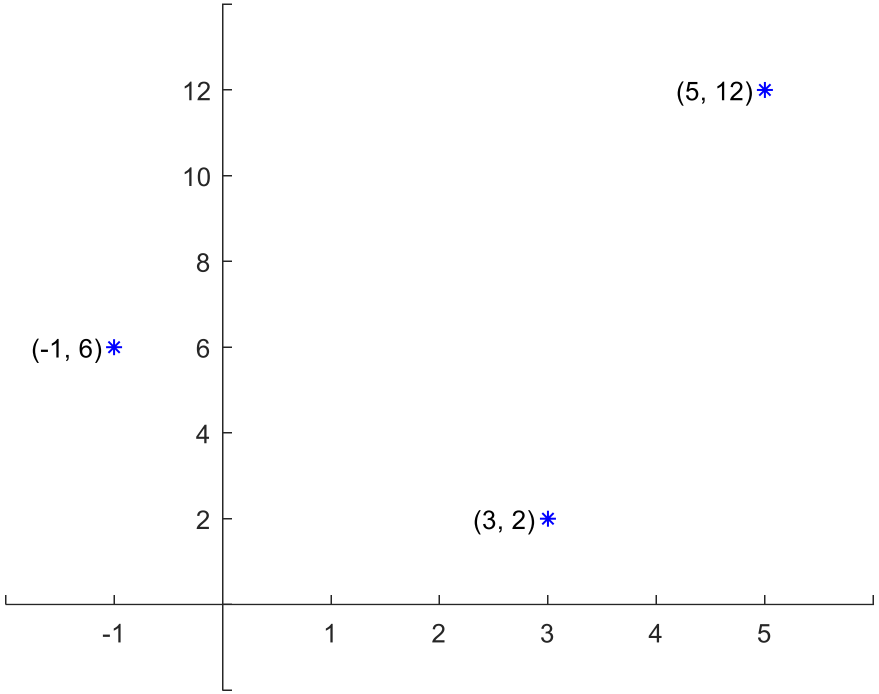
Q3) (1,2), (3,4), (5,5)를 <u>**"가장 가깝게"**</u> 지나는 직선의 방정식을 구하시오.

Q4) (1,5), (3,1), (5,11), (7,19)를 <u>**"가장 가깝게"**</u> 지나는 2차 함수를 구하시오.

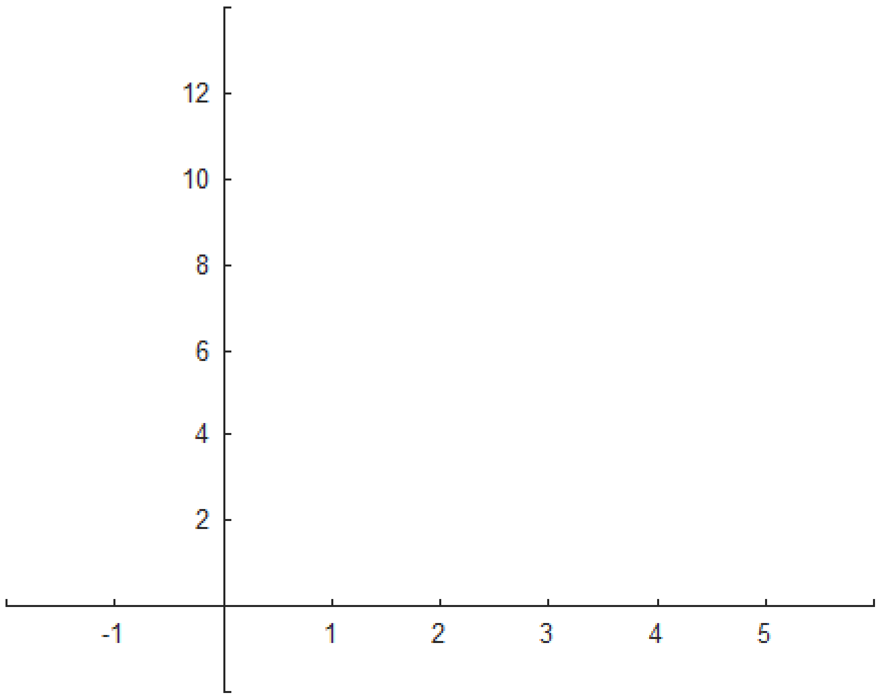
두 점을 지나는 직선의 방정식



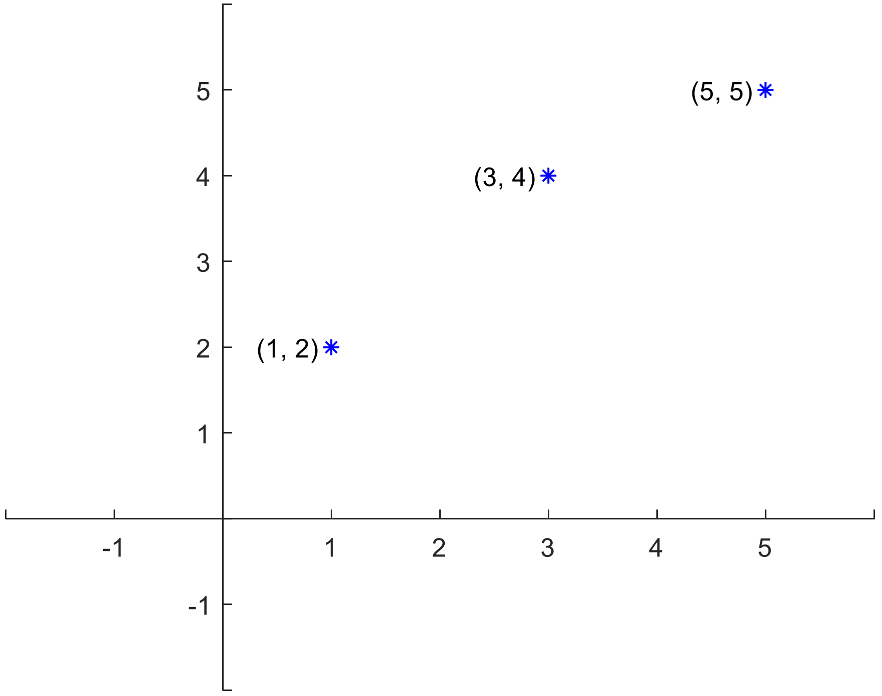
세 점을 지나는 2차 함수



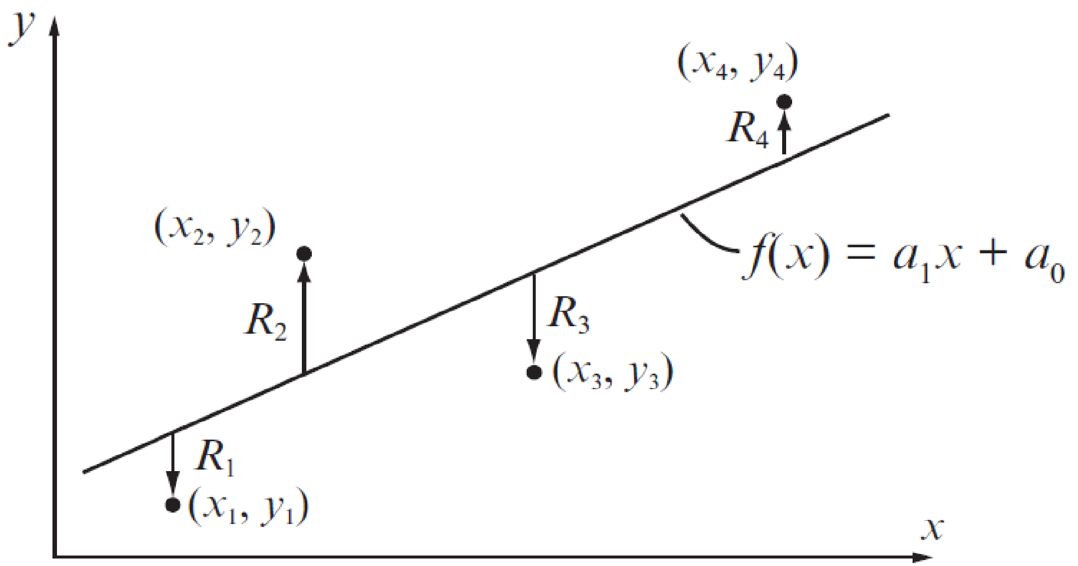
n+1개의 점을 지나는 n차 함수



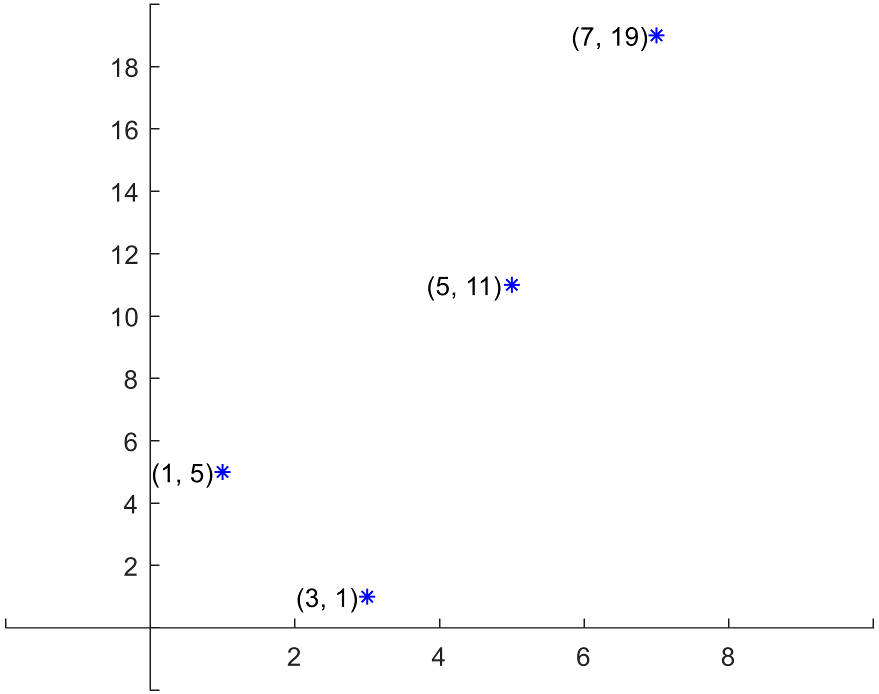
3개의 점을 "가장 가깝게" 지나는 직선?



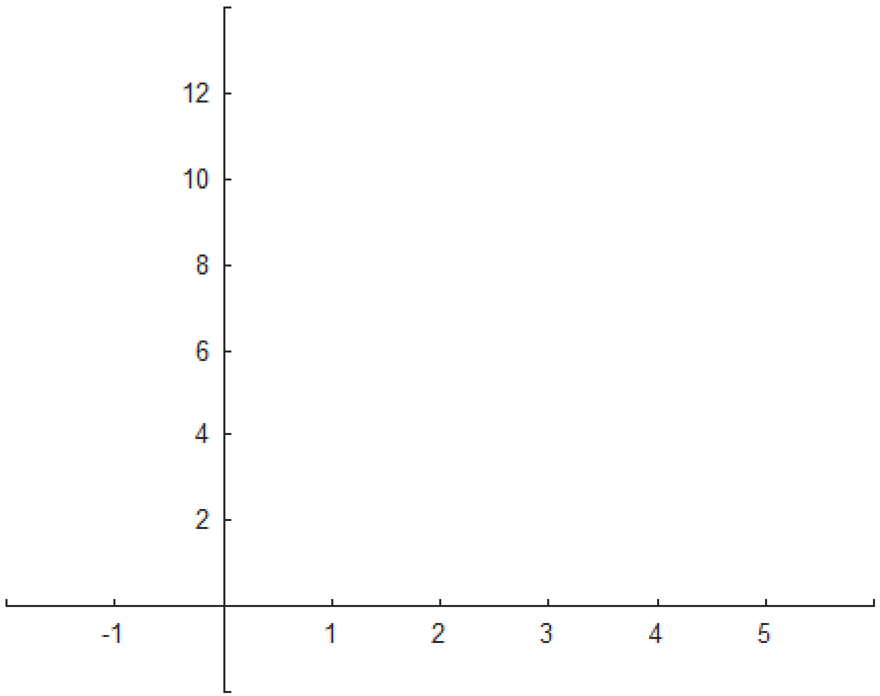
최소자승법 (least-square method)



4개의 점을 "가장 가깝게" 지나는 2차 함수



n개의 점을 지나는 1변수 p차 함수



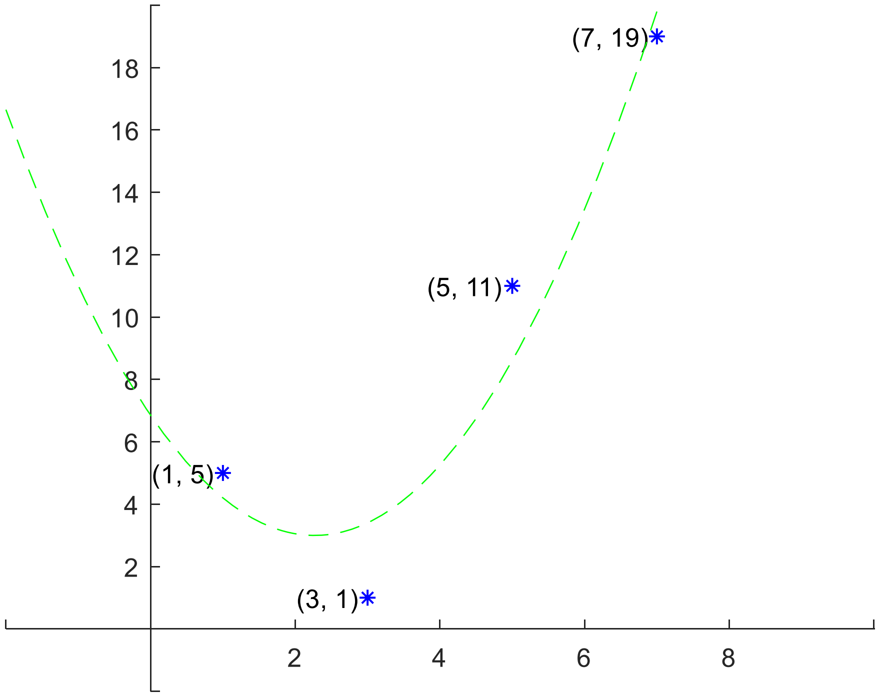
polyfit

```
x = [1, 3];
y = [2, 4];
p = polyfit(x, y, 1);

x = [-1, 3,  5];
y = [ 6, 2, 12];
p = polyfit(x, y, 2);

x = [1, 3, 5];
y = [2, 4, 5];
p = polyfit(x, y, 1);

x = [1, 3, 5, 7];
y = [5, 1, 11, 19];
p = polyfit(x, y, 2);
```

```
p = polyfit(x, y, n);
```

그 계수를
p에 대입해라.

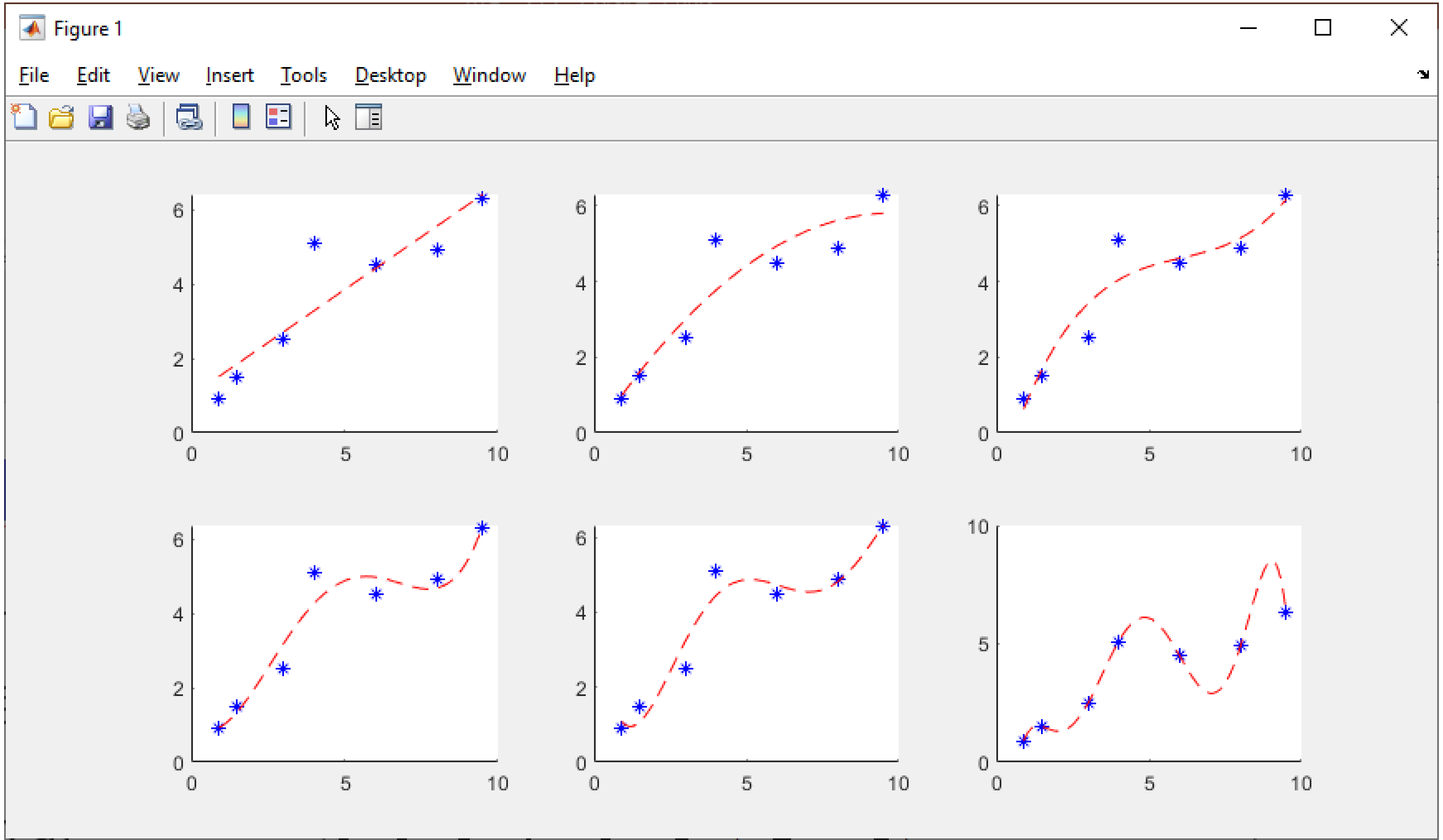
(x, y)를 지나는 n차 polynomial을
최소자승법으로 구하고



polyfit 예제

Q) 실험 측정 결과치가 아래와 같이 나왔다.
결과로부터 1차~6차 polynomial로 모델을 구하시오.

(0.9, 0.9)
(1.5, 1.5)
(3.0, 2.5)
(4.0, 5.1)
(6.0, 4.5)
(8.0, 4.9)
(9.5, 6.3)



polynomial이 아닌 경우는?

$$y = be^{mx}$$

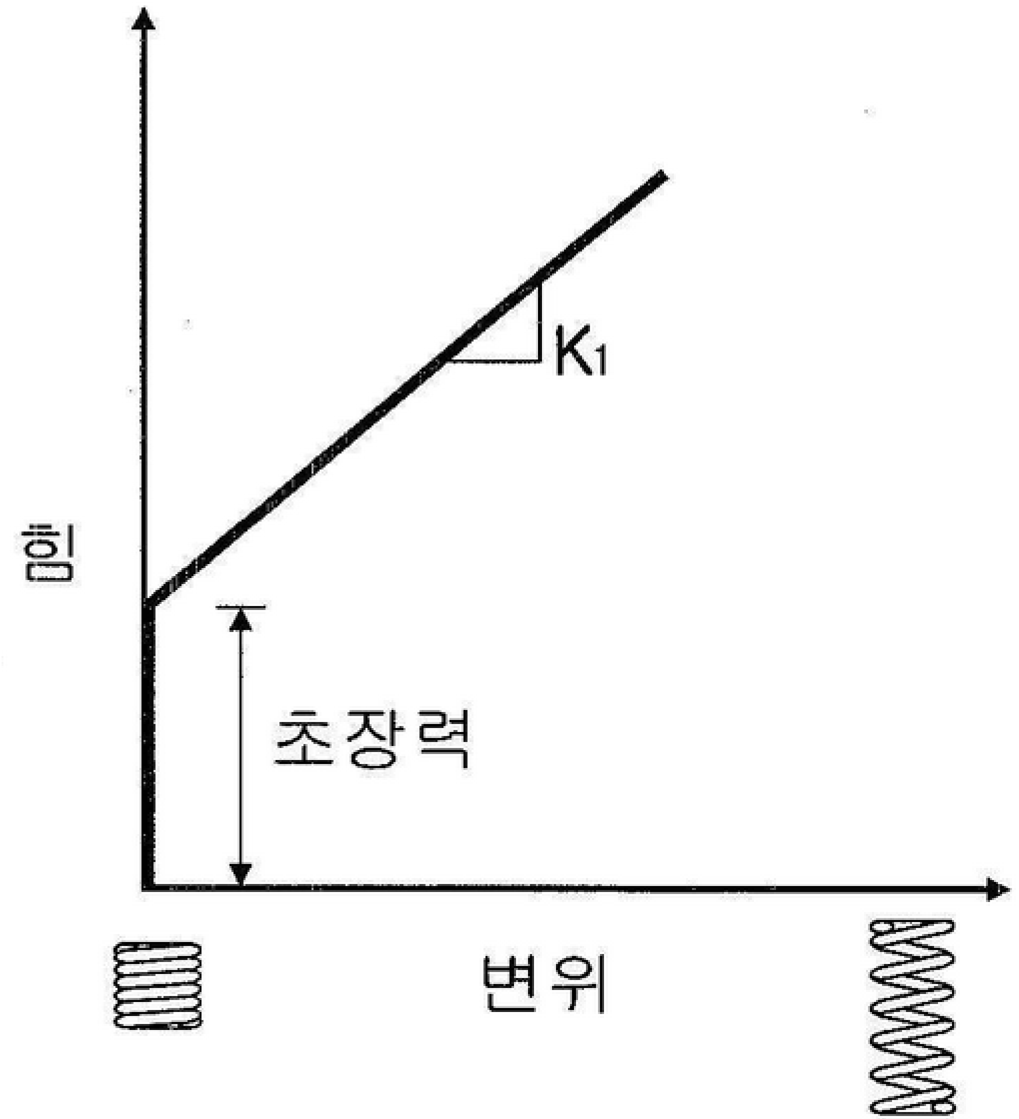
$$y = m\ln(x) + b$$

$$y = \frac{1}{mx + b}$$

$$y = \frac{nx}{mx + b}$$

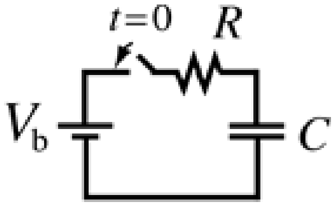
※ 모델 선정 -> polynomial fitting

예제 – 스프링 상수, 초장력



https://patents.google.com/patent/KR100973986B1/ko

예제 – RC 회로



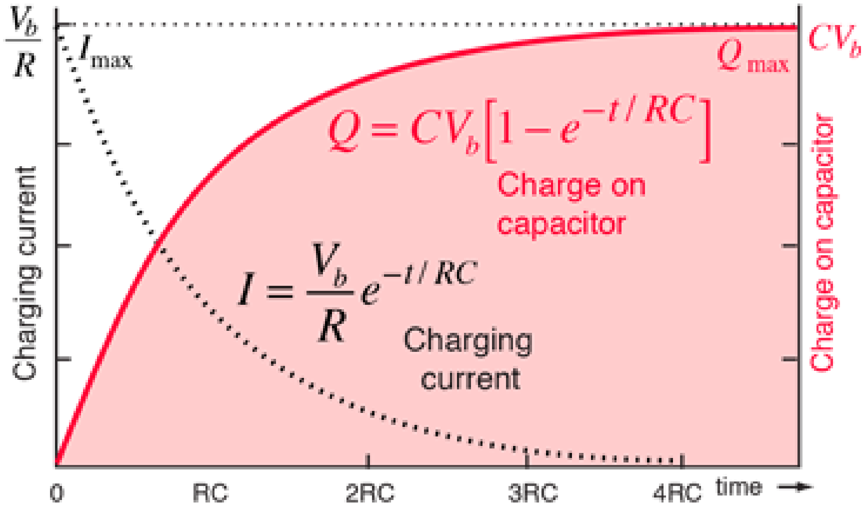
$$V_b = V_R + V_C$$

$$V_b = IR + \frac{Q}{C}$$

As charging progresses,

$$V_b = IR + \frac{Q}{C}$$

current decreases and charge increases.



At $t = 0$

$Q = 0$

$V_C = 0$

$I = \frac{V_b}{R}$

As $t \to \infty$

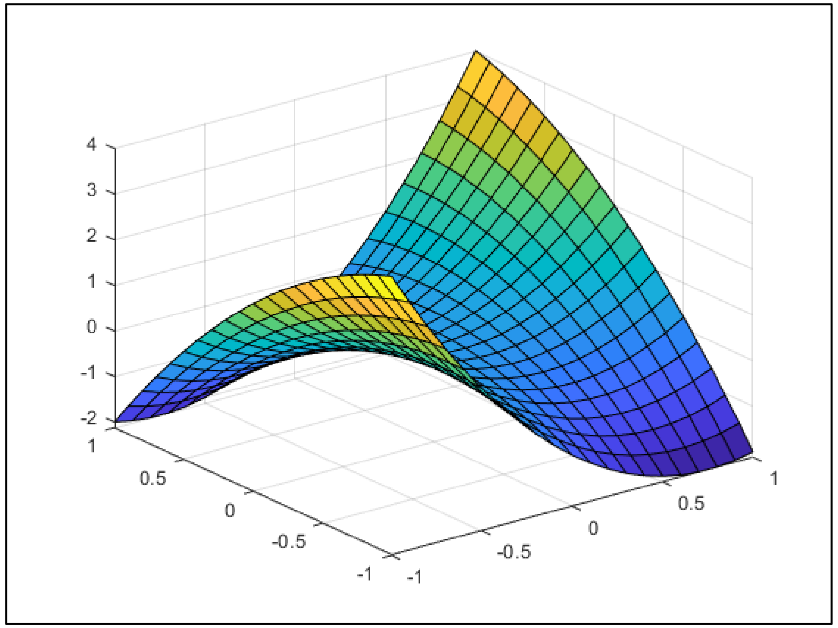
$Q \to CV_b$

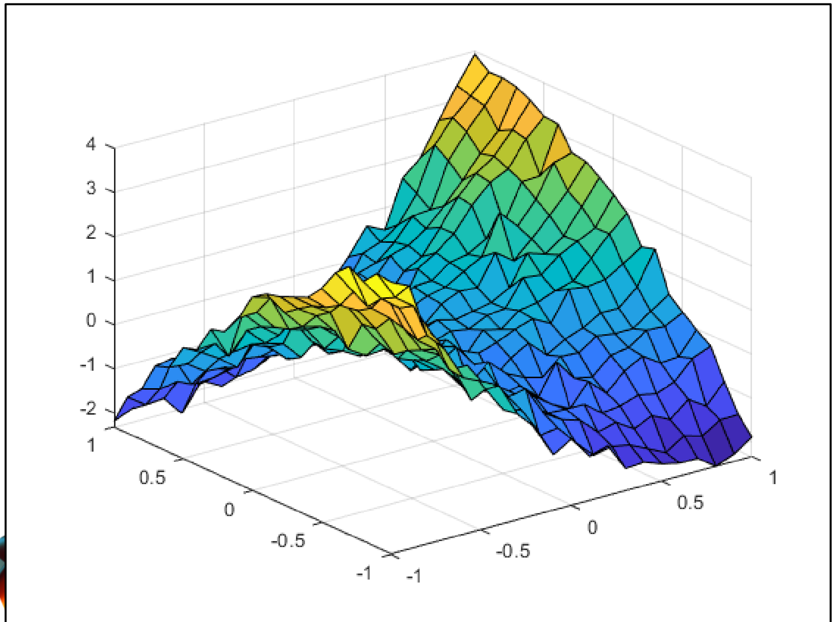
$V_C \to V_b$

$I \to 0$

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/capchg.html

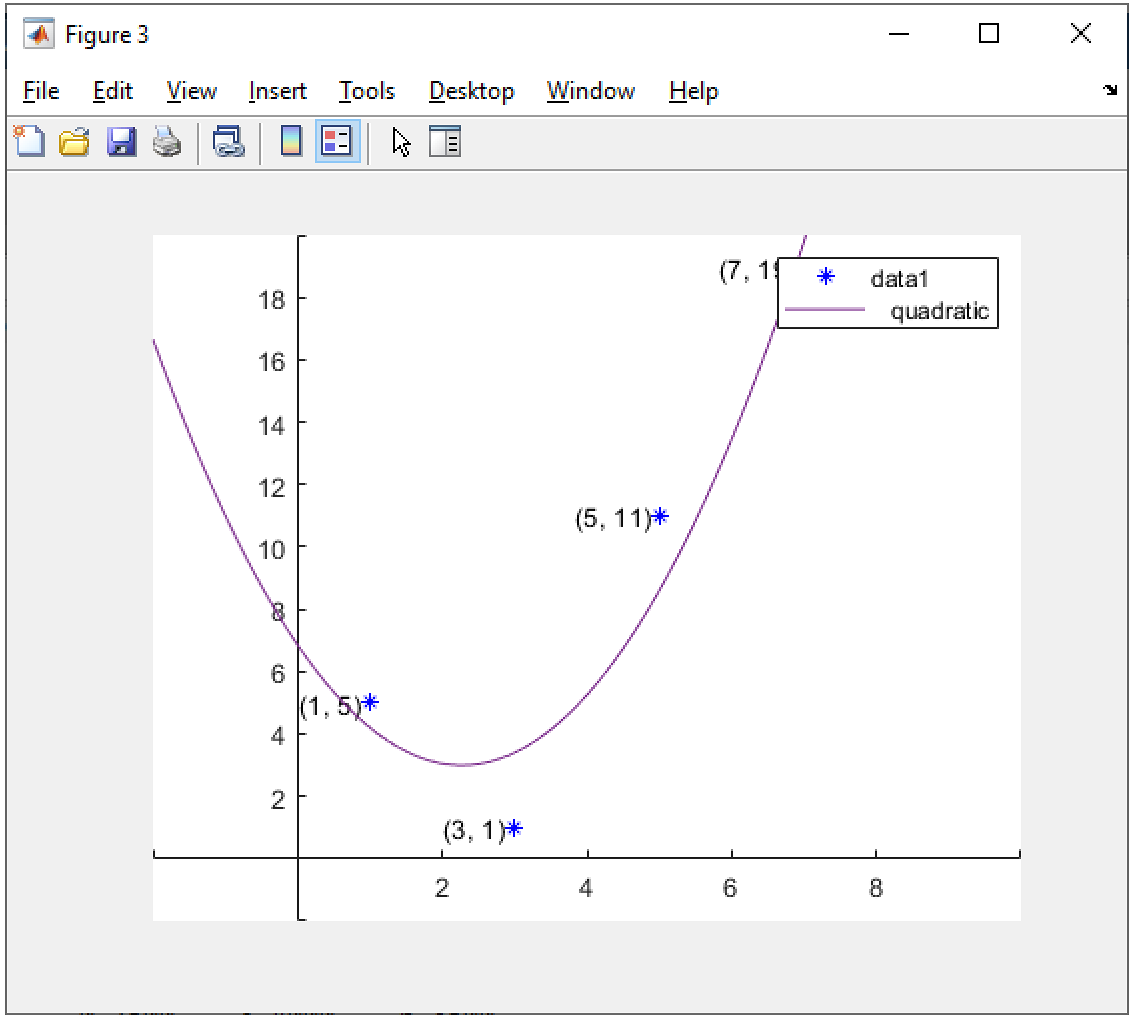
다변수 polynomial의 경우는?

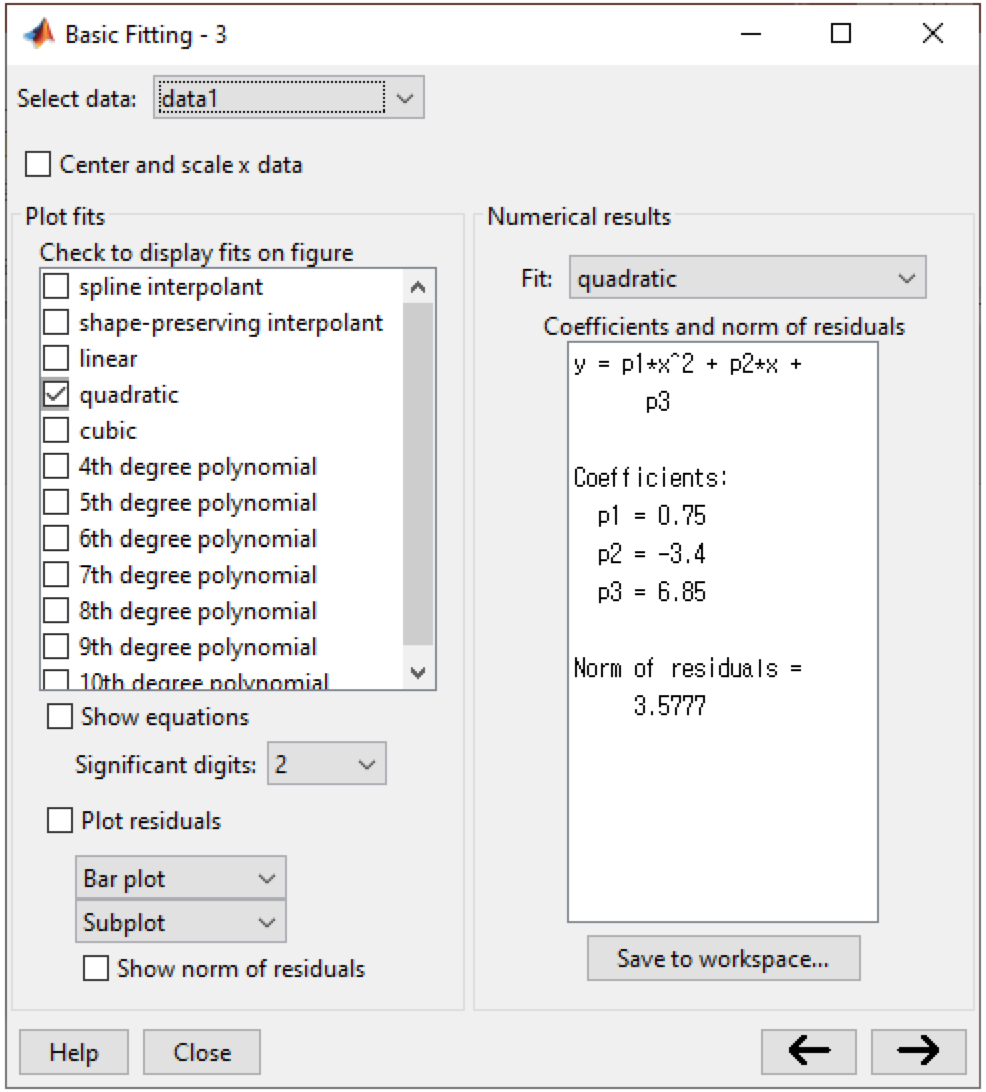




$$z = px^2 + qxy + ry^2 + s$$

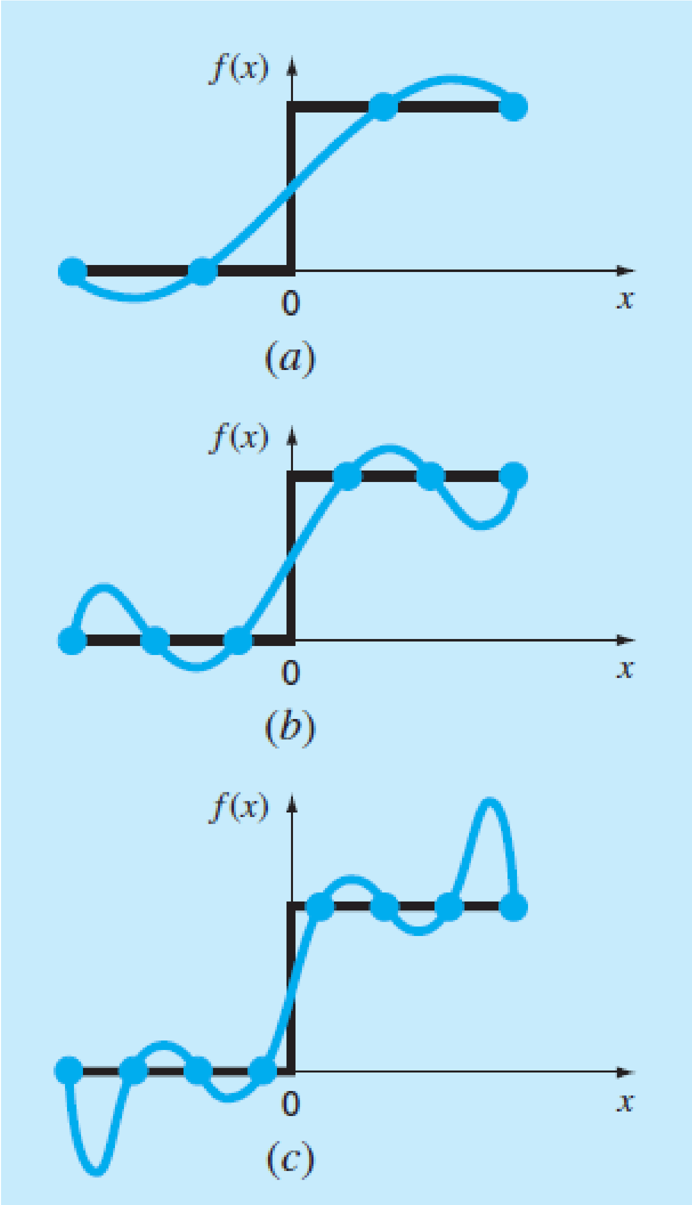
basic fitting tool



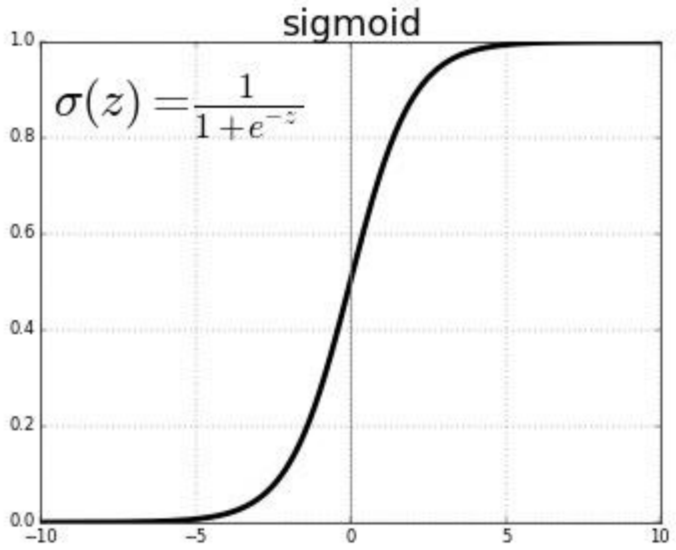


interpolation

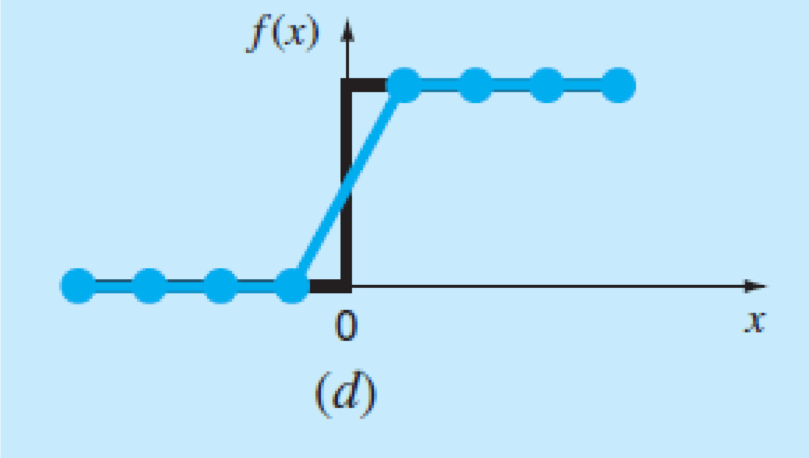
motivation



$f(x)$
0
x
(a)
$f(x)$
0
x
(b)
$f(x)$
0
x
(c)

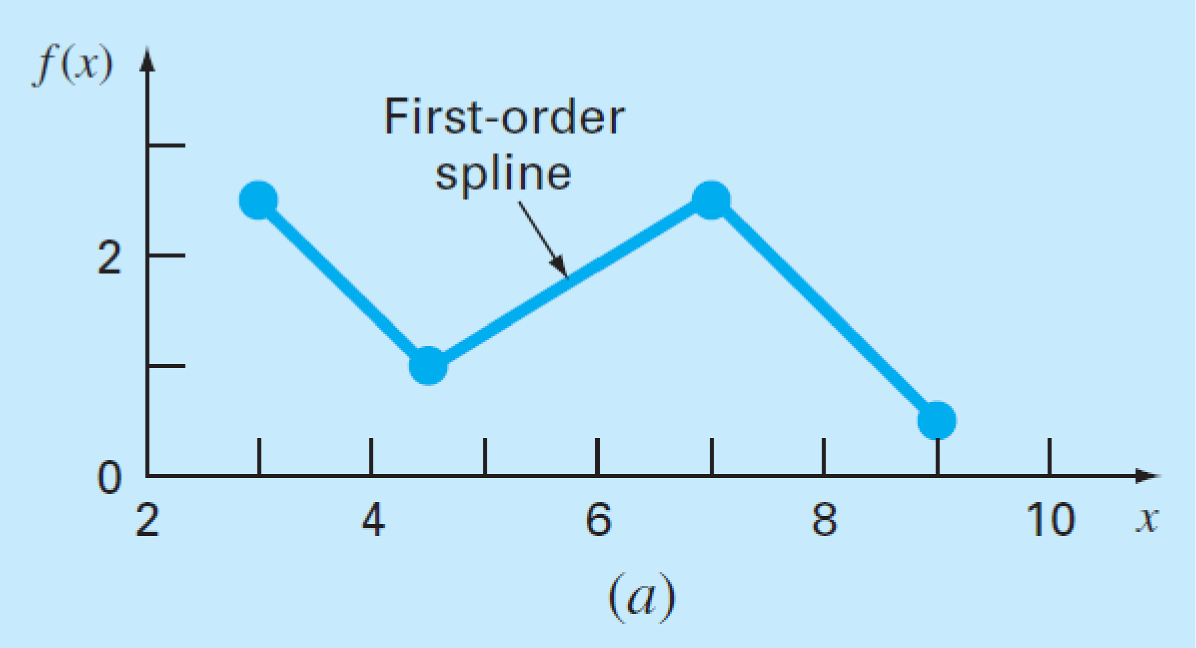


sigmoid
$\sigma(z) = \frac{1}{1+e^{-z}}$
1.0
0.8
0.6
0.4
0.2
0.0
−10
−5
0
5
10

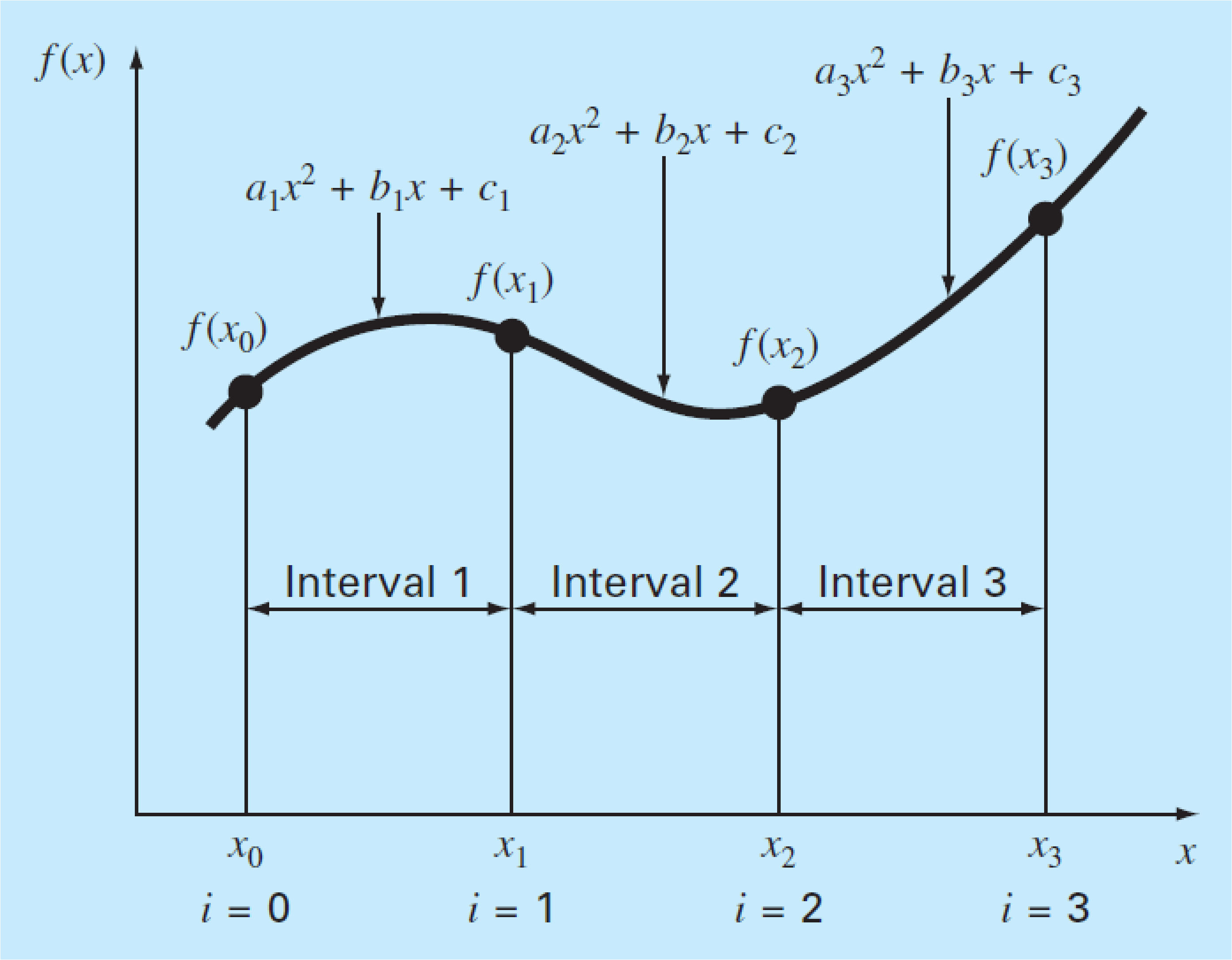


$f(x)$
0
x
(d)

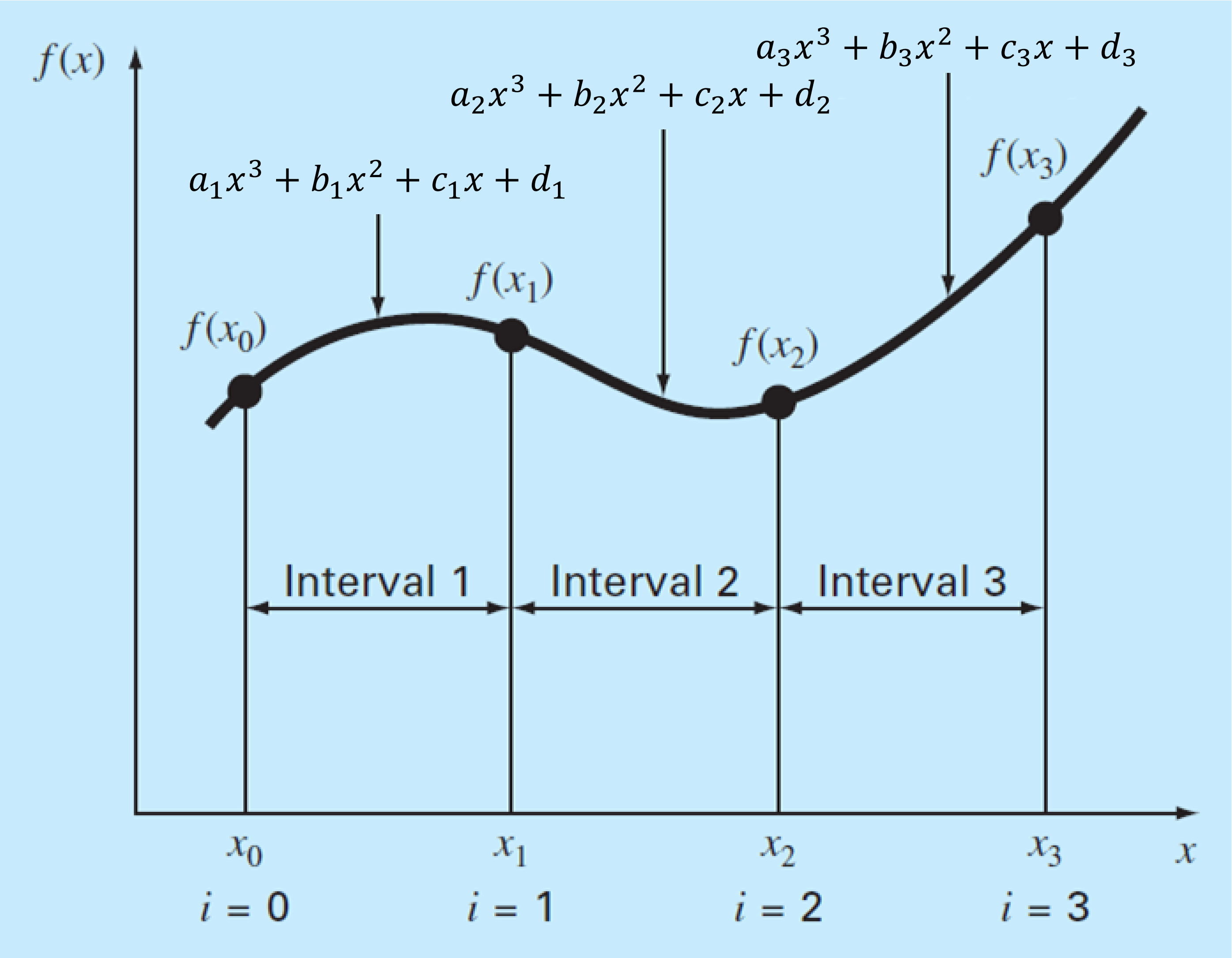
linear interpolation (linear spline)



quadratic spline

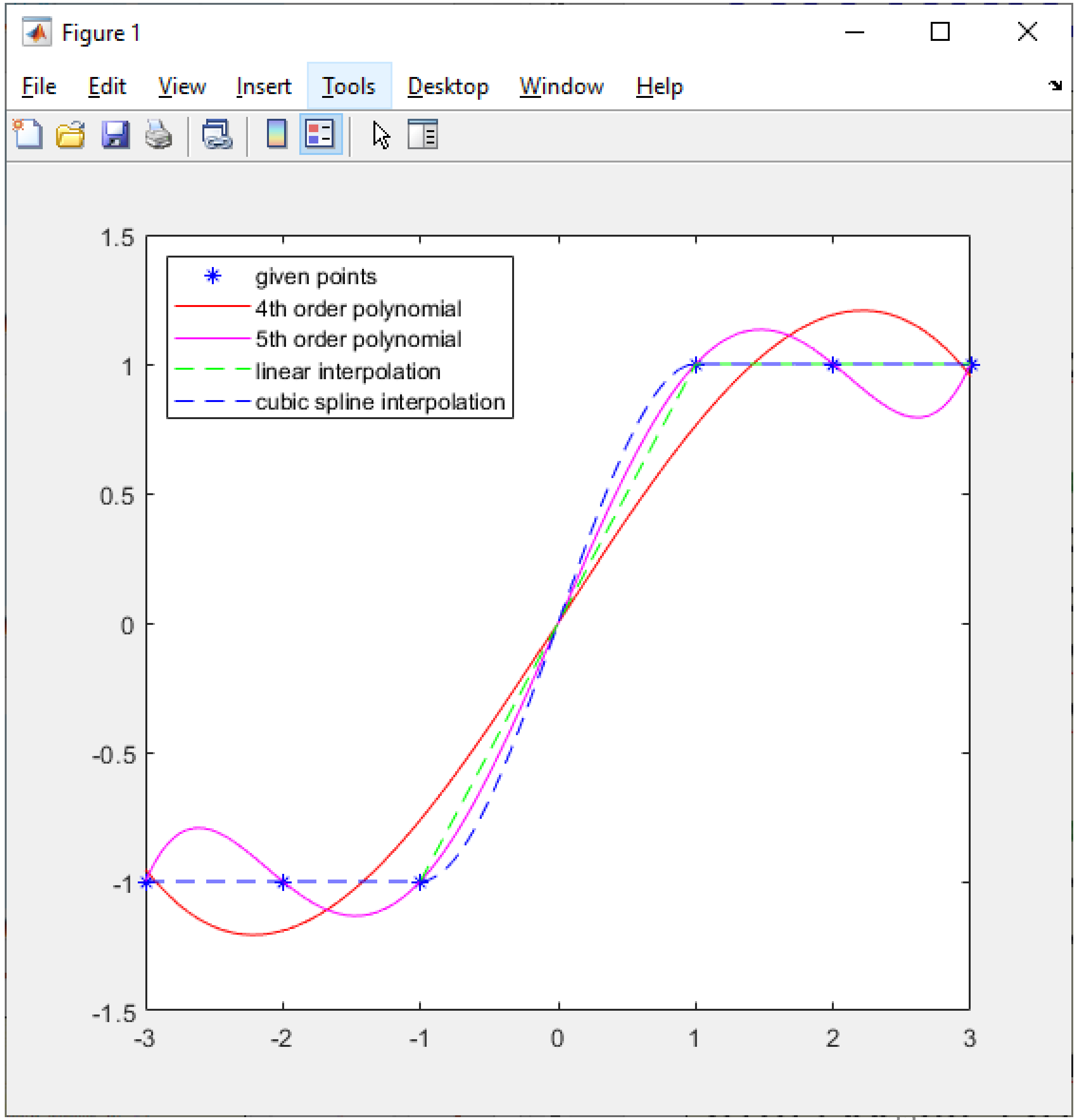


cubic spline

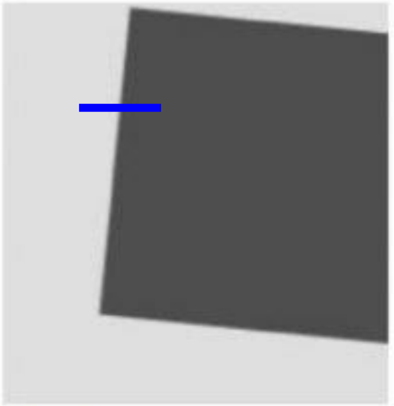


interp1

```
x = [-3, -2, -1, 1, 2, 3];
y = [-1, -1, -1, 1, 1, 1];
xx = linspace(x(1),x(end));

figure, plot(x,y,'b*'), hold on

% high order polynomial fitting
p4 = polyfit(x,y,4);
p5 = polyfit(x,y,5);
plot(xx, polyval(p4, xx), 'r')
plot(xx, polyval(p5, xx), 'm')

% interpolation
yy_lin = interp1(x,y,xx); % 'linear' is default.
yy_spl = interp1(x,y,xx,'cubic');

plot(xx,yy_lin,'g--')
plot(xx,yy_spl,'b--')
```

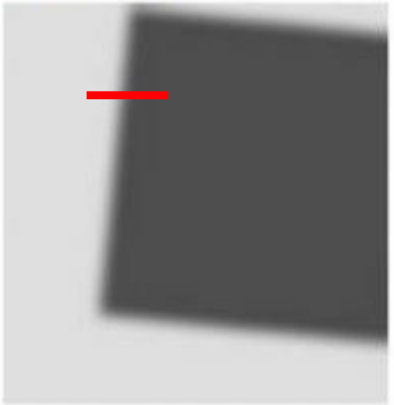


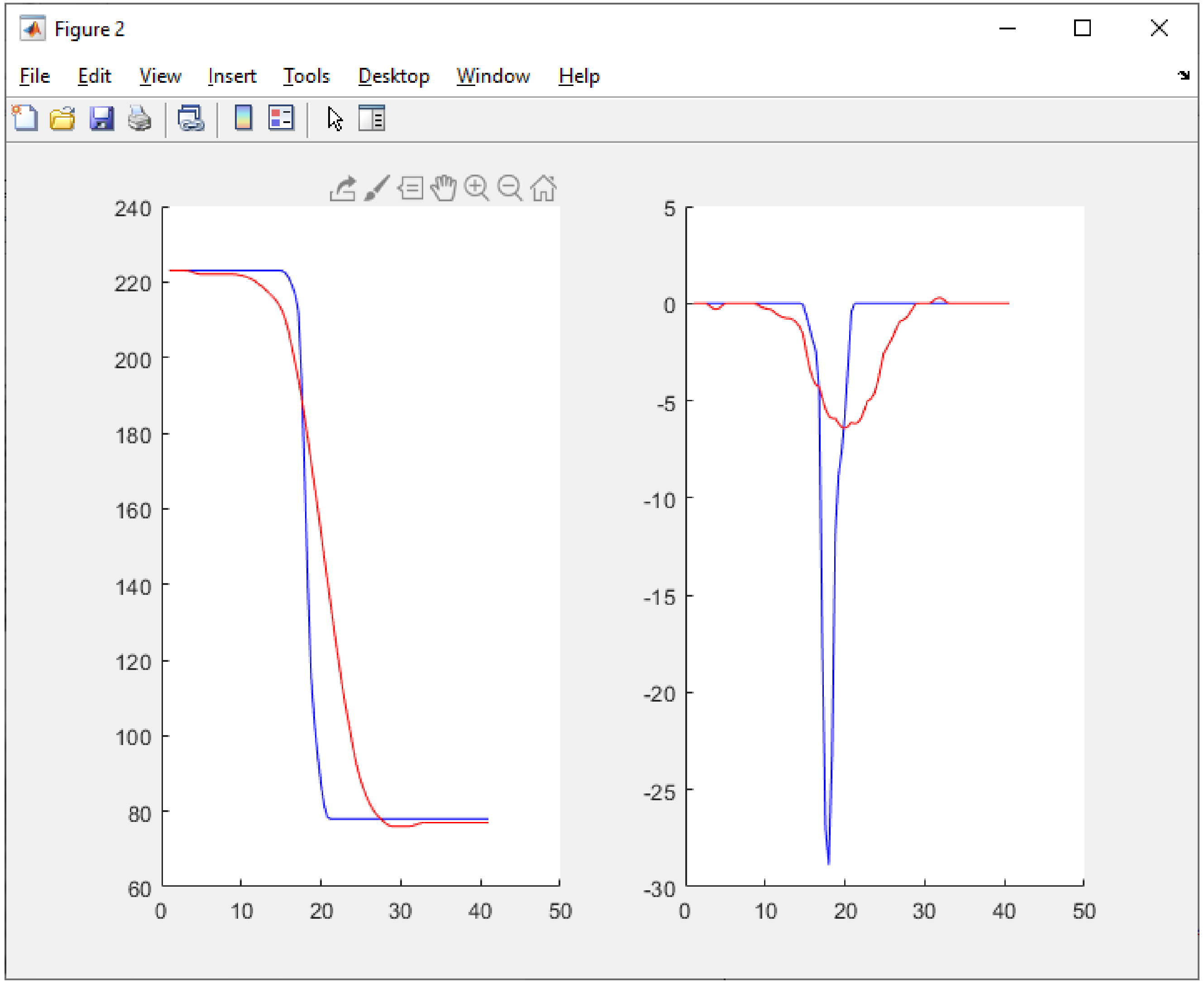
예제 – line spread function (LSF)



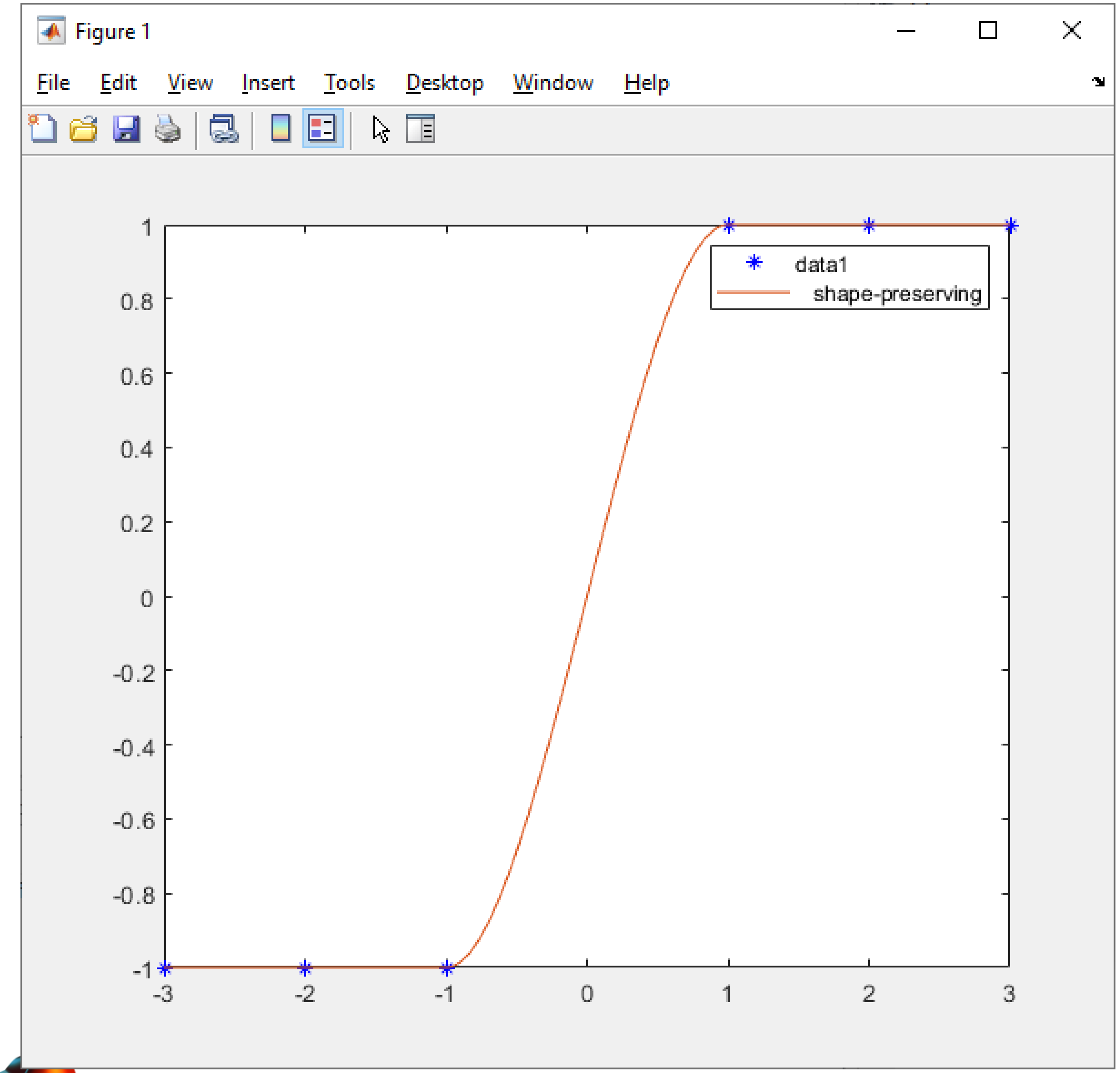
Sharp

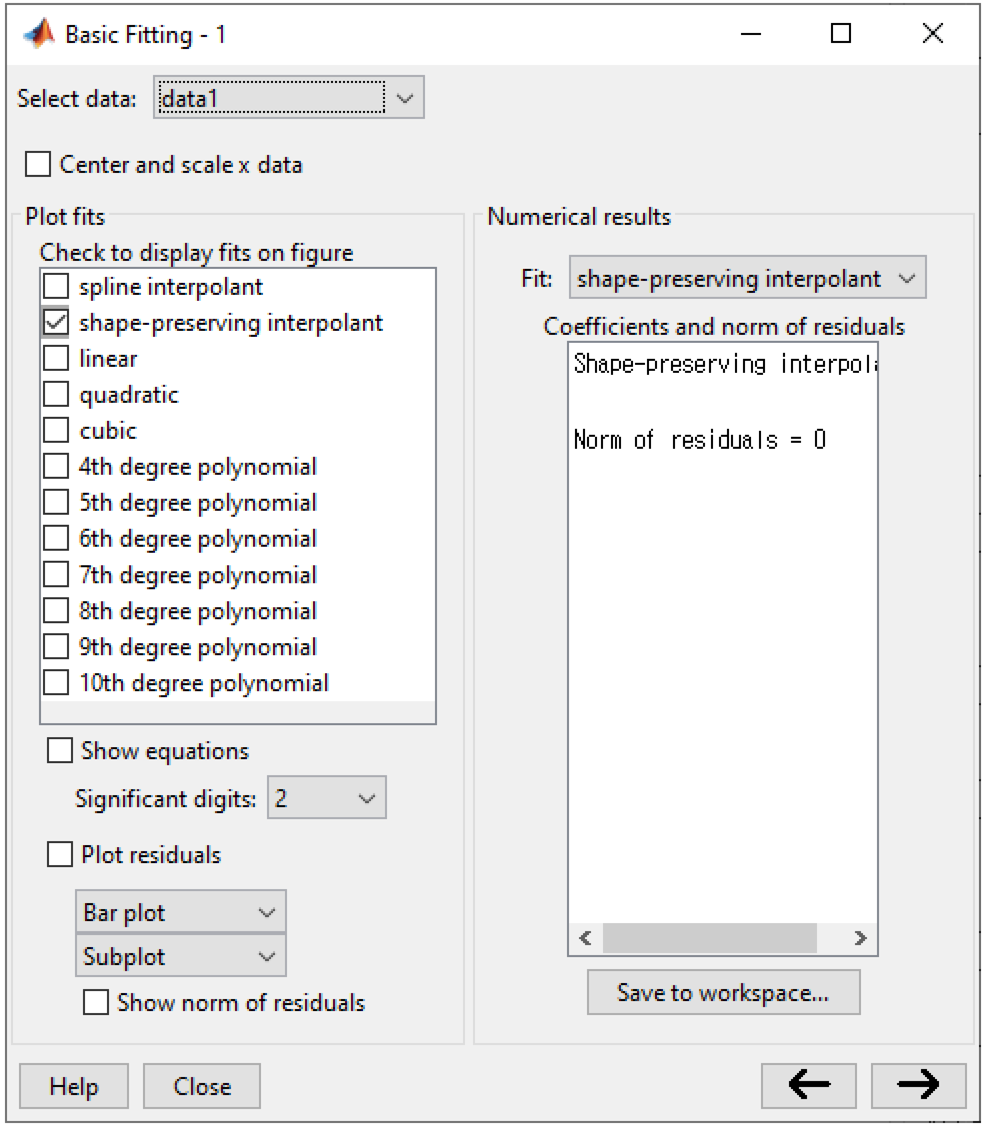
Blurred



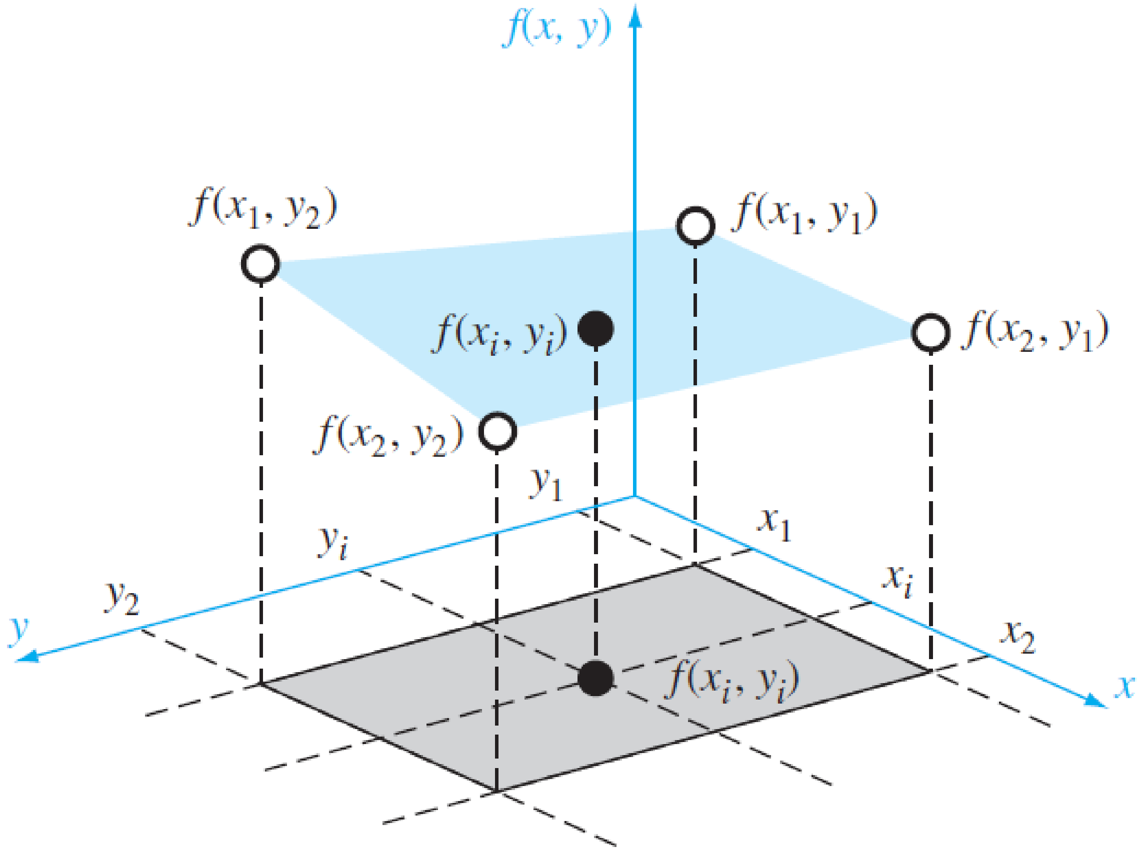


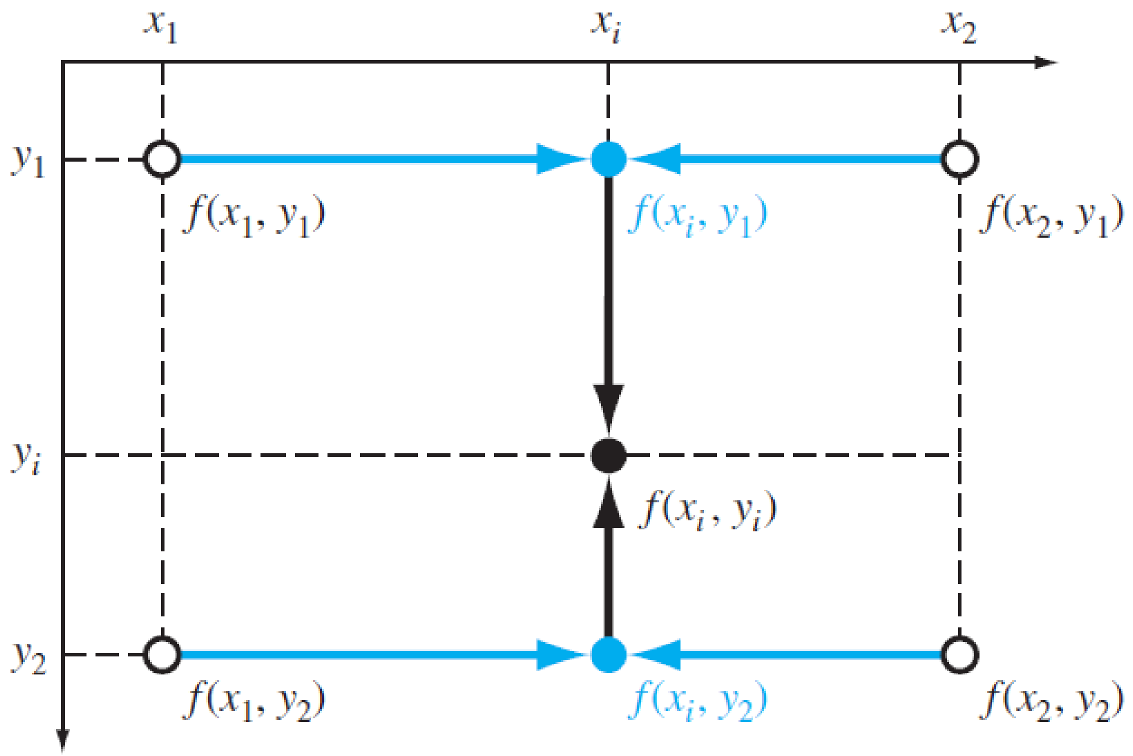
basic fitting tool





2차원 interpolation





interp2 예제

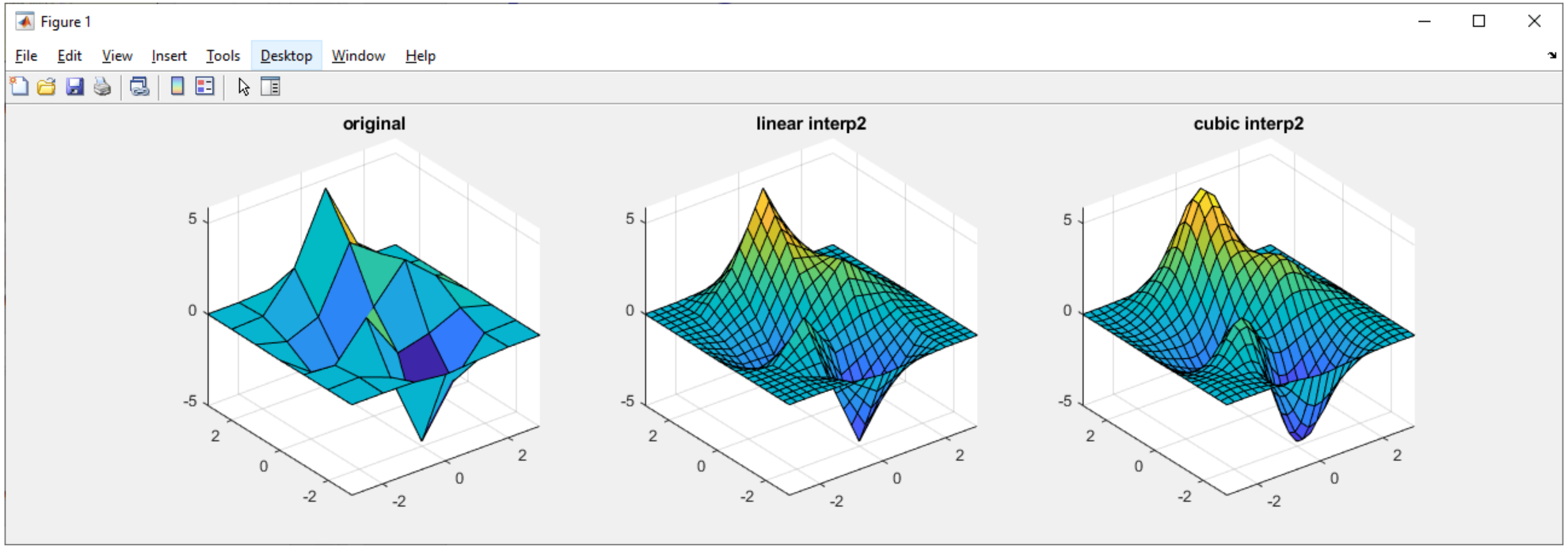


Figure 1
File Edit View Insert Tools Desktop Window Help
original
linear interp2
cubic interp2
5
0
-5
2
0
-2
-2
0
2

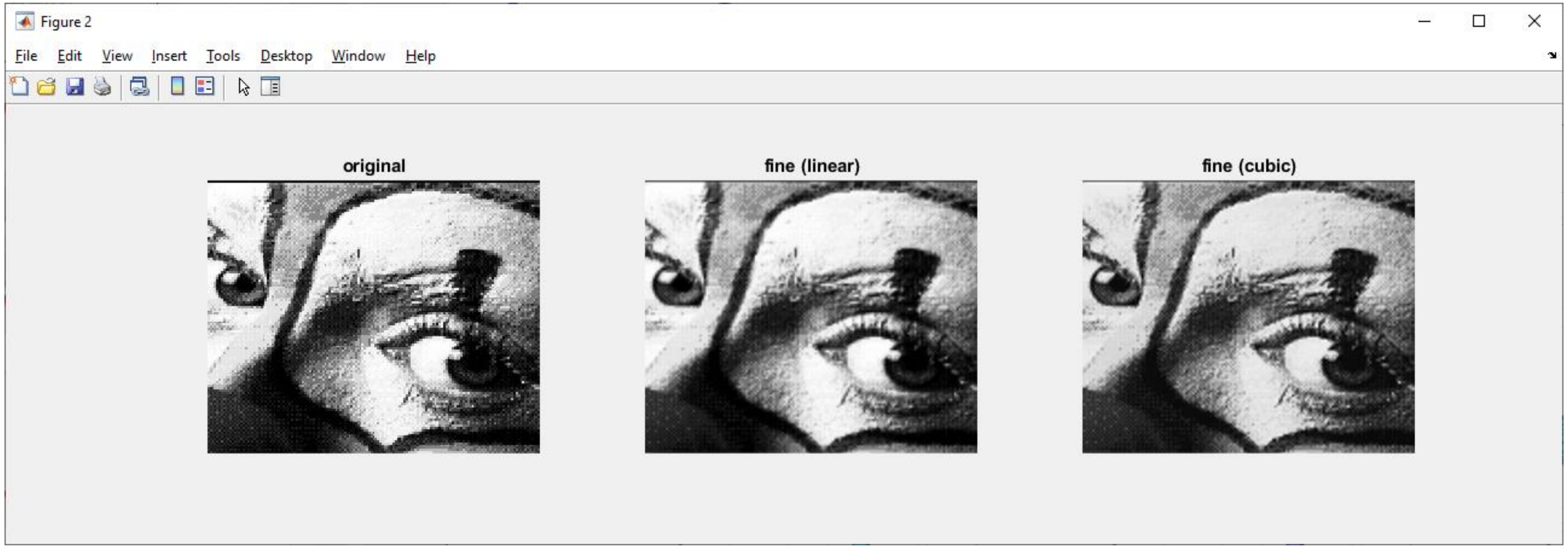


Figure 2
File Edit View Insert Tools Desktop Window Help
original
fine (linear)
fine (cubic)

Q&A