

# MSiA-400 Everything Starts with Data

## Lab Assignment 1

**Due date: Thursday November 1, 12 pm**

EXERCISE INSTRUCTIONS: Please submit one report file that includes: short answer, related code and print for each problem if necessary. Push your answers to Github (required) and Canvas (optional).

### Problem 1

In *Markov100.txt*, the one step transition probability matrix for a Markov chain with 100 states (State 1 to State 100) is given. Note that the data has no heading.

Name of the data set	Markov100
Number of rows	100
Number of columns	100

#### Problem 1(a)

Suppose we are at State 1 now. Find and display the probability of being in State 5 after 10 transitions.

#### Problem 1(b)

Suppose we are at one of States 1, 2, and 3 with equal probabilities. Find and display the probability of being in State 10 after 10 transitions.

#### Problem 1(c)

Find the steady state probability of being in State 1.

#### Problem 1(d)

Find the mean first passage time from State 1 to State 100.

### Problem 2

You are asked to analyze the data from an website with 8 pages (Page 1 - Page 8). Let us assume that there is a virtual page (Page 9) that a visitor must automatically visit when the visitor leaves the website. The visitors always start their visit from Page 1. Let us formulate a Markov chain for this website. The states are defined as

$$S_i = \text{visitor is at Page } i, i = 1, \dots, 9.$$

For example, suppose that a visitor enters the website (hence visit Page 1), moves to Page 3, Page 5, and then leave the website, sequentially. Then, the user visits States  $S_1, S_3, S_5$ , and  $S_9$ , sequentially.

Please find the attached data *webtraffic.txt*. The data includes the record of 1000 visitors (rows). The data has 81 columns labeled as  $t_{11}, t_{12}, \dots, t_{19}, t_{21}, t_{22}, \dots, t_{29}, \dots, t_{91}, t_{92}, \dots, t_{99}$ . The label  $t_{ij}$  represents the transition from State  $i$  to State  $j$ , for  $i = 1, \dots, 9$  and  $j = 1, \dots, 9$ . For example,  $t_{12}$  is the transition from State 1 to State 2, and  $t_{84}$  is the transition from State 8 to State 4. For each visitor (row), it has 1 for column  $t_{ij}$  if the visitor makes transition from State  $i$  to State  $j$ , and it has 0 elsewhere. For example, if a visitor

visits States  $S_1, S_3, S_5$ , and  $S_9$ , sequentially, then the corresponding row has 1 for columns  $t_{13}, t_{35}, t_{59}$  and 0 elsewhere.

The summary of the data set is below.

Name of the data set	webtraffic
Type of data	binaries (0,1)
Number of rows	1000
Number of columns	81

### Problem 2(a)

Construct 9 by 9 matrix **Traffic** that counts total traffic between State  $i$  to State  $j$  for all  $i = 1, \dots, 9$  and  $j = 1, \dots, 9$ . Display **Traffic**.

*Hint* `colSums()` adds all rows for each column.

### Problem 2(b)

Observe that **Traffic** has 0's in row 9 and 0's in column 1. Set **Traffic**[9,1]=1000. Construct the one step transition probability matrix **P** and display it.

### Problem 2(c)

Calculate and display the steady state probability vector **Pi**.

### Problem 2(d)

The following table presents the average time that the visitors spend on each page.

Page	1	2	3	4	5	6	7	8
Avg(minute)	0.1	2	3	5	5	3	3	2

Calculate and display the average time a visitor spend on the website (until she leaves).

### Problem 2(e)

In the output of Problem 2(c), observe that Pages 3 and 4 are one of the most crowded pages except Pages 1 and 9. To balance the traffic, the owner of the website decided to create links from Page 2 to Pages 6,7 (hence, from State 2 to States 6,7). By adding the links, the owner anticipates that, from Page 2, 30% of the current outgoing traffic to State 3 would move to State 6, and 20% of the current outgoing traffic to State 4 would move to State 7. Calculate new steady state probability vector **Pi2** to check the effect of the new links. Decide if the link helped balancing the traffic by comparing the variance of **Pi** and **Pi2**.

*Hint* Start with matrix **Traffic** from Problem 2(a).