

# Korygowanie harmonogramów z uwzględnieniem awarii maszyn

Kamil Niemczyk

Politechnika Wrocławska  
Automatyka i Robotyka, Wydział Elektroniki  
Technologie informacyjne w systemach automatyki



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Zaproponowano algorytm Tabu Search z nawrotami, który przystosowano do rozwiązywania elastycznych problemów gniazdowych.

# Elastyczny problem gniazdowy

Let us consider

- $J = \{1, 2, \dots, n\}$  – a set of jobs,
- $M = \{1, 2, \dots, m\}$  – a set of machines,
- $O = \{1, 2, \dots, o\}$  – a set of operations.

A job  $j$  consists of the sequence of  $o_j$  operations. An operation  $i$  has to be executed on the dedicated machine without interruptions in time  $p_i > 0$ ,  $i \in O$ . The solution is a vector of times  $S = (S_1, S_2, \dots, S_o)$  of operations beginning such, that:

- a job begins its execution on the next machine if it is finished on the previous one,
- a job begins on the machine if the previous job executed on this machine is finished,
- beginning times are not negative.

# The job shop problem

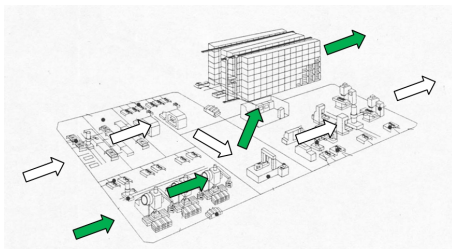
A feasible solution: a vector  $S = (S_1, S_2, \dots, S_o)$  such, that:

$$S_{l_{j-1}+1} \geq 0, \quad j = 1, 2, \dots, n,$$

$$S_i + p_i \leq S_{i+1}, \quad i = l_{j-1} + 1, \quad l_{j-1} + 2, \dots, l_j - 1, \quad j = 1, 2, \dots, n,$$

$$S_i + p_i \leq S_j \quad \text{or} \quad S_j + p_j \leq S_i, \quad i, j \in O, \quad v_i = v_j, \quad i \neq j.$$

where  $C_j = S_j + p_j$ , a machine number  $v_i$  of a job  $i$ .



# The job shop problem

An appropriate criterion function has to be added to the problem constraints:

- minimization of the time of finishing all the jobs

$$C_{\max}(S) = \max_{1 \leq j \leq n} C_{l_j},$$

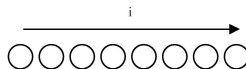
- minimization of the sum of job finishing times

$$C(S) = \sum_{j=1}^n C_{l_j}.$$

Both problems described are strongly NP-hard and although they are similarly modelled, the second one is found to be harder because of the lack of some specific properties (so-called block properties).

## Neighborhood

In the considered neuro-tabu search algorithm *NTS* each move is represented by its neuron. For the adjacent swap neighborhood a network of neurons formed of  $o - 1$  neurons. Let  $i$ -th neuron represents a move consisting in swap of two adjacent elements on the positions  $i$  and  $i + 1$  in a solution  $\pi$ .



If  $i$ -th neuron fire



$$\pi : (\pi(1), \dots, \pi(i), \pi(i + 1), \dots, \pi(o))$$



$$\pi' : (\pi(1), \dots, \pi(i + 1), \pi(i), \dots, \pi(o))$$

# Tabu effect

In a proposed neural network architecture a history of each neuron is stored as its internal state (*tabu effect*). Each neuron is defined by the following equations:

$$\eta_i(t+1) = \alpha \Delta_i(t),$$

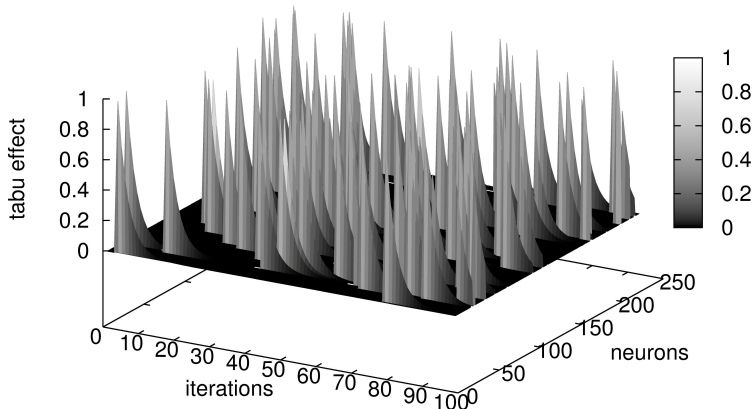
$$\Delta_i(t) = \frac{C_{max}(\pi_v^{(t)}) - C_{max}^*}{C_{max}^*},$$

$$\gamma_i(t+1) = \sum_{d=0}^{s-1} k^d x_i(t-d),$$

where  $x_i(t)$  is an output of the neuron  $i$  in the iteration  $t$ .

## Tabu effect

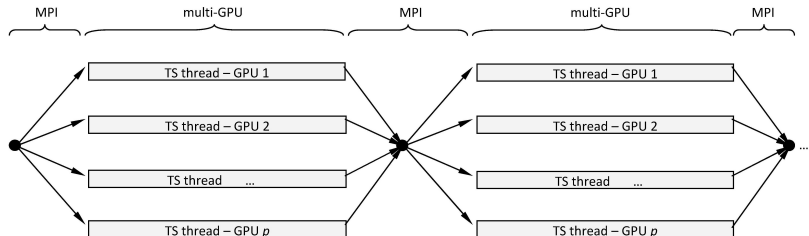
A neuron  $i$  is activated if it has the lowest  $(\eta_i(t+1) + \gamma_i(t+1))$  value of all the neurons.





## Parallel neuro-tabu search

Here we propose a solution method to the job shop problem in the distributed computing environments, such as multi-GPU clusters. Neuro-tabu search algorithm is executed in concurrent working threads.



Algorithm 1.  $NIS(\gamma, \delta, C^R)$

Input:  $\gamma, \delta$  – two processing orders;  $C^R$  – reference makespan;

Output:  $\varphi$  – processing order; update reference makespan  $C^R$ ;

$\pi \leftarrow \gamma$ ;  $iter \leftarrow 0$ . Find  $\delta^{-1}$  and  $D(\gamma, \delta)$

repeat

$iter \leftarrow iter + 1$ ; Find  $N(\pi)$ ;

    For any  $v \in N(\pi)$  calculate and store  $C_{max}(\pi(v))$ ;

    Find  $N^+ = \{v = (x, y) \in N(\pi) : \delta^{-1}(y) < \delta^{-1}(x)\}$ ;

    if  $N^+ \neq \emptyset$  than  $K \leftarrow N^+$  else  $K \leftarrow N(\pi)$ ;

    Select the move  $w \in K$  such, that

$$C_{max}(\pi(w)) = \min_{v \in K} C_{max}(\pi(v));$$

    Denote  $\pi(w)$  by  $\alpha$ ;  $\pi \leftarrow \alpha$ ;  $\varphi \leftarrow \pi$ ;

    if  $C_{max}(\pi) < C^R$  than  $C^R \leftarrow C_{max}(\pi)$  and exit;

until  $iter \geq maxV \cdot D(\gamma, \delta)$ ;  $\{maxV \in (0, 1)$  – parameter}

### Algorithm 2. *iNTS*

Input:  $\pi^0$  – processing orders provided by INSA;

Output:  $\pi^*$  – the best found processing order  
and its makespan  $C^*$ ;

$(\pi^1, C^1) \leftarrow NTS(\pi^0)$ ;  $C^* \leftarrow C^1$ ;

for  $i \leftarrow 2, \dots, \max E$  do

$\varphi \leftarrow NIS(\pi^{i-1}, \pi^0, C^*)$ ;  $(\pi^i, C^i) \leftarrow NTS(\varphi)$ ;

$C^* = \min\{C^*, C^i\}$ ;

repeat

Find  $1 \leq l \leq \max E$  so that

$D(\pi^k, \pi^l) = \max\{D(\pi^k, \pi^i) : 1 \leq i \leq \max E\}$ ;

$\varphi \leftarrow NIS(\pi^k, \pi^l, C^*)$ ;  $(\pi^l, C^l) \leftarrow \mathbf{NeuroTS}(\varphi)$ ;

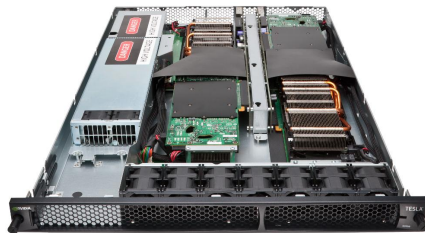
if  $C^l < C^k$  than set  $(\pi^*, C^*) \leftarrow (\pi^l, C^l)$  and  $k \leftarrow l$ ;

until  $\max\{D(\pi^k, \pi^i) : 1 \leq i \leq \max E\} < \max D$ .

## Computational experiments

Proposed algorithms were ran on the server based on 6-cores Intel Core i7 CPU X980 (3.33GHz) processor equipped with nVidia Tesla S2050 GPU (1792 cores).

- *sNTS* – sequential neuro-tabu Search algorithm,
- *pNTS* – parallel (for  $p = 16$ ) neuro-tabu Search algorithm,
- *iNTS* – advanced *NTS* algorithm based on the diversification and intensification methodology.



## Results of experiments

problem	$n \times m$	$sNTS$	$pNTS(p = 16)$	$iNTS$
TA01-10	$15 \times 15$	0.4948	0.3141	0.0652
TA11-20	$20 \times 15$	1.1691	0.9129	0.4412
TA21-30	$20 \times 20$	1.2486	0.7033	0.4803
TA31-40	$30 \times 15$	1.0592	0.7965	0.3764
TA41-50	$30 \times 20$	1.8565	1.5634	0.8328
TA51-60	$50 \times 15$	0.0915	0.0915	0.0520
TA61-70	$50 \times 20$	0.1479	0.0210	0.0140
TA71-80	$100 \times 20$	0.0090	0.0090	0.0090
<b>average</b>		<b>0.7596</b>	<b>0.5515</b>	<b>0.2839</b>

**Table:** Percentage relative deviations (PRD) to the best known solutions.

## Conclusions

- We propose an approach designed to solve difficult problems of combinatorial optimization in distributed parallel architectures without shared memory, such as clusters of nodes equipped with GPU units (i.e. multi-GPU clusters).
- The methodology can be especially effective for large instances of hard to solve optimization problems, such as flexible scheduling problems as well as discrete routing and assignment problems.