CS 2400 : Assignment 1

Group 7

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Introduction/Problem Description

In this assignment, we generate and sample two sinusoidal signals, and observe their properties in the frequency domain using the DFT. In this process, we study the properties and intricacies associated with the DFT.

Sinusoidal Signals are parameterized by amplitude A and angular frequency ω . The Fourier transform of a sinusoid consists of two impulses at $+\omega$ and $-\omega$. In our experiments, we sample two sinusoids and use the discrete time signal for further operations. The sampled signal has as envelope the original sinusoid.

Problem 1 : Experiment/Output/Discussion

Part A:

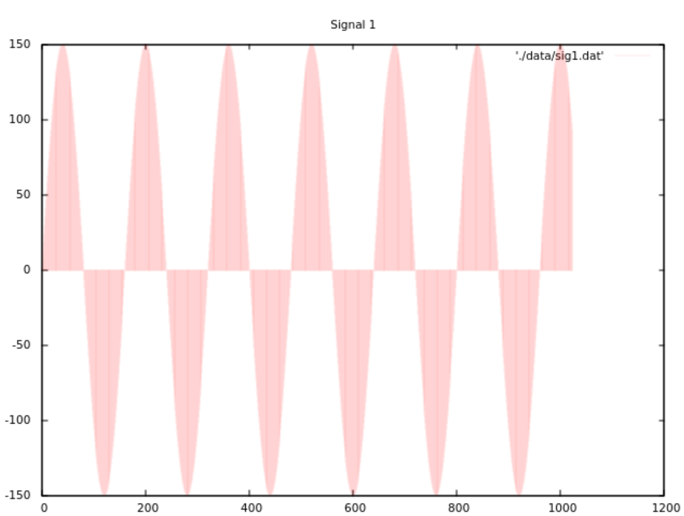
We generate 4 random number: two amplitudes A1, A2 and two angular frequencies $\omega 1$, $\omega 2$.

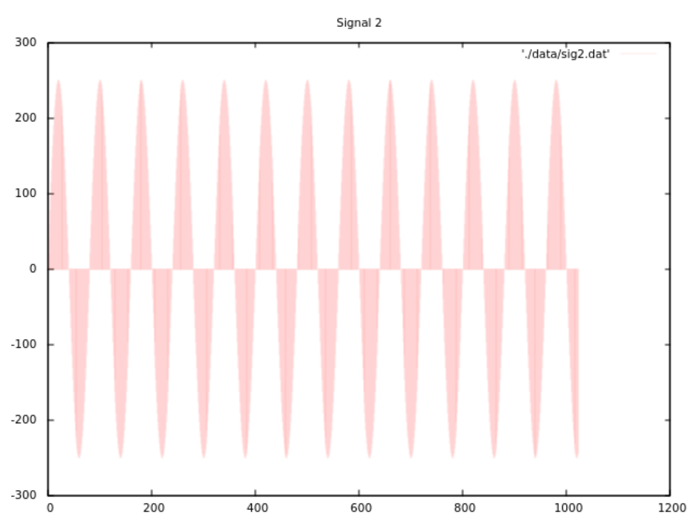
For the given set, we use A1 = 150, A2 = 250, $\omega 1 = 25$, $\omega 2 = 50$, and R = 2000 and 4000.

We pass a sampling rate R(in rad/s) to the sampling function along with the above values. The function computes $T = 2\pi \div R$ and finds the value of $y = Asin(\omega t)$ at $t = nT$ $where\ 0 \leq n \leq 1023$ and writes them to an output file.

We use a plotting function to plot this data. The plot contains 1024 impulses of magnitude $y = Asin(n\omega T)$ against sample number on the x-axis.

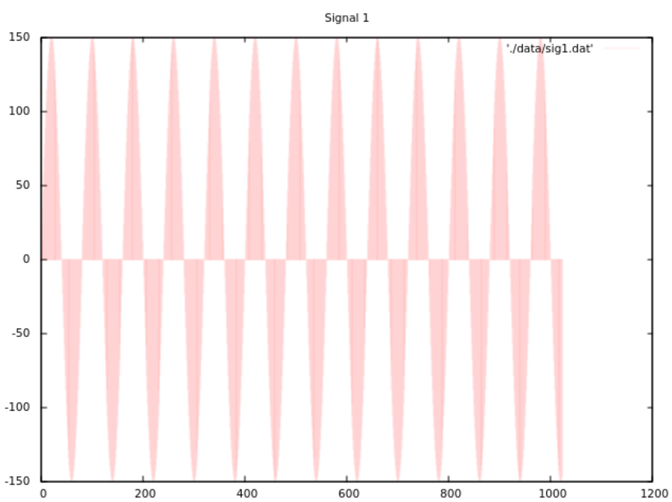
For R = 4000:

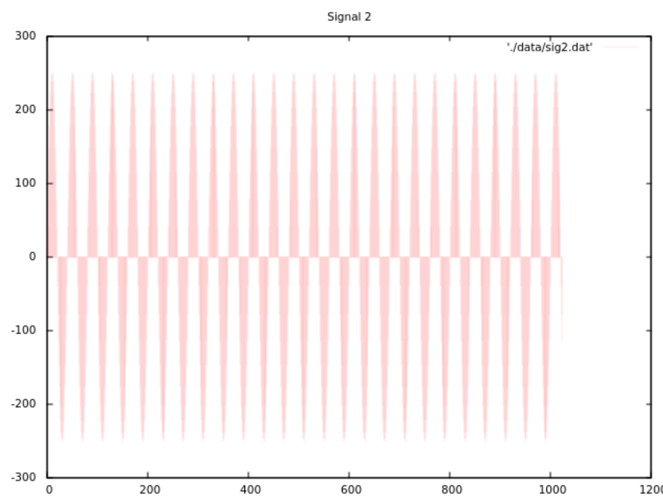




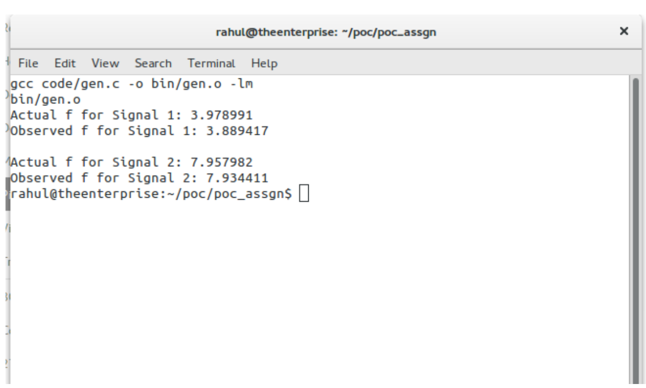
We also find the frequency of the above signals from their samples using zero crossings. If no of zero crossings is n, then frequency, $f = n \div (2 \times N \times T) \; = \; (n \times R) \div (2 \times 1023 \times 2\pi)$. As can be seen from the following screenshot, the observed frequency is fairly accurate.

For R = 2000:



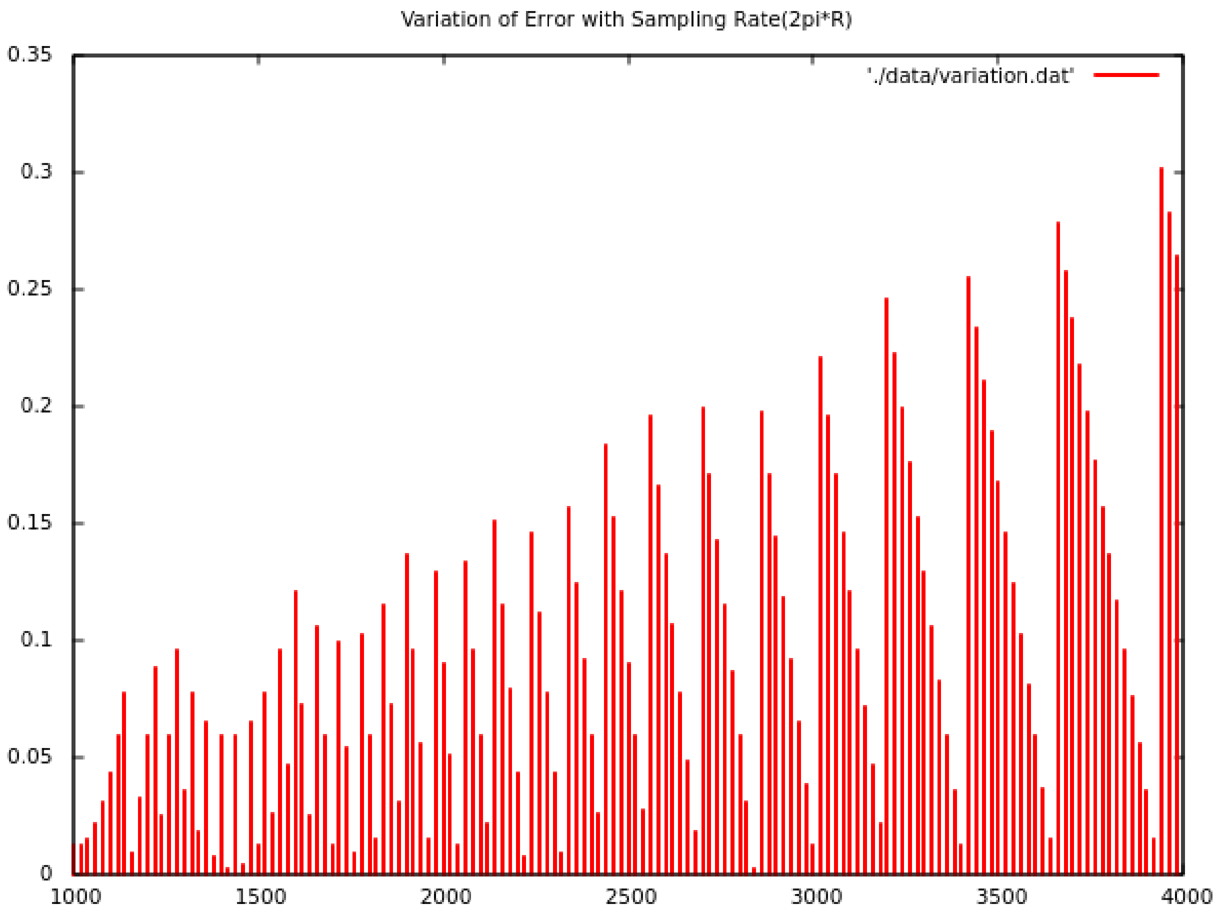


And computed frequency:



By increasing R, we get larger number of samples per cycle. Since, the number of samples is fixed, larger number of cycles are plotted for R = 2000 than 4000.

Then, we use a random value of A (=150) and ω (= 25) and vary the R value and plot the error in frequency estimation with respect to R(in rad/s).

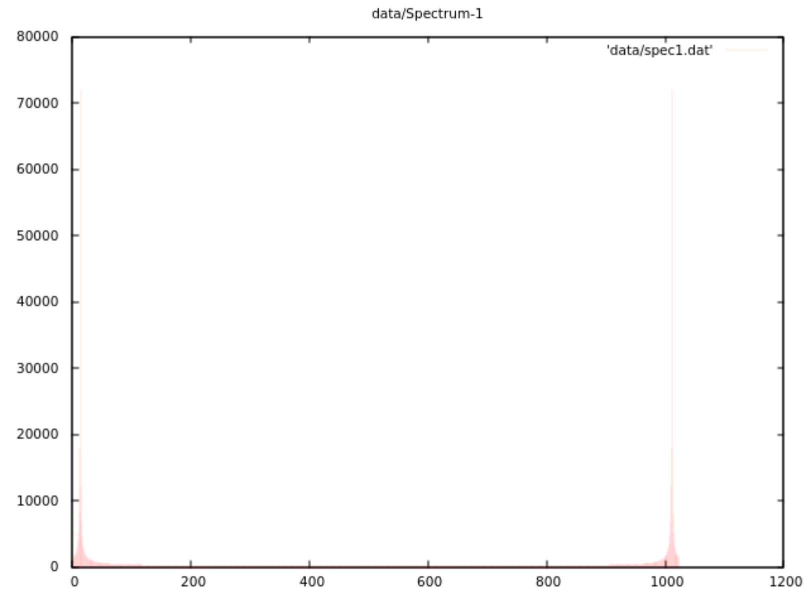


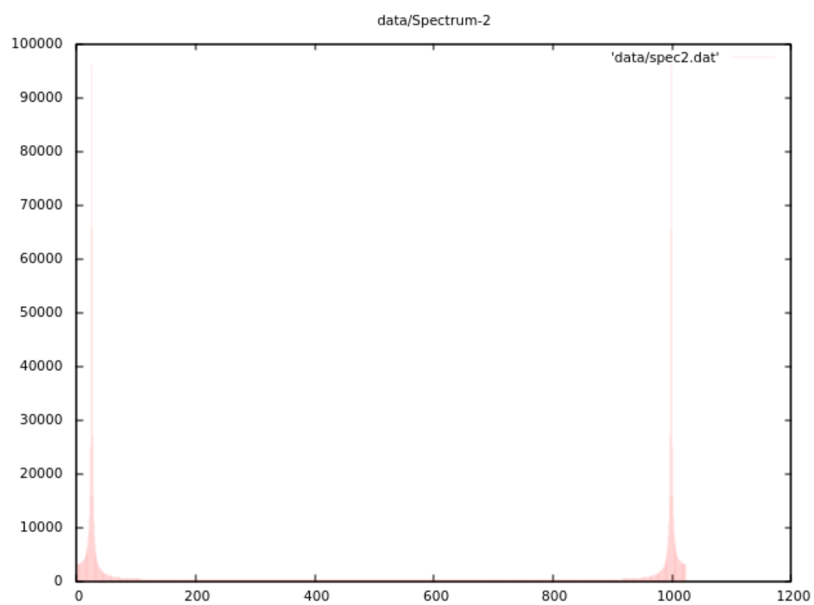
With increasing R, the maximum error has a general increasing trend.

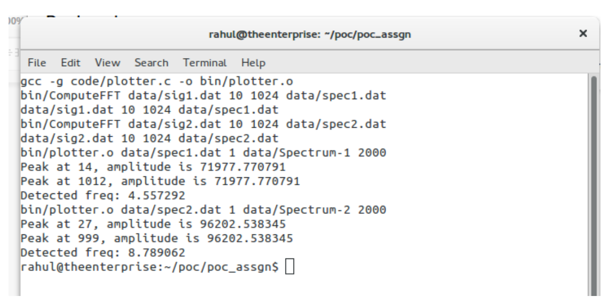
Part B:

Following are the DFT spectra of signal 1 and signal 2 along with the detected peaks and observed frequency.

For R = 2000, N = FFTSize = 1024:







rahul@theenterprise: ~/poc/poc_assgn

File Edit View Search Terminal Help

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gcc -g code/plotter.c -o bin/plotter.o
bin/ComputeFFT data/sig1.dat 10 1024 data/spec1.dat
data/sig1.dat 10 1024 data/spec1.dat
bin/ComputeFFT data/sig2.dat 10 1024 data/spec2.dat
data/sig2.dat 10 1024 data/spec2.dat
bin/plotter.o data/spec1.dat 1 data/Spectrum-1 2000
Peak at 14, amplitude is 71977.770791
Peak at 1012, amplitude is 71977.770791
Detected freq: 4.557292
bin/plotter.o data/spec2.dat 1 data/Spectrum-2 2000
Peak at 27, amplitude is 96202.538345
Peak at 999, amplitude is 96202.538345
Detected freq: 8.789062
rahul@theenterprise:~/poc/poc_assgn$
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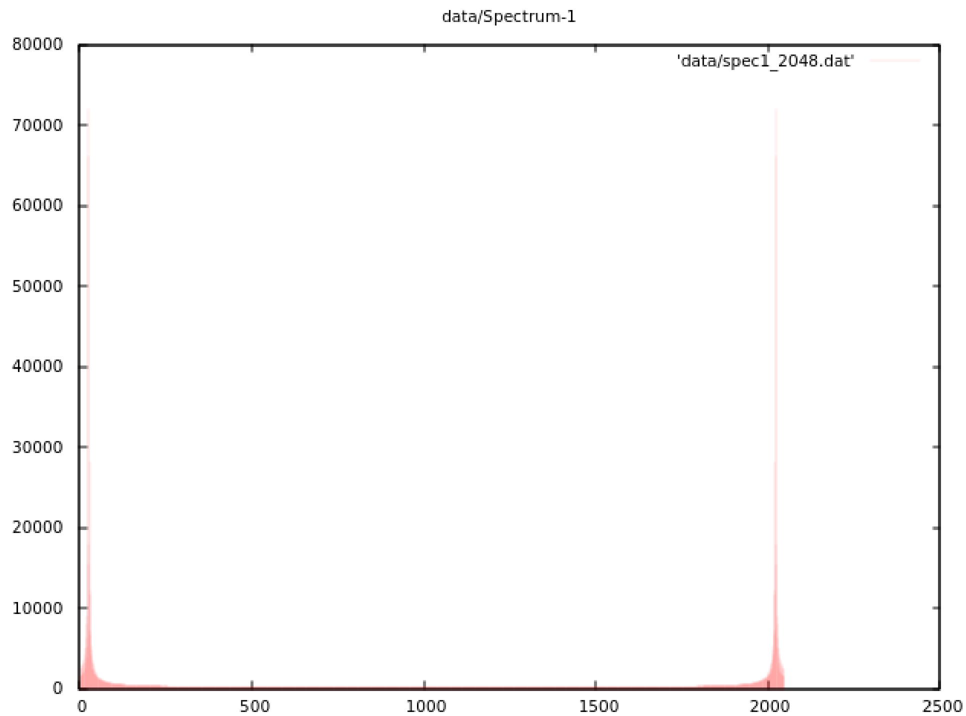
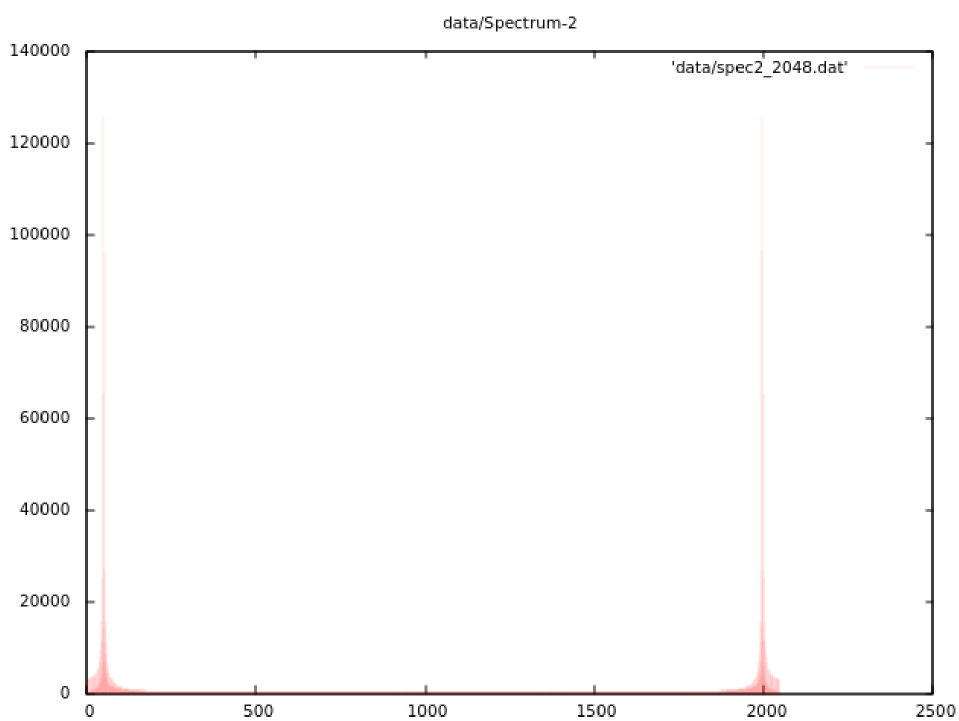
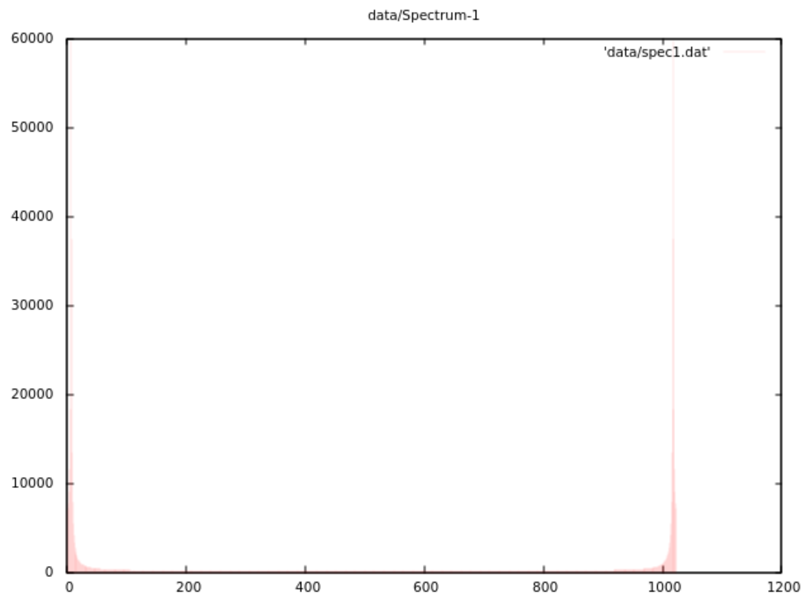
Following are the DFT spectra of signal 1 and signal 2 both padded with 1024 zeroes along with the detected peaks and observed frequency.

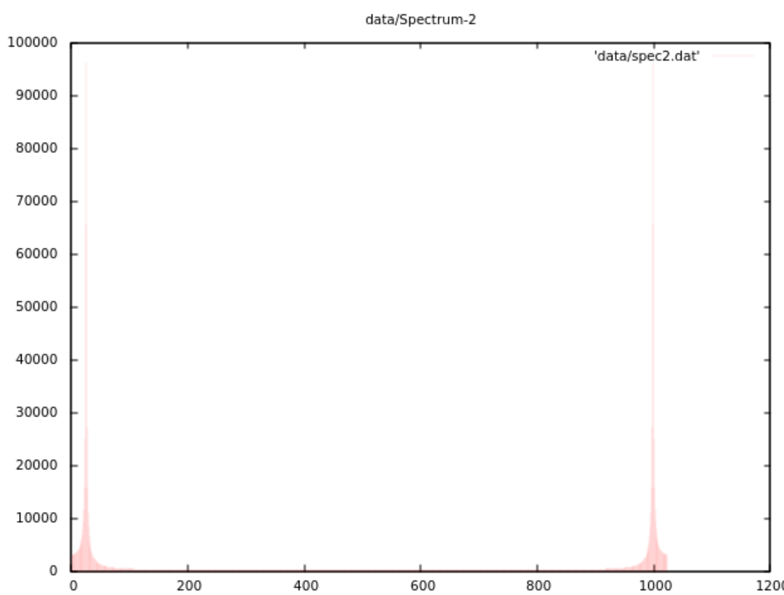
For R = 2000, N = FFTSize = 2048:

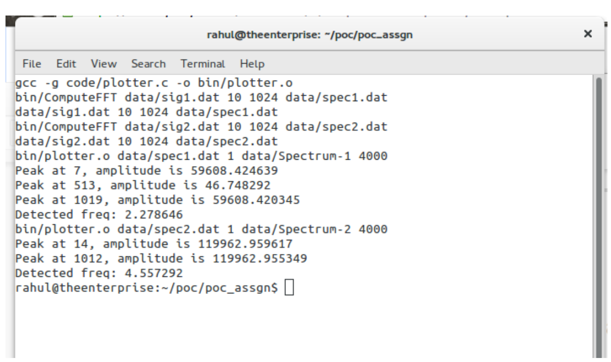




Following are the DFT spectra of signal 1 and signal 2 along with the detected peaks and observed frequency.
For R = 4000, N = FFTSize = 1024








```
gcc -g code/plotter.c -o bin/plotter.o
bin/ComputeFFT data/sig1.dat 10 1024 data/spec1.dat
data/sig1.dat 10 1024 data/spec1.dat
bin/ComputeFFT data/sig2.dat 10 1024 data/spec2.dat
data/sig2.dat 10 1024 data/spec2.dat
bin/plotter.o data/spec1.dat 1 data/Spectrum-1 4000
Peak at 7, amplitude is 59608.424639
Peak at 513, amplitude is 46.748292
Peak at 1019, amplitude is 59608.420345
Detected freq: 2.278646
bin/plotter.o data/spec2.dat 1 data/Spectrum-2 4000
Peak at 14, amplitude is 119962.959617
Peak at 1012, amplitude is 119962.955349
Detected freq: 4.557292
rahul@theenterprise:~/poc/poc_assgn$
```

We use the provided ComputeFFT binary to perform the discrete Fourier Transform and then, use a function to find all local maximas in the graph(shown in above screenshot). Then, we compute frequency of signal,f as

$$f = (i \times R) \div N,\ R\ in\ Hz\ .$$

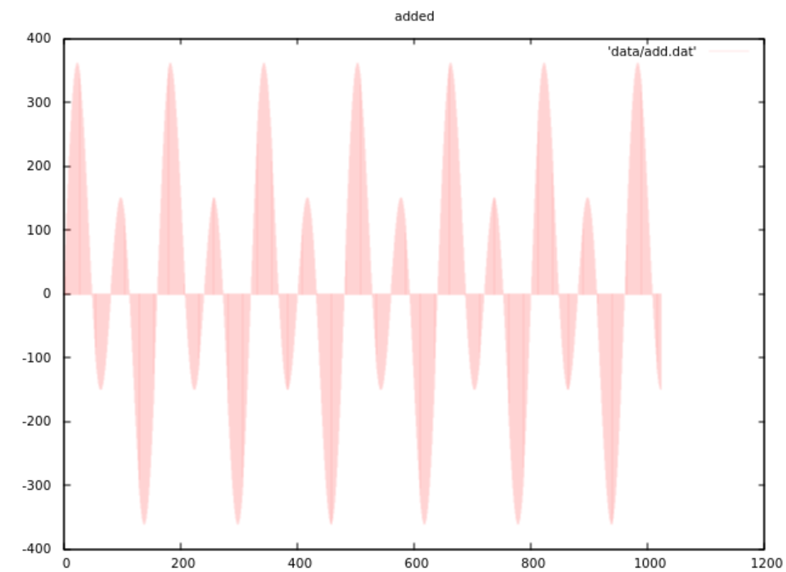
As can be seen from the spectrum, we get 2 main peaks in the DFT spectrum. We can extract the frequency of the sinusoid from this graph.

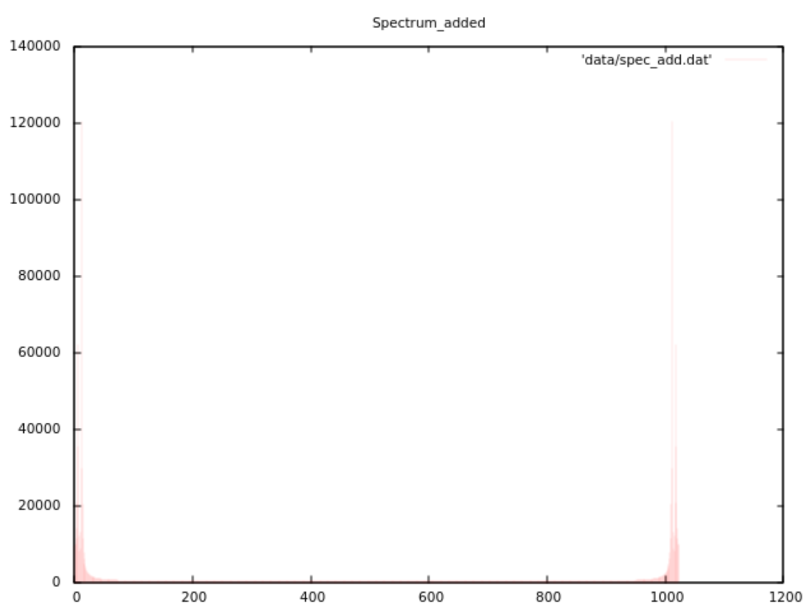
Problem 2 : Experiment/Output/Discussion

Part A:

We compute the added signal, z(n) from x(n) and y(n) as $z(m) = x(m) + y(m)$, where m is the sample number. Following images are the added signal and its DFT respectively.

For R =4000 and FFTSize = 1024,





Part B:

If f and g are the two signals of size n each, then their convolution is given by:

$$(f * g)[m] = \sum_{i=0}^{n-1} f[i] \times g[m-i]$$

Here, g has samples from 0 to $n-1$ only but the above formula requires g to have samples from $-(n-1)$ to $n-1$. Hence we using zero padding and create samples of g from $-(n-1)$ to -1 each of which is a zero.

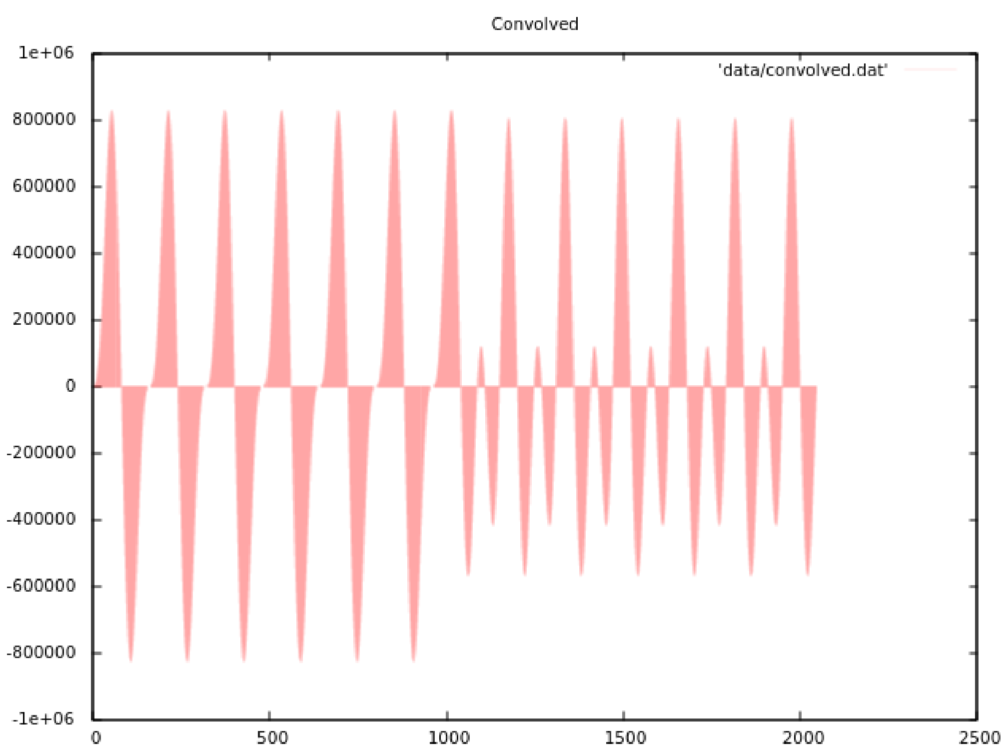
After zero padding g in the above manner, we can directly use the above formula to compute the convolution.

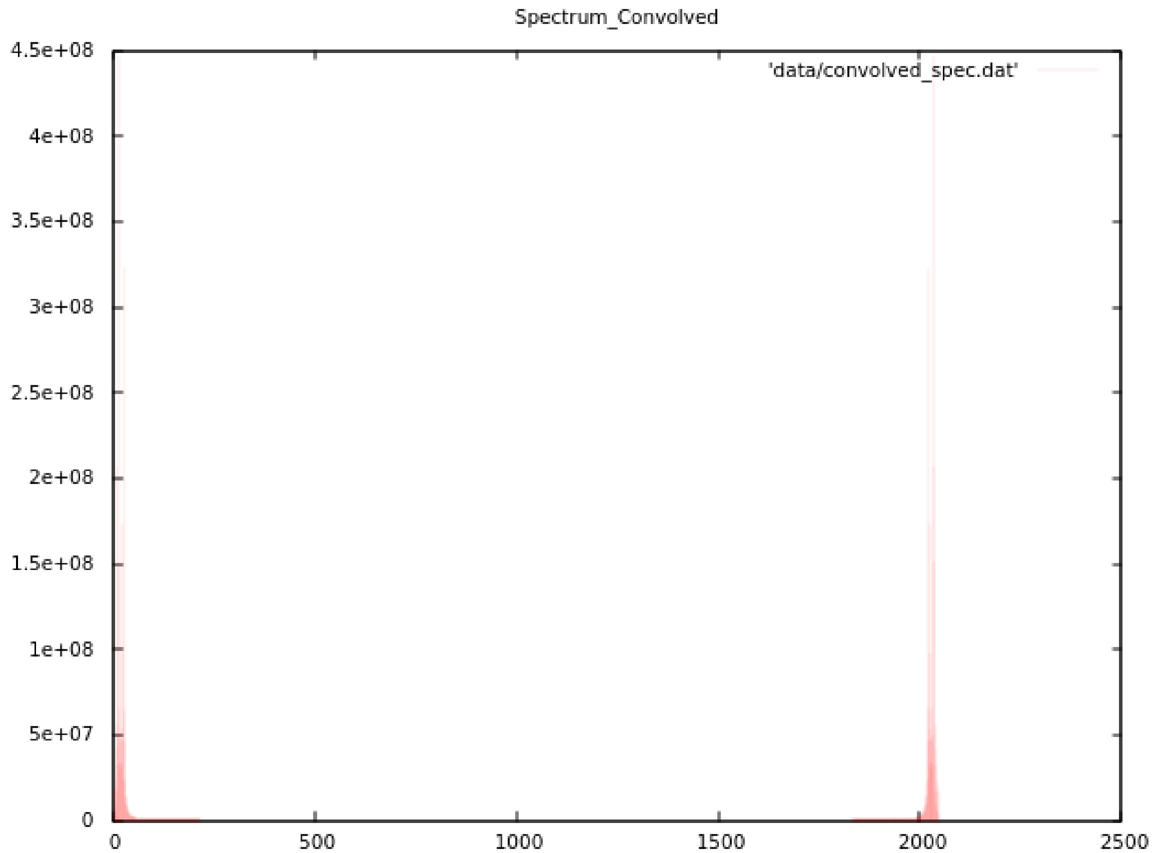
The size of the convolved signal is $2n-1$. We know that FFT requires the signal size to be in powers of 2. Therefore to calculate DFT of the convolved signal, we pad it with a 0 at the end to get a signal of size $2n$ which is a power of 2 (we know that n is a power of 2). Since n = 1024, size of convolved signal (along with a single zero padding) will be 2048 and the same will be FFTSize.

Once everything is done, we can use the ComputeFFT binary to generate the DFT.

The convolved signal (of the two signals in problem 1a) and its DFT are given below:

For R = 4000, FFTSize = 2048:





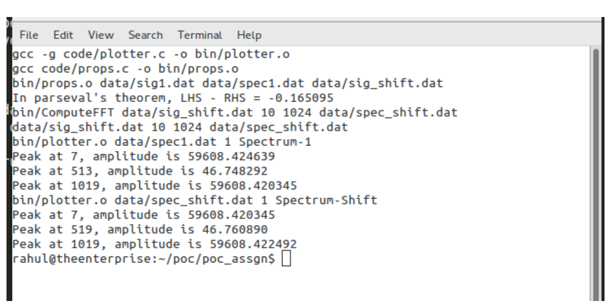
Part D:

We verify the following properties :

1) Parseval's theorem:

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \times \sum_{k=0}^{N-1} |X[k]|^2$$

We used the signal and it's DFT from problem 1.
As we can see from the image below, LHS - RHS = -0.165095 which is quite accurate, considering all the approximations.



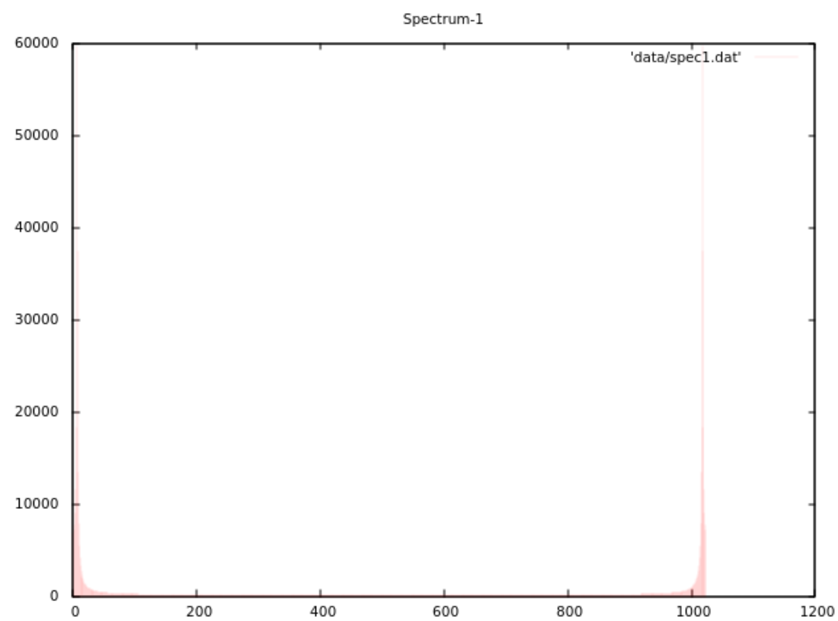
2) <u>Circular time shift:</u>

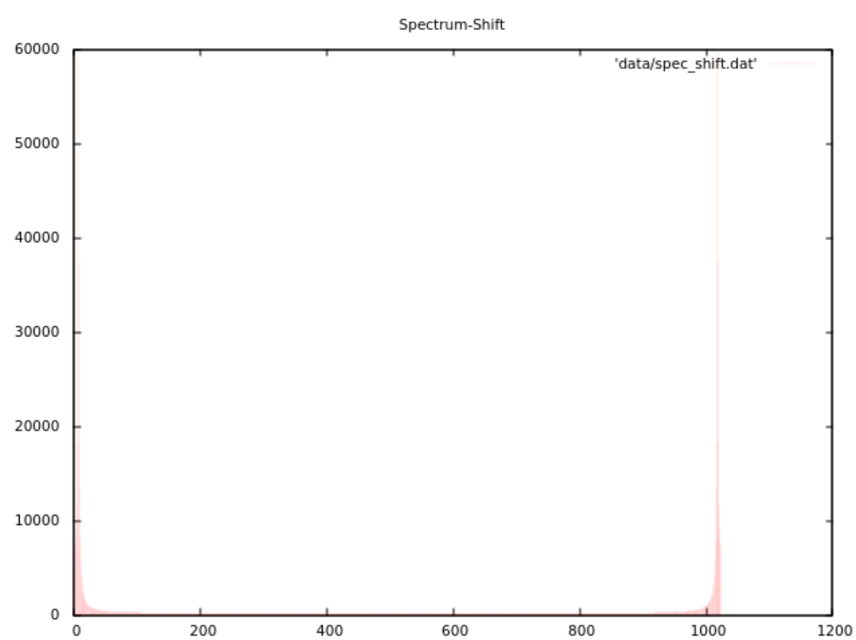
$$x[(n - n_0)_N] \leftrightarrow e^{-j2\pi k n_0/N} \times X[k]$$

Even when we shift the time domain by n_0, the magnitude spectrum of DFT will remain the same because $|e^{-j2\pi k n_0/N}| = 1$.

We use the signal 1 from problem 1 and shift by n_0 units where n_0 is chosen randomly.

Following are the DFT spectrums of both signal 1 and the shifted signal 1 are the same. As we can see, both are the same.





General observations/conclusions:

1. In the DFT spectrum, if there is a peak at i, there will be a peak of same size at $n - i$. This means that the DFT spectrum is symmetric about $y = N/2$.
2. Zero padding increases the resolution in frequency domain.
3. From problem 2a, we can conclude that principle of superposition holds. This can be extended to scaling as well.
4. We have also verified Parseval's theorem and the time shift property.