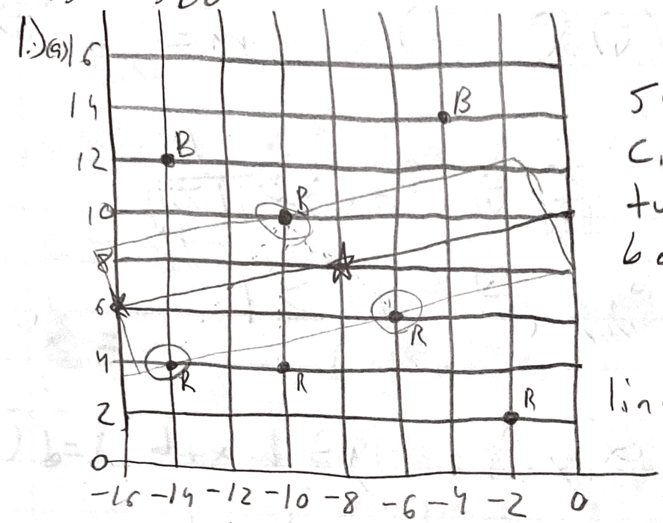
CS 4780

1.) (a)



Support vectors circled

two points on decision boundary:

$\{(-8, 8), (-16, 6)\}$

line: $y = \frac{1}{4}x + 10$ with margin $\frac{5}{2}$

$y = wx + b$ with c margin

$y - (wx + b) = c$ for blue

$y - (wx + b) = -c$ for red

$6 - (-16w + b) = -c$

$10 - (-10w + b) = c$

$4 - (-14w + b) = -c$

$6 + 6w - b = -c$

$10 + 10w - b = c$

$4 + 14w - b = -c$

$6 + 6w - b = -10 - 10w + b$

$16 + 16w = 2b$

$8 + 8w = b$

$10 + 10w - (8 + 8w) = c$

$2 + 2w = c$

$4 + 14w - (8 + 8w) = -(2 + 2w)$

$-4 + 6w = -2 - 2w$

$-2 = -8w$

$\boxed{\frac{1}{4} = w \qquad b = 10 \qquad c = \frac{5}{2}}$

1b) $\frac{5}{2} = -10w_1 + 10w_2 + b$

$-\frac{5}{2} = -14w_1 + 4w_2 + b$

$-\frac{5}{2} = -6w_1 + 6w_2 + b$

$-\frac{5}{2} + 6w_1 - 6w_2 = b$

$\cancel{-\frac{5}{2}} = -14w_1 + 4w_2 \cancel{-\frac{5}{2}} + 6w_1 - 6w_2$

$0 = -8w_1 - 2w_2$

$w_2 = -4w_1$

$\frac{5}{2} = -10w_1 + 10(-4w_1) - \frac{5}{2} + 6w_1 - 6(-4w_1)$

$5 = -20w_1$ $\qquad b = -\frac{5}{2} + 6(-\frac{1}{4}) - 6(1)$

$-\frac{1}{4} = w_1$ $\qquad w_2 = 1$ $\qquad b = -10$

$w = \langle -\frac{1}{4}, 1 \rangle$ $\qquad b = -10$

normalized $w = \frac{-\frac{1}{4}}{\sqrt{\frac{1}{4}^2 + 1^2}}, \frac{1}{\sqrt{\frac{1}{4}^2 + 1^2}}$

normalized $w = \langle -\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \rangle$

Geometric margin $= \frac{\text{margin}}{\|w\|} = \frac{\frac{5}{2}}{\sqrt{(\frac{1}{4})^2+1^2}}$

$= \frac{10}{\sqrt{17}}$ or 2.4254

All dual variables α_i other than those for $((-10,10),1)$, $((-14,4),-1)$, and $((-6,6),-1)$ are equal to zero.

		positive	negative
1(c)(i)	P: 10 features	max 10 1's	max 3 1's
	N: 10 features	max 3 1's	max 10 1's
	Q 99980 features	20 1's max	20 1's max
	max 1's for x_i	33 1's max	33 1's max

$\max_S R = \sqrt{33}$

1c(ii) Let x_i be form $\langle P, N, Q \rangle$

margin $= \frac{y_i}{\|\vec{w}\|}(\vec{w}\,\vec{x}_i + b)$

let $b = 0$ $\quad w = \langle 1,1,1,1,1,1,1,1,1,1,-1,-1,-1,-1,-1,-1,-1,-1,-1,-1,0,\ldots,0\rangle$

pos (at least 5 1's)(at most 3 1's), 0 0's ...

$\to$ for positive $\vec{x}_i$ $\vec{w}\cdot\vec{x}_i \geq 2$

thus $y_i = 1$ $\quad \|\vec{w}\| = \sqrt{20}$

thus min geometric is when $\vec{x}_i$ 5 1's in P, 3 1's in N,

then $b + \vec{w}\,\vec{x}_i = 2 + b = 2$

thus geometric margin $= \frac{2}{\sqrt{20}} = \frac{1}{\sqrt{5}}$

negative similarly, $y_i = -1$ x_i has 3 1's in P 5 1's in N

$b + \vec{w}\,\vec{x}_i = -2 + b = -2$ $\quad -2 \cdot y_i = 2$

geometric margin $= \frac{2}{\sqrt{20}} = \frac{1}{\sqrt{5}}$

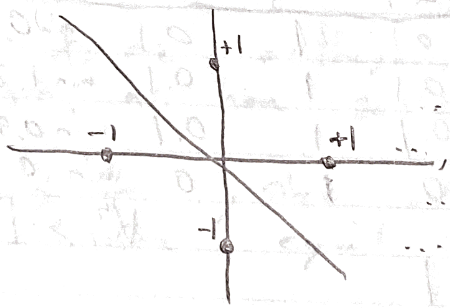
$\min_S \gamma = \frac{1}{\sqrt{5}}$

(c)(iii) $\frac{1}{m}\frac{R^2}{\gamma^2} = \frac{1}{100000}\cdot\frac{\sqrt{33}^2}{\left(\frac{1}{\sqrt{5}}\right)^2} = \frac{33}{\left(\frac{10000}{25}\right)}$

$= \frac{33}{4000}$

$\frac{33}{4000}$ is the upper bound on the RHS and thus the upper bound overall on the expected generalization error

2(a)



x_i	$\cos\frac{\pi}{2}x_i$	$\sin\frac{\pi}{2}x_i$
$\ldots,-4,0,4,\ldots$	1	0
$\ldots,-3,1,5,\ldots$	0	1
$\ldots,-2,2,6,\ldots$	-1	0
$\ldots,-1,3,7,\ldots$	0	-1

2(b) $K(x,x') = \phi(x)\cdot\phi(x')$

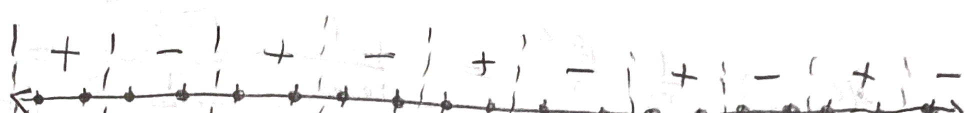
$= \cos\left(\frac{\pi}{2}x\right)\cdot\cos\left(\frac{\pi}{2}x'\right) + \sin\left(\frac{\pi}{2}x\right)\cdot\sin\left(\frac{\pi}{2}x'\right)$

$= \frac{\cancel{\cos\left(\frac{\pi}{2}x + \frac{\pi}{2}x'\right)} + \cos\left(\frac{\pi}{2}x - \frac{\pi}{2}x'\right)}{2} + \frac{\cos\left(\frac{\pi}{2}x - \frac{\pi}{2}x'\right) - \cancel{\cos\left(\frac{\pi}{2}x + \frac{\pi}{2}x'\right)}}{2}$

$= \cos\left(\frac{\pi}{2}x - \frac{\pi}{2}\cdot x'\right)$

$K(x,x') = \cos\left(\frac{\pi}{2}(x - x')\right)$

2(c)



Lie on margin: All points lie on margin.

2(d) $\phi(x) = \left(\cos\frac{\pi}{4}x, \ \sin\frac{\pi}{4}x\right)$

3(a):

i	y_i	$w^T x_i + b$	α_i	$\max_s R$	$2\alpha R^2$	ξ_i
1	1	0.4	0.1	1	0.1	min 0.6
2	−1	0.2	0.1	1	0.1	min 1.2
3	1	0.1	0.1	1	0.1	min 0.9
4	1	1	0	1	0	min 0

$\forall_{i=1}^{n}, y_i[\vec{w}\cdot\vec{x}_i + b] \geq 1 - \xi_i$

$2\alpha_i R^2 + \xi_i \geq 1$

$2(0.1)(1)^2 + 0.6 \geq 1$

$0.7 \geq 1$ false

$2(0.1)(1)^2 + 1.2 \geq 1$ true

$2(0.1)(1)^2 + 0.9 \geq 1$ true

$2(0)(1)^2 + 0 \geq 1$ false

Thus, these are our two guaranteed leave out errors

3(b) maximize $\sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} y_i y_j \alpha_i \alpha_j (\vec{x}_i \cdot \vec{x}_j)$

we know $\forall\, i \neq j$ $\quad x_i^T x_j = 0$

thus

we want to maximize $\sum_{i=1}^{n} \alpha_i \qquad \sum_{i=1}^{n} y_i \alpha_i = 0$

all x_i are orthogonal, thus lie on margin, all have same α_i

$0 \leq \alpha_i \leq C$, since all $\vec{x}_i$ are same length, all will have equal influence

thus $\frac{1}{n} = \alpha_i$ for all $i \in 1 \ldots n$