8 Feb 2019

ANNOUNCEMENT: Homework #3 will be released Monday night,
due Tuesday morning 2/19 at 11:59 am
max of one slip day.
Stay tuned for announcements about shifting office hrs.

DYNAMIC PROGRAMMING

The Weighted Interval Scheduling Problem (§6.1 in the book)

An input instance is a list of intervals numbered by $i = 1, 2, \ldots, n$ such that

- each interval has
  - start time s_i
  - finish time $f_i > s_i$
  - weight $w_i > 0$

- assume intervals ordered by increasing finish time:
  $f_1 \leq f_2 \leq \cdots \leq f_n$.

  (Can always satisfy this with $O(n \log n)$ preprocessing.)

Goal. Find a non-conflicting subset of intervals with maximum total weight.

Greedy strategies?

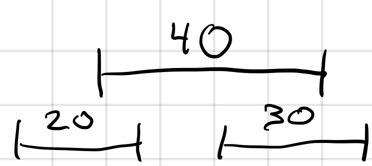
[1] Earliest finish time?

$f_1 \to f_n$

$w_1 = w_2 = \cdots = w_n = 1$

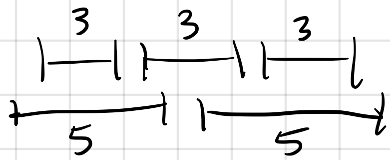
$w_{n+1} = n+1$.

[2] Greatest weight first?



[3] Greedy followed by local search:

1. run the "earliest finish time" algorithm.
2. while $\exists$ a "exchange one interval for two that conflict with it" move that improves total weight, make this move.



(An example where this doesn't work.)

<u>Def.</u> A <u>valid solution</u> of a weighted interval scheduling problem instance is a set of non-conflicting intervals.

<u>LEMMA.</u> For any valid solution that is non-empty, the set consists of an interval i and a valid subset of $\{$the intervals that finish before $s_i\}$.

<u>Proof.</u> If S is valid and non-empty, let i = highest numbered interval in S. Then $S \setminus \{i\}$ is valid and every interval j in $S \setminus \{i\}$ finishes before s_i because

(i) j can't finish after f_i (i is highest-numbered in S)

(ii) j can't finish in the time interval $[s_i, f_i]$ (no two intervals in S conflict).

<u>Def.</u> Let $p(i)$ denote the "predecessor of interval i", i.e. the highest-number interval j such that $f_j < s_i$.
Note that $\{$intervals that finish before $s_i\} = \{1, 2, \ldots, p(i)\}$.
If $\nexists$ an interval j that finishes before s_i, let $p(i) = 0$.

<u>COROLLARY.</u> Among all the valid solutions whose highest-numbered interval is i, the one that maximizes combined weight is $\{i\} \cup \{$max-weight valid subset of $1, \ldots, p(i)\}$.
When $p(i) = 0$ the set $1, \ldots, p(i)$ refers to $\emptyset$.