15th April, 2019

Universal SJava Program /

An interpreter for SJava.

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* An SJava program

UniversalSJava (string M , string x)

* it executes the SJava program
M on input x , and
returns the output (of M on x).

→ Refer to handout for some
details on the skeleton code.

Convention: If M is not a valid
SJava program, output of

UniversalSJava on M, x is
'False'.

Uncomputability

* Will deal with decision problems.

If M is an SJava program for a decision problem. D, then on input n :

    (i)   M 'accepts' n :   M halts and outputs 'True' on input n.

    (ii)   M 'rejects' u :   M halts, outputs 'False' on input n.

    (iii)   M does not halt on u.

Def^m: (Language of a program M)

$$L_M = \{ n : M \text{ accepts } n \}.$$

Convention: If M is not a valid SJava program or does not return Boolean outputs, then
$$L = \phi.$$

$$M$$

Def^n: M 'recognizes' a language L

if $L_M = L$.

Def^n: A language L is recognizable.

if $\exists$ an S Java program M

s.t M recognizes L.

Question: Suppose M recognizes L.

Behavior of M on x, where $x \notin L$.

* M either rejects x or
M does not halt on x.

Def^n: M 'decides' a language L
if $L_M = L$, and M

rejects all $x \notin L$.

Def^n: A language L is decidable

if $\exists$ program M that decides L?

Example: $IS = \{(G, k) : G$ has
                       an IS of
                       size at least
                       $k\}$.

Is   IS   decidable?

Yes.

Pf: Boolean   IS (int a[ ][ ],
                          int k)
   {

   // enumerate   over all   subsets
   // of size   k   of vertices   and
   // check   if   there is an IS.
   // return Yes   if so
   // else   return No.

   }

Running time : $O(n^k)$

Lemma: If $L \in NP$, then L is decidable.

Hint: $L \in NP \Rightarrow \exists$ verifier V
and constant C s.t

$x \in L$ iff $\exists y, \ |y| \leq |x|^C ,$

$$V(x, y) = 1 .$$

$\square$,