CS 4820, Spring 2019 **Homework 1, Problem 1**

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(1) *(5 points)* For each positive integer n, let t_n denote the number of distinct ways to cover a rectangular $2 \times n$ grid with non-overlapping dominoes. What is the value of t_n? Prove the correctness of your answer using mathematical induction.

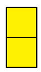


Figure 1: $t_1 = 1$

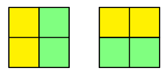


Figure 2: $t_2 = 2$

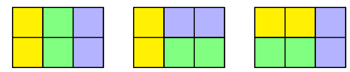
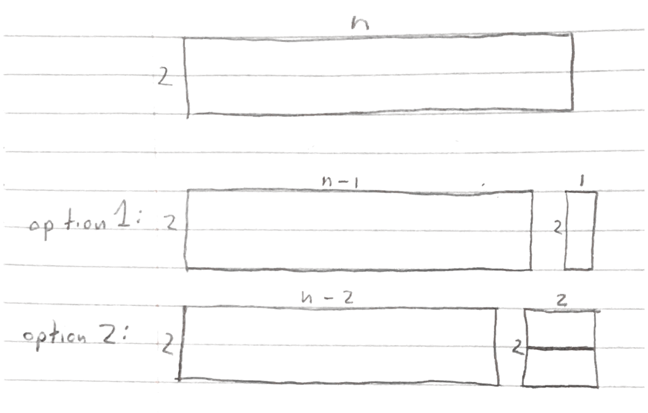


Figure 3: $t_3 = 3$

Solution
Claim:
$t_n = t_{n-1} + t_{n-2}$
Proof:
This proof will be done by strong induction on n. Let P(n)="$t_n = t_{n-1} + t_{n-2}$".
Base Case: The values of t_1 and t_2 must be calculated as t_0 does not exist. This is done simply by inspection and it can be seen that $t_1 = 1$ and $t_2 = 2$. Thus, the first value for which the claim can be made is t_3. We want to show that P(3) as our base case thus that $t_3 = t_1 + t_2$. As stated above $t_1 = 1$ and $t_2 = 2$ and by inspection there are 3 ways to arrange dominoes to fill a 2x3 rectangle thus as 3=2+1 the claim holds in the base case.
Inductive Step: We want to show that P(n) holds given P(n-1)...P(1) thus that $t_n = t_{n-1} + t_{n-2}$. We have a 2xn rectangle to be filled and this is equivalent to a 2x(n-1) rectangle with any additional domino in the upright position added on the end shown labelled option 1 in the image below. The only other possibility that this expansion over by one makes possible is that the 2xn rectangle is equivalent to a 2x(n-2) rectangle with two stacked horizontal dominoes occupying an attached 2x2 rectangle shown labelled option 2 in the image below.



We know that there are t_{n-1} distinct arrangements of dominoes for a 2x(n-1) rectangle and we also know that there are t_{n-2} distinct domino arrangements for a 2x(n-2) rectangle. Together these two cases represent all possible distinct domino arrangements for a 2xn rectangle and thus put together they total t_n thus it holds that $t_n = t_{n-1} + t_{n-2}$.