Introduction to Algorithms	**Problem Set 9**
CS 4820, Spring 2019	**Due 11:59pm Thursday, April 25**

Hand in your solutions electronically using CMS. Each solution should be submitted as a separate file. Collaboration is encouraged while solving the problems, but:

1. **list the names of those with whom you collaborated;**
2. **you must write up the solutions in your own words;**
3. **you must write your own code.**

On this assignment, unlike other assignments in CS 4820, we are not concerned with running times of algorithms. In particular, algorithms that you design on this problem set are not required to run in polynomial time.

(1) *(10 points)* Let $\mathcal{L} \subseteq \{0,1\}^*$ be a language, and let $\overline{\mathcal{L}}$ be its complement, i.e. the set of all finite binary strings that don't belong to $\mathcal{L}$. Prove that $\mathcal{L}$ is decidable if and only if both $\mathcal{L}$ and $\overline{\mathcal{L}}$ are recognizable.

(2) *(10 points)* As you probably know, buffer overflows are a common security flaw in computer programs that can be exploited by attackers to overwrite areas of memory that should have been protected from access. This exercise explores why it's so difficult to perform static analysis of programs to discover potential buffer overflows. In each of the sub-questions below, we use the term *buffer program* to refer to an SJava program whose base function contains a variable named "buffer" of string type. The *length of the buffer* refers to the number of characters in this string.

For each of the folloiwng decision problems, determine whether the problem is decidable, recognizable but not decidable, or not recognizable. Substantiate your answer with a proof. If one part of your solution uses material from an earlier part, it is fine to refer back to that earlier part of your solution rather than repeating the material.

(2a) *(5 points)* $\mathcal{L}_{\mathrm{A}}$ is the set of all ordered triples $\langle M, x, k \rangle$ such that M is a buffer program, and when M runs on input x the length of the buffer exceeds k at least once during its execution.

(2b) *(5 points)* $\mathcal{L}_{\mathrm{B}}$ is the set of all ordered pairs $\langle M, x \rangle$ such that M is a buffer program, and when M runs on input x the length of the buffer grows unboundedly large. (Meaning: M runs forever on input x, and for all k there is a time during its execution when the length of the buffer exceeds k.)

(3) *(10 points)* Recall that the execution state of an SJava program is defined to be a stack of function states. Further, recall that this is encoded as a string called `execState` by the universal SJava program `interpreter`. (See Section 5 of the Computability handout for more details.)

Define $\mathcal{L}_{\mathrm{SPACE}}$ to be the set of all ordered triples $\langle M, x, C \rangle$ such that in the execution of `interpreter`(M,x), the string `execState` never exceeds length C. Either prove that there exists a SJava program that decides $\mathcal{L}_{\mathrm{SPACE}}$, or prove that $\mathcal{L}_{\mathrm{SPACE}}$ is undecidable.