CS 4820, Spring 2019	**Homework 8, Problem 3**
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(3) *(10 points)* The transactions in a blockchain ledger can be modeled as a directed acyclic graph $G = (V, E)$ whose vertex set is partitioned into subsets $V_1, V_2, \ldots, V_p$, where V_i represents the set of transactions pertaining to user i, and an edge (u, v) can be interpreted as meaning that transaction u is a predecessor of transaction v. The graph G and its partition $V_1, \ldots, V_p$ are assumed to satisfy the following property:

($*$) *For $i = 1, \ldots, p$, V_i contains a node r_i that has no incoming edges in G. For every other $v \in V_i$ there is at least one $u \in V_i$ such that $(u, v) \in E$.*

A set of transactions, S, is called *compatible* if it satisfies the following two properties.

1. For all $(u, v) \in E$, if $v \in S$ then $u \in S$.
2. For all $i = 1, 2, \ldots, p$, if V_i contains three distinct nodes u, v, w such that u has edges to both v and w in G, then v and w cannot both belong to S.

The first constraint can be interpreted as stating that a transaction cannot be accepted unless all of its predecessors are accepted. The second constraint prevents each user i from "double-spending."

Consider the decision problem COMPAT defined as follows. An input instance consists of a directed acyclic graph $G = (V, E)$, a partition of V into subsets $V_1, \ldots, V_p$ satisfying property ($*$), and a positive integer $k \leq |V|$. It is a 'Yes' instance of COMPAT if and only if there exists a compatible set of at least k transactions. Prove that COMPAT is NP-complete.

Not sure.