

20 Feb 2018

Prelim 1 review

Thurs, 2/21, Statler Auditorium (Not Statler 196)

7:30 - 9:00.

closed book, closed notes.

bring a pen/pencil (+ spare)

Topics. Chapters 1, 4, 6. (Not RNA, not recurrences)  
Boruvka & Floyd-Warshall in lectures, not book.  
 $\therefore$  not on prelim!

Algorithms we taught

Stable Matching

Greedy

Gale - Shapley

Earliest Finish Time (int'l sched)

Boruvka  
Kruskal } MST  
Prim

Huffman (prefix coding)

Dynamic Programming

Weighted interval scheduling  
Knapsack

Bellman-Ford

Floyd-Warshall

RNA secondary structure

(\* read about CYK some time!)  
(but not for Prelim 1...)

Types of questions.

- Straight "recall facts from your memory"

E.g. Running time of Bellman-Ford on  
a graph with  $n$  vertices,  $m$  edges is:

(A)  $O(m+n)$  (C)  $O(mn)$

(B)  $O(\ln \log n)$  (D)  $O(n^3)$

- Run a "named algorithm" on a sample input.
- We present an incorrect algorithm. You find a counterexample.
- We present a correct algorithm. You figure out its running time.
- We give you a simple fact that we want you to prove or give counterexample.

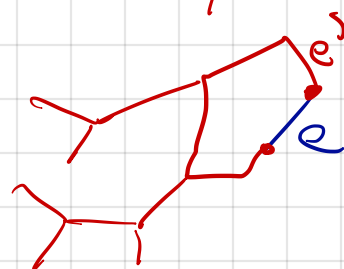
Ex 4. Prove or disprove. If  $G$  is a <sup>connected</sup> graph with distinct edge costs, the minimum cost edge belongs to every M.S.T. <sup>with at least one edge.</sup>

(A) Prove (B) Disprove

Proof 1. If  $e = (u, v)$  is the min cost edge, then  $e$  is the min cost edge crossing the cut from  $\{v\}$  to  $V - \{v\}$ .  
By Cut Property,  $e \in \text{MST}$ .

Proof 2. By contradiction. Assume  $e \notin T$ .  
Let  $e'$  be any other edge in the cycle that  $e$  forms with  $T$ .

Then  $T - \{e'\} \cup \{e\}$  is a spanning tree and its cost is less than that of  $T$ .



Proof 3. Kruskal's Algorithm works. It always selects the min-cost edge. Hence that edge belongs to one MST. That is the only MST because  $G$  has distinct costs.

Uniqueness? When  $G$  has distinct edge costs, the MST is always unique.

- Design & analyze an algorithm.

Use if you want.  
↓  
Prelim won't require it.

(Greedy)

(Dy n Prog)

(Reduction)

Bellman-Ford is a  
great target for  
reductions.

(Problem 6.7 from K&T)

String of words:  $w_1, w_2, \dots, w_n$   
character counts:  $c_1, c_2, \dots, c_n$

Print text in a format with  $\leq c_{\max}$  characters per line.  
(with 1 space btw adj words on a line).

Avoid "ragged right edge".

Line of text with  $c_{\text{tot}}$  characters is penalized  
by  $(c_{\max} - c_{\text{tot}})^2$ .

Minimize sum of line penalties.

Keep words in same order, just insert line breaks.

Greedy: keep inserting line breaks at latest  
possible time until you run out of  
words.

ALGORITHMS ARE COOL.

$$c_1 = 10$$

$$c_2 = 3$$

$$c_3 = 4$$

$$c_{\max} = 14.$$

ALGORITHMS ARE//  
COOL

$$\begin{array}{r} 0^2 \\ + 10^2 \\ \hline 100 \end{array}$$

ALGORITHMS //  
ARE COOL

$$\begin{array}{r} 4^2 \\ + 6^2 \\ \hline 52 \end{array}$$

