CS 4820, Spring 2019	**Homework 8, Problem 2**
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(2) *(10 points)* Consider the problem of NDPATH defined in the following way: An input instance is of the form (G, S, T), where G is an undirected graph on n vertices, $S = \{s_1, \ldots, s_k\}, T = \{t_1, \ldots, t_k\}$ are disjoint subsets of nodes of equal cardinality k (for any integer $k \in \{1, \ldots, \lfloor n/2 \rfloor\}$). It is an 'Yes' instance of NDPATH if there are node disjoint paths from s_i to t_i, for all $i = 1, \ldots, k$, and is a 'NO' instance otherwise. Prove that NDPATH is NP-complete.

Hint: Reduce from 3SAT. For each clause and each variable, add a pair of s_i's and t_i's. For each clause, introduce a few intermediate nodes, corresponding to the variables appearing in the clause.

To prove NDPATH is Np-complete, we will reduce 3SAT to an instance of NDPATH. Suppose we have an instance of 3SAT with m clauses and n variables. for each clause c we can label the 3 variables in this clause x_1c, x_2c, and x_3c. We can create new nodes for each v in 1...n and c in 1...m, x_{vcF} and x_{vcT}. Thus, we will have a true and a false node for each original variable in each clause. In additions we will create a begin and end node for each clause thus for each c in 1...m we create nodes b_c and e_c. Similarly, for each v in 1...n we create a begin and end node b_v and e_v. If in clause c x_ij is negated then we will connect b_c to x_{vcF} to e_c. Similarly, if it is not negated, then we will connect b_c to x_{vcT} to e_c.

For each variable x_v, we will connect b_v to to the lowest numbered clause where the variable x_v appears. b_v will be connected to x_{vcT} and then x_{vcT} is connected to the next occurrence of variable x_v and so on until there are no other occurrences and the last instance is connected to e_v. The same is done for each x_{vcF} for each time variable x_v. This creates two paths one in which the variable is evaluated to true and the other where it is evaluated to false.

3SAT is satisfiable, then each variable can be assigned to either true or false. If false is to be assigned for x_v 3SAT to be satisfied, then the path for x_v from b_v to e_v should select the path that goes through all the x_{vcT} nodes leaving only the false nodes available for clause paths. Similarly, if true is to be assigned x_v for 3SAT to be satisfied, then the path for x_v from b_v to e_v should select the path that goes through all the x_{vcF} nodes leaving only the true nodes available for clause paths. This is done for each variable. Then there exists a path from each clauses begining to its end such that each path is disjoint and paths never cross and thus the set of clauses beginning nodes and the set of end nodes make up the disjoint node sets on this undirected graph and thus the reduction from 3SAT to NDPATH is complete and thus, NDPATH is NP-complete.