Networks hw 5

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1. A research study says that buyers on eBay are willing to pay a premium for buying from a seller with 2000 positive ratings and 1 negative rating, compared to a seller with 10 positive and no negative ratings. Let's try to understand why the total number of ratings being larger, even with a larger number of negative ratings, might make such a difference—in other words, you can think of this problem as a step towards quantifying the value of information. Suppose a buyer has no idea whether a seller is good (G) or bad (B), and models this by assuming that there is a 50% chance the seller is bad. Suppose a bad seller cheats buyers 30% of the time. A good seller never cheats buyers, but buyers might still have a bad experience with the seller 1% of the time due to circumstances outside the seller's control—the package really was lost in the post, for example. (We'll assume that no such additional causes occur with the bad seller, so that you have a bad experience with the bad seller only if he is cheating you.)

1. (a) (5pts.) Suppose there are 10 ratings for a seller, and they are all pos- itive. What is the probability that the seller being rated is actually bad?

50% bad sellers 50% good sellers

P(10 positive ratings|good seller)=(.99)^10=0.904

P(10 positive ratings| bad seller)=(.7)^10=0.0282

P(bad seller| 10 positive ratings)=bayes theorem= P(10 positive ratings| bad seller)P(bad seller)/P(10 positive ratings)

=(0.0282)*(0.5)/((0.904*0.5)+(0.0282*0.5))=0.0282/0.9322=0.03

Thus, there is a 3% chance that given 10 positive ratings, the seller is bad.

(b) (5pts.) Now suppose there are 1000 positive ratings and 10 nega- tive ratings. Now what is the probability that the seller being rated is actually bad? What if there are 1000 positive and 50 negative ratings?

P(these ratings|good seller)= ${}_{1010}C_{10}$(.99)^1000(.01)^10= 2.91*10^23(1*10^-20)(4.317*10^-5)=12.56*10^-2=0.1256

P(these ratings|bad seller)= ${}_{1010}C_{10}$(.70)^1000(.30)^10= 2.91*10^23(1*10^-20)(4.317*10^-5)=2.1535*10^-137

P(bad seller| 1000 positive ratings 10 negative)=bayes theorem= P(10 positive ratings| bad seller)P(bad seller)/P(10 positive ratings)= =(2.1535*10^-137)*(0.5)/((0.1256*0.5)+(2.1535*10^-137*0.5))=1.7145*10^-136

P(bad seller| 1000 positive ratings 50 negative)=(.7)^1000*(.3)^50/((.7)^1000*(.3)^50+(.99)^1000*(0.01)^50)=2.084*10^-77

(c) (15pts.) Now consider these two sellers, call them sellers A and B, where A has 10 positive and no negative ratings, and B has 1000 pos- itive and 10 negative ratings. Suppose A and B are selling identical items that are worth v to the buyer. The buyer derives this value v if she has a good experience with the seller, and derives value 0 otherwise. How much more would the buyer be willing to pay to buy from seller B than A? (Continue with the assumption that the buyer believes sellers are equally likely to be good or bad.)

P(bad seller| 1000 positive ratings 10 negative)= 1.7145*10^-136

P(bad seller| 10 positive ratings)= 0.03

$V_{1000 \text{ and } 10}$=(1-1.7145*10^-136)*v+(1.7145*10^-136)*0=v

V_{10}=(1-0.03)*v+(0.03)*v=0.97v

$V_{1000 \text{ and } 10}$- V_{10}=0.03v

Thus, the buyer should be willing to pay 3% of the v more when buying from B vs buying from A.

2. (When does the market fail?) We said in class that whether information asymmetry leads to market failure depends on the distribution of qualities in the seller population, and the buyer and seller valuations for items— informally, markets with a greater proportion of low-quality items, and a smaller gap between buyer and seller values, are more likely candidates for market failure. In this problem, we're going to explore how market failure depends on these parameters, for the same toy market that we've been using with just two types of items (which we'll continue calling cars), and address the following question: How does the existence of an efficient self-fulfilling expectations equilibrium (SFEE) (i.e., an equilibrium with h = g) depend on the buyer and seller values, and the fraction of good cars in the population)?

Specifically, suppose there are good cars and bad cars on the market; and a fraction g of used cars are good; good cars are valued at s_H by sellers and b_H by buyers, and bad cars are valued at s_L by sellers and b_L by buyers. (As always, there is information asymmetry in the market, where sellers know the quality of the cars they sell, whereas buyers do not; also, we'll continue to assume that there are more buyers than sellers.)

(a) First, let's warm up with a concrete example: suppose s_H = 10, b_H =15, s_L =3and b_L =6. Forthesevalues:

i. (5pts.) Is h = g a self-fulfilling expectations equilibrium (SFEE) if g = 0.8?

v_b=(0.8)*15+(0.2)*6= 13.2 Thus, as 13.2> s_H>s_L all buyers and sellers participate and this is an SFEE.

ii. (5pts.) What if g = 0.2?

v_b=(0.2)*15+(0.8)*6= 7.8, which is not greater than s_H and thus no good seller sell so h-> 0 which does not equal g which is 0.2 so we do not have an SFEE.

(b) Now let's use abstract values s_H , s_L , b_H and b_L for the sellers' and buyers' valuations of good and bad cars as defined above, and under- stand how the non-existence of an efficient equilibrium depends on the fraction of good cars in the population (i.e., the distribution of qualities in the market), and on the buyer and seller valuations.

i. (5pts.) (Dependence on distribution of qualities.) Explain why the set of values of g—the fraction of good cars in the population— for which there is an equilibrium with h = g (i.e., with the efficient outcome in which both good and bad cars are sold on the market) is an interval, of the form $[g^*,1]$.
ii. (10pts.) (Dependence on items' valuations.) This threshold g^* describes the minimum fraction of good cars that the market must contain for it to retain an efficient equilibrium h = g with information asymmetry (the lower the threshold, the more bad sellers the market can tolerate without failure despite the information asymmetry). How does this threshold g^* depend on the values s_H,s_L,b_H,b_L?

    (To answer this question, it is enough to derive an expression for the value of g^* as a function of these parameters.)

i. This is true as in order for h=g to be an equilibrium, the value of buyers must be greater than that of both the good sellers and the bad sellers. Thus, b_{H^*}(g)+ b_{L^*}(1-g)>= s_h. As g decreases the value on the left hand side of this inequality will decrease until it is equal to the right hand side of the inequality. Any g lower than this will indicate the buyers will only be willing to pay prices lower than that of the s_h which will in turn mean that good sellers will no longer participate meaning h-> 0 meaning this is not a SFEE anymore.
ii. b_{H^*}(g*)+ b_{L^*}(1-g*)= s_h. The minimum fraction of good cars the market must contain for an efficient h=g equilibrium will be governed by this equation b_{H^*}(g*)+ b_{L^*}(1-g*)= s_h. This means that multiplication of the buyer good value and g*(the proportion of good cars) added to the proportion of bad cars * the buyer value of bad cars gives us the price that buyers are willing to pay. When this amount is equal to the seller value of good cars, this will result in all sellers entering the market and h=g. Any lower g* and the buyer price to pay will be lower than the seller good car value and none will sell on the market meaning g will not equal h.

3. (Repeated Prisoners' Dilemma: Discount factors and equilibria.) Consider the infinitely repeated prisoners' dilemma, where the stage game has pay- offs (r, r), (t, s), (s, t), and (p, p), and both agents have a discount factor of δ (all terminology is as defined in class; see Lecture 23 if necessary). To start with, let's consider a specific example with payoffs (2, 2), (3, 0), (0, 3), and (1, 1). Let's consider the grim-trigger strategy G that we discussed in class. We'll now reason about when (i.e., for what values of the discount factor δ) it is a Nash equilibrium for both agents to play according to the grim-trigger strategy in the infinitely repeated game.

(a) (5pts.) Suppose player P1 is playing the grim-trigger strategy G. If player P2 also plays G, then since the game starts in state (C,C) at time n = 0, applying the strategy G will keep both players in the same states (C,C) for all steps n ≥ 0. What is the total payoff to player P2 playing G in the infinitely repeated Prisoners' Dilemma game, when the discount factor is δ?

$$P2\ Total\ Payoff = \sum_{t=0}^{\infty}(\delta^{t})*2$$

(b) (7pts.) Now suppose P1 plays according to the strategy G, but P2 deviates at some step N ≥ 0 by selecting D. Given that P1 (contin- ues to) play the grim-trigger strategy G,
(i) What is player P2's best response in steps N + 1 onwards (i.e., in steps N +1,N +2,...): that is, at what steps n ≥ N +1 should she choose to play C, and when should she choose D (having deviated at step N)?

P1\P2	C	D
C	(2,2)	(0,3)
D	(3,0)	(1,1)

From steps N+1 onward P2 should always choose to defect. Given that P2 defects for the first time at step N and P1 is using the grim-trigger strategy, we know that on step N+1 and all steps after, P1 will choose to defect. Given that P1 defects, P2. Should also choose defect as the benefit is δ^{step} whereas the benefit of cooperation is zero. As steps goes to infinity, the benefit of defecting will go to zero but for all steps it remains positive and thus larger than zero, the benefit of cooperation.

(ii) Does your answer to this question depend on the value of δ?

No, as δ can never be negative and thus the benefit of defecting can never be lower than the benefit of cooperating given P1 defects.

(c) (8pts.) What is the total payoff to P2 if she decides to play according to the grim-trigger strategy for n < N, and then play D for n ≥ N, while P1 continues to play the grim-trigger strategy G throughout?

$$\sum_{t=0}^{N-1}(\delta^{t})*2 + 3\delta^{N} + \sum_{t=N+1}^{\infty}(\delta^{t})$$

(d) (10pts.) From parts (a), (b) and (c), can you now reason for what values of δ is it a Nash equilibrium for both agents to play according to the grim-trigger strategy G? (You should both derive the condition on δ, as well as explain how you arrived at the conclusion that the grim-trigger strategy is an equilibrium for these values of δ.)

$(1-\delta)(3+\delta+\delta^{2}+\ldots)=(1-\delta)(3+\delta/(1-\delta))=3(1-\delta)+\delta$

So long as $3(1-\delta)+\delta<=2$ we know that the player cannot increase their payoff by deviating. Thus $\delta>=1/2$.

This conclusion is driven by the calculation of a players payoff when they deviate vs when they cooperate. So long as cooperation payoff is greater than the payoff of deviating, then the grim trigger strategy for both agents will yield a nash equilibrium.

4. (20pts.) (Repeated Prisoners' Dilemma: Payoffs and discount factors.) In the previous question, we saw how to reason about whether the grim- trigger strategy is a Nash equilibrium, and how the discount factor plays a role in determining whether or not G is an equilibrium set of strategies in the infinitely repeated prisoners' dilemma.

In this problem, we're going to try to understand how the payoffs r, s, t and p in the stage game affect how patient players need to be to sustain cooperation via the grim-trigger strategy—i.e., how the minimum value of the discount factor δ at which both players playing according to G is an equilibrium depends on the payoffs in the game.

Hint: For this problem, think about how to derive an expression for δ^* in terms of the payoffs in the stage game (r, s, t, p), such that the grim-trigger strategy is an equilibrium for all $\delta \geq \delta^*$. You can use that expression to answer the questions below; this will save you the effort of doing the same calculations with different numbers again and again for each part.

(a) Suppose the reward r from cooperating goes up to 2.5 from 2, whereas s, t, p remain unchanged (i.e., s, t, p are the same as in Q2). Following the same arguments as in Q2, for what values of δ is the grim-trigger strategy now a Nash equilibrium?

C: $(1-\delta)(1+\delta+\delta^2+\cdots) = 2.5$

D: $(1-\delta)(3+ \delta+ \delta^2+\ldots)=(1- \delta)(3+ \delta/(1- \delta))=3(1- \delta)+ \delta$

$3(1- \delta)+ \delta<=2.5$

$3-2 \delta<=2.5$

$1/4<= \delta$

Thus when the discount δ is greater than or equal to ¼, G will be a Nash equilibrium strategy.

(b) Now let's change the values of the temptation t from 3 to 5, while r=2,s=0andp=1remainasbefore. What is the new minimum value of δ required for G to be an equilibrium set of strategies?

D: $(1-\delta)(5+ \delta+ \delta^2+\ldots)=(1- \delta)(5+ \delta/(1- \delta))=5(1- \delta)+ \delta$

$5(1- \delta)+ \delta<=2$

$5-4 \delta<=2$

$3<=4 \delta$

$3/4<= \delta$

Thus when the discount δ is greater than or equal to3/4, G will be a Nash equilibrium strategy.

(c) Let'schangepnext—sayp=[1] whiler=2,s=0andt=3remain 2

as before. Now what is the new minimum value of δ required for G to be an equilibrium?

D:(½)(1-δ)(6+ δ+ δ^2+...)=(1/2)(1- δ)(3+ δ/(1- δ))=3/2(1- δ)+ 1/2δ

3/2(1- δ)+ 1/2δ<=2

3/2-δ<=2

-1/2<= δ

Thus, for all discount factors greater than or equal to -1/2 this strategy will be a nash equilibrium.

(d) Finally, supposing that r, t, p remain the same as in Q2, how does a change in s, say from 0 to −1, affect the minimum discount factor δ beyond which G is an equilibrium?

The changing of s will not effect the inequality and thus the discount factor must remain greater than or equal to ½.