CS 4820, Spring 2019	**Homework 2, Problem 1**
Name: Kevin Klaben	
NetID: kek228	
Collaborators:Quinn Lui	

(1) *(10 points)* Suppose $n (\geq 1)$ people are stranded on a freeway due to a particularly heavy snowstorm. Let us model the freeway as the real line. The locations of the stranded people are given by real numbers. An emergency rescue operation found n hotels to potentially accommodate the n people. Assume that each hotel has the capacity to shelter 1 person. However, due the prevailing road conditions access to these hotels are severely limited. For each hotel H, there is a specific segment of the freeway such that only people stranded in this segment can make it to H. Given as input the locations of the stranded people, and a set of n segments of the freeway (each segement corresponding to a particular hotel), your task is to design an efficient algorithm to decide if it is possible for all n people to find accommodation for the night. (Assume that the input contains real numbers of finite precision, so that any arithmetic operation on two real numbers takes constant time. Neither the list of people's locations nor the list of segments are assumed to be sorted in any particular order.)

while there is an unmatched person **do**
  for each unmatched person P **do**
    if no unmatched person has been stored or the absolute location of the stored unmatched person is higher than the location of P **then**
      store P
    end if
  end for
  for each unmatched segment S **do**
    if the start of S is lower than the stored absolute location of the unmatched person and the end of S is higher than the stored absolute location of the unmatched person **then**
      if there is no stored segment or the stored segments end bound is higher than S's end bound **then**
        store S
      end if
    end if
  end for
  if a segment S has been stored corresponding to hotel H **then**
    (P,H) become matched meaning P and S are marked as matched
  else
    return False
  end if
end while
return True

If True is returned then this means that all people can find lodging for the night while false indicates that not all people can be accommodated.

Runtime Analysis
The while loop can run at a maximum n times as each loop always results in either one fewer

unmatched person or the while loop ending. For each loop of the while loop there are two for each loops which iterate over a maximum of n elements. Within each while loop there are several comparison operations among numbers which have been established as taking constant time. Inside the while loop there is also an if statement which will run in constant time. Thus over all each for loop runs in O(n) time thus each iteration of the while loop runs in O(n)+O(n)+O(1) time which is equivalent to O(n) time and the while loop will run a maximum of n times and thus the overall run time is O(n^2).

Proof of Correctness Claim: If in fact all people can find a place to stay the algorithm will output True, if this is impossible the algorithm will output False.

Case 1: The algorithm out puts True, then there is a way for all people to be lodged Proof: For the sake of contradiction suppose that there in fact is no way for all people to be lodged. Therefore, there exists some set of people for which there are not enough segments that can accommodate all of them. Taking this subset of people and segments which can possibly reach them we begin running the algorithm pairing the person in this subset with the lowest number with segments of road which have the lowest end bound. As there are more people than segments which can possibly accommodate them, at least one person will be left unmatched within the running of the algorithm and thus no segment will be stored. For this person outside of for loops within the while loop the if statement will be reached and return False as there is no segment stored which can be paired with this person. However, the algorithm had to have returned true thus this is a contradiction.

Case 2: The algorithm outputs False then there does not exist a way to accommodate all people. Proof: For the sake of contradiction suppose that there does in fact exist a possible way to accommodate all people. Therefore there exists a way for each person to be paired with a different hotel. In order for False to be produced, there must be at least one person who cannot find a match. As all people with lower scores have already been paired with corresponding hotels with the lowest possible end bounds. As there are no unmatched hotels that are possible to be matched with the unmatched person who produced the output of False, this unmatched person can only be matched with a previously matched hotel. As some x amount of people have already been matched with x amount of hotels and there remains one additional unmatched person who must match with one of these matched hotels as no unmatched hotels can accommodate it. There is no possible way for x+1 people to be matched with x hotels thus this is a contradiction as there is no way for all people to be accommodated. Thus, the claim holds.