Bird's eye view of complexity theory.

'Yes' instances

NP: let A be a decision problem.

$A \in NP$ if :

For $x \in I_A$,

there is a short certificate which indicates x is a 'Yes' instance.

Formal version: $\exists$ a polytime verifier

V s.t $x \in A$ iff $\exists\ y$, certificate witness

$|y| \le |x|^{c}$ constant, s.t $V(x,y) = 1$.

e.g. 1 $3SAT = \{ \phi : \phi$ is a satisfiable 3CNF $\}$.

ϕ : certificate (if $\phi \in 3SAT$)

is a satisfying truth assignment.

eg 2 $IS = \{ (G, k) :$ G has an IS of size at least $k \}$.

certificate of a 'Yes' instance:

$S \subseteq V$ s.t all nodes in S do not have any edge between them.

Def^n Let A be a decision problem.

Define $\overline{A} = \{ x^{\in IA} : x \notin A \}$

decision problem.

all 'Yes' instances of $\overline{A}$

are exactly the 'No' instances of A.

<u>CoNP:</u> All decision problems A s.t $\overline{A}$ is in NP.

<u>e.g1</u> TAUTOLOGY $= \{ \phi : \phi$ is a 3CNF s.t every truth assignment satisfies $\phi \}$.

$\overline{\text{TAUTOLOGY}} = \{ \phi : \phi$ is a 3CNF and $\exists$ some assig. which falsifies $\phi \}$.

Clearly, $\overline{\text{TAUTOLOGY}} \in NP$

$\Rightarrow$ TAUTOLOGY $\in$ CoNP.

<u>e.g2</u> $\overline{IS} = \{ (G, k) :$ the largest IS in G is of size atmost $k-1 \}$.

Def^n $A \in coNP$ if $\exists$ a polytime verifier V s.t. $x \in A$ iff

$\forall\, y,\ |y| \le |x|^c,\ V(x,y) = 1.$

Can a decision problem A be in both NP and coNP?

(a) Yes ✓ (b) No.

Thm: $P \subseteq NP \cap coNP$.

Pf sketch. Let A be a decision problem in P.

$\Rightarrow \overline{A} \in P$.

We know $P \subseteq NP \Rightarrow A \in NP$,
$\overline{A} \in NP$.

$\Rightarrow A \in coNP$.

Thus, $P \subseteq NP \cap coNP$.

It is not known if P is strictly contained in $NP \cap coNP$.

A problem in $NP \cap coNP$ which we do not know to be in P is:

FACTORING = $\{(n, k)$: n has a factor less than k and more than $1\}$.