Networks II: Market Design—Lecture 13
Matching Markets with Non-transferable Utilities: Two-sided preferences

ARPITA GHOSH

Dept. Of Information Science,
Cornell University

Logistics

- Today: Hello project! (Details: Next few slides)
- Immediate action item: Project groups
  - Due next Thursday (March 21) 12 noon on CMS: Weight of 2%
  - Groups of **size 4**: Start looking for groupmates!
  - Search for groupmates on Piazza
  - Only need to declare group, title, and one-line description now
    - Title and one-line description can be changed later, if necessary
  - Next activity: Preliminary report, due Thursday April 11

- Topic: *Anything* related to networked behavior
- Can pick topic of interest from Networks I, II, or out-of-class
- Your project **must**:
  - Make connection to networked behavior
  - Reflect underlying theme of this class (and Networks I): Value of abstract mathematical model in understanding real world settings

Rewinding back to Lecture 1 (and Lecture 12!)

Three aspects:

- Modeling: How do you identify and *model* a real-world setting as an instance of networked behavior?
- Analysis: What are the general principles that apply to this instance?
- Design: Can we use our model and analysis to *design* for desirable outcomes?

- Ideal project: Depth on *at least* one dimension, touches on other two

Project: Content

- For example: (i) Identify a real-world networks setting; provide possible formal models/abstractions for it; identify assumptions and issues (depth on identification and modeling) (ii) Summarize literature to suggest analysis, design solutions
  - Important note: What is the point/value of an abstract model?
- Another example: (i) Identify (simple but well-motivated) extension on existing model (ii) Provide (progress towards) analysis or design solution
- Literature surveys are fine, provided they meet evaluation criteria (coming up next)

Project: Evaluation

- Evaluation criteria
  - Significance (to course): How is your project related to networked behavior and how do you benefit from an abstract model?
  - Clarity: What exactly are you doing, and what are you doing about it?
  - Coherence: How do the various "what are you doing's" connect to each other?
    - Especially important for literature survey!
  - Value-add: Original contribution (insight, perspective, modeling, problem-solving, . . .)
- FAQ: How do projects and blog posts relate?
  - OK for project to further explore blog post of any group member
  - Distinction from blog: Fleshing out ideas fully—extent of effort and depth much higher!

Where are we now?

- Matching markets, without money ('non-transferable utility')
- Central question: How to pair entities(who gets what)?
- One-sided preferences: Only one side has *preferences* over other
  - Many real-world applications: Scheduling, college housing assignments, school choice, organ exchange, ...
- Investigate this question in a range of models:
  - No initial endowments: (i) Binary (ii) Strict rank-order preferences
  - Initial endowments: Strict rank-order preferences
  - Binary preferences with initial endowments (kidney exchange markets)

Recap: What we've seen so far

How to pair entities: What are 'good' matchings?

- No initial endowments: Binary preferences
  - Easy to identify 'best' allocations: Perfect, maximum matchings
  - Hall's theorem: Existence of perfect matching
- No initial endowments: (Strict) rank-order preferences
  - 'Best' difficult to define: Pareto-efficiency as goodness
  - HW2: Maximum-size matchings are Pareto-efficient **with binary preferences**

Recap: What we've seen so far

- Next in 'what are good matchings': *Initial endowments*
  - Every agent comes to market *with* an item: $((a_k, h_k), \succ)$
  - Note contrast with no initial-endowment market: $(A, H, \succ)$
- Now what is a good matching?
  - Pareto-efficiency is not adequate
  - Individual rationality: No agent unhappier 'after market than before'
  - Asking for even more: No *group* of agents can do better by not participating in market
- Core: No **subset** of agents can 'do better outside market'
  - Matching in core must be Pareto-efficient (PE) and individually rational (IR)
  - But a PE-and-IR matching need not be in core!

Recap: What we've seen so far

How do we *find* these 'good' matchings?

- Mechanisms: Choosing a matching for any input market
- 'Goodness': Always return a matching that is 'good' wrt reported input
  - Pareto-efficiency: Matching $M = \varphi(A, H, \succ)$ Pareto-efficient wrt $\succ$ for all inputs
  - Core: Matching is in core wrt input market $((a_k, h_k), \succ)$
- Strategyproofness: Get *true* input!
  - Informally: Agents *never* have reason to lie about preferences to φ
  - Revisit formal definition!

- Strict rank-order preferences with no initial endowments:
  - Serial dictatorship: Pareto-efficient and strategyproof mechanism
  - Characterization: Any strategyproof, non-bossy, neutral mechanism is a serial dictatorship
- Strict rank-order preferences with initial endowments:
  - Existence of a core matching: Top Trading Cycles (TTC) algorithm
  - Uniqueness of core matching
  - Strategyproofness of core *mechanism*
  - Characterization: Any IR, PE, SP mechanism must return core matching
- An application: Kidney exchange design

What have we **learnt**?

- What we've **learnt**: Concepts and ideas which apply well beyond one-sided markets
  - Pareto-efficiency: Any setting with no total ordering over outcomes
  - Core: No group of agents can create higher value without coming to market
  - Strategyproofness: Incentives and preference reporting
- Even more broadly:
  - Abstract reasoning (arguing via contrapositives, . . .)
  - Asking the right questions: What constitutes good, not just how to achieve it
  - Process of abstract modeling, analysis and design

Matching markets without money

- Markets so far: Agents on one side, objects on the other
- Matching markets: Two sets of *entities*
  - Value created only when entities from opposite sides are paired
  - Entities: Agents, items
  - Agents have preferences, but items don't
- What happens where we have agents on **both** sides?
  - What are "good" matchings?
  - Do they always exist? Can we find them?
  - If there's more than one good matching, can we *compare* good matchings?
  - What about incentives?

Markets with one-sided preferences: Check-in!

- Which of these—Pareto-efficiency (PE), individual rationality (IR), strategyproofness (SP) and core—is a property of a matching, and which is a property of a mechanism?

  A All four are properties only of matchings
  B PE, SP, IR: Only matchings; Core: Only mechanisms
  C PE, IR, Core: Both mechanisms and matchings; SP: Only mechanisms
  D PE, SP: Both mechanisms and matchings; IR, core: Only mechanisms
  E All four are properties only of mechanisms

Markets with two-sided preferences

- Two-sided preferences: Agents on **both** sides of market
  - Markets with ordinal (not cardinal) preferences: 'Markets without money'
- Many instances in real life, both offline and online:
  - College admissions: Schools and applicants
  - Labor markets: Firms and workers
  - Sports drafts: Teams and players
  - Mentor matching: Mentors and mentees
  - Online crowdsourcing platforms (Amazon Mechanical Turk, oDesk, . . .): Requesters and workers
  - Online dating: Men and women
  - Online accomodation (Couchswap): Hosts and guests
  - . . .

- Market with A,B,C on one side, and x,y,z, on the other
- Preferences on one side are A: x $\succ$ z; B: y $\succ$ x $\succ$ z; C: z $\succ$ x $\succ$ y
- Matchings $M_1 =$ (A-x, B-y, C-z), $M_2 =$ (A-z, B-y, C-x)
- If x, y, z are *items*, which matching is better?
  A M_1
  B M_2
- Now suppose x, y, z are *agents*, with preferences x: B $\succ$ C, y: B $\succ$ A $\succ$ C, z: B $\succ$ A $\succ$ C. Which matching is better?
  A M_1
  B M_2

Markets with two-sided preferences

- ‘What happens’ when *both* sides of market have preferences over other side?
  - Recall: Central question is who to pair up with whom
- As we saw in markets with one-sided preferences:
  - Agents with preferences: Cannot always make everybody happy
  - No money: Cannot make up for unhappiness with money!
- What are ‘good’ allocations (matchings)?
  - Do ‘good’ matching always exist when both sides have preferences over the other side?
  - Can we find them?

An abstract model

- Set of 'men': M; set of 'women': W
- To begin with: Assume $|M| = |W| = n$
- Each man $m \in M$ has rank-order list specifying (strict) preferences $\succ_m$ over women in W
- Each woman $w \in W$ has rank-order list specifying (strict) preferences $\succ_w$ over men in M
- The 'Stable Marriage' model: Gale-Shapley'62

The Marriage Problem

- *Matching*: Set of pairs (m, w) such that each agent $(m \text{ or } w)$ appears in at most one pair
- Formally: μ is a function from $M \cup W$ to $M \cup W$ such that
  1. $\mu(m) \in W$ and $\mu(w) \in M$, and
  2. $\mu(m) = w \iff \mu(w) = m$, for every $m \in M$ and woman $w \in W$
- What is a good matching?
  - How to *match* men and women so all agents are "happy"?

An example

- Two men (m_1, m_2) and two women (w_1, w_2)
- Preferences:

m_1	m_2	w_1	w_2
w_1	w_2	m_2	m_1
w_2	w_1	m_1	m_2

- Matchings $M_1 = (m_1 - w_1, m_2 - w_2)$, $M_2 = (m_1 - w_2, m_2 - w_1)$
- Which matching is better?
  - A M_1
  - B M_2
  - C I can't say

Another example

- Again, two men (m_1, m_2) and two women (w_1, w_2)
- Now suppose preferences are:

m_1	m_2	w_1	w_2
w_1	w_1	m_1	m_1
w_2	w_2	m_2	m_2

- Matchings $M_1 = (m_1 - w_1, m_2 - w_2)$, $M_2 = (m_1 - w_2, m_2 - w_1)$
- Which matching is better with these preferences?
  - A M_1
  - B M_2
  - C I can't say

Blocking pairs

- Question: How to *match* men and women so that everyone is "happy"?
- Let's start by trying to define 'unhappy'
- **Blocking pair**: Man and woman who prefer each other to their current partners
- Formally: Man-woman pair (m, w) is a blocking pair for (or *blocks*) matching μ if $w \succ_m \mu(m)$ and $m \succ_w \mu(w)$
- Recall: Strict preferences

‘Happiness’: Stability

- A matching is good if there are no ‘unhappy’ agents
- **Stable** matchings: Matchings with no blocking pairs
- Formally: A matching μ is *stable* if there is no pair (m, w) such that $w \succ_m \mu(m)$ and $m \succ_w \mu(w)$
- Notice analogy with ideas in one-sided markets

An example

- 3 men (m_1, m_2, m_3) and 3 women (w_1, w_2, w_3)
- Preferences:

m_1	m_2	m_3	w_1	w_2	w_3
w_1	w_1	w_3	m_2	m_3	m_1
w_2	w_3	w_2	m_3	m_1	m_3
w_3	w_2	w_1	m_1	m_2	m_2

- Is the matching $m_1 - w_1, m_2 - w_2, m_3 - w_3$ stable?
  - A Yes
  - B No
- What about matching $m_1 - w_3, m_2 - w_1, m_3 - w_2$?
  - A Yes
  - B No

- Immediate questions:
  - How does stability relate to Pareto-efficiency, core in two-sided markets?
  - Does a stable matching always *exist*? Can we find one if it does exist?
  - What are the properties of stable matchings?
  - Stability is theoretically appealing, but does it matter in real life?