Networks II: Market Design—Lecture 7 Matching Markets with Non-transferable Utilities: One-sided preferences

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Coming up: Outline

- Strategyproofness: Recap
- Strategyproofness and serial dictatorship
- How general are serial dictatorships?

Revisiting strategyproofness

- Recall: Mechanism is a *function* mapping input markets $(A, H, \succ)$ to matchings M
- Strategyproofness: A desirable property for a mechanism to have (recall inputs come from agents with preferences)
  - Mechanism φ returns matching $M = \varphi(\succ)$
  - Agent a has true preferences $\succ_a$; other agents report $\succ_{-a}$
  - $M = \varphi(\succ_a, \succ_{-a})$; $M' = \varphi(\succ'_a, \succ_{-a})$
  - Mechanism φ is *strategyproof* if $M(a) \succeq_a M'(a)$ **for all** $\succ_{-a}$, **for all agents** a **and (corresponding preferences)** $\succ_a$
    - Alert: For all agents a and corresponding preferences $\succ_a$
    - To repeat: $\succ_a$ can be *any* ranking over items in H

Revisiting strategyproofness

- To say a mechanism $\varphi(\succ)$ is *not* strategyproof: One instance of 'game-ability'
  - **One** agent a with **one** ("true") preference $\succ_a$, in one market $(A, H, \succ)$, who can benefit (w.r.t. $\succ_a$) from reporting **one other** profile $\succ'_a$, for **one** specific $\succ_{-a}$
  - (One amongst all possible markets of all sizes, all possible $\succ_{-a}$, $\succ_a, \succ'_a$!)
  - Ingredients: Two data points, $\varphi(\succ_{-a}, \succ_a)$ and $\varphi(\succ_{-a}, \succ'_a)$
- To say mechanism *is* strategyproof: Not 'game-able' for *any* instance
  - Argument must apply to all possible market sizes, all possible input preference profiles $\succ$ (*i.e.*, all possible true preferences $\succ_a$ and all possible deviations $\succ'_a$, for all possible reports $\succ_{-a}$ from remaining agents, for all agents a)

Checking in: Pareto-efficiency and strategyproofness

- If a mechanism is strategyproof (SP), does that guarantee it is Pareto-efficient (PE)?
  - A Yes
  - B No

- If a mechanism is Pareto-efficient (PE), does that guarantee it is strategyproof (SP)?
  - A Yes
  - B No

- Suppose a prefers her allocation in $M = \varphi(\succ'_a, \succ'_{-a})$ to that in $M' = (\succ_a, \succ_{-a})$. Is φ strategyproof?
  - A Yes
  - B No, by definition of strategyproofness
  - C I can't be sure
  - — News you can't use: Does not say φ is not strategyproof because $\succ_{-a}$ changed!

Recall: Incentive properties of serial dictatorship

- Serial dictatorship specifies order amongst agents; assigns top remaining choice in order
- Serial dictatorship mechanism is Pareto-efficient with respect to *reported inputs*
- What about with respect to true preferences?

Theorem

The serial dictatorship mechanism is strategyproof and therefore Pareto-efficient with respect to true preferences.

Incentive properties of serial dictatorship

Theorem

The serial dictatorship mechanism is strategyproof.

Why?

- Let f be the priority order
- Agent $f(1)$ obtains her favorite item when she tells the truth, so she has no incentives to lie
- Agent $f(2)$ gets her favorite among remaining items, so she has no incentives to lie, ... and so on

How does this argument rely on the serial dictatorship property?

Incentive properties of serial dictatorship: Questions

- Sequential reports and strategyproofness:
  - Assumption so far: Agents report preferences $\succ_a$ in advance to centralized allocation mechanism
  - Suppose instead: Agent a only reports her preferences $\succ_a$ at step $f^{-1}(a)$
  - Is the corresponding mechanism still strategyproof?
    [A] Yes [B] No

Incentive properties of serial dictatorship: Questions

- Suppose preferences can be *incomplete*: Is serial dictatorship mechanism still strategyproof?
  [A] Yes [B] No
  - Incomplete preferences: Agents may not find all items acceptable
  - Observe: Changing mechanism versus changing (allowed) preferences
- Serial dictatorship with incomplete preferences:
  - Pareto-efficient, strategyproof
  - However, can produce a 'small' matching

Upgrading serial dictatorship

- Serial dictatorship with incomplete preferences:
  - Pareto-efficient, strategyproof
  - But can produce a 'small' matching (if incomplete preferences)
- 'Serial dictatorship 2.0': Amongst all Pareto-efficient matchings, choose one with maximum size
  - (Break ties amongst matchings in prespecified order)
- Is Serial dictatorship 2.0 strategyproof?
  - A Yes
  - B No
- Agents a, b, Houses x, y, preferences: $a : x \succ y$; $b : x$
  - Priority order (for tiebreaking): a, b
  - a can benefit by lying and claiming that she only likes x

Revisiting: Incentive properties of serial dictatorship

Theorem

The serial dictatorship mechanism is strategyproof.

Why?

- Let f be the priority order: Agent $f(1)$ obtains her favorite item when she tells the truth, $f(2)$ gets her favorite among remaining items, ... and so on
- Formally: For any $\succ_{-a}$, no agent a prefers outcome from misreporting her preferences:
  - Let $M = \varphi(\succ)$ denote output of mechanism
  - Let a be ith ranked agent: i is s.t. $f(i) = a$
  - Critical: *Value of i is* **independent** *of report $\succ_a$*
  - Allocation $M(b)$ for all agents $b \in \{f(1), \ldots, f(i-1)\}$ is **independent** of report $\succ_a$
  - Fix $\succ_{-a}$: a never strictly prefers allocation in $\varphi(\succ'_a)$ to that in $\varphi(\succ_a)$

Serial dictatorship: Putting it all together

- What we've seen so far:
  - What are 'good' mechanisms? Produce 'good' (Pareto-efficient) matchings on all inputs
  - Theorem 1: Serial dictatorship is Pareto-efficient
  - Agents may lie about preferences: Strategyproof mechanisms
  - Theorem 2: Serial dictatorship is strategyproof

Theorem

The serial dictatorship mechanism is strategyproof and therefore Pareto-efficient with respect to true preferences.

How general are serial dictatorship mechanisms?

- **Theorem** (Abdulkadiroglu & Sonmez'98): For any Pareto-efficient matching of a given house allocation problem $(A, H, \succ)$, there exists a serial dictatorship that achieves this matching.
  - Is this saying that every Pareto-efficient mechanism is a serial dictatorship?
- **Theorem** (Svensson'98): A mechanism that is non-bossy, neutral, and strategyproof is a serial dictatorship.
  - Definitions of non-bossy, neutral (and proof of theorem) not part of syllabus!