Networks II: Market Design—Lecture 4 Matching Markets with Non-transferable Utilities: Rank-order preferences

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Bulletin board

- HW1 released: Due Thursday Feb 7, by 12 noon on CMS
  - **Homework guidelines** handout on CMS : Deadline, late policy, how much to explain, . . .
  - Beware!: The dysfunctional-Internet phenomenon
- Office hours: Start tomorrow, Friday Feb 1
  - See course website: https://courses.cit.cornell.edu/info4220/
- Piazza sign-up: piazza.com/cornell/spring2019/info4220
  - **Important: See pinned post for guidelines on use**
    - Content-questions only, no logistics
    - No discussion of policies, no ranting of any kind
    - Individual responsibility

Recap: What we're doing

- Last week: Matching markets without money
- One-sided preferences: Agents only on one side
- Simplest form of preference: Binary (0-1)
  - Canonical setting: Allocating rooms to students
  - Well-defined 'ideal' allocation in market: Every agent (student) gets acceptable item (room)
- *Markets to graphs*:
  - Market with binary preferences ⇔ Bipartite graph
  - Ideal allocation in market iff perfect matching in bipartite graph

Recap: What we did

- Asked how to decide if we can achieve ideal allocation
- Existence of perfect matchings (ideal allocations): The *Matching Theorem*
- Proof illustrated:
  - Augmenting paths: Growing (improving) matchings
  - How to *construct* a perfect matching, if one exists
  - How to build a *maximum* matching, if one doesn't

Coming up next

- So far: One-sided markets with *binary* preferences
- Extending to richer preferences: Ranked preference lists
- What is a 'good' allocation?
- How do we find it?

Rank-order preferences in markets without money

- Binary $(0 - 1)$ preferences: All acceptable items are 'equal'
- Generalization: *Rank* order amongst acceptable alternatives
- Real-life (centralized) systems with rank-order inputs:
  - Undergraduate, graduate housing allocation
  - Office space allocation
  - Preferences over reviewing articles: Refereeing in academic conferences

Markets without money: Rank-order preferences


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Feb 09, 2014
Graduate Housing IAP Sublets last day available February 9, 2014

Mar 26, 2014
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Mar 26, 2014
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CAMPUS MAP »

More dates & deadlines »

WELCOME

Graduate & Family Housing

- Residences
- Get housing
  - How to apply
    - Your Housing Options
    - Who is Eligible for Housing
    - > How We Assign Housing
      - Enter the Fall Allocation
    - Enter the November Allocation
    - Why Live on Campus
    - Campus Resources
    - WHOI Housing Information
  - Renew or Reapply
  - Change or Cancel
  - Waiting List Information
  - Sublicense Center
  - Roommates
  - Supplemental Housing Requests
- Your Assignment & License Agreement

HOME >> GRADUATE & FAMILY HOUSING >> GET HOUSING >> HOW TO APPLY >> HOW WE ASSIGN HOUSING

How We Assign Housing

It starts with a mission...and an algorithm

MIT has invented a computerized model to make the process of deciding who lives where as efficient and equitable as possible—and to create residential communities that feel like home.

Most assignments are made by running both the student applications and the housing vacancies through a series of algorithms. Algorithms are used in two separate allocations. The May allocation assigns students to housing in the fall. The November allocation makes assignments for the spring. These complicated systems achieve a simple goal—housing the maximum number of graduate students who want to live on campus in their first choice of assignment.

How the algorithm works

- First priority is to give the greatest number of graduate students the opportunity to live in graduate housing.
- Second priority is to get students into their top choices. More weight is given to moving someone onto campus than moving someone from their second to their first choice.
- The more places you rank among your preferences, the more likely you are to receive an on-campus housing assignment.

What happens next

- You will be sent one—and only one—residence assignment.
- You must accept the assignment or pay a $250 cancellation fee.
- Review the step-by-step process, from application to move in.

Is the allocation process a lottery?

- The process for assigning graduate housing is less of a lottery than ever before.
- Previously, students were randomly assigned a number and placed into numerical assignment until there were no more vacancies. This could take weeks to complete.
- Our current process is extremely fast and efficient. Using algorithms, a computer program takes into account all preferences

Choosing a 'good' allocation

- Distinction: Markets with versus without money
- Values versus rank-order preferences:
  - Recall: v_{ij} in (one-sided) matching markets *with* money
  - v_{ij} uniquely specify rank-order preference
  - But rank-order preferences do not similarly specify v_{ij}: Strictly less information!
- Binary preferences: Well-defined ideal allocation
  - Good allocation: Maximum matching
- What is a 'good' allocation with rank-order preferences?

Choosing a ‘good’ allocation: More complexity

- ‘Initial property rights’, or ‘initial endowments’
- Market where each agent owns, or otherwise has ‘rights’ to an entity to start with
    - Dorm room allocation
    - Organ donation exchanges
    - School assignments
- Fairness, individual rights protection: ‘Individual rationality’, no ‘justified envy’

- Start out simple(r): No initial endowments
- Canonical language: Agents and houses
  - A: Set of agents
  - H: Set of goods ("houses")
  - Assume $|H| = |A|$
- Each agent $a \in A$ has preferences over houses, $\succ_a$
- $\succ = (\succ_a)_{a \in A}$ is 'preference profile'
- **Assume: Preferences are strict** (amongst acceptable houses)
  - Does this model include our binary preferences model?
  - Also assume preferences are **complete**, unless otherwise specified: All houses are acceptable

- 'House allocation problem': Tuple $(A, H, \succ)$
  - Hylland and Zeckhauser (1979)
- Matching M:
  - Function specifying who gets what good
  - $M : A \rightarrow H$
  - $M(a)$ is the house that agent a receives in M
- Tuple $(A, H, \succ)$: Where did the preferences $\succ$ go?
  - Market more complex: Cannot just be described by bipartite graph!
  - *Matching* in bipartite graph captures *allocations*, but edge set does *not* capture preferences!

Outline

- One-sided preferences: The 'House Allocation' problem
- Pareto-efficient matchings
- Mechanisms and Pareto-efficiency

Rank-order preferences: What is a ‘good’ matching?

- Consider market with 3 agents A,B,C and 3 houses x,y,z
- Preferences are A: x $\succ$ y; B: x $\succ$ z; C: z $\succ$ x
- Three matchings: $M_1 =$ (A-y, B-z, C-x), $M_2 =$ (A-x, B-z), $M_3 =$ (A-x, C-z)
  - Note: These are incomplete preferences!
- Which of these is ‘best’?
  - M_1: Perfect matching in underlying bipartite graph
  - But size is not only criterion!
  - M_3: All matched agents are matched to top preferences
  - M_2 matches B, who may value being matched (at all) more than C values z
  - What about $M_4 =$ (A-y, B-x, C-z)?

Matchings and goodness

- Another example: 3 agents A,B,C and 3 houses x,y,z
- Preferences A: x $\succ$ y $\succ$ z; B: x $\succ$ y $\succ$ z; C: x $\succ$ z $\succ$ y
- Matchings $M_1 =$ (A-x, B-y, C-z), $M_2 =$ (A-y, B-x, C-z)
  - Both matching have same number of agents matched to their first and second choices
  - Which of M_1 and M_2 is better?
    - [A] M_1 [B] M_2 [C] Can't say
- Matching $M_3 =$ (A-x, B-z, C-y), $M_4 =$ (A-z, B-x, C-y)
  - Which of M_3 and M_4 is better?
- Can you compare (i) M_1 with M_3? (ii) M_2 with M_4?

Matchings and goodness

- Recall our question: Given preferences $\succ$, what is a 'good' matching?
- Rank-order preferences $\succ$: Not enough information to choose 'best' matching
- A different question: Which are **bad** matchings?
  - And from there: Which are **not-bad** matchings?
  - The new 'Good' matchings: 'Not-Bad' matchings
- Note **change in question**: Situation changes, 'good' question changes!

Matchings and goodness: Formalizing not-bad matchings

- Consider a market, or tuple, $(A, H, \succ)$
- A matching $\hat{M}$ **Pareto-dominates** matching M if
  - $\hat{M}(a) \succeq_a M(a)$ for *every* agent a, and
  - $\hat{M}(b) \succ_b M(b)$ for *some* agent b
- A matching M is **Pareto-efficient** if there is no other matching $\hat{M}$ that Pareto-dominates it
- Pareto-efficient matchings: 'Not bad' matchings
- Note: Pareto-efficiency of matching defined with respect to *specific* preference profile $\succ$

Example

- Example: Agents a,b,c; Houses x,y,z
- Preferences $\succ$: a: x $\succ$ z $\succ$ y; b: x $\succ$ z $\succ$ y; c: x $\succ$ y $\succ$ z (Use [A] Yes [B] No)

  1. Matching 1: a-x, b-z, c-y
  2. Matching 2: a-x, b-y, c-z
  3. Matching 3: a-y, b-x, c-z
  4. Matching 4: a-z, b-x, c-y

- New preferences $\succ'$ are a: z $\succ$ x $\succ$ y; b: x $\succ$ z $\succ$ y; c: x $\succ$ y $\succ$ z

  1. Matching 1: a-x, b-z, c-y
  2. Matching 4: a-z, b-x, c-y

- Consider two matchings $M, \hat{M}$, and some agent a
- $\hat{M}$ is strict improvement over M, **for agent** a if: $\hat{M}(a) \succ_a M(a)$
- $\hat{M}$ is weak improvement for a if: $\hat{M}(a) \succeq_a M(a)$
  - $(A, H, \succ)$ has strict preferences: Weak improvement not strict iff $\hat{M}(a) = M(a)$
  - Aside: Not true if preferences are not strict
- To claim M is not Pareto-efficient: Produce a matching $\hat{M}$ which is a weak improvement for all agents and a strict improvement for at least one agent

Pareto-efficiency

- Pareto-efficiency: Comparisons between *matchings*, not just single agents
- Food for thought: How does Pareto-efficiency for matching relate to definition from homework?