Networks II: Market Design—Lecture 8
Markets with Initial Endowments: The Core

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Recap: Last time

- Theorem 1: Serial dictatorship is strategyproof and therefore Pareto-efficient wrt true preferences
  - Note: Ordering of agents f is **independent** of preference profile $\succ$!
  - Serial dictatorship 2.0: Pareto-efficient, but **not** strategyproof!
- How general are serial dictatorships?
  - Theorem 2: Every Pareto-efficient matching in $(A, H, \succ)$ is the output of **some** serial dictatorship.
  - Theorem 3: A mechanism is non-bossy, neutral, and strategyproof if and only if it is a serial dictatorship.

Coming up

- Looking back: What we've seen so far
- Initial endowments: The 'Housing Market' problem
- The core
- Existence of a core allocation: Gale's TTC algorithm
- Properties of core allocations

So where are we now?

- Markets without money: Choosing a matching
  - One-sided preferences: Agents only on one side
  - Lots of real-world applications: Scheduling, college housing assignments, school choice, organ exchange, ...
  - Binary preferences: Easy to define 'ideal', 'best' allocations
  - Rank-order preferences: 'Best' difficult to define
    - Good matchings: Pareto-efficient (*i.e.*, not-bad) matchings
    - Recall: *Concept* of Pareto-efficiency applies more generally
    - *Not* restricted to strict rank-order preferences!

So where are we now?

- Mechanisms: Choosing a matching given market $(A, H, \succ)$
  - Pareto-efficiency of mechanism φ: Matching $M = \varphi(A, H, \succ)$ Pareto-efficient wrt $\succ$ for all inputs
  - Strategyproofness of φ: Agents *never* have reason to lie about preferences to φ
  - Existence and characterization:
    - Serial dictatorship: A strategyproof, Pareto-efficient mechanism
    - Any strategyproof, non-bossy, neutral mechanism must be a serial dictatorship

Before we go on, though

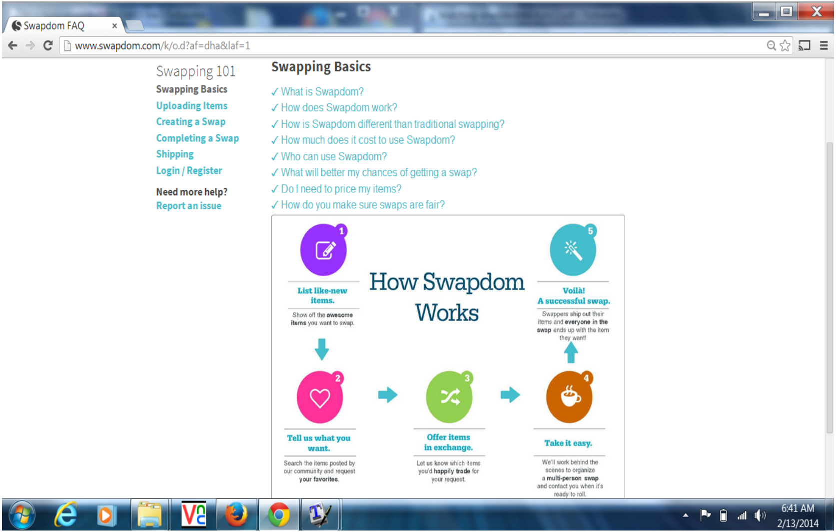
- M is for: Models, matchings, mechanisms,... and mix-ups!
- Models:
  - How do agents express preferences over objects?
  - Binary versus rank-order preferences
  - Initial endowments or not
  - Agents on one versus both sides
  - Centralized allocation or decentralized, . . .
- Matchings:
  - Allocation: Which agent is assigned which item
- Mechanisms (in our settings)
  - Choosing a matching
  - Takes as input a *particular instance* of preference profiles

So:

- Pareto-efficiency is a property of
  - A The strict rank-order preference model
  - B Matchings
  - C Mechanisms
  - D Matchings and mechanisms
  - E All three: Model, matchings and mechanisms
- Strategyproofness is a property of
  - A The strict rank-order preference model
  - B Matchings
  - C Mechanisms
  - D Matchings and mechanisms
  - E All three: Model, matchings and mechanisms

Moving on: One-sided markets with initial endowments

- Markets so far: No 'initial endowments', or 'initial property rights'
  - No agent pre-owned (or had claims to) any item
  - So: No property rights issues
- What if agents own an item to start with?
  - Dorm room allocation
  - Organ donation exchanges
  - School assignments
  - Online used-good, service exchanges (Swapdom, Swapace, ...)


Swapdom FAQ
www.swapdom.com/k/o.d?af=dha&laf=1
Swapping 101
Swapping Basics
Uploading Items
Creating a Swap
Completing a Swap
Shipping
Login / Register
Need more help?
Report an issue
Swapping Basics
✓ What is Swapdom?
✓ How does Swapdom work?
✓ How is Swapdom different than traditional swapping?
✓ How much does it cost to use Swapdom?
✓ Who can use Swapdom?
✓ What will better my chances of getting a swap?
✓ Do I need to price my items?
✓ How do you make sure swaps are fair?
How Swapdom Works
1
List like-new items.
Show off the awesome items you want to swap.
2
Tell us what you want.
Search the items posted by our community and request your favorites.
3
Offer items in exchange.
Let us know which items you'd happily trade for your request.
4
Take it easy.
We'll work behind the scenes to organize a multi-person swap and contact you when it's ready to roll.
5
Voilà! A successful swap.
Swappers ship out their items and everyone in the swap ends up with the item they want!
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2/13/2014

One sided matching markets with initial endowments

- General markets with initial endowments: Some agents have initial endowments while others do not
  - Undergraduate and graduate housing: First years versus remainder
  - Organ donation exchanges: Patients with live donors, and those without
- Study simpler version: *All* agents have an initial endowment
  - So far: Note *no* agents had initial endowments
- Central question: Allocation—who should get what?
  - Fairness, individual rights protection: 'Individual rationality', no 'justified envy'

Initial endowments: What are good matchings?

What properties must a 'good' matching have?

- Is Pareto efficiency 'enough'?
  - Example: Agents a,b,c; Houses x,y,z
  - Preferences $\succ$: a: x $\succ$ z $\succ$ y; b: x $\succ$ z $\succ$ y; c: x $\succ$ y $\succ$ z
  - Initial endowments: a owns x, b owns y, c owns z
  - Matching $M_1 =$ (a-z, b-x, c-y)
  - Is M_1 Pareto-efficient?
    - A Yes
    - B No
  - Is M_1 a 'good' matching?
- *'Individual rationality'*: No agent is better off not participating in the market

What properties must a ‘good’ matching have?

- Another example:
  - Agents a,b,c,d; Houses x,y,z,w
  - Preferences: a: $z \succ y \succ w \succ x$; b: $z \succ x \succ w \succ y$; c: $y \succ w \succ x \succ z$; d: $z \succ x \succ y \succ w$
  - Initial endowments: a owns x, b owns y, c owns z, d owns w
  - Matching $M_1 =$ (a-y, b-x, c-w, d-z)
  - Is M_1 Pareto-efficient?
    - A Yes
    - B No
  - What about individually rational? [A] Yes [B] No
  - Is M_1 a ‘good’ matching?
    - Hint: What if b and c met on their way to the market?
- No *group* of agents can do better by not participating in the market and trading *the items they own* amongst themselves

One sided matching markets with initial endowments

- Markets with initial endowments: From allocation to *trade*
  - The 'housing market' model (Shapley and Scarf 1974)
- **Housing market**: $((a_k, h_k)_{k \in \{1,\ldots,n\}}, \succ)$
  - $\{a_1, \ldots, a_n\}$ is a set of agents and $\{h_1, \ldots, h_n\}$ is a set of houses, where agent a_k owns house h_k
  - Each agent a has strict preferences $\succ_a$ over houses
  - Contrast market with no initial endowments: $(A, H, \succ)$
- Matching M: Function specifying who gets what
  - a receives $M(a)$ under M

Initial endowments: What are good matchings?

- What properties should a good matching have?
  - Pareto efficiency
  - *'Individual rationality'*: No agent is better off not participating in the market
$$M(a_k) \succeq h_k$$
  - No *group* of agents can do better by not participating in the market and trading their initial endowments amongst themselves

Solution Concept: The Core

- **Core**: A central concept in game theory
- Matching M is in the **core** if there is **no** coalition of agents $C \subseteq A$, and a matching $\hat{M}$, such that
  - For any $a \in C$, $\hat{M}(a)$ is the *initial house* of some $a' \in C$
  - $\hat{M}(a) \succeq_a M(a)$ for all $a \in C$
  - $\hat{M}(b) \succ_b M(b)$ for some $b \in C$

Core: An Example

(A matching M is in the **core** if there is no coalition of agents $B \subseteq A$, and a matching $\hat{M}$, such that

- For any $a \in B$, $\hat{M}(a)$ is the *initial house* of some $b \in B$, and
- $\hat{M}(a) \succeq_a M(a)$ for all $a \in B$ and $\hat{M}(b) \succ_b M(b)$ for some $b \in B$)

- Agents a,b,c; Houses x,y,z
- Preferences $\succ$ are a: y $\succ$ z $\succ$ x; b: z $\succ$ y $\succ$ x; c: y $\succ$ x $\succ$ z
- Initial endowments: a owns x, b owns y, c owns z
- Matching M_1 = (a-z, b-x, c-y), M_2= (a-x, b-z, c-y)
  [A] Yes [B] No
  - Is M_1 Pareto-efficient?
  - Is M_1 in the core?
  - Is M_2 in the core?

(A matching M is in the **core** if there is no coalition of agents $B \subseteq A$, and a matching $\hat{M}$, such that

- For any $a \in B$, $\hat{M}(a)$ is the *initial house* of some $b \in B$, and
- $\hat{M}(a) \succeq_a M(a)$ for all $a \in B$ and $\hat{M}(b) \succ_b M(b)$ for some $b \in B$)

- Agents a,b,c; Houses x,y,z
- Preferences $\succ$ are a: y $\succ$ z $\succ$ x; b: x $\succ$ y $\succ$ z; c: z $\succ$ y $\succ$ x
- Initial endowments: a owns z, b owns y, c owns x
- Matching $M =$ (a-z, b-x, c-y) [A] Yes [B] No
  - Is M individually rational?
  - Is M in the core?
  - Does coalition $\{a, c\}$ demonstrate M is not in the core?