Networks II: Market Design—Lecture 21 Information and Networked Behavior

ARPITA GHOSH

Dept. Of Information Science,
Cornell University

Logistics

- Reminder: **Midterm** next class
  - April 18th, 2:55-4:10pm, Olin 155
  - (All details in previous lecture: Please revisit as necessary!)

- Details on logistics
  - Make sure to *be there* (in time)!
  - Arrive early to settle down
  - Please start filling up seats from middle outwards
  - Leave a seat's gap between each other
  - *Academic integrity: No communication with neighbors*
  - Sign-out sheet: At time of turning in your exam
    - Yes, this does mean exit is slowed down
  - Turn in **both** questions and answerbook

Recap: Information asymmetry and inefficient outcomes

- Exploring information asymmetry ('hidden information'): Model
  - Items of different qualities in market
  - Sellers informed, buyers uninformed
  - Buyers **know** that sellers know
- Consequence of information asymmetry: Two questions

1. What 'should' happen?
   - Identifying efficient outcome; achievability with complete information: Existence of market-clearing prices (April 9th)
2. What does happen with asymmetric information?
   - Reasoning about outcomes: Self-fulfilling expectations equilibria (SFEE) (April 11th)

Recap: Self-fulfilling expectations equilibria

- Outcome in market with sellers (rather than items) who make choices:
  - Prices buyers are willing to offer depends on *which sellers sell*
  - Which sellers are willing to sell depends on prices buyers are willing to offer!
- How to reason about 'what happens'?
- Self-fulfilling expectations equilibria: A belief about actions of agents that is 'self-consistent'
  - Expectation, or belief, about distribution induces exactly that distribution: A distribution of qualities (here h) such that if buyers 'expect' this distribution (here, fraction h of cars for sale that are good), offered (*i.e.*, market-clearing) price *induces* that distribution of sellers in market (here, a fraction h of cars offered for sale will indeed be good)

Recap: Equilibrium outcomes

- h (fraction of cars on market that are good) is SFEE if there is a (market-clearing) price p^* 'supporting' fraction h:
  - Sellers willing to sell at p^*, *i.e.*, with $v_s \leq p^*$, are such that fraction h of cars for sale are good
  - Expected value of cars to buyers *with 'distribution' h* is no smaller than price:

$$h \cdot b_H + (1 - h) \cdot b_L \geq p^*$$

  - (Using $\geq$ instead of $=$: SFEE *definition* must allow for $n \leq m$)
- To put it another way:
  - Buyers do not want to change decision (price offered) given sellers' decisions
  - No seller wants to change decision—*either to sell* **or** *not sell*—given buyers' decision

Recap: Identifying SFEE—an example

Recap and a question:

- Consider market with n buyers and m sellers, $n > m$
  - Seller values: $s_H = 10$, $s_L = 4$
  - Buyer values: $b_H = 12$, $b_L = 6$
  - Fraction of good car sellers in population $g = \frac{4}{5}$
- Which of the following is true?
  (Recall: h is fraction of cars for sale that are good)
  - $h = \frac{4}{5}$ is an SFEE
  - $h = \frac{3}{4}$ is an SFEE
  - Both $h = \frac{4}{5}$ and $h = \frac{3}{4}$ are SFEE
  - None of the above
- *Why specify value of g?*

Identifying SFEE: More on this example

- Example: Market with n buyers and m sellers, $n > m$
- Seller values: $s_H = 10$, $s_L = 4$
- Buyer values: $b_H = 12$, $b_L = 6$
- Fraction of good car sellers in population $g = \frac{4}{5}$

- How many distinct values of $h \in [0, 1]$ can be supported in a self-fulfilling expectations equilibrium (SFEE) in this market?
  - A One: There is a unique equilibrium at $h = \frac{4}{5}$
  - B Two
  - C Three
  - D Every value of $h \leq g$ is an SFEE
  - E None of the above

Identifying SFEE: An example

Solving for equilibria in this market (arbitrary g):

- Question equivalent to: For what $h \in [0, 1]$ can we find price p 'supporting' h?
- First: Recall we must have $h \leq g$
- Next: Sellers have identical valuations for (good) cars!
  - If $p > 10$, *all* good sellers want to sell (as also bad sellers): $h = g$
  - If $4 < p < 10$, *all* good sellers don't want to sell (and bad sellers do): $h = 0$
- So two candidates for h: g and 0 except: At $p = 10$, good sellers are *indifferent!*
  - $n > m$: So price $p = 12 \cdot h + 6 \cdot (1 - h)$ (for market-clearing)
  - So h must be $\frac{2}{3}$ if $p = 10$

Identifying SFEE: An example

Solving for equilibria in this market:

- So three candidate values for equilibria: $h = g, h = 0, h = \frac{2}{3}$
- $h = 0$ is always an equilibrium
  - $p = b_L > s_L$: All bad sellers are willing to sell and no good sellers want to sell
- $h = g$ is an equilibrium if and only if:

$$12g + 6(1 - g) \geq 10 \Rightarrow g \geq \frac{2}{3}$$

- Finally: $h = \frac{2}{3}$ is also an equilibrium if and only if $g \geq \frac{2}{3}$
  - Unstable equilibrium: Contrast with $h = 0, g$

Equilibrium outcomes with asymmetric information

Recap of example: Values $s_H = 10, s_L = 4$; $b_H = 12, b_L = 6$

- For $g \geq \frac{2}{3}$: Two kinds of possible equilibria
  - (i) Low prices, bad cars
    - Buyers expect used cars to be bad: Unwilling to pay high prices
    - Sellers of good cars unwilling to sell at low prices
  - (ii) Higher prices, mix of cars
    - Buyers expect higher value cars on average
    - Prices are high enough to attract good cars

- If $g < 2/3$: $h = 0$ is (only) equilibrium (with $p = 6$)
  - No equilibrium possible with $h = g$:
    - Maximum price buyers are willing to pay is $6 + 6g$
    - $g < 2/3$, so maximum possible price $6 + 6g < 10$
    - Sellers value good cars at 10: No seller of a good car wants to sell

Equilibrium outcomes with asymmetric information: Inefficiency

- Trading good cars benefits both sellers and buyers: Buyer value v_b exceeds seller value v_s for both good and bad cars
- But market **only** trades bad cars in equilibrium
- Inefficiency—due to information asymmetry!

Information asymmetry and inefficiency

- Main takeaway so far: Information asymmetry can lead to inefficiency
  - Efficient outcome is full trade: All cars are traded
  - With symmetric information: Efficient outcome is realized
  - Information *asymmetry*: When g is low enough ($g < 2/3$), **only** bad cars are sold on market
- *Asymmetric* information, enough bad cars:
  - Good cars driven out of market, only bad cars sold
  - As a market designer: Not a very nice market!

Asymmetric versus incomplete information

- Suppose **neither** buyers nor sellers can distinguish car quality
- Quality of car traded is random: Let probability of car on market being good be h
- Suppose $g < 2/3$: Is $h = g$ an equilibrium when *both* sides lack information about quality? ([A] Yes [B] No)
  - Expected value of a car to buyer: $12g + 6(1 - g)$
  - Expected value of car to seller: $10g + 4(1 - g)$
  - Trade at price $p = 12g + 6(1 - g)$: p is higher than all sellers' *expected* values from car, so all sellers are willing to sell!
- Inefficiency is **not** due to *incomplete* information (*i.e.*, uncertainty) *alone*: **Asymmetry** is source of inefficiency!

Asymmetric information: Market failure

- So far: Market is inefficient, but still realizes some positive gains from trade ($b_L - s_L > 0$)
- Situation can get even worse: Enter the lemons!
- (The market for lemons, Akerlof 1970)
  - So far: Good cars ($b_H > s_H$) and bad cars ($b_L > s_L$)
  - Introduce third kind of car: 'Lemons' ($b_0 = s_0 = 0$)
  - Distribution of qualities in population: p_H, p_L, p_0 ($p_H + p_L + p_0 = 1$; all probabilities positive)
  - What happens with asymmetric information?

Asymmetric information: Market failure

- Set up: Good cars, bad cars, and lemons
  - Values: $b_H > s_H$, $b_L > s_L$, $b_0 = s_0 = 0$
  - Distribution of qualities in population: p_H, p_L, p_0
- Reasoning about outcome with asymmetric information:
  - SFEE: Buyer belief about distribution of qualities that induces same distribution of sellers in market (with offered price)
  - Here: SFEE is distribution h_H, h_L, h_0
- Suppose $p_H = p_L = p_0 = 1/3$. Which of the following h **cannot** occur in equilibrium for *any* agent values?
  - A $h_H = h_L = h_0 = 1/3$
  - B $h_H = h_L = 1/2, h_0 = 0$
  - C $h_H = 0, h_L = h_0 = 1/2$
  - D All of these could occur in equilibrium

Asymmetric information: Market failure

An example with specific buyer and seller values:

- Three types of cars: Good (G), bad (B), and 'lemons' (L)
  - *Sellers' values*: G at 10, B at 4, L at 0
  - *Buyers' values*: G at 12, B at 6, L at 0
  - *Distribution*: Equal fractions (1/3) of each type
- What is market outcome with symmetric information?
  - Complete information: 'All' cars are traded (Why?)
  - Incomplete information on both sides: Again, all cars are traded (Why?)
- What happens with asymmetric information?

The market for lemons

Equilibria: Belief about distribution of qualities *in market*

- (i) Belief: All cars on market
  - Expected value of a car to buyers is $\frac{1}{3}(12+6+0)=6$
  - Price p cannot be greater than 6 in equilibrium
  - $p<10$: So sellers of good cars will not sell
  - All cars on market: Not equilibrium
- (ii) Belief: Only bad cars and lemons on market
  - Expected value of a car is $\frac{1}{2}(6+0)=3<4$
  - So sellers of bad cars will not sell: Not equilibrium
- (iii) Belief: Only lemons on market
  - Value and price both 0: Equilibrium!
- Why only three candidates for equilibria?

Complete market failure: No gains from trade in market!

Information asymmetry: Adverse selection and market failure

- Information asymmetry with 'hidden information' leads to *adverse selection*:
  - Buyers cannot distinguish quality: Uniform price
  - 'Lower half' of market participates: Further drive down average quality, prices
- **Uniform prices 'select'** worst traders into market
- Market can completely unravel and collapse!

Information asymmetry: A richer example

- A richer example: Continuous distribution of item qualities
- Consider market for used cars with spectrum of qualities
  - Sellers' values V_s uniformly distributed between 0 and 1
  - Buyer's value, *if* quality were discernible: $V_b = 1.5V_s$
- With complete information: All cars would be sold
- Again: What happens with asymmetric information?
- SFEE is belief about distribution of quality: What are candidate distributions?
  - Threshold V_s^* with sellers in $[0, V_s^*]$ in market
  - Why?: If seller with value v_s is willing to sell at p, all sellers with values $v_s' < v_s$ are also willing to sell

Information asymmetry: A richer example

- Market for used cars with spectrum of qualities
  - Sellers' values V_s uniformly distributed between 0 and 1
  - Buyer's value, *if* quality were discernible: $V_b = 1.5V_s$
- What is equilibrium price p in this market with asymmetric information?

A $p = 1$

B $p = \frac{1}{2}$

C $p = \frac{3}{4}$

D $p = \frac{2}{3}$

E None of the above

Information asymmetry: A richer example

- SFEE: Distribution of qualities in market can only be 'uniform below threshold'
  - Threshold V_s^* with sellers in $[0, V_s^*]$ in market: If seller with value v_s is willing to sell at p, all sellers with values $v_s' < v_s$ are also willing to sell
- Solving for equilibrium threshold V_s^*
  - At p: Sellers with values $V_s \leq p$ will sell; rest don't
  - Expected value to buyers, **given** sellers who sell at p:

$$E[V_b|p] = 1.5E[V_s|p] = 1.5 * p/2 = 0.75p$$

- For equilibrium, $E[V_b|p] \geq p \Rightarrow 0.75p \geq p$: Only solution is $p = 0$!