A bipartite graph (=num nodes on L&R) has no perfect matching iff it has a constricted set on R>N(R) (ie 3 nodes on R, matched to 2 nodes on L). Perfect matching includes all nodes in graph.
Alternating path: path alternates btwn matched & um nodes
Augmenting paths: Alternating paths with unmatched endpoints. Matching can be enlarged if Augment path exists.
Prove: Constricted Set exists, then no Perf Match . easy. PM then no CS-- take a max matching, try to use Alt BFS. Will fail. This means either PM or constricted set.
Alternating BFS: UM RNode, look for non matched edge from R, matched edge from L. If produces layer with UM node from L, then keep going or Aug path. Max Match:cant be enlarged. ABFS will fail

Strategyproof: mechanism is sp if agents cant benefit by lying about their preferences, no matter what other agents report
Serial Dictatorship Mech is SP even when pref are incomplete
Thrm Svensson'98:mech that is non-bossy, neutral & SP is SD.
PE = matching or mech ; SP = mech; IR is quality of mech
PE, IR, Core: Both mechanisms and matchings; SP: Only mechanisms
Indiv rationality: No agent better off not participating in the market
PE w init endowments: IR and no grp of agents can do better by not participating in the market and trading amongst themselves
In **Core** if M has no subset of agents C w a M' where:
1- for any a in C, that agent gets matched some a' in C's item
2,3- m'(a)>=m(a) for all a in c & m'(b)>m(b) for at least one b in

Pareto Dominate: Matching M PD M' IF,
1: M(a)>= M'(a) for EVERY agent A
2: 1 agent is strictly better off in M
Pareto Efficient: matching is PE if no other matching can PD it. WRT specific pref profile. This is our not bad matching.
Comparison btwn matching not agents
Matching: frm set of agents to set of obj
Mechanism: frm set of preference profiles to set of matchings
Serial dictatorship mech is strategy proof & thr4 PE WRT strict rank preference

Thm: Assume agents have strict preferences. There exists a core matching for any housing market. using Top Trading Cycles algo
P: set of all agent-item pairs in the market graph G(P): Each agent points to (pair corresponding to) her most-pref item in P
Lema: If P is non-empty there exists at least one cycle in G(P) and no cycles intersect
Assign each agent item pointing to
Remove the agents who got shit from P
Keep going till P empty. TCC= CORE MATCH
THM: The matching produced by Gale's TTC algorithm is the unique core matching.
THM: core mechanism is the only mech that is IR, PE, and SP for a 'housing market'

Serial Dictatorship Mech: Agent f(i) gets item ranked highest in her pref list amongst all item except those already taken by agents f(1) to f(i−1), where i is some rank -- ----- HW1
Bayes: P(B|A) = P(A|B)P(B) / P(A)
P(A)=P(A| B)P(B) + P(A|B')P(B'))
Stmnt: if P then Q; Conv: If Q then P
Inverse: If not P then not Q
Contrapos: If not Q then not P
Statement = T = contrapositive
Converse = T = inverse

Serial Dictatorship WEAK pref
- not PE: a: x=y, b:x>y
rank a then b, if a:x b:y not PE
- is SP- no agent has an incentive to lie abt pref, since receives a remaining item that she most prefed according to her pref profile at given turn

-Is every maximum matching in a graph Pareto-efficient with respect to the preferences represented by the underlying graph? YES conta:
max matching M*, and WTS M PD M*
i. Binary pref so strictly better off = nothin in M*to smthing in M (2), no worse off means stay matched in both (3)
ii. bc M PD M* Eagent not matched in M*, but matched in M
3 bc M PD M* all mtched in M* must be matched in M- not worse off
iv. 2, 3 size M > M* bc 1 more agent is matched in M than M*
v. contradiction, M* is a max matching, and size of matching M cannot be bigger than the size of M*.
So it is impossible for a matching M to PD M* hence M* is PE
- pref profile that creates unique PE match - all have diff top choices
- If all agts has same cmplete pref then all matching possible is PE

iClicker: 1 sided market w init endowments:
-There is exactly 1 cycle in G(P) at every step of algorithm: FALSE (two people point to their own item)
- at any step: Each agent in P belongs to exactly one cycle in G(P): FALSE (what if no cycle at all)
*Suppose that at step l, a1's second choice is h2 and a2's top choice is h1.
- If a1 does not belong to a cycle in step l, then 1 a2 does not belong to a cycle either: TRUE (bc a2 would point to a1 and a1 not in cycle so a2

- bipartite graph has more than one max matching. Then, there must be more than one constricted set. FALSE..
R:(A, B) L:(x, y) edges = (A−x, B−x).
1 const set {A,B}, 2 max matchings:A−x & B−x
- Max matching in such a graph. Does an ABFS starting from UM R node on the R always yield a constricted set containing all UM RHS nodes. NO Take two of graph above, then CS wont have all
-- each student who is UM in a maximum matching sure to belong to some constricted set of R nodes
-Binary preferences, graph G has match M1& M2 (m2 larger than m1) will M2 always PD M1? NO
M1 = (a − x, c − z), M2 = (b − x, c − w, d − z). Then matching M2 is larger than M1 but does not Pareto-dominate it, since agent a is unmatched in M2 and therefore unhappier in M2 than in M1.

iClicker: 2 sided pref

Blocking pair: M W who prefer each other to their current partners
Stable matchings: Matchings w no blocking pairs
- not all PE matches are stable (M 1>2) (W 2>1) M1:W1, M2:W2. This is PE but not stable
- if pref struct and stable match exists, then PE
Stability: no pair can improve
Core: no group of agents can improve
-if pref are strict, core of the marriage market equals the set of stable matchings.
- if matching U' PD U then it dominates U
- if matcin U' dom U doesnt necessarily dom U
- PD, everyone >=
- dominates, some subset better off
- any instance of the stable marriage problem, there always exists a stable matching
Achievable partner: w is achievable for m if there is some stable matching μ where μ(m) = w

Stability: No college-student pair exists that prefer each other to current allocation
Matching μ is stable in M if no mutually acceptable pair (s, c) such that
- s is um & c havnt filled its quota
- s um & c pref s to at least 1 of its students
- c hasnt filled quota & s pref c to its college
- s prefers c to μ(s), and c prefers s to at least one student in μ(c)
Thm: NRMP algo ret a stable matching & same outcome as the hospital-proposing DA algo
Thm: The set of s & c that are um is same for all stable match: s um in some stble match is um in all stable matching; all c fill the same num of positions across stable matchings

DA each round:
-Each 'free' man m proposes to his top-ranked woman w who has not rejected him yet
- Each woman chooses amongst her current 'provisional' partner, and the best proposal from the current round
-Woman is engaged to this chosen man at the end of the round, and rejected men become free
Algo terminates in n rounds given n M and n W
- DA gives stable matching
Incomplete pref, then stability = no blockng pairs AND IR (no agent better off in another matching
Comparing matchings- matching M better if all agents on one side like it more than M'
DA algo returns a M-op stable matching, that is, a SM every M weakly prefers to all other SM
- No M is rejected by his most-preferred achievable w in M DA algorithm

Marriage model questions
-does every stable matching have a pair (m,w) where both ranked eo as top choice. NO switch prefs
- if m and w are at top of eo priority list will they match in stable match? YES if not blocking pair
- man and women at bottom of eo lists can they match, n>=2? YES m and w same preferences
- an arbitrary pair of stable matchings may not be comparable, but DA creates a stable that is.
unique stable matching, Mp & WP return same outcom
- if Mp and Wp produce same outcome, is outcome the only stable matching? YES
Say there is some diff stable match V other than U
MP says all men weakly prefer v(m) to given u(m)
Wp says w weakly prefer v(w) to u(w)
U and v are diff so at least one pair are mtched to diff
BUT pref are strict, m must prefer w to v(m), and w must also prefer m to v(w). (m, w) is a blocking pair in v, so that v cannot be stable.

For any two stable matchings μ and μ',
μ >=M μ' iff μ' >= W μ.
The M op stable matching is W pessimal. Similarly, the W op stable matching is M pessimal.
The set of men and women that are unmatched is the same in all stable matchings
-M cannot be simultaneously SP and a stable matching mechanism
- The M DA algo SP for M

THM: The college-proposing DA algorithm is not SP for students. Also, no stable mechanism is SP for colleges: in particular, the college-proposing DA algorithm is not SP for colleges
-NRMP algorithm is not strategyproof
- exists a stable matching in any many-to-one matching market
- matching μ is stable in the many-to-one market M iff the corresp matching μˆ is stable in Mˆ .
A student is either unmatched in all stable matchings or matched in all stable matchings; the number of students assigned to a given college is the same across all stable matchings.

Information asymmetry: Some subset of agents in market have more information about goods or services exchanged
- now we have cardinal values ($$) rather than ordinal
- price vector is market clearing if we have perfect matching
- No matching which is Pareto-dominated can be welfare-max
- For every PE matching M, there is a valuation profile such that M is a welfare-maximizing allocation for vij .
- Outcome (p, vb − p) Pareto-dominates (vs, 0) if vs<p<vb
Self-fulfilling expectations equilibrium- If buyers 'expect' this distribution (fraction h of good cars /cars), then the offered (market-clearing) price induces that distribution of sellers in market
- ie buyers nor sellers will change v based on other
Sfee h : h *bH + (1 − h) * bL ≥ p