Networks II: Market Design—Lecture 12 Matching markets without money: Kidney Exchange

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Recap: Last time

- Revisiting theorem: TTC algorithm produces a core matching
- Uniqueness: Exactly one matching in the core!
- Core *mechanism* is strategyproof
- Characterization: *Any* mechanism that is individually rational, Pareto efficient and strategyproof must be the core mechanism
- A note: Markets with mixed initial endowments
  - 'House allocation with existing tenants': YRMH-IGYT mechanism

Coming up today

- A real-world application of one-sided markets without money: Kidney exchange design
  - Why kidney exchanges?
  - A formal model for kidney exchange
  - Market design
  - Strategic issues and open problems

Kidney Donation

- Kidney transplantion: Most preferred treatment for end-stage renal disease
- National Organ Transplant Act (1984): Illegal to buy or sell a kidney in US
  - "It shall be unlawful for any person to knowingly acquire, receive or otherwise transfer any human organ for valuable consideration for use in human transplantation."
- *Donation* only viable option for kidney transplants

Kidney Donation

- Kidney transplants: An important problem
  - $\sim$16,500 kidney donation transplants conducted in 2008
  - 4,200 patients died while waiting for a kidney
- Two types of transplants: Deceased or live donor
  - 2008: $\sim$ 10,500 from deceased donors, $\sim$ 6,000 from living donors
  - Significant shortage of deceased donor kidneys:
    - March 2009: $\sim$79,000 patients in (US) waitlist

Living donors

- One healthy kidney typically adequate for body's functioning: Feasible to donate other kidney
  - Live donor: Typically relative or friend of recipient
  - Kidney from live donor survives significantly longer than from deceased donor
- Increase in number of live donations with advances in medical technology

Donor Types	2008	1998	1988
All donors	10,920	9,761	5,693
Deceased donors	5,992	5,339	3,876
Live donors	4,928	4,422	1,817

Kidney *exchange*

- Recipient often cannot receive willing live-donor's kidney due to incompatibility (blood-type, antibodies)
- Donors typically willing to give kidney only if particular recipient can receive transplant
- Incompatibility: No kidney for patient, 'wasted' live donor
- How to increase number of transplants?

Kidney exchange

- Live-donor (paired) kidney exchanges: Donor in each pair gives kidney to the other pair's recipient
  - Proposed by medical doctor F. T. Rapaport, 1986
- A paired exchange: Two patient-donor pairs (say pair 1 and 2) such that
  - Donor in pair 1 is incompatible with patient in pair 1, but compatible with patient in pair 2
  - Donor in pair 2 is incompatible with patient in pair 2, but compatible with patient in pair 1
  - Exchange: Donor 1's kidney transplanted into patient 2; donor 2's kidney transplanted to patient 1

Kidney exchanges: A history

- 1990s: Korea, Netherlands start building databases to organize swaps
  - 2005: Exchanges $>$ 10% of the live-donor transplants in both countries
- 2000: Kidney exchange deemed ethical by medical community in US
- New England, Ohio, Johns Hopkins transplant programs start live-donor kidney exchange operations

Kidney exchanges: A history

- Initial hurdle in organizing kidney exchanges: Lack of mechanisms to clear market in efficient, incentive-compatible way
- 2004: Renal Transplant Oversight Committee of New England approved establishment of *clearinghouse* for kidney exchange
- Design of clearinghouse *jointly* by economists (Roth, Sonmez, Unver) and doctors

Kidney exchange and markets

- Fundamental issue: Allocations (who gets what?)
  - How to decide who gets kidney donations, and which kidneys go to whom?
- Any mechanism for allocation must address:
  - Efficiency: Pareto efficiency; maximizing number of transplants
  - Incentives (strategyproofness)
  - Fairness

Rewinding back to Lecture 1

Three aspects:

- Modeling: How do you identify and *model* a real-world setting as an instance of networked behavior?
- Analysis: What are the general principles that apply to this instance?
- Design: Can we use our model and analysis to *design* for desirable outcomes?

Developing a model for kidney exchanges: The facts

Why do we need exchanges?

- Donor must be both blood-type and tissue type compatible with patient for transplant to be feasible
  - Blood-type compatibility: Recipient must have all blood-type proteins that donor possesses
  - Tissue compatibility (crossmatch): Antibodies cause tissue rejection, determined by HLA proteins
- Amongst feasible kidneys, likelihood of graft survival depends on various factors:
  - HLA match [Opelz 1997], donor age, kidney size, . . .

Developing a model for kidney exchanges

- A formal model: A kidney exchange consists of
  - Set of donor-patient pairs $\{(k_1, p_1), \ldots, (k_n, p_n)\}$
  - Set of feasible kidneys for each patient
  - A preference ordering over feasible kidneys $\{k_1, \ldots, k_n\}$ for each patient p_i

- *Matching*: Function specifying which patient obtains which kidney

Developing a model

- Does this scenario look similar to any model we've seen?
  - One sided preferences with initial endowments: The 'housing market' model!
  - TTC mechanism: Pareto-efficient, individually rational, strategyproof
- Does housing market model miss any (important) aspects of kidney exchanges?
  - Strict, complete preferences
  - Kidneys from deceased donors, and patients without live donors

Recall, from Lecture 1

Networks: Three aspects

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First design for centralized kidney exchange (RSU 2004)

- First design for centralized kidney exchange (Roth, Sonmez, Unver 2004):
  - Model: One-sided preferences with **mixed** initial endowments
    - Strict preference rankings
  - Mechanism: TTC-like mechanism, TTCC, has good efficiency, incentive properties
- Simulations using real population parameters show dramatic improvement in efficiency with TTCC

Design 2: Roth, Sonmez, Unver (2005)

Roth, Sonmez, and Unver discuss design in RSU'04 with doctors, who note that:

- Surgeries must be conducted simultaneously: Contracting is illegal
  - Only pairwise exchanges are easily feasible
- Binary (0-1) preferences better approximation to reality than strict preferences
  - All compatible kidneys equally good; all incompatible kidneys equally bad
- Compatible pairs unlikely to participate in an exchange

Three aspects to networks

Recall from Lecture 1: Three aspects to networks

- Modeling: How do you identify and *model* a real-world setting as an instance of networked behavior?
- Analysis: What are the general principles that apply to this instance?
- Design: Can we use our model and analysis to *design* for desirable outcomes?
- Iterate: Refine models (and correspondingly, analysis and design) based on feedback

Reformulating the model and design

- RSU'05: Redo kidney exchange design to account for input from doctors
- **Change in model**: Indifference amongst all acceptable options, instead of strict rank-order preferences
  - Can this setting be captured a model you have seen before?
    - A Yes!
    - B No, I haven't seen a model for this so far
  - A new twist on two prior models: Binary preferences, but *with* initial endowments
- In addition, **restriction on allowable mechanisms**: Two-way exchanges only
  - Cannot remove cycles of length greater than 2

The second iteration: A formal model

- N: Number of incompatible donor-patient pairs
  - Assume only incompatible pairs participate
- $R = (r_{ij})_{i,j \in N}$ is *mutual compatibility* matrix:

$$r_{ij} = \begin{cases} 1 & \text{if } \textbf{pairs } i \text{ and } j \text{ are mutually compatible,} \\ 0 & \text{otherwise} \end{cases}$$

- Note: Matrix R simply encodes (**undirected, non-bipartite**) graph G_R of feasible two-way exchanges
  - Nodes in graph: (Patient, donor) tuples (p_i, k_i)
  - Edge between nodes i and j iff p_i is compatible with k_j and p_j is compatible with k_i

The second iteration: A formal model

- **Matching** is now a **set of edges in G_R with no common endpoints**
  - Note carefully: Matching in **non-bipartite** graph G_R!
- Formally: Matching is function μ from N to itself s.t.
  - $\mu(i) = j$ if and only if $\mu(j) = i$: Only pairwise exchanges are possible
  - If $\mu(i) = j$ $(i \neq j)$, then $r_{ij} = 1$: Only mutually compatible exchanges are possible
- Matching μ is Pareto efficient if there is no other matching in G_R that makes every patient weakly better off and at least one patient strictly better off

An example

- Exchange with 4 patient-donor pairs: Patient p_1 is compatible with donors k_2, k_3, k_4, patient p_2 is compatible with donor k_1, patient p_3 is compatible with donor k_2, and patient p_4 is compatible with donor k_1
- Consider the following exchanges:
    - E_1: k_1 donates to p_2, k_2 donates to p_1
    - E_2: k_1 donates to p_4, k_4 donates to p_1
    - E_3: k_1 donates to p_2, k_2 donates to p_3, k_3 donates to p_1
- Which of these constitute a *Pareto-efficient matching* in our new setting?
    A E_1 only
    B E_1 and E_2 only
    C E_2 and E_3 only
    D E_3 only

Recall, from Lecture 1

Networks: Three aspects

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An efficiency result

Lemma (RSU (2005))

The same number of patients are matched at each Pareto-efficient matching; this is the maximum number of patients that can be matched.

- Note: Result does not hold if exchanges larger than two-way exchanges are possible!
- Why?: What are the Pareto-efficient matchings with multi-way exchanges in our previous example?
  [A] E_1 only [B] E_1 and E_2 only [C] E_2 and E_3 only [D] E_3 only
- Patient p_1 is compatible with donors k_2, k_3, k_4, patient p_2 is compatible with donor k_1, patient p_3 is compatible with donor k_2, and patient p_4 is compatible with donor k_1
  - E_1: k_1 donates to p_2, k_2 donates to p_1
  - E_2: k_1 donates to p_4, k_4 donates to p_1
  - E_3: k_1 donates to p_2, k_2 donates to p_3, k_3 donates to p_1

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Clearing the market

- Finding a Pareto-efficient matching with only two-way exchanges: 'Cardinality matching problem'
  - Maximum matching in a non-bipartite, undirected graph
  - Polynomial-time algorithms (Edmonds 1965, ...)
- Mechanism: Needs to also incentivize truthful reporting!
  - A mechanism is strategy-proof if no pair benefits by misreporting who is mutually compatible
  - Not all *algorithms* may be equally good as *mechanisms*

A priority mechanism

- A **priority mechanism**: Choose some ordering of pairs (random, waiting time based, medical considerations, ...)
  - If there is any matching in which the top priority pair is matched, then match that pair. Otherwise, skip that pair
  - Match the second-top priority pair if there is such a matching that also matches the first pair (if they were matched in the previous step), then match the pair. Otherwise, skip that pair

    ⋮

  - Match the k^{th} top priority pair if there is such a matching that also match all the pairs that were matched in previous steps, then match the pair. Otherwise, skip that pair

Properties of the priority mechanism

Theorem (RSU 2005)

The priority mechanism is Pareto-efficient, and strategy-proof.

- Note: Priority mechanism not same as greedy algorithm to find a matching
  - Greedy algorithm only returns maximal, not maximum matching
- Entire matching may change from step to step
  - Fact: Updating the matching is computationally efficient

Going forward: Efficiency gain from multi-way exchanges

- Evolution in medical technology: An exchange involving more than two pairs may be difficult, but may not be infeasible
- Logistical constraints nonetheless remain real
  - Two-way (pairwise) exchanges are easier than three-way, which are easier than four-way exchanges, and so on
- How much efficiency can really be gained through larger cycles?

Back to analysis

- Roth, Sonmez, Unver 2007: Investigating benefits from multi-way exchanges
  - Theoretical results: Bounding number of exchanges in terms of patient-donor blood type pairs
  - Simulations to evaluate improvement: Parameters based on US population
    - Three-way exchanges add lot of transplants, majority of improvement
    - Four-way exchanges add only a little
    - But what about five, six, . . . -way exchanges?

Back to analysis: Efficiency gain from larger cycles

Theorem (RSU 2007)

Consider a patient population for which certain reasonable assumptions hold, and let μ be any maximum matching (when there is no restriction on the size of the exchanges). Then there exists a maximum matching $\hat{\mu}$ that consists only of two-way, three-way, and four-way exchanges, under which the same set of patients receive transplants as in matching μ.

- Four-way exchanges may suffice: All efficient matchings can be achieved just using cycles of length two, three, and four

Many other challenging problems

- Computation: Larger cycles; chains
- Incentives: Transplant centers
- Dynamics: Optimizing over time
- Weighted matchings: Likelihood of transplant success
- Patients in multiple exchanges

Summary

- Markets for organs illegal: Difficulty in efficiently allocating organs
- Matching markets without money: Matching theory to design allocation
- Design considerations: Efficiency, incentives, fairness
- Existing matching theory is useful, but practicalities also motivate new models and problems: Plenty of new research problems!

Do patients and doctors behaves strategically? Here is one example indicating they do.

A news report by Reuters (2003-7-29)

> *Three Chicago hospitals were accused of fraud by prosecutors on Monday for manipulating diagnoses of transplant patients to get them new livers. Two of the institutions paid fines to settle the charges.*
> *"By falsely diagnosing patients and placing them in intensive care to make them appear more sick than they were, these three highly regarded medical centers made patients eligible for liver transplants ahead of others who were waiting for organs in the transplant region," said Patrick Fitzgerald, the U.S. attorney for the Northern District of Illinois.*

Transplant center incentives

- Priority mechanism is strategyproof with respect to reporting edges: Each pair reports mutual compatibilities accurately
- But what about reporting the *nodes* themselves?
- *Transplant center* incentives: Hiding nodes may be beneficial
  - Consider compatibility $v_1 - v_2 - v_3 - v_4 - v_5 - v_6 - v_7$
  - Hospital 1 'owns' v_1, v_4, v_5, v_6
  - Maximum matching could leave v_1 unmatched
  - 'Hide' v_5, v_6: Unique maximum matching $v_1 - v_2$, $v_3 - v_4$
- Hospitals may not declare all their pairs to the exchange!
- Loss in welfare: Active research area