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Networks II homework 1

(1) The solutions to 1a and 1b both can be solved using Bayes' Theorem. Bayes theorem is that the probability of event A given event B has happened is equal to the probability of B given A times the probability of event A divided by the probability of B. As a formula it is: $P(A|B) = P(B|A)P(A)/P(B)$
We are given that: P(Negative Experience (NE)|Bad Doctor (BD))=0.96
P(Negative Experience (NE)|Good Doctor (GD))=0.02
(a)We are additionally given that: P(BD)=0.05
Find P(BD|NE).
By applying Bayes Theorem we have: $P(BD|NE) = P(NE|BD)P(BD)/P(NE)$
The only piece that is missing is P(NE). This can be found by breaking down this probability into P(NE)=P(NE|BD)P(BD)+P(NE|GD)P(GD).
As P(BD)=0.05 then P(GD)=1-P(BD)=0.95
Thus, P(NE)= (0.96)(0.05)+(0.02)(0.95)=0.067
In turn $P(BD|NE) = \frac{(0.96)(0.05)}{0.067} = 0.716$
Now, Find the probability that a doctor who's patients have two negative experiences in a row are in fact bad.
In the first part of this question we found that after the first negative experience there is a 71.6% chance that the doctor is bad and in turn there is a 28.4% chance that the doctor is good. Now applying Bayes' Theorem we have P(BD| the second bad experience)=P(the second bad experience |BD)P(BD after the first negative experience)/P(second NE)=(0.96)(0.716)/((0.716)(0.96)+(0.02)(0.284))=0.992
(b) We are additionally given that: P(BD)=0.6
Find P(BD|NE).
Applying Bayes' Theorem we have $P(BD|NE) = P(NE|BD)P(BD)/P(NE)$
Thus as before P(NE)=P(NE|BD)P(BD)+P(NE|GD)P(GD) So overall,

$$P(BD|NE) = \frac{(0.96)(0.6)}{(0.96)(0.6) + (0.4)(0.02)} = 0.986$$

(2) (a) i. This statement is not necessarily true. It is the converse of the original statement and thus is not always necessarily true. Simply, the person's sleep problems may not be the result of blue light and could instead be the result of another ailment.

ii. This statement is true. This is the contrapositive of the original statement. This statement is necessarily true as in all cases if blue light then there will be sleep problems thus if there are no sleep problems then there cannot be blue light as the statement says.

iii. This statement is not necessarily true as it is the inverse of the original statement. A simple counterexample is that a person could not be using blue light and they could still have sleep problems due to another ailment.

(b) David could certainly be used to disprove the statement. As David is an avid user of blue light, Carla could ask him if he sleeps well. If he answers that he sleeps well then the claim is disproven. If he answers that he doesn't sleep well it proves nothing as the blue light may not necessarily be causing the sleep problems. Jessica cannot be used to prove anything as whether or not she has sleep problems cannot say anything about the claim as she does not use blue light before bed. Kim can be used to help disprove the claim. Carla must ask Kim whether or not she uses blue light before bed. If Kim does in fact use blue light before bed then the claim has been disproven as she uses blue light but still sleeps well. If Kim says she does not use blue light then nothing can be said about the claim. Ethan cannot be used for the claim as he has sleep problems. Even if he says that he uses blue light before bed, it may not necessarily be related as his sleep problems could be the result of some other cause.

(3) (a) App G does not Pareto-dominate B as G has more friends but it has a lower usability score. In order to Pareto dominate B, G would have to have at least one higher score and the other score would have to be at least the same as the score of B but in this case G's usability score is lower and so fails the first condition of Pareto-domination. B does not Pareto-dominate G as the first condition of Pareto-domination is failed. B has a lower score in terms of number of friends than G and thus it does not Pareto-dominate G.

(b) App G Pareto-dominates only app A. G has both a higher usability score and a higher number of friends and thus satisfies both conditions of Pareto-dominance. In all other cases G does not Pareto-dominate any of the other apps. This is because G's usability score of 3 is lower than all the other apps usability scores other than app A. Thus, G with all other apps fails the first condition of Pareto-dominance. App G is not Pareto-optimal as App G is dominated by both App C and App D and thus App G cannot be Pareto-optimal.

(c) App E is not Pareto-optimal as App E is dominated by App C. App C Has a higher number of friends and a higher usability score than App E meaning that App C dominates App E and therefore App E is not Pareto-optimal. App E Pareto-dominates App A only. App E has a higher number of friends and a higher usability score than App A and thus satisfies both conditions of Pareto-dominance. App E has a lower number of friends than all other apps except B, and for app B, App E has a lower usability score. Thus, App E fails the first condition of Pareto-dominance with all other apps than App A which is Pareto-dominates.

(d) Several Apps are Pareto-optimal these are, App B, App C, and App D.

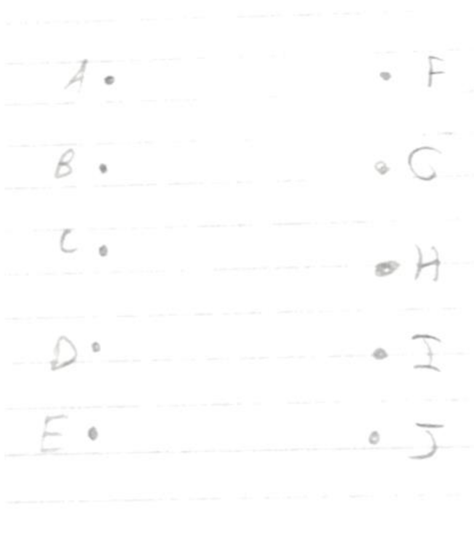
(4) (a) The statement is false by counter example. Using the bipartite graph laid out below, the C and D side being the agent side as constricted sets are limited to the agent side in this class.

Possible maximum matchings are the set of edges of {B-C} and the set of edges {B-D}. In both cases the constricted set is {C,D}. The cardinality of this set is 2 which is larger than the cardinality of the set of neighbors which is 1 so therefore the set of agents {C,D} is the only constricted set on the graph for two possible maximum matchings thus the statement is false.

(b) (i) Claim: Each student who is unmatched in a maximum matching sure to belong to some constricted set of RHS nodes.

Proof: Suppose for the sake of contradiction that one of the unmatched nodes does not belong to a constricted set. Then this means that it is not part of a set which is larger than the neighbors of this set. Thus, for all sets for which this unmatched student is a member is not a constricted set and has more neighbors than members of the set. Thus, there exists a matching within this set such that this unmatched student can be matched as the set is not constricted. If the unmatched node can be matched then this is not a maximum matching which is a contradiction. Thus, the claim holds.

(ii) This statement is false by the following counter example.



A maximum matching for the above bipartite graph is the empty set. Starting an ABFS starting from an unmatched RHS node for example F yields the set of {F} as F has no neighbors. As all RHS nodes are unmatched yet only one is yielded by the ABFS. Thus the claim is false.