ELEC ENG – 2CJ4

Laboratory Experiments (Set 5)

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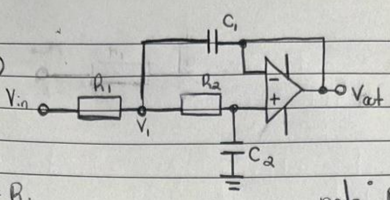
A) Derive an expression for the transfer function of the filter.

a)



$Z_1 = R_1$

$Z_2 = R_2$

$Z_3 = 1/sC_1$

$Z_4 = 1/sC_2$

note: $R_1 = R_2 = R$

$C_1 = C_2 = C$

$V^+ = V^-$

$V_o = V_1 \cdot \frac{Z_4}{Z_2 + Z_4}$

cond: $C = \frac{1}{sC}$

$V_1 = V_o \cdot \frac{Z_2 + Z_4}{Z_4}$

applying KCL at V_1: $\frac{V_1 - V_{in}}{Z_1} + \frac{V_1 - V_o}{Z_3} + \frac{V_1}{Z_2 + Z_4} = \emptyset$

$$\frac{V_o \cdot \frac{Z_2+Z_4}{Z_4} V_{in}}{Z_1} + \frac{V_o \cdot \frac{Z_2+Z_4}{Z_4} - V_o}{Z_3} + \frac{V_o \cdot \frac{Z_2+Z_4}{Z_4}}{Z_2+Z_4} = \emptyset$$

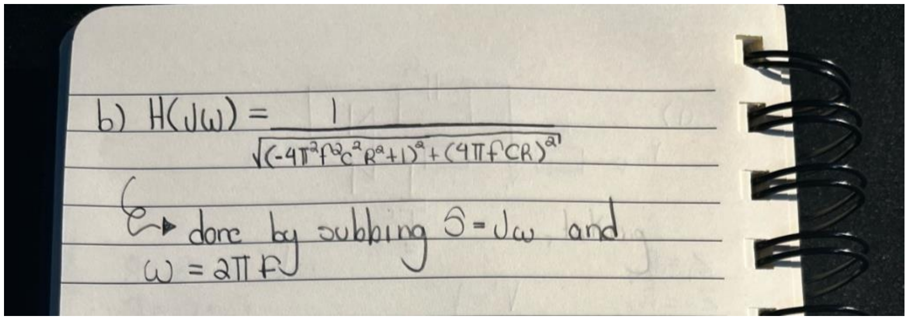
$$V_o \left(\frac{Z_2+Z_4}{Z_4 Z_1} + \frac{Z_2+Z_4}{Z_4 Z_3} - \frac{1}{Z_3} + \frac{1}{Z_4} \right) = \frac{V_{in}}{Z_1}$$

$$\frac{V_o}{V_{in}} = \frac{1}{\frac{Z_2+Z_4}{Z_4} + \frac{Z_1(Z_2+Z_4)}{Z_4 Z_3} - \frac{Z_1}{Z_3} + \frac{Z_1}{Z_4}}$$

$$\frac{V_o}{V_{in}} = \frac{1}{sC_2(R_2 + 1/sC_2) + s^2 C_1 C_2 R_1 (R_2 + \frac{1}{sC_2}) - sC_1R_1 + sC_1R_1 + sC_2R_2}$$

$$H(s) = \frac{1}{s^2C^2R^2 + 2sCR + 1} = \frac{1}{10^{-6}\omega^2 + j\omega(2\times10^{-3}) + 1}$$

B) Evaluate the filter transfer function abs($\frac{V_O}{V_{IN}}$) using the transfer function derived in part (a) for the frequencies shown in the table.



b) $H(j\omega) = \frac{1}{\sqrt{(-4\pi^2 f^2 C^2 R^2 + 1)^2 + (4\pi f C R)^2}}$

→ done by subbing $s = j\omega$ and $\omega = 2\pi f$

Frequency	abs ($\frac{V_O}{V_I}$) (analytical)	abs ($\frac{V_O}{V_I}$) (measured)
50 Hz	0.91	
100 Hz	0.72	
200 Hz	0.39	
500 Hz	0.092	
1 kHz	0.025	
1.1 kHz	0.020	
1.2 kHz	0.017	
1.3 kHz	0.014	
1.4 kHz	0.013	
1.5 kHz	0.011	
1.6 kHz	0.0097	
1.7 kHz	0.0087	
1.8 kHz	0.0078	
1.9 kHz	0.0070	
2 kHz	0.0062	
5 kHz	0.0010	

Figure 1: Frequency, Analytical, and Measured

C) Measure the transfer function using the AD2 board and fill the corresponding components of the table below. Use a sine wave with an amplitude of 2V and offset of 0V (Vcc = ±5F).

Frequency	abs $(\frac{V_O}{V_I})$ (analytical)	abs $(\frac{V_O}{V_I})$ (measured)
50 Hz	0.91	0.913
100 Hz	0.72	0.736
200 Hz	0.39	0.427
500 Hz	0.092	0.111
1 kHz	0.025	0.030
1.1 kHz	0.020	0.0249
1.2 kHz	0.017	0.0210
1.3 kHz	0.014	0.0178
1.4 kHz	0.013	0.0154
1.5 kHz	0.011	0.0134
1.6 kHz	0.0097	0.0118
1.7 kHz	0.0087	0.0105
1.8 kHz	0.0078	0.00914
1.9 kHz	0.0070	0.00819
2 kHz	0.0062	0.00740
5 kHz	0.0010	0.000685

Figure 2: Frequency, Analytical, and Measured

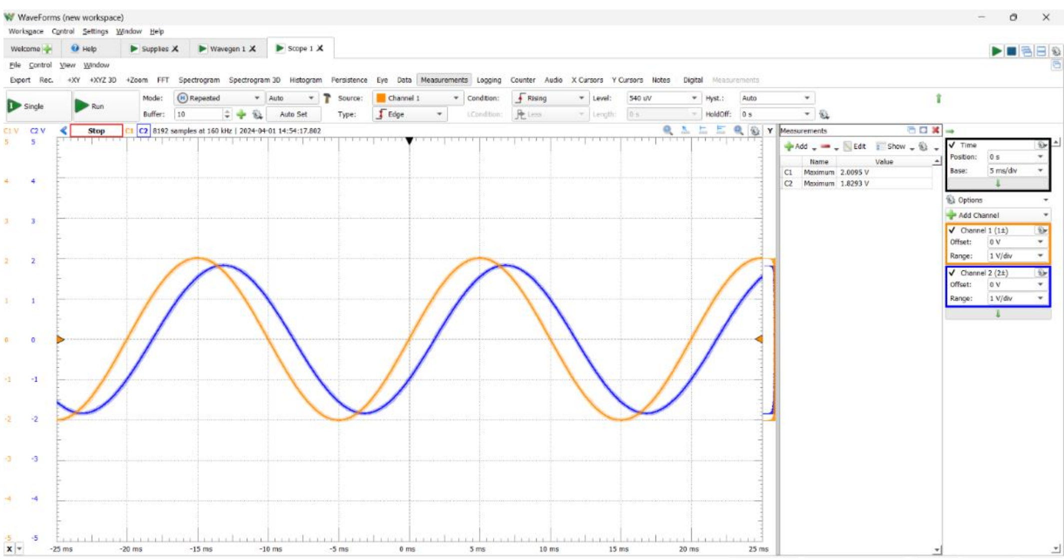


Figure 3: Waveforms Input (Orange) VS Output (Blue): 50Hz

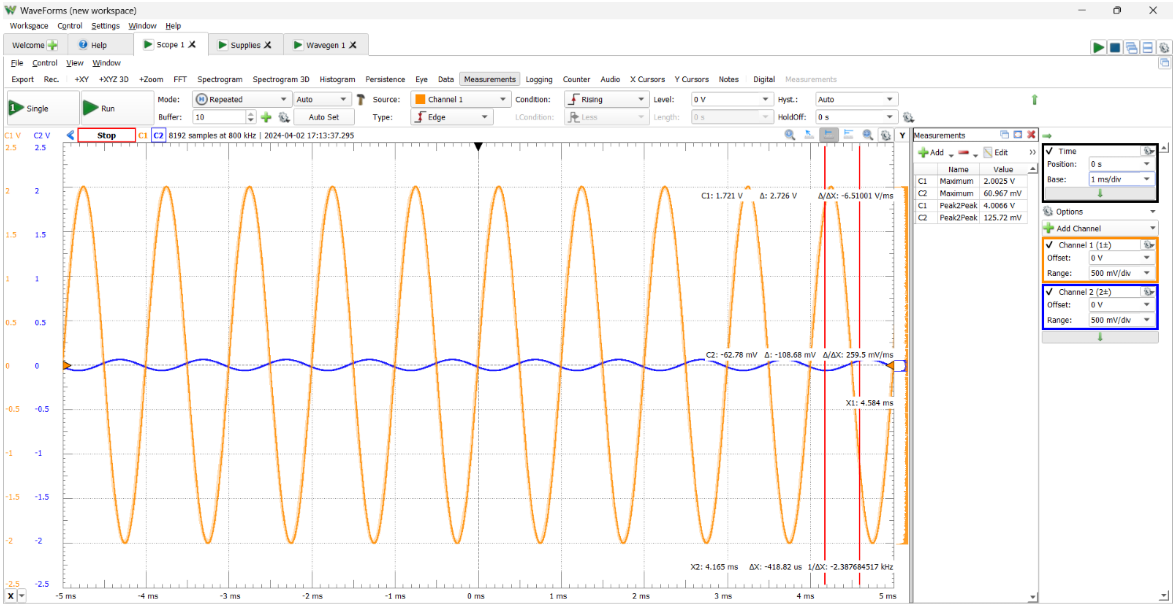


Figure 4: Waveforms Input (Orange) VS Output (Blue): 1kHz

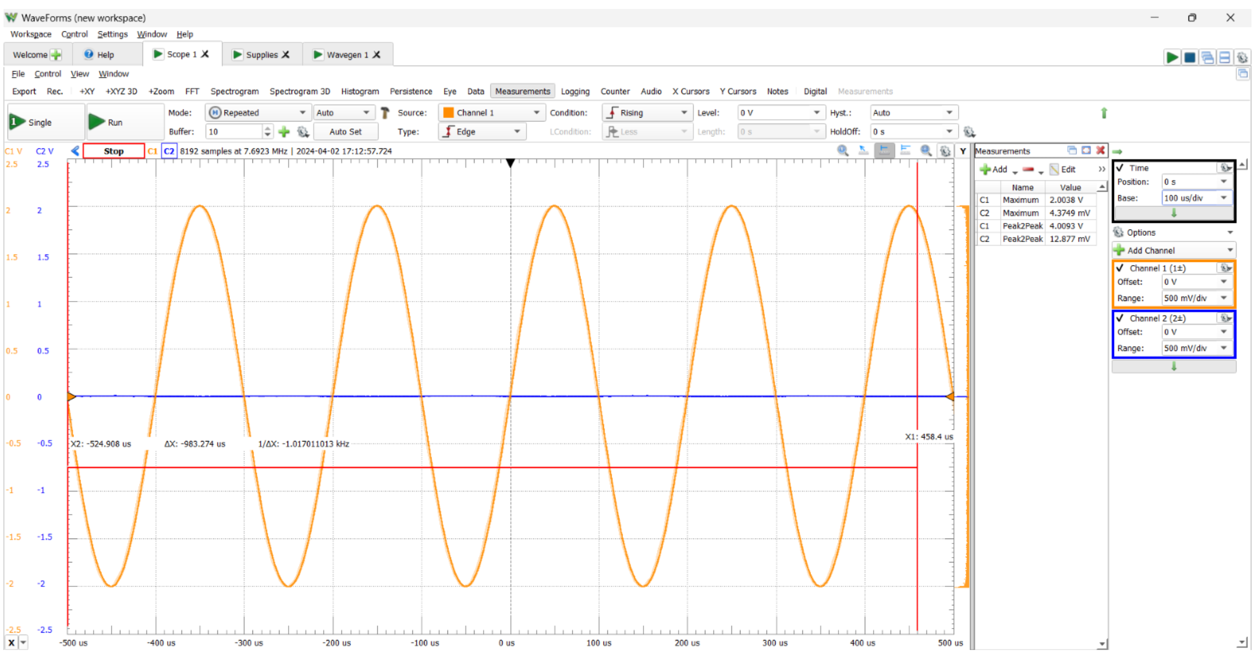


Figure 5: Waveforms Input (Orange) VS Output (Blue): 5 kHz

D) What is the cut-off frequency of this filter?

$$f_C = \frac{1}{2\pi \mathrm{RC}} = \frac{1}{2\pi(10\mathrm{k}\Omega)(100\mathrm{nF})} = 159.15\ Hz$$

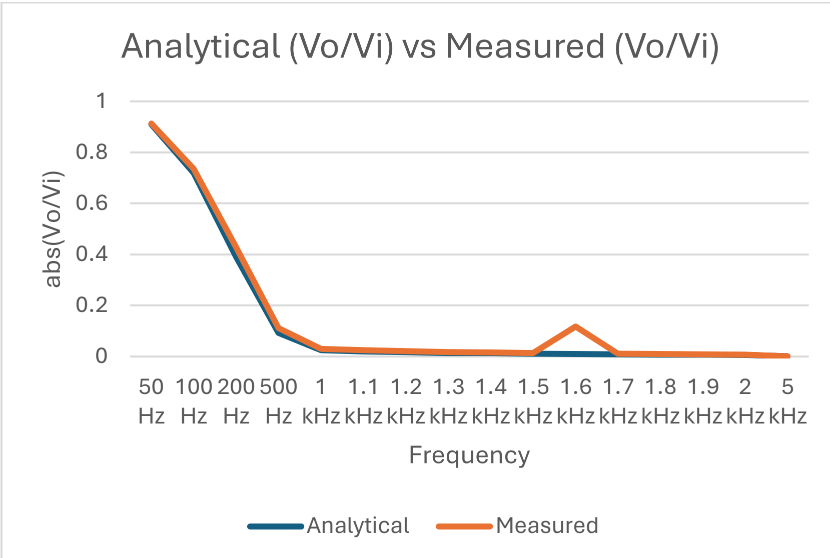


Figure 6: Analytical (Vo/Vi) vs Measured (Vo/Vi)

As demonstrated in both Figures 2 and 6, the analytical and the measured values are very similar for the lower levels of frequency; however, they become less accurate as the frequency is increased. This is expected as the further from the cut-off frequency, the gain will decrease thus, the difference amongst the values is noticed.