EE2FH4
Electromagnetics I

Term II, January – April 2024

MATLAB Examples and Exercises (Set 1)

Prepared by: Dr. M. H. Bakr and C. He

Example: Given the points M(0.1,-0.2,-0.1), N(-0.2,0.1,0.3) and P(0.4,0,0.1), find: a) the vector $\mathbf{R}_{NM}$, b) the dot product $\mathbf{R}_{NM} \cdot \mathbf{R}_{PM}$, c) the projection of $\mathbf{R}_{NM}$ on $\mathbf{R}_{PM}$ and d) the angle between $\mathbf{R}_{NM}$ and $\mathbf{R}_{PM}$. Write a MATLAB program to verify your answer.

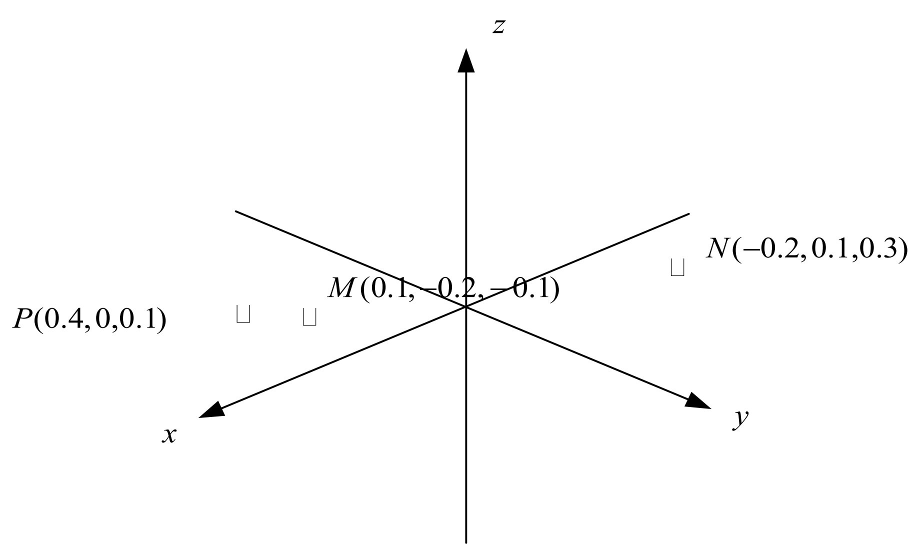


Figure 1.1 The points used in the example of Set 1.

Analytical Solution:

a) $\mathbf{R}_{NM} = \mathbf{R}_{MO} - \mathbf{R}_{NO}$

$$= (0.1\mathbf{a}_x - 0.2\mathbf{a}_y - 0.1\mathbf{a}_z) - (-0.2\mathbf{a}_x + 0.1\mathbf{a}_y + 0.3\mathbf{a}_z)$$
$$= 0.3\mathbf{a}_x - 0.3\mathbf{a}_y - 0.4\mathbf{a}_z$$

b) $\mathbf{R}_{PM} = \mathbf{R}_{MO} - \mathbf{R}_{PO}$

$$= (0.1\mathbf{a}_x - 0.2\mathbf{a}_y - 0.1\mathbf{a}_z) - (0.4\mathbf{a}_x + 0.1\mathbf{a}_z)$$
$$= -0.3\mathbf{a}_x - 0.2\mathbf{a}_y - 0.2\mathbf{a}_z$$

$$\mathbf{R}_{NM} \cdot \mathbf{R}_{PM} = (0.3\mathbf{a}_x - 0.3\mathbf{a}_y - 0.4\mathbf{a}_z) \cdot (-0.3\mathbf{a}_x - 0.2\mathbf{a}_y - 0.2\mathbf{a}_z)$$
$$= 0.3 \times (-0.3) + (-0.3) \times (-0.2) + (-0.4) \times (-0.2)$$
$$= -0.09 + 0.06 + 0.08 = 0.05$$

c) $\text{proj}_{\mathbf{R}_{PM}} \mathbf{R}_{NM} = \dfrac{\mathbf{R}_{NM} \cdot \mathbf{R}_{PM}}{\mathbf{R}_{PM} \cdot \mathbf{R}_{PM}} \mathbf{R}_{PM}$

$$= \frac{0.05}{(-0.3)^2 + (-0.2)^2 + (-0.2)^2} (-0.3\mathbf{a}_x - 0.2\mathbf{a}_y - 0.2\mathbf{a}_z)$$
$$= -0.088\mathbf{a}_x - 0.059\mathbf{a}_y - 0.059\mathbf{a}_z$$

d) $\cos\theta = \dfrac{\mathbf{R}_{NM} \cdot \mathbf{R}_{PM}}{|\mathbf{R}_{NM}||\mathbf{R}_{PM}|}$

$$= \frac{0.05}{\sqrt{(0.3)^2 + (-0.3)^2 + (-0.4)^2} \sqrt{(-0.3)^2 + (-0.2)^2 + (-0.2)^2}}$$
$$= 0.208$$

$$\theta = \cos^{-1} 0.208 = 1.36$$

Definition
Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be two nonzero vectors in $\boldsymbol{R}^n$. The cosine of the angle θ between these vectors is $\cos\theta = \dfrac{\boldsymbol{v} \cdot \boldsymbol{v}}{|\boldsymbol{u}| \cdot |\boldsymbol{v}|}$, $0 \le \theta \le \pi$

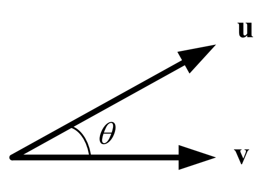


Figure 1.2 The angle between two vectors.

This problem is a direct application to vector algebra. It requires clear understanding of the basic definitions used in vector analysis.

The vector $\mathbf{R}_{NM}$ can be obtained by subtracting the vector $\mathbf{R}_{NO}$ from vector $\mathbf{R}_{MO}$, where O is the origin.

Definition
Let $\boldsymbol{u}=(u_1,\ldots,u_n)$ and $\boldsymbol{v}=(v_1,\ldots,v_n)$ be two vectors in $\boldsymbol{R}^n$. The **dot product** of $\boldsymbol{u}$ and $\boldsymbol{v}$ is defined by

$\boldsymbol{u} \cdot \boldsymbol{v} = u_1 v_1 + \cdots + u_n v_n$

The dot product assigns a real number to each pair of vectors.

Definition
The projection of a vector $\boldsymbol{v}$ onto a nonzero vector $\boldsymbol{u}$ in $\boldsymbol{R}^n$ is denoted $\text{proj}_u \boldsymbol{v}$ and is defined by

$$\text{proj}_u \boldsymbol{v} = \frac{\boldsymbol{v} \cdot \boldsymbol{u}}{\boldsymbol{u} \cdot \boldsymbol{u}} \boldsymbol{u}$$

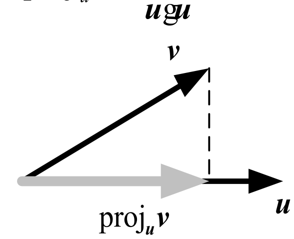


Figure 1.3 The projection of one vector onto another.

MATLAB SOLUTION:

```
clc; %clear the command line
clear; %remove all previous variables

O=[0 0 0]%the origin
M=[0.1 -0.2 -0.1];%Point M
N=[-0.2 0.1 0.3];%Point N
P=[0.4 0 0.1]%Point P

R_MO=M-O;%vector R_MO
R_NO=N-O;%vector R_NO
R_PO=P-O;%vector R_PO

R_NM=R_MO-R_NO;%vector R_NM
R_PM=R_MO-R_PO;%vector R_PM

R_PM_dot_R_NM=dot(R_PM,R_NM);%the dot
product of R_PM and R_NM
R_PM_dot_R_PM=dot(R_PM,R_PM);%the dot product of R_PM and R_PM

Proj_R_NM_ON_R_PM=(R_PM_dot_R_NM/R_PM_dot_R_PM)*R_PM;%the projection of R_NM ON R_PM

Mag_R_NM=norm(R_NM);%the magnitude of R_NM
Mag_R_PM=norm(R_PM);%the magnitude of R_PM

COS_theta=R_PM_dot_R_NM/(Mag_R_PM*Mag_R_NM);%this is the cosine value of the angle
between R_PM and R_NM
theta=acos(COS_theta);%the angle between R_PM and R_NM
```

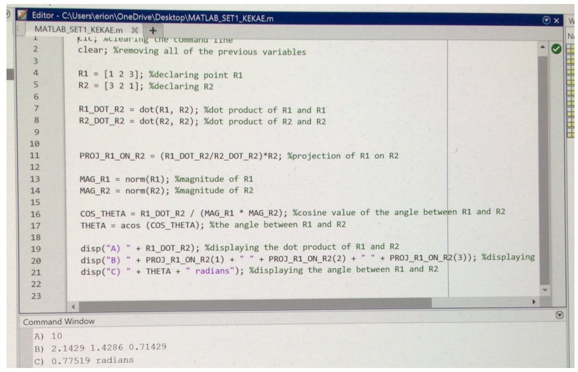
To declare and initialize vectors or points in a MATLAB program, we simply type N=[-0.2 0.1 0.3] for example, and MATLAB program will read this as $N=-0.2a_1+0.1a_2+0.3a_3$, a 3-D vector. If we type N=[-0.2 0.1] MATLAB program will read this as $N=-0.2a_1+0.1a_2$, a 2-D vector.
Some of the functions are already available in the MATLAB library so we just need to call them. For those not included in the library, the variables are utilized in the formulas we derived in the analytical part.

```
R_NM =

   0.3000  -0.3000  -0.4000

R_PM_dot_R_NM =

   0.0500

Proj_R_NM_ON_R_PM =

  -0.0882  -0.0588  -0.0588

theta =

   1.3613

>>
```

The running result is shown in the left, note that θ is given in radians here.

Exercise: Given the vectors $\mathbf{R}_1 = \boldsymbol{a}_x + 2\boldsymbol{a}_y + 3\boldsymbol{a}_z$, $\mathbf{R}_2 = 3\boldsymbol{a}_x + 2\boldsymbol{a}_y + \boldsymbol{a}_z$. Find a) the dot product $\mathbf{R}_1 \square \mathbf{R}_2$, b) the projection of $\mathbf{R}_1$ on $\mathbf{R}_2$, c) the angle between $\mathbf{R}_1$ and $\mathbf{R}_2$. Write a MATLAB program to verify your answer.

matlab output:



$$\underline{R}_1 = \underline{a}_x + 2\underline{a}_y + 3\underline{a}_z$$

$$\underline{R}_2 = 3\underline{a}_x + 2\underline{a}_y + \underline{a}_z$$

a) $\underline{R}_1 \cdot \underline{R}_2 = \underline{a}_{x_1}\underline{a}_{x_2} + \underline{a}_{y_1}\underline{a}_{y_2} + \underline{a}_{z_1}\underline{a}_{z_2}$

$$= (1)(3) + (2)(2) + (3)(1)$$

$$= 3 + 4 + 3$$

$$= 10$$

$\therefore$ the dot product of R_1 and R_2 is 10.

b) $\text{proj}_{\underline{R}_2}\underline{R}_1 = \dfrac{\underline{a}\cdot\underline{b}}{|\underline{b}|^2}\underline{b}$

$$= \frac{\underline{R}_1 \cdot \underline{R}_2}{|\underline{R}_2|^2}\underline{b}$$

$$= \frac{10}{\left(\sqrt{3^2+2^2+1^2}\right)^2}(3, 2, 1)$$

$$= \frac{10}{14}(3, 2, 1)$$

$$= \frac{5}{7}(3, 2, 1)$$

$$= \left(\frac{15}{7}, \frac{10}{7}, \frac{5}{7}\right)$$

$$\therefore \text{proj}_{\underline{R}_2}\underline{R}_1 = \frac{15}{7}\underline{a}_x + \frac{10}{7}\underline{a}_y + \frac{5}{7}\underline{a}_z$$

c) $\cos\theta = \dfrac{\underline{R}_1 \cdot \underline{R}_2}{|\underline{R}_1||\underline{R}_2|}$

$$|\underline{R}_1| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$|\underline{R}_2| = \sqrt{3^2+2^2+1^2} = \sqrt{14}$$

$$\theta = \cos^{-1}\left(\frac{10}{\sqrt{14}\sqrt{14}}\right)$$

$$\theta = \cos^{-1}\left(\frac{10}{14}\right)$$

$$\theta = 0.775 \text{ rad}$$

$\therefore$ the angle between R_1 and R_2 is 0.775 rad.