EE2FH4
Electromagnetics I

Term II, January – April 2024

MATLAB Examples and Exercises (Set 2)

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Example: The open surfaces $\rho = 2.0$ m and $\rho = 4.0$ m, $z = 3.0$ m and $z = 5.0$ m, and $\phi = 20^\circ$ and $\phi = 60^\circ$ identify a closed surface. Find a) the enclosed volume, b) the total area of the enclosed surface. Write a MATLAB program to verify your answers.

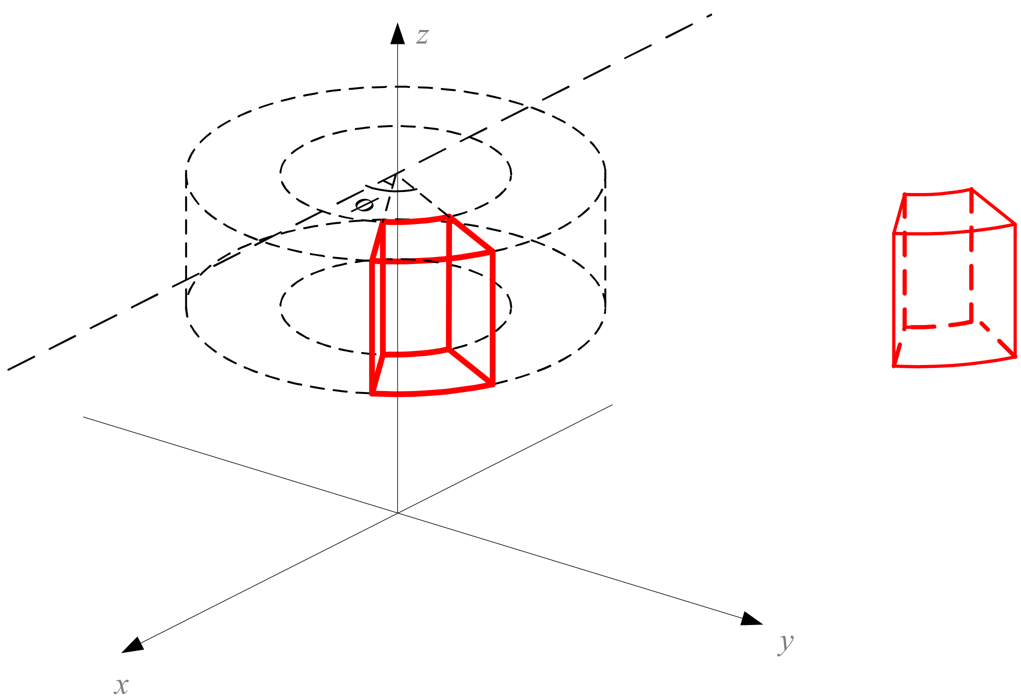


Figure 2.1 The enclosed volume for the example of Set 2.

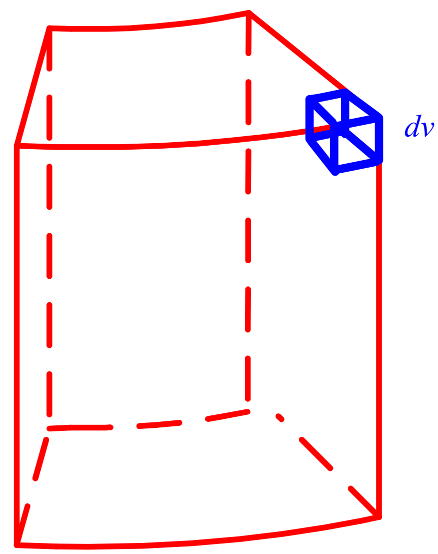
Analytical Solution:

The closed surface in this problem is shown in Figure 2.1 and Figure 2.2. To find the volume v of a closed surface we first find out dv, the volume element. In cylindrical coordinates, dv is given by $dv = \rho d\phi d\rho dz$ as shown in Figure 2.2**.** Once we get the expression of dv, we integrate dv over the entire volume.

$$dv = \rho d\phi d\rho dz$$

$$
\begin{aligned}
v &= \iiint_v dv \\
&= \iiint_v \rho d\phi d\rho dz \\
&= \int_{\rho=2}^{\rho=4}\int_{\phi=20^\circ}^{\phi=60^\circ}\int_{z=3}^{z=5} \rho d\phi d\rho dz \\
&= \int_{\rho=2}^{\rho=4} \rho d\rho \int_{\phi=\frac{20}{180}\pi}^{\phi=\frac{60}{180}\pi} d\phi \int_{z=3}^{z=5} dz \\
&= \frac{1}{2}\rho^2\Big|_{\rho=2}^{\rho=4} \times \phi\Big|_{\phi=\frac{2}{18}\pi}^{\phi=\frac{6}{18}\pi} \times z\Big|_{z=3}^{z=5} \\
&= \frac{1}{2}\times(4^2-2^2)\times(\frac{6}{18}\pi-\frac{2}{18}\pi)\times(5-3) \\
&= \frac{8}{3}\pi = 8.378
\end{aligned}
$$

When evaluating an integral, we have to convert degree to radian for all angles, otherwise this will result in a wrong value for the integral.



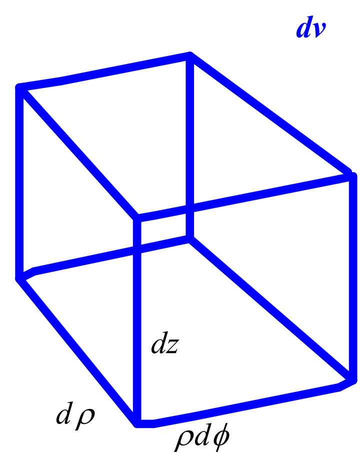


Figure 2.2. The unit volume.

The area of the closed surface is given by

$S_{enclosed} = S_1 + S_2 + S_3 + S_4 + S_5 + S_6$

We need to find dS_1, dS_2, …, and dS_6 and to integrate them over their boundary. It is obvious that $S_3 = S_4$ and $S_5 = S_6$. The surfaces S_1 and S_2 have similar shapes as we can see from the following expressions,

$$dS_1 = \rho d\phi dz\big|_{\rho=2} = 2\, d\phi dz \quad \text{and}$$

$$dS_2 = \rho d\phi dz\big|_{\rho=4} = 4\, d\phi dz$$

The steps to evaluate the area of each surface are executed as follows:

$$dS_1 = \rho d\phi dz$$

$$S_1 = \iint_s dS_1$$

$$= \iint_s \rho d\phi dz$$

$$= 2\int_{\phi=\frac{20}{180}\pi}^{\phi=\frac{60}{180}\pi} d\phi \int_{z=3}^{z=5} dz$$

$$= 2\,\phi\Big|_{\phi=\frac{2}{18}\pi}^{\phi=\frac{6}{18}\pi} \times z\Big|_{z=3}^{z=5}$$

$$= 2\times(\frac{6}{18}\pi - \frac{2}{18}\pi)\times(5-3)$$

$$= \frac{8}{9}\pi \text{ m}^2$$

$$S_2 = \iint_S dS_2$$

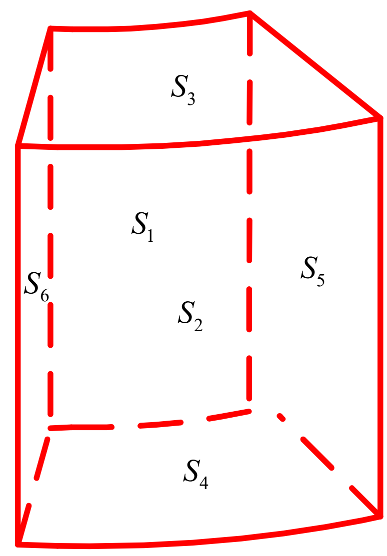
$$= \iint_S \rho d\phi dz$$

$$= 4\int_{\phi=\frac{20}{180}\pi}^{\phi=\frac{60}{180}\pi} d\phi \int_{z=3}^{z=5} dz$$

$$= 4\,\phi\Big|_{\phi=\frac{2}{18}\pi}^{\phi=\frac{6}{18}\pi} \times z\Big|_{z=3}^{z=5}$$

$$= 4\times(\frac{6}{18}\pi - \frac{2}{18}\pi)\times(5-3)$$

$$= \frac{16}{9}\pi \text{ m}^2$$



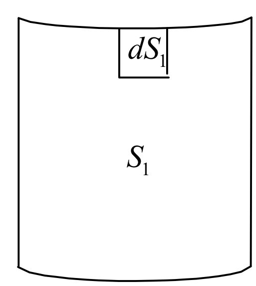


Figure 2.3 The closed surface.

$$dS_3 = \rho\, d\phi\, d\rho$$

$$\begin{aligned}
S_3 &= \iint_S dS_3 \\
&= \iint_S \rho\, d\phi\, d\rho \\
&= \int_{\rho=2}^{\rho=4} \rho\, d\rho \int_{\phi=\frac{2}{18}\pi}^{\phi=\frac{6}{18}\pi} d\phi \\
&= \frac{1}{2}\rho^2\Big|_{\rho=2}^{\rho=4} \times \phi\Big|_{\phi=\frac{2}{18}\pi}^{\phi=\frac{6}{18}\pi} \\
&= \frac{1}{2}\times(4^2-2^2)\times\left(\frac{6}{18}\pi-\frac{2}{18}\pi\right) \\
&= \frac{4}{3}\pi \ \text{m}^2
\end{aligned}$$

$$dS_5 = d\rho\, dz$$

$$\begin{aligned}
S_5 &= \iint_S dS_5 \\
&= \iint_S d\rho\, dz \\
&= \int_{\rho=2}^{\rho=4} d\rho \int_{z=3}^{z=5} dz \\
&= \rho\Big|_{\rho=2}^{\rho=4} \times z\Big|_{z=3}^{z=5} \\
&= (4-2)\times(5-3) \\
&= 4 \ \text{m}^2
\end{aligned}$$

$$\begin{aligned}
S_{closed} &= S_1 + S_2 + 2S_3 + 2S_5 \\
&= \frac{8}{9}\pi + \frac{16}{9}\pi + 2\times\frac{4}{3}\pi + 2\times \\
&= 24.755 \ \text{m}^2
\end{aligned}$$

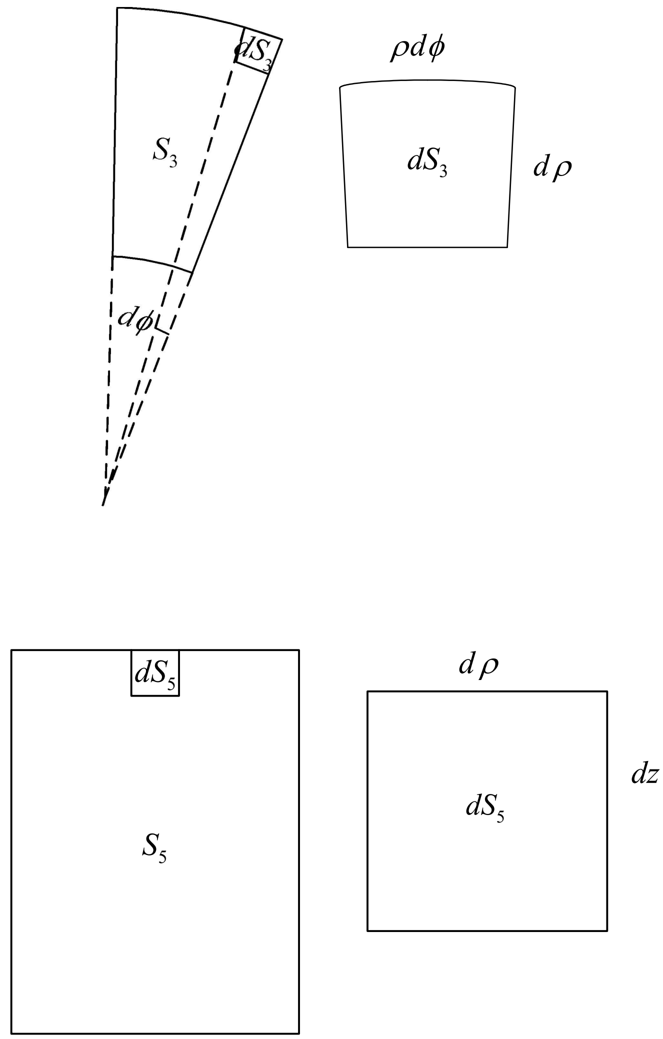


Figure 2.4 The surfaces S_3 and S_5 and their incremental elements.

MATLAB SOLUTION:
As shown in the figure to the right, the approximate value of the enclosed volume is

$$v \approx \sum_{k=1}^{p}\sum_{j=1}^{m}\sum_{i=1}^{n}\Delta v_{i,j,k} = \sum_{k=1}^{p}\sum_{j=1}^{m}\sum_{i=1}^{n}(\rho_{i,j,k}\Delta\phi)\times(\Delta\rho)\times(\Delta z)$$

We write a MATLAB program to evaluate this expression. To do this, our program evaluates all element volumes $\Delta v_{i,j,k}$, and increase the total volume by $\Delta v_{i,j,k}$ each time. We cover all elements $\Delta v_{i,j,k}$ through 3 loops with counters i in the inner loop, j in the middle loop and k in the outer loop. The approach used to evaluate the surfaces is similar to that of the volume. The MATLAB code is shown in the next page.

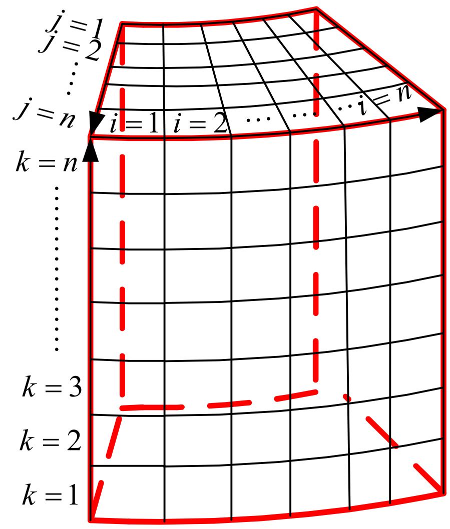


Figure 2.5 The discretized volume.

MATLAB code:

```
clc; %clear the command line
clear; %remove all previous variables

V=0;%initialize volume of the closed surface to 0
S1=0;%initialize the area of S1 to 0
S2=0;%initialize the area of S1 to 0
S3=0;%initialize the area of S1 to 0
S4=0;%initialize the area of S1 to 0
S5=0;%initialize the area of S1 to 0
S6=0;%initialize the area of S1 to 0
rho=2;%initialize rho to the its lower boundary
z=3;%initialize z to the its lower boundary
phi=pi/9;%initialize phi to the its lower boundary

Number_of_rho_Steps=100; %initialize the rho discretization
Number_of_phi_Steps=100;%initialize the phi discretization
Number_of_z_Steps=100;%initialize the z discretization

drho=(4-2)/Number_of_rho_Steps;%The rho increment
dphi=(pi/3-pi/9)/Number_of_phi_Steps;%The phi increment
dz=(5-3)/Number_of_z_Steps;%The z increment

%%the following routine calculates the volume of the enclosed surface
for k=1:Number_of_z_Steps
    for j=1:Number_of_rho_Steps
        for i=1:Number_of_phi_Steps
            V=V+rho*dphi*drho*dz;%add contribution to the volume
        end
        rho=rho+drho;%p increases each time when z has been traveled from its lower boundary to its
upper boundary
    end
    rho=2;%reset rho to its lower boundary
end
```

```
%%the following routine calculates the area of S1 and S2
rho1=2;%radius of S1
rho2=4;%radius of s2
for k=1:Number_of_z_Steps
    for i=1:Number_of_phi_Steps
       S1=S1+rho1*dphi*dz;%get contribution to the the area of S1
       S2=S2+rho2*dphi*dz;%get contribution to the the area of S2
    end
end
%%the following routing calculate the area of S3 and S4
rho=2;%reset rho to it's lower boundaty
for j=1:Number_of_rho_Steps
    for i=1:Number_of_phi_Steps
       S3=S3+rho*dphi*drho;%get contribution to the the area of S3
    end
    rho=rho+drho;%p increases each time when phi has been traveled from it's lower boundary to it's
upper boundary
end
S4=S3;%the area of S4 is equal to the area of S3

%%the following routing calculate the area of S5 and S6
for k=1:Number_of_z_Steps
    for j=1:Number_of_rho_Steps
       S5=S5+dz*drho;%get contribution to the the area of S3
    end
end
S6=S5;%the area of S6 is equal to the area of S6
S=S1+S2+S3+S4+S5+S6;%the area of the enclosed surface
```

Running Result

Command Window

```
>> V

V =

    8.3497

>> S

S =

   24.7272

>>
```

By comparing, we see that the result of our analytical solution is close to the result of our MATLAB solution.

Exercise: The surfaces $r = 0$ and $r = 2$, $\phi = 45^\circ$, $\phi = 90^\circ$, $\theta = 45^\circ$ and $\theta = 90^\circ$ define a closed surface. Find the enclosed volume and the area of the closed surface S. Write a MATLAB program to verify your answer.

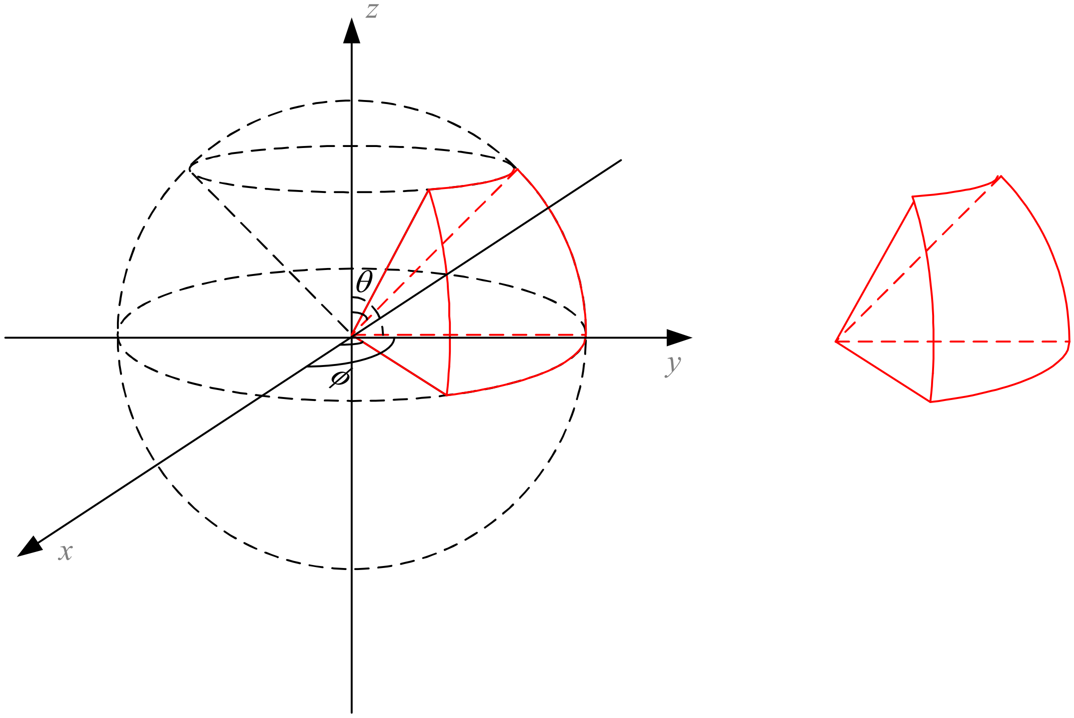
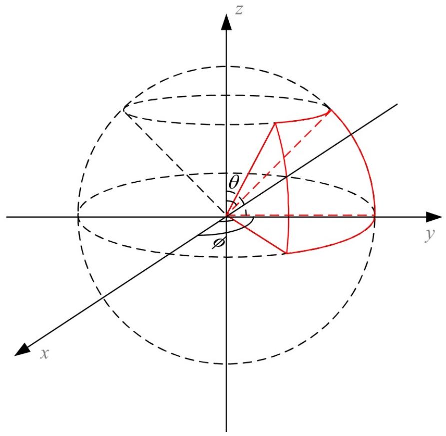
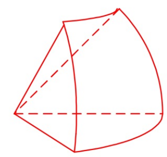


Figure 2.6 The surface of the exercise of Set 2.





Exercise: The surfaces $r = 0$ and $r = 2$, $\phi = 45^\circ$, $\phi = 90^\circ$, $\theta = 45^\circ$ and $\theta = 90^\circ$ define a closed surface. Find the enclosed volume and the area of the closed surface S. Write a MATLAB program to verify your answer.

} assuming meters used

Volume: $dV = r^2 \sin\theta \, dr\, d\theta\, d\phi$

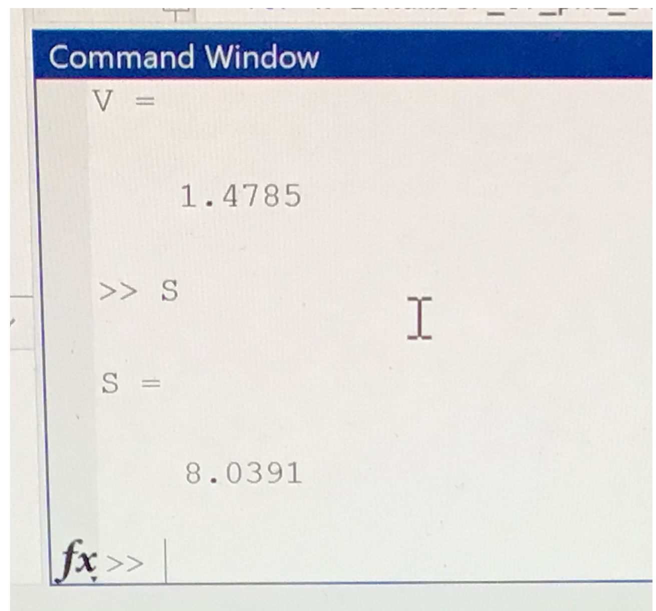
$$V = \iiint r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= \int_0^2 r^2 dr \int_{\pi/4}^{\pi/2} \sin\theta\, d\theta \int_{\pi/4}^{\pi/2} d\phi$$

$$= \left[\frac{r^3}{3}\Big|_0^2\right]\left[-\cos(\pi/2) + \cos(\pi/4)\right]\left[\pi/2 - \pi/4\right]$$

$$= (8/3)\left(\tfrac{\sqrt{2}}{2}\right)\left(\tfrac{\pi}{4}\right) \Rightarrow \underline{1.4809\ m^3}$$

matlab output:



Surface Area:

$$dS = \begin{cases} r^2 \sin\theta\, d\theta\, d\phi\, \mathbf{a_r} \\ r \sin\theta\, dr\, d\phi\, \mathbf{a_\theta} \\ r\, dr\, d\theta\, \mathbf{a_\phi} \end{cases}$$

$$dS_1 = r^2 \sin\theta\, d\theta\, d\phi$$

$$S_1 = \iint_S r^2 \sin\theta\, d\theta\, d\phi$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} r^2 \sin\theta\, d\theta\, d\phi$$

$$= \underbrace{\cancel{r}^2}_{\emptyset} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta\, d\theta \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\phi$$

↳ everything becomes ∅

$$= \emptyset$$

$$S_2 = \iint_S r^2 \sin\theta\, d\theta\, d\phi$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} r^2 \sin\theta\, d\theta\, d\phi$$

$$= r^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\theta\, d\theta \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\phi$$

$$= 2^2 \times \left[-\cos\theta\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}\right] \times \left[x\Big|_{\frac{\pi}{4}}^{\pi/2}\right]$$

$$= 4\left[-\cos\left(\tfrac{\pi}{2}\right) + \cos\left(\tfrac{\pi}{4}\right)\right]\left[\tfrac{\pi}{2} - \tfrac{\pi}{4}\right]$$

$$= \cancel{4}\left(\tfrac{\sqrt{2}}{2}\right)\left(\tfrac{\pi}{\cancel{4}}\right) \Rightarrow \tfrac{\sqrt{2}}{2}\pi$$

$$S_3 = \iint_3 r \sin\theta\, dr\, d\phi$$

$$= \sin\theta \int_0^2 \int_{\pi/4}^{\pi/2} r\, dr\, d\phi$$

$$= \sin\theta \int_0^2 r\, dr \int_{\frac{\pi}{4}}^{\pi/2} d\phi$$

$$= \sin(\pi/4) * \left[\tfrac{r^2}{2}\Big|_0^2\right] * \left[\pi/4\right]$$

$$= \left(\tfrac{\sqrt{2}}{2}\right)(4/2)(\pi/4)$$

$$= \frac{\sqrt{2}\pi}{4}$$

$$S_4 = \iint_3 r \sin\theta\, dr\, d\phi$$

$$= \sin\theta \int_0^2 \int_{\pi/4}^{\pi/2} r\, dr\, d\phi$$

$$= \sin\theta \int_0^2 r\, dr \int_{\frac{\pi}{4}}^{\pi/2} d\phi$$

$$= \sin(\pi/2) * \left[\tfrac{r^2}{2}\Big|_0^2\right] * \left[\pi/4\right]$$

$$= (1)(4/2)(\pi/4)$$

$$= \frac{4\pi}{8} \Rightarrow \pi/2$$

$$S_5 = \iint r\, dr\, d\theta$$

$$= \int_0^2 r\, dr \int_{\pi/4}^{\pi/2} d\theta$$

$$= \left[\tfrac{r^2}{2}\Big|_0^2\right]\left[\pi/4\right]$$

$$= \pi/2$$

$$S_6 = S_5$$

$$\therefore S = S_1 + S_2 + S_3 + S_4 + S_5 + S_6$$

$$= \emptyset + \frac{\sqrt{2}}{2}\pi + \frac{\sqrt{2}\pi}{4} + \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}$$

$$= \underline{8.045\ m^2}$$