

## EE2FH4 Electromagnetics I

Term II, January – April 2024

### MATLAB Examples and Exercises (Set 1)

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**Example:** Given the points  $M(0.1, -0.2, -0.1)$ ,  $N(-0.2, 0.1, 0.3)$  and  $P(0.4, 0, 0.1)$ , find: a) the vector  $\mathbf{R}_{NM}$ , b) the dot product  $\mathbf{R}_{NM} \cdot \mathbf{R}_{PM}$ , c) the projection of  $\mathbf{R}_{NM}$  on  $\mathbf{R}_{PM}$  and d) the angle between  $\mathbf{R}_{NM}$  and  $\mathbf{R}_{PM}$ . Write a MATLAB program to verify your answer.

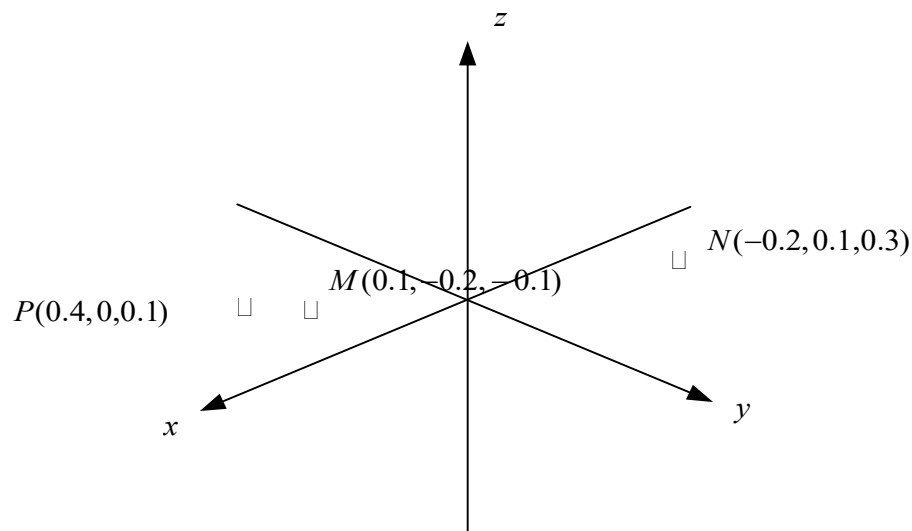


Figure 1.1 The points used in the example of Set 1.

**Analytical Solution:**

$$\begin{aligned} \text{a) } \mathbf{R}_{NM} &= \mathbf{R}_{MO} - \mathbf{R}_{NO} \\ &= (0.1\mathbf{a}_x - 0.2\mathbf{a}_y - 0.1\mathbf{a}_z) - (-0.2\mathbf{a}_x + 0.1\mathbf{a}_y + 0.3\mathbf{a}_z) \\ &= 0.3\mathbf{a}_x - 0.3\mathbf{a}_y - 0.4\mathbf{a}_z \end{aligned}$$

$$\begin{aligned} \text{b) } \mathbf{R}_{PM} &= \mathbf{R}_{MO} - \mathbf{R}_{PO} \\ &= (0.1\mathbf{a}_x - 0.2\mathbf{a}_y - 0.1\mathbf{a}_z) - (0.4\mathbf{a}_x + 0.1\mathbf{a}_z) \\ &= -0.3\mathbf{a}_x - 0.2\mathbf{a}_y - 0.2\mathbf{a}_z \end{aligned}$$

$$\begin{aligned} \mathbf{R}_{NM} \cdot \mathbf{R}_{PM} &= (0.3\mathbf{a}_x - 0.3\mathbf{a}_y - 0.4\mathbf{a}_z) \cdot (-0.3\mathbf{a}_x - 0.2\mathbf{a}_y - 0.2\mathbf{a}_z) \\ &= 0.3 \times (-0.3) + (-0.3) \times (-0.2) + (-0.4) \times (-0.2) \\ &= -0.09 + 0.06 + 0.08 = 0.05 \end{aligned}$$

$$\begin{aligned} \text{c) } \text{proj}_{\mathbf{R}_{PM}} \mathbf{R}_{NM} &= \frac{\mathbf{R}_{NM} \cdot \mathbf{R}_{PM}}{\mathbf{R}_{PM} \cdot \mathbf{R}_{PM}} \mathbf{R}_{PM} \\ &= \frac{0.05}{(-0.3)^2 + (-0.2)^2 + (-0.2)^2} (-0.3\mathbf{a}_x - 0.2\mathbf{a}_y - 0.2\mathbf{a}_z) \\ &= -0.088\mathbf{a}_x - 0.059\mathbf{a}_y - 0.059\mathbf{a}_z \end{aligned}$$

$$\begin{aligned} \text{d) } \cos \theta &= \frac{\mathbf{R}_{NM} \cdot \mathbf{R}_{PM}}{|\mathbf{R}_{NM}| |\mathbf{R}_{PM}|} \\ &= \frac{0.05}{\sqrt{(0.3)^2 + (-0.3)^2 + (-0.4)^2} \sqrt{(-0.3)^2 + (-0.2)^2 + (-0.2)^2}} \\ &= 0.208 \\ \theta &= \cos^{-1} 0.208 = 1.36 \end{aligned}$$

**Definition**

Let  $\mathbf{u}$  and  $\mathbf{v}$  be two nonzero vectors in  $\mathbf{R}^n$ . The cosine of the angle  $\theta$  between these vectors is  $\cos \theta = \frac{\mathbf{v} \cdot \mathbf{u}}{|\mathbf{u}| |\mathbf{v}|}$ ,  $0 \leq \theta \leq \pi$

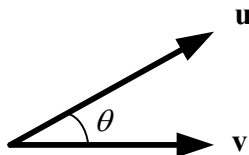


Figure 1.2 The angle between two vectors.

This problem is a direct application to vector algebra. It requires clear understanding of the basic definitions used in vector analysis.

The vector  $\mathbf{R}_{NM}$  can be obtained by subtracting the vector  $\mathbf{R}_{NO}$  from vector  $\mathbf{R}_{MO}$ , where O is the origin.

**Definition**

Let  $\mathbf{u} = (u_1, \dots, u_n)$

and  $\mathbf{v} = (v_1, \dots, v_n)$

be two vectors in  $\mathbf{R}^n$ . The **dot product** of  $\mathbf{u}$  and  $\mathbf{v}$  is defined by

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + \dots + u_n v_n$$

The dot product assigns a real number to each pair of vectors.

**Definition**

The projection of a vector  $\mathbf{v}$  onto a nonzero vector  $\mathbf{u}$  in  $\mathbf{R}^n$  is denoted  $\text{proj}_{\mathbf{u}} \mathbf{v}$  and is defined by

$$\text{proj}_{\mathbf{u}} \mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}} \mathbf{u}$$

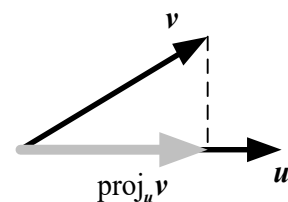


Figure 1.3 The projection of one vector onto another.

## MATLAB Examples and Exercises (Set 1)

## MATLAB SOLUTION:

```
clc; %clear the command line
clear; %remove all previous variables
```

```
O=[0 0 0]; %the origin
M=[0.1 -0.2 -0.1]; %Point M
N=[-0.2 0.1 0.3]; %Point N
P=[0.4 0 0.1]; %Point P
```

```
R_MO=M-O; %vector R_MO
R_NO=N-O; %vector R_NO
R_PO=P-O; %vector R_PO
```

```
R_NM=R_MO-R_NO; %vector R_NM
R_PM=R_MO-R_PO; %vector R_PM
```

```
R_PM_dot_R_NM=dot(R_PM,R_NM); %the dot
product of R_PM and R_NM
R_PM_dot_R_PM=dot(R_PM,R_PM); %the dot product of R_PM and R_PM
```

```
Proj_R_NM_ON_R_PM=(R_PM_dot_R_NM/R_PM_dot_R_PM)*R_PM; %the projection of R_NM ON R_PM
```

```
Mag_R_NM=norm(R_NM); %the magnitude of R_NM
Mag_R_PM=norm(R_PM); %the magnitude of R_PM
```

```
COS_theta=R_PM_dot_R_NM/(Mag_R_PM*Mag_R_NM); %this is the cosine value of the angle
between R_PM and R_NM
theta=acos(COS_theta); %the angle between R_PM and R_NM
```

```
R_NM =
```

```
0.3000 -0.3000 -0.4000
```

```
R_PM_dot_R_NM =
```

```
0.0500
```

```
Proj_R_NM_ON_R_PM =
```

```
-0.0882 -0.0588 -0.0588
```

```
theta =
```

```
1.3613
```

```
>>
```

To declare and initialize vectors or points in a MATLAB program, we simply type  $N = [-0.2 \ 0.1 \ 0.3]$  for example, and MATLAB program will read this as  $N = -0.2a_1 + 0.1a_2 + 0.3a_3$ , a 3-D vector. If we type  $N = [-0.2 \ 0.1]$  MATLAB program will read this as  $N = -0.2a_1 + 0.1a_2$ , a 2-D vector.

Some of the functions are already available in the MATLAB library so we just need to call them. For those not included in the library, the variables are utilized in the formulas we derived in the analytical part.

The running result is shown in the left, note that  $\theta$  is given in radians here.

## MATLAB Examples and Exercises (Set 1)

**Exercise:** Given the vectors  $\mathbf{R}_1 = a_x + 2a_y + 3a_z$ ,  $\mathbf{R}_2 = 3a_x + 2a_y + a_z$ . Find a) the dot product  $\mathbf{R}_1 \cdot \mathbf{R}_2$ , b) the projection of  $\mathbf{R}_1$  on  $\mathbf{R}_2$ , c) the angle between  $\mathbf{R}_1$  and  $\mathbf{R}_2$ . Write a MATLAB program to verify your answer.

$$\underline{\mathbf{R}}_1 = a_x + 2a_y + 3a_z$$

$$\underline{\mathbf{R}}_2 = 3a_x + 2a_y + a_z$$

matlab output:

```

1  % MATLAB SET1_KEKAE.m
2  % Clearing the command line
3  clear; % Removing all of the previous variables
4  R1 = [1 2 3]; % Declaring point R1
5  R2 = [3 2 1]; % Declaring R2
6
7  R1_DOT_R2 = dot(R1, R2); % Dot product of R1 and R1
8  R2_DOT_R2 = dot(R2, R2); % Dot product of R2 and R2
9
10
11  PROJ_R1_ON_R2 = (R1_DOT_R2/R2_DOT_R2)*R2; % Projection of R1 on R2
12
13  MAG_R1 = norm(R1); % Magnitude of R1
14  MAG_R2 = norm(R2); % Magnitude of R2
15
16  COS_THETA = R1_DOT_R2 / (MAG_R1 * MAG_R2); % Cosine value of the angle between R1 and R2
17  THETA = acos(COS_THETA); % The angle between R1 and R2
18
19  disp('A) ' + R1_DOT_R2); % Displaying the dot product of R1 and R2
20  disp('B) ' + PROJ_R1_ON_R2(1) + ' ' + PROJ_R1_ON_R2(2) + ' ' + PROJ_R1_ON_R2(3)); % Displaying
21  disp('C) ' + THETA + ' radians'); % Displaying the angle between R1 and R2
22
23

```

Command Window

```

A) 10
B) 2.1429 1.4286 0.71429
C) 0.77519 radians

```

$$\begin{aligned}
 \text{a) } \underline{\mathbf{R}}_1 \cdot \underline{\mathbf{R}}_2 &= \underline{a}_x \underline{a}_{x2} + \underline{a}_y \underline{a}_{y2} + \underline{a}_z \underline{a}_{z2} \\
 &= (1)(3) + (2)(2) + (3)(1) \\
 &= 3 + 4 + 3 \\
 &= 10
 \end{aligned}$$

$\therefore$  the dot product of  $\mathbf{R}_1$  and  $\mathbf{R}_2$  is 10.

$$\begin{aligned}
 \text{b) } \text{proj}_{\underline{\mathbf{R}}_2} \underline{\mathbf{R}}_1 &= \frac{\underline{\mathbf{R}}_1 \cdot \underline{\mathbf{R}}_2}{|\underline{\mathbf{R}}_2|^2} \underline{\mathbf{R}}_2 \\
 &= \frac{10}{|\underline{\mathbf{R}}_2|^2} \underline{\mathbf{R}}_2 \\
 &= \frac{10}{(\sqrt{3^2 + 2^2 + 1^2})^2} (3, 2, 1) \\
 &= \frac{10}{14} (3, 2, 1) \\
 &= \frac{5}{7} (3, 2, 1) \\
 &= \left( \frac{15}{7}, \frac{10}{7}, \frac{5}{7} \right)
 \end{aligned}$$

$$\therefore \text{proj}_{\underline{\mathbf{R}}_2} \underline{\mathbf{R}}_1 = \frac{15}{7} a_x + \frac{10}{7} a_y + \frac{5}{7} a_z$$

$$\begin{aligned}
 \text{c) } \cos \theta &= \frac{\underline{\mathbf{R}}_1 \cdot \underline{\mathbf{R}}_2}{|\underline{\mathbf{R}}_1| |\underline{\mathbf{R}}_2|} & |\underline{\mathbf{R}}_1| &= \sqrt{1^2 + 2^2 + 3^2} \\
 & & &= \sqrt{14} \\
 \theta &= \cos^{-1} \left( \frac{10}{\sqrt{14} \sqrt{14}} \right) & |\underline{\mathbf{R}}_2| &= \sqrt{3^2 + 2^2 + 1^2} \\
 & & &= \sqrt{14}
 \end{aligned}$$

$$\theta = \cos^{-1} \left( \frac{10}{14} \right)$$

$$\theta = 0.775 \text{ rad}$$

$\therefore$  the angle between  $\mathbf{R}_1$  and  $\mathbf{R}_2$  is 0.775 rad.