Lab 5: Active Filter Circuits

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1. **(20 Points)** (1) Find the transfer function of the first-order LPF, its low-frequency gain, and its - 3dB frequency f c. (2) Compare the calculated low-frequency gain and the -3dB frequency f c with the simulated data from Step 1.3 and the measured data from Step 1.8, respectively. Justify/discuss the observation and comparison.

   a. $T(s) = \frac{V_o}{V_{in}} \rightarrow \ For\ an\ ideal\ op-amp: V^+ = V^-$

   $$V^- = V_O(s)\frac{R_3}{R_2 + R_3} \rightarrow \ \frac{V_{in} - V^+}{R_1} = \ V^+(sC_1)$$

   $$\frac{V_{in} - V_O(s)\frac{R_3}{R_{2\ +\ R_3}}}{R_1} = \left(V_O(s)\frac{R_3}{R_{2\ +\ R_3}}\right)(sC_1)$$

   $$T(s) \ = \frac{V_O}{V_{in}} = \frac{R_2 + \ R_3}{R_3(R_1 * sC_1 + 1)}$$

   $\rightarrow \ Low - Frequency\ Gain:$

   $$T(0) = \frac{R_3 + R_2}{R_3} \rightarrow \frac{200k\Omega + 100k\Omega}{200k\Omega} = \frac{3}{2} \rightarrow 20log_{10}\left(\frac{3}{2}\right) = 3.522\ dB$$

   $\rightarrow 3dB - Frequency:$

   $$f_c = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi(100k\Omega)(1nF)} = \ 1591\ Hz$$

   b. For Step 1.3:

   $\rightarrow \ Low - Frequency\ Gain:$

   $$T(0) = \frac{V_O}{V_{in}} = \frac{150mV}{100mV} = 1.5 \rightarrow 20log_{10}(1.5) = 3.522\ dB$$

   $\rightarrow 3dB - Frequency:$

   $$20log_{10}\left(\frac{V_O}{100mV}\right) = 3.522 - 3 \rightarrow V_O = 0.1062\ V$$

   $The\ corresponding\ 3dB - frequency\ to\ this\ V_O\ is\ 1577.68$ Hz

For Step 1.8:

$\rightarrow\ Low - Frequency\ Gain$:

$$T(0) = \frac{V_O}{V_{in}} = \frac{149.93\ mV}{100mV} = 1.4993 \rightarrow 20log_{10}(1.4993) = 3.518\ dB$$

$\rightarrow 3dB - Frequency$:

$$20log_{10}\left(\frac{V_O}{100mV}\right) = 3.518 - 3 \rightarrow V_O = 0.1061\ V$$

$The\ corresponding\ 3dB - frequency\ to\ this\ V_O\ is$ 1659.587 Hz

When comparing the calculated low-frequency gain and 3-dB frequency with both the measured and simulated data obtained in Steps 1.3 and 1.8 respectively, the results align closely. With Step 1.3, the low-frequency gain, features no percent difference with the calculated low-frequency gain; however, the 3-dB frequencies feature a negligible percent difference of 0.84%. With Step 1.8, the low-frequency gain features a negligible percent difference of 0.11% when being compared to the calculated value. The 3-dB frequency also features a percent difference of 4.22% between the measured and calculated values. Overall, upon comparing the low-frequency gain and the 3-dB frequency, there is a very minimal percent difference expressing that the measured and simulated data closely align the data obtained through calculations. These percent differences may have arisen due to measurement inaccuracies with the Analog Discovery 3, ideality assumptions made in the calculations, and component tolerances.

2. **(20 Points)** Derive the transfer function and calculate the low-frequency gain. Verify the calculated gain using the simulated data obtained in Step 2.2 and the measured data obtained in Step 2.6, respectively.

   a. $T(s) = \frac{V_o}{V_{in}} \rightarrow$ *For an ideal op − amp*: $V^+ = V^-$

   Let V_1 *be the node between* R_1 *and* R_2: $0 = \frac{V_1 - V_{in}}{R_1} + \frac{V_1}{R_2 + sC_2} + \frac{V_1 - V_O}{sC_1}$

   $$V^+ = V_O \frac{R_3}{R_3 + R_4}$$

   $$T(s) = \frac{V_O}{V_{in}} = \frac{R_3 + R_4}{s^2(C_1{*}C_2{*}R_1{*}R_3{*}R_3) + s(C_2{*}R_2{*}R_3 + C_2{*}R_1{*}R_3 - C_1{*}R_1{*}R_4) + R_3}$$

   $$T(s) = \frac{V_O}{V_{in}} = \frac{91{*}10^6}{s^2 + (15.5{*}10^3)s + 45.5{*}10^6}$$

   $\rightarrow$ *Low − Frequency Gain*:

   $$T(0) = \frac{R_3 + R_4}{R_3} = \frac{100k\Omega + 100k\Omega}{100k\Omega} = 2 \rightarrow 20log_{10}(2) = 6.021\ dB$$

   In Step 2.2, the low-frequency gain is found to be 6.021 dB whereas, in Step 2.6, the low-frequency gain is found to be 6.024 dB. Upon comparing these results with the calculated low-frequency gain, we can see that they closely align and that the measured and simulated values agree with the calculated values.

3. **(20 Points)** Calculate (1) the pole frequency fo, (2) the cut-off frequency (or -3dB frequency) fc, (3) the pole quality factor Q, (4) the peak value of the magnitude of the transfer function, |T(s)|max, and (5) the frequency fmax where the peak value of the magnitude of the transfer function happens. Verify the calculated fc using the simulated data obtained in Step 2.2 and the measured data obtained in Step 2.6, respectively.

   a. *The Pole Frequency*, $f_O = \frac{1}{2\pi\omega_O} = \frac{1}{2\pi\sqrt{C_1{*}C_2{*}R_1{*}R_2}}$

   $$f_O = \frac{1}{2\pi\sqrt{(1nF)(2.2nF)(100k\Omega)(100k\Omega)}} = 1073\ Hz$$

   b. *The cut − off frequency*, f_c:

   $$|T(j\omega_c)| = \frac{91{*}10^6}{\sqrt{\left(45.5{*}10^6 - \omega_c^2\right)^2 + (15.5{*}10^3)^2{*}\omega_c^2}} = \sqrt{2}$$

   From this,

   $$\omega_c = 3574.2\frac{rad}{m} \rightarrow f_c = \frac{\omega_c}{2\pi} = \frac{3574.2\frac{rad}{m}}{2\pi} = 568.9\ Hz$$

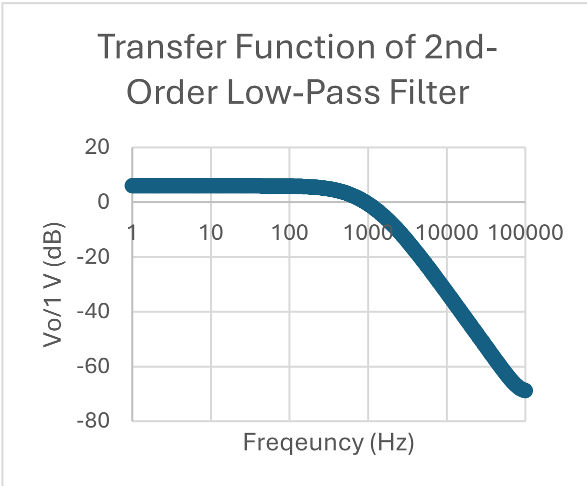

Figure 2: Step 2.2 -Transfer Function of 2nd-Order Low-Pass Filter

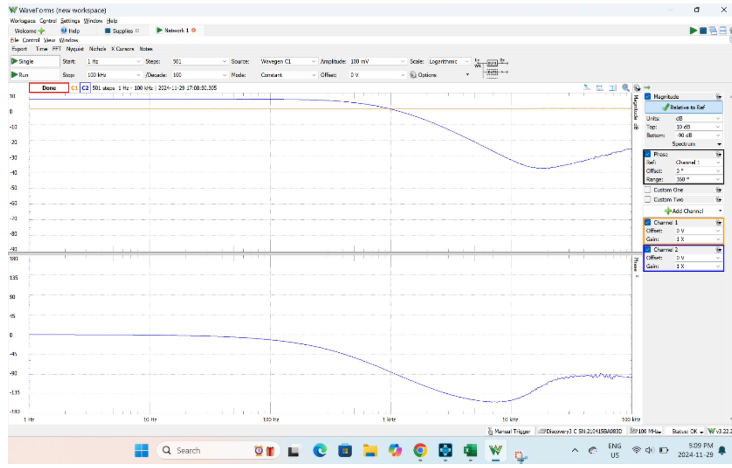

Figure 1: Graphs from Step 2.6

As we can see from Figure 1 and Figure 2, our calculated cut-off frequency of 568.9 Hz matches with both outputs. We see a cut-off at this frequency within both graphs. In addition, in Step 2.2, we obtain a frequency of 565.5 Hz, a magnitude of 3.036 dB and a phase of -59.17 degrees. In comparison, in Step 2.6, we obtained a value of 562.3 Hz for the frequency, a magnitude of 3.245 dB and -57.09 degrees for the phase. These values align with our calculated findings.

c. $Finding\ the\ pole\ quality\ factor, Q$:

$$\frac{\omega_o}{Q} = 15.5 * 10^3 \rightarrow Q = \frac{6745.37}{15.5 * 10^3} = 0.435$$

d. Since in previous part, we found Q to be less than $\frac{1}{\sqrt{2}}$ which tells us that the system is overdamped; so the peak value of the magnitude can be found through using the equation, $1 + \frac{R_3}{R_4} = 1 + \frac{100k\Omega}{100k\Omega} = 2 \rightarrow 20\log_{10} 2 = 6.021\ dB$.

e. The frequency f_{max} where the peak value of the magnitude of the transfer function happens occurs within a range. This range is from 0 Hz to 1073 Hz which is the pole frequency previously found.

4. **(20 Points)** Derive the transfer function and calculate the center frequency gain. Verify the calculated gain using the simulated data obtained in Step 4.2 and the measured data obtained in Step 4.6, respectively.

   a. $T(s) = \frac{V_o}{V_{in}} \rightarrow \ For\ an\ ideal\ op - amp: V^+ = V^-$

   $Through\ KCL, we\ find:$

$$0 = \frac{V_1 - V_{in}}{R_1} + \frac{V_1}{\frac{1}{sC_2}} + \frac{V_1 - V_O}{\frac{1}{sC_2}}$$

   $From\ this, we\ get:$

$$T(s) = \frac{V_O}{V_{in}} = \frac{\frac{-s}{C_1R_1}}{s^2 + s\left(\frac{1}{C_1R_2} + \frac{1}{C_2R_2}\right) + \frac{1}{C_1C_2R_1R_2}}$$

$$= -\frac{s(C_2R_2)}{s^2(C_1C_2R_1R_2) + s(C_2R_1 + C_1R_1) + 1}$$

   $Finding\ Center - Frequency:$

$$f_O = \frac{1}{2\pi\omega_O} = \frac{1}{2\pi\sqrt{C_1 * C_2 * R_1 * R_2}} = \frac{1}{2\pi(0.00024)} = 663.15\ Hz$$

   $Now, the\ Center - Frequency\ Gain:$

   $T(\frac{1}{\sqrt{C_1 * C_2 * R_1 * R_2}} = 4166.7\ rad/s)$ = -6.02 dB and -0.5 dB

   In Step 4.2, the center-frequency gain is found to be -6.021 dB at a frequency of 663.4 Hz whereas, in Step 4.6, the center-frequency gain is found to be -6.329 dB at a frequency of 660 Hz. Upon comparing these results with the calculated center-frequency gain, we can see that they closely align; however, there is a minor percent difference present with the measured result due to similar errors mentioned in Q1.

5. **(20 Points)** Calculate (1) the center frequency ωo (2) the pole quality factor Q, (3) the two 3dB frequencies ω1 and ω2, and (4) the 3-dB bandwidth BW = ω2 - ω1. Verify the calculated results using the simulated data obtained in Step 4.2 and the measured data obtained in Step 4.6, respectively.

   a. The Center Frequency was found to be 663 Hz using a value of 4166.7 rad/s for ω_o which was also found in the previous question.

   b. The pole quality factor, Q, can be determined through the following equation:

$$\frac{\omega_O}{Q} = \frac{1}{C_1R_2} + \frac{1}{C_2R_2} \rightarrow \frac{4166.7\ rad/s}{Q} = \frac{1}{(1nF)(240k\Omega)} + \frac{1}{(1nF)(240k\Omega)} \rightarrow Q = 0.5$$

   c. To find the two 3-dB frequencies, ω_1 and ω_2:

$$\omega = \omega_O\sqrt{1+\frac{1}{4Q^2}} \pm \frac{\omega_O}{2Q}$$

$$\rightarrow \omega_2 = \omega_O\sqrt{1+\frac{1}{4Q^2}} + \frac{\omega_O}{2Q}$$

$$\omega_2 = \left(4166.7\frac{rad}{s}\right)\sqrt{1+\frac{1}{4(0.5)^2}} + \left(4166.7\frac{rad}{s}\right) = 10059.3\frac{rad}{s}$$

$$\rightarrow \omega_1 = \omega_O\sqrt{1+\frac{1}{4Q^2}} - \frac{\omega_O}{2Q}$$

$$\omega_1 = \left(4166.7\frac{rad}{s}\right)\sqrt{1+\frac{1}{4(0.5)^2}} - \left(4166.7\frac{rad}{s}\right) = 1725.9\frac{rad}{s}$$

$$f_1 = \frac{\omega_1}{2\pi} = \frac{1725.9\frac{rad}{s}}{2\pi} = 274.8\ Hz \rightarrow f_2 = \frac{\omega_2}{2\pi} = \frac{10059.3\frac{rad}{s}}{2\pi} = 1600.9\ Hz$$

   d. To find the 3-dB bandwidth, BW:

$$BW = \omega_2 - \omega_1 = \left(10059.3\frac{rad}{s}\right) - \left(1725.9\frac{rad}{s}\right) = 8333.4\frac{rad}{s}\ or\ 1326.3\ Hz$$

Table 1: Step 3.2

Frequency	DB(V(Vout))	P(V(Vout))
Hz	dBV	Degrees
272.6738966	-9.07597690	-134.708069
1614.064152	-9.08188415	-225.357865

Table 2: Step 3.6

Frequency	Magnitude	Phase
Hz	dB	Degrees
275.4228703	-9.50381927	-133.716819
1584.893192	-9.07814002	136.5746131

Upon comparing this to the simulated and measured data found from Steps 3.2 and 3.6, we can see that they verify the calculations performed in these parts of the questions. The data and calculations closely align with one another.