Exercise 12 - Multiple regression

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Abstract

This exercise will show you how multiple predictors can be used in the same regression model to achieve better prediction efficiency.

Data management and descriptive statistics

Load data about housing prices in King County, USA

In this exercise we will predict the price of apartments and houses.

We use a dataset from Kaggle containing data about housing prices and variables that may be used to predict housing prices. The data is about accomodations in King County, USA (Seattle and sorrounding area). You can find info about the dataset here: https://www.kaggle.com/harlfoxem/housesalesprediction

We only use a portion of the full dataset now containing information about N = 200 accomodations.

The .sav file can be downloaded from here: https://github.com/kekecsz/SIMM32/blob/master/2021/Lab_2/House%20price%20King%20County.sav

Check the dataset

You should always check the dataset for coding errors or data that does not make sense.

View data in the data editor and display simple descriptive statistics and plots. We are going to predict price of the apartment using the variables sqft_living (the square footage of the living area), and grade (overall grade given to the housing unit, based on King County grading system), so let's focus on these variables. You can find the commands for data exploration in the **Analyze > Descriptive Statistics tab**

Analyze > Descriptive Statistics tab > Frequencies

Analyze > Descriptive Statistics tab > Descriptives

Analyze > Descriptive Statistics tab > Explore

Later we are also going to use a categorical variable, has_basement (whether the apartment has a basement or not) as well.

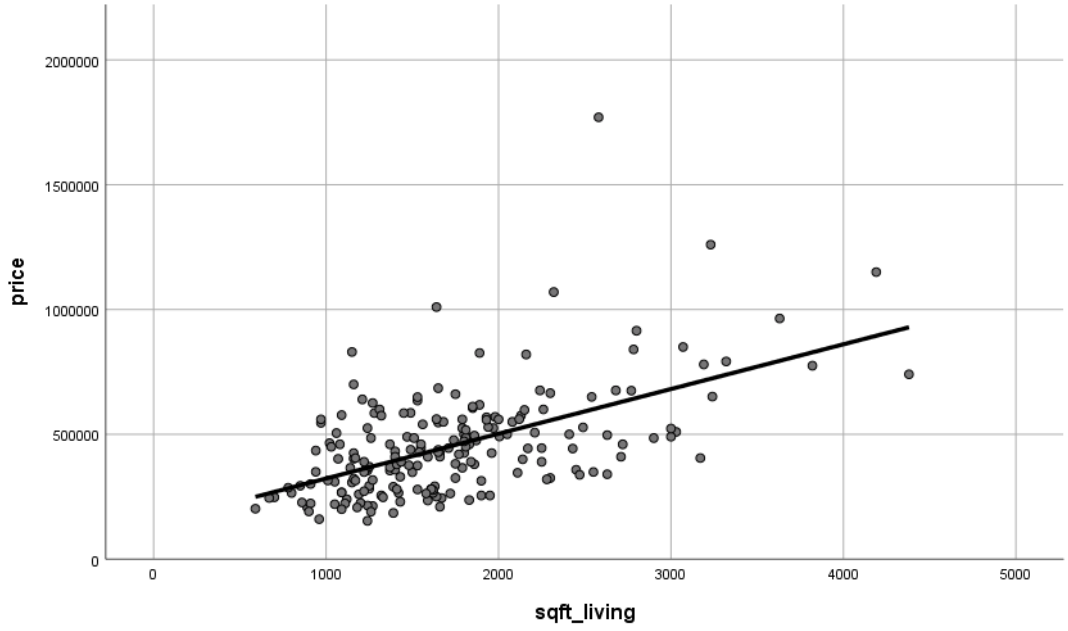
```
* Descriptives


FREQUENCIES VARIABLES=price sqft_living grade basement
  /ORDER=ANALYSIS.


DESCRIPTIVES VARIABLES=price sqft_living grade
  /STATISTICS=MEAN STDDEV MIN MAX KURTOSIS SKEWNESS.


EXAMINE VARIABLES=price sqft_living grade
  /PLOT BOXPLOT HISTOGRAM NPPLOT
  /COMPARE GROUPS
  /STATISTICS DESCRIPTIVES
  /CINTERVAL 95
  /MISSING LISTWISE
  /NOTOTAL.
```

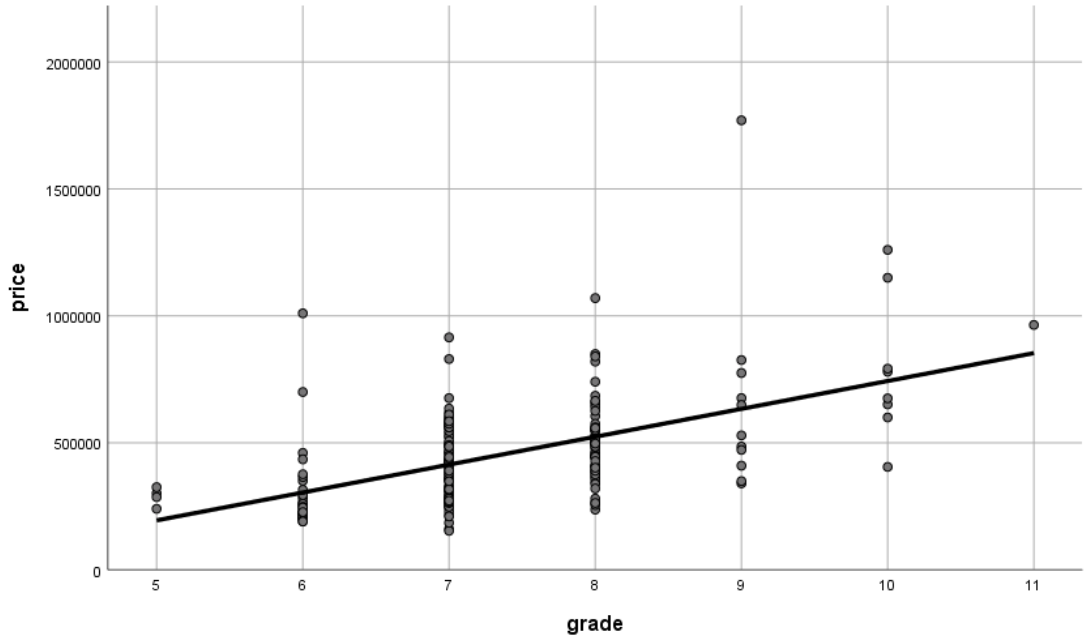
Multiple regression

Visualization

First we will fit a regression model to predict **price** by using multiple predictors: **sqft_living** and **grade**. It is always a good idea to visualize the relationship between the predictors and the outcome before fitting the regression model.

It is not trivial to visualize the regression equation in multiple regression. You can plot every pairwise relationship separately, but that is not an accurate depiction of the prediction model.





Because in a multiple regression the regression "line" is actually a multidimensional plane.

This is just to demonstrate how a 3d scatterplot looks like with the regression plane overlayed, this plot is not a depiction of the data we use here.

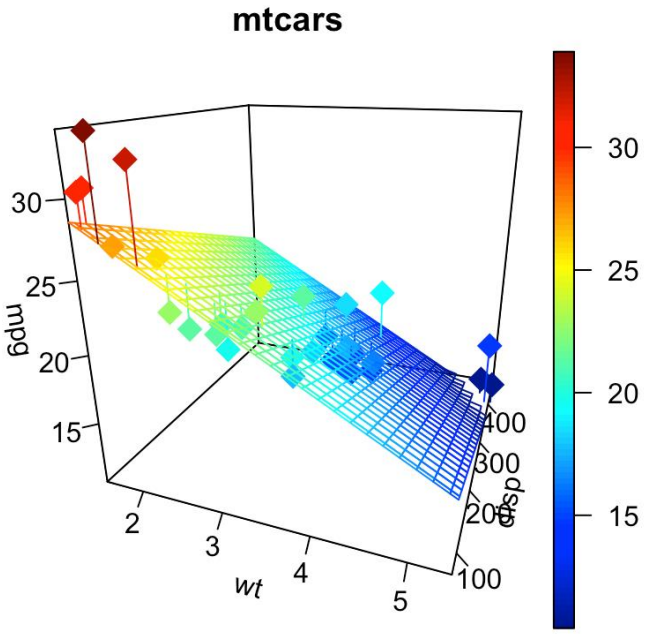


Image from: http://www.sthda.com/sthda/RDoc/figure/3d-graphics/plot3d-regression-plane-1.png.

Fitting the regression model

Now we fit a regression model to predict **price** by using multiple predictors: **sqft_living** and **grade**. **Anaysis > Regression > Linear,** and let's ask for confidence intervals of regression coefficients in the **Statistics…** button.

```
* Multiple regression


        REGRESSION
          /MISSING LISTWISE
          /STATISTICS COEFF OUTS CI(95) R ANOVA
          /CRITERIA=PIN(.05) POUT(.10)
          /NOORIGIN
          /DEPENDENT price
          /METHOD=ENTER sqft_living grade.
```

Prediction

Again, we can compute predictions for specific values of predictors (new data), but we need to specify all predictor values (in this case, both sqft_living and grade of the apartment) to get a prediction. You can compute the predicted values in **Transform > Compute variable…** , by entering the regression formula based on the coefficients in the regression output.

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			95,0% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	-174389,862	95255,171		-1,831	,069	-362240,588	13460,864
	sqft_living	119,173	24,762	,374	4,813	,000	70,341	168,005
	grade	57352,786	16052,790	,278	3,573	,000	25695,416	89010,156

a. Dependent Variable: price

Based on this output, you can provide the following formula, given that you entered the new values for which you want to get a prediction to the variables called new_sqft_living and new_grade: -174389.86 + new_sqft_living * 119.17 + new_grade * 57352.79.

We can do this for multiple apartments at the same time if we enter the parameters of the apartments we want to get the prediction for in new variables, just like we did in the previous exercise. Note that here we need to enter values for two variables, sqft_living and grade, since we have two predictors in the model. In the example below I calculate the estimated price for four different apartments: one that is 1000sqft in size with a grade of 5, one that is 1000sqft in size with a grade of 8, one that is 1500sqft in size with a grade of 5, and one that is 1500sqft in size with a grade of 8. I enter the values of the predictor variables in new columns named new_sqft_living and new_grade, and I compute the estimated prices into a new variable called predicted_values using the Compute function.

```
COMPUTE predicted_value=-174389.86 + new_sqft_living * 119.17 + new_grade *
57352.79.

EXECUTE.
```

long	sqft_living15	sqft_lot15	basement	new_sqft_living	new_grade	predicted_value
-121,834000000000	1560	11700	no basement	1000,00	5,00	231544,09
-122,359000000000	2150	19000	no basement	1000,00	8,00	403602,46
-122,327000000000	1300	1169	has basement	1500,00	5,00	291129,09
-122,404000000000	2330	6022	has basement	1500,00	8,00	463187,46
-122,267000000000	980	5650	no basement	.	.	.
-122,355000000000	1429	5400	has basement	.	.	.
-122,081000000000	1620	9690	no basement	.	.	.
-122,316000000000	1690	5800	has basement	.	.	.
-122,046000000000	1980	13664	has basement	.	.	.
-122,100000000000	1910	8000	no basement			

What to report in a publication

In a publication (and in the home assignment) you will need to report the following information about most types of regression analysis:

First of all, you will have to **describe the regression model** you built. For example:

"In a linar regression model we predicted housing price (in USD) with square footage of living area (in ft) and King County housing grade as predictors."

Next you will have to indicate **the effectiveness of the model as a whole**. You can do this by after a text summary of the results, giving information about the F-test of the whole model listed in the ANOVA table of the output, specifically, the F value, the degrees of freedom, and the p-value. Note that there are two degrees of freedom for the F test. You will need to provide the df listed in the "regression" and the "residual" lines within the ANOVA table. Also provide information about the model fit using the adjusted R squared from the Model Summary table.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Selection Criteria: Akaike Information Criterion	Selection Criteria: Amemiya Prediction Criterion	Selection Criteria: Mallows' Prediction Criterion	Selection Criteria: Schwarz Bayesian Criterion
1	,598[a]	,358	,352	170071,376	4820,567	,662	3,000	4830,462

a. Predictors: (Constant), grade, sqft_living

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	3177917994185,416	2	1588958997092,708	54,935	,000[b]
	Residual	5698081797844,145	197	28924273085,503		
	Total	8875999792029,560	199			

a. Dependent Variable: price

b. Predictors: (Constant), grade, sqft_living

Don't forget to use APA guidelines when determining how to report these statistics and how many decimal places to report (2 decimals for every number except for p values, which should be reported up to 3 decimals).

"The multiple regression model was significantly better than the null model, explaining 35.15% of the variance in housing price (F (2, 197) = 54.98, p < .001, Adj. R^2 = 0.35)."

Furthermore, you will have to provide information about **statistics for individual predictors**. **the regression equation** and the **predictors' added value** to the model. You can do this by creating a table with the following information:

Regression coefficients with confidence intervals, and standardized beta values for each predictor, together with the p-values of the t-test. You can get all this information from the Coefficients table:

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			95,0% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	-174389,862	95255,171		-1,831	,069	-362240,588	13460,864
	sqft_living	119,173	24,762	,374	4,813	,000	70,341	168,005
	grade	57352,786	16052,790	,278	3,573	,000	25695,416	89010,156

a. Dependent Variable: price

The final table should look something like this:

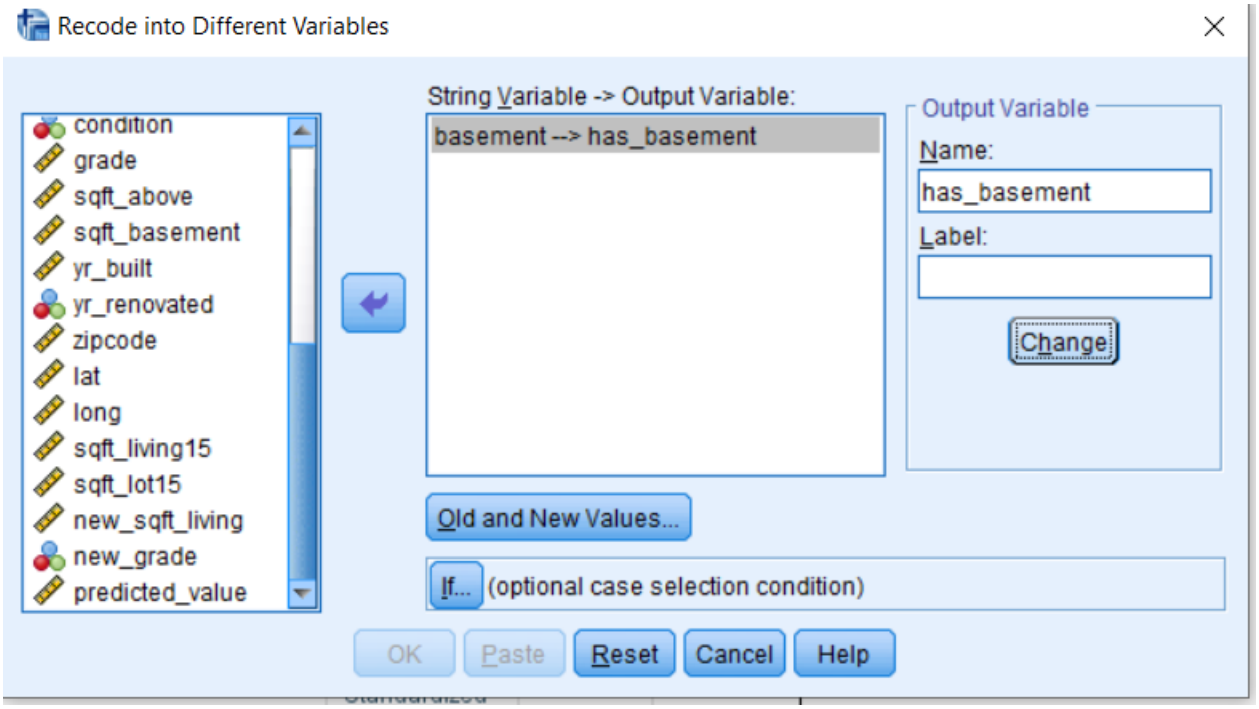
Table 1. Regression coefficitions

	b	95% CI lb	95% CI ub	Std.Beta	p-value
Intercept	-174389,862	-362240,588	13460,864		0,069
sqft_living	119,173	70,341	168,005	0,374	>0,001
grade	57352,786	25695,416	89010,156	0,278	>0,001

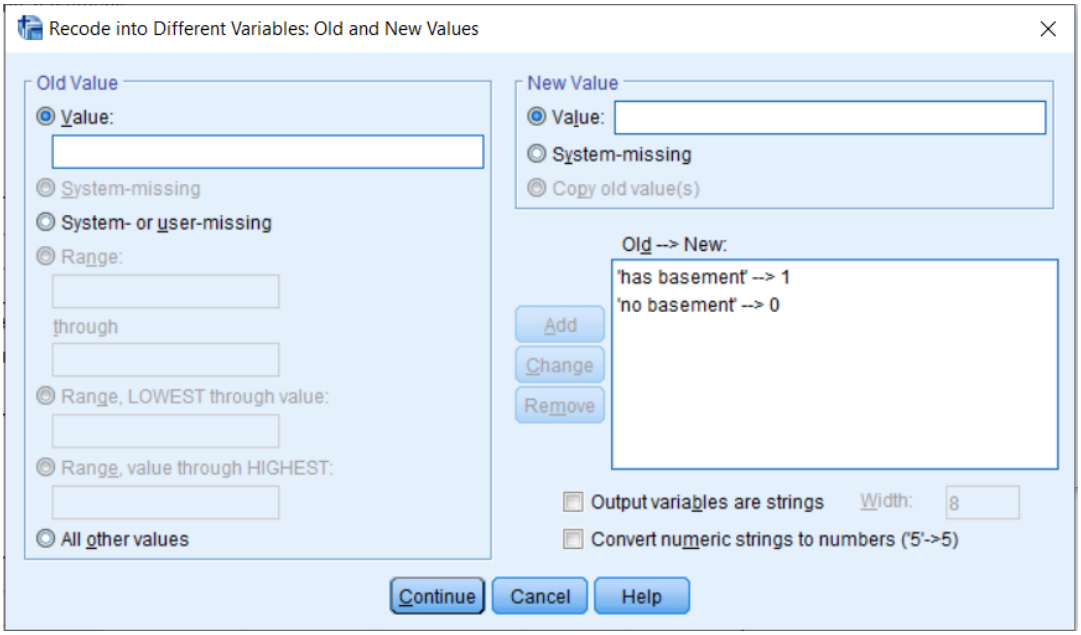
Adding categorical predictors to the model

So far we have worked with numerical variables in our regression models, but you might want to add categorical predictors to you regression model as well. This is possible to do, but SPSS does not accept string (text) variables in linear regression models as predictors, so we need to assign numerical values to the different categories/levels of the variable. The database contains a variable called **basement**. This is a string variable with two levels: “has basement” and “no basement”. We need to recode this variable into a new variable where one of the levels is recoded as 0 (best to assign this to no basement), and the other is

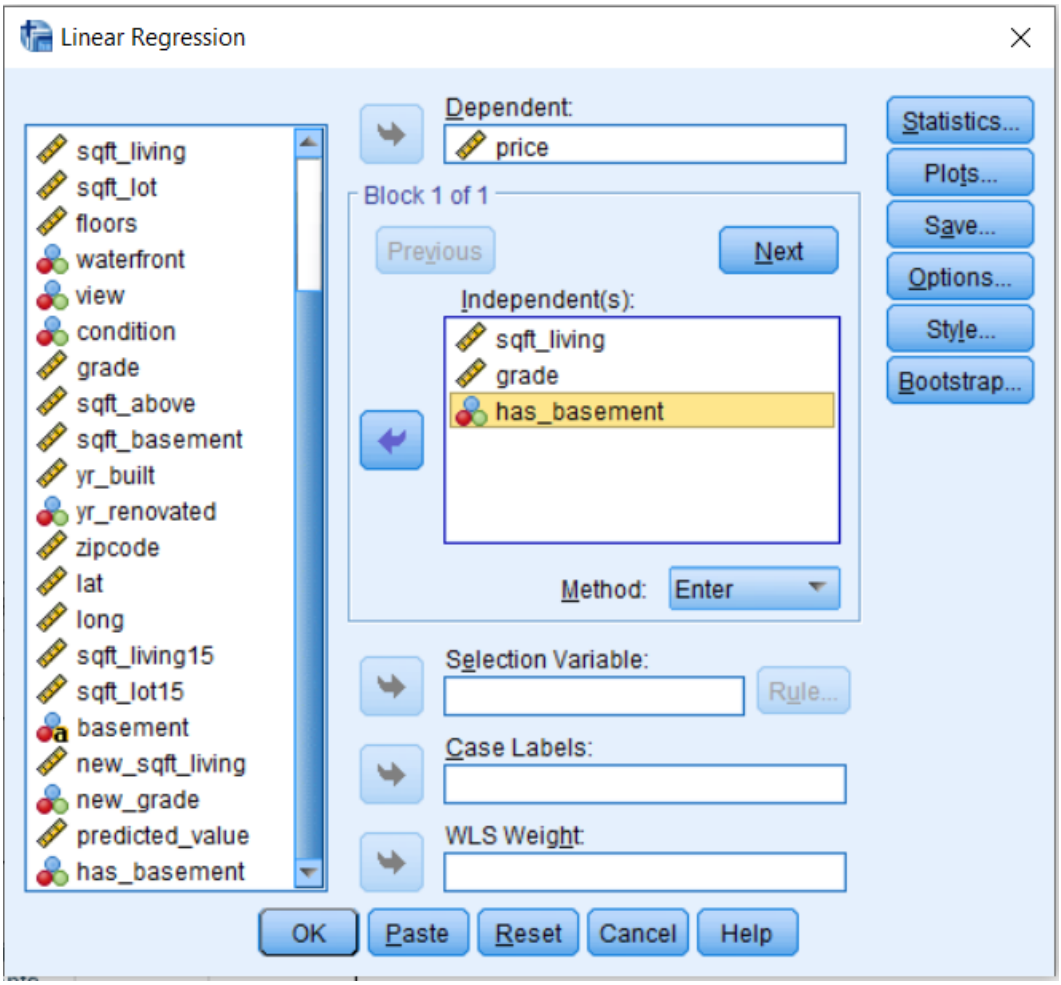
recoded as 1 (has basement). There are multiple ways you can do this in SPSS, maybe the best is to use the **Transform > Recode into different variables…** function. Here I selected basement as the String variable, then I entered the name of the Output variable as "has_basement" and pressed Change. This specifies the variable to be recoded and the name of the new variable which will contain the recoded data.



Now pressing the Old and new values button will let us specify which values in the old variable to recode into which new values in the recoded variable.



By pressing continue and then OK will finalize these changes, and now we can enter the new recoded variable into our regression model as a predictor.



Note that if you have a categorical predictor with more than two levels, you may need to create multiple "dummy" variables to be able to enter the predictor correctly into the model, but this will be discussed in more detail in the exercise about special predictors, along with the correct interpretation of the output of such regression analysis.

Interpreting the output

Interpretation of the regression coefficients

The interpretation of the regression coefficients of the predictors is: this is the amount by which the outcome variable's estimate would change if the predictor's value is increased by 1.

In our example the regression coefficient linked to sqft_living is 119,173. This means that an increas in the area of the apartment by 1 sqft results in an increased estimate in the price of the apartment by 119,173.

Interpretation of the estimate of the intercept

The coefficient of the intercept is a contant (different for each regression model) that is not dependent on the values of the predictors. It can be interpreted as if all the predictors in the model would have the value of zero (0), this would be the estimated value for the outcome. (Be careful that this often does not represent a true physical reality, if a 0 predictor value is meaningless, nevertheless, the matematical interpretation stays the same.)

Interpretation of the standard beta

The benefit of the regression coefficient is that it is on the same metric as the predicted variable, making the influence of each predictor easy to interpret. However, we need to realize that this value is also dependent on the scale of the predictor. This makes it hard to directly compare the influence of predictors that use different scales just using the regression coefficient.

In order to be able to directly compare the predictive value contained by each predictor in the model, we can use the standardized Beta coefficient. This value is computed by refitting the model with the standardized predictors. Using the standardized Beta coefficient we can directly compare the predictive value of predictors within the context of the whole model. It is important to note that the predictive value of any given predictor in a model might be very different from its individual correlation with the outcome. This is because multiple predictors can explain the same portion of the variance, this way, the predictive value of any single predictor can be "masked" in the model by the predicitve value of other predictors explaining the same portion of the variance.

Practice exercise

1. Run the regression analysis predicting the price of the apartment/house where you use any predictors in the dataset which you think could have an influence on price.

2. Determine whether your model is significantly better than the null model based on the p-value of the F-test.

3. What is the percentage of variance explained by the model (according to the adjusted R^2).

4. Which predictor is the most influential in the model (which predictor has the most predictive value in the model)?

5. By experimenting with different models, try to raise the adjusted R^2 above 52%. If you want to get access to the whole dataset or get ideas on which model works best, go to Kaggle, check out the top kernels, and download the data.
https://www.kaggle.com/harlfoxem/housesalesprediction/activity