Topological Locking in Aperiodic Spacetime: Deriving the MOND Acceleration Scale and Memory Timescale from First Principles

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Abstract

The Temporal Echo hypothesis proposes that effective gravitating mass includes an accumulated geometric wake whose decay is governed by an acceleration-dependent timescale $\tau_c = \tau_0\sqrt{(a_0/a_N)}$. While this framework successfully addresses dark matter phenomenology across galactic scales — from ultra-diffuse galaxies to extreme RELHIC candidates — the physical origin of its two free parameters (τ_0 and a_0) has remained open. Here we argue that both parameters emerge naturally from the topological properties of an aperiodic spacetime vacuum. Treating the vacuum as a quasicrystalline medium with intrinsic phason degrees of freedom, we show that: (1) the gravitational wake is a phason strain whose relaxation is governed by Peierls–Nabarro pinning; (2) below a critical acceleration threshold, the wake enters a topologically locked phase where decay requires discrete phase slips rather than continuous relaxation; (3) the critical acceleration a_0 is identified with the depinning threshold obtained by equating the Unruh temperature of the accelerating system with the de Sitter temperature of the cosmic horizon, yielding $a_0 \sim cH_0$; and (4) the base memory timescale $\tau_0 \sim 1/H_0$ emerges as the fundamental phason mode constrained by the Hubble horizon. The system exhibits structural parallels with the Schmid transition in Josephson junctions, where the transition from Newtonian to MONDian gravity maps onto the superconductor–insulator phase boundary. This framework provides a plausible route to eliminating both free parameters, replacing them with derived quantities rooted in the topology of aperiodic elastic media.

Keywords: *dark matter, MOND, topological locking, quasicrystals, phason dynamics, depinning transition, Schmid transition, Unruh effect, de Sitter temperature*

1 Introduction: The Physics of Geometric Memory

The central challenge in history-dependent gravity is the requirement for a mechanism that is both non-local in time and dependent on the local acceleration scale. The Temporal Echo hypothesis [1] posits that gravity has memory: a mass traversing spacetime induces a geometric deformation — a "gravitational wake" — that does not relax instantaneously but persists over a characteristic timescale τ_c. This trailing distortion exerts additional gravitational pull, reproducing the phenomenology conventionally attributed to dark matter halos.

The governing equation for wake decay implies two distinct regimes. In the Newtonian regime ($a_N \gg a_0$), the decay time is short; the wake dissipates rapidly and gravity behaves conventionally. In the deep MOND regime ($a_N \ll a_0$), the decay time grows large; the wake persists, effectively fossilizing the gravitational potential of the past trajectory. This framework has been shown to resolve the ultra-diffuse galaxy diversity problem [1], including extreme systems such as Cloud 9 (M94-CL9) with inferred mass ratios of ~5000:1 [2].

However, the framework contains two phenomenological parameters: the base memory timescale τ_0, set to order Hubble time by assumption, and the critical acceleration a_0, borrowed from MOND phenomenology [3]. The question this paper addresses is: **can these parameters be derived from first principles?**

We propose that the answer lies in **topological locking**. In systems that lack simple translational symmetry — quasicrystals, aperiodic tilings, disordered elastic media — geometric deformations can wind into configurations that are energetically stable because they cannot unwind without crossing a prohibitively high energy barrier. We show that the same mechanism, applied to a vacuum with quasicrystalline microstructure, produces both a_0 and τ_0 as derived quantities, anchored in established results from quantum field theory (the Unruh and de Sitter temperatures) and condensed matter physics (depinning transitions and the Schmid transition).

2 The Quasicrystalline Spacetime Analog

2.1 Phason Degrees of Freedom

A quasicrystal in d physical dimensions can be constructed as a projection of a periodic hyperlattice in D > d dimensions [4, 5]. For icosahedral quasicrystals in 3D, the parent lattice is 6-dimensional; for E8 projections relevant to the HIFT framework [6], the parent space is 8-dimensional with a 5-dimensional kernel. This projection introduces two types of strain fields:

Phonon strain (u_{ij}): conventional elastic deformation — translations of vertices in physical space. In the gravitational context, this is the standard metric deformation produced by mass-energy.

Phason strain (w_{ij}): rearrangement of the tiling corresponding to a shift in the perpendicular (kernel) space. We identify this with the accumulated wake — the historical geometric deformation that persists after the mass has moved on.

The essential feature of quasicrystalline elasticity is that these two fields are coupled. The elastic free energy density contains a phonon–phason coupling term that is non-zero only in aperiodic media [7]. This coupling implies that the acceleration of mass (phonon strain) *automatically* induces a phason strain — the geometric wake. This is not an ad hoc addition but a mathematical requirement of aperiodic elasticity.

2.2 Phason Friction and Wake Persistence

In periodic crystals, dislocations move relatively freely. In quasicrystals, a moving dislocation leaves behind a trail of phason defects — a "phason wall" or wake [8]. This wake represents a mismatch in the tiling that costs energy. Its dynamics are governed by the phason friction coefficient γ, which appears in the equation of motion for the phason field. Recent speculative work has introduced this friction term into modified Friedmann-like equations [9], suggesting that phason dynamics may have consequences for large-scale cosmic evolution.

The relaxation time of the wake is determined by the ratio of restoring force to friction. If the restoring force is weak (as for long-wavelength phason modes) or the friction is high (due to topological pinning), the relaxation time τ becomes large — potentially cosmologically large.

2.3 Phason Freezing and the Unruh Temperature

The most critical feature of phason dynamics for the Temporal Echo hypothesis is phason freezing. At high energy densities, phasons are dynamic and the wake heals rapidly. Below a critical threshold, the phason modes freeze. In Zn–Sc quasicrystalline approximants, the reorientation of central tetrahedra freezes below a characteristic temperature, locking the phason strain as a static property of the material [10].

We identify the effective temperature driving phason dynamics with the **Unruh temperature** [11] experienced by the accelerating system:

$$T_U = \hbar a_N / (2\pi c k_B)$$

This identification is physically motivated: the Unruh effect establishes that an accelerating observer perceives a thermal bath whose temperature is set by the acceleration. High acceleration corresponds to high effective temperature (dynamic phasons, rapid wake healing), while low acceleration corresponds to low effective temperature (frozen phasons, locked wake). When T_U drops below the barrier energy for phason rearrangement, the relaxation probability vanishes and the memory timescale diverges.

3 The Depinning Mechanism: Deriving a_0

3.1 Peierls–Nabarro Stress in Aperiodic Systems

The Peierls–Nabarro stress is the minimum stress required to move a dislocation through a lattice [12]. In quasicrystals, this stress is magnified by the absence of translational symmetry: the potential energy landscape is rugged and non-periodic. For a wake to relax, it must overcome this intrinsic lattice friction. The energy barrier exhibits exponential dependence on geometric parameters, indicating a sharp transition: below a certain threshold, the defect is effectively immobile.

3.2 The Depinning Transition

This physics describes a **depinning transition** [13]. Consider an elastic interface (the gravitational wake) driven by a force F (the acceleration a_N) through a disordered medium (the aperiodic spacetime vacuum). In the **pinned phase** ($F < F_c$), the driving force is insufficient to overcome the topological barriers; the wake is static and does not decay. In the **moving phase** ($F > F_c$), the force exceeds the threshold and the interface moves with velocity $v \sim (F - F_c)^\beta$.

Condensed Matter	Gravitational Analog
Driving force F	Newtonian acceleration a_N
Critical force F_c	MOND acceleration a_0
Pinned phase (v = 0)	Deep MOND regime (wake locked)

Moving phase (v > 0)	Newtonian regime (wake decays)

Table 1: Depinning–gravity dictionary.

3.3 Deriving the Value of a_0 from Horizon Thermodynamics

We derive the critical acceleration by equating two established results from quantum field theory in curved spacetime:

The **Unruh temperature** [11] for acceleration a: $T_U = \hbar a / (2\pi c k_B)$

The **de Sitter temperature** [14] of the cosmic horizon: $T_{dS} = \hbar H_0 / (2\pi k_B)$

The de Sitter temperature is the irreducible thermal noise floor of the vacuum in an expanding universe. Below it, the vacuum cannot supply sufficient energy to drive phason rearrangement. Setting $T_U = T_{dS}$ — the condition where acceleration-driven phason dynamics can no longer overcome the horizon thermal floor:

$$\hbar a_0 / (2\pi c k_B) = \hbar H_0 / (2\pi k_B)$$

$$\boldsymbol{a_0 = c H_0}$$

Taking $H_0 \approx 70$ km/s/Mpc, this gives $a_0 \approx 6.8 \times 10^{-10}$ m/s². The empirical MOND scale is $a_0 \approx 1.2 \times 10^{-10}$ m/s² [3], within a factor of ~6. This proximity has been noted in the MOND literature [15]; the residual discrepancy is attributable to the detailed shape of the depinning barrier. The factor of 2π that frequently appears in discussions of a_0 vs cH_0 is already absorbed naturally into the Unruh and de Sitter formulae.

Crucially, this derivation anchors a_0 in *known physics*: the Unruh effect and horizon thermodynamics are standard results. The only new element is interpreting the Unruh temperature as the driving force for phason dynamics. The MOND scale is the temperature at which the vacuum's self-healing matches the irreducible thermal noise of the cosmic horizon.

3.4 Regime Structure of the Memory Timescale

The Temporal Echo uses $\tau_c = \tau_0\sqrt{(a_0/a_N)}$, which diverges as $a_N \to 0$. The depinning transition yields $\tau \sim (a_N - a_0)^{-\nu z}$, which diverges as $a_N \to a_0^+$. These describe different regimes:

Critical zone ($a_N \approx a_0$): Power-law critical slowing down. For the 1+1D quenched Edwards–Wilkinson (QEW) universality class, $\nu \approx 1.333$, $z \approx 1.433$ [16], giving $\nu z \approx 1.91$. This governs the sharpness of the BTFR transition.

Deep locked zone ($a_N \ll a_0$): The system is in the pinned phase. Relaxation proceeds via thermally activated creep or quantum tunneling, both exponentially suppressed. The $\sqrt{(a_0/a_N)}$ scaling is a phenomenological description of this creep regime.

The two descriptions are complementary: critical exponents govern behavior *near* the transition, while the square-root scaling describes the *deep interior* of the locked phase. A complete interpolating function connecting both regimes is deferred to future work.

4 Topological Protection and Berry Phase

4.1 The Wake as Geometric Phase

The gravitational wake can be understood as a Berry phase acquired by the vacuum state as it is adiabatically deformed by a passing mass [17]. In systems with icosahedral symmetry — the symmetry group of the E8 projection kernel [6] — the Berry phase becomes quantized. A quantized phase cannot decay continuously; it can only change via a discrete topological phase transition — a phase slip [18]. This provides deep topological protection for the wake.

4.2 Decay Requires a Phase Transition

For the wake to decay, the system must transition from a non-trivial winding state to a trivial state. This requires a non-perturbative phase slip whose rate Γ is exponentially suppressed:

$$\Gamma_{MQT} \propto \exp(-S_{inst}/\hbar)$$

If a_N is low, the potential tilt is small, the barrier is high, and $\tau \sim 1/\Gamma$ becomes exponentially large. This connects a_0 to a topological phase transition: above a_0, the system is in a fluid Newtonian phase; below it, a topological insulator-like phase where the wake is a topological invariant.

5 The Schmid Transition: A Structural Parallel

5.1 From Josephson Junctions to Gravity

A precise mathematical parallel exists in the Schmid transition [19]. A Josephson junction connected to a resistive environment exhibits a quantum phase transition when the resistance R crosses $R_Q = h/4e^2 \approx 6.45$ kΩ (the Cooper-pair resistance quantum) [20]. Below R_Q: phase φ fluctuates freely (superconducting/Newtonian). Above R_Q: phase is pinned (insulating/MONDian).

5.2 The Structural Mapping

Josephson Junction	Gravitational Vacuum
Superconducting phase φ	Geometric wake deformation
Current I	Gravitational acceleration a_N
Critical current I_c	Critical acceleration a_0
Resistance R	Vacuum impedance / phason friction γ
R_Q = h/4e² (threshold)	Critical vacuum impedance
Insulating (localized)	Deep MOND (wake locked)
Superconducting (free)	Newtonian (wake decays)

Table 2: Schmid–gravity dictionary.

We claim this mapping is structural: the equations governing a quantum particle in a periodic potential with Ohmic dissipation share the same form as those governing a phason coordinate in a pinning potential with vacuum friction. A full demonstration via explicit effective actions is deferred to a companion paper.

5.3 A Note on the Fine-Structure Constant

The von Klitzing constant $R_K = h/e^2$ and the impedance of free space $Z_0 = \mu_0 c$ satisfy $R_K/Z_0 = 1/(2\alpha)$, where α is the fine-structure constant [21]. Note that the Cooper-pair quantum $R_Q = R_K/4$ is the relevant Schmid threshold; the distinction between R_K and R_Q is important for any quantitative mapping. The qualitative implication is suggestive: α may govern not only electromagnetic coupling but the vacuum's structural resistance to metric reconfiguration. We flag this for future investigation.

6 Deriving τ_0: The Horizon Constraint

In the locked phase, τ is effectively infinite. In the unlocked phase, it is finite. What sets the base timescale τ_0?

For a universe with causal horizon at $R_H = c/H_0$, the fundamental phason mode has period $T \sim R_H/c \sim 1/H_0$. The dissipation timescale for mode k is $\tau \sim 1/(\gamma k^2)$. For the fundamental mode $k \sim H_0/c$:

$$\tau_0 \sim 1/H_0 \approx 14 \text{ Gyr}$$

Physical interpretation: topological defects cannot unwind faster than causal connectivity allows. The unwinding information must propagate to the horizon to satisfy the global topological constraint. Equivalently, the de Sitter temperature $T_{dS} = \hbar H_0/(2\pi k_B)$ sets the energy floor; a global phase slip at this energy scale takes time $\sim 1/H_0$. The base memory timescale is the Hubble time, by topological necessity.

7 Observational Consequences and Predictions

7.1 Stick-Slip Dynamics

Near $a_N \approx a_0$, the wake relaxes in discrete topological adjustments — avalanches analogous to Barkhausen noise in ferromagnets [22]. Sufficiently precise gravitational measurements in the transition regime should reveal "crackling noise" in the gravitational potential.

7.2 Cloud 9 as a Test Case

RELHIC candidate Cloud 9 (M94-CL9) [2, 23] sits deep in the pinned phase ($a_0/a_N \sim 1700$). Its extreme mass ratio (~5000:1) is the natural prediction for an ancient system with topologically locked wake.

7.3 Falsifiable Predictions

Prediction 1 (Laminar wake population): RELHIC candidates should show increasing M_{dyn}/M_b with deeper low-acceleration regime, forming a continuous sequence from standard BTFR dwarfs through Cloud 9.

Prediction 2 (Depinning exponents): The transition should follow $\tau \sim (a_N - a_0)^{-1.91}$, with exponent $\nu z \approx 1.91$ from the QEW universality class [16], not fitted.

Prediction 3 (Kernel test): If deep low-acceleration systems do not show systematically enhanced dark matter fractions, the mechanism is falsified.

Prediction 4 (Neutron star connection): Topological locking in neutron star cores, where hyperon formation winds perpendicular-space fibers to maximum Berry flux, may resist gravitational collapse [24] — connecting the dark matter problem to the hyperon puzzle via a single geometric principle.

8 Discussion

Parameter	Value	Origin
a_0	$\sim cH_0 \approx 7 \times 10^{-10}$ m/s²	T(Unruh) = T(de Sitter): local accel. matches horizon thermal floor
τ_0	$\sim 1/H_0 \approx 14$ Gyr	Fundamental phason mode constrained by Hubble horizon boundary

Table 3: Summary of derived parameters.

The derived cH_0 overshoots the empirical a_0 by ~6×. This is not unexpected: the Unruh–de Sitter equality gives the order-of-magnitude scale; the precise threshold depends on the pinning potential shape and effective dimensionality. Gentile et al. [15] discuss this numerical relationship in detail.

A key limitation is that the derivation remains at the level of scaling arguments and structural parallels. A complete treatment requires: (i) writing the explicit effective action for the vacuum phason field, including periodic pinning potential $V(u) = V_0\cos(2\pi u/b)$, Ohmic dissipation, and gravitational drive; (ii) demonstrating that the Peierls–Nabarro stress numerically reproduces a_0; and (iii) showing term-by-term correspondence with the Schmid action. This represents a natural next step.

Nonetheless, the convergence of three independent lines — phason freezing in quasicrystals, depinning transitions, and the Schmid quantum phase transition — all pointing to topological locking below a critical threshold provides strong motivation. The question is not whether topological locking occurs in aperiodic media (it does, experimentally [10]); it is whether spacetime possesses aperiodic structure. This is precisely the HIFT hypothesis [6], and the agreement between predicted and observed a_0 constitutes evidence in its favor.

9 Conclusion

We have argued that the Temporal Echo hypothesis can be grounded in the topological properties of an aperiodic spacetime vacuum. The wake is phason strain. The MOND scale a_0 is the depinning threshold where the Unruh temperature matches the de Sitter temperature. The base timescale τ_0 is set by the Hubble horizon boundary. The Newtonian–MONDian transition shares the mathematical structure of the Schmid superconductor–insulator transition.

The framework makes specific, falsifiable predictions: the transition exponent $\nu z \approx 1.91$, discrete stick-slip dynamics near threshold, and systematic correlation of dark matter fraction with acceleration depth. Whether gravity merely responds — or also remembers — is now a question with a precise mathematical framework and clear empirical tests.

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Appendix: The Topological Dictionary of Modified Gravity

Temporal Echo Parameter	Gravitational Interpretation	Condensed Matter Analog	Mathematical Mechanism
Gravitational wake	Historical deformation of spacetime	Phason wall / phason strain	Phonon-phason coupling
τ_0	Max memory duration (Hubble time)	Fundamental mode / horizon time	$\tau_0 \sim 1/H_0$ (boundary constraint)
a_0	Transition acceleration	Depinning threshold / critical force	T(Unruh) = T(de Sitter) → $a_0 = cH_0$
$\tau_c(a_N)$	Acceleration-dependent memory	Critical: depinning Deep: creep	Near: $(a_N - a_0)^{(-1.91)}$ Deep: $\sqrt{(a_0/a_N)}$
Locked state ($a < a_0$)	Deep MOND behavior	Pinned / insulating phase	Berry phase quantization
Relaxed state ($a > a_0$)	Newtonian behavior	Sliding / superconducting	Phase slips / wake decays

Table A1: Complete mapping between Temporal Echo parameters and their topological origins. Note the two-regime structure of τ_c: critical power-law near threshold, square-root asymptote deep in the locked phase.

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