2.23 卡车 $\xleftarrow{a=7m/s^2}$ $\vec{v}$

$f \leftarrow \boxed{\quad} \rightarrow ma$

$f = \mu_k mg$

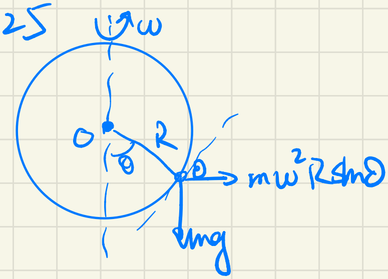
以车厢参考系：

$$a_{箱} = \frac{ma - \mu_k mg}{m} = a - \mu_k g.$$

$$v^2 = 2al$$

$$\Rightarrow v = \sqrt{2al} \approx 2.9 m/s$$

2.25



切线方向受力平衡：

$$m\omega^2 R\sin\theta\cos\theta = mg\sin\theta$$

$$\cos\theta = \frac{g}{\omega^2 R} \quad \text{or} \quad \sin\theta = 0$$

$\Downarrow$

$\theta = 0$ or π

$\Downarrow$

在轴上的两点

① $\cos\theta = \frac{g}{\omega^2 R}$

$$\frac{dF_t}{d\theta} = m\omega^2 R(\cos^2\theta + \sin^2\theta) - mg\cos\theta$$

$$= m\omega^2 R - \frac{mg^2}{\omega^2 R}$$

若 $\frac{dF_t}{d\theta} < 0$，即 $\omega > \sqrt{\frac{g}{R}}$，稳定

若 $\frac{dF_t}{d\theta} \geq 0$，即 $\omega \leq \sqrt{\frac{g}{R}}$，不稳定

② $\theta = 0$

$$\frac{dF_t}{d\theta} = m\omega^2 R - mg \Rightarrow \begin{cases} \omega < \sqrt{\frac{g}{R}} \text{ 稳定} \\ \omega \geq \sqrt{\frac{g}{R}} \text{ 不稳定} \end{cases}$$

③ $\theta = \pi$

$$\frac{dF_t}{d\theta} = m\omega^2 R - 0$$

$$= m\omega^2 R > 0$$

不稳定

3.4

$$\overline{F}_{N} = \frac{Nmv}{t} = 120\,\mathrm{min}^{-1} \cdot 7.90\,g \cdot 735\,m/s \div 60\,s$$

$$= (120 \times 7.9 \times 10^{-3} \cdot 735 \div 60)\,N$$

$$= 11.6\,N$$

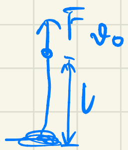
3.5

$$I = mv = (\rho S v t) v$$

$$= \rho S v^2 t$$

$$F = \frac{I}{t} = \rho S v^2 \approx 1.1 \times 10^5\,N$$

3.7



Δt时间内，有Δm质量的链被提起，

即速度 $0 \to v_0$

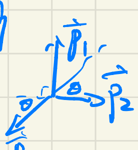
$$\Delta I = \Delta m\, v_0 = (\rho_l \cdot v_0 \Delta t) v_0$$

$$= \rho_l v_0^2 \Delta t$$

$$\begin{cases} F_1 = \dfrac{\Delta I}{\Delta t} = \rho_l v_0^2 \\ F_0 = \rho_l \cdot l \cdot g \end{cases} \Rightarrow F = F_0 + F_1$$

$$= \rho_l l g + \rho_l v_0^2$$

3.9



$$\vec{P} = \vec{P_1} + \vec{P_2}$$

$$|\vec{P}| = \sqrt{|\vec{P_1}|^2 + |\vec{P_2}|^2} = 1.07 \times 10^{-20}\,kg \cdot m/s$$

方向与$\vec{P_1}$夹角为$90° + \theta$

与$\vec{P_2}$夹角为$180° - \theta$

其中$\tan\theta \approx 1.73$

3.11

$$\begin{cases} v^2 = 2s\cdot a \\ a = \dfrac{2m\mu_k g}{2m} = \mu_k g \end{cases} \Rightarrow v = \sqrt{2s\mu_k g} = 19.8\,m/s$$

考虑，$v_0 = v_1 = v_m = 14m/s$

此时 $v' = \frac{\sqrt{2}}{2} v_m = 9.9m/s < v$

因此至少有一位司机的话不可信

3.14

(1)

$$F = u\frac{dm}{dt} = Mg$$

$$\Rightarrow u = \frac{Mg}{\dot{m}(t)} = 2.07\times10^4\ m/s$$

(2)

$$\begin{cases} dp = Mdv - u\,dm \\ dM = -\,dm \end{cases} \Rightarrow dp = Mdv + u\,dM = -Mg\,dt$$

$$\Rightarrow dv = -u\frac{dM}{M} - g\,dt$$

$$\Rightarrow v = u\ln\left|\frac{M}{M-m_{燃}}\right| - gt$$

$$= 2.68\times10^3\ m/s$$

(3)

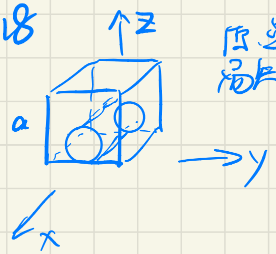
$$h = \int_0^t v\,dt = \int_0^t \left(u\ln\left|\frac{M}{M-\frac{dm}{dt}t}\right| - gt\right)dt$$

$$= u\ln M\cdot t - \frac{1}{2}gt^2 - u\int_0^t \ln\left|M - \frac{dm}{dt}\cdot t\right| dt$$

$$= u\ln M\,t - \frac{1}{2}gt^2 - u\left(t - \frac{M}{\frac{dm}{dt}}\right)\ln\left(t - \frac{M}{\frac{dm}{dt}} - 1\right) + u\ln\frac{dm}{dt}\,t$$

$$\approx 1.72\times10^5\ m$$

3.18



原点位于底面中心

易知质心在 $x=0,\ y=0$ 上

挖前质心：$z_0=\frac{a}{2}$，质量：ρa^3

挖去部分质心：$z_1=\frac{a}{4}$，质量：$\rho\cdot a\cdot\pi\left(\frac{a}{4}\right)^2$

$=\rho\cdot\frac{\pi}{16}a^3$

现质心：$z=\frac{M_0z_0-M_1z_1}{M_0-M_1}=0.561a$

3.19

$t_0=1s$

$$v=\frac{m_1v_1+m_2v_2}{m_1+m_2}=\frac{m_1 gt+m_2 g(t-t_0)}{m_1+m_2}$$

$$=\left(t-\frac{5}{7}\right)g$$

$$a=\frac{dv}{dt}=g$$

附加题：

惯性离心力：$F=m\omega^2 r$ 提供向心力