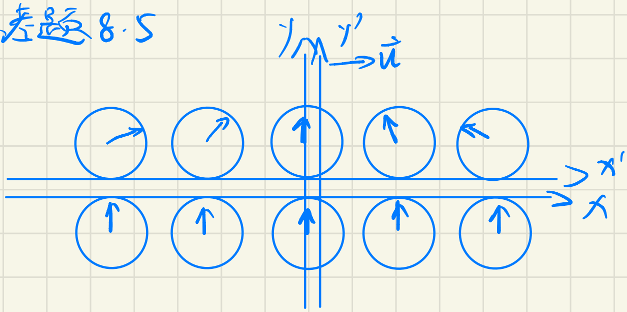
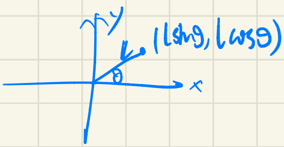
思考题 8.5



$$t' = \frac{t - \frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}}$$

8.1



$$x' = x\sqrt{1-\frac{u^2}{c^2}}$$

$$y' = y$$

$$l' = \sqrt{(x')^2 + (y')^2}$$

$$= \sqrt{l^2\sin^2\theta\left(1-\frac{u^2}{c^2}\right) + l^2\cos^2\theta}$$

$$= l\sqrt{1-\sin^2\theta\cdot\frac{u^2}{c^2}}$$

$$\Rightarrow \tan\theta' = \frac{y'}{x'}$$

$$= \tan\theta \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\Rightarrow \theta' = \arctan\left(\tan\theta\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}\right)$$

8.3

$$\Delta x = 1\,\mathrm{m}$$

$$\Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1-\frac{u^2}{c^2}}} \quad (\Delta t = 0)$$

$$= \frac{\Delta x}{\sqrt{1-\frac{u^2}{c^2}}} > 1\,\mathrm{m}$$

对于S'系中的观察者，两枪并非同时响起，而是远离O点的枪先响

8.4

$$t_1' = \frac{t_1 - \frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}} \qquad t_2' = \frac{t_2 - \frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{u^2}{c^2}}} \Rightarrow \sqrt{1-\frac{u^2}{c^2}} = \frac{\Delta t}{\Delta t'} = \frac{2}{3} \Rightarrow u = \frac{\sqrt{5}}{3}c$$

$$d = u\cdot\Delta t' = 3\sqrt{5}\times10^{8}\ m/s \approx 6.71\times10^{8}\ m$$

8.5

$$\Delta x' = \frac{\Delta x - u\Delta t}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{\Delta x}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\Rightarrow \sqrt{1-\frac{u^2}{c^2}} = \frac{\Delta x}{\Delta x'} = \frac{1}{2} \Rightarrow u = \frac{\sqrt{3}}{2}c$$

$$\Delta t' = \left|\frac{\Delta t - \frac{u}{c^2}\Delta x}{\sqrt{1-\frac{u^2}{c^2}}}\right| = \frac{\frac{u}{c^2}\Delta x}{\sqrt{1-\frac{u^2}{c^2}}} = 5.77\times10^{-9}\ s$$

8.6

(1) $d = c\cdot\frac{t_0}{2} = 9\times10^{9}\ m$

(2) $L_0 = d - u\cdot\frac{t_0}{2} = 1.8\times10^{9}\ m$

$$L_{0地} = \frac{L_0}{\sqrt{1-\frac{u^2}{c^2}}} = 3\times10^{9}\ m$$

从发射到接收: $\Delta t_{地} = \frac{t_0 - u\cdot0}{\sqrt{1-\frac{u^2}{c^2}}} = 100s$

$$l = L_{0地} + \Delta t_{地}\cdot u = 2.7\times10^{10}\ m$$

8.7

$$t = \frac{t' + \frac{u}{c^2}x'}{\sqrt{1-\frac{u^2}{c^2}}} = -2\times10^{8}\ s \qquad y = y' = 1.2\times10^{17}\ m \qquad z = z' = 0$$

$$x = \frac{x' + ut'}{\sqrt{1-\frac{u^2}{c^2}}} = 6\times10^{16}\ m$$

8.8

(1) $t_1' = t' + \Delta t_1' = t' + \frac{\sqrt{(x')^2+(y')^2+(z')^2}}{c} = 1.2\times10^{8}\ s$

(2) $t_2 = t_1 + \Delta t_2 = \frac{t_1' + \frac{u}{c^2}x_2'}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{x_2}{c} = \frac{t_1' + \frac{u}{c^2}x_2'}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{x_2' + \frac{u}{c}t_1'}{\sqrt{1-\frac{u^2}{c^2}}}$

$$= \frac{t_1'}{\sqrt{1-\frac{u^2}{c^2}}} + \frac{\frac{u}{c}t_1'}{\sqrt{1-\frac{u^2}{c^2}}} = t_1'\cdot\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}} = t_1'\sqrt{\frac{u+c}{c-u}} = 3t_1' = 3.6\times10^{8}\ s$$

(3) $t_3 = t + \frac{\sqrt{x^2+y^2}}{c} = 2.5\times10^{8}\ s$

8.9

(1) 以飞船为S系，地球为S'系.

$$v' = -0.8c$$

$$v = \frac{v'+u}{1+\frac{uv'}{c^2}} = \frac{-0.8c-0.6c}{1+0.8\times 0.6} \approx -0.95c$$

(2)

$$\Delta t = \Delta t'\sqrt{1-\frac{u^2}{c^2}} = 0.8\Delta t' = 0.8\times 5s = 4s$$

8.10

$$v_x' = c\cos\theta' \qquad v_y' = c\sin\theta'$$

$$v_x = \frac{v_x'+u}{1+\frac{uv_x'}{c^2}} = \frac{\cos\theta' c+u}{1+\frac{\cos\theta' u}{c}} \qquad v_y = \frac{v_y'}{1+\frac{uv_x'}{c^2}}\sqrt{1-\frac{u^2}{c^2}} = \frac{\sin\theta' c}{1+\frac{\cos\theta' u}{c}}\sqrt{1-\frac{u^2}{c^2}}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{\sin\theta' c}{\cos\theta' c+u}\sqrt{1-\frac{u^2}{c^2}}$$

$$v^2 = v_x^2+v_y^2 = \frac{(u+\cos\theta' c)^2}{(c+\cos\theta' u)^2}c^2 + c^2\cdot\frac{\sin^2\theta'(c^2-u^2)}{(c+\cos\theta' u)^2} = c^2$$

8.12

$$\left.\begin{aligned} \vec{P} &= m\vec{v} = \vec{F}t \\ m &= \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned}\right\} \Rightarrow \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = Ft$$

$$m_0^2 v^2 = F^2t^2\left(1-\frac{v^2}{c^2}\right)$$

$$\left(m_0^2+\frac{F^2t^2}{c^2}\right)v^2 = F^2t^2$$

$$v = \frac{Ftc}{\sqrt{m_0^2c^2+F^2t^2}}$$

$$x = \int_0^t v\,dt = \frac{c}{F}(m_0^2c^2+F^2t^2)^{\frac{1}{2}} - \frac{c}{F}m_0c$$

$$t \ll \frac{m_0c}{F} \Rightarrow \begin{cases} v \approx \frac{Ft}{m_0c}\cdot c = \frac{F}{m_0}t = at \\ x \approx \frac{1}{2}\frac{F}{m_0}t^2 = \frac{1}{2}at^2 \end{cases}$$

$$t \gg \frac{m_0c}{F} \Rightarrow \begin{cases} v = \frac{c}{\sqrt{1+\left(\frac{m_0c}{Ft}\right)^2}} \approx c \\ x = \frac{c}{F}\cdot Ft\left(\left(\frac{m_0c}{Ft}\right)^2+1\right)^{\frac{1}{2}} - \frac{m_0c^2}{F} = ct\left(1+\frac{1}{2}\frac{m_0^2c^2}{F^2t^2}\right) - \frac{m_0c^2}{F} \\ \approx ct \end{cases}$$

8.14

(1) $E_K = E - E_0 = mc^2 - m_e c^2 = \frac{m_e c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_e c^2$

$\Rightarrow \frac{E_k}{m_e c^2} + 1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$1-\frac{v^2}{c^2} = \left(\frac{m_e c^2}{E_k + m_e c^2}\right)^2$

$v^2 = c^2 - c^2 \frac{m_e^2 c^4}{(E_k + m_e c^2)^2}$

$= \frac{E_k^2 + 2E_k m_e c^2}{(E_k + m_e c^2)^2} c^2$

$= \frac{(2.8\times10^9)^2 + 5.6\times10^9 \times 5.1\times10^5}{(2.8\times10^9 + 5.1\times10^5)^2} c^2$

$\approx 0.9999999668\, c^2$

$\Rightarrow v = 0.9999999834\, c$

$= (3\times10^8 - 5)\ m/s$

8.16

$E_{结} = E_{p0} + E_{n0} - E_{D0}$

$= c^2(m_p + m_n - m_D) = 3.55\times10^{-13}\ J$

$\eta = \frac{E_{结}}{m_D c^2} = 0.12\%$

8.17

(1) $\Delta E = c^2(4\times m_p - m_{He} - m_e)$

$= 4.15\times10^{-12}\,J$

(2) $\eta = \dfrac{\Delta E}{4\times m_p c^2} = 0.69\%$

(3) $E_m = \dfrac{\Delta E}{4m_p} = 6.2\times10^{14}\,J/kg$

(4) $m_t = \dfrac{P}{E_m} = 6.3\times10^{11}\,kg/s$

(5) $T = \dfrac{m}{m_t} = 2.4\times10^{18}\,s$

8.21

$E_{av} = 2.8\times2\,GeV = 5.6\,GeV$

$= \sqrt{2mc^2E_k + (2mc^2)^2}$

$\Rightarrow E_k = \dfrac{E_{av}^2 - 4m^2c^4}{2mc^2} \approx 3.1\times10^4\,GeV$