These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Two

Budget Constraint

Where are We in the Course?

We are working on the 1st of the 3 components of microeconomics: Consumer behavior, production theory, and market.

There are three elements of consumer behavior: budget constraint, preference, and choices.

Consumption Choice Sets

A **consumption choice set** is the collection of all consumption choices available to the consumer.

What constrains consumption choice?

- Money
- And something else?

Budget Constraints

Consumer plans to spend money on n goods

Bundle: $(x_1, \dots, x_n)$

Q: When is a bundle $(x_1, \dots, x_n)$ affordable at prices $p_1, \dots, p_n$?

A: When

$$p_1x_1 + \dots + p_nx_n \leq m$$

where m is the consumer's income.

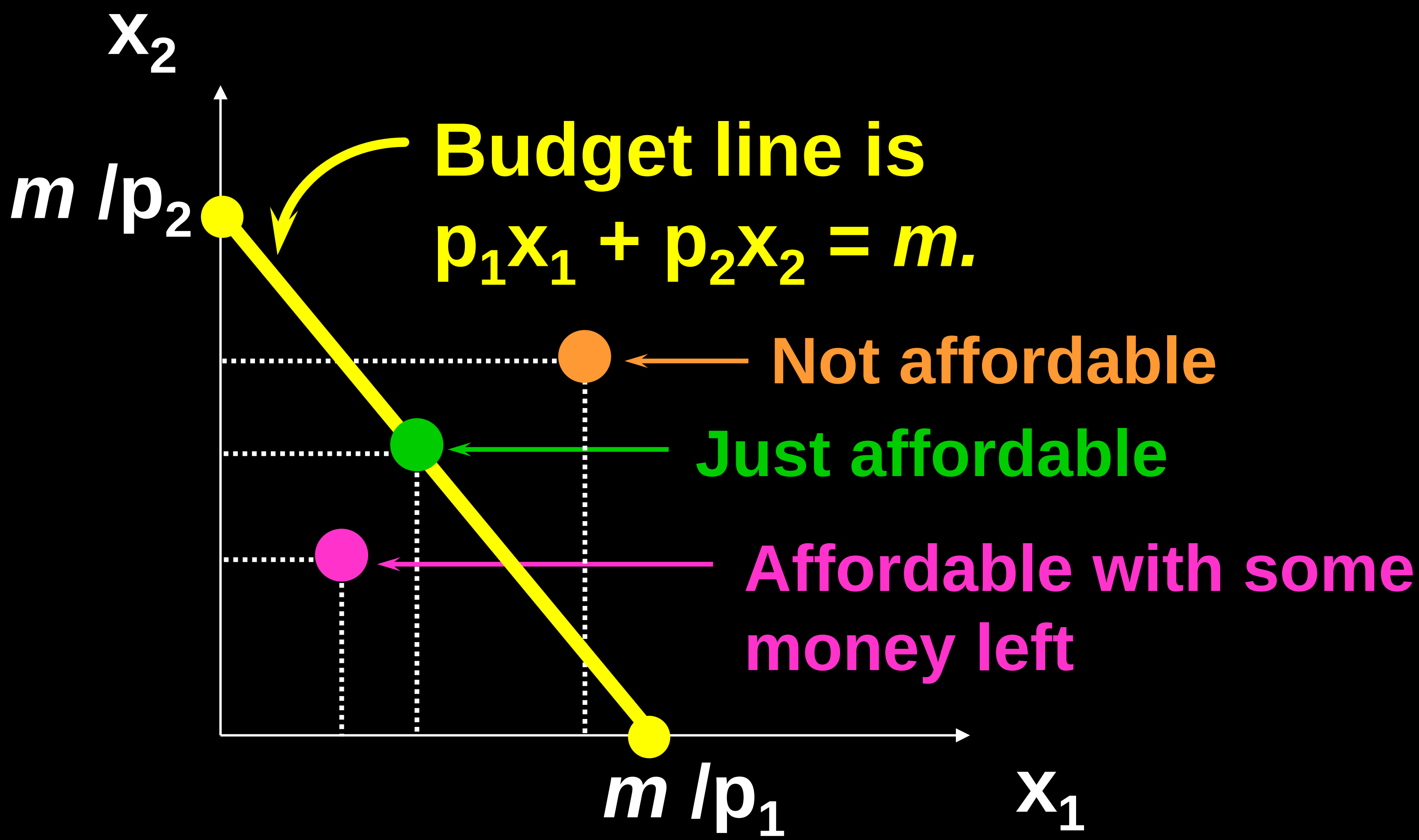
Budget Constraints

The consumer's budget set is the set of all affordable bundles;

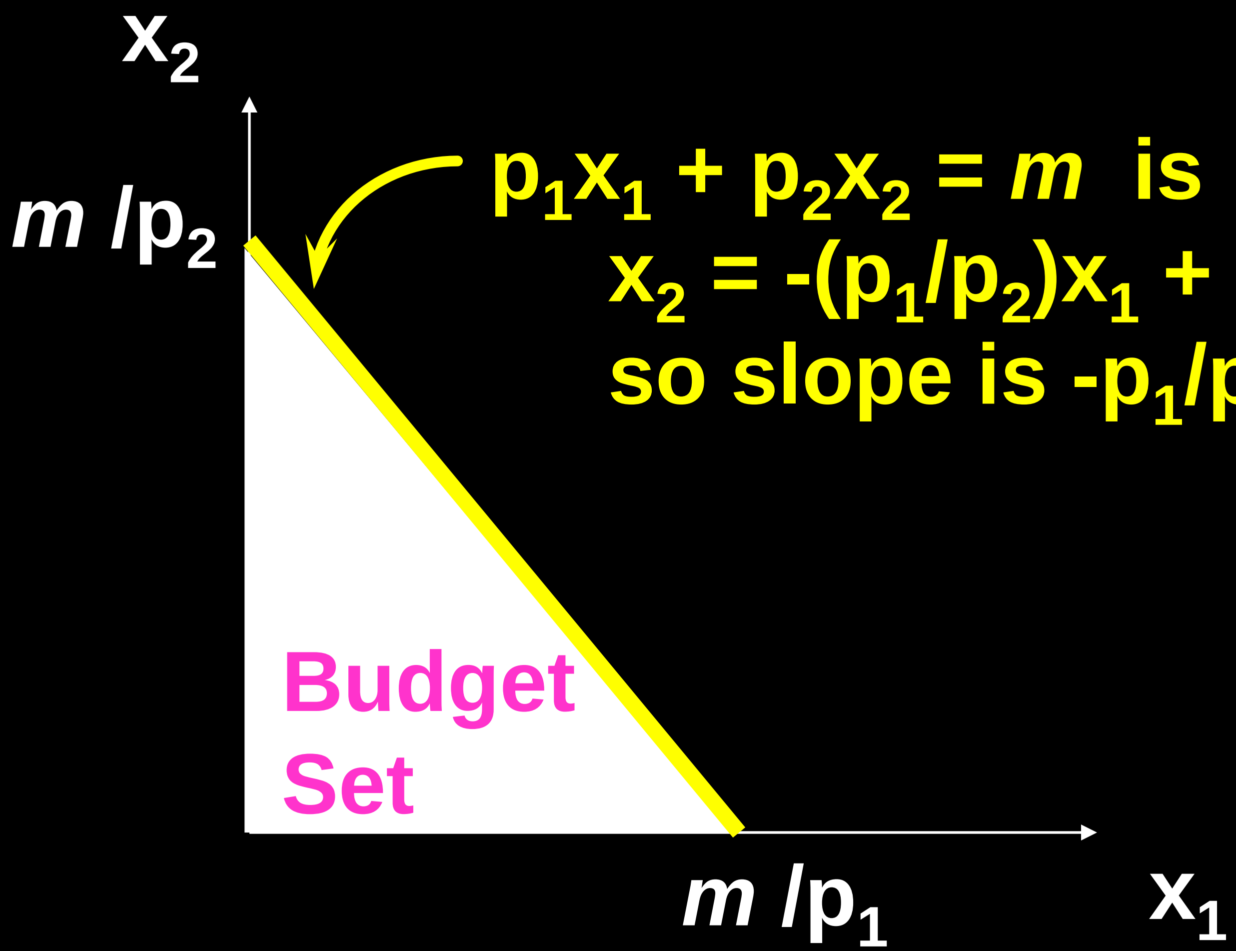
$$B(p_1, \ldots, p_n, m) =$$
$$\{ (x_1, \ldots, x_n) \mid x_1 \geq 0, \ldots, x_n \geq 0 \text{ and } p_1x_1 + \ldots + p_nx_n \leq m \}$$

The budget line is the upper boundary of the budget set.

Budget Set for Two Commodities



Budget Set for Two Commodities

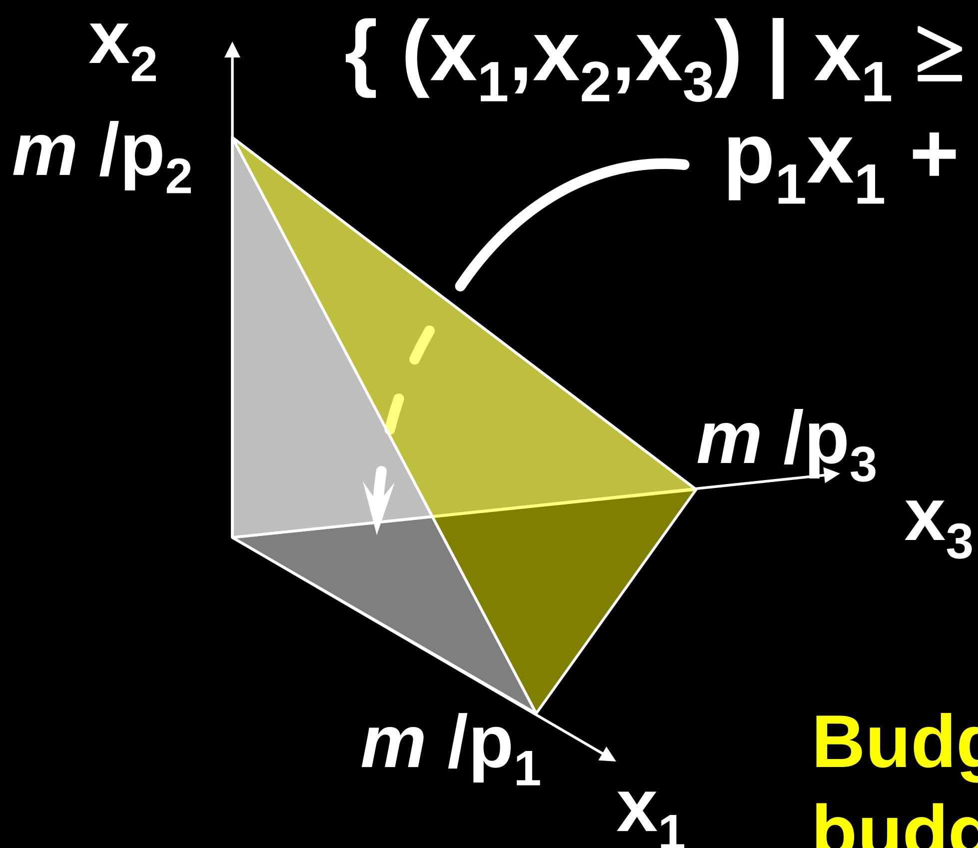


$p_1x_1 + p_2x_2 = m$ is
$x_2 = -(p_1/p_2)x_1 + m/p_2$
so slope is $-p_1/p_2$.

Budget Constraints

If n = 3 what do the budget line and the budget set look like?

Budget Set for Three Commodities



$\{ (x_1,x_2,x_3) \mid x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$ and $p_1x_1 + p_2x_2 + p_3x_3 \leq m \}$

Budget line becomes budget "plane"

Budget Constraints

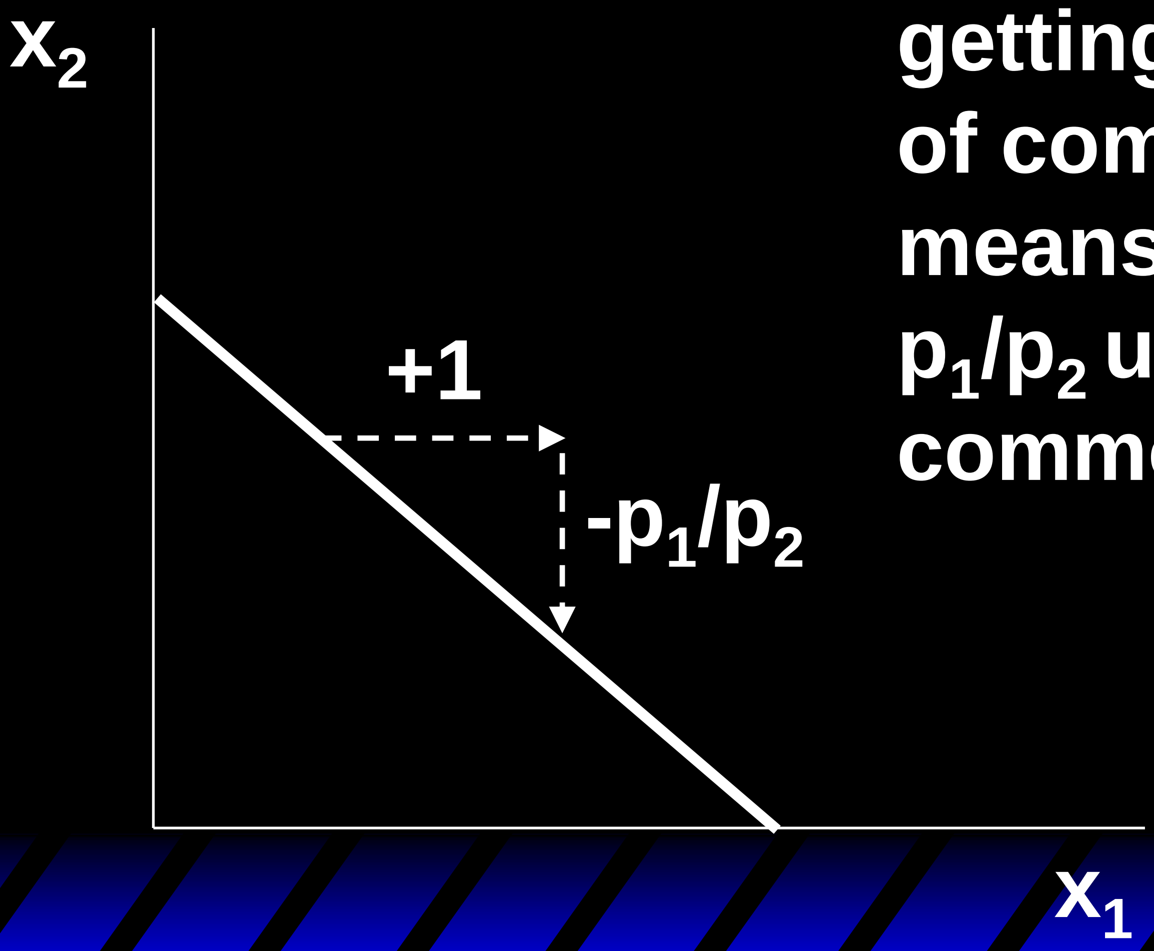
For $n = 2$ and x_1 on the horizontal axis, the budget line's slope is $-p_1/p_2$. What does it mean?

$$x_2 = -\frac{p_1}{p_2} x_1 + \frac{m}{p_2}$$

Increasing x_1 by 1 must reduce x_2 by p_1/p_2.

– Opportunity cost / trade-off

Budget Constraints



On the budget line, getting an extra unit of commodity 1 means forgoing p_1/p_2 units of commodity 2

Notes on Units of Measure

Economists tend to be sloppy in terms of units of measure.

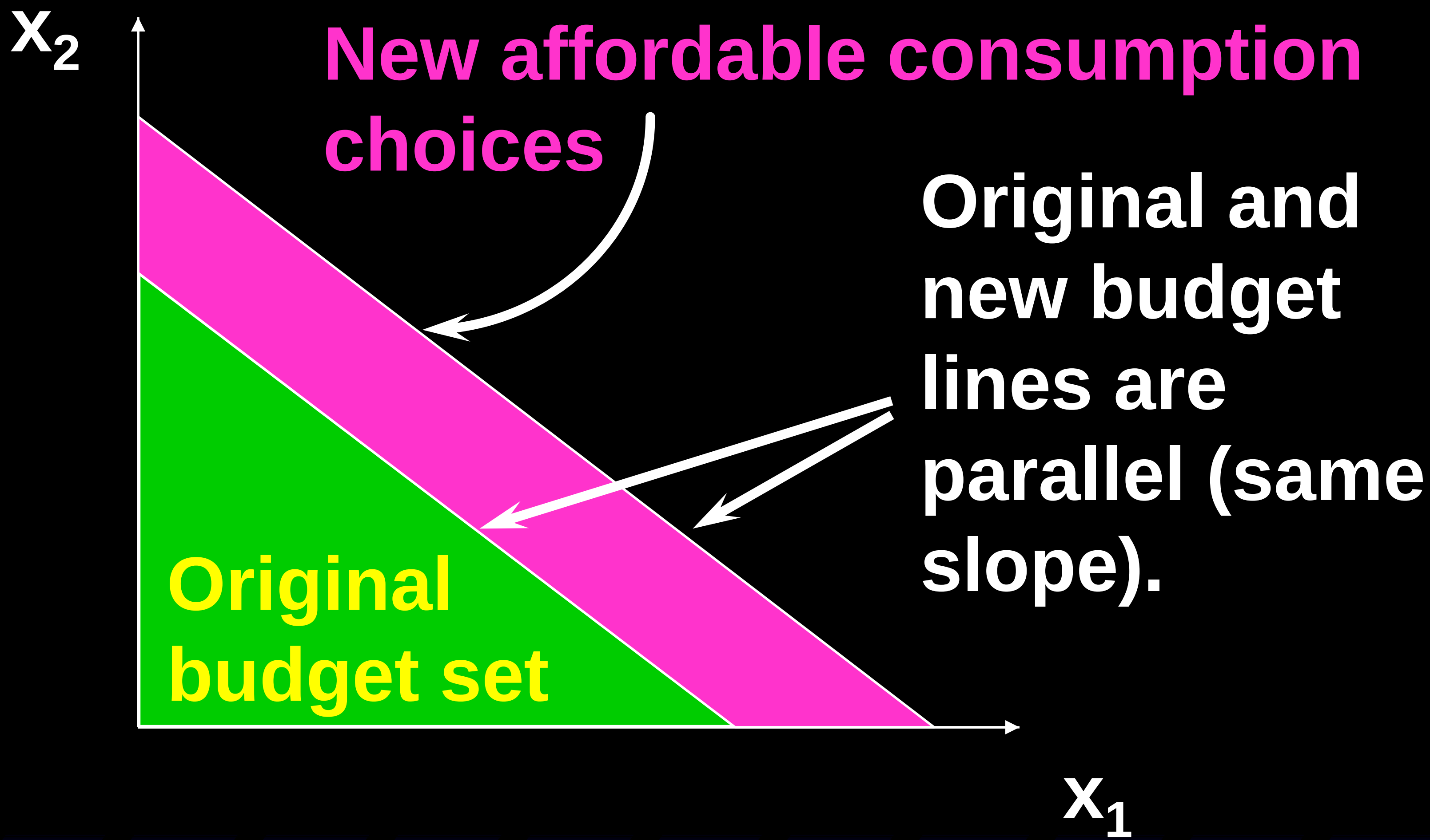
It depends on specific applications:

- **15 kg of apples**
- **1 hour of massage**
- **…**

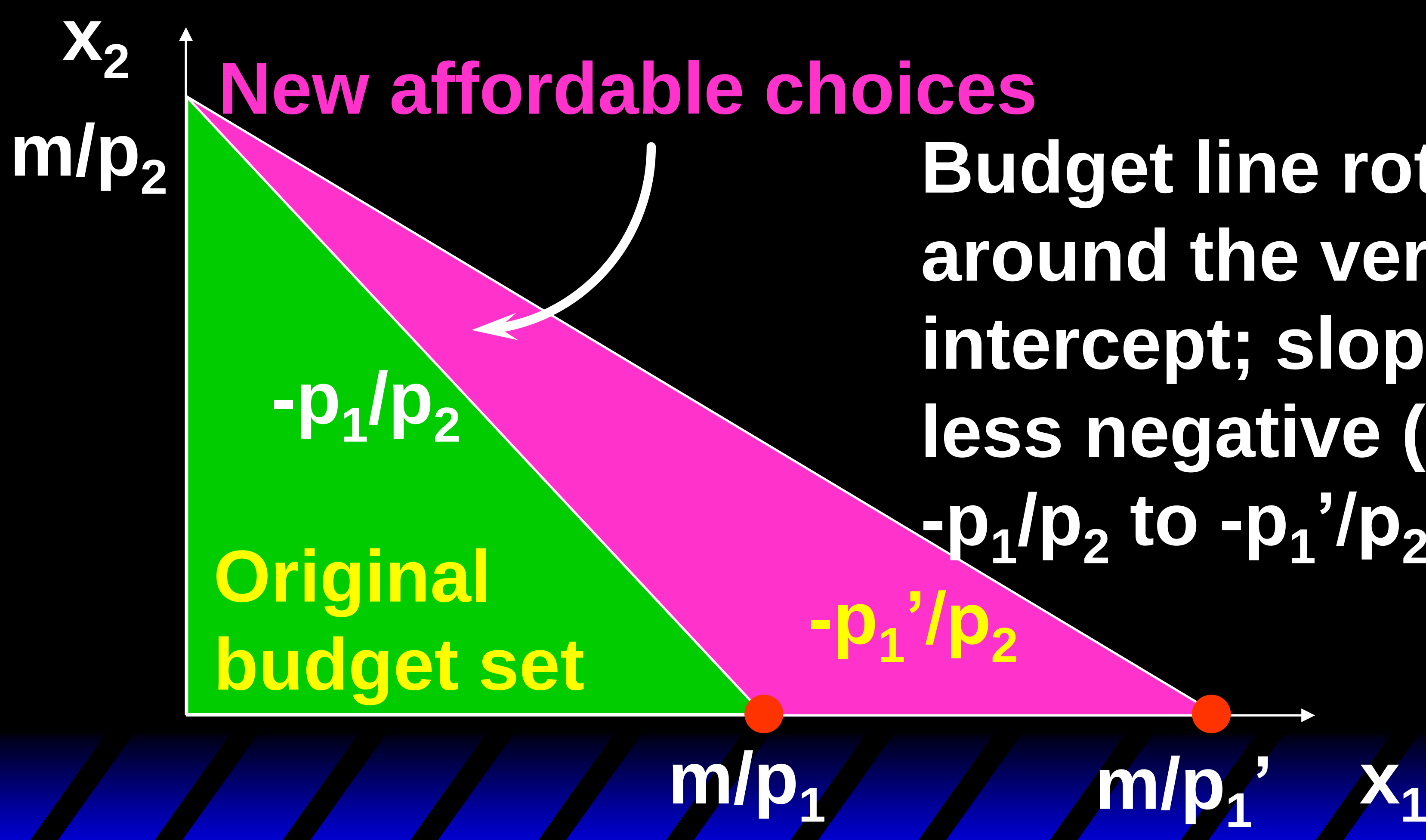
Income and Price Changes

What happens as prices or income change?

Higher income gives more choice



How do the budget set change as $\mathbf{p_1}$ decreases from $\mathbf{p_1}$ to $\mathbf{p_1}$'?



Budget line rotates around the vertical intercept; slope is less negative (from $-p_1/p_2$ to $-p_1'/p_2$.

Uniform *Ad Valorem* Sales Taxes in the US

- An *ad valorem* sales tax levied at a rate of 5% increases all prices by 5%, from p to (1+0.05)p = 1.05p.
- An *ad valorem* sales tax levied at a rate of *t* increases all prices by *t*p from p to (1+*t*)p.
- A uniform sales tax is applied uniformly to all commodities.

Uniform *Ad Valorem* Sales Taxes

A uniform sales tax levied at rate *t* changes the constraint from

$$p_1x_1 + p_2x_2 \leq m$$

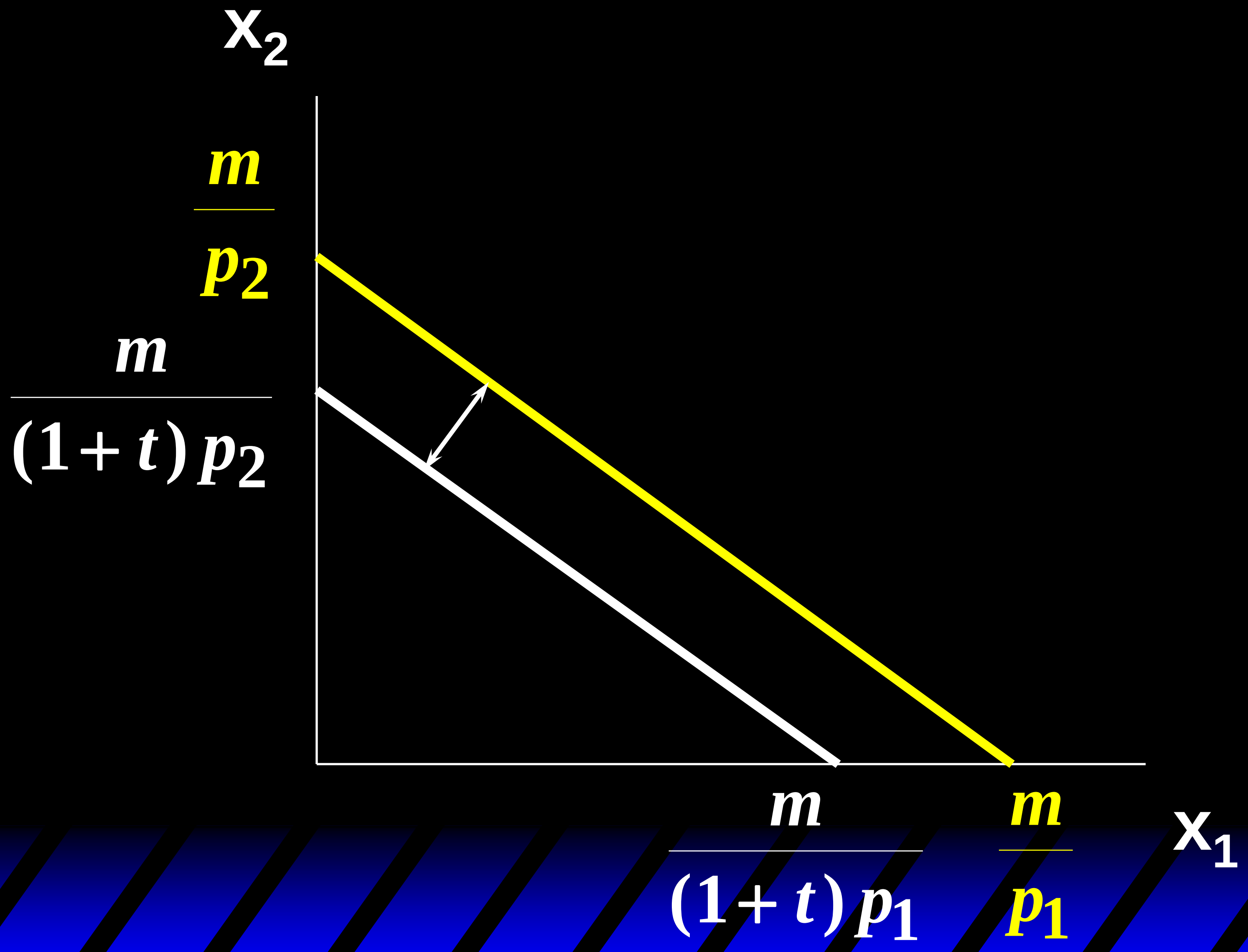
to

$$(1+t)p_1x_1 + (1+t)p_2x_2 \leq m$$

i.e.

$$p_1x_1 + p_2x_2 \leq m/(1+t).$$

Uniform *Ad Valorem* Sales Taxes

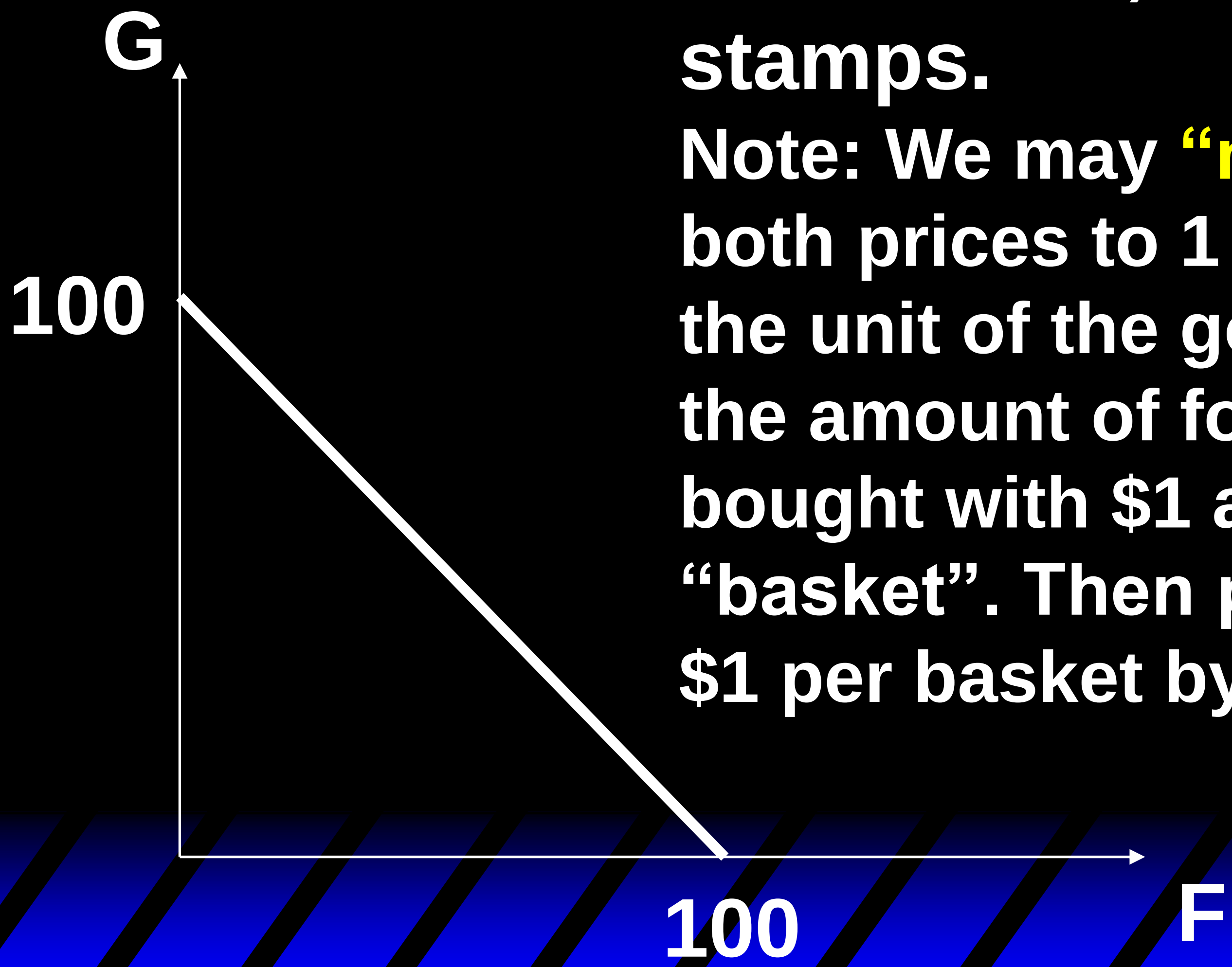


The Food Stamp Program in the US

Food stamps are coupons that can be legally exchanged only for food.

How do food stamps change a family’s budget set?

The Food Stamp Program



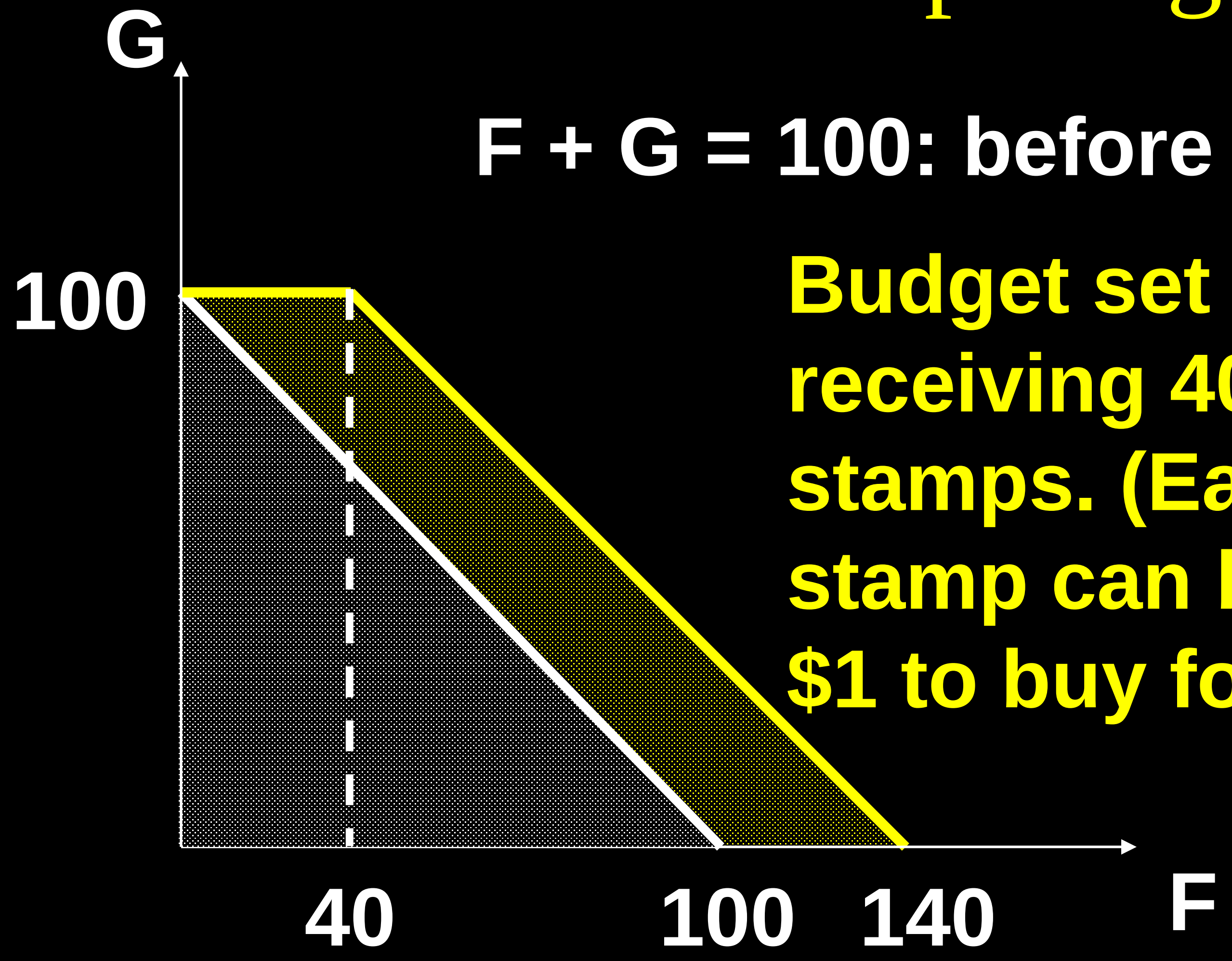
F + G = 100; without stamps.

Note: We may "normalize" both prices to 1 by changing the unit of the goods: Define the amount of food that can be bought with $1 as one "basket". Then price of food is $1 per basket by definition.

The Food Stamp Program

F + G = 100: before stamps.

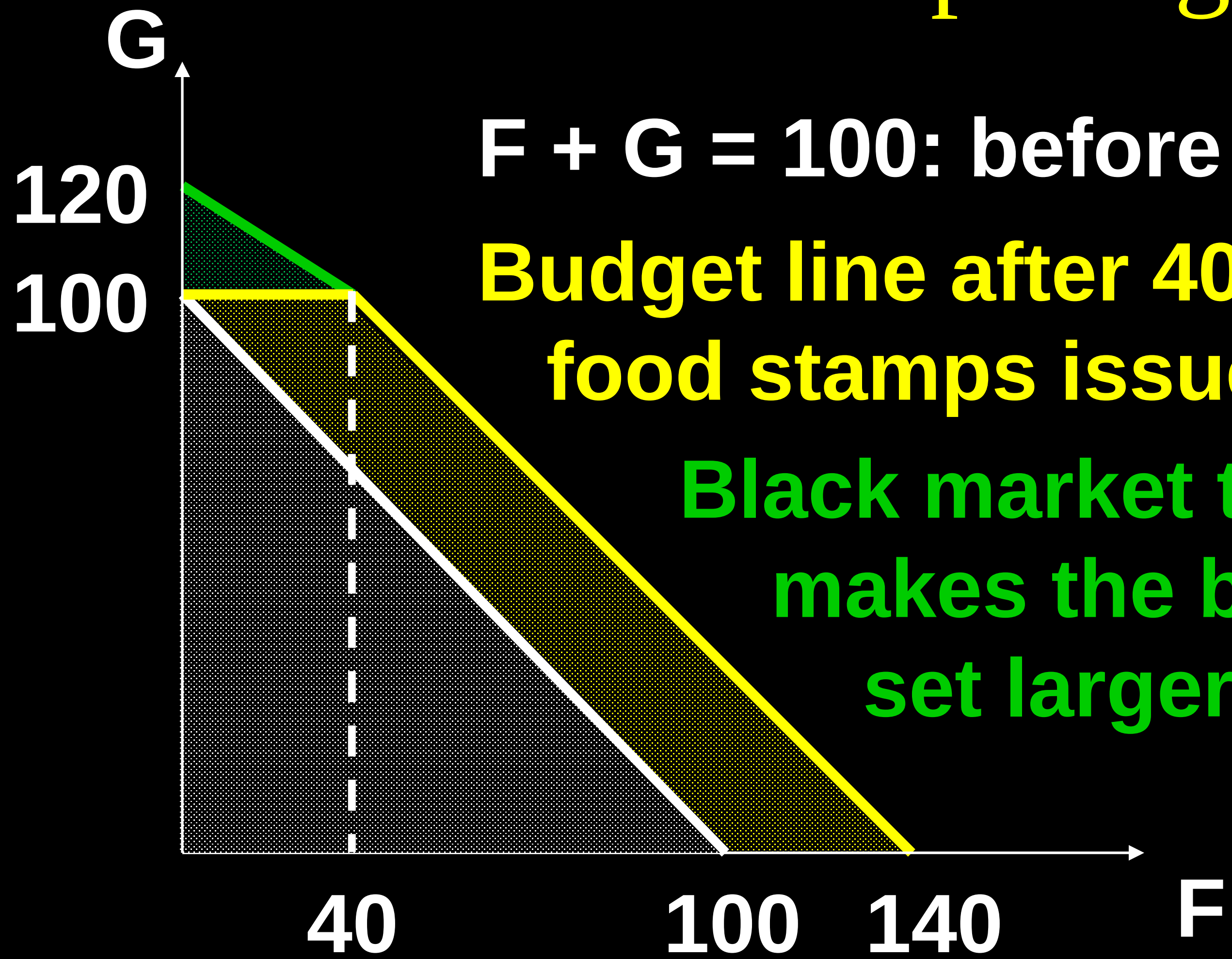
Budget set after receiving 40 food stamps. (Each food stamp can be used as \$1 to buy food only.)



The Food Stamp Program

What if food stamps can be traded on a black market at the price of $0.5?

The Food Stamp Program



F + G = 100: before stamps.

Budget line after 40 food stamps issued.

Black market trading makes the budget set larger again.

Relative Prices

Suppose prices and income are measured in dollars. Say p_1=\$2, p_2=\$3, *m* = \$12. Then the constraint is

$$2x_1 + 3x_2 \leq 12.$$

Relative Prices

The constraint for $p_1=2$, $p_2=3$, $m=12$

$$2x_1 + 3x_2 \leq 12$$

is also $x_1 + (3/2)x_2 \leq 6$,

the constraint for $p_1=1$, $p_2=3/2$, $m=6$.

Setting $p_1=1$ makes commodity 1 the **numeraire** and defines all prices in terms of commodity 1.

Numeraire

Any commodity can be chosen as the numeraire without changing the budget set.

– Dividing the budget constraint by p_k will make commodity k the numeraire.

Shapes of Budget Set

But what if prices are not constants?

- ***E.g.* bulk buying discounts**
- **or the opposite, price penalties for buying “too much”.**

Then lines will be curved.

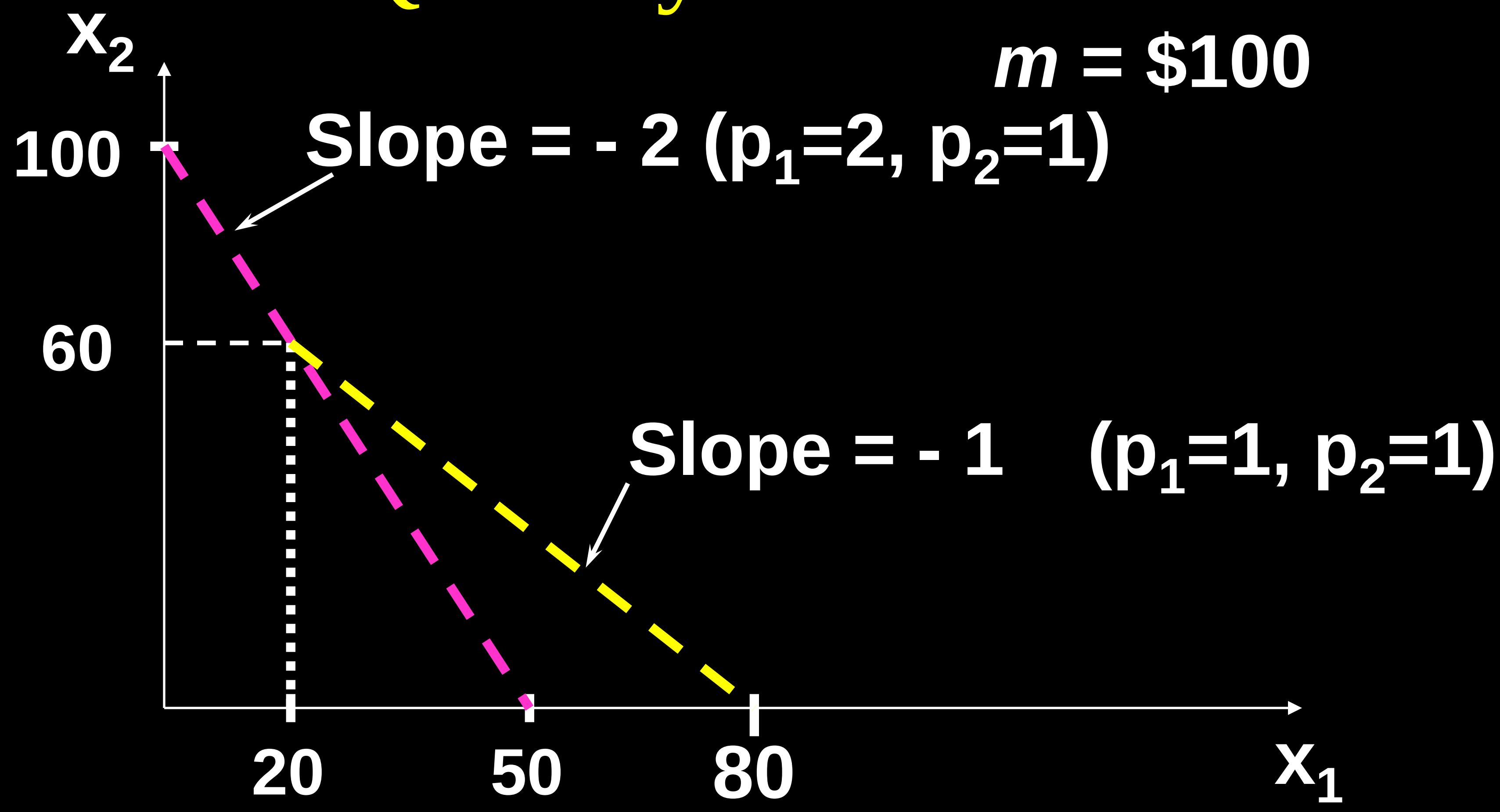
Shapes of Budget Sets - Quantity Discounts

Suppose p_2 is constant at \$1 but that p_1=\$2 for $0 \le x_1 \le 20$ and p_1=\$1 for x_1>20.

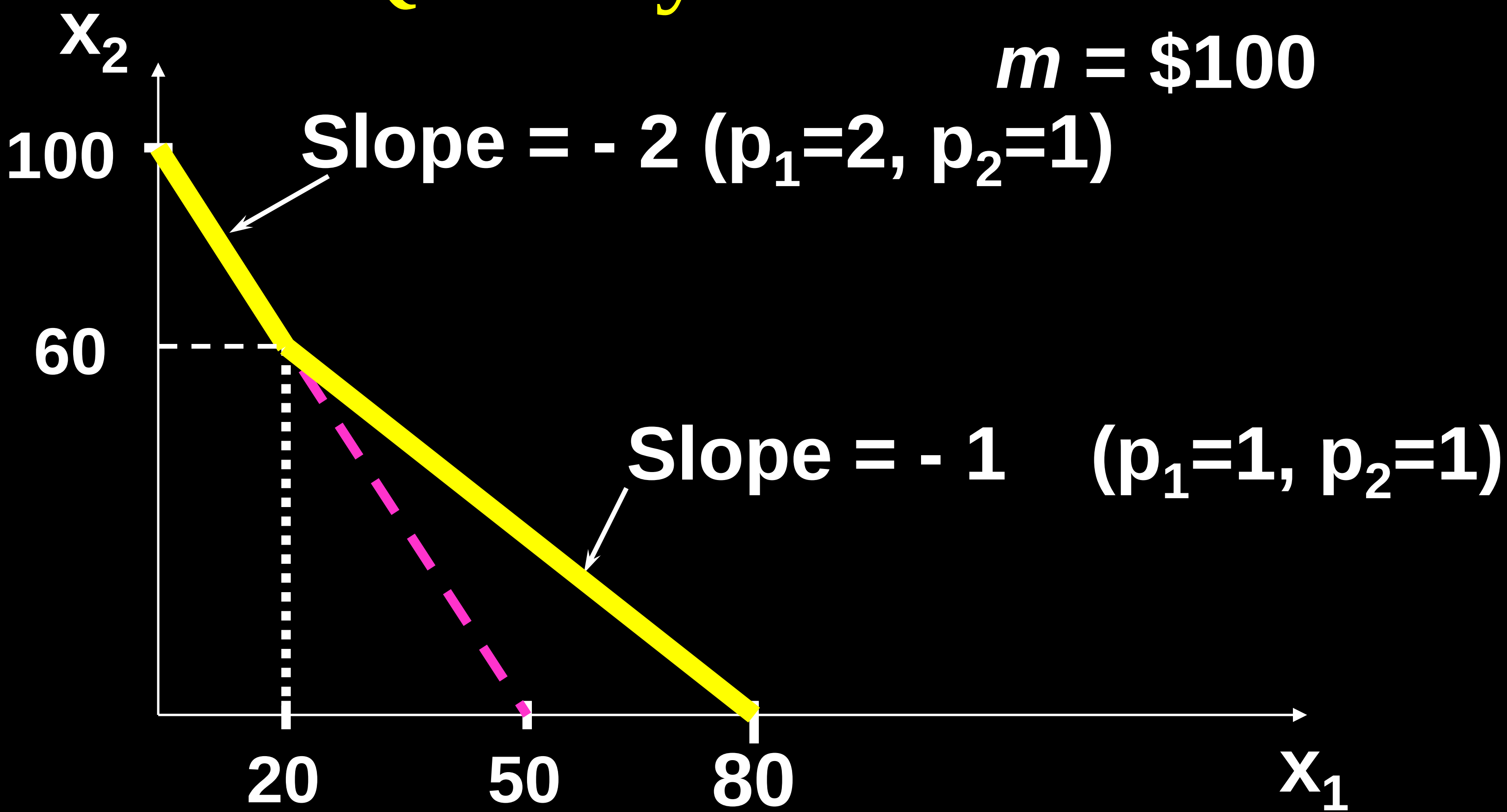
Then budget line's slope is

$$-p_1/p_2 = \begin{cases} -2, & \text{for } 0 \le x_1 \le 20 \\ -1, & \text{for } x_1 > 20 \end{cases}$$

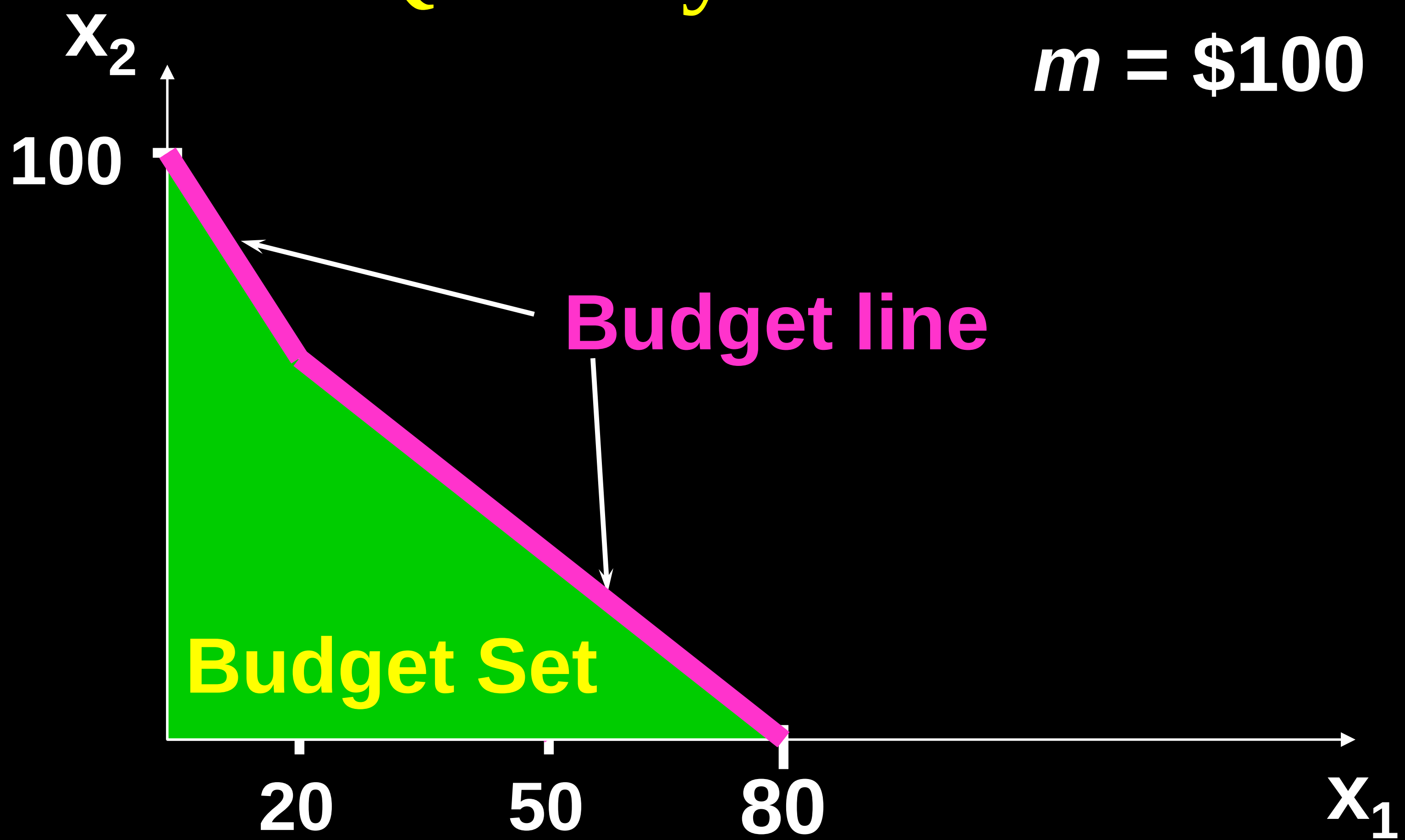
Shapes of Budget Sets with a Quantity Discount



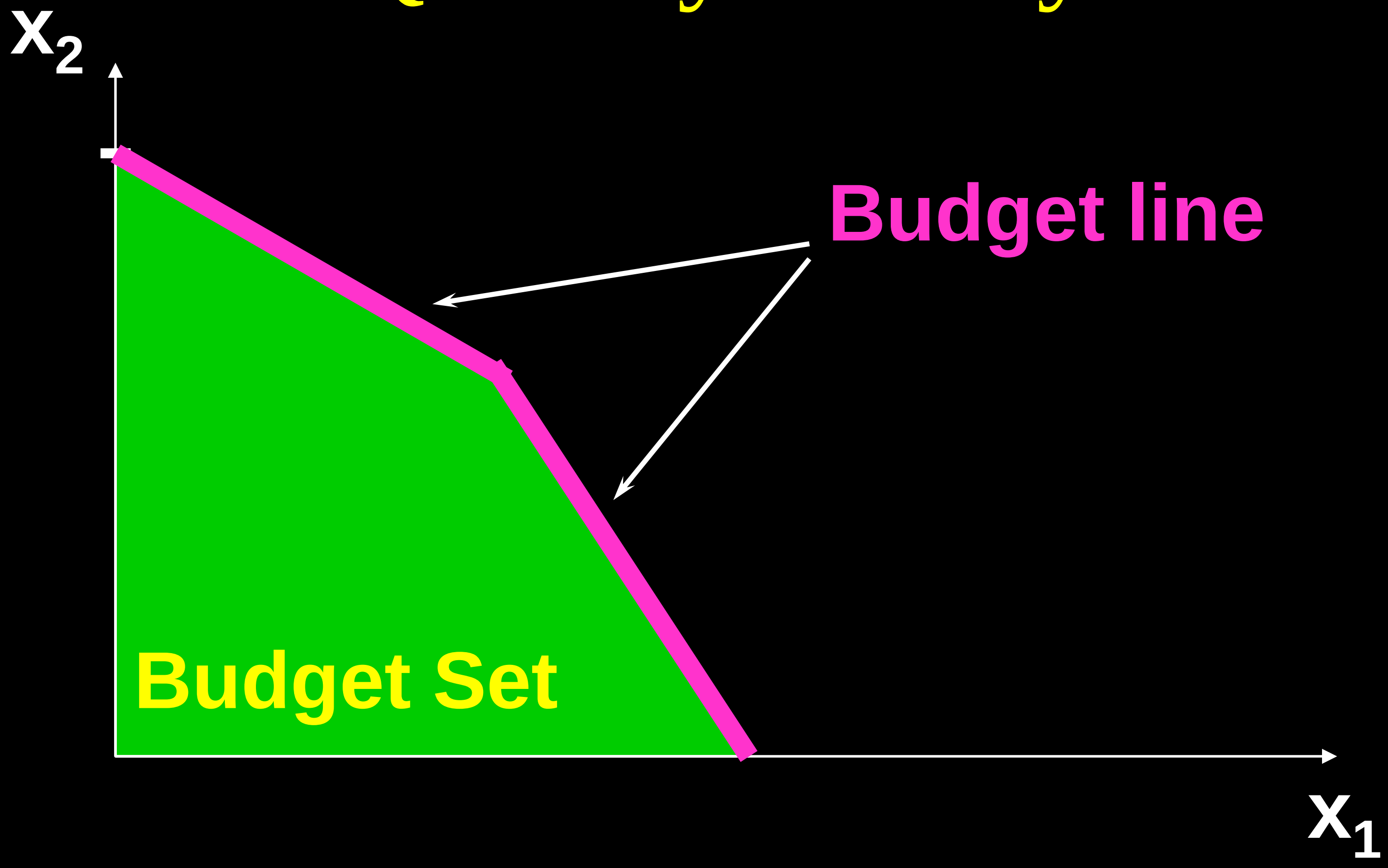
Shapes of Budget Sets with a Quantity Discount



Shapes of Budget Sets with a Quantity Discount



Shapes of Budget Sets with a Quantity Penalty

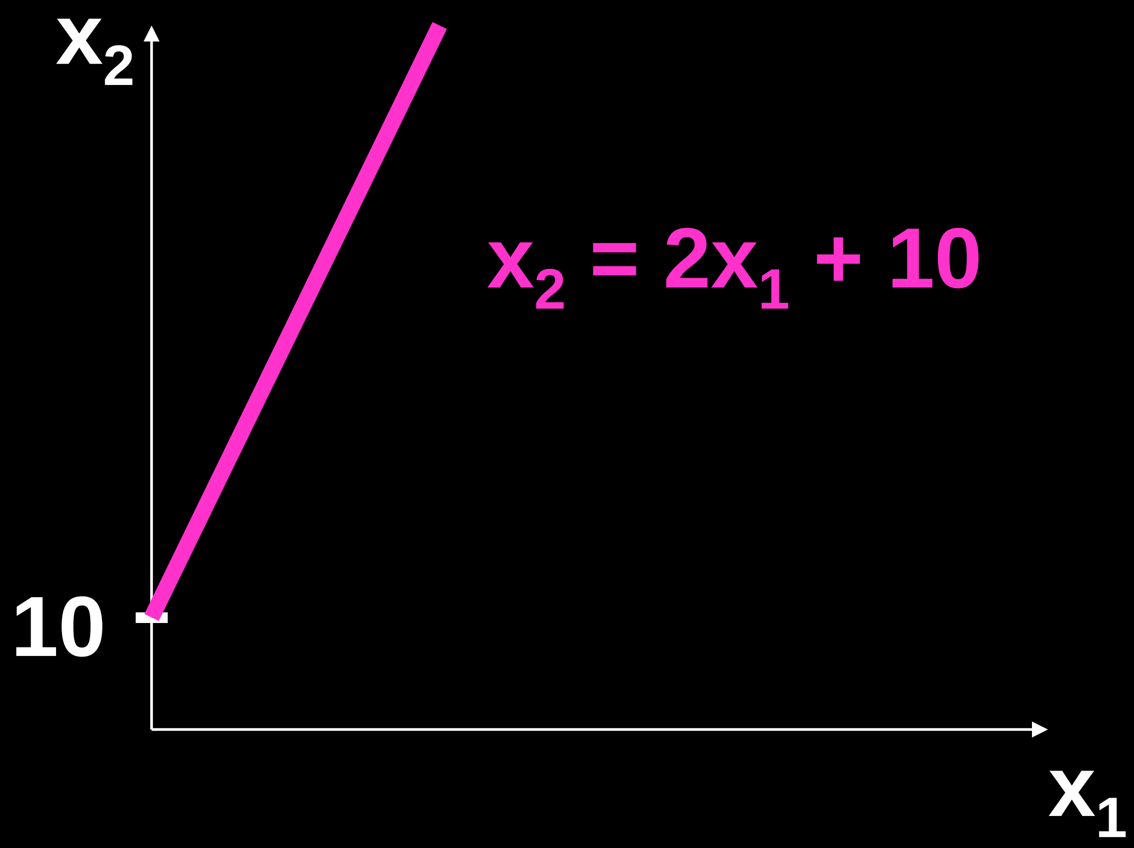


Shapes of Budget Sets - One Price Negative

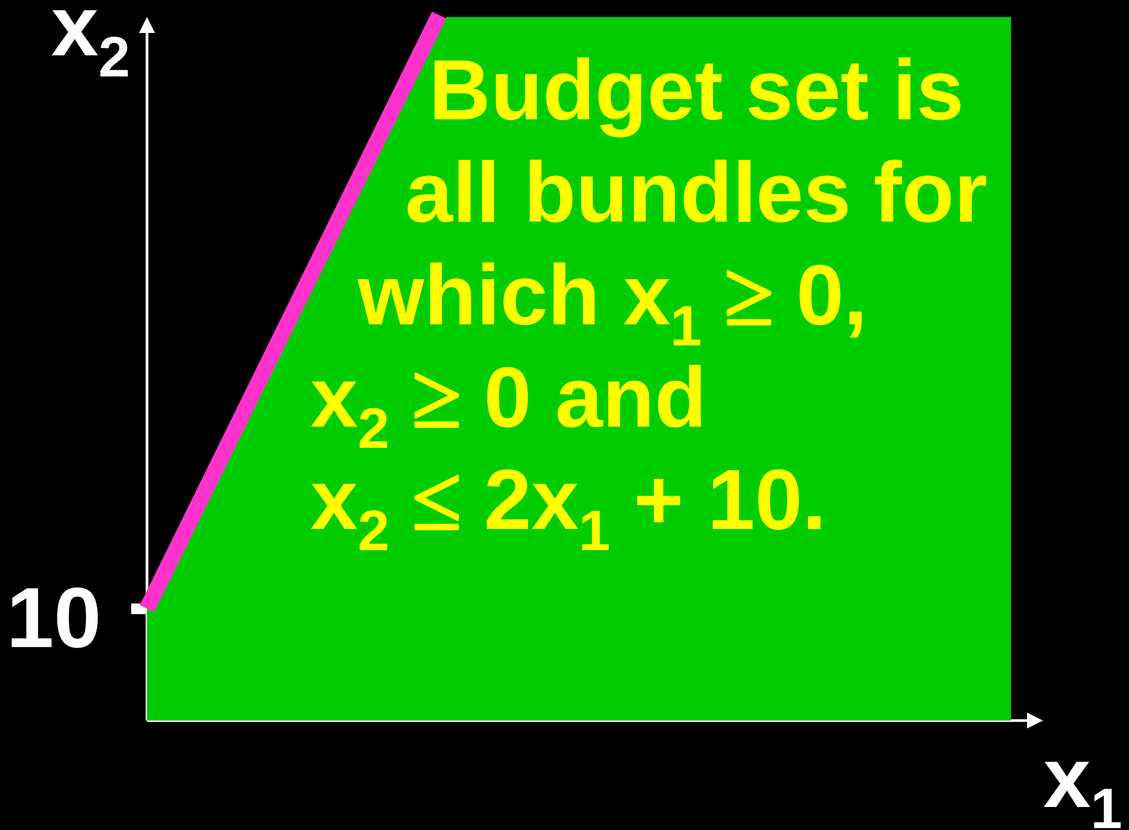
- **Commodity 1 is stinky garbage. You are paid \$2 per unit to accept it; *i.e.* p_1 = - \$2. p_2 = \$1. Income, other than from accepting commodity 1, is *m* = \$10.**
- **Then the constraint is**

$$-2x_1 + x_2 \leq 10 \quad \text{or} \quad x_2 \leq 2x_1 + 10.$$

Shapes of Budget Sets - One Price Negative



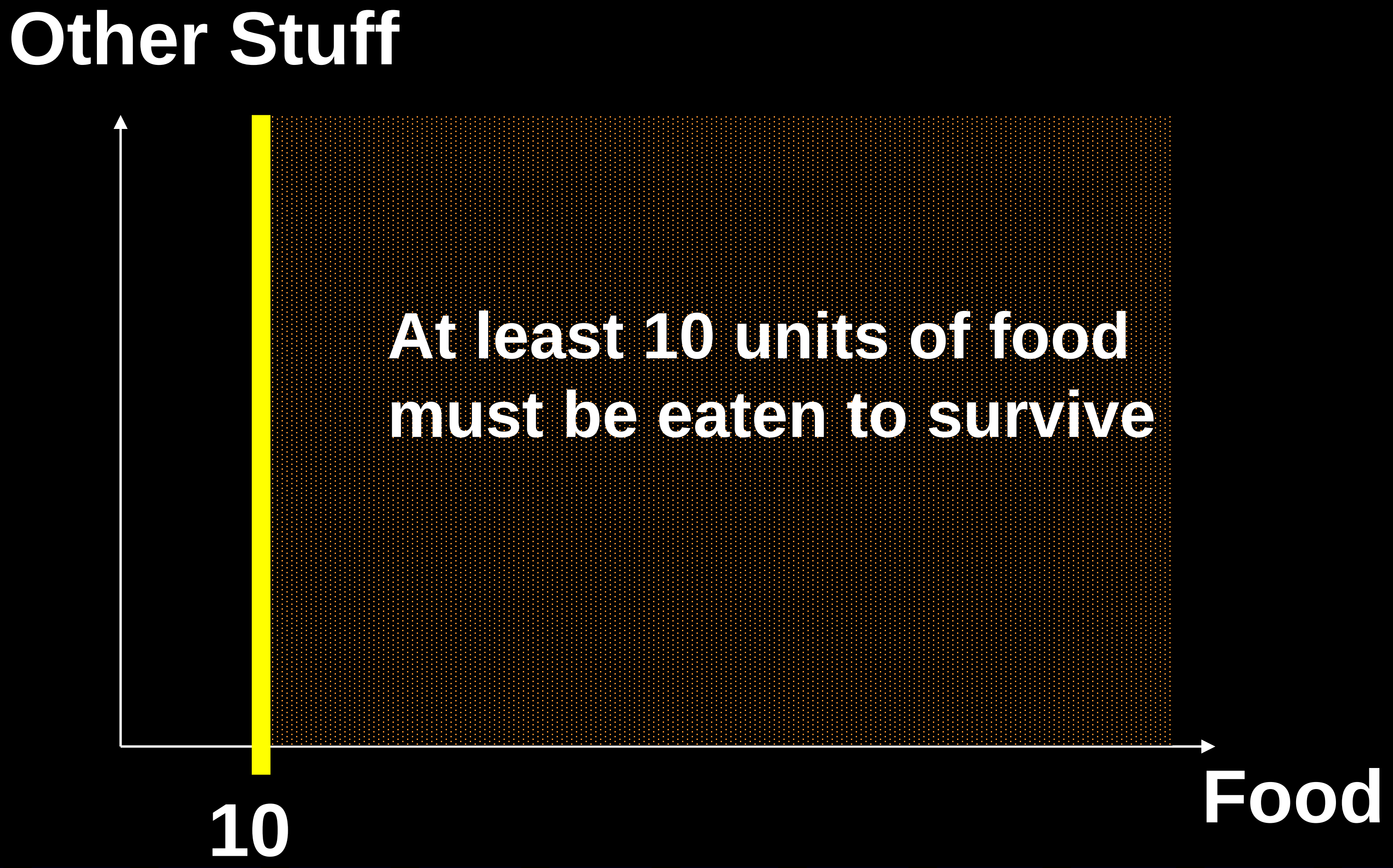
Shapes of Budget Sets - One Price Negative



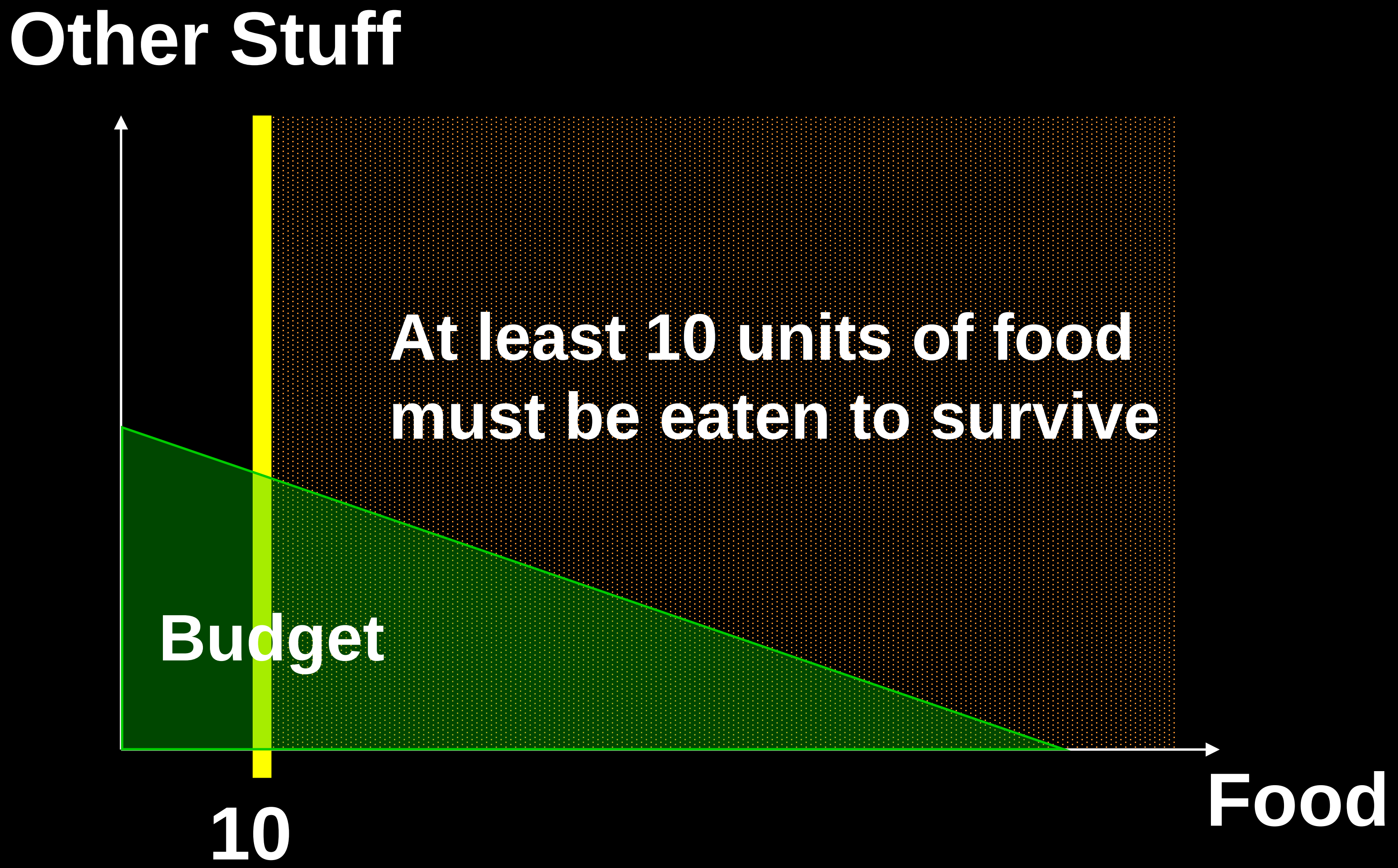
More General Choice Sets

- Choices are usually constrained by more than a budget; e.g. time constraints and other resources constraints.
- A bundle is available only if it meets **every** constraint.

More General Choice Sets



More General Choice Sets



The set of feasible bundles is the intersection.

A Quick Summary

1. Basic concepts
   1. Budget set
   2. Numeraire and relative price

2. Basic skills

   To work out a budget set