3.19

$$t_0 = 1\,s$$

$$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m_1 g t + m_2 g (t - t_0)}{m_1 + m_2}$$

$$= \left(t - \frac{5}{7}\right) g$$

$$a = \frac{dv}{dt} = g$$

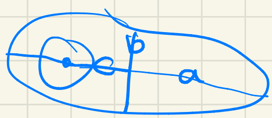
3.22

(1) 角动量守恒

$$\Rightarrow (h_{\text{远}} + r_E) \cdot v_{\text{远}} = (h_{\text{近}} + r_E) v_{\text{近}}$$

$$\Rightarrow v_{\text{远}} = 1.59\ km/s$$

(2)



$$S = \pi a b$$

$$= \pi \cdot \frac{(h_{\text{远}} + h_{\text{近}} + 2 r_E)}{2} \cdot \sqrt{(h_{\text{远}} + r_E)(h_{\text{近}} + r_E)}$$

$$\frac{ds}{dt} = \frac{1}{2} (h_{\text{近}} + r_E) v_{\text{近}}$$

$$T = \frac{S}{\frac{ds}{dt}} = \frac{\pi (h_{\text{远}} + h_{\text{近}} + 2 r_E)}{v_{\text{近}}} \sqrt{\frac{h_{\text{远}} + r_E}{h_{\text{近}} + r_E}}$$

$$\approx 38038.5\,(s)$$

3.23

$$m r_0 v_0 = m r v$$

$$v = \frac{r_0}{r} v$$

3.24

(1)

$$\frac{m}{m+m+m} a = \frac{1}{3} a$$

$\Rightarrow$ 质心距碰撞点 $\frac{1}{3}a$

$$v_c = \frac{m \cdot \omega \frac{a}{2} + m \cdot 0 + m \cdot (-\omega) \frac{a}{2}}{3m} = 0$$

(2)

$$L = m \cdot 0 \cdot \frac{a}{3} + m \cdot (\omega \frac{a}{2}) \cdot \frac{a}{3} + m \cdot (\omega \frac{a}{2}) \frac{2a}{3}$$

$$= \frac{1}{2} m \omega a^2$$

$$L' = 2m \cdot \frac{\omega a}{4} \cdot \frac{a}{3} + m \cdot \omega \frac{a}{2} \cdot \frac{2a}{3}$$

$$= \frac{1}{2} m \omega a^2$$

(3)

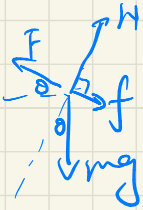
$$L' = \frac{1}{2} m \omega a^2$$

$$= 2m \cdot \frac{\omega' a}{3} \cdot \frac{a}{3} + m \cdot \frac{2\omega' a}{3} \cdot \frac{2a}{3}$$

$$= \frac{2}{3} m \omega' a^2$$

$$\omega' = \frac{3}{4} \omega$$

4.2



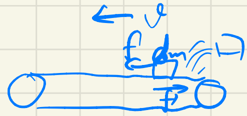
$$F = f + mg\sin\theta$$
$$= \mu_k N + mg\sin\theta$$
$$= \mu_k mg\cos\theta + mg\sin\theta$$

$$W_F = \int_0^{\frac{\pi}{4}} F\, dl(\theta)$$
$$= \int_0^{\frac{\pi}{4}} mg(\mu_k\cos\theta + \sin\theta)\, R\, d\theta$$
$$= mgR(\mu_k\sin\theta - \cos\theta)\Big|_0^{\frac{\pi}{4}}$$
$$= mgR\left(\frac{\sqrt{2}}{2}\mu_k - \frac{\sqrt{2}}{2} + 1\right)$$

$$W_G = -mg\cdot R\left(1 - \cos\frac{\pi}{4}\right)$$
$$= mgR\left(\frac{\sqrt{2}}{2} - 1\right)$$

$$W_f = -\int_0^{\frac{\pi}{4}} f\, dl(\theta)$$
$$= -\int_0^{\frac{\pi}{4}} \mu_k mg\cos\theta\cdot R\, d\theta$$
$$= -\frac{\sqrt{2}}{2}\mu_k mgR$$

4.4



$$dm = q\,dt$$

$$F = f' = f = \frac{dm \cdot v}{dt} = qv$$

$$P = Fv = qv^2 = 112.5\ \text{W}$$

$$\frac{dE_k}{dt} = \frac{1}{2}\frac{dm \cdot v^2}{dt} = \frac{1}{2}qv^2 \neq P$$

原因：矿砂从0加速至v的过程中，与传送带产生了相对位移，摩擦力做负功，产生内能

4.6

(1) $mv = (M+m)v_0$

$$\Delta E_{km} = \frac{1}{2}m(v_0^2 - v^2) = \frac{1}{2}mv^2 \cdot \left(\frac{m^2}{(M+m)^2} - 1\right) = \frac{-Mmv^2(M+2m)}{2(M+m)^2} = W_{f'}$$

$$\Delta E_{kM} = \frac{1}{2}Mv_0^2 - 0 = \frac{1}{2}M \cdot \frac{m^2}{(m+M)^2}v^2 = W_f$$

(2)

$$\Delta E = \Delta E_{km} + \Delta E_{kM}$$
$$= W_{f'} + W_f$$
$$= -f' \cdot (s_1 + s') + f s_1$$
$$= -f \cdot s_1$$

4.7

动能定式(4.9)： $A_{AB} = E_{kB} - E_{kA}$

$$\int_A^B dA = \int_A^B \vec{F} d\vec{r}$$

$$\begin{cases} \vec{v}_A' = \vec{v}_A + \vec{u} \\ \vec{v}_B' = \vec{v}_B + \vec{u} \\ \vec{F}' = \vec{F} \\ d\vec{r}' = d\vec{r} + \vec{u}\,dt \end{cases}$$

$$A'_{AB} = \int_A^B dA' = \int_A^B \vec{F}' d\vec{r}' = \int_A^B \vec{F}(d\vec{r} + \vec{u}\,dt)$$

$$= A_{AB} + \left(\int_A^B \vec{F} dt\right) \cdot \vec{u}$$

$$\int_A^B \vec{F} dt = m(\vec{v}_B - \vec{v}_A)$$

$$\Rightarrow A'_{AB} = A_{AB} + m(\vec{v}_B - \vec{v}_A)\vec{u}$$

$$= \frac{1}{2}m\vec{v}_B^2 - \frac{1}{2}m\vec{v}_A^2 + m\vec{v}_B\vec{u} - m\vec{v}_A\vec{u}$$

$$= \frac{1}{2}m(\vec{v}_B^2 + 2\vec{v}_B\vec{u} + \vec{u})^2$$

$$- \frac{1}{2}m(\vec{v}_A^2 + 2\vec{v}_A\vec{u} + \vec{u})^2$$

$$= \frac{1}{2}m(\vec{v}_B + \vec{u})^2 - \frac{1}{2}m(\vec{v}_A + \vec{u})^2$$

$$= \frac{1}{2}m\vec{v}_B'^2 - \frac{1}{2}m\vec{v}_A'^2$$

4.10

(1) v_{max} 时,

$$F = k(x - l_0) = mg$$

$$\Rightarrow x = \frac{mg}{k} + l_0 = 31.76\ m$$

$$E_{km} - 0 = mgx - \frac{1}{2}k(x-l_0)^2$$

$$\parallel$$

$$\frac{1}{2}m v_{max}^2$$

$$\Rightarrow v_{max} = \sqrt{2gx - \frac{k(x-l_0)^2}{m}} = 22.52\ m/s$$

(2) 在最低点

$$0 - 0 = mg x_m - \frac{1}{2}k(x_m - l_0)^2$$

$$\Rightarrow \frac{1}{2}k x_m^2 - (mg + k l_0) x_m + \frac{1}{2} l_0^2 = 0$$

$$\Rightarrow x_m = 58\ m < 60\ m$$

不会角出到水面

4.11

$$v_{A0} = \frac{m v_0}{m+M}$$

$$v_{B0} = 0$$

当 $v_A = v_B$ 时，弹簧压缩量最大

$$(M+m)v_A + M v_B = m v_0$$

$$\Rightarrow v_A = v_B = \frac{m v_0}{2M+m}$$

$$\frac{1}{2}k x_m^2 = \frac{1}{2}(M+m) v_{A0}^2 - \frac{1}{2}(2M+m) v_A^2$$

$$= \frac{1}{2} \cdot \frac{m^2 v_0^2}{m+M} - \frac{1}{2} \frac{m^2 v_0^2}{2M+m}$$

$$= \frac{1}{2} \cdot \frac{M}{(m+M)(m+2M)} m^2 v_0^2$$

$$\Rightarrow x_m = \sqrt{\frac{M}{k(m+M)(m+2M)}} \cdot m v_0$$