1.

(a) Firm1: $\pi_1(p_1, p_2) = p_1 y_1 - \frac{c}{2} y_1$

$$= (p_1 - \frac{c}{2})(a - b(p_1 + p_2))$$

Firm2: $\pi_2(p_1, p_2) = (p_2 - \frac{c}{2})(a - b(p_1 + p_2))$

(b)
$$\begin{cases} \frac{\partial \pi_1}{\partial p_1} = -2bp_1 + a - bp_2 + \frac{bc}{2} = 0 \\ \frac{\partial \pi_2}{\partial p_2} = -2bp_2 + a - bp_1 + \frac{bc}{2} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} p_1 = \frac{2a - 2bp_2 + bc}{4b} \\ p_2 = \frac{2a - 2bp_1 + bc}{4b} \end{cases}$$

(c) $p_1 = p_2 = \frac{2a + bc}{6b}$

$P_J = p_1 + p_2 = \frac{2a + bc}{3b}$

(d) $\frac{\partial \pi_i}{\partial p_i} = -2bp_i + a - b\sum_{j \neq i} p_j + \frac{bc}{n} = 0$

sum up $\Rightarrow$ $P_J = \sum_{i=1}^{n} p_i = \frac{na + bc}{(n+1)b}$

(e) If n increases, P_J increases. $(a > bc)$

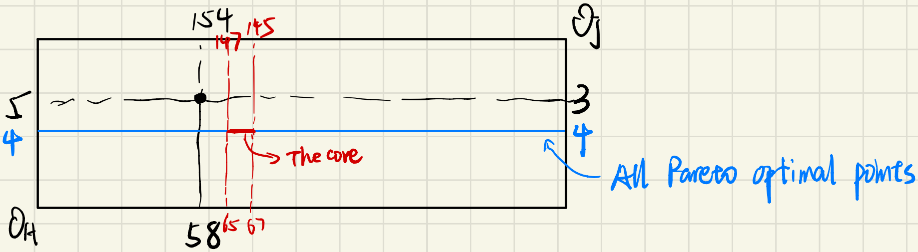
$\lim_{n \to \infty} P_J = \frac{a}{b}$.

2.

(a)

$$\begin{cases} \dfrac{\partial b_H}{\partial a_H} = -\dfrac{\frac{\partial U_H}{\partial a_H}}{\frac{\partial U_H}{\partial b_H}} = -\dfrac{1}{-2(b_H-8)} = \dfrac{1}{2(b_H-8)} \\[2ex] \dfrac{\partial b_J}{\partial a_J} = -\dfrac{\frac{\partial U_J}{\partial a_J}}{\frac{\partial U_J}{\partial b_J}} = -\dfrac{1}{-2(b_J-8)} = \dfrac{1}{2(b_J-8)} \end{cases} \Rightarrow b_H = b_J$$

$$\begin{cases} b_H = b_J \\ b_H + b_J = 5+3 = 8 \end{cases} \Rightarrow b_H = b_J = 4$$



(b)

$$U_H(58, 5) = 58 - 3^2 = 49$$

$$U_J(154, 3) = 154 - 5^2 = 129$$

$\Rightarrow$ The core: $a_H \in [65, 67]$

$a_J \in [145, 147]$

(c)

$$\frac{\partial b_H}{\partial a_H} = \frac{1}{2(b_H^*-8)} = -\frac{b_H^* - b_H^0}{a_H^* - a_H^0} = \frac{5-b_H^*}{a_H^*-58}$$

$$\parallel$$

$$\frac{\partial b_J}{\partial a_J} = \frac{1}{2(b_J^*-8)} = \frac{3-b_J^*}{a_J^*-154}$$

$\Rightarrow b_H^* = b_J^* = 4$

$a_H^* = 50$, $a_J^* = 162$

$$\frac{P_a}{P_b} = -\frac{\partial a_J}{\partial b_J} = 8$$

3.

(a) $\Pi_a(X) = 48X - X^2$

$$\frac{\partial \Pi_a}{\partial X} = 48 - 2X = 0 \Rightarrow X = 24, \quad \Pi_a|_{max} = 576$$

(b) $\Pi_d(X, Y) = 60Y - Y^2 - XY$

$$\stackrel{(a)}{\Longrightarrow} \Pi_d(24, Y) = 60Y - Y^2 - 24Y$$

$$= 36Y - Y^2$$

$$\frac{\partial \Pi_d}{\partial Y} = 36 - 2Y \Rightarrow Y = 18, \quad \Pi_d(24, Y)|_{max} = 324$$

(c) $\Pi_a'(X, Y) = 48X - X^2 - XY$

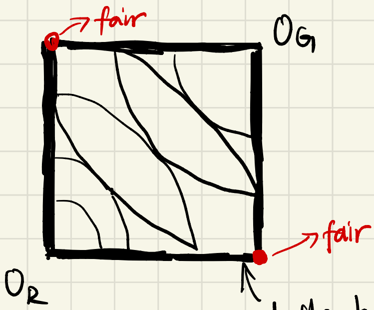
$$\Pi_d'(X, Y) = 60Y - Y^2 \Rightarrow \frac{\partial \Pi_d'}{\partial Y} = 0 \Rightarrow Y = 30$$

$$\Rightarrow \frac{\partial \Pi_a'(X, 30)}{\partial X} = 16 - 2X \Rightarrow X = 8$$

$$\Rightarrow \begin{cases} \Pi_a'(8, 30) = 64 \\ \Pi_d'(8, 30) = 900 \end{cases}$$

4.

(a)



all the boundaries of the box are the Pareto optimal allocations.

(b) Give each of them 10 units of x or 10 units of y.

5.

(a) suppose there're n consumers

$\theta s_1 - p_1$
$\vee$
$\theta s_2 - p_2$

$p_2 - p_1 > \Delta s \cdot \theta$

$\theta < \frac{p_2 - p_1}{\Delta s}$

$$q_1(p_1, p_2) = \frac{p_2 - p_1 - \underline{\theta}}{\bar{\theta} - \underline{\theta}} n = (p_2 - p_1 - \frac{1}{2})n$$

$$q_2(p_1, p_2) = (\frac{3}{2} + p_1 - p_2)n$$

(b) $\pi_1(y_1, y_2) = p_1 y_1 - \frac{1}{3} y_1$

$\pi_2(y_1, y_2) = p_2 y_2 - \frac{1}{3} y_2$

(c) $\pi_1(p_1, p_2) = (p_1 - \frac{1}{3}) \cdot (p_2 - p_1 - \frac{1}{2})n$

$\pi_2(p_1, p_2) = (p_2 - \frac{1}{3})(\frac{3}{2} + p_1 - p_2)n$

$\Rightarrow \frac{\partial \pi_1}{\partial p_1} = -2np_1 + n(p_2 - \frac{1}{6}) = 0$

$\frac{\partial \pi_2}{\partial p_2} = -2np_2 + \frac{11}{6}n + np_1 = 0$

$$\Rightarrow \begin{cases} p_1 = \frac{1}{2}p_2 - \frac{1}{12} \\ p_2 = \frac{1}{2}p_1 + \frac{11}{12} \end{cases}$$

(d) $\Rightarrow \begin{cases} p_1 = \frac{1}{2} \\ p_2 = \frac{7}{6} \end{cases}$ $\quad q_1 = \frac{1}{6}n \quad \pi_1 = \frac{1}{36}n$

$q_2 = \frac{5}{6}n \quad \pi_2 = \frac{25}{36}n$

(e)

$$q_1(p_1, p_2) = (\frac{p_2 - p_1}{\Delta s} - \underline{\theta})n$$

$$q_2(p_1, p_2) = (1 + \underline{\theta} + \frac{p_1 - p_2}{\Delta s})n$$

$$\Rightarrow \pi_1(p_1, p_2) = (p_1 - c)(\frac{p_2 - p_1}{\Delta s} - \underline{\theta})n$$

$$\pi_2(p_1, p_2) = (p_2 - c)(1 + \underline{\theta} + \frac{p_1 - p_2}{\Delta s})n$$

$$\Rightarrow \frac{\partial \pi_1}{\partial p_1} = -\frac{2n}{\Delta s}p_1 + n(\frac{c}{\Delta s} + \frac{p_2}{\Delta s} - \underline{\theta}) = 0$$

$$\frac{\partial \pi_2}{\partial p_2} = -\frac{2n}{\Delta s}p_2 + n(\frac{c}{\Delta s} + 1 + \underline{\theta} + \frac{p_1}{\Delta s}) = 0$$

$$\Rightarrow p_1 = \frac{c + p_2 - \Delta s \underline{\theta}}{2}$$

$$p_2 = \frac{c + p_1 + \Delta s + \Delta s \underline{\theta}}{2}$$

$$\Rightarrow \begin{cases} p_1 = c + \frac{1}{3}\Delta s - \frac{\Delta s \underline{\theta}}{3} \\ p_2 = c + \frac{2}{3}\Delta s + \frac{\Delta s \underline{\theta}}{3} \end{cases}$$

$$\Rightarrow \begin{cases} q_1 = (\frac{1}{3} - \frac{1}{3}\underline{\theta})n \\ q_2 = (\frac{2}{3} + \frac{1}{3}\underline{\theta})n \end{cases}$$

$$\Rightarrow \begin{cases} \pi_1 = (\frac{1}{3}\Delta s - \frac{\Delta s}{3}\underline{\theta})(\frac{1}{3} - \frac{1}{3}\underline{\theta}) \\ \pi_2 = (\frac{2}{3}\Delta s + \frac{\Delta s}{3}\underline{\theta})(\frac{2}{3} + \frac{1}{3}\underline{\theta}) \end{cases}$$