10.1

1. $T_pM = \operatorname{span}\begin{pmatrix}\sqrt{2}\\-\sqrt{3}\\0\end{pmatrix}, \begin{pmatrix}\sqrt{2}\\0\\-\sqrt{6}\end{pmatrix}$

2. $(2dx_p + 3dy_p)(\vec{V})$

$= 2\times\sqrt{2} + 3\times(-\sqrt{3}) = 2\sqrt{2} - 3\sqrt{3}$

3. $\forall \vec{v} \in T_pM,$

$\vec{V} = \begin{pmatrix}(a+b)\sqrt{2}\\-\sqrt{3}a\\-\sqrt{6}b\end{pmatrix}, \forall a, b \in \mathbb{R}$

$dx_p(\vec{V}) = (a+b)\sqrt{2}$

$dy_p(\vec{V}) = -\sqrt{3}a \qquad \Rightarrow \sqrt{3}dx_p(\vec{V}) + \sqrt{2}dy_p(\vec{V}) + dz_p(\vec{V}) = 0$

$dz_p(\vec{V}) = -\sqrt{6}b \qquad \Rightarrow \sqrt{3}dx_p + \sqrt{2}dy_p + dz_p = 0$

4. dx_p & dy_p are linearly dependent

$\Rightarrow \forall \vec{v} \in T_pM, \vec{v} = \begin{pmatrix}?\\?\\0\end{pmatrix}$

$\Rightarrow$ all points on $\begin{cases} z=0 \\ x^2+y^2=1 \end{cases}$

5. $\int_\gamma dx = \int_\gamma dy = 0, \quad \int_\gamma dz = 2$

6. $\int_\gamma (yz\,dx + zx\,dy + xy\,dz) = \int_\gamma d(xyz) = \frac{\sqrt{2}}{8} - \frac{1}{6}$

10.2

1. $f(p) = \begin{pmatrix} \frac{2a}{2-c} \\ \frac{2b}{2-c} \end{pmatrix}$

2. $\vec{p} \cdot \vec{v} = 0 \Rightarrow \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} y_0 + (1+\frac{1}{\sqrt{3}}) z_0 = 0$

$\vec{v} = \begin{pmatrix} 1 \\ y_0 \\ z_0 \end{pmatrix}$

since $\vec{v}$ parallels to the ground : $z_0 = 0$

$\Rightarrow y_0 = -1$

$\Rightarrow \vec{v} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$$f_*(\vec{v}) = \begin{pmatrix} \frac{2}{1-\frac{1}{\sqrt{3}}} \\ \frac{-2}{1-\frac{1}{\sqrt{3}}} \end{pmatrix} = \begin{pmatrix} 3+\sqrt{3} \\ -3-\sqrt{3} \end{pmatrix}$$

3. $dg = y\,dx + x\,dy$, $f(p) = \begin{pmatrix} \sqrt{3}+1 \\ -\sqrt{3}-1 \end{pmatrix}$

$\Rightarrow dg_{f(p)} = \begin{pmatrix} -\sqrt{3}-1 & \sqrt{3}+1 \end{pmatrix}$

$\Rightarrow f^*(dg_{f(p)}) \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$

$$= dg_{f(p)}\left(f_*\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}\right)$$

$$= \begin{pmatrix} -\sqrt{3}-1 & \sqrt{3}+1 \end{pmatrix} \begin{pmatrix} \frac{2\sqrt{3}x_0 - (3+\sqrt{3})z_0}{\sqrt{3}-1} \\ \frac{2\sqrt{3}y_0 - (3+\sqrt{3})z_0}{\sqrt{3}-1} \end{pmatrix}$$

$$= (2+\sqrt{3})\left((\sqrt{3}+3)z_0 - 2\sqrt{3}x_0 + 2\sqrt{3}y_0 - (3+\sqrt{3})z_0\right)$$

$$= (-4\sqrt{3}-6)x_0 + (4\sqrt{3}+6)y_0$$

$$\Rightarrow f^*(dg_{f(p)}) = \begin{pmatrix} -4\sqrt{3}-6 & 4\sqrt{3}+6 & 0 \end{pmatrix}$$