1.

证：不妨设 G 是可平面的

则 $m_G \le 3n-6$

$d_G \le 2n-4$

$$\Rightarrow m_{\bar{G}} = \frac{n(n-1)}{2} - m_G \ge \frac{1}{2}n^2 - \frac{1}{2}n - 3n + 6$$

$$= \frac{1}{2}n^2 - \frac{5}{2}n + 6$$

$$> 3n-6 \ (n>10)$$

因此 $\bar{G}$ 不可平面

得证

2. 假设存在题干所述的平面图 G，

则其对偶图 G^* 有6个顶点，

由于 G 中5个面两两之间至少有一个公共边界，

再加上 G 的无限域与这5个面的边界

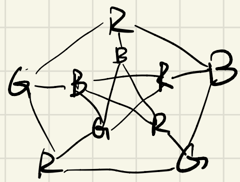
则 G^* 的边数 $d^* \ge C_5^2 + 5 = 15$

但由于 G^* 为平面图，$d^* \le 2n^* - 4 = 2\times6-4=8$

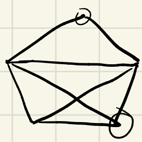
矛盾，故不存在 G

$\Rightarrow$ 得证

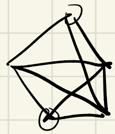
3.



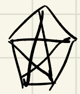
4.



$\xrightarrow{\bar{G}}$

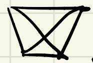


$\xrightarrow{\bar{G}}$



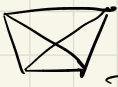
$f(t) = \frac{t!}{(t-5)!}$

$\xrightarrow{\mathring{G}}$



$f(t) = \frac{t!}{(t-4)!}$

$\xrightarrow{\mathring{G}}$



$f(t) = \frac{t!}{(t-4)!}$

$$\Rightarrow f(t) = \frac{t!}{(t-5)!} + \frac{t! \times 2}{(t-4)!}$$