These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Fourteen

Consumer’s Surplus

Welfare Measure

- We already have the notion of utility to measure consumer welfare, although ordinally.
- However, in this chapter we want to develop a measure of consumer welfare **in the unit of money**.

Monetary Measure of Welfare

- Consider a consumer with quasi-linear utility function

$$u(x, y) = v(x) + y$$

- x: the amount of rice consumption (in kg)
- y: the amount of money left for all other goods

Monetary Measure of Welfare

- For the purpose of illustration, let's consider the case where x is restricted to 0,1,2,…
- The monetary equivalent of the 1st kg of rice is $v(1) - v(0)$ because

$$u(1, m) = u(0, m + v(1) - v(0))$$

Monetary Measure of Welfare

- **The monetary equivalent of the 2nd kg of rice is $v(2) - v(1)$ because**

$$u(2, m) = u(1, m + v(2) - v(1))$$

- **The monetary equivalent of the 3rd kg of rice is $v(3) - v(2)$ because**

$$u(3, m) = u(2, m + v(3) - v(2))$$

Monetary Measure of Welfare

- In general, the monetary equivalent of the n^{th} kg of rice is

$$r_n := v(n) - v(n-1)$$

because

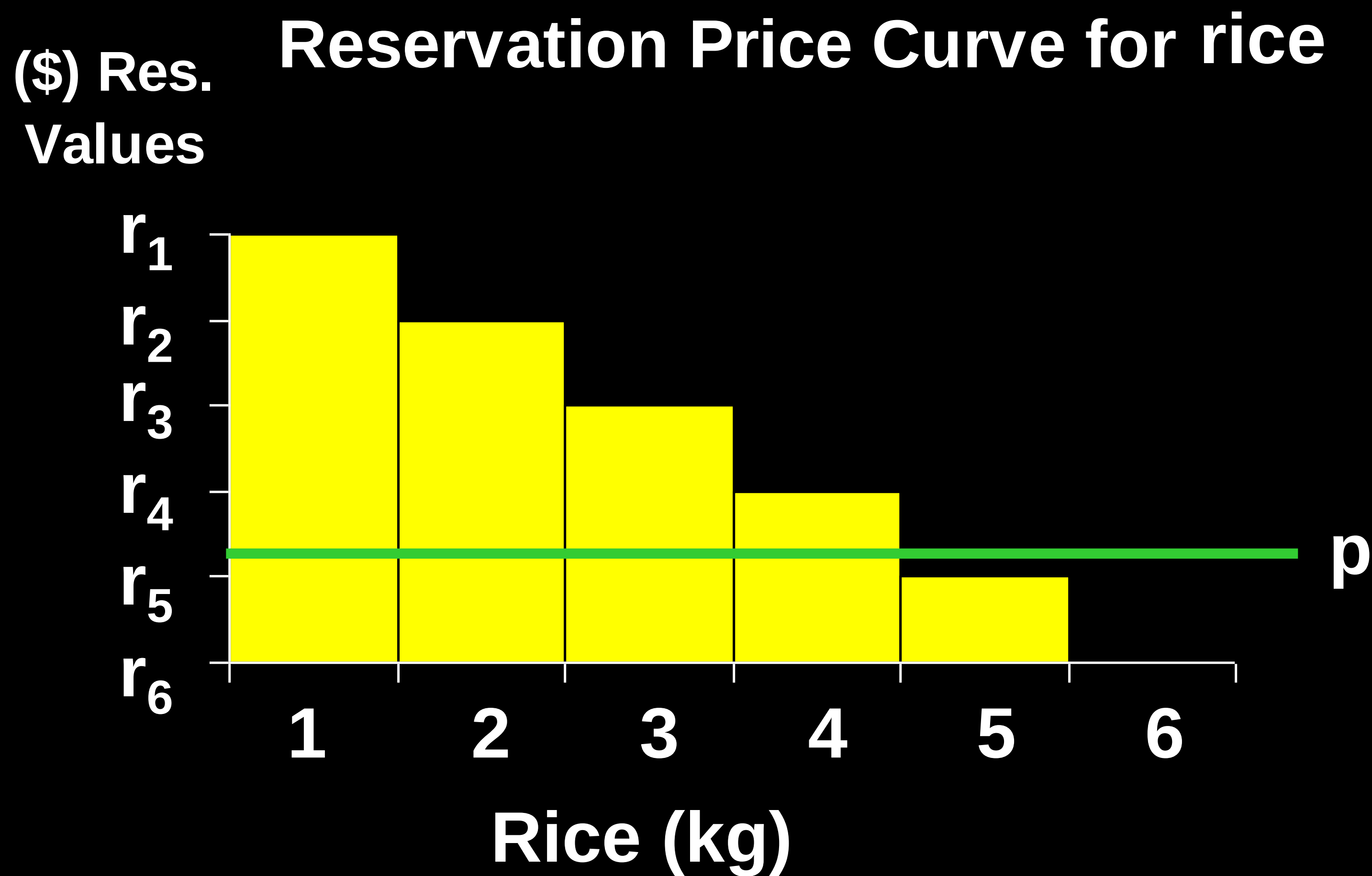
$$u(n, m) = u(n-1, m + r_n)$$

- Let's call r_n the consumer's **reservation price** of the n^{th} kg of rice.

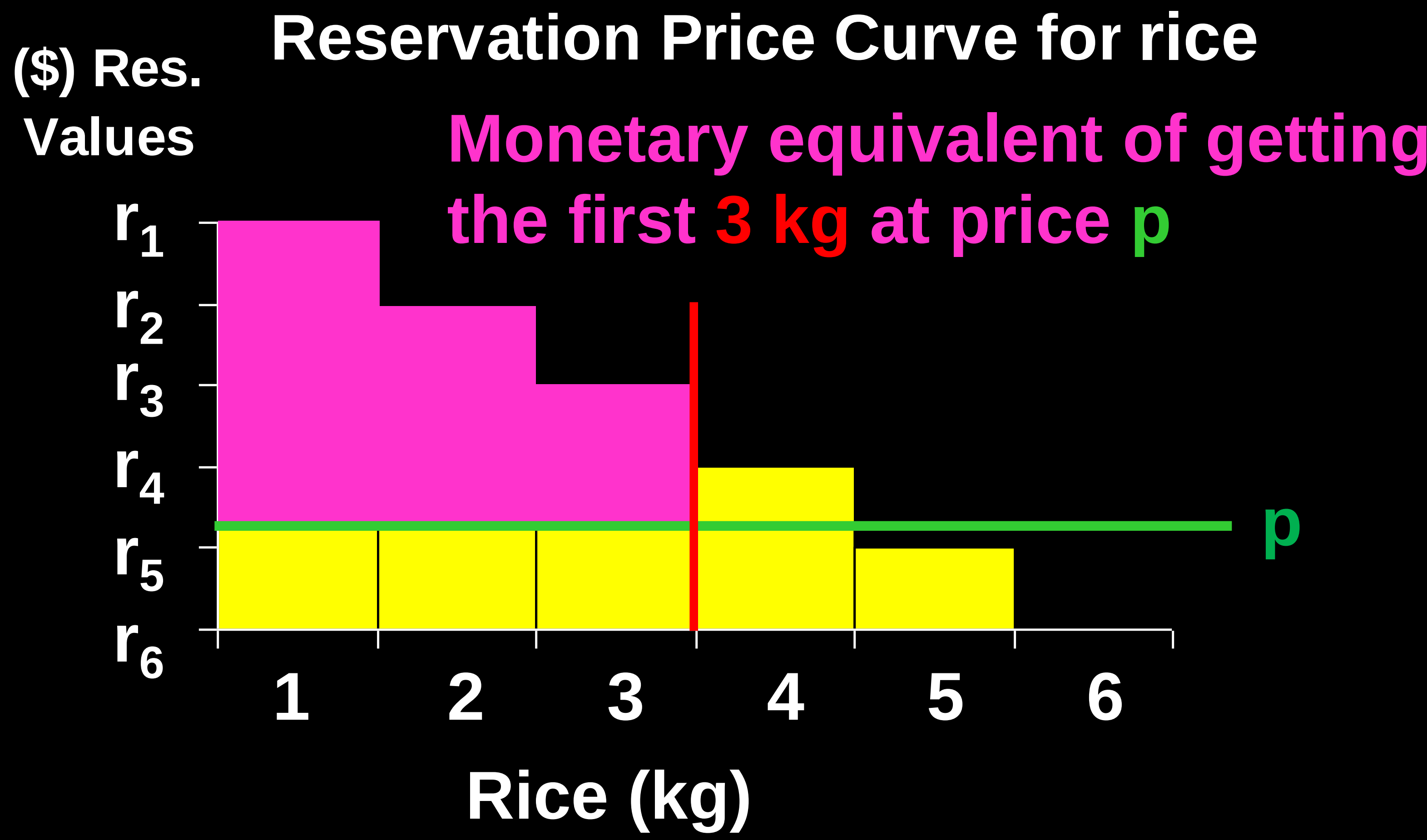
Monetary Measure of Welfare

- $r_1 + \ldots + r_n = v(n)-v(0)$ is therefore the monetary equivalent of getting n kg of rice for free.
- So $r_1 + \ldots + r_n - pn$ the monetary equivalent of getting n kg of rice at the price \$p/kg.

Graphical Illustration



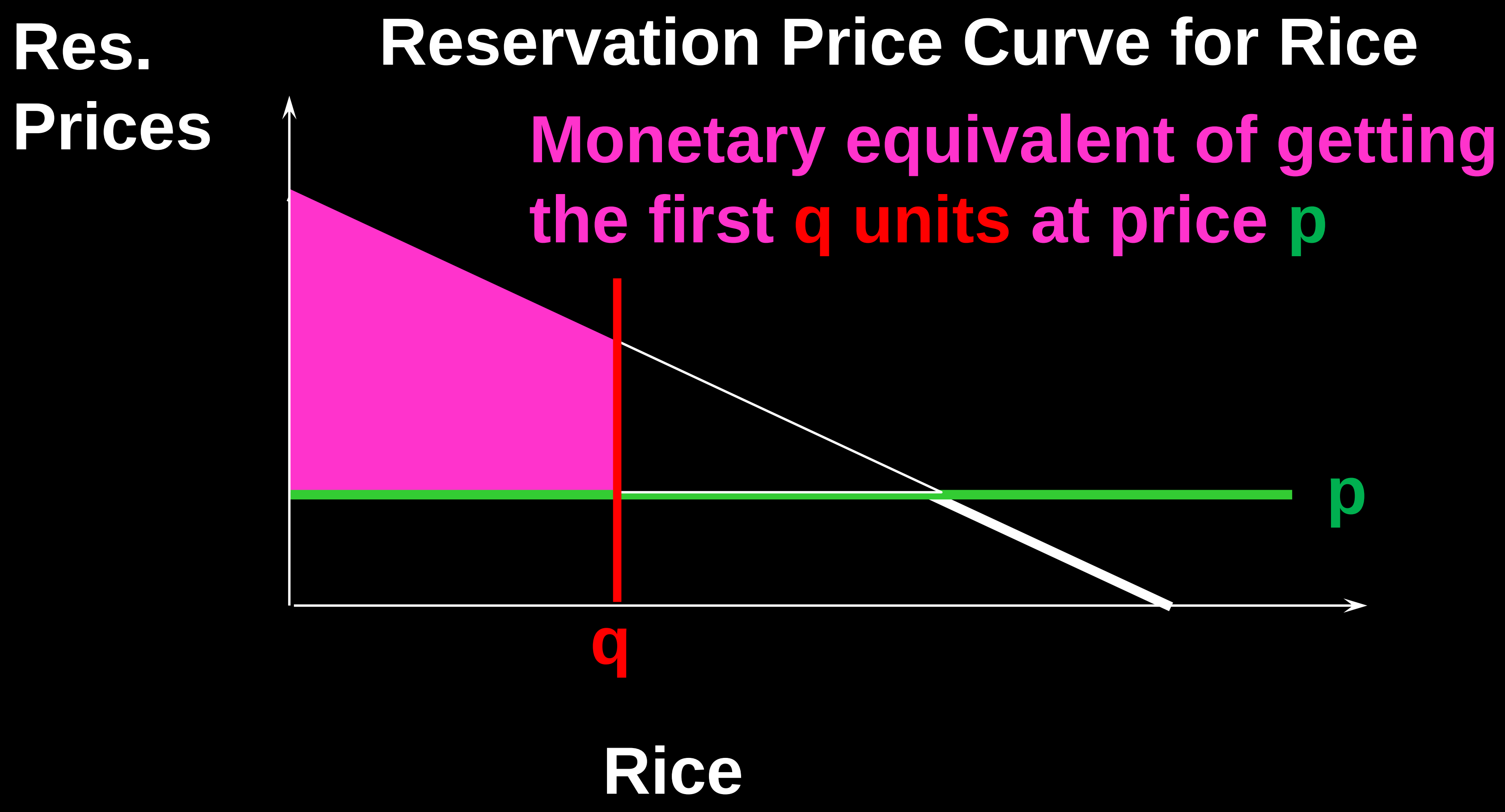
Graphical Illustration



Continuous Quantity?

Suppose rice can be purchased in any continuous quantity, then ...

Monetary Measure of Welfare



Consumer Surplus for Quasi-linear Utility

- **Define consumer surplus (CS) as the area of the region surrounded by the vertical axis, quantity line, reservation price curve, and price line.**

An Observation

- **Under the assumption of quasi-linear utility function, the reservation price curve is the same as the demand curve, both are given by $p = v'(q)$.**

Consumer Surplus

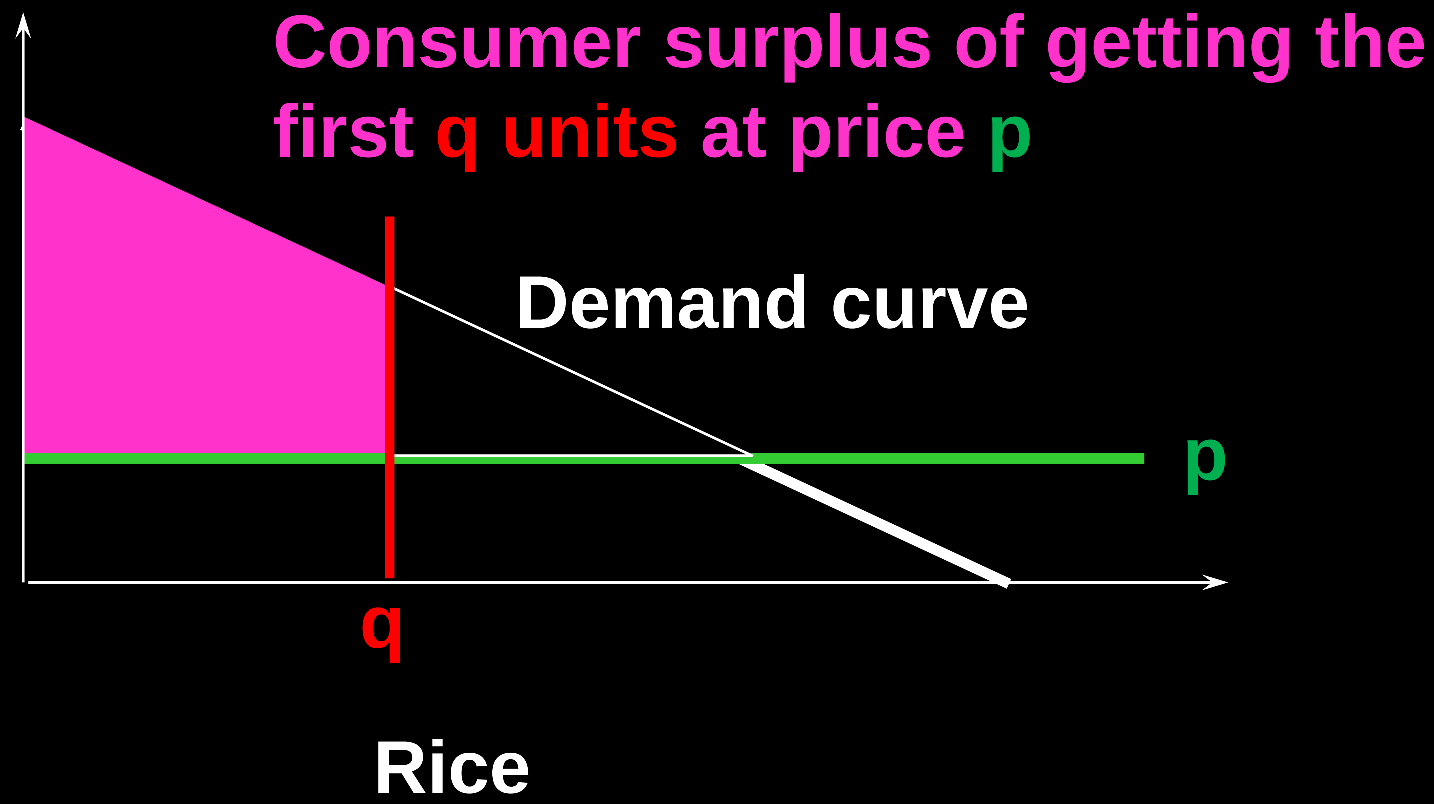
- Without the quasi-linear assumption, the notion of reservation price is controversial, since the consumer's willingness to pay for a unit of rice usually depends on the consumer's income level due to income effect.

- But, demand is always well-defined, so…

Consumer Surplus

- So, even without the quasi-linear assumption, we can define **consumer surplus** as the area of the region surrounded by the vertical axis, quantity line, **demand curve**, and price line.

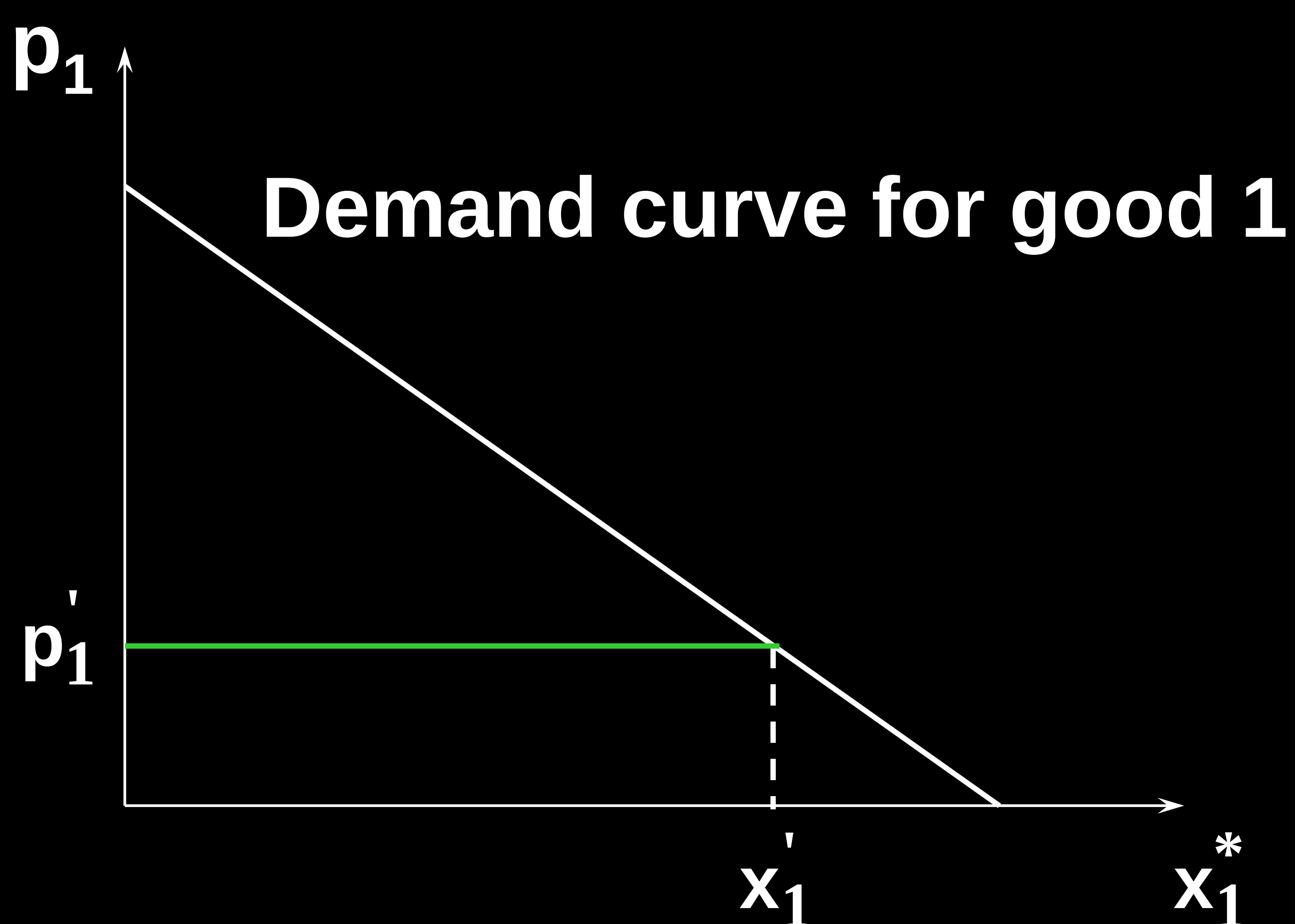
Consumer Surplus



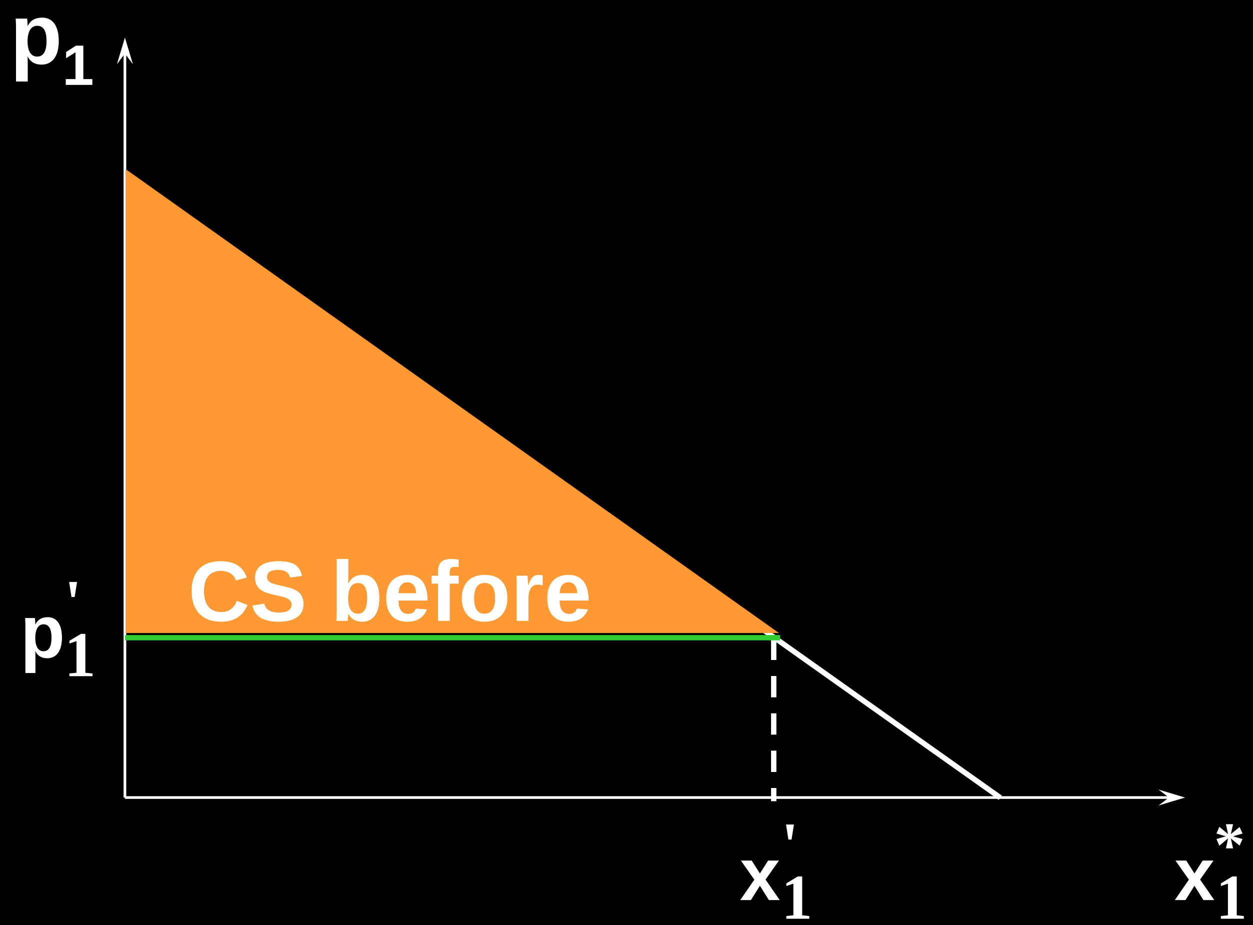
Interpretation of CS

- Under the quasi-linear assumption, CS is equivalent to the consumer's utility gain from purchasing the good.
- In general, CS can be regarded as a monetary measure of welfare change that is reasonably accurate if the income effect is not that significant.

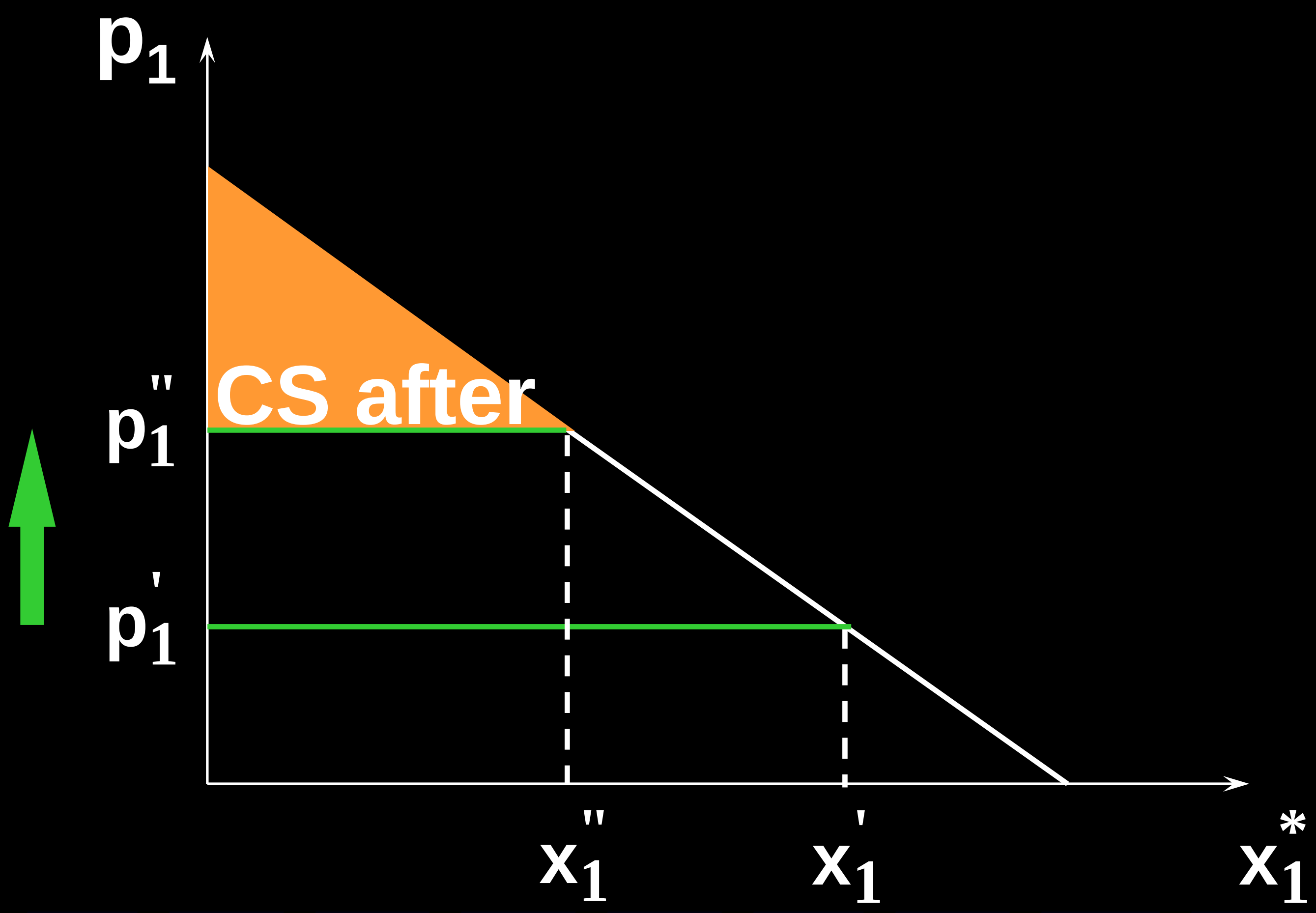
Consumer's Surplus



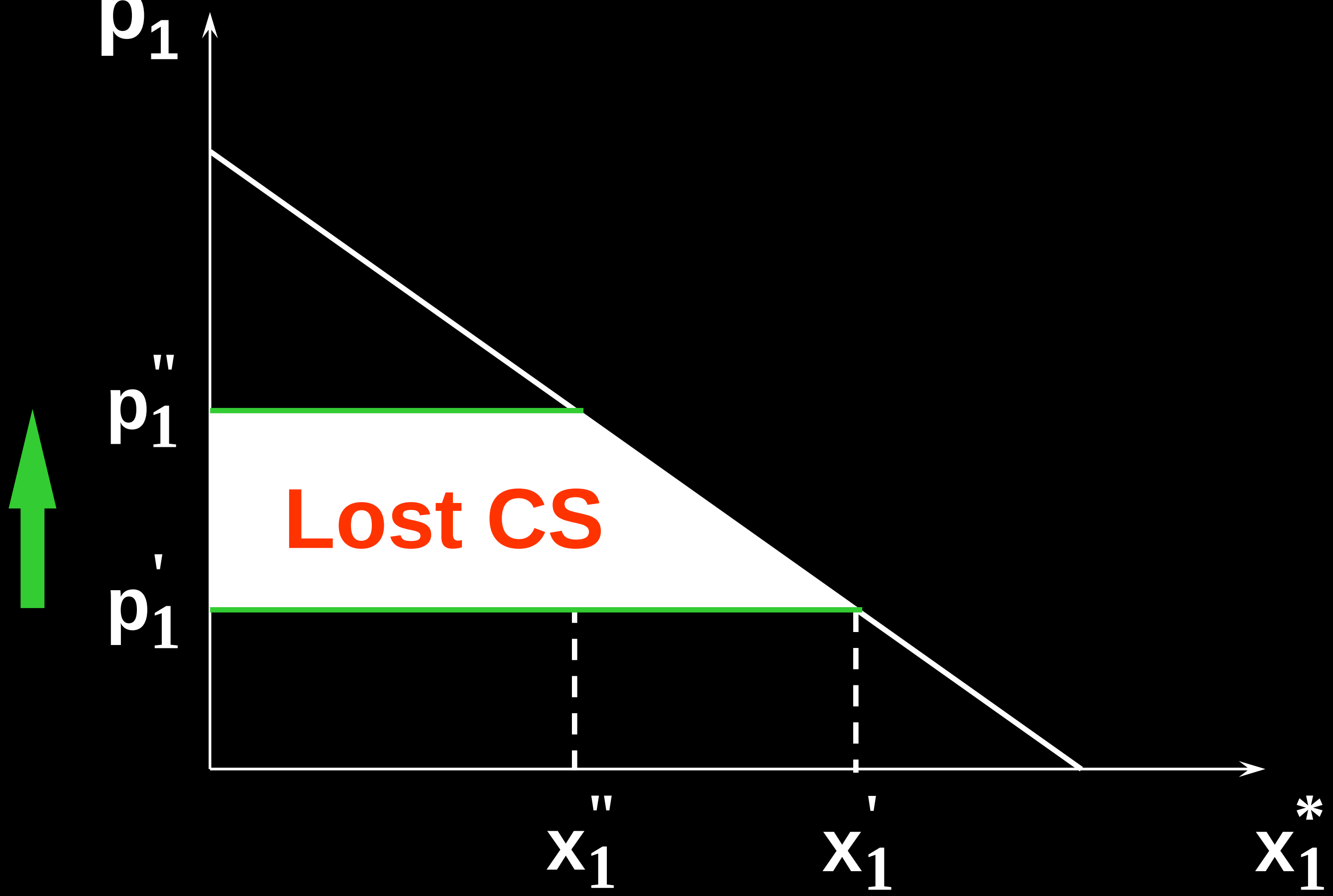
Consumer’s Surplus



Consumer's Surplus



Consumer's Surplus



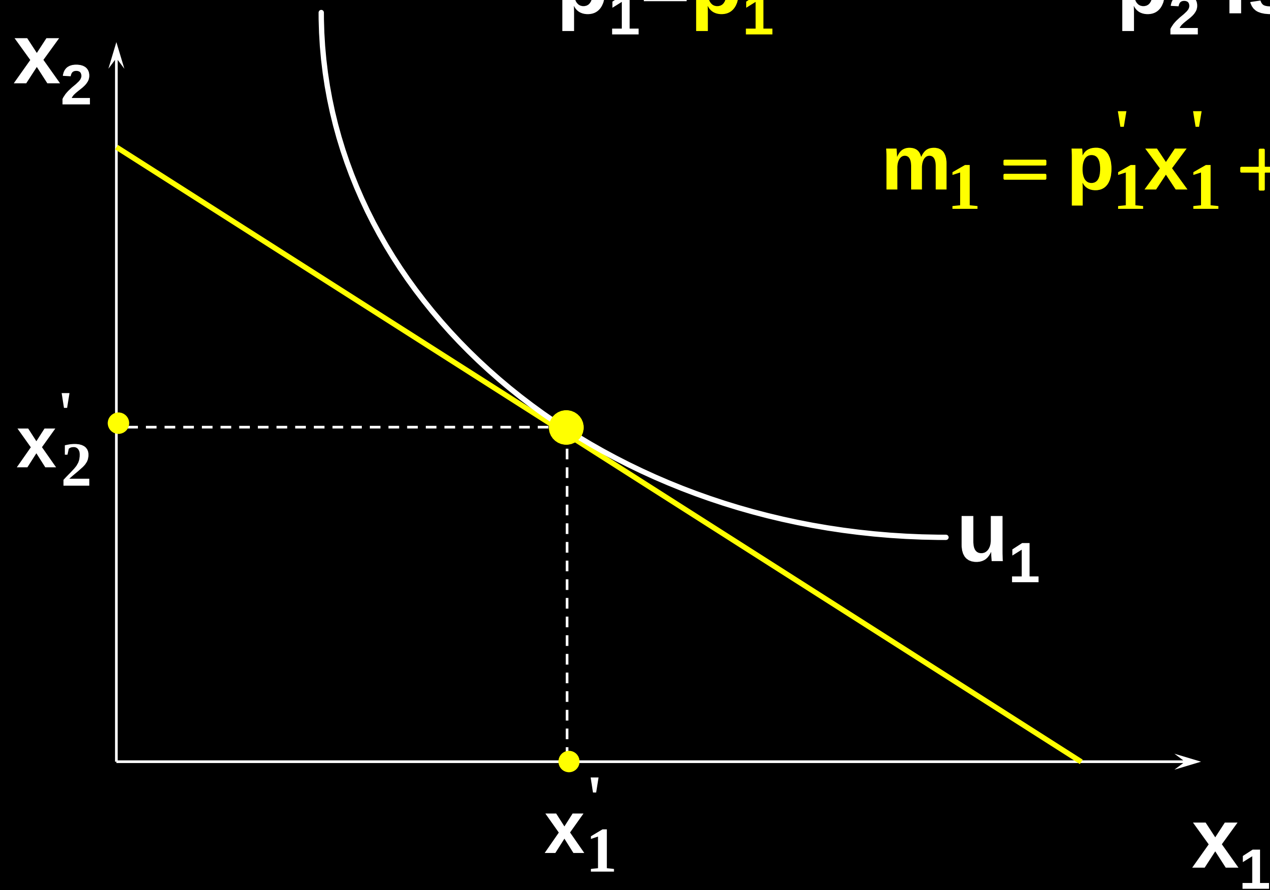
Compensating Variation (CV)

- Another monetary measure of welfare change is known as CV.
- Consider an increase in p_1
- CV is level of income increase under the new price that will make the consumer as well off as before.

Compensating Variation

$p_1 = p_1'$ p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$



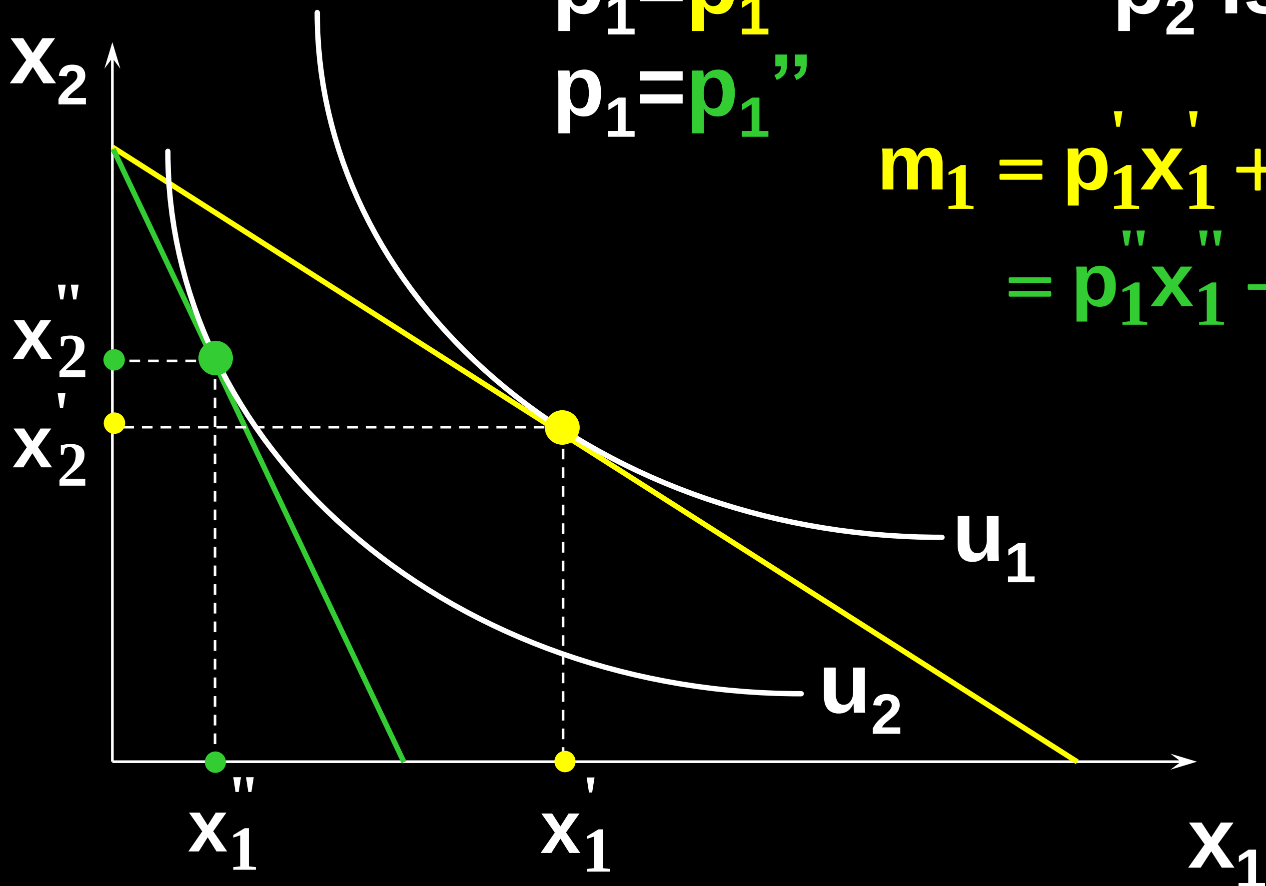
Compensating Variation

$p_1 = p_1'$

$p_1 = p_1''$

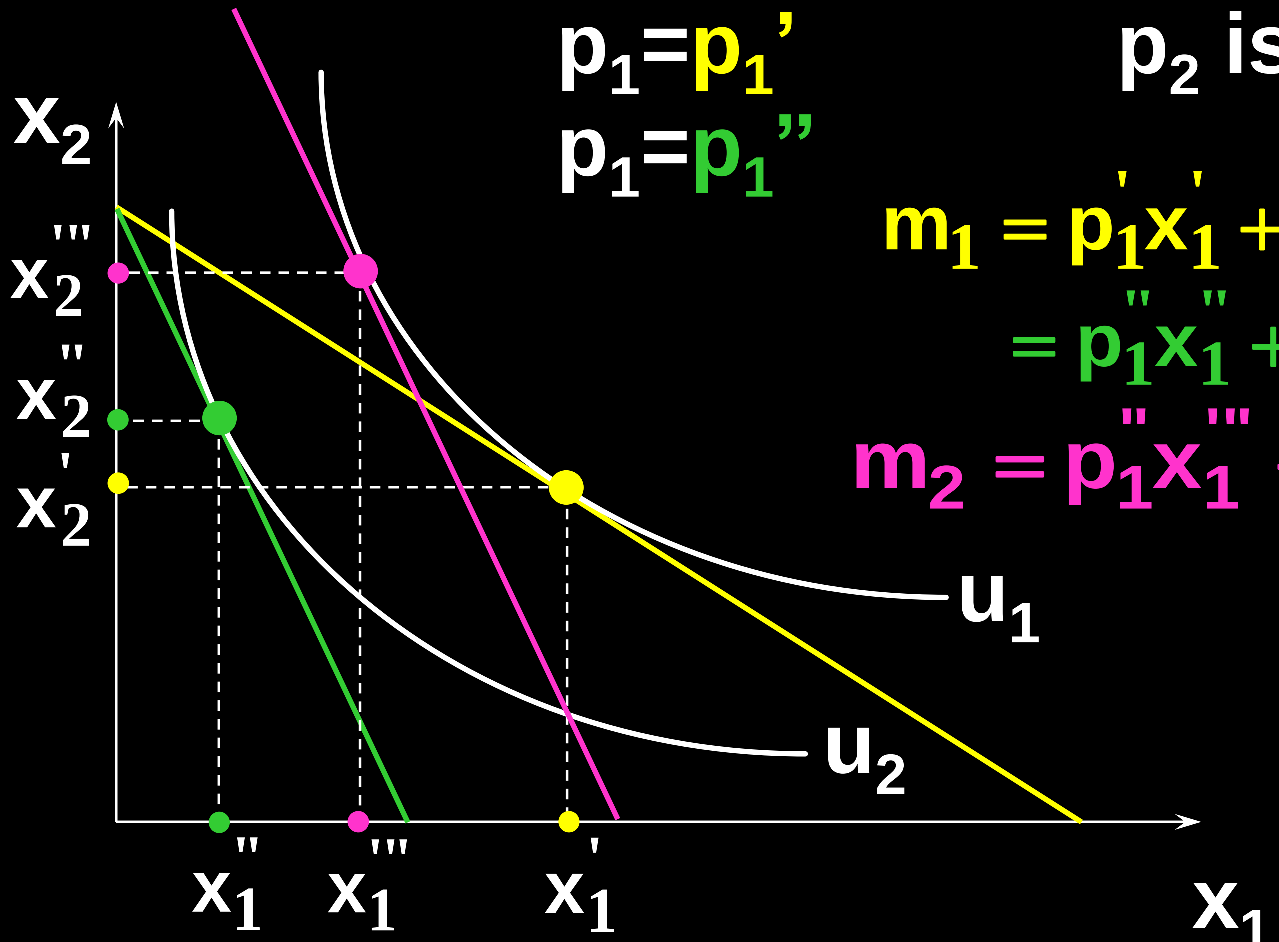
p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$
$$= p_1'' x_1'' + p_2 x_2''$$



Compensating Variation

p₂ is fixed.



$$m_1 = p_1' x_1' + p_2 x_2'$$

$$= p_1'' x_1'' + p_2 x_2''$$

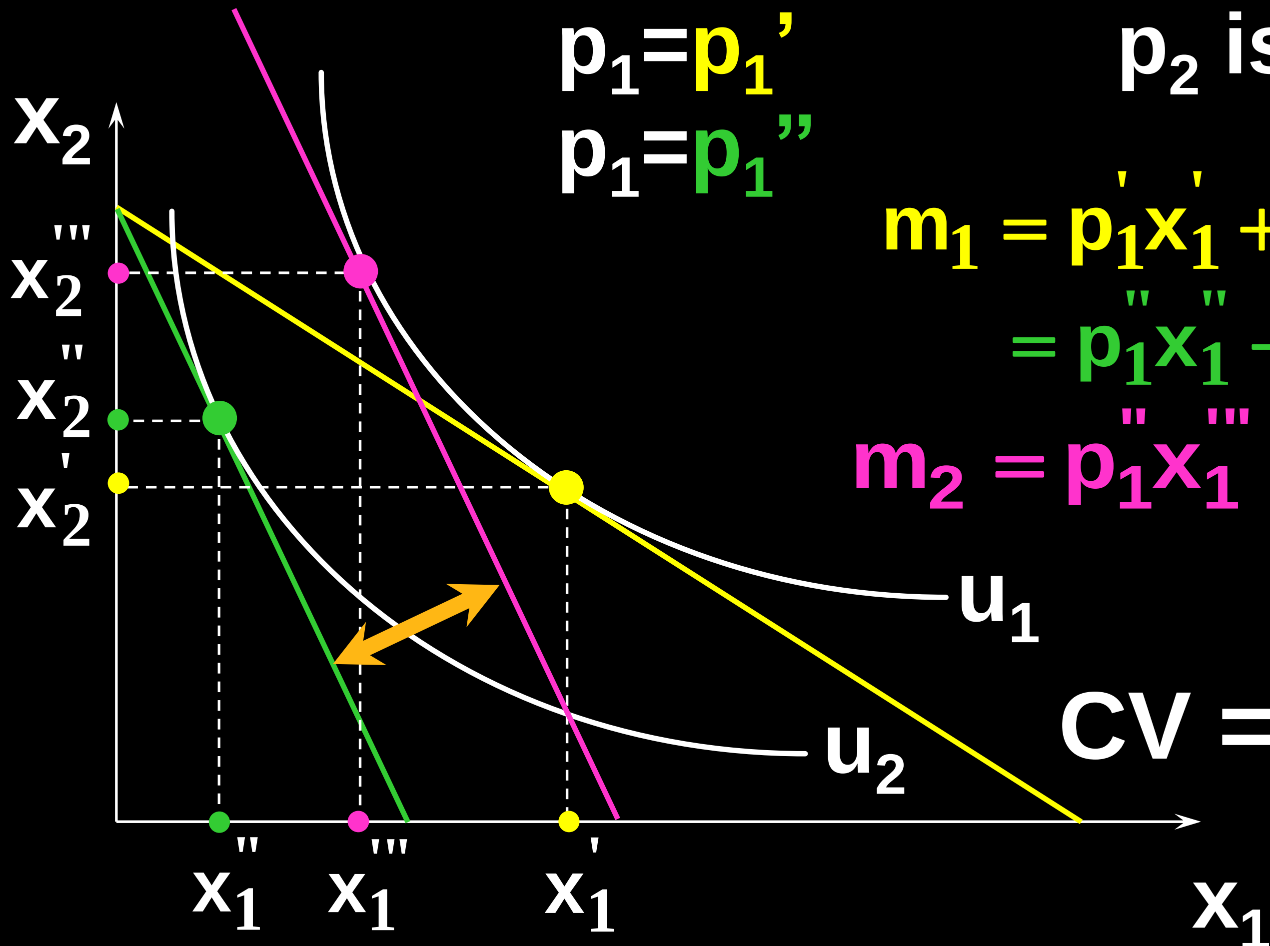
$$m_2 = p_1'' x_1''' + p_2 x_2'''$$

Compensating Variation

$p_1 = p_1'$

$p_1 = p_1''$

p_2 is fixed.



$$m_1 = p_1' x_1' + p_2 x_2'$$

$$= p_1'' x_1'' + p_2 x_2''$$

$$m_2 = p_1'' x_1''' + p_2 x_2'''$$

CV = m_2 - m_1.

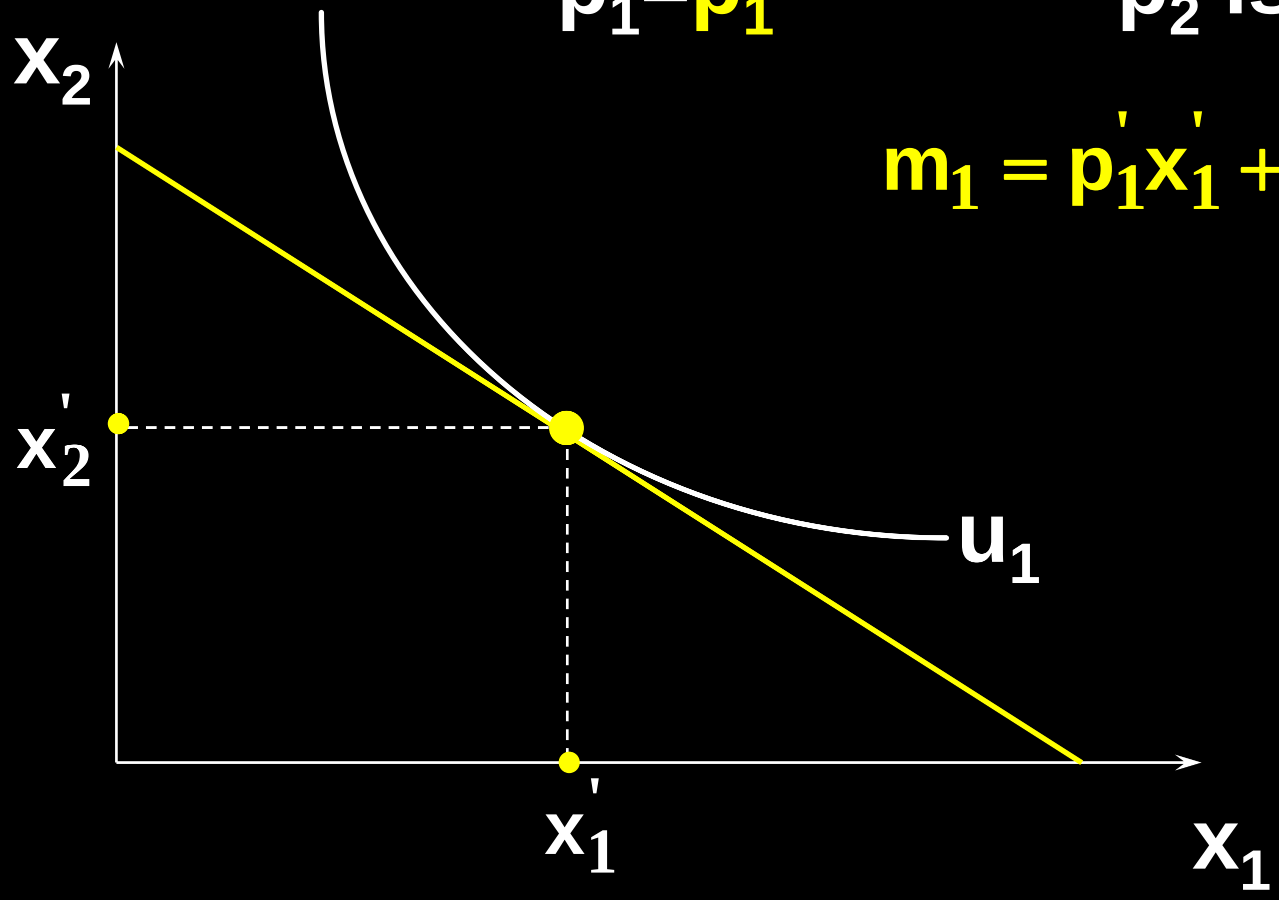
Equivalent Variation (EV)

- Consider an increase in p_1
- EV is level of income decrease under the old price that will make the consumer as well off as under the new price.

Equivalent Variation

$p_1 = p_1'$ $\quad p_2$ is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$

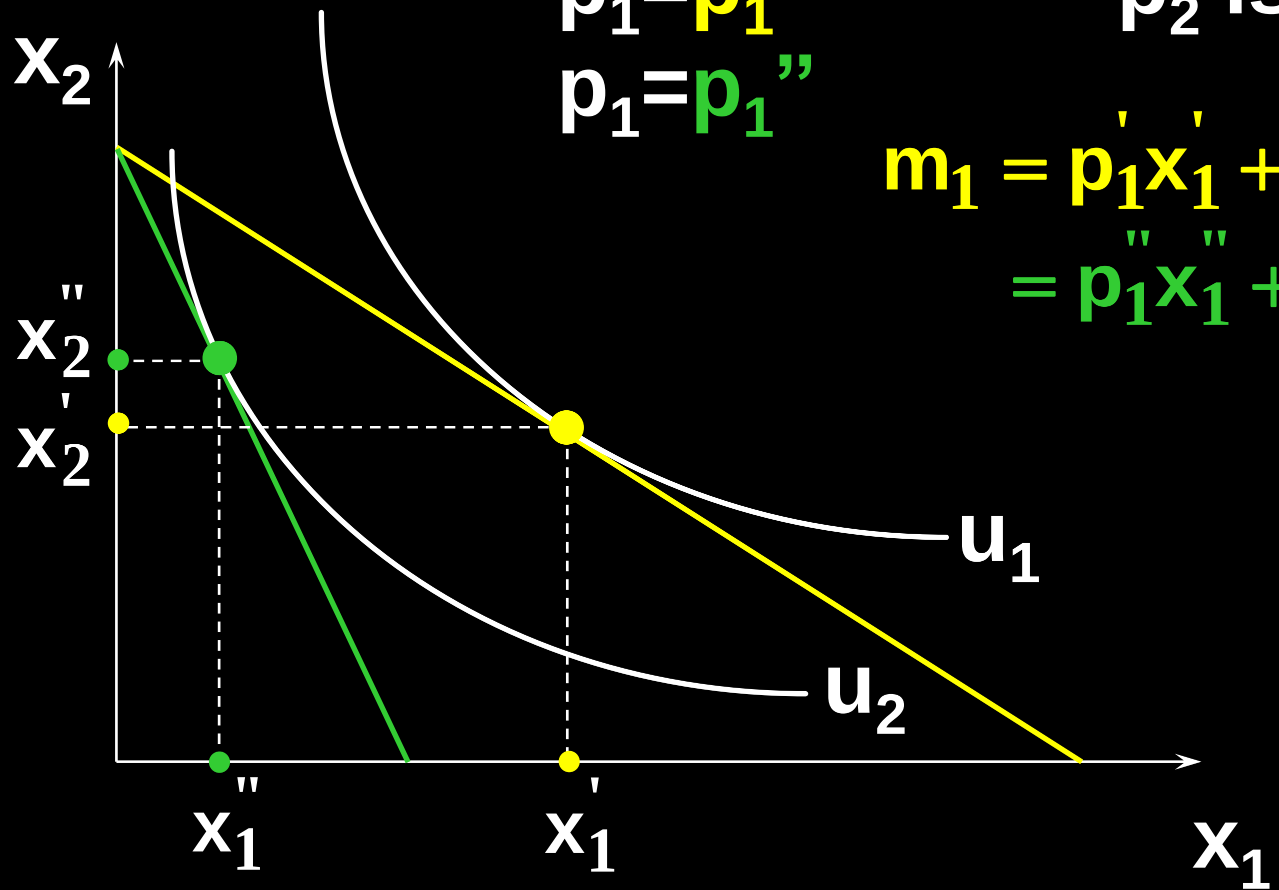


Equivalent Variation

$p_1 = p_1'$
$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2' = p_1'' x_1'' + p_2 x_2''$$



Equivalent Variation

$p_1 = p_1'$

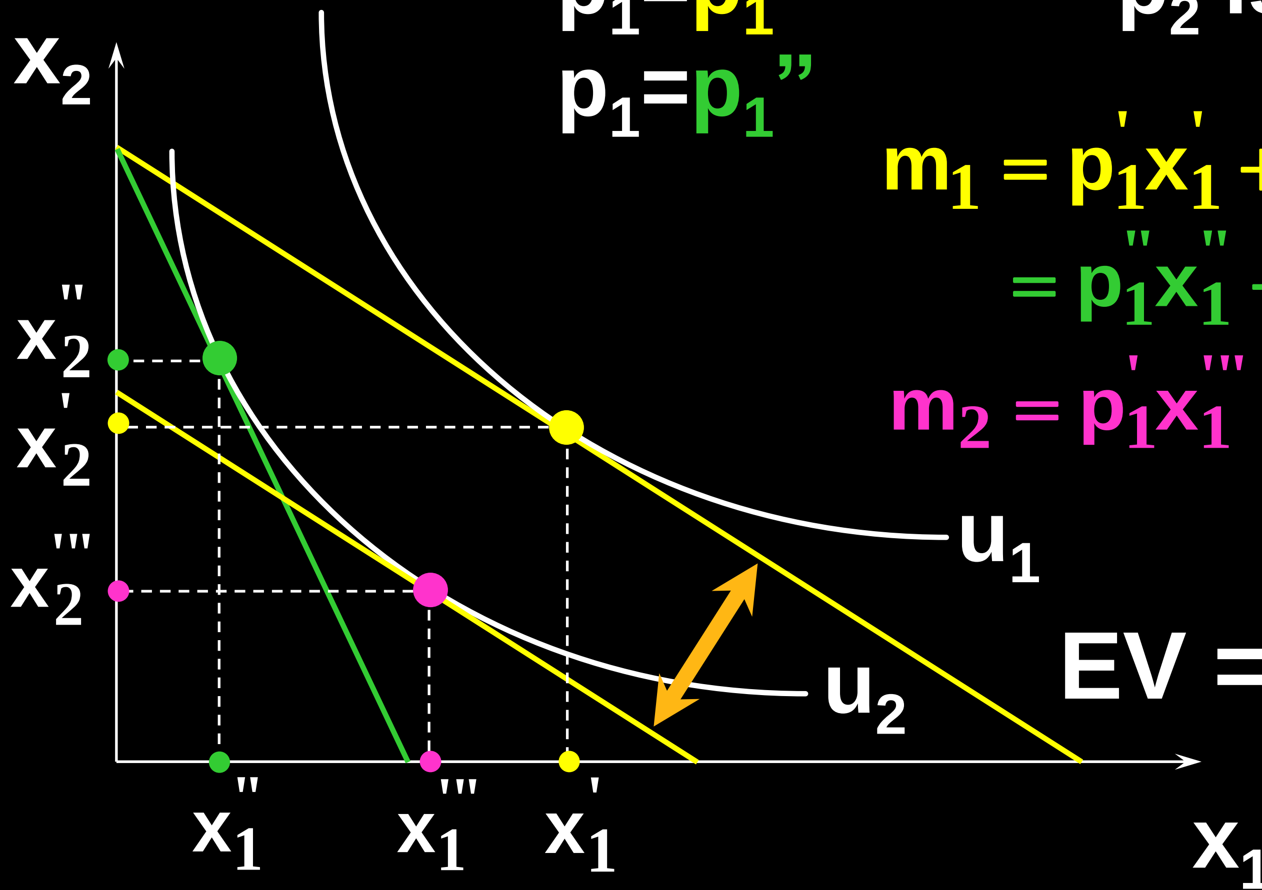
$p_1 = p_1''$

p_2 is fixed.

$$m_1 = p_1' x_1' + p_2 x_2'$$

$$= p_1'' x_1'' + p_2 x_2''$$

$$m_2 = p_1' x_1''' + p_2 x_2'''$$



EV = m_1 - m_2.

CS, CV, and EV

- Under a quasi-linear utility function and interior solution, the three concepts are equivalent.

CS, CV, and EV

- Consider first the change in Consumer's Surplus when p_1 rises from p_1' to p_1".

CS, CV, and EV

If $U(x_1, x_2) = v(x_1) + x_2$ then

$$CS(p_1') = v(x_1') - v(0) - p_1'x_1'$$

The monetary equivalent of welfare loss measured by CS is

$$-\Delta CS = CS(p_1') - CS(p_1'')$$

$$= v(x_1') - v(0) - p_1'x_1' - \left[v(x_1'') - v(0) - p_1''x_1''\right]$$

$$= v(x_1') - v(x_1'') - (p_1'x_1' - p_1''x_1'').$$

CS, CV, and EV

- Now consider the monetary equivalent of welfare loss measured by CV.
- The consumer's utility for given p_1 is

$$v(x_1^*(p_1)) + m - p_1 x_1^*(p_1)$$

and CV is the extra income which, at the new prices, makes the consumer's utility the same as at the old prices. That is, ...

CS, CV, and EV

$$v(x_1') + m - p_1' x_1'$$
$$= v(x_1'') + m + CV - p_1'' x_1''.$$

So

$$CV = v(x_1') - v(x_1'') - (p_1' x_1' - p_1'' x_1'')$$
$$= -\Delta CS.$$

CS, CV, and EV

- Now consider the monetary equivalent of welfare loss measured by EV.
- The consumer's utility for given p_1 is

$$v(x_1^*(p_1)) + m - p_1 x_1^*(p_1)$$

and EV is the reduction in income which, at the old prices, makes the consumer's utility the same as at the new prices.   That is, ...

CS, CV, and EV

$$v(x_1^{'}) + m - EV - p_1^{'}x_1^{'}$$
$$= v(x_1^{''}) + m - p_1^{''}x_1^{''}.$$

That is,

$$\mathbf{EV = v(x_1^{'}) - v(x_1^{''}) - (p_1^{'}x_1^{'} - p_1^{''}x_1^{''})}$$
$$\mathbf{= -\Delta CS.}$$

CS, CV, and EV

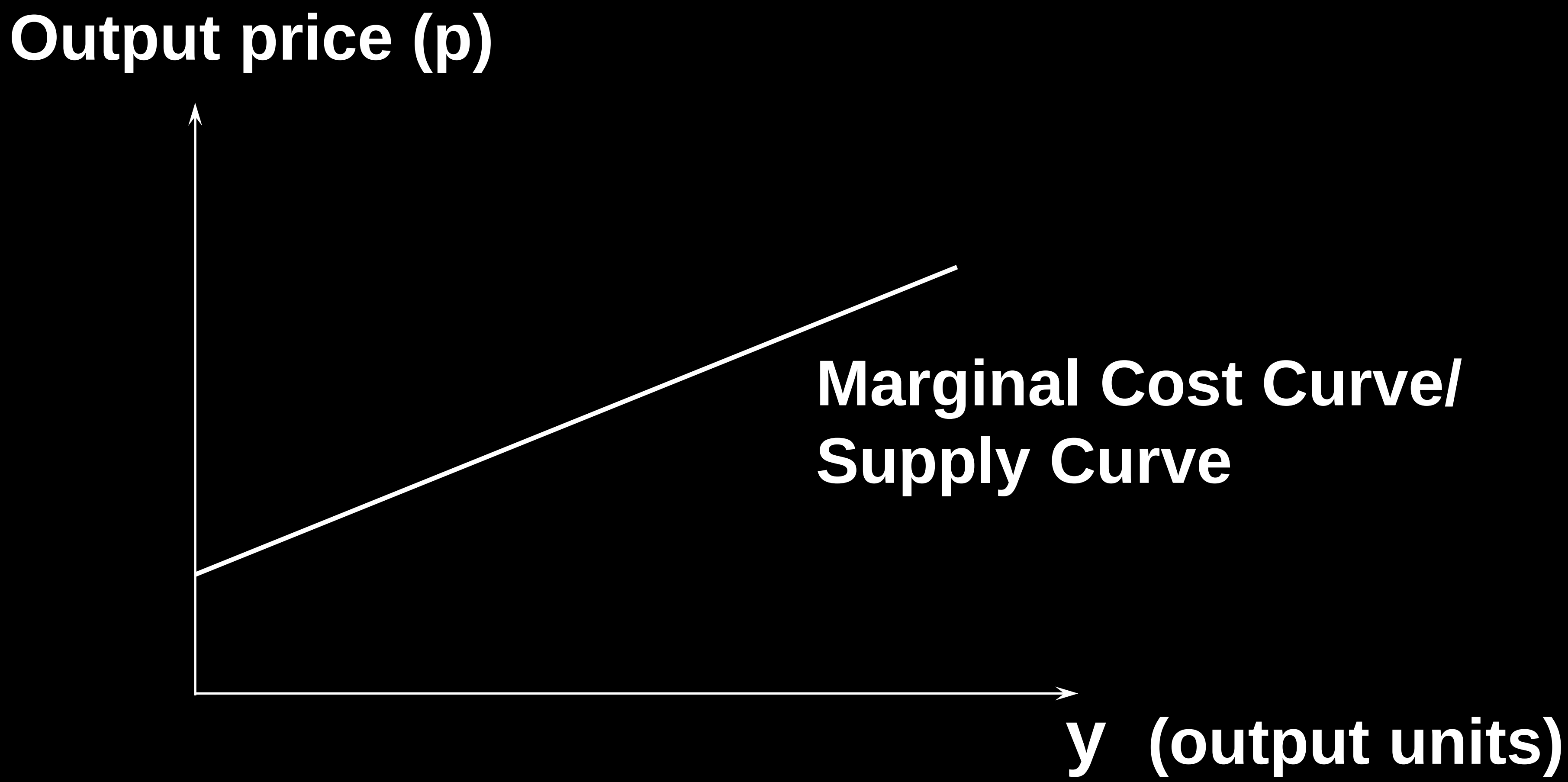
To sum up, with quasilinear utility, we have **CV = EV =-ΔCS**.

Without quasilinear utility, it can be shown that the welfare change measured in CS lies between CV and EV, and the difference is usually relatively small. (c.f. Robert Willig, "Consumer's Surplus without Apology," American Economic Review, 66 (1976), 589–597.)

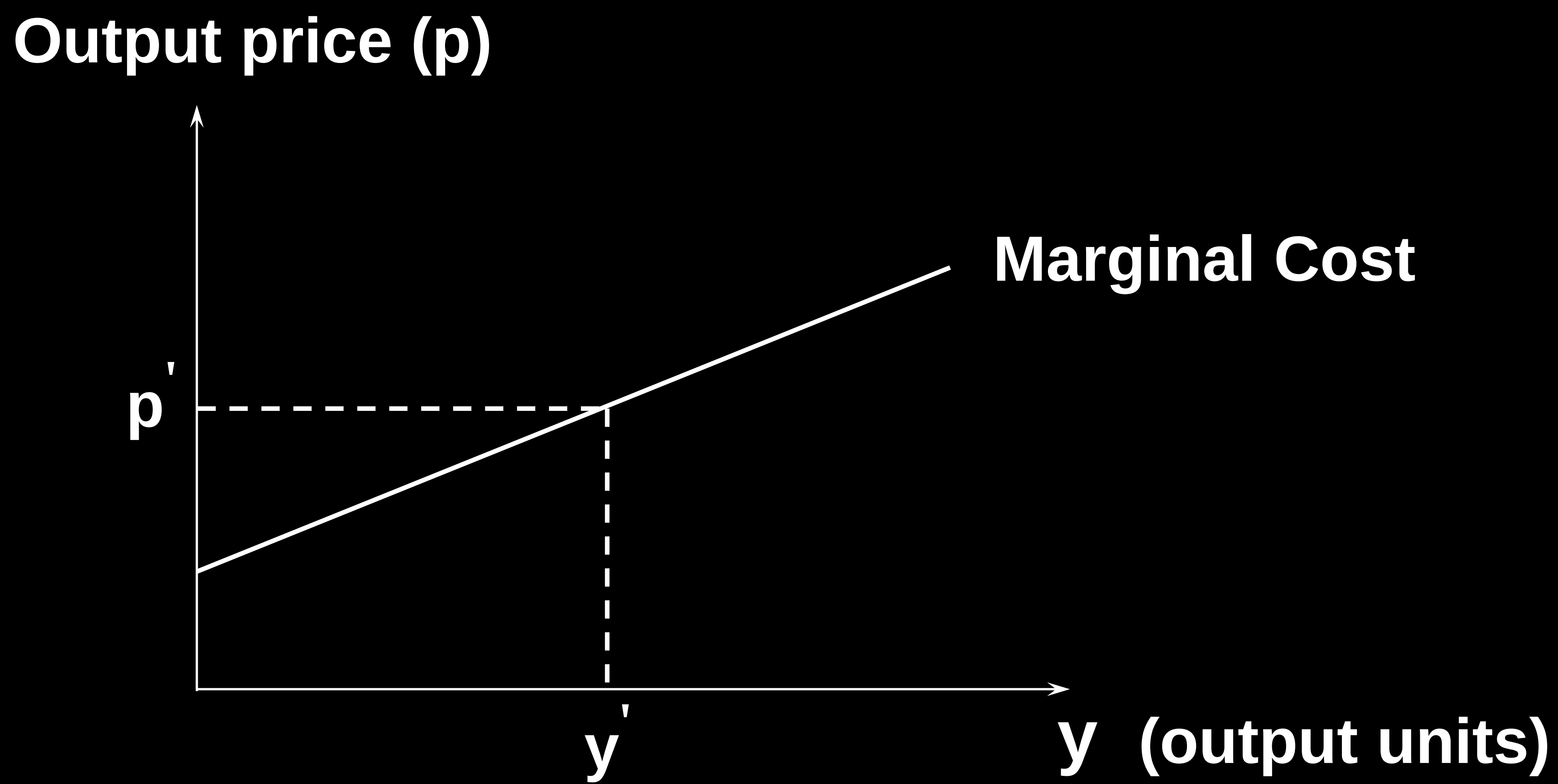
Producer's Surplus

- Changes in a firm's welfare can be measured in the unit of money similarly to a consumer's welfare.

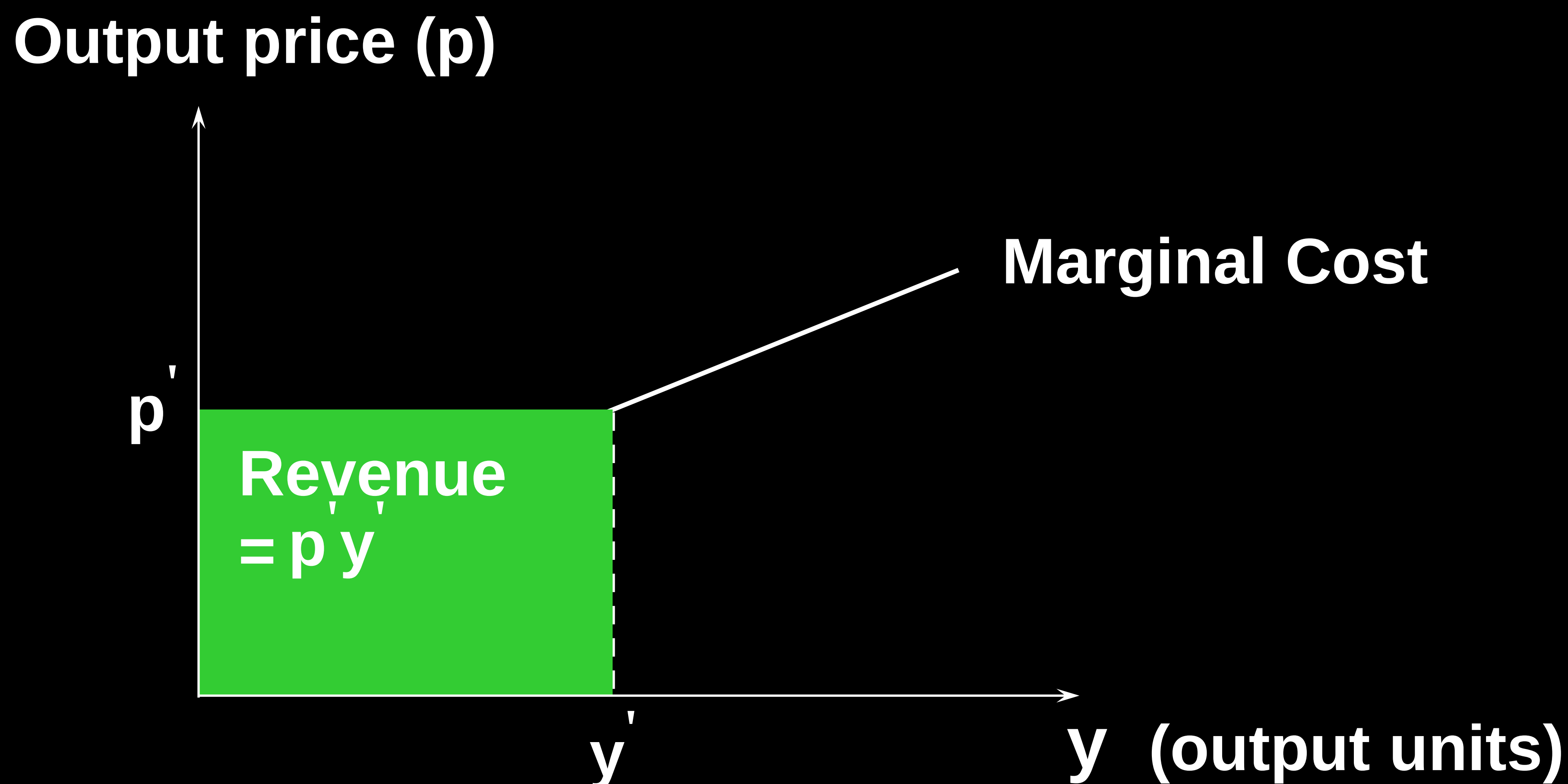
Producer's Surplus



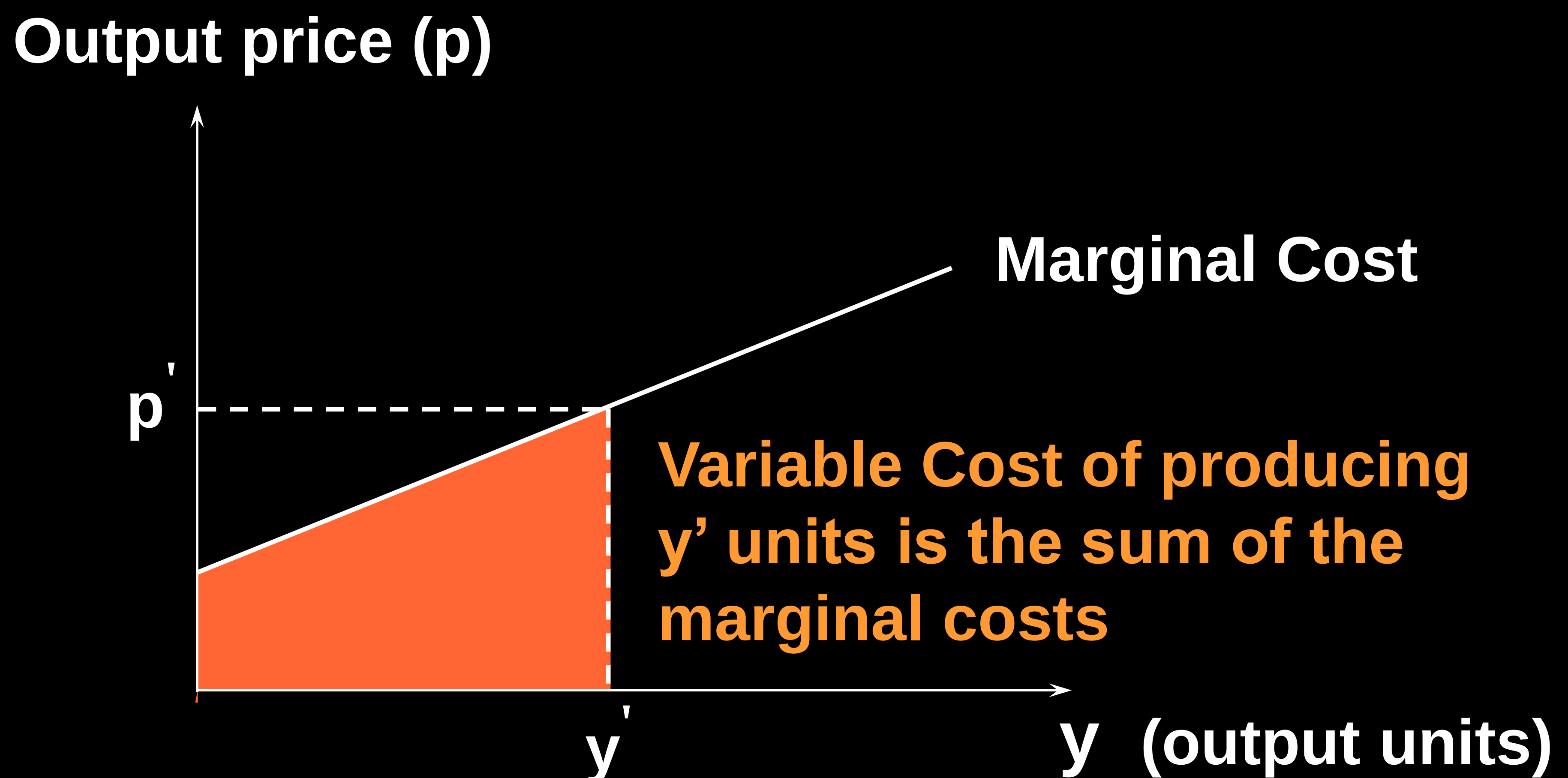
Producer's Surplus



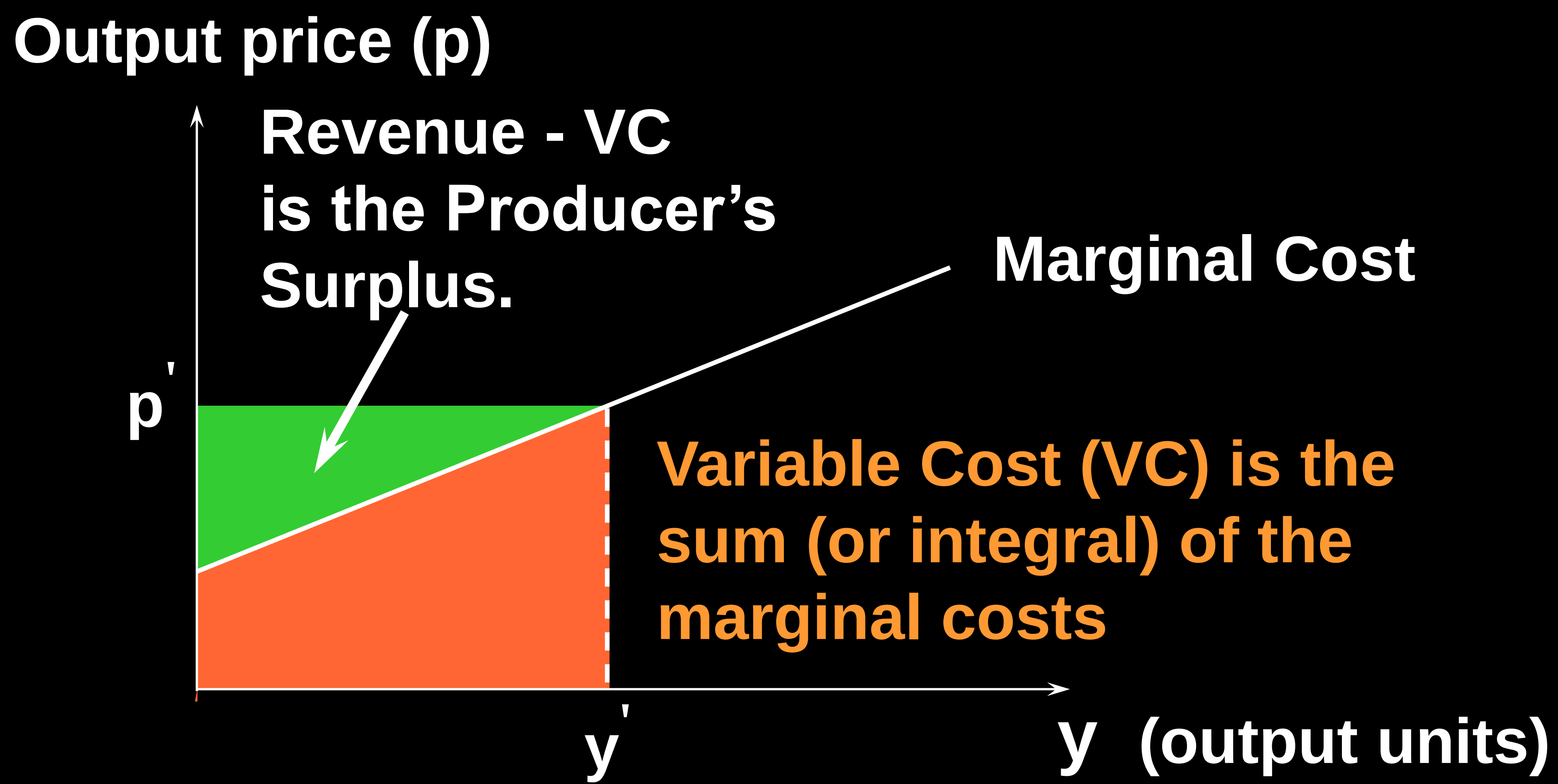
Producer's Surplus



Producer's Surplus



Producer's Surplus



Summary

- **Key concepts:**
  - **Consumer surplus;**
  - **Compensation variation;**
  - **Equivalent variation.**