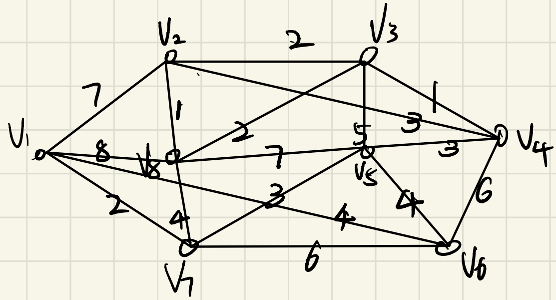
1.



	1	2	3	4	5	6	7	8
(1)	0	7	∞	∞	∞	4	2	8
(2)	0	7	∞	∞	5	4	2	6
(3)	0	7	∞	10	5	4	2	6
(4)	0	7	10	8	5	4	2	6
(5)	0	7	8	8	5	4	2	6
(6)	0	7	8	8	5	4	2	6
(7)	0	7	8	8	5	4	2	6
(8)	0	7	8	8	5	4	2	6

1. 7. 6. 5. 8. 2. 3. 4

2.

(1)

a. 关联矩阵 $B=\begin{array}{c} \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \begin{pmatrix} -1 & 1 & & & & & \\ 1 & & -1 & & -1 & & \\ & -1 & 1 & -1 & & -1 & \\ & & & 1 & 1 & & 1 \\ & & & & & 1 & -1 \end{pmatrix} \end{array} \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}$

$$\begin{pmatrix} -1 & & & & & & \\ & & -1 & & -1 & & \\ & -1 & & -1 & & -1 & \\ & & & & & & -1 \end{pmatrix}$$

$$B_3=\begin{pmatrix} -1 & 1 & & & & & \\ 1 & & -1 & & -1 & & \\ & & & 1 & 1 & & 1 \\ & & & & & 1 & -1 \end{pmatrix}$$

支撑树数量 $=\det(B_3B_3^T)=\det\begin{pmatrix} 2 & -1 & & \\ -1 & 3 & -1 & \\ & -1 & 3 & -1 \\ & & -1 & 2 \end{pmatrix}=21$

b. 将 V_3, V_4 合并为 $V_{3,4}$

$B'=\begin{array}{c} \begin{pmatrix} -1 & 1 & & & & \\ 1 & & -1 & -1 & & \\ & -1 & 1 & 1 & -1 & 1 \\ & & & & 1 & -1 \end{pmatrix} \\ \begin{array}{cccccc} e_1 & e_2 & e_3 & e_5 & e_6 & e_7 \end{array} \end{array} \begin{array}{c} v_1 \\ v_2 \\ v_{3,4} \\ v_5 \end{array}$ $\quad B_{3,4}=\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_5 & e_6 & e_7 \end{array} \\ \begin{pmatrix} -1 & 1 & & & & \\ 1 & & -1 & -1 & & \\ & & & & 1 & -1 \end{pmatrix} \end{array} \begin{array}{c} v_1 \\ v_2 \\ v_5 \end{array}$

含 e_4 的支撑树数量 $=\det(B_{3,4}B_{3,4}^T)=\det\begin{pmatrix} 2 & -1 & \\ -1 & 3 & \\ & & 2 \end{pmatrix}=10$

c. 将 e_4 去掉:

$B_3'=\begin{array}{c} \begin{pmatrix} -1 & 1 & & & & \\ 1 & & -1 & -1 & & \\ & & & 1 & & 1 \\ & & & & 1 & -1 \end{pmatrix} \\ \begin{array}{cccccc} e_1 & e_2 & e_3 & e_5 & e_6 & e_7 \end{array} \end{array} \begin{array}{c} v_1 \\ v_2 \\ v_4 \\ v_5 \end{array}$

不含 e_4 的支撑树数量 $=\det(B_3'B_3'^T)$

$=\det\begin{pmatrix} 2 & -1 & & \\ -1 & 3 & -1 & \\ & -1 & 2 & -1 \\ & & -1 & 2 \end{pmatrix}=11$

d. $B_4=\begin{pmatrix} -1 & 1 & & & & & \\ 1 & & -1 & & -1 & & \\ & -1 & 1 & -1 & & -1 & \\ & & & & & 1 & -1 \end{pmatrix} \Rightarrow \det(B_4B_4^T)=\det\begin{pmatrix} 1 & -1 & & \\ & 2 & 1 & \\ -1 & & 3 & -1 \\ & & & 1 \end{pmatrix}=5$

(2) $S_f = \begin{pmatrix} S_{f11} & I \end{pmatrix}$, $B_K = \begin{pmatrix} B_{11} & B_{12} \end{pmatrix}$

$$S_{f11} = B_{12}^{-1} B_{11}$$

$T = \{e_1, e_4, e_5, e_7\}$

$$\text{取} B_3 = \begin{array}{c} \begin{pmatrix} -1 & 1 & & & & & \\ 1 & & -1 & & -1 & & \\ & & & 1 & 1 & & 1 \\ & & & & & 1 & -1 \end{pmatrix} \\ \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{matrix} \end{array} = \begin{array}{c} \left(\begin{array}{ccc|cccc} 1 & & & -1 & & & \\ & -1 & & 1 & & -1 & \\ & & & & 1 & 1 & 1 \\ & & 1 & & & & -1 \end{array}\right) \\ \begin{matrix} e_2 & e_3 & e_6 & e_1 & e_4 & e_5 & e_7 \end{matrix} \end{array}$$

$$\Rightarrow B_{11} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & \\ & & 1 \end{pmatrix} \qquad B_{12} = \begin{pmatrix} -1 & & & \\ 1 & & -1 & \\ & 1 & 1 & 1 \\ & & & -1 \end{pmatrix}$$

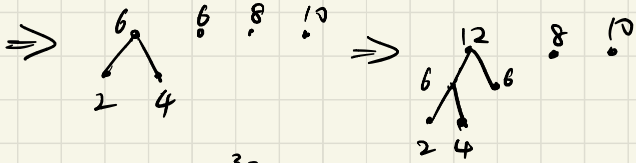
$$\Rightarrow B_{12}^{-1} = \begin{pmatrix} -1 & & & \\ 1 & 1 & 1 & 1 \\ -1 & -1 & & \\ & & & -1 \end{pmatrix}$$

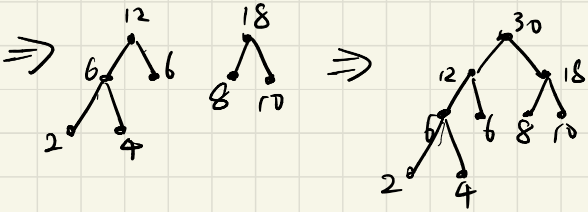
$$\Rightarrow S_{f11} = \begin{pmatrix} -1 & & \\ 1 & -1 & 1 \\ -1 & 1 & \\ & & -1 \end{pmatrix}$$

$$\Rightarrow S_f = \begin{array}{c} \left(\begin{array}{ccc|cccc} -1 & & & 1 & & & \\ 1 & -1 & 1 & & 1 & & \\ -1 & 1 & & & & 1 & \\ & & -1 & & & & 1 \end{array}\right) \\ \begin{matrix} e_2 & e_3 & e_6 & e_1 & e_4 & e_5 & e_7 \end{matrix} \end{array}$$

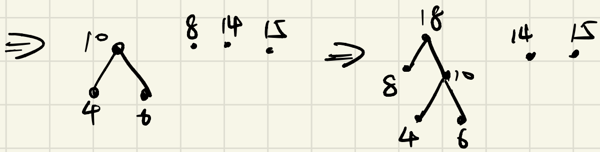
3.

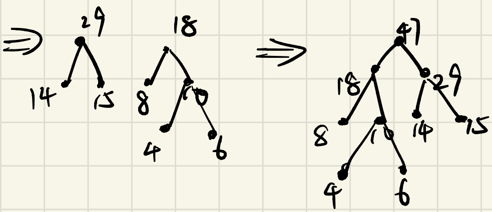
(1) 2, 4, 6, 8, 10



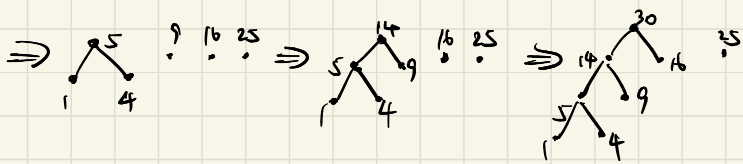


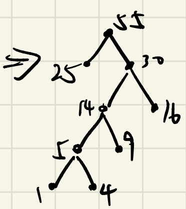
(2) 4, 6, 8, 14, 15





(3) 1, 4, 9, 16, 25





(4) 10, 12, 13, 16, 17, 17