

Assignment 4

Intermediate Microeconomics

Due on Dec. 22. 2023

I hope everyone enjoys this assignment and learns something from it. The questions may be more flexible than what you typically encounter in textbooks, designed to exercise your ability to understand the model in a more flexible manner. Don't worry too much about grades of this assignment. Please send emails to haoting_thu@163.com if you find any errors in the questions.

1 Short-Run Decisions (10')

Suppose that Howard's short-run total cost of repairing s cars per week is $c(s) = 2s^2 + 18$. If the price he receives for repairing a car is 8, then in the short run, how many cars will he fix per week if he maximize profits? Briefly outline the calculation process.

2 Firm Supply without Free Entry (10')

Consider a firm's total cost function: $c(y) = y^2 + 10$ for $y > 0$ and $c(0) = 0$.

- a) Write down its MC function, AVC function and ATC function. (6')
- b) In a competitive market, what is the lowest price at which he will supply a positive quantity in long-run equilibrium? (4')

3 Comprehensive Cost in Industry Cost (10')

Suppose that the cost of capturing a cockatoo and transporting him to the United States is about \$40 per bird. Each smuggled cockatoo has a 10% probability of being discovered, in which case the smuggler is fined. If the fine imposed for each smuggled cockatoo is \$1,000. Cockatoos are drugged and smuggled in suitcases to the United States. Half of the smuggled cockatoos die in transit. Suppose the market is competitive and involves a large number of smugglers, who only care about their expected profit, then the equilibrium price of cockatoos in the United States will be?

4 Industry Supply without Free Entry (10')

Consider a competitive industry with several firms all of which have the same cost function $c(y) = y^2 + 4$ for $y > 0$ and $c(0) = 0$. The demand curve for this industry is $D(p) = 50 - p$, where p is the price. Calculate the long run equilibrium number of firms in this industry.

5 Education with Encouragement (40')

Howard, a graduate of HT University, becomes a cram school teacher. Howard finds that students' academic achievements stem from both knowledge teaching and encouragement. A student i 's score, s_i , depends on T and e_i , where T is the number of hours of knowledge taught in class and e_i is the number of hours of private encouragement Howard offers her. (i.e. if a student attends a total of 16 hours of Howard's classes and received 2 hours of encouragement, she is expected to score $4 \times 16 + 2 = 66$ points). Interacting with students comes with a cost, manifested as varying degrees of fatigue. Suppose teaching costs w_T per hour and encouragement costs w_e .

5.1 A good start: cost minimization (10')

Suppose Howard only has one student, whose score will be $s = 4T + e$.

- a) If Howard doesn't encourage, how many hours of teaching must he devote if his goal is for the student to achieve a score of 80 points? (2')
- b) If $w_T > 4w_e$, would it be cheaper for Howard to use teaching or encouragement to raise one point of score? What if $w_T < 4w_e$? (2')
- c) Write down his cost function $c(w_T, w_e; s)$. (3')
- d) Write down his conditional factor demand for encouragement $e, e(w_T, w_e; s)$. (3')

5.2 Making a name: profit maximization with a public good (20')

Hypothesis modification. After a period of exploration, Howard discovers that, for himself, $w_T = 2$ and $w_e = 1$. He also finds that a student's score follows $s_i = 2\sqrt{T} + \sqrt{e_i}$. He decides to improve student grade regardless of fatigue.

- e) Suppose Howard still only has one student. Howard has a total of h hours available to allocate for the student. Calculate the optimal allocation of time between instruction and encouragement to maximize the student's performance. (5')
- f) The exceptional results as a teacher have garnered Howard a certain level of reputation, leading to an increase in the number of students. Now there are n students in Howard's class, with the same score function: $s_i = 2\sqrt{T} + \sqrt{e_i}$, for $i \in \{1, 2, \dots, n\}$. Howard has a total of h hours available to allocate for the students and aims at maximizing $\sum_{i=1}^n s_i$. Calculate the optimal allocation of time among (T, e_1, \dots, e_n) . (10')

*(Hint1: Efforts on teaching contributes simultaneously to **ALL** students.)*

(Hint2: If you find it challenging, you can start with the scenario where $n = 2$.)

- g) What discoveries have you made? Try relating them to real-life situation. (5')

5.3 Distinction: asset allocation across sectors. (10')

Building upon the framework discussed in the previous Section 5.2, we introduce the heterogeneity among students. For student $i \in \{1, 2, \dots, n\}$, his score function is $s_i = 2\sqrt{T} + A_i\sqrt{e_i}$, where $A_1 > A_2 > \dots > A_n > 0$. This can be interpreted as stronger students, under the influence of encouragement, being able to ignite stronger motivation for learning, resulting in higher scores.

h) Assuming Howard has prior knowledge of each student's type (i.e., A_i), briefly discuss how the differences among students would affect his decision-making. What if his compensation is determined by the top-performing students in the class? And what if it is determined by the lowest score in class? Only **qualitative** discussion is required. (10')

i) (*Optional*) In reality, it is difficult for teachers to have precise knowledge of students' characteristics in advance. How can this impact teachers' strategies? What methods can teachers utilize to measure student characteristics? (0')

6 Monopoly (20')

A monopolist produces a good using only one factor, labor. There are constant returns to scale in production, and the demand for the monopolist's product is described by a downward-sloping straight line with slope -1. The monopolist faces a horizontal labor supply curve. The monopolist chooses output to maximize profits. Please write down:

a) Marginal cost of producing one more unit of the good. (5')

b) Marginal revenue of producing one more unit of the good. (5')

c) The optimal quantity and price set by the monopolist. (5')

And discuss:

d) The relationship between wage and marginal product of labor times price of output. (5')

You may use notations of your own choice for the parameters in the model, but please be clear about the meaning of the notations you use.