Tutorial: Assignment 4

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Question 1

Q1

Suppose that Howard's short-run total cost of repairing s cars per week is $c(s) = 2s^2 + 18$, If the price he receives for repairing a car is 8, then in the short run, how many cars will he fix per week if he maximize profits?

Short term profit $\Pi(s) = 8s - (2s^2 + 18)$.
Solving for $\max_s \Pi(s)$ we derive to the F.O.C.: $\frac{\mathrm{d}}{\mathrm{d}s}\Pi(s) = 8 - 4s = 0$.
$s^* = 2$.

Q2

Q2.a

Consider a firm's total cost function: $c(y) = y^2 + 10$ for $y > 0$ and $c(0) = 0$.
a) Write down its MC function, AVC function and ATC function.

$MC = 2y$; $AVC = y$; $ATC = y + \frac{10}{y}$

Q2.b

In a competitive market, what is the lowest price at which he will supply a positive quantity in long-run equilibrium?

Lowest ATC, $2\sqrt{10}$

Q3

Suppose that the cost of capturing a cockatoo and transporting him to the United States is about \$40 per bird. Each smuggled cockatoo has a 10% probability of being discovered, in which case the smuggler is fined. If the fine imposed for each smuggled cockatoo is \$1,000. Cockatoos are drugged and smuggled in suitcases to the United States. Half of the smuggled cockatoos die in transit. Suppose the market is competitive and involves a large number of smugglers, who only care about their expected profit, then the equilibrium price of cockatoos in the United States will be?

Consider carrying 20 smuggled cockatoos initially.

$20 \xrightarrow{\textit{Caught}} 18 \xrightarrow{\textit{Dead}} 9$; Cost: $20 \times 40 + 1000 \times 2 = 2800$.

Equilibrium price: $\frac{2800}{9} = 311$

Q4

Q4

Consider a competitive industry with several firms all of which have the same cost function $c(y) = y^2 + 4$ for $y > 0$ and $c(0) = 0$. The demand curve for this industry is $D(p) = 50 - p$, where p is the price. Calculate the long run equilibrium number of firms in this industry.

When n firms, facing the price p each firm produces y, $MC = 2y = p$.
Industry supply curve: $S(p) = \frac{n}{2}p$.
In equilibrium, $D = S$ and $p^* = \frac{100}{n+2}$.
New firms will not enter if and only if $\min ATC = y + \frac{4}{y} \geq p^*$.
$\min ATC = 4$, and $n = 23$.

Q5

Environment

A student i's score, s_i, depends on T (hours of knowledge) and e_i (hours of private encouragement).
Teaching costs w_T per hour and encouragement costs w_e.

Q5.1

One student; $s = 4T + e$.

- If $w_T > 4w_e$, encouragement is cheaper. Else is teaching.
- $c(w_T, w_e; s) = \min\{\frac{w_T}{4}, w_e\}s$
- $e(w_T, w_e; s) = s$ if $w_T > 4w_e$ and 0 if $w_T \leq 4w_e$

Q5 Cont'd

Environment

- $w_T = 2$ and $w_e = 1$ (redundant)
- $s_i = 2\sqrt{T} + \sqrt{e_i}$
- Regardless of fatigue

Q5.2 e

One student. Calculate the optimal allocation of time h to maximize the student's performance.

Lagrange's theorem: $L = 2\sqrt{T} + \sqrt{e} - \lambda(T + e - h)$
F.O.C. $\frac{\partial}{\partial T}L = \frac{1}{\sqrt{T}} - \lambda = 0$, $\frac{\partial}{\partial e}L = \frac{1}{2\sqrt{e}} - \lambda = 0$
$T^* = \frac{1}{\lambda^2}$, $e^* = \frac{1}{4\lambda^2}$, so $T^* = 4e^* = \frac{4}{5}h$ and $e^* = \frac{1}{5}h$

Q5 Cont'd

Environment

- $s_i = 2\sqrt{T} + \sqrt{e_i}$
- $T + \sum_i e_i \leq h$

Q5.2 f

n students. Howard has h hours to allocate for maximizing $\Sigma_{i=1}^n s_i$. Calculate the optimal allocation of time among $(T, e_1, ..., e_n)$.

Problem: max $S = (2n)\sqrt{T} + \Sigma_{i=1}^n \sqrt{e_i}$ s.t. $T + \Sigma_{i=1}^n e_i \leq h$
Again, $L = (2n)\sqrt{T} + \Sigma_{i=1}^n \sqrt{e_i} - \lambda(T + \Sigma_{i=1}^n e_i - h)$
F.O.C. $\frac{\partial}{\partial T} L = 0 = \frac{n}{\sqrt{T}} - \lambda$, $\frac{\partial}{\partial e_i} L = 0 = \frac{1}{2\sqrt{e_i}} - \lambda$
Then we derive that $T^* = \frac{n^2}{\lambda^2}$ and $e_i^* = \frac{1}{4\lambda^2}$ for $i = 1, 2, ..., n$.
$T^* = 4n^2 e^* = \frac{4n^2}{4n^2+n} h = \frac{4n}{4n+1} h$, while $e_1 = ... = e_n = \frac{1}{4n^2+n} h$

Q5 Cont'd

Qualitative Discussion

- How encouragement changes when n increases?
- What if students are heterogeneous?
- How incentive designs distorts behavior?
- How teachers overcome information asymmetry?

- While larger classes may enhance the efficiency of education per unit, they can sometimes lead to a decrease in the quality of individualized teaching.
- If teachers have the ability to observe the individual characteristics of students, they can provide tailored levels of encouragement to each student based on their specific needs.
- Motivating teachers to attend to students with lower learning abilities could be achieved by assigning greater weight to those students' performance appraisals, especially if they have lower scores.
- Diagnostic tests?

Q6

Q6 a-c

A monopolist produces a good using only one factor, labor. There are constant returns to scale in production, and the demand for the monopolist's product is described by a downward-sloping straight line with slope -1. The monopolist faces a horizontal labor supply curve. The monopolist chooses output to maximize profits.

Settings: wage w, labor input l, quantity of output q, price p.
Production function: $q = kl$; Reverse demand function: $p = P - q$
a) Marginal cost of one more unit of good $= \frac{w}{k}$
b) Marginal revenue of one more unit $= \frac{\mathbf{d}}{\mathbf{d}q}[q(P-q)] = P - 2q$
c) Optimal quantity and price: solve q^* from MC = MR.
Or we maximize profit $= pq - wl = kl(P - kl) - wl$, $l^* = \frac{kP-w}{2k^2}$. So:
$q^* = \frac{kP-w}{2k}$; $p^* = P - \frac{kP-w}{2k} = \frac{kP+w}{2k}$

Q6 Cont'd

Q6 d

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- Discuss the relationship between wage and marginal product of labor times price of output.

Marginal product of labor is k, $kp^* = \frac{kP+w}{2}$.
Recalling $q^* = kl^* = \frac{kP-w}{2k} > 0$, we know $kp|_{l=l^*} > w$.

- Try exploring by yourselves and identifying the assumptions that can be released as the relationship holds!

The End