Homework V

January 9, 2024

Name **Student's ID**

I hope everyone enjoys this assignment and learns something from it. The questions may be more flexible than what you typically encounter in textbooks, designed to exercise your ability to understand the model in a more flexible manner. Don't worry too much about grades of this assignment. Please send emails to *lijch@sem.tsinghua.edu.cn* if you find any errors in the questions.

Oligopoly model with complementary goods (20 points)

Consider a commodity J that has 2 essential components, each of which is produced by a separate firm. We assume that the component 1 is produced by firm 1 and component 2 is produced by firm 2. Both firms has the same marginal cost $\frac{C}{2}$ for producing its component, and hence the total production cost of the commodity (with the mentioned 2 parts) is C. In the market consumers value the good as a bundle of perfect complementary goods. Denote the prices of the two components as p_1 and p_2 respectively, and the total price of commodity J is p_J ($p_J = p_1 + p_2$). The market demand function hence is $Q(p_J) = a - b \times p_j$ (We assume $a > bC$ for simplicity consideration). The following subproblems are designed you to discover the market equilibrium price p_J step by step.

(a) Write the profit function of each firm. (4 points)

(b) What are the best pricing response of each firm? (4 points)

(c) Finding the pricing strategy in equilibrium of the 2 firms by combining your solutions in (b). And write out the price p_J of the commodity J. (4 points)

(d) Suppose commodity J consists of n identical complementary components produced by n different firms. Each firm has same constant marginal production cost $\frac{c}{n}$. What is the final price p_J of commodity J? (4 points)

(e) Notably, in our lecture slides, we learned that the product's price in the n firms Cournot model with substitues is $n(p - c) = -\frac{Q(p)}{Q'(p)}$, implying that market competition leads to lower price. However, in this problem with complementary goods, what's your observation? (4 points)

Exchange economy (20 points)

Consider a standard Edgeworth box economy with two consumers Miss.Hu and Mr.Jin, where Miss.Hu's utility function is given by $U_H(a_H, b_H) = a_H - (8 - b_H)^2$ and Mr.Jin's utility function is given by $U_J(a_J, b_J) = a_J - (8 - b_J)^2$. Notation a and b represent the consumption of apples and bananas for Miss.Hu and Mr.Jin. What's more Miss.Hu has an endowment of 58 apples and 5 bananas, while Mr.Jin has 154 apples and 3 bananas.

(a) Find ALL Pareto optimal points and show them in the Edgeworth box. (Hint: try to figure out how indifference curves behave at the Pareto optimal points) (6 points)

(b) Find the core and show it in the Edgeworth box. (Hint: Core is related to the initial endowments, with no agent blocking the allocation.)(6 points)

(c) Find the competitive equilibrium. (Hint: You need to specify the value of a_w, b_w, a_J, b_J, and the relative price of apples to bananas P_a/P_b in the equilibrium). (8 points)

Externality (20 points)

An airport is located next to a large tract of land owned by a housing developer. The developer would like to build houses on this land, but noise from the airport reduces the value of the land. The more planes that fly, the lower is the amount of profit that the developer makes. Let X be the number of planes that fly per day and let Y be the number of houses that the developer builds. The airport's total profit is $48X - X^2$ and the developer's total profit is $60Y - Y^2 - XY$. Let us consider the outcome under various assumptions about institutional rules and about bargaining between the airport and the developer.

"Free to Choose with No Bargaining"

(a) Suppose that no bargains can be struck between the airport and the developer and that each can decide on its own level of activity. No matter how many houses the developer builds, what is the number of planes per day that maximizes profit for the airport? Write out the profit of the airport. (6 points)

(b) Given that the airport is landing this number of planes, what is the number of houses that maximizes the developer's profit? Write our the profit of the developer. (6 points)

"Lawyer's Paradise"

(c) Suppose that a law is passed that makes the airport liable for all damages to the developer's property values. Since the developer's profit is $60Y - Y^2 - XY$ and his profit would be $60Y - Y^2$ if no planes were flown, the total amount of damages awarded to the developer will be XY. Therefore if the airport flies X planes and the developer builds Y houses, then the airport's profit after it has paid damages will be $48X - X^2 - XY$. The developer's profit including the amount he receives in payment of damages will be $60Y - Y^2 - XY + XY = 60Y - Y^2$. Then how many houses will be built by the developer? How many planes will be flown by the airport? How many profit will they make? (8 points)

Fair and exchange economy (20 points)

Roger and Gordon have identical utility functions, $U(x, y) = x^2 + y^2$. There are 10 units of x and 10 units of y to be divided between them.

(a) Draw an Edgeworth box showing some of their indifference curves and mark the Pareto optimal allocations with bold black ink. (Hint: Notice that the indifference curves are nonconvex.) (10 points)

(b) What are the fair allocations in this case? (10 points)

Price Competition With Different Qualities (20 points)

Suppose the consumer's utility is given by $\theta s - p$, where θ is the consumer's type, s is the quality of the product, which the consumer can choose to buy or not buy, and p is the price of that product. θ is assumed to be uniformly distributed between $[\underline{\theta}, \overline{\theta}]$, and $\overline{\theta} = \underline{\theta} + 1$. Consider the case where there are two firms, indexed by $i = 1, 2$, each of which provides a product of quality s_i. Without loss of generality, let's assume $s_2 \geq s_1$. The marginal cost is c for both firms. Denote $\Delta s \equiv s_2 - s_1$.

We assume $\underline{\theta} = \frac{1}{2}$, $\overline{\theta} = \frac{3}{2}$, $s_1 = 1$, $s_2 = 2$, $c = \frac{1}{3}$ throughout questions below.

(a) Derive the demand for each firm, i.e., $q_i(p_1, p_2), i = 1, 2$. (4 points)

(b) Write the profit function for firm i. (4 points)

(c) Derive each firm's best-response function, that is, its optimal price, taking as given the (arbitrary) price of the competing firm. (4 points)

(d) Solve for the Nash equilibrium profile (price, demand, profit) of each firm. (4 points)

(e) Interpret these results and their relationship to Δs. (4 points)