

Problem Set 3

Intermediate Microeconomics

Date: November 24, 2023

1. Suppose that the inverse demand curve is given by $P(q) = 150 - 30q$. The consumer's utility function is quasi-linear and he is purchasing the 3 units at a price of \$60 per unit. If you require him to reduce his purchases to zero, how much money would be necessary to compensate him? (Hint: Compute *net* surplus.)
2. Suppose that a consumer has a utility function $u(x_1, x_2) = x_1 + x_2$. Initially the consumer faces prices $(1, 2)$ and has income 10. If the prices change to $(4, 2)$, calculate the compensating and equivalent variations.
3. Wilson consumes bread, and his demand function for bread is given by $D(p) = 100 - p$, where p is the price of bread.
 - (a) If the price of bread is \$50, how much bread will he consume?
 - (b) How much gross consumer's surplus does he get from this consumption?
 - (c) How much money does he spend on bread?
 - (d) What is his net consumer's surplus from bread consumption?
4. For each demand function, find an expression for the price elasticity of demand. The answer will typically be a function of the price, p .
 - (a) $D(p) = 40 - p$
 - (b) $D(p) = 20p^{-3}$
 - (c) $D(p) = (p + 4)^{-2}$
5. Consider a monopoly firm. If the demand function is $P = 10 - Q$.
 - (a) Write down the inverse demand function.
 - (b) Write expressions for the firm's total revenue, average revenue and marginal revenue as a function of the number of goods sold.
 - (c) At what price will total revenue realized from its sale be at a maximum?

- (d) How many goods will be sold at that price? At this quantity, what is the price elasticity of demand?
6. The demand curve for bread is given by $D(p_D) = 100 - 2p_D$ and the supply curve is given by $S(p_S) = 3p_S$
- What is the equilibrium price and quantity?
 - A tax of \$10 per loaf of bread is imposed on consumers. Write an equation that relates the price paid by demanders to the price received by suppliers. Write an equation that states that supply equals demand (market clearing condition).
 - Solve above two equations for the two unknowns p_S and p_D . With the \$10 tax, what is the equilibrium price p_D paid by consumers? What is the total amount of bread?
7. The demand curve for bread is given by $D(p_D) = 200 - 5p_D$ and the supply curve is given by $S(p_S) = 5p_S$
- What is the equilibrium price and quantity?
 -) A quantity subsidy of \$2 **per loaf sold** is placed on bread. What is the new equilibrium price paid by the demanders? What is the new price received by the suppliers? What is the new equilibrium quantity sold?
8. Suppose that the production function is $f(x_1, x_2) = Cx_1^a x_2^b$, where a , b , and C are **positive** constants.
- For what positive values of a , b , and C are there decreasing returns to scale? Constant returns to scale? Increasing returns to scale?
 - For what positive values of a , b , and C is there decreasing marginal product for x_1 ?
 - For what positive values of a , b , and C is there diminishing technical rate of substitution?
9. A firm uses a single input to produce a commodity according to a production function $f(x) = 4\sqrt{x}$, where x is the number of units of input. The commodity sells for \$100 per unit. The input costs \$50 per unit.
- Write down a function that states the firm's profit as a function of the amount of input.
 - What is the profit-maximizing amount of input? How much profits does it make when it maximizes profit?
 - Suppose that the commodity sells for \$80 per unit and the input costs \$40 per unit. What is its new input level? What is its new output level? How much profit does it make now?
 - Suppose that instead of these taxes and subsidies, the firm is taxed at 50% of its profits. Write down its after-tax profits as a function of the amount of input. What is the profit-maximizing amount of output? How much profit does it make after taxes?

10. A firm uses two kinds of inputs to produce a commodity according to a production function $f(x_1, x_2) = x_1^{1/2} x_2^{1/4}$, where x_1 and x_2 are the number of units of input. The commodity sells for \$4 per unit. The input x_1 costs w_1 per unit and the input x_2 costs w_2 per unit.
- (a) Write equations which say that the value of the marginal product of inputs is equal to the prices of inputs.
 - (b) Solve above equations in the two unknowns, x_1 and x_2 , to give the amounts of inputs 1 and 2 that maximize the firm's profits as a function of w_1 and w_2 .