Solution for Problem Set 3

Intermediate Microeconomics

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1. Suppose that the inverse demand curve is given by $P(q) = 150 - 30q$. The consumer's utility function is quasi-linear and he is purchasing the 3 units at a price of \$60 per unit. If you require him to reduce his purchases to zero, how much money would be necessary to compensate him? (Hint: Compute *net* surplus.)

   $3 \times (60 + 150)/2 - 3 \times 60 = 315 - 180 = 135.$

2. Suppose that a consumer has a utility function $u(x_1, x_2) = x_1 + x_2$. Initially the consumer faces prices $(1, 2)$ and has income 10. If the prices change to $(4, 2)$, calculate the compensating and equivalent variations.

   Compensating variation is the amount of extra income that the consumer would need at the new prices to be as well off as she was facing the old prices. **Equivalent variation** is the amount of money that it would be necessary to take away from the consumer at the old prices to make her as well off as she would be, facing the new prices. Although different in general, the change in consumer's surplus and the compensating and equivalent variations will be the same if preferences are **quasi-linear**.

   Since the two goods are perfect substitutes, the consumers will initially consume the bundle $(10, 0)$ and get utility 10. After the prices change, she will consume the bundle $(0, 5)$ and get utility 5. **After the price change** she would need \$20 to get a utility of 10; therefore the compensating variation is $20 - 10 = 10$. **Before the price change**, she would need an income of 5 to get a utility of 5. Therefore the equivalent variation is $10 - 5 = 5$.

3. Wilson consumes bread, and his demand function for bread is given by $D(p) = 100 - p$, where p is the price of bread.

   (a) If the price of bread is \$50, how much bread will he consume?

   $D = 100 - 50 = 50.$

   (b) How much gross consumer's surplus does he get from this consumption?

$TS = (50 + 100) \times 50/2 = 3750$ or $= \int_0^{50} (100 - p)dp = 3750$

(c) How much money does he spend on bread?

$C = 50 \times 50 = 2500$

(d) What is his net consumer's surplus from bread consumption?

$NS = TS - C = 1250$

4. For each demand function, find an expression for the price elasticity of demand. The answer will typically be a function of the price, p.

The price elasticity of demand is defined to be the percent change in quantity divided by the percent change in price.

(a) $D(p) = 40 - p$

$\varepsilon = \frac{dD}{dp}\frac{p}{D} = -\frac{p}{40-p}$

(b) $D(p) = 20p^{-3}$

$\varepsilon = \frac{d \ln D}{d \ln p} = -3$

(c) $D(p) = (p + 4)^{-2}$

$\varepsilon = \frac{dD}{dp}\frac{p}{D} = -2\frac{(p+4)^{-3}p}{(p+4)^{-2}} = -2\frac{p}{p+4}$

5. Consider a monopoly firm. If the demand function is $D(P) = 10 - P$.

(a) Write down the inverse demand function.

Answer: $P = 10 - Q$.

(b) Write expressions for the firm's total revenue, average revenue and marginal revenue as a function of the number of goods sold.

Answer: $TR = PQ = (10 - Q)Q$. $AR = TR/Q = 10 - Q$. $MR = dTR/dQ = 10 - 2Q$.

(c) At what price will total revenue realized from its sale be at a maximum?

Answer: We need $MR = 0$ with $dMR/dQ < 0$. $MR = 0 \Rightarrow Q = 5$ and $P = 5$.

(d) How many goods will be sold at that price? At this quantity, what is the price elasticity of demand?

Answer: $Q = 5$. $\varepsilon = \frac{dD}{dP}\frac{P}{D} = -1 \times \frac{5}{5} = -1$

6. The demand curve for bread is given by $D(p_D) = 100 - 2p_D$ and the supply curve is given by $S(p_S) = 3p_S$

(a) What is the equilibrium price and quantity?

$100 - 2p = 3p \Rightarrow p = 20$ and $q = 60$

(b) A tax of \$10 per loaf of bread is imposed on consumers. Write an equation that relates the price paid by demanders to the price received by suppliers. Write an equation that states that supply equals demand (market clearing condition).

$p_D - 10 = p_S$ and $100 - 2p_D = 3p_S$

(c) Solve above two equations for the two unknowns p_S and p_D. With the \$10 tax, what is the equilibrium price p_D paid by consumers? What is the total amount of bread?

$100 = 5p_D - 30 \Rightarrow p_D = 26$, $p_S = 16$, $q = 100 - 2p_D = 3p_S = 48$

7. The demand curve for bread is given by $D(p_D) = 200 - 5p_D$ and the supply curve is given by $S(p_S) = 5p_S$

(a) What is the equilibrium price and quantity?

$200 - 5p = 5p \Rightarrow p = 20$ and $q = 100$

(b)) A quantity subsidy of \$2 **per loaf sold** is placed on bread. What is the new equilibrium price paid by the demanders? What is the new price received by the suppliers? What is the new equilibrium quantity sold?

The equilibrium can be determined by $p_D + 2 = p_S$ and $200 - 5p_D = 5p_S$. Then $p_D = 19$, $p_S = 21$ and $q = 5p_S = 105$.

8. Suppose that the production function is $f(x_1, x_2) = Cx_1^a x_2^b$, where a, b, and C are **positive** constants.

(a) For what positive values of a, b, and C are there decreasing returns to scale? Constant returns to scale? Increasing returns to scale?

Decreasing returns to scale: $f(tx_1, tx_2) < tf(x_1, x_2)$ for all $t > 1$. Check the definition: $C(tx_1)^a(tx_2)^b < tCx_1^a x_2^b \Rightarrow a + b < 1$ and $a, b, C > 0$

(b) For what positive values of a, b, and C is there decreasing marginal product for x_1?

Decreasing marginal product: Marginal product of a factor will diminish as we get more and more of that factor. Check the definition: $MP_1 = Cax_1^{a-1}x_2^b$. $dMP_1/dx_1 = Ca(a-1)x_1^{a-2}x_2^b < 0 \Rightarrow a < 1$and $a, b, C > 0$

(c) For what positive values of a, b, and C is there diminishing technical rate of substitution?

Roughly speaking, the assumption of diminishing TRS means that the slope of an isoquant must decrease in absolute value as we move along the isoquant in the direction of increasing x_1, and it must increase as we move in the direction of increasing x_2. This means that the isoquants will have the same sort of convex shape that well-behaved indifference curves have. Chech the definition: $TRS(x_1, x_2) = \frac{dx_2}{dx_1} = -\frac{MP1}{MP2} = -\frac{ax_2}{bx_1} \Rightarrow a, b, C > 0$

9. A firm uses a single input to produce a commodity according to a production function $f(x) = 4\sqrt{x}$, where x is the number of units of input. The commodity sells for \$100 per unit. The input costs \$50 per unit.

(a) Write down a function that states the firm's profit as a function of the amount of input.

$\pi = pf(x) - 50x = 400\sqrt{x} - 50x$

(b) What is the profit-maximizing amount of input? How much profits does it make when it maximizes profit?

$d\pi/dx = 0 \Rightarrow 200/\sqrt{x} - 50 = 0 \Rightarrow x^* = 16$, $\pi^* = 400 \times 4 - 50 \times 16 = 800$

(c) Suppose that the commodity sells for \$80 per unit and the input costs \$40 per unit. What is its new input level? What is its new output level? How much profit does it make now?

$\pi = 320\sqrt{x} - 40x$, $d\pi/dx = 0 \Rightarrow 160/\sqrt{x} - 40 = 0 \Rightarrow x^* = 16$, $\pi^* = 320 \times 4 - 40 \times 16 = 640$

(d) Suppose that instead of these taxes and subsidies, the firm is taxed at 50% of its profits. Write down its after-tax profits as a function of the amount of input. What is the profit-maximizing amount of output? How much profit does it make after taxes?

$\pi' = 0.5\pi$, x^* is unchanged and $\pi'^* = 0.5\pi^*$.

10. A firm uses two kinds of inputs to produce a commodity according to a production function $f(x_1, x_2) = x_1^{1/2} x_2^{1/4}$, where x_1 and x_2 are the number of units of input. The commodity sells for \$4 per unit. The input x_1 costs w_1 per unit and the input x_2 costs w_2 per unit.

(a) Write equations which say that the value of the marginal product of inputs is equal to the prices of inputs.

$pMP_i = \omega_i$, $i \in \{1, 2\}$. Or the first-order conditions of $\pi = 4x_1^{1/2} x_2^{1/4} - \omega_1 x_1 - \omega_2 x_2$, which are $2x_1^{-1/2} x_2^{1/4} = \omega_1$ and $x_1^{1/2} x_2^{-3/4} = \omega_2$.

(b) Solve above equations in the two unknowns, x_1 and x_2, to give the amounts of inputs 1 and 2 that maximize the firm's profits as a function of w_1 and w_2.

$x_1 = \frac{8}{\omega_1^3 \omega_2}$ and $x_2 = \frac{4}{\omega_1^2 \omega_2^2}$