

4.

$$(1) (\forall x)(P(x) \vee Q(x))$$

$$= (P(1) \vee Q(1)) \wedge (P(2) \vee Q(2))$$

$$= \top$$

$$(2) (\forall x)(P \rightarrow Q(x)) \vee R(a)$$

$$= (\forall x)(Q(x)) \vee R(3)$$

$$= (\forall x)(Q(x))$$

$$= Q(-2) \wedge Q(1) \wedge Q(2) \wedge Q(3) \wedge Q(5) \wedge Q(6)$$

$$= \top$$

$$(3) (\exists x)(P(x) \rightarrow Q(x))$$

$$= (P(0) \rightarrow Q(0)) \vee (P(1) \rightarrow Q(1)) \vee (P(2) \rightarrow Q(2))$$

$$= \top$$

5.

$$(7) P(x): x \text{ 是在北京工作的人}$$

$$Q(x): x \text{ 是北京人}$$

$$(\exists x)(P(x) \wedge \neg Q(x))$$

$$(9) P(x): x \text{ 是金属}$$

$$Q(x, y): x \text{ 可溶解在 } y \text{ 中}$$

$$S(x): x \text{ 是液体}$$

$$(\forall x)(P(x) \rightarrow (\exists y)(S(y) \wedge Q(x, y)))$$

6.

(4) 任一实数不是有理数就是无理数

(6) 任一正整数不是偶数就是奇数

(7) 没有同时是正整数, 偶数, 奇数的数

(9) 任一正整数都是有理数并且不是任一有理数都是正整数

7.

$$(3) (\forall x)P(x) \wedge (\exists x)Q(x)$$

$$= P(a) \wedge P(b) \wedge P(c) \wedge (Q(a) \vee Q(b) \vee Q(c))$$

$$(5) (\forall x) \neg P(x) \vee (\forall x)P(x)$$

$$= (\neg P(a) \wedge \neg P(b) \wedge \neg P(c)) \vee (P(a) \wedge P(b) \wedge P(c))$$

$$(6) (\exists x)(\forall y)P(x, y)$$

$$= (\forall y)P(a, y) \vee (\forall y)P(b, y) \vee (\forall y)P(c, y)$$

$$= (P(a, a) \wedge P(a, b) \wedge P(a, c))$$

$$\vee (P(b, a) \wedge P(b, b) \wedge P(b, c))$$

$$\vee (P(c, a) \wedge P(c, b) \wedge P(c, c))$$

$$(7) (\forall x)(\exists y)(P(x, y) \rightarrow Q(x, y))$$

$$= (\exists y)(P(a, y) \rightarrow Q(a, y)) \wedge$$

$$(\exists y)(P(b, y) \rightarrow Q(b, y)) \wedge$$

$$(\exists y)(P(c, y) \rightarrow Q(c, y))$$

$$= ((P(a, a) \rightarrow Q(a, a)) \vee (P(a, b) \rightarrow Q(a, b)) \vee (P(a, c) \rightarrow Q(a, c))) \wedge$$

$$((P(b, a) \rightarrow Q(b, a)) \vee (P(b, b) \rightarrow Q(b, b)) \vee (P(b, c) \rightarrow Q(b, c))) \wedge$$

$$((P(c, a) \rightarrow Q(c, a)) \vee (P(c, b) \rightarrow Q(c, b)) \vee (P(c, c) \rightarrow Q(c, c)))$$

$$(9) (\exists x)(\exists y)P(x, y)$$

$$= (\exists y)P(a, y) \vee (\exists y)P(b, y) \vee (\exists y)P(c, y)$$

$$= P(a, a) \vee P(a, b) \vee P(a, c) \vee P(b, a) \vee P(b, b) \vee P(b, c) \vee P(c, a) \vee P(c, b) \vee P(c, c)$$

8.

(2) 普遍有效的

(3) 可满足的

(6) 普遍有效的

(7) 可满足的

9.

$$P(x): x=2$$

$$(\exists x)P(x)$$

10.

$$(7) (\forall x)(\forall y)(P(x,y) \rightarrow P(y,x))$$

$$= (\forall y)(P(a,y) \rightarrow P(y,a)) \wedge$$

$$(\forall y)(P(b,y) \rightarrow P(y,b))$$

$$= (P(a,a) \rightarrow P(a,a)) \wedge (P(a,b) \rightarrow P(b,a))$$

$$\wedge (P(b,a) \rightarrow P(a,b)) \wedge (P(b,b) \rightarrow P(b,b))$$

$$= \top$$

$$(8) (\exists y) | \forall x) P(x,y)$$

$$= (\forall x) P(x,a) \vee (\forall x) P(x,b)$$

$$= (P(a,a) \wedge P(b,a)) \vee (P(a,b) \wedge P(b,b))$$

$$= \text{F}$$