These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Twenty

Profit-Maximization

You may skip 20.11

Profit

The **profit** generated by $(x_1,\ldots,x_m,y_1,\ldots,y_n)$ is

$$\Pi = p_1y_1 + \cdots + p_ny_n - w_1x_1 - \cdots w_mx_m.$$

Before Chapter 25, we maintain the assumption that the firm is competitive, and therefore **takes prices** $p_1,\ldots,p_n$ $w_1,\ldots,w_m$ **as given** constants;

Moreover, assume that the firm only has one output (n=1).

Short-run Profit

Suppose the firm is in a short-run circumstance in which $x_2 \equiv \tilde{x}_2$.

Its short-run production function is

$$y = f(x_1, \tilde{x}_2).$$

The firm's fixed cost is $FC = w_2\tilde{x}_2$ and its profit function is

$$\Pi = py - w_1x_1 - w_2\tilde{x}_2.$$

Short-Run Iso-Profit Lines

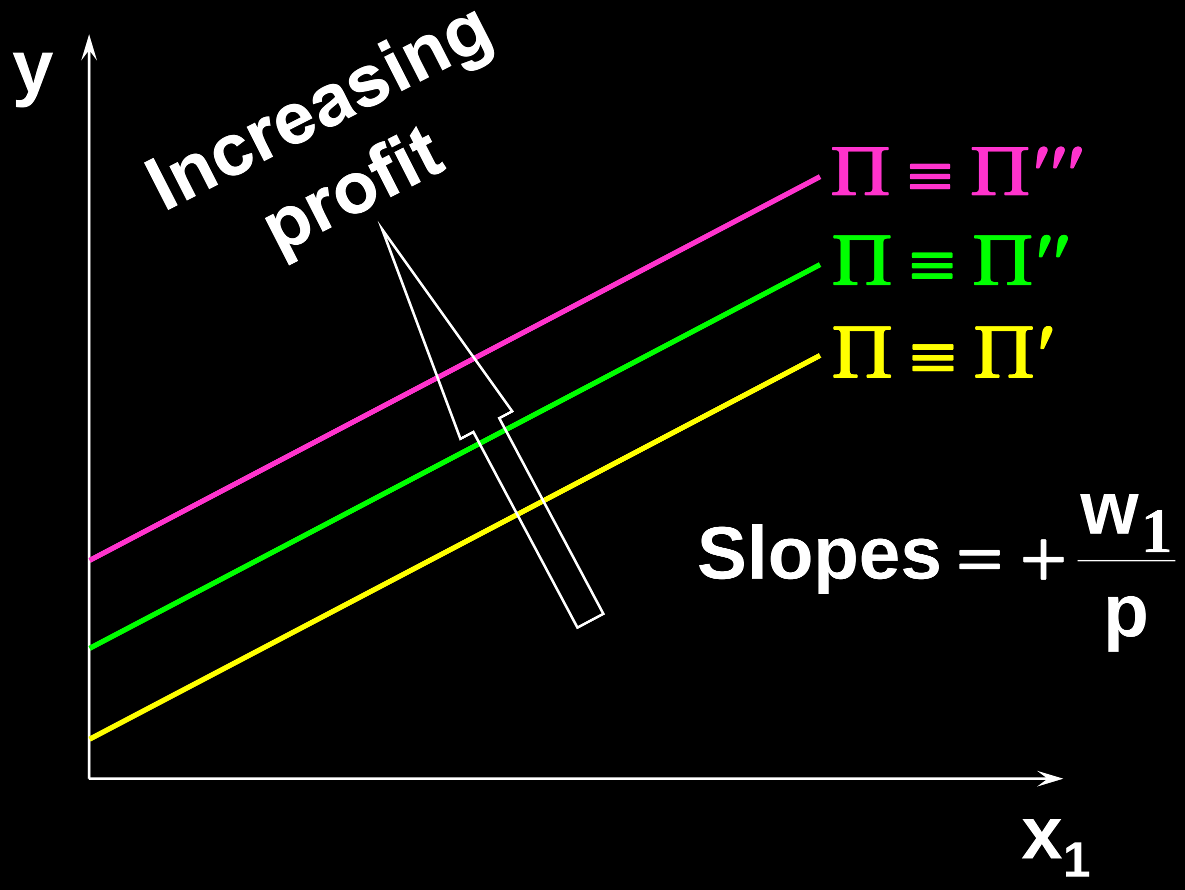
- An **iso-profit line** in the x_1-y plane contains all the production plans that yield the same profit level.
- The equation of an iso-profit line is

$$\Pi \equiv py - w_1x_1 - w_2\tilde{x}_2.$$

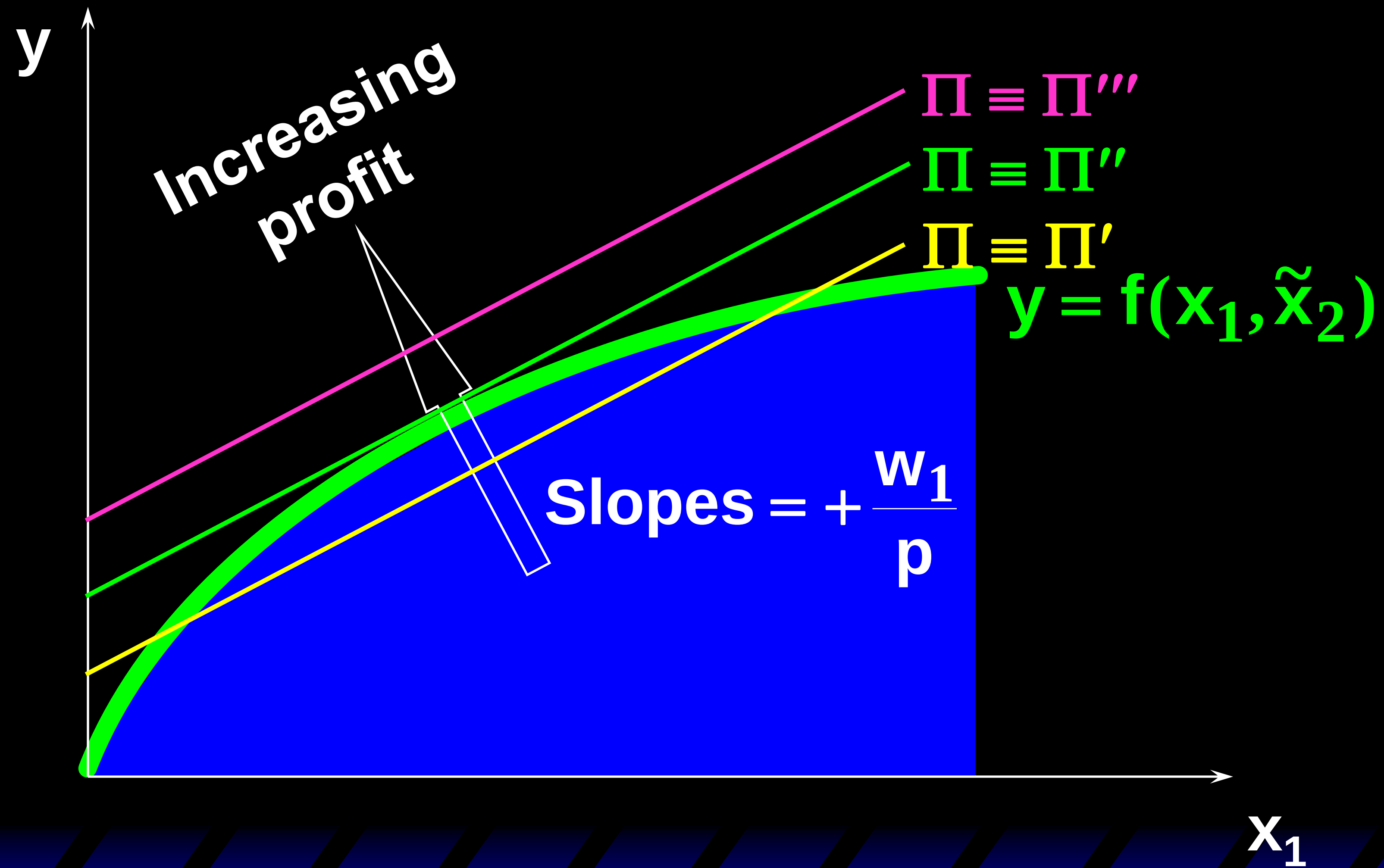
- I.e.

$$y = \frac{w_1}{p}x_1 + \frac{\Pi + w_2\tilde{x}_2}{p}.$$

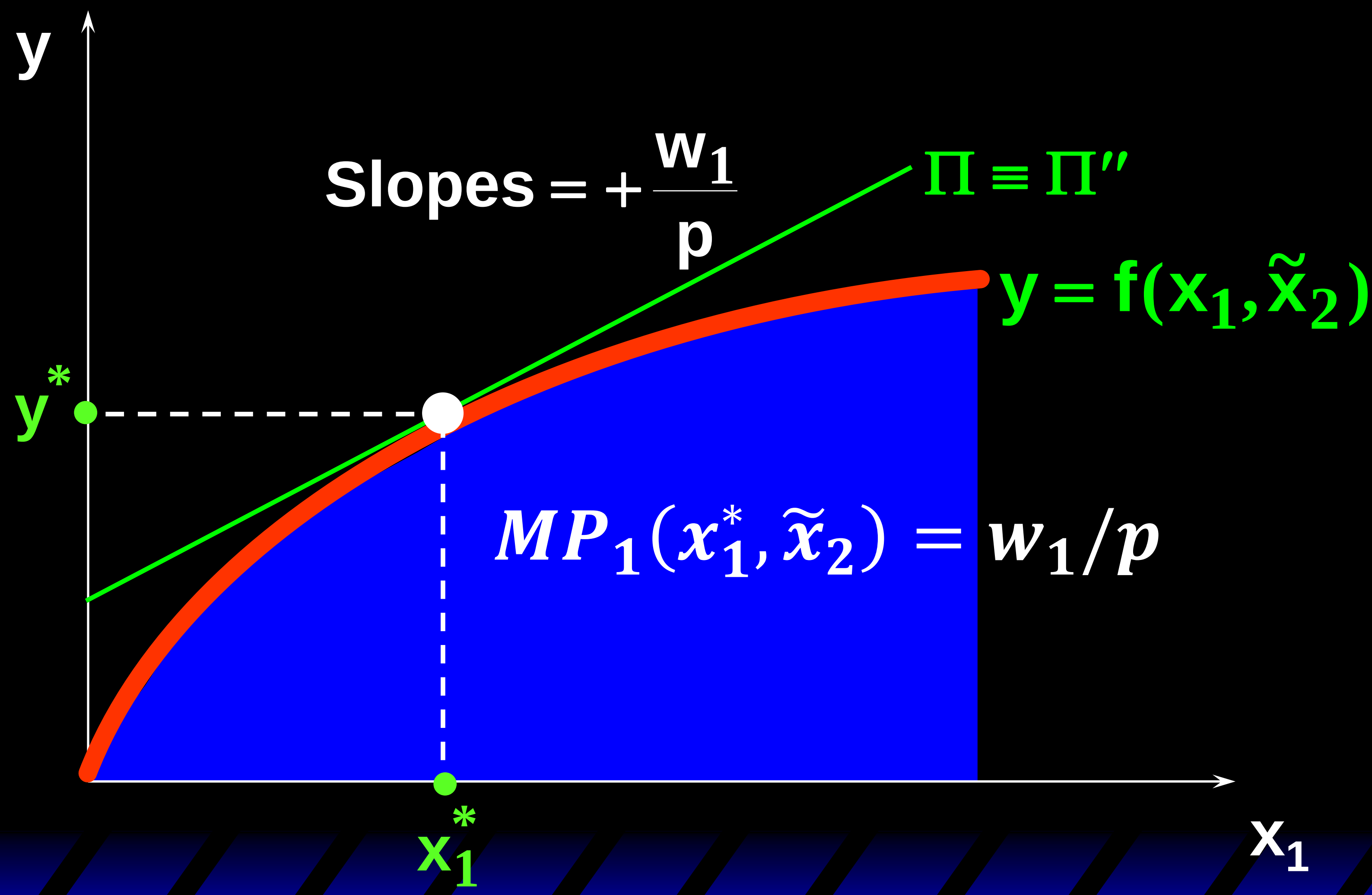
Short-Run Iso-Profit Lines



Short-Run Profit-Maximization



Short-Run Profit-Maximization



Short-Run Profit-Maximization

$$MP_1 = \frac{w_1}{p} \quad \Leftrightarrow \quad p \times MP_1 = w_1$$

$p \times MP_1$: **marginal revenue** of increasing x_1

w_1: **marginal cost** of increasing x_1

If $p \times MP_1 > w_1$ then profit increases with x_1.

If $p \times MP_1 < w_1$ then profit decreases with x_1.

Short-Run Profit-Maximization

Algebraically, the firm's short run problem is:

Maximize $\Pi = py - w_1 x_1 - w_2 \tilde{x}_2.$

Subject to $y = f(x_1, \tilde{x}_2).$

FOC: $pMP_1 = w_1$

FOC will give us the solution if f is a concave function.

Comparative Statics of Short-Run Profit-Maximization

What happens to the short-run profit-maximizing production plan as the output price p changes?

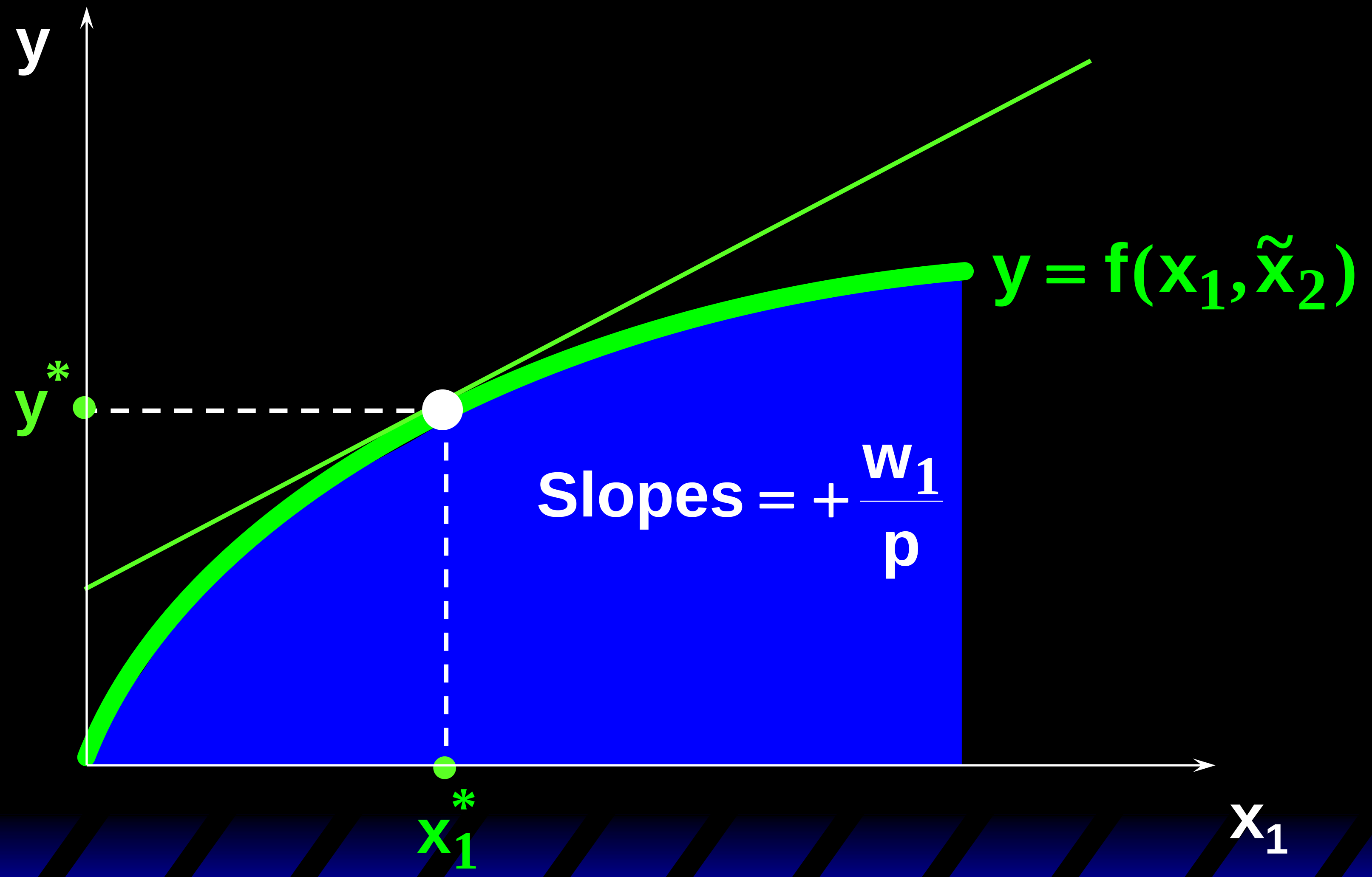
Comparative Statics of Short-Run Profit-Maximization

The equation of a short-run iso-profit line is

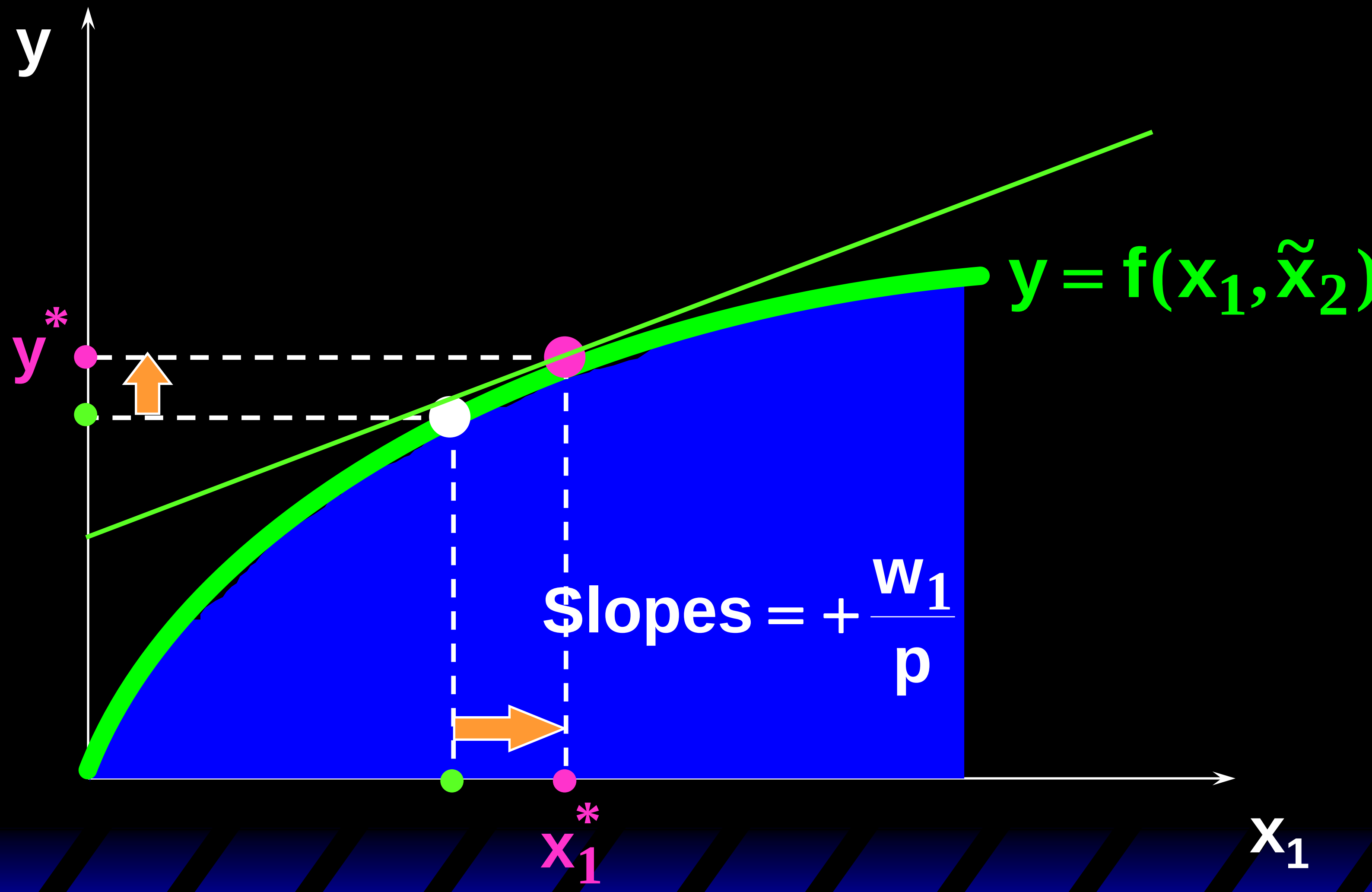
$$y = \frac{w_1}{p} x_1 + \frac{\Pi + w_2 \tilde{x}_2}{p}$$

so an increase in p causes a reduction in the slope of the family of iso-profit lines

Comparative Statics of Short-Run Profit-Maximization



Comparative Statics of Short-Run Profit-Maximization



Comparative Statics of Short-Run Profit-Maximization

An increase in p causes (weak) increases in

- **optimal input level;**
- **optimal output level;**
- **optimized profit.**

$$\begin{cases} py - w_1x_1 - w_2\tilde{x}_2 \geq py' - w_1x_1' - w_2\tilde{x}_2 \\ p'y' - w_1x_1' - w_2\tilde{x}_2 \geq p'y - w_1x_1 - w_2\tilde{x}_2 \end{cases} \Rightarrow \underbrace{(p' - p)}_{>0}(y' - y) \geq 0$$

$$\Rightarrow y' \geq y$$

Comparative Statics of Short-Run Profit-Maximization

What happens to the short-run profit-maximizing production plan as the variable input price w_1 changes?

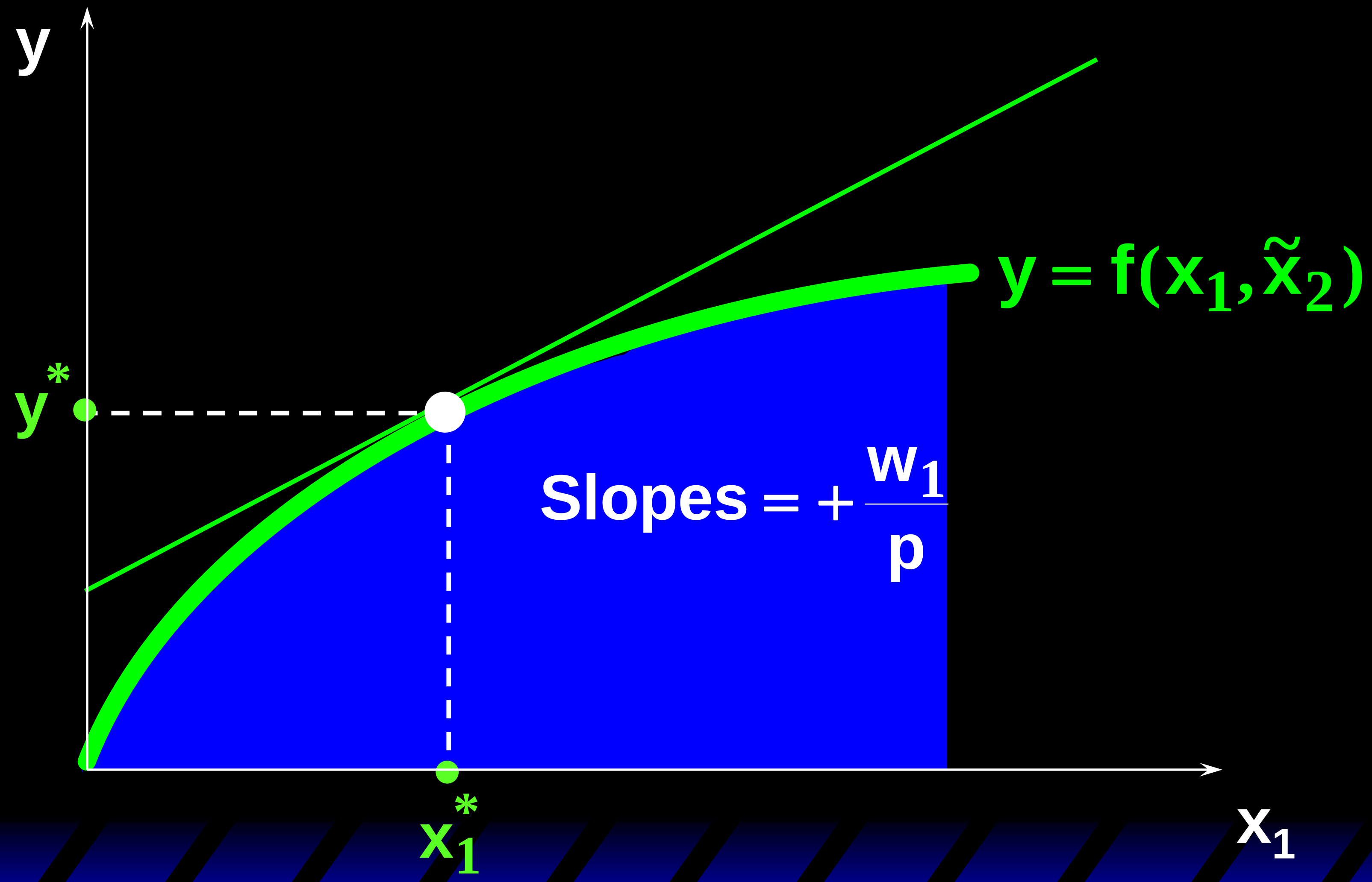
Comparative Statics of Short-Run Profit-Maximization

The equation of a short-run iso-profit line is

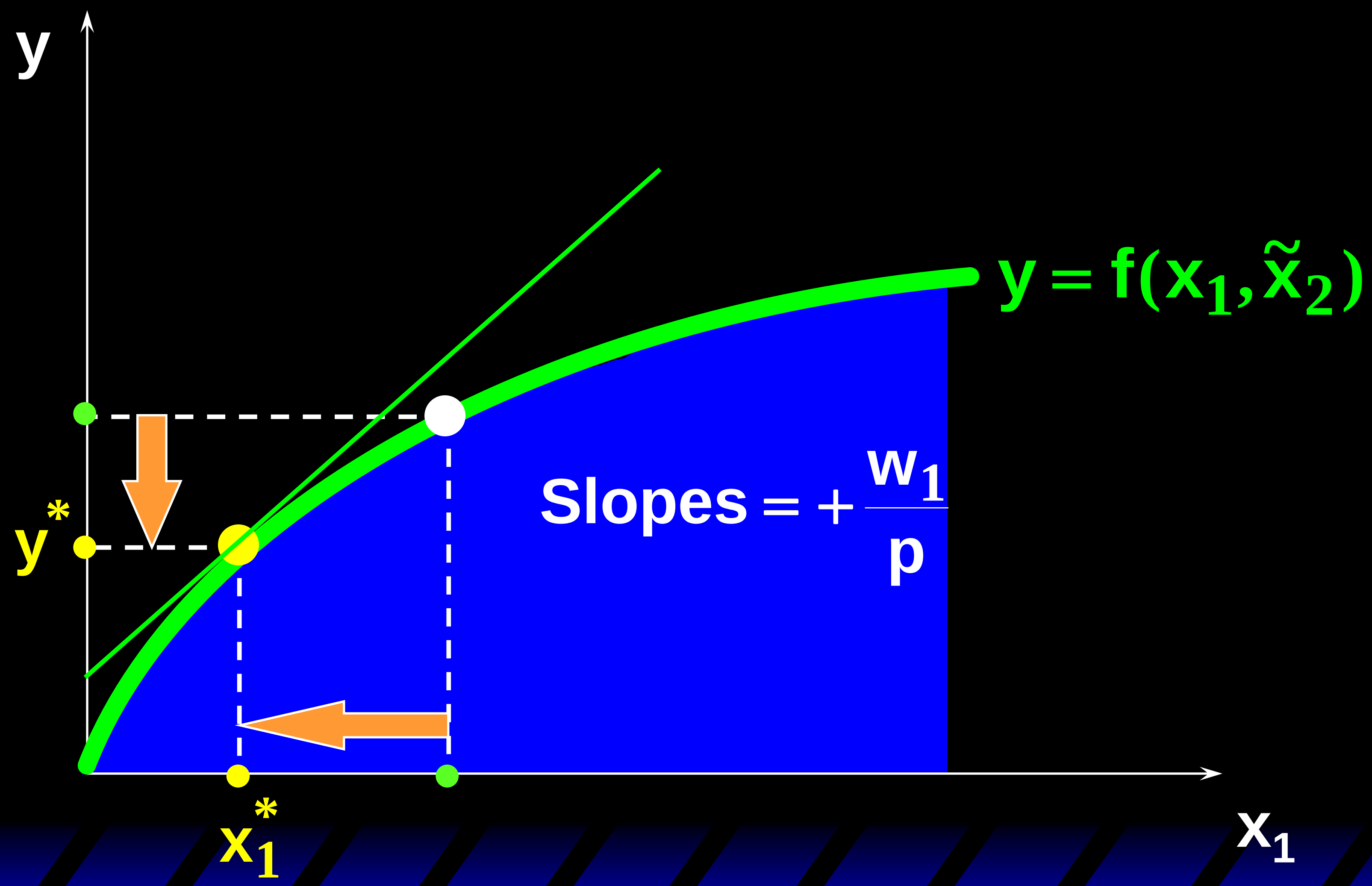
$$y = \frac{w_1}{p} x_1 + \frac{\Pi + w_2 \tilde{x}_2}{p}$$

so an increase in w_1 causes an increase in the slope, and

Comparative Statics of Short-Run Profit-Maximization



Comparative Statics of Short-Run Profit-Maximization



Comparative Statics of Short-Run Profit-Maximization

- An increase in w_1 causes (weak) decreases in
  - optimal input level;
  - optimal output level;
  - optimized profit.

Long-Run Profit-Maximization

- Now allow the firm to vary both input levels.
- Since no input level is fixed, there are **no fixed costs**.
- Fixed cost vs. **quasi-fixed cost**
  - See Varian 21.5

Long-Run Profit-Maximization

In the long run, both x_1 and x_2 are variable.

Firm's problem is

Maximize $py - w_1x_1 - w_2x_2$

Subject to $y = f(x_1, x_2)$

Long-Run Profit-Maximization

Firm's problem now becomes:

Maximize $pf(x_1, x_2) - w_1x_1 - w_2x_2$

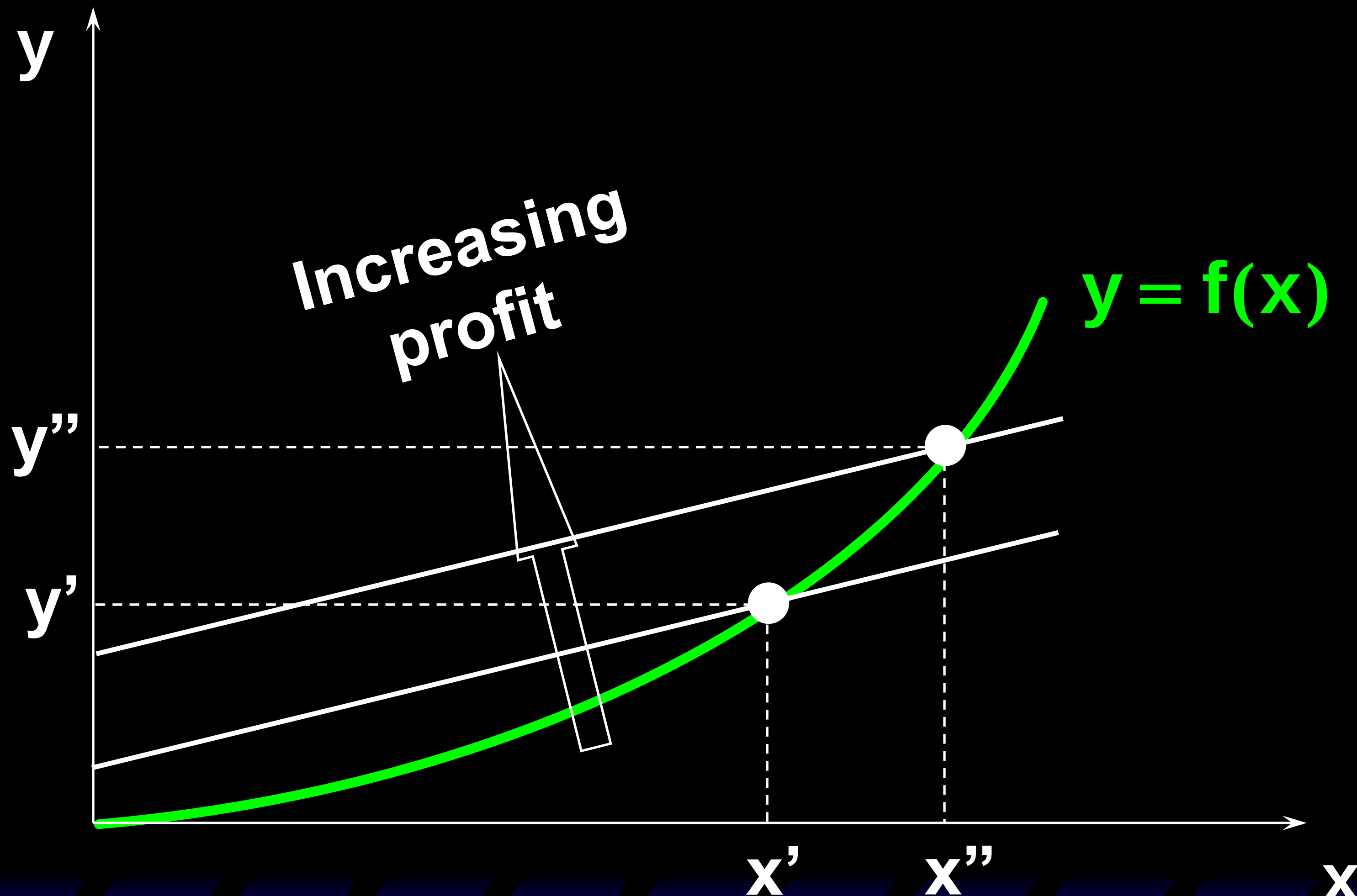
FOC:

$$p \times MP_1 - w_1 = 0 \qquad p \times MP_2 - w_2 = 0.$$

That is, marginal revenue = input price

FOC will give us the solution if f is a concave function.

What if We Have Increasing Returns to Scale?



Infinite Profit?

I would interpret this as a misspecification of the production function in the model.

In reality, increasing returns to scale usually happens at relatively low production scale.