These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Twenty-Eight

Oligopoly

What Is Oligopoly?

- A monopoly is an industry consisting a single firm.
- A duopoly is an industry consisting of two firms.
- An **oligopoly** is an industry consisting of a few firms.

Cournot: Quantity Competition

- Assume that firms compete by choosing output levels.
- Suppose firm 1 takes firm 2's output level choice y_2 as given. Then firm 1 sees its profit function as

$$\Pi_1(y_1;y_2) = p(y_1 + y_2)y_1 - c_1(y_1).$$

- Given y_2, what output level y_1 maximizes firm 1's profit?

Quantity Competition

Generally, given firm 2's chosen output level y_2, firm 1' maximizes its profit

$$\Pi_1(y_1;y_2) = p(y_1 + y_2)y_1 - c_1(y_1)$$

by choosing y_1. FOC:

$$\frac{\partial \Pi_1}{\partial y_1} = p(y_1 + y_2) + y_1 \frac{\partial p(y_1 + y_2)}{\partial y_1} - c_1{}'(y_1) = 0.$$

The solution, $y_1 = R_1(y_2)$, is firm 1's Cournot-Nash reaction to y_2.

Quantity Competition

Similarly, given firm 1's chosen output level y_1, firm 2' maximizes its profit

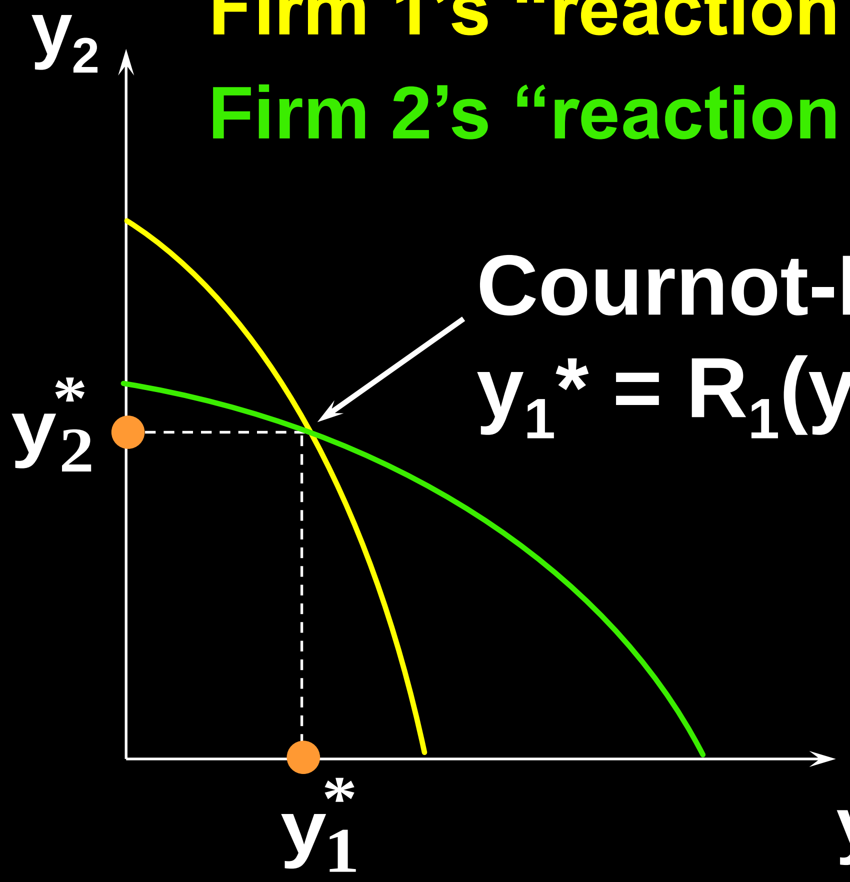
$$\Pi_2(y_2; y_1) = p(y_1 + y_2)y_2 - c_2(y_2)$$

by choosing y_2. FOC:

$$\frac{\partial \Pi_2}{\partial y_2} = p(y_1 + y_2) + y_2 \frac{\partial p(y_1 + y_2)}{\partial y_2} - c_2{}'(y_2) = 0.$$

The solution, $y_2 = R_2(y_1)$, is firm 2's **Cournot-Nash reaction** to y_1.

Quantity Competition



Firm 1's "reaction curve" $y_1 = R_1(y_2)$.

Firm 2's "reaction curve" $y_2 = R_2(y_1)$.

Cournot-Nash equilibrium
$y_1^* = R_1(y_2^*)$ and $y_2^* = R_2(y_1^*)$

Quantity Competition; An Example

- The market inverse demand function is $p = 60 - y_T$. The firms' cost functions are $c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2$.
- Reaction function

$$y_1 = R_1(y_2) = \frac{60 - y_2}{4}$$

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}$$

Quantity Competition; An Example

- **Solution:** $y_1^* = 13, y_2^* = 8$

A Symmetric Example with n Firms

- Suppose there are n firms, whose MC is constant at c. No fixed or quasi-fixed cost.
- Market inverse demand: p = a – bQ
- Taking $y_2, y_3, \dots, y_n$ as given, Firm 1 chooses y_1 to maximize

$$[a - b(y_1 + y_2 + \cdots + y_n)]y_1 - cy_1$$

- FOC: $a - b\sum_{j\neq 1} y_j - 2by_1 - c = 0$

A Symmetric Example with n Firms

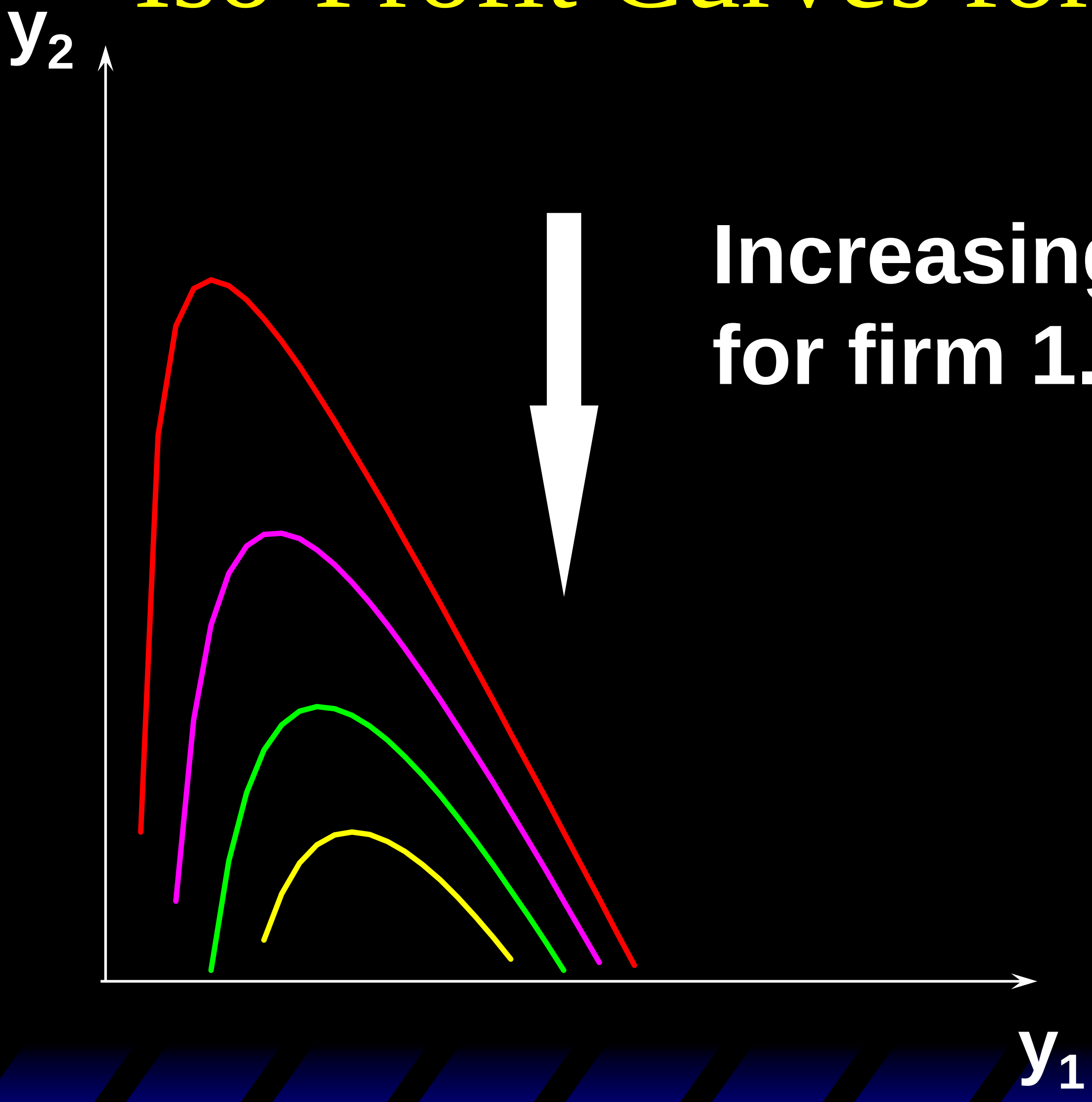
- Symmetrically, Firm i chooses y_i to maximize

$$[a - b(y_1 + y_2 + \cdots + y_n)]y_i - cy_i$$

- FOC: $a - b\sum_{j\neq i} y_j - 2by_i - c = 0$
- Therefore, the unique Cournot-Nash equilibrium is $y_i = \frac{a-c}{b(n+1)}$ for all i, and the equilibrium price is

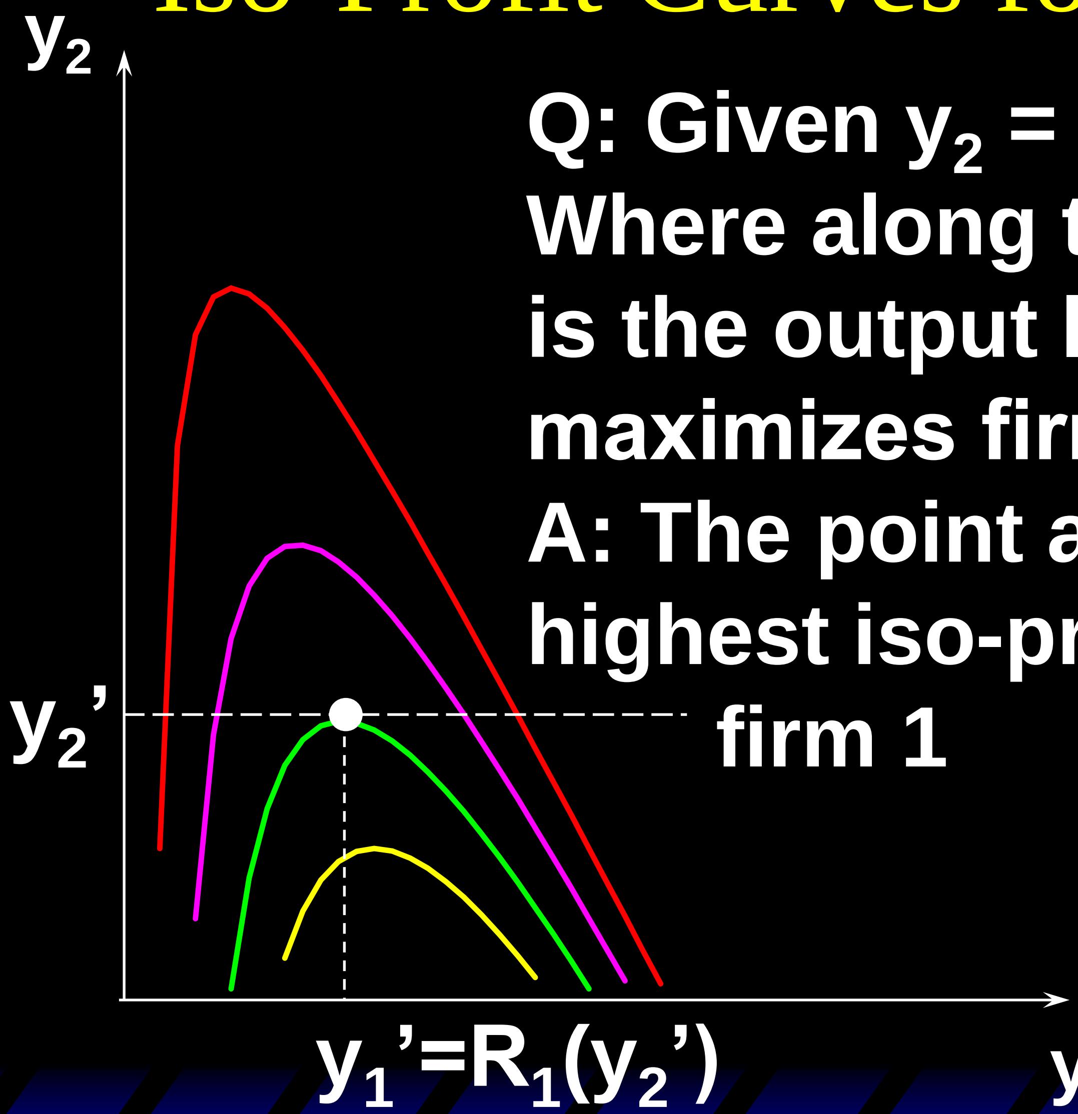
$$p = c + \frac{a - c}{n + 1}$$

Iso-Profit Curves for Firm 1



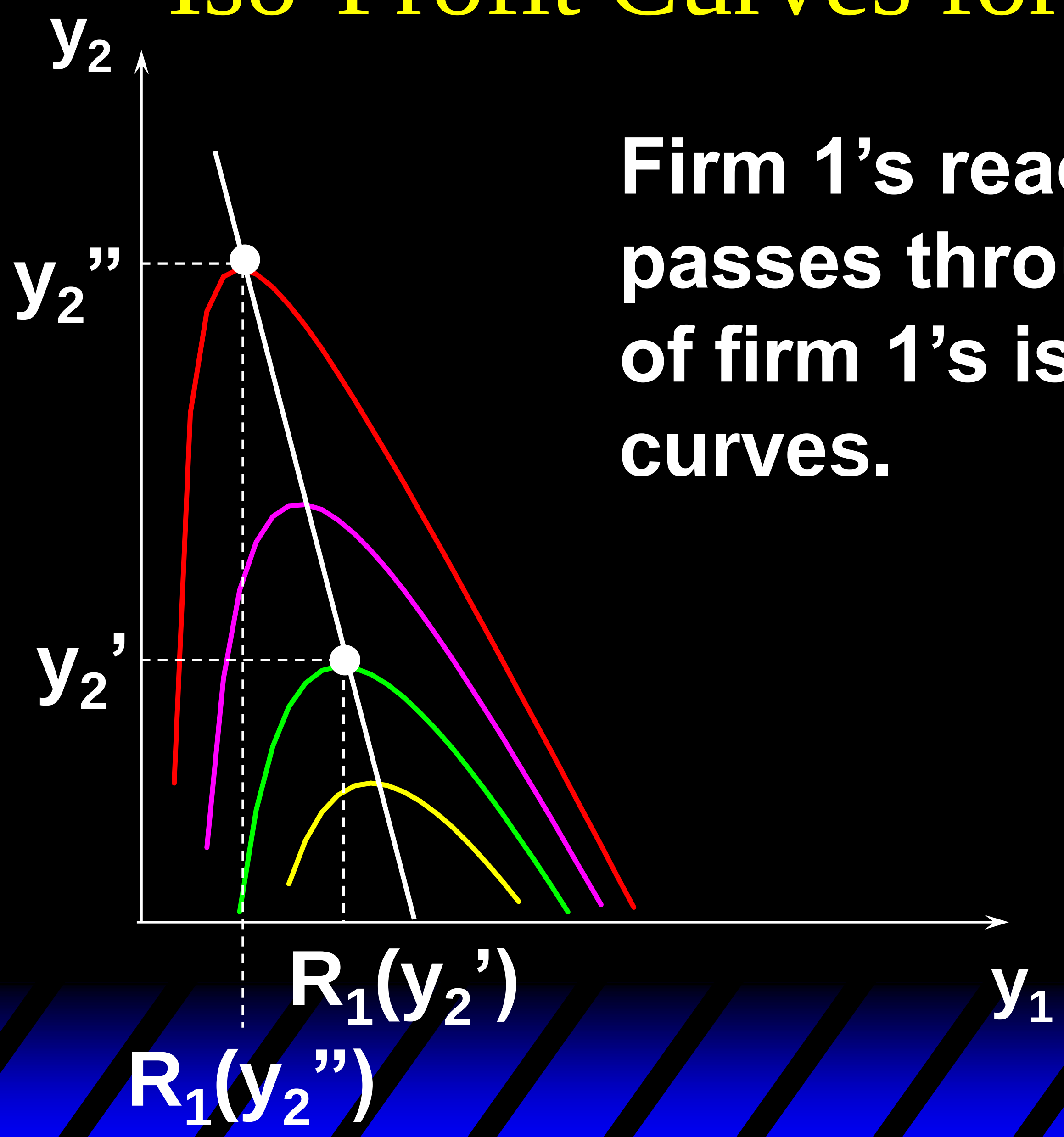
Increasing profit for firm 1.

Iso-Profit Curves for Firm 1



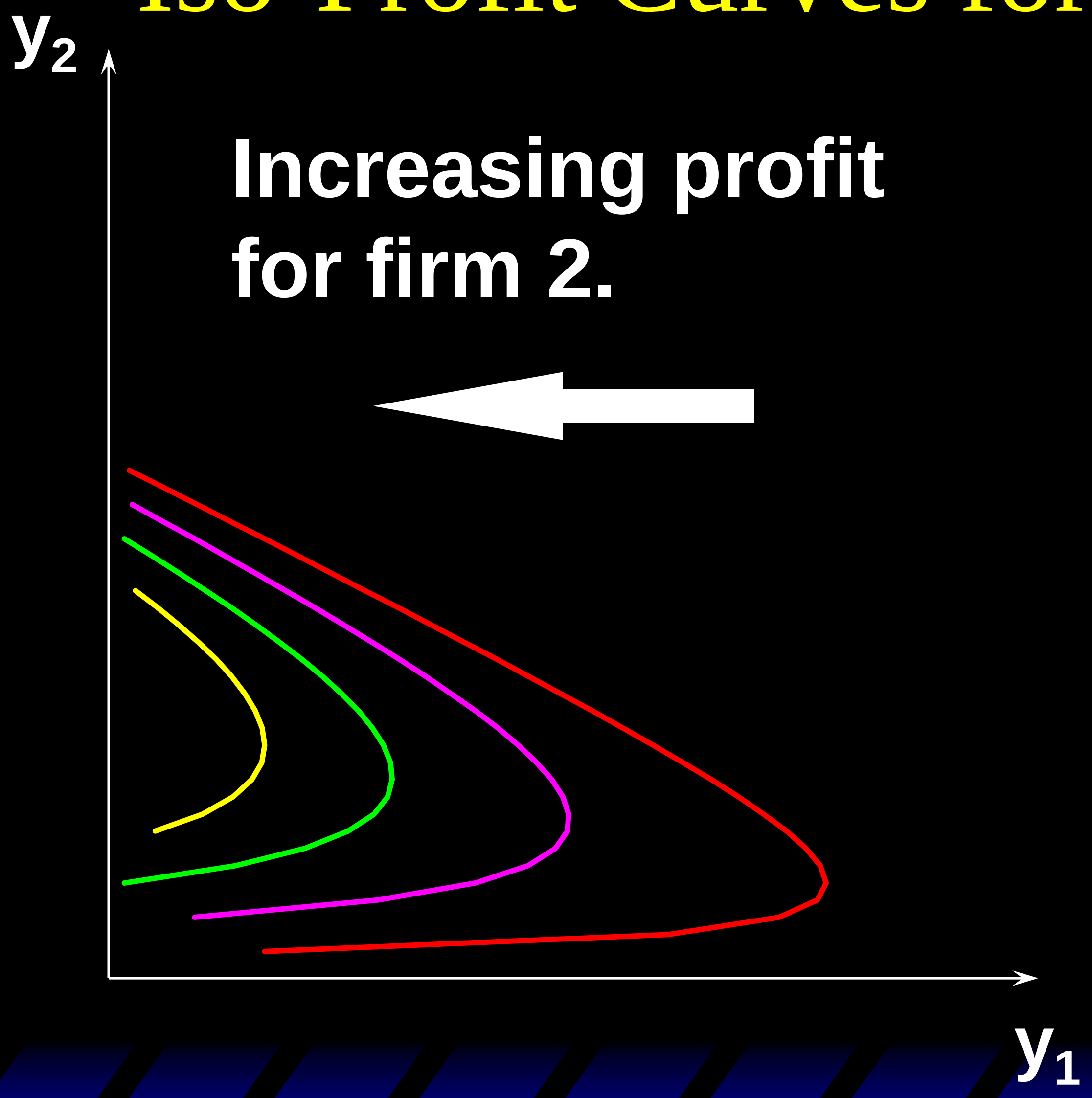
Q: Given $y_2 = y_2'$,
Where along the line $y_2 = y_2'$
is the output level that
maximizes firm 1's profit?
A: The point attaining the
highest iso-profit curve for
firm 1

Iso-Profit Curves for Firm 1



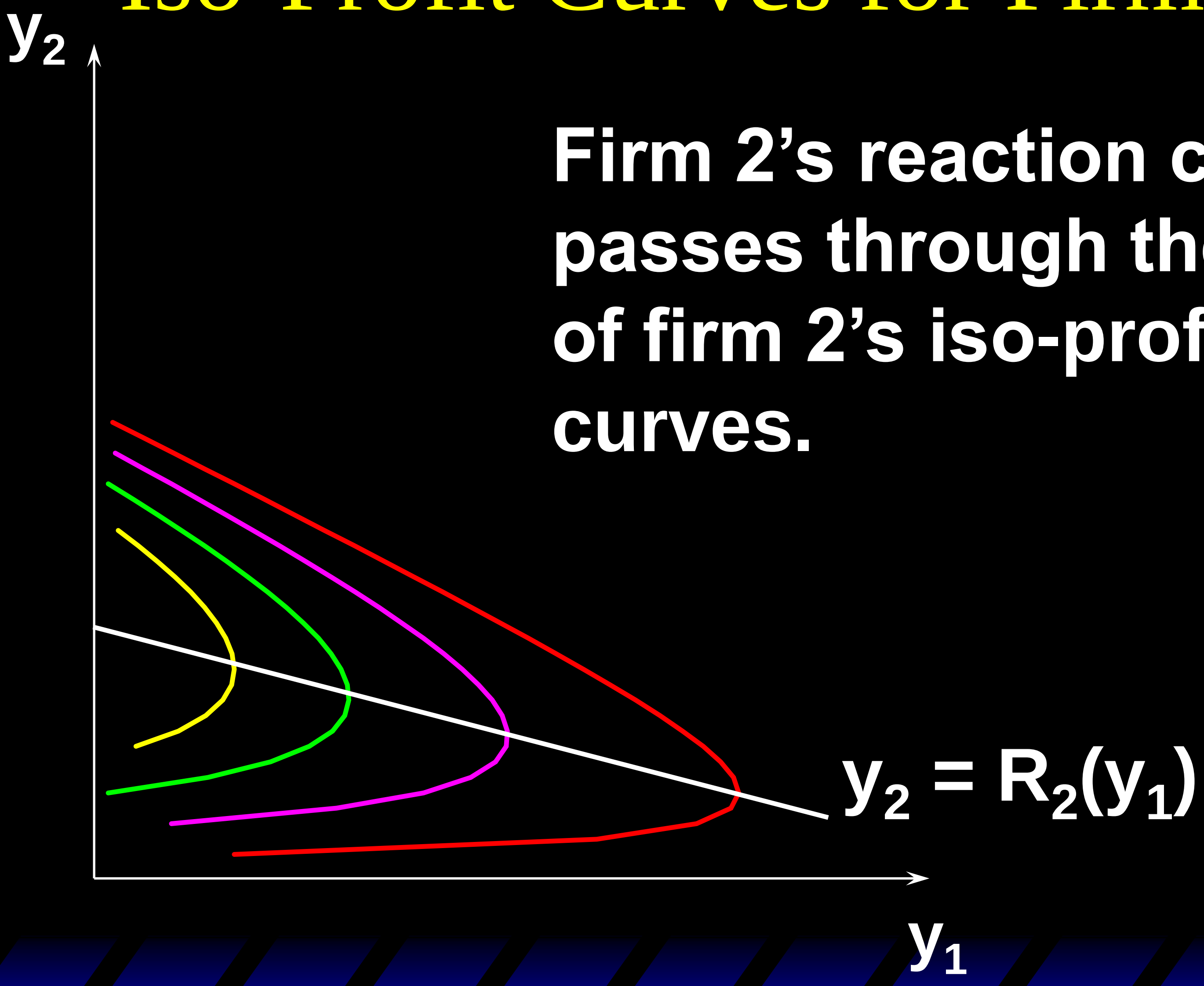
Firm 1's reaction curve passes through the "tops" of firm 1's iso-profit curves.

Iso-Profit Curves for Firm 2



Iso-Profit Curves for Firm 2

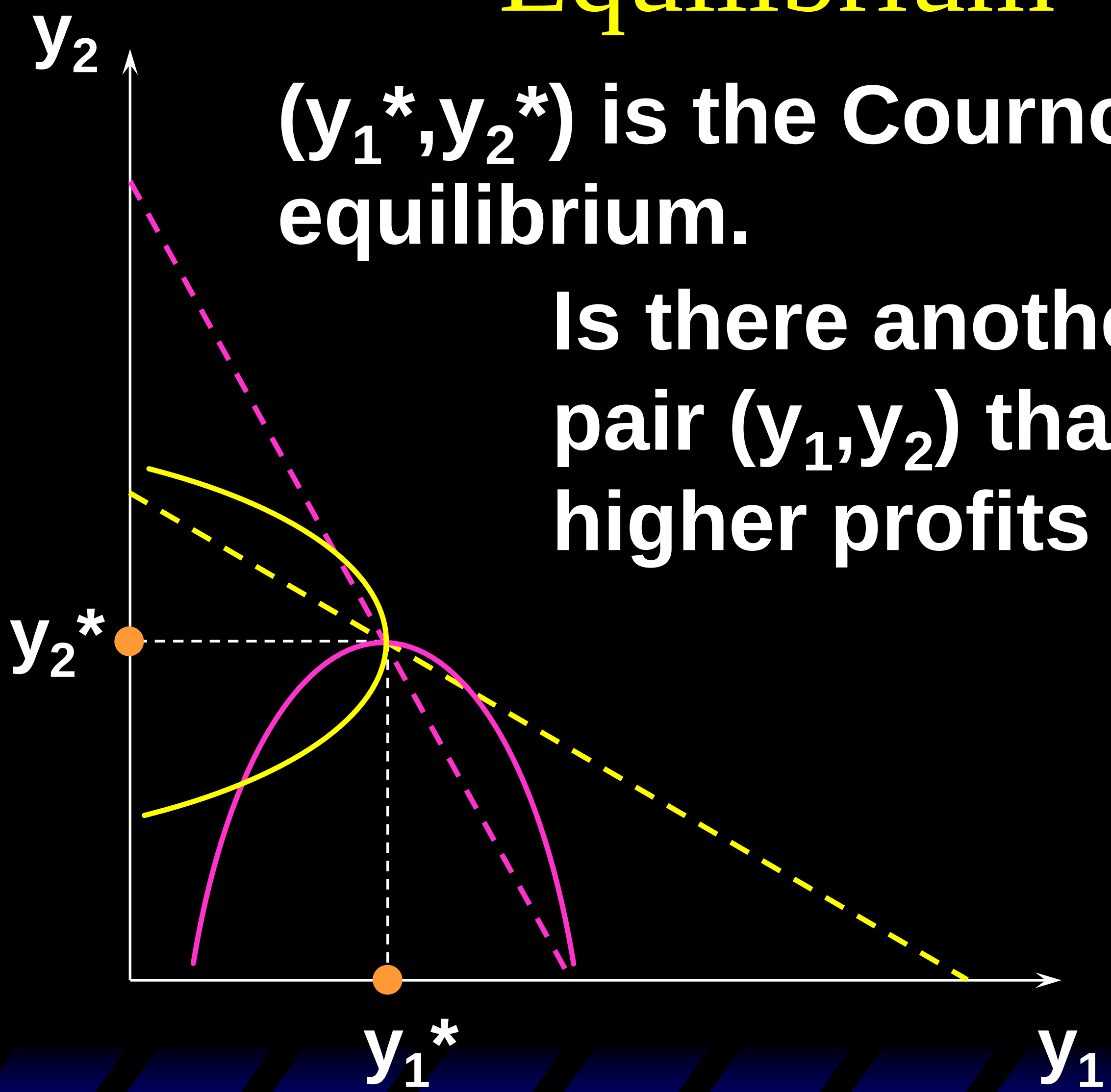
Firm 2's reaction curve passes through the "tops" of firm 2's iso-profit curves.



Equilibrium

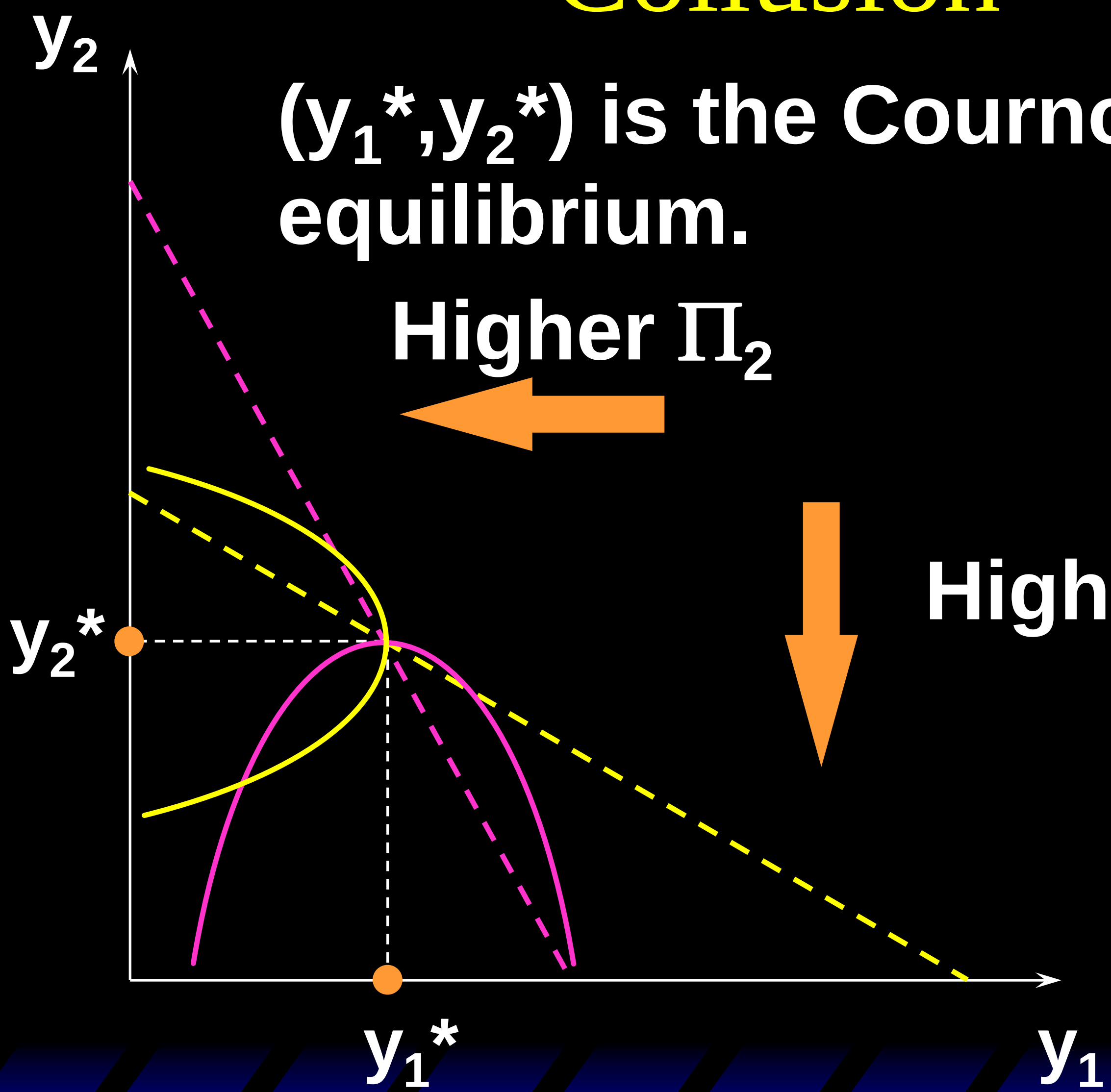
(y_1*,y_2*) is the Cournot-Nash equilibrium.

Is there another output level pair (y_1,y_2) that gives higher profits to both firms?

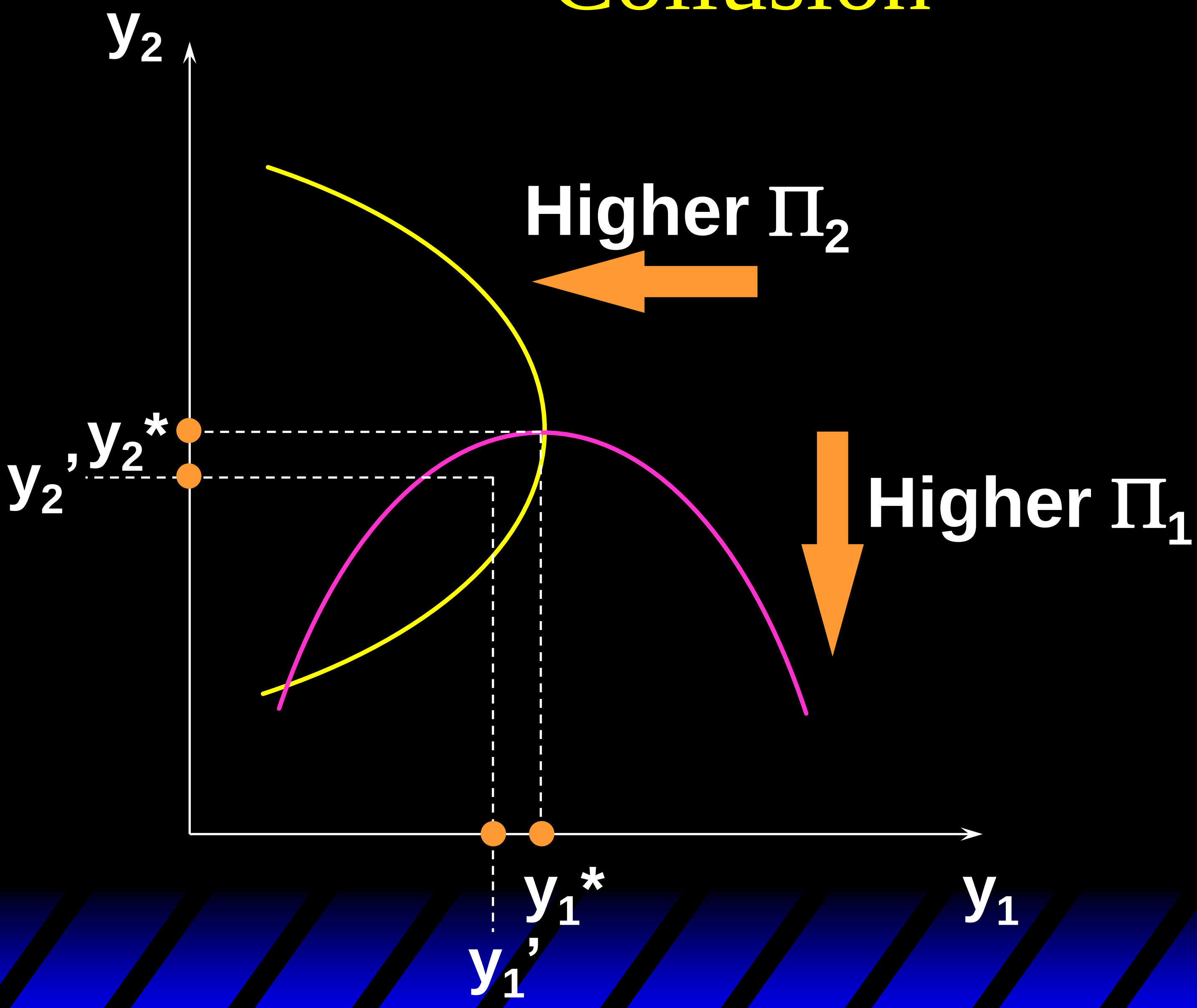


Collusion

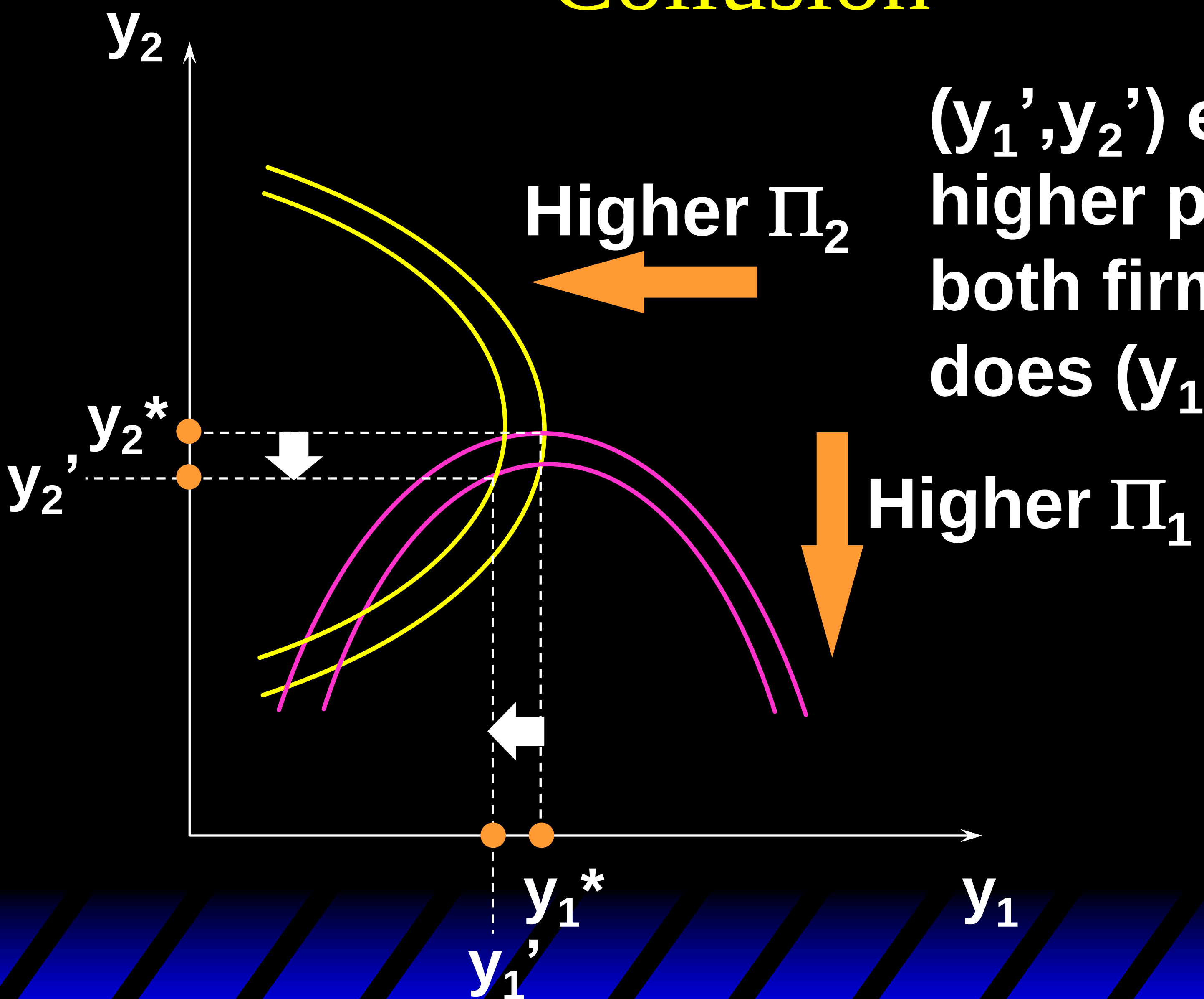
(y_1^*, y_2^*) is the Cournot-Nash equilibrium.



Collusion



Collusion



(y_1',y_2') earns higher profits for both firms than does (y_1^*,y_2^*).

Collusion

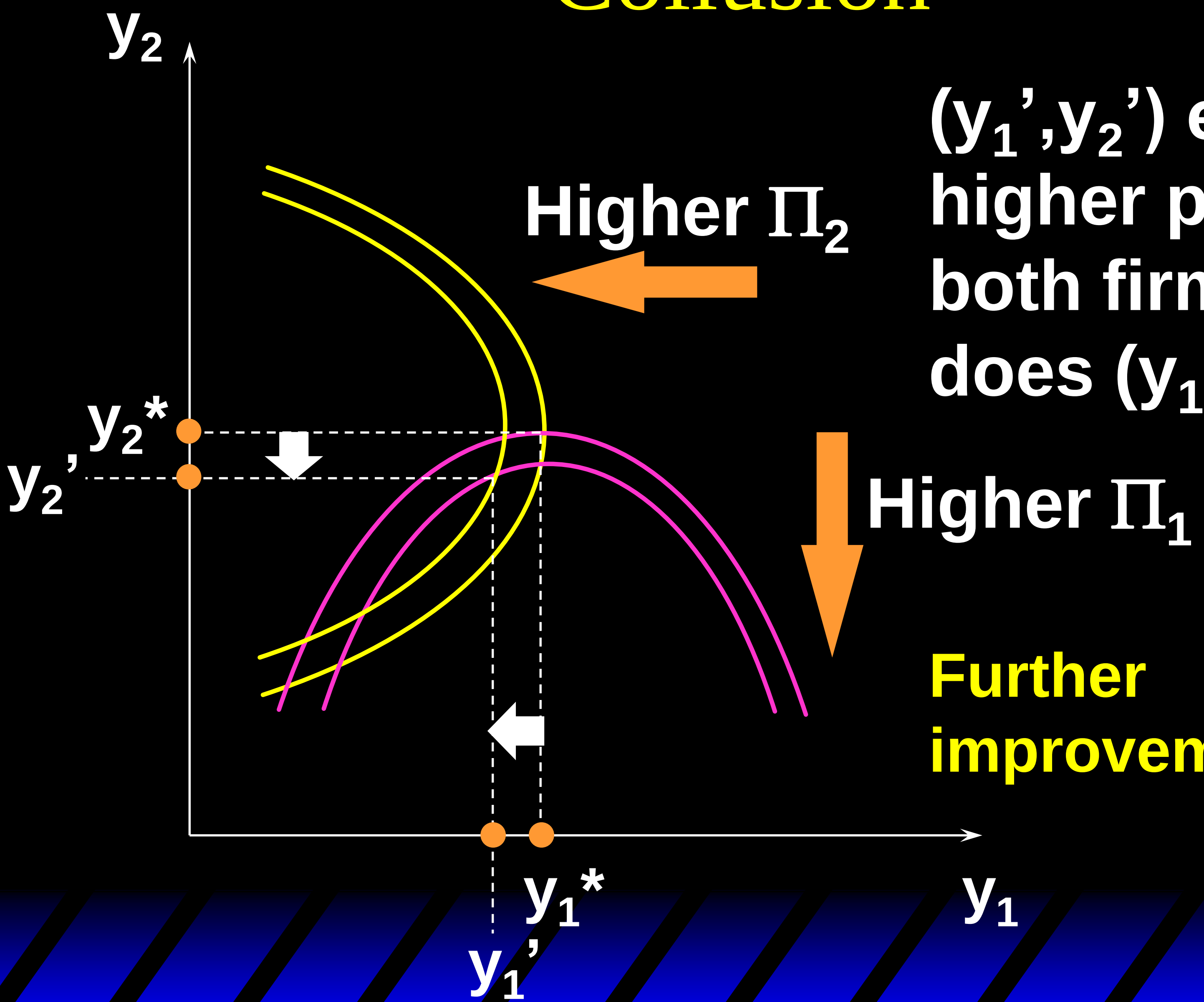
- So there are incentives for both firms to “cooperate” by lowering their output levels.
- Such cooperation is called **collusion**.
- Firms that collude are said to have formed a **cartel**.
- If firms form a cartel, how should they do it?

Collusion

- Suppose the two firms want to maximize their total profit and divide it between them. Their goal is to choose cooperatively output levels y_1 and y_2 that maximize

$$\Pi^m(y_1, y_2) = p(y_1 + y_2)(y_1 + y_2) - c_1(y_1) - c_2(y_2).$$

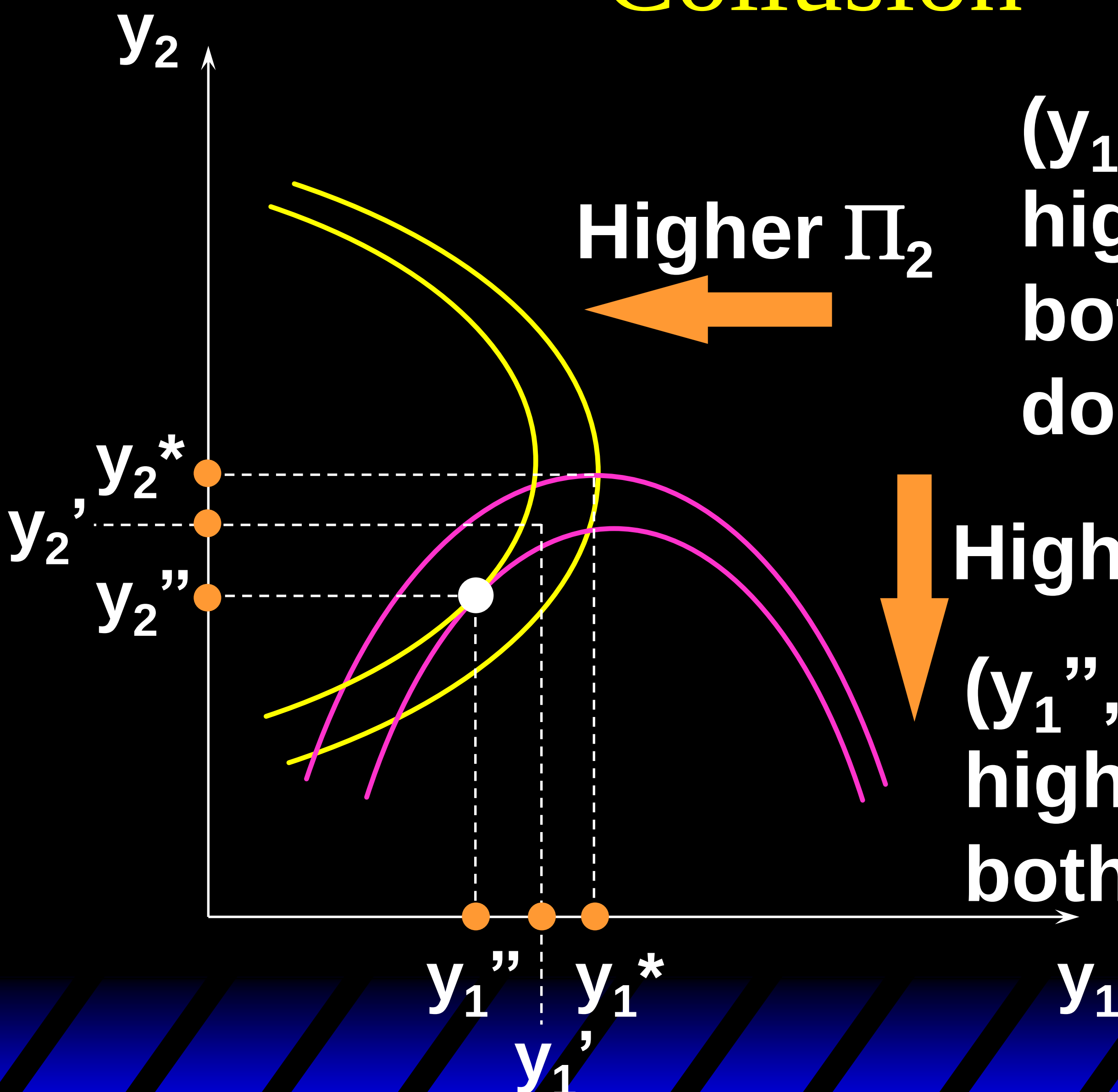
Collusion



(y_1', y_2') earns higher profits for both firms than does (y_1^*, y_2^*).

Further improvement?

Collusion

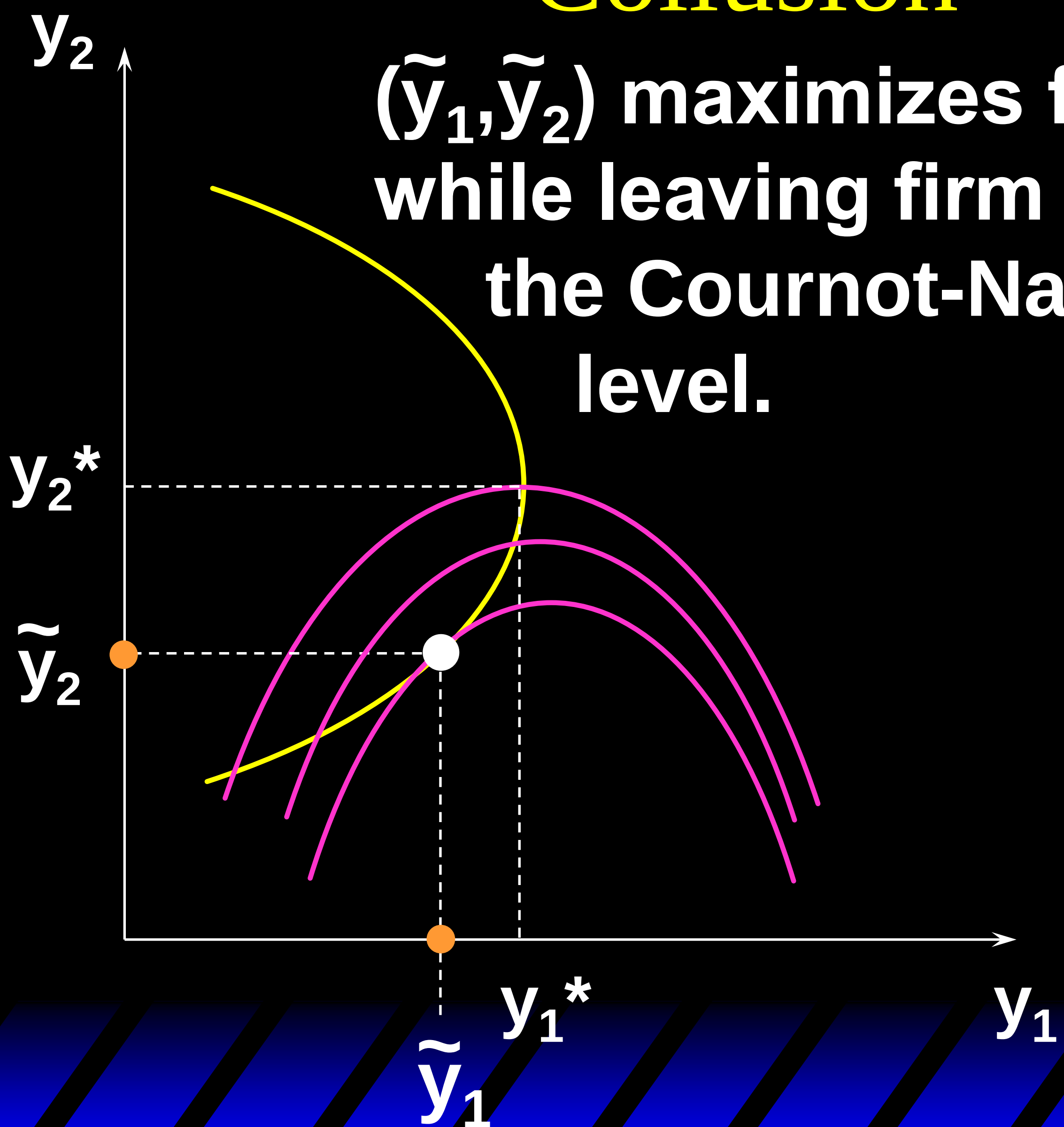


(y_1', y_2') earns higher profits for both firms than does (y_1^*, y_2^*).

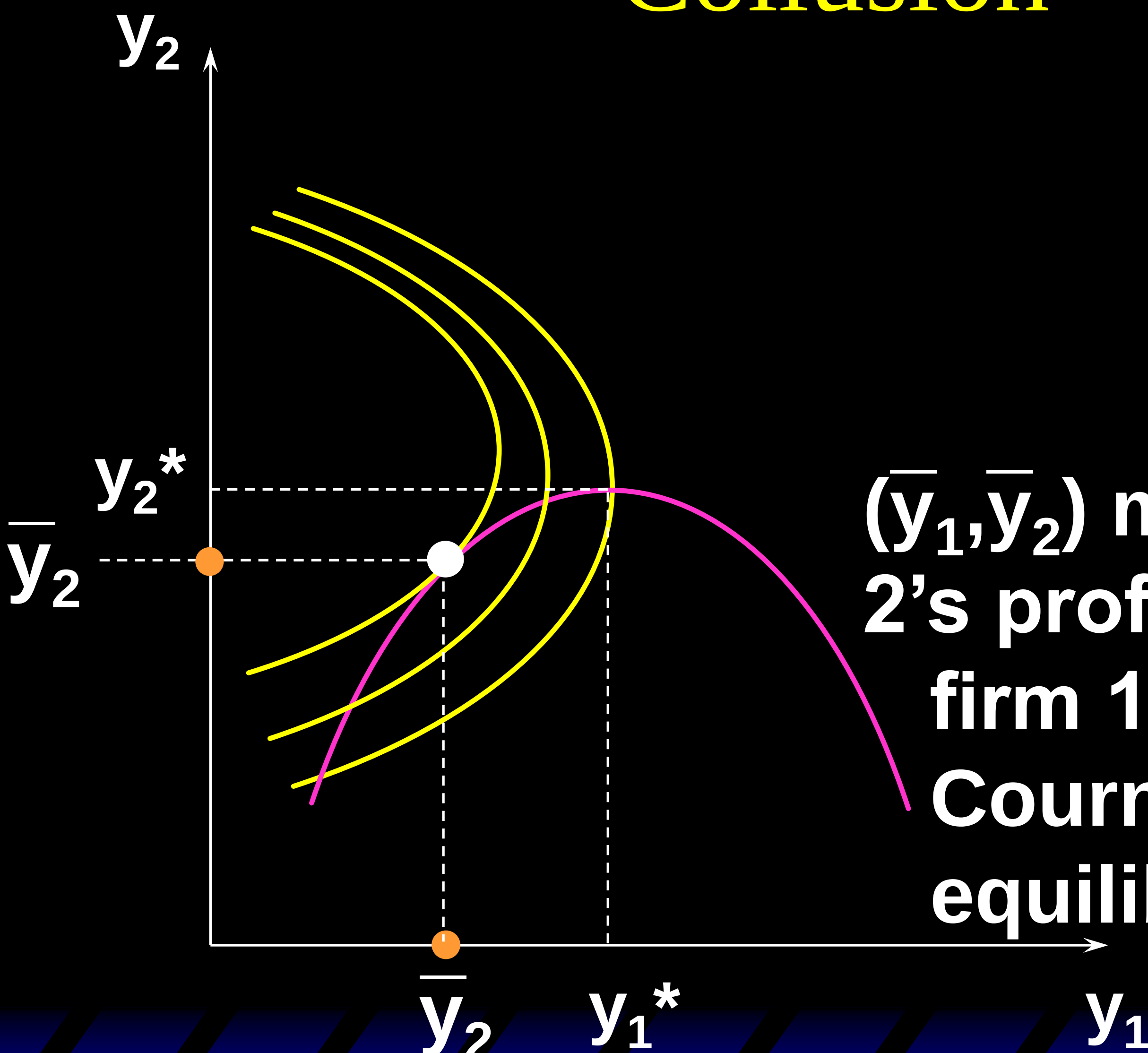
(y_1'', y_2'') earns even higher profits for both firms.

Collusion

$(\tilde{y}_1,\tilde{y}_2)$ maximizes firm 1's profit while leaving firm 2's profit at the Cournot-Nash equilibrium level.

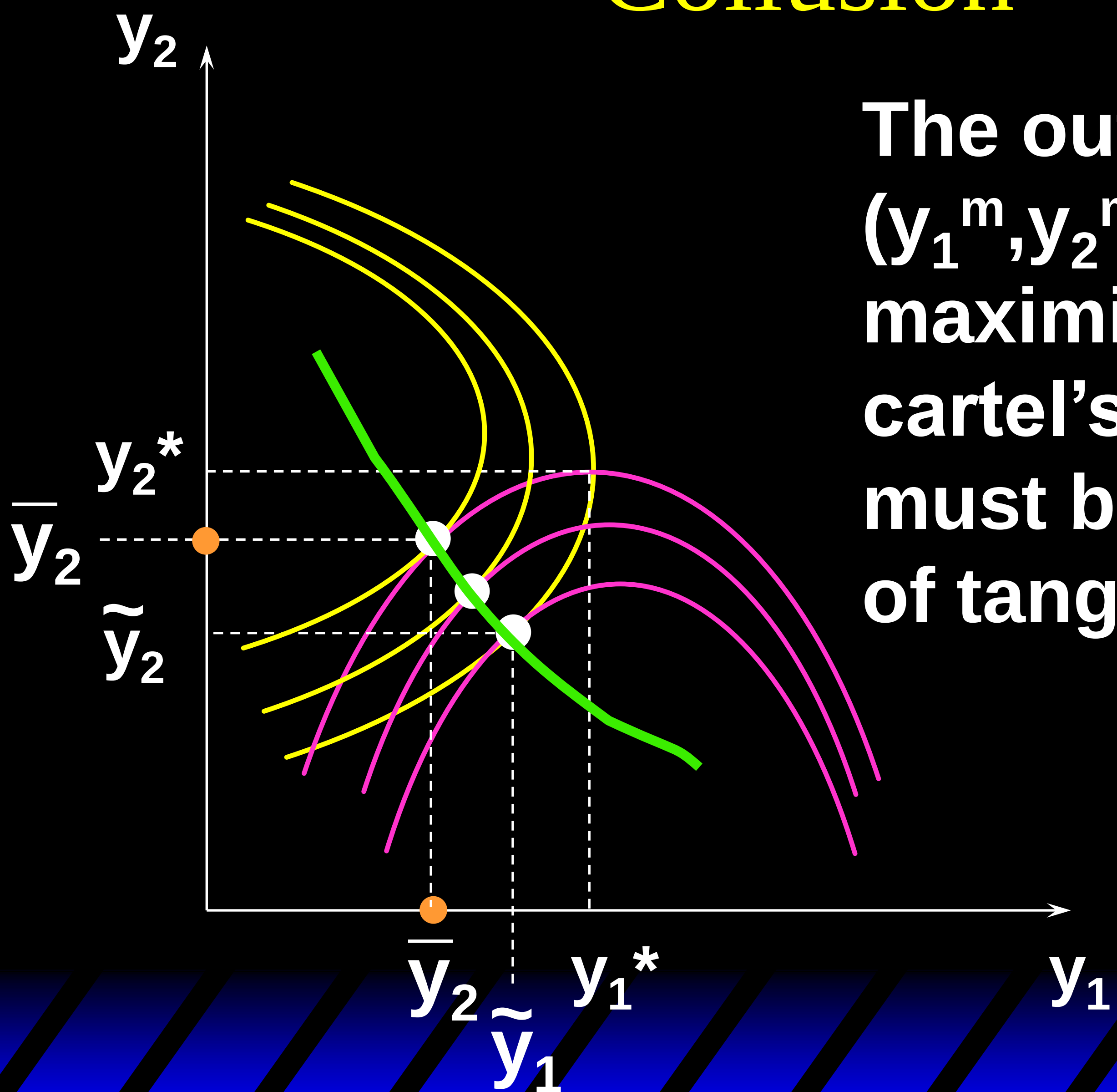


Collusion



$(\bar{y}_1,\bar{y}_2)$ maximizes firm 2's profit while leaving firm 1's profit at the Cournot-Nash equilibrium level.

Collusion



The output pair (y_1^m, y_2^m) that maximizes the cartel's joint profit must be on the path of tangent points.

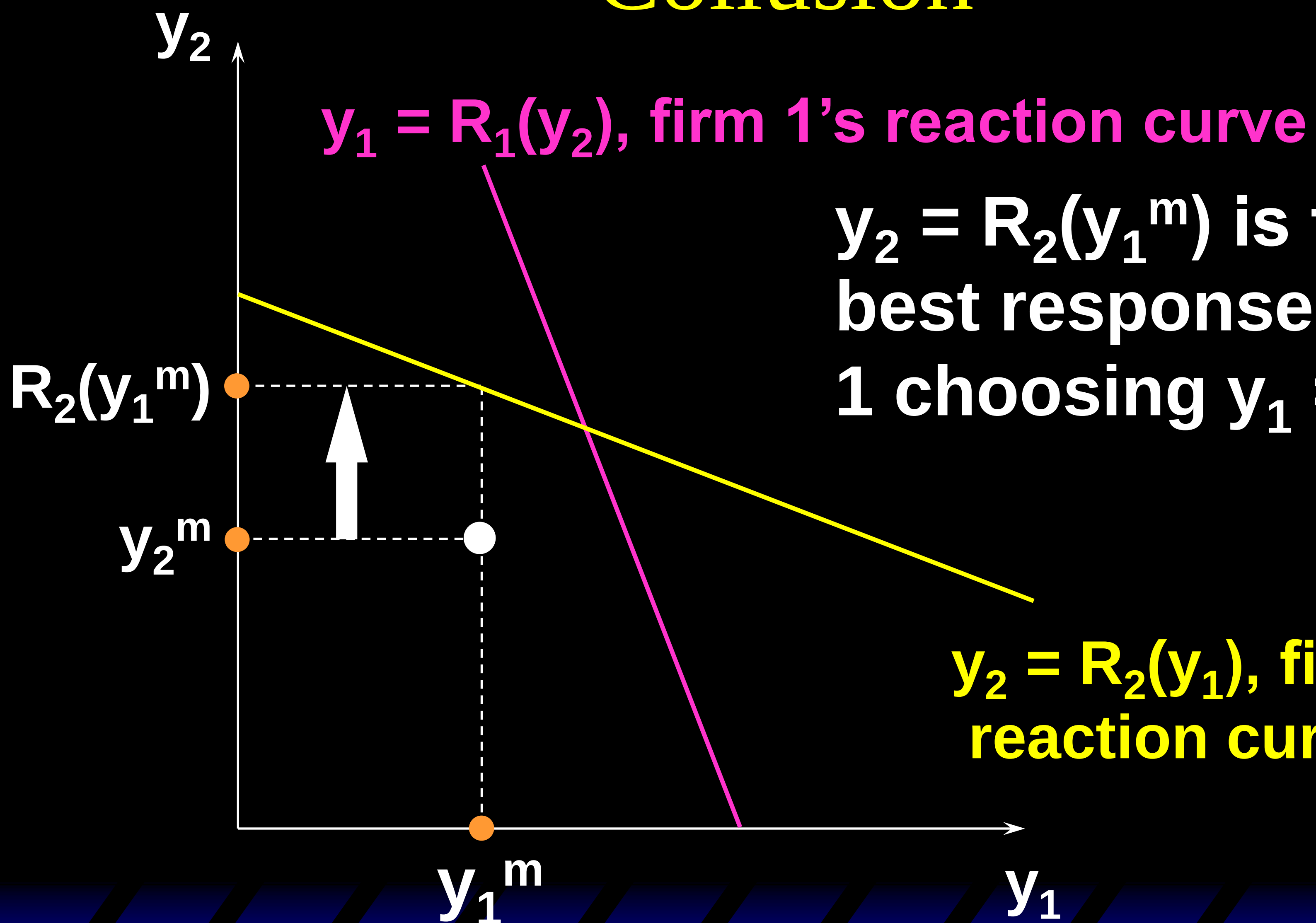
Collusion

- **Is such a collusion "stable"?**
- **Does one firm have an incentive to cheat on the other?**
  - **Suppose they agree on producing y_1^m and y_2^m. If firm 1 comply with their agreement to produce y_1^m units, does firm 2 have incentive to deviate from producing y_2^m units?**

Collusion

- Firm 2's profit-maximizing response to $y_1 = y_1^m$ is $y_2 = R_2(y_1^m)$.

Collusion



$y_2 = R_2(y_1^m)$ is firm 2's best response to firm 1 choosing $y_1 = y_1^m$.

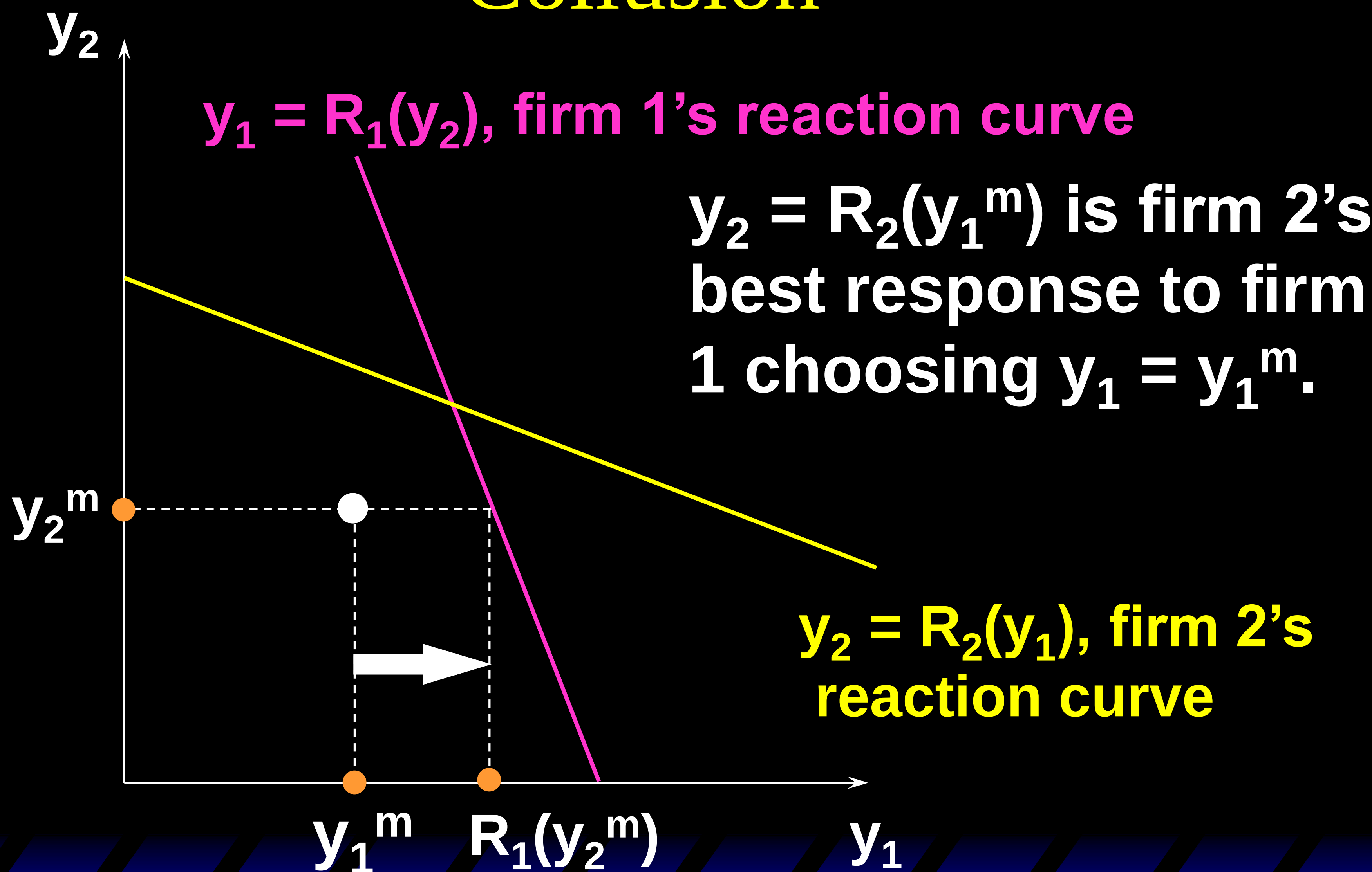
Collusion

- Firm 2's profit-maximizing response to $y_1 = y_1^m$ is $y_2 = R_2(y_1^m) > y_2^m$.
- Firm 2's profit increases if it cheats on firm 1 by increasing its output level from y_2^m to $R_2(y_1^m)$.

Collusion

- **Similarly, firm 1's profit increases if it cheats on firm 2 by increasing its output level from y_1^m to $R_1(y_2^m)$.**

Collusion



y_2
$y_1 = R_1(y_2)$, firm 1's reaction curve
$y_2 = R_2(y_1^m)$ is firm 2's best response to firm 1 choosing $y_1 = y_1^m$.
y_2^m
$y_2 = R_2(y_1)$, firm 2's reaction curve
y_1^m
$R_1(y_2^m)$
y_1

Collusion

- **So a profit-maximizing cartel in which firms cooperatively set their output levels tend to be unstable.**
- **In reality, oligopolists tends to find ways to sustain collusion, which may raise anti-trust concerns.**

The Order of Play

- So far it has been assumed that firms choose their output levels **simultaneously**.
  - Simultaneous move games

The Order of Play

- What if firm 1 chooses its output level first and then firm 2 responds to this choice?
  - Sequential games, a.k.a. Stackelberg games
- Firm 1 is then a **leader**. Firm 2 is a **follower**.

Stackelberg Games

- **Backward induction:**
  - **Observing firm 1's decision y_1, the best response that firm 2 can make is to choose $y_2 = R_2(y_1)$.**
  - **Firm 1 expect this reaction, and therefore choose y_1 to maximize**

$$\Pi_1^s(y_1) = p(y_1 + R_2(y_1))y_1 - c_1(y_1).$$

Stackelberg Games

- **Q: Will the leader make a profit at least as large as its Cournot-Nash equilibrium profit?**

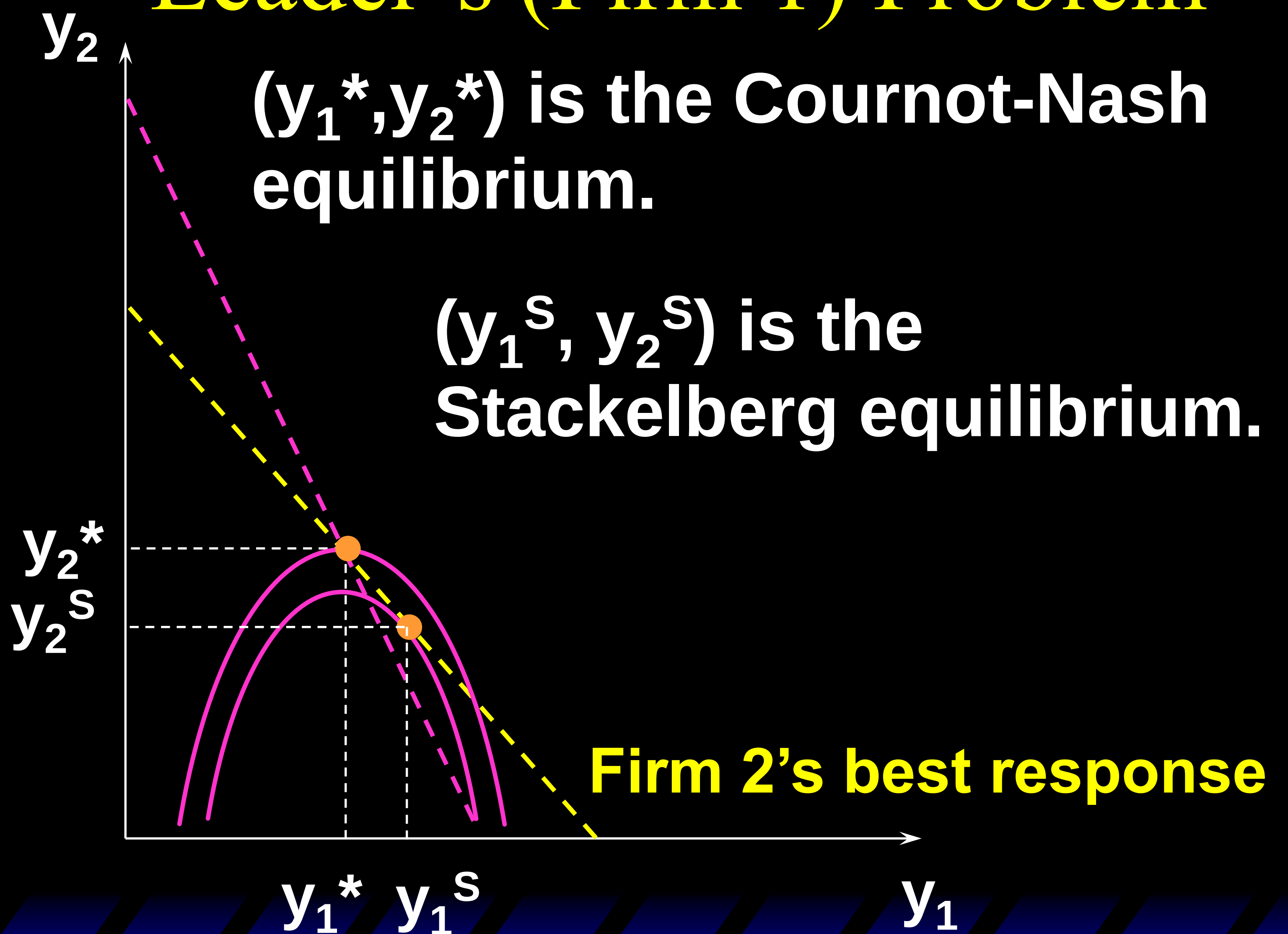
Stackelberg Games

- A: Yes. A feasible strategy for the leader is to choose its Cournot-Nash output level, which will lead to the follower also choosing its Cournot-Nash output level.
- The leader may do even better than that.

Leader's (Firm 1) Problem

(y_1^*, y_2^*) is the Cournot-Nash equilibrium.

(y_1^S, y_2^S) is the Stackelberg equilibrium.



Stackelberg Games; An Example

- The market inverse demand function is $p = 60 - y_T$. The firms' cost functions are $c_1(y_1) = y_1^2$ and $c_2(y_2) = 15y_2 + y_2^2$.
- Firm 2 is the follower. Its reaction function is

$$y_2 = R_2(y_1) = \frac{45 - y_1}{4}.$$

Stackelberg Games; An Example

The leader's profit function is therefore

$$\Pi_1^s(y_1) = (60 - y_1 - R_2(y_1))y_1 - y_1^2$$

$$= (60 - y_1 - \frac{45 - y_1}{4})y_1 - y_1^2$$

$$= \frac{195}{4}y_1 - \frac{7}{4}y_1^2.$$

For a profit-maximum, $y_1^s = 13.9$

Stackelberg Games; An Example

Q: What is firm 2's response to the leader's choice $y_1^s = 13.9$

A: $y_2^s = R_2(y_1^s) = \frac{45-13.9}{4} = 7.8$

Recall that the C-N output levels are (y_1*,y_2*) = (13,8)

Price Competition

- What if firms compete by **simultaneously setting price** instead of quantity?
  - **Bertrand** games.

Bertrand Duopoly

- Two firms simultaneously set their prices.
- Assume that both firms' MC is constant at c.

- Equilibrium is unique: both firms set their prices equal to the marginal cost c.

Cournot vs. Bertrand

- **According to Cournot model of oligopoly, equilibrium price will gradually converge to MC as the number of firms increases.**
- **But in Bertrand model, two firms are sufficient to bring the price down to MC.**

Sequential Price Competition

- Suppose firm 1 sets p_1, observed by firm 2, and then firm 2 sets p_2.
- Again assume that both firms' MC is constant at c.
- Same equilibrium as Bertrand.
- Varian's book assumes that the follower must follow the price set by the leader, which leads to different results.

Game Theory

- Gibbons, Robert S. *Game theory for applied economists*. Princeton University Press, 1992.
- Fudenberg, Drew, and Jean Tirole. *Game theory*. MIT press, 1991.
- Maschler, Michael, Shmuel Zamir, and Eilon Solan. *Game theory*. Cambridge University Press, 2020.

Summary

- **Key concept**
  - **Equilibrium in Cournot, Stackelberg, and Bertrand model.**
- **Key technique**
  - **Reaction functions.**