These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Twenty-One

Cost Minimization

Why Cost Minimization?

The firm's problem is profit maximization.

We can break down the problem into two steps.

1. Given output y, what is the smallest possible cost to produce y? Let the answer be c(y)
2. Choose the optimal y to maximize profit py-c(y).

Step 1 is cost minimization.

Cost Minimization

- Given output level $y \geq 0$.
- Let $c(y)$ denote the firm's smallest possible total cost of producing y units of output.
- $c(y)$ is the firm's **total cost function**.

Cost Minimization

Clearly, cost also relies on input prices, so in principle c is a function

$c(w_1,\ldots,w_n,y)$.

The Cost-Minimization Problem

- Consider a firm using two inputs to produce one output.
- The production function is
  $y = f(x_1,x_2)$.
- Take the output level $y \geq 0$ as given.
- Given the input prices w_1 and w_2, the cost of an input bundle (x_1,x_2) is $w_1x_1 + w_2x_2$.

The Cost-Minimization Problem

For given w_1, w_2 and y, the firm's cost-minimization problem is to solve

$$\min_{x_1, x_2 \geq 0} w_1 x_1 + w_2 x_2$$

subject to $f(x_1, x_2) = y.$

The solution $x_1^*(w_1,w_2,y)$ and $x_2^*(w_1,w_2,y)$ are the firm's **conditional demands for the two inputs**.

Iso-cost Lines

A curve that contains all of the input bundles that cost the same amount is an iso-cost curve.

Iso-cost Lines

Generally, given w_1 and w_2, the equation of the \$c iso-cost line is

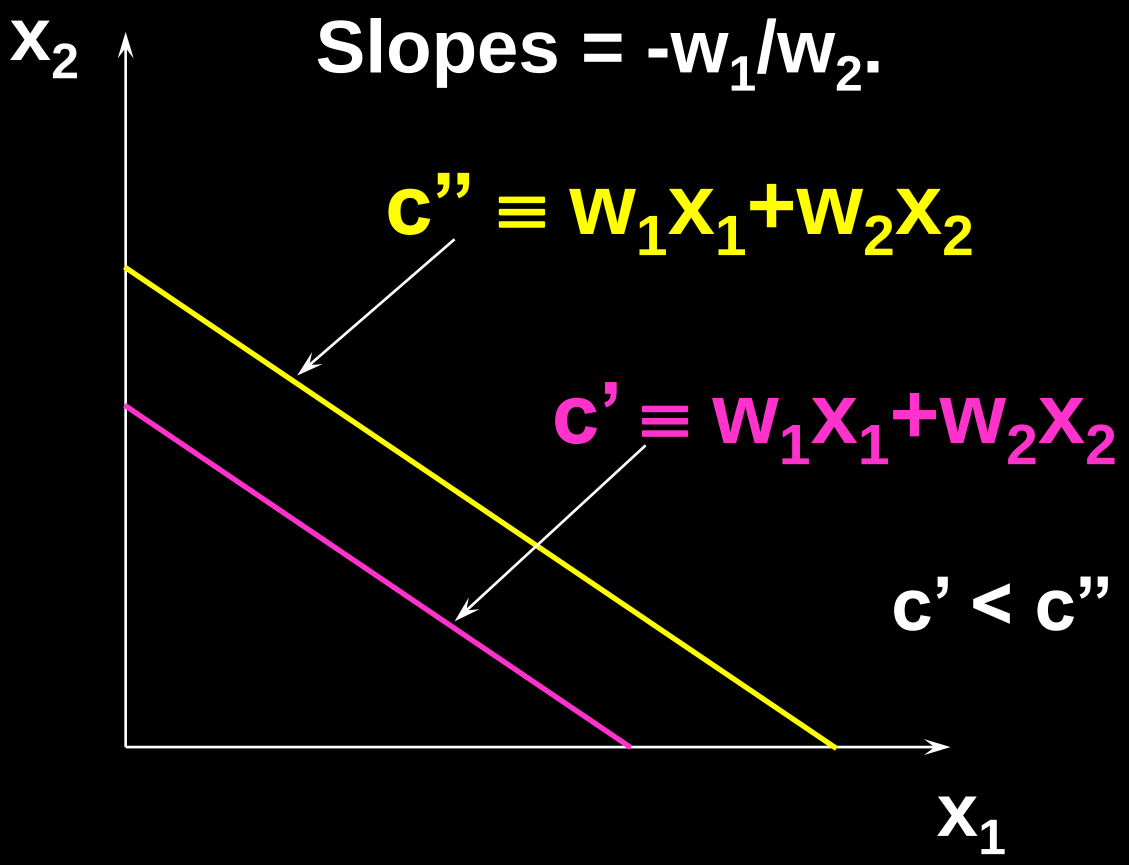
$$w_1x_1 + w_2x_2 = c$$

i.e.

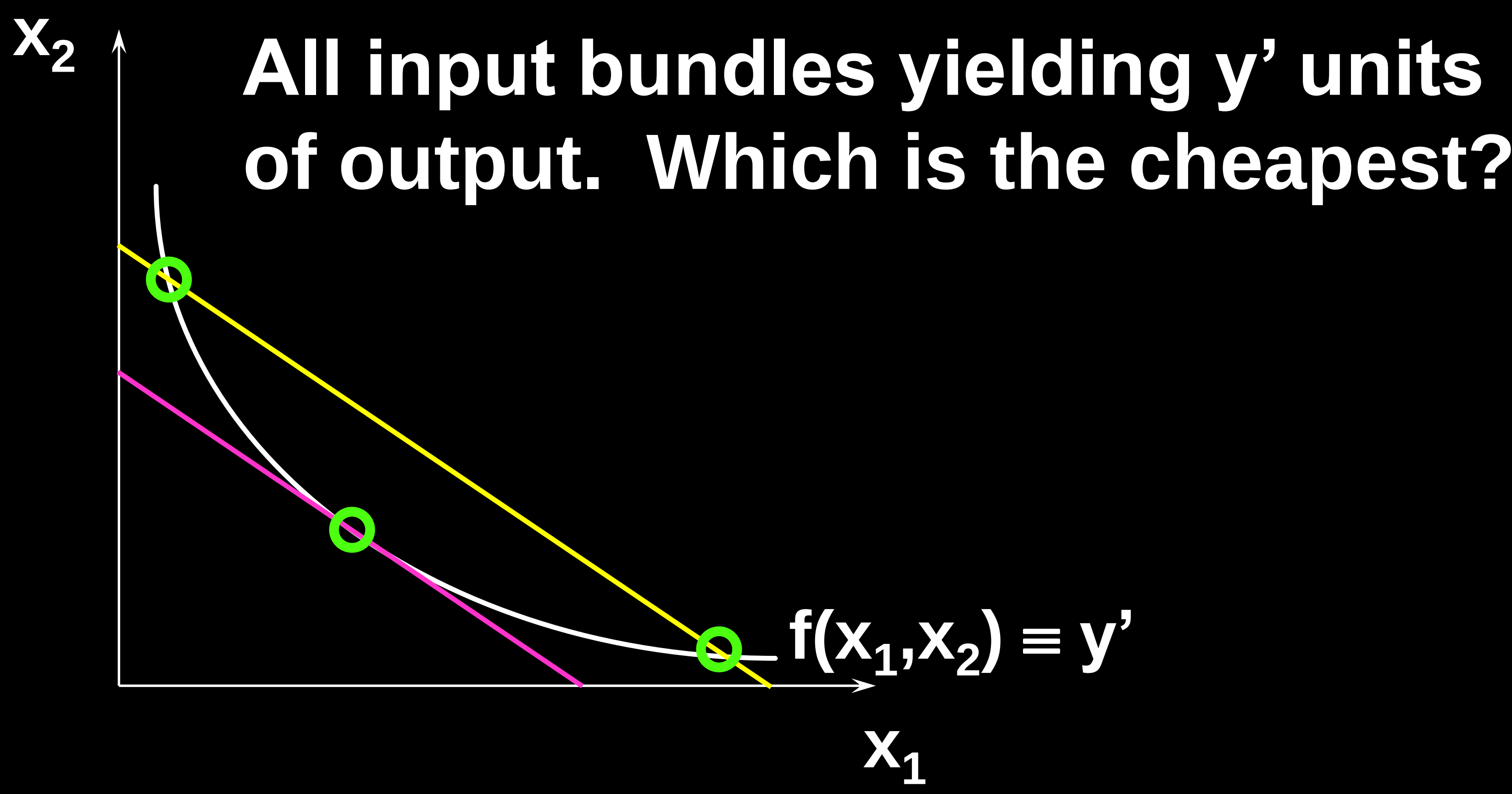
$$x_2 = -\frac{w_1}{w_2}x_1 + \frac{c}{w_2}.$$

Slope is - w_1/w_2.

Iso-cost Lines



The Cost-Minimization Problem



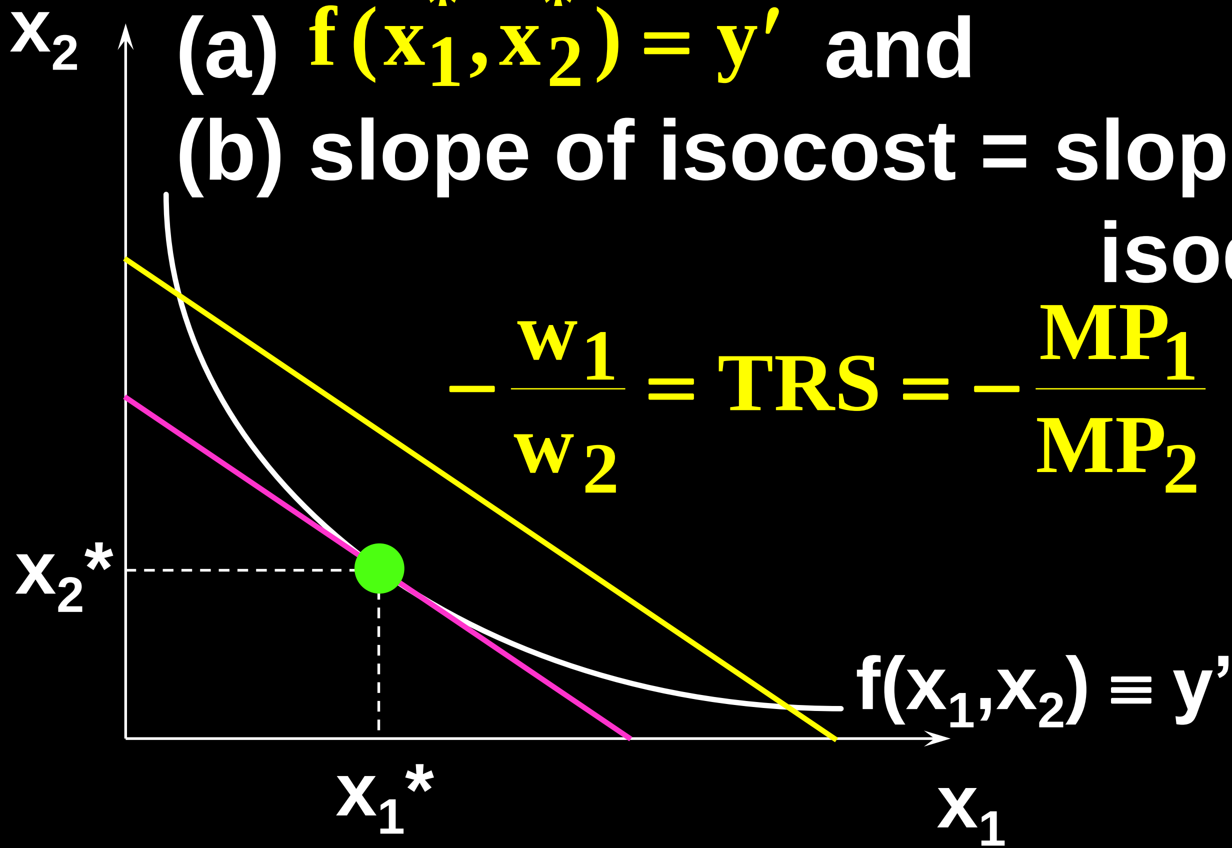
The Cost-Minimization Problem

At an interior cost-min input bundle:

(a) $f(x_1^*, x_2^*) = y'$ and

(b) slope of isocost = slope of isoquant; i.e.

$$-\frac{w_1}{w_2} = TRS = -\frac{MP_1}{MP_2} \text{ at } (x_1^*, x_2^*).$$



A Cobb-Douglas Example of Cost Minimization

- A firm's Cobb-Douglas production function is
$$y = f(x_1, x_2) = x_1^{1/3} x_2^{2/3}.$$
- Input prices are w_1 and w_2.
- What are the firm's conditional input demand functions?

A Cobb-Douglas Example of Cost Minimization

At the input bundle (x_1^*,x_2^*) which minimizes the cost of producing y output units:

(a) $$y = (x_1^*)^{1/3}(x_2^*)^{2/3}$$ **and**

(b)

$$-\frac{w_1}{w_2} = -\frac{\partial y/\partial x_1}{\partial y/\partial x_2} = -\frac{(1/3)(x_1^*)^{-2/3}(x_2^*)^{2/3}}{(2/3)(x_1^*)^{1/3}(x_2^*)^{-1/3}}$$

$$= -\frac{x_2^*}{2x_1^*}.$$

A Cobb-Douglas Example of Cost Minimization

(a) $y = (x_1^*)^{1/3}(x_2^*)^{2/3}$ (b) $\dfrac{w_1}{w_2} = \dfrac{x_2^*}{2x_1^*}.$

Note (b) implies:
$w_1 x_1 / w_2 x_2 = ½$,
which is the ratio the exponential term in the Cobb-Douglas production function.

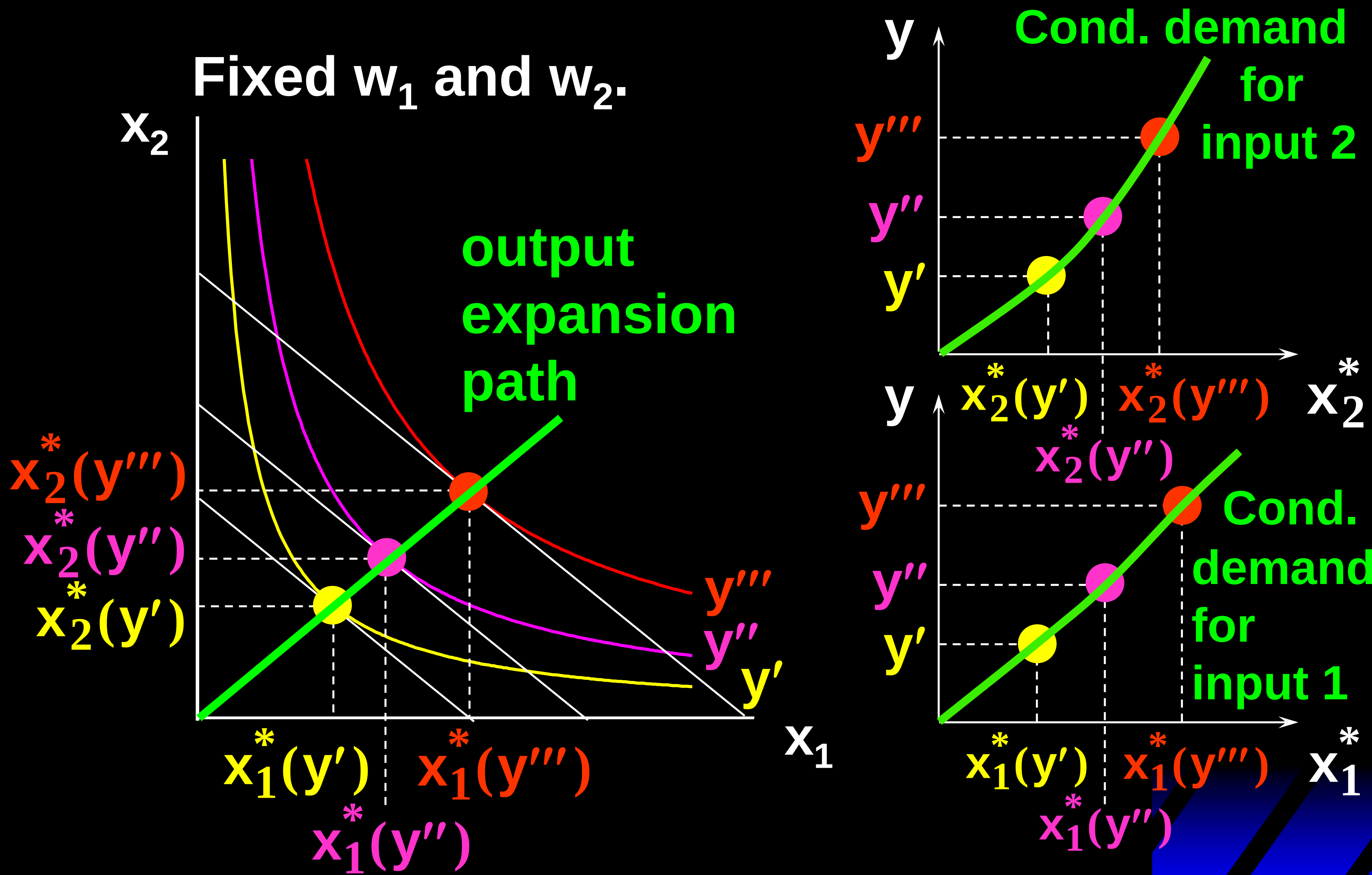
A Cobb-Douglas Example of Cost Minimization

(a) combined with (b) gives:

$$\left(x_1^*(w_1, w_2, y), x_2^*(w_1, w_2, y)\right)$$

$$= \left(\left(\frac{w_2}{2w_1}\right)^{2/3} y, \left(\frac{2w_1}{w_2}\right)^{1/3} y\right).$$

Conditional Input Demand Curves



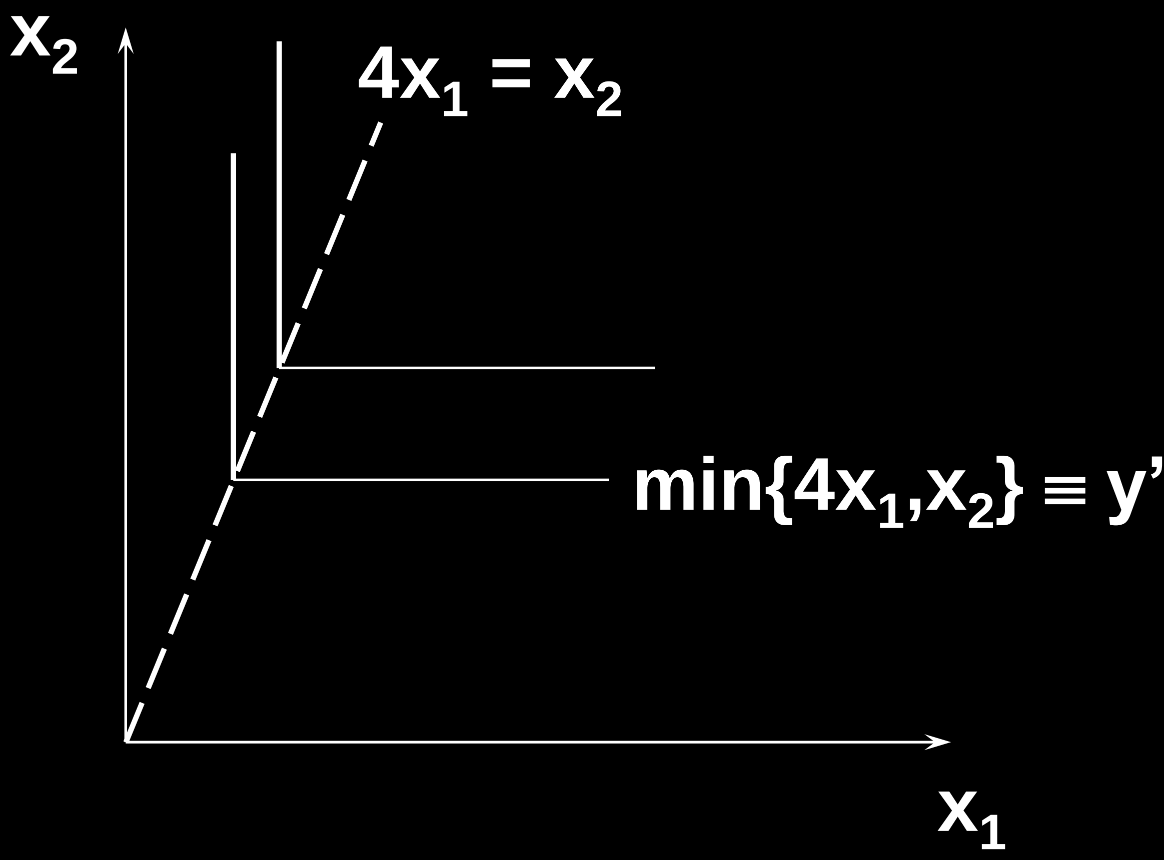
A Perfect Complements Example of Cost Minimization

- The firm's production function is

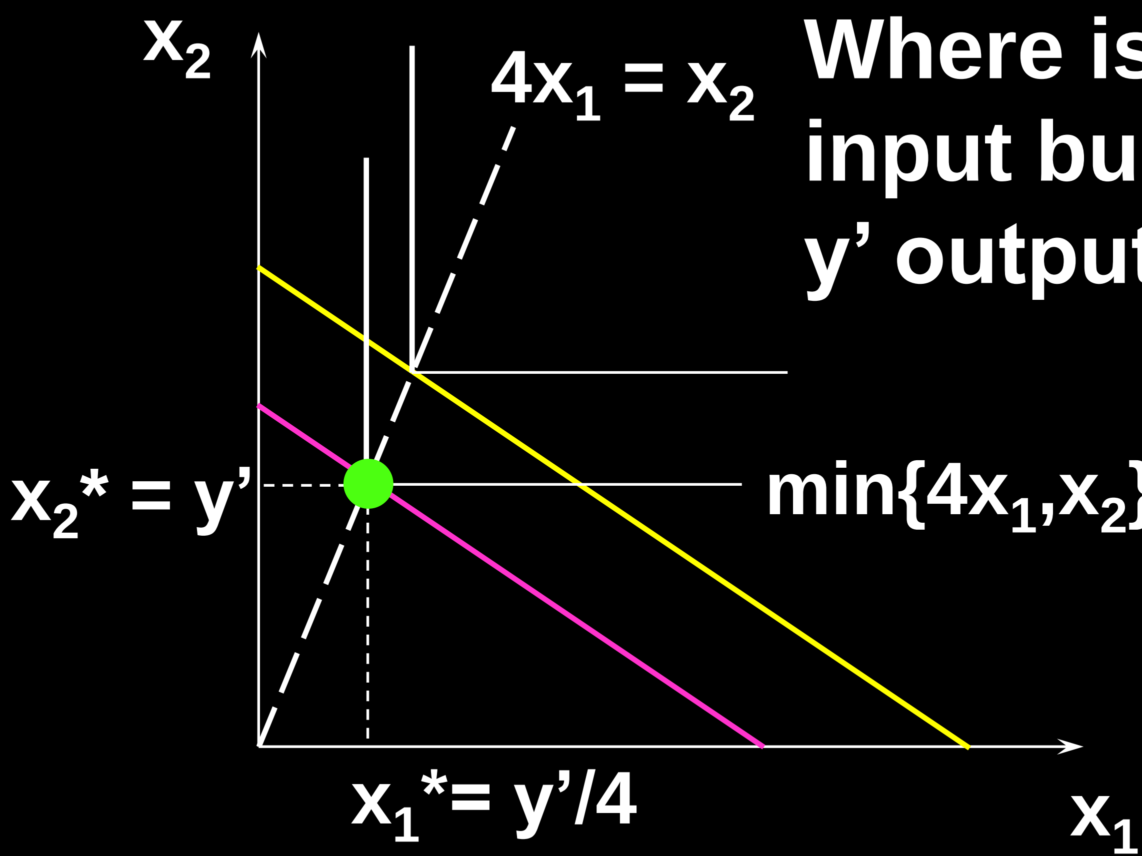
$$y = \min\{4x_1, x_2\}.$$

- Input prices w_1 and w_2 are given.
- What are the firm's conditional demands for inputs 1 and 2?

A Perfect Complements Example of Cost Minimization



A Perfect Complements Example of Cost Minimization



Where is the least costly input bundle yielding y' output units?

A Perfect Complements Example of Cost Minimization

The firm's production function is

$$y = \min\{4x_1, x_2\}$$

and the conditional input demands are

$$x_1^*(w_1, w_2, y) = \frac{y}{4} \quad \text{and} \quad x_2^*(w_1, w_2, y) = y.$$

So the firm's total cost function is

$$c(w_1, w_2, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$$

$$= w_1 \frac{y}{4} + w_2 y = \left(\frac{w_1}{4} + w_2\right) y.$$

Average Total Production Costs

- For positive output levels y, a firm's average total cost of producing y units is

$$AC(w_1, w_2, y) = \frac{c(w_1, w_2, y)}{y}.$$

Constant Returns-to-Scale and Average Total Costs

- If a firm's technology exhibits constant returns-to-scale then doubling its output level from y' to 2y' requires doubling all input levels.
- Total production cost doubles.
- Average production cost does not change.

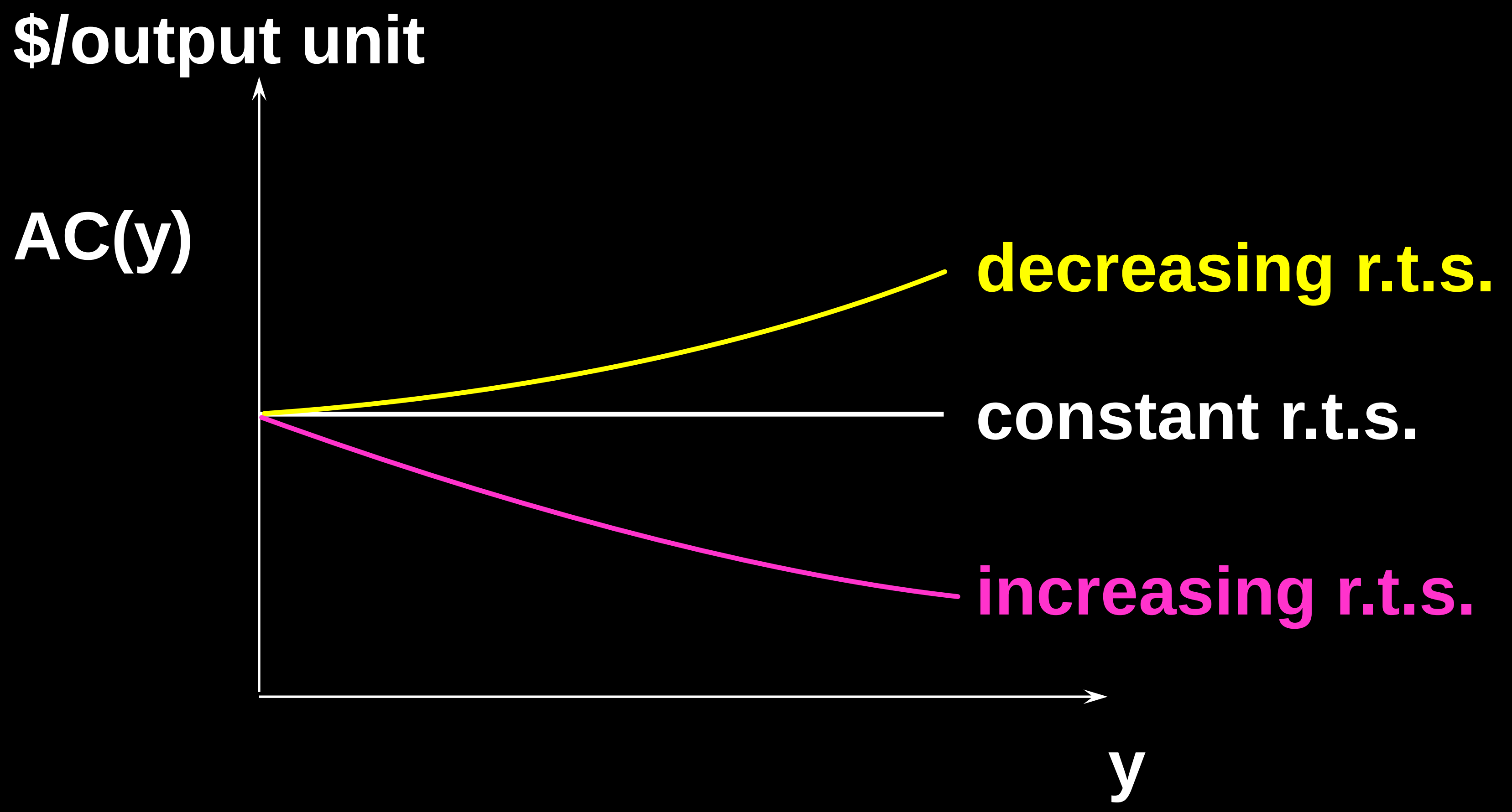
Decreasing Returns-to-Scale and Average Total Costs

- If a firm's technology exhibits decreasing returns-to-scale then doubling its output level from y' to 2y' requires more than doubling all input levels.
- Total production cost more than doubles.
- Average production cost increases.

Increasing Returns-to-Scale and Average Total Costs

- If a firm's technology exhibits increasing returns-to-scale then doubling its output level from y' to 2y' requires less than doubling all input levels.
- Total production cost less than doubles.
- Average production cost decreases.

Returns-to-Scale and Av. Total Costs

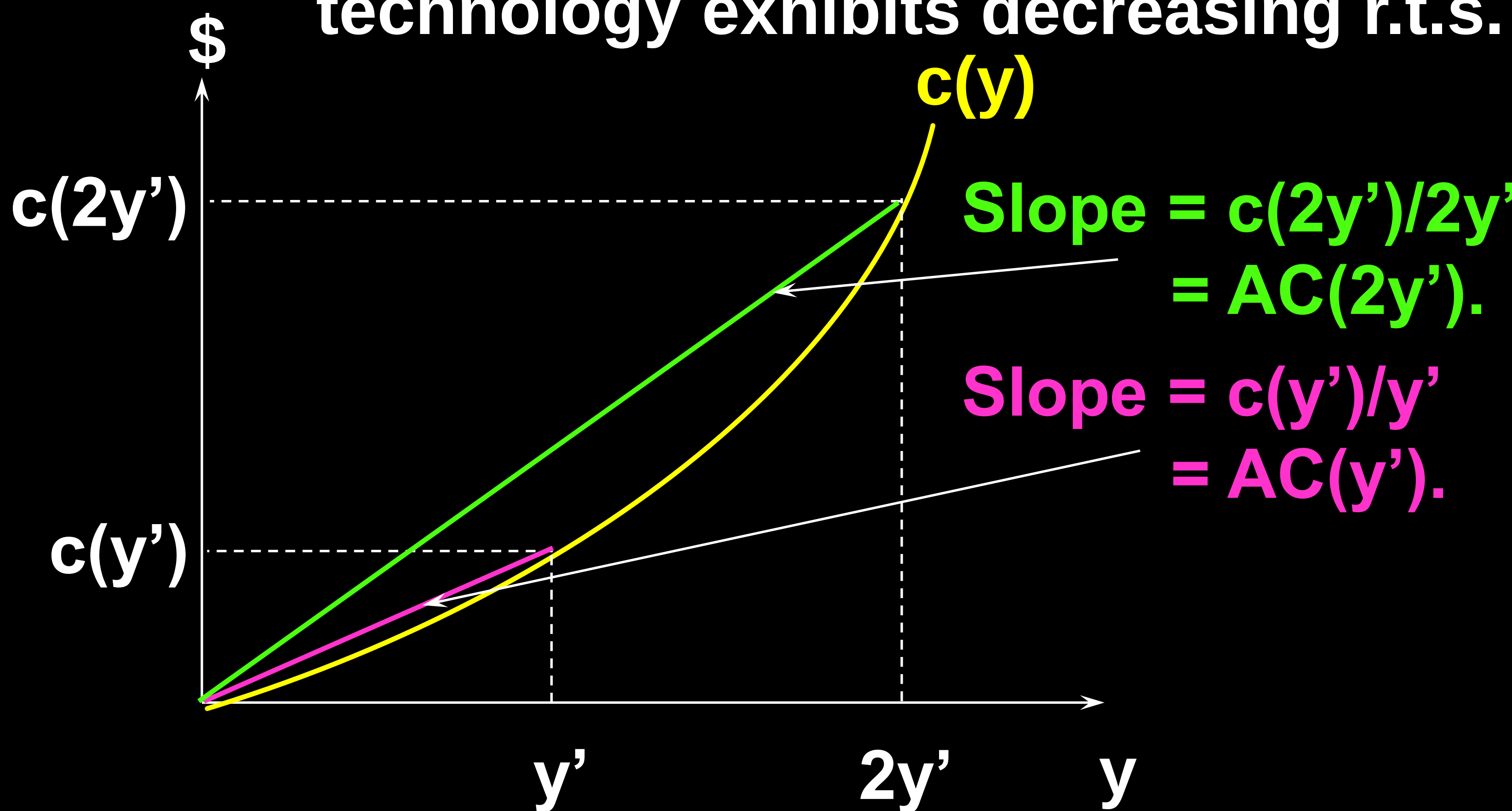


Returns-to-Scale and Total Costs

What does this imply for the shapes of total cost functions?

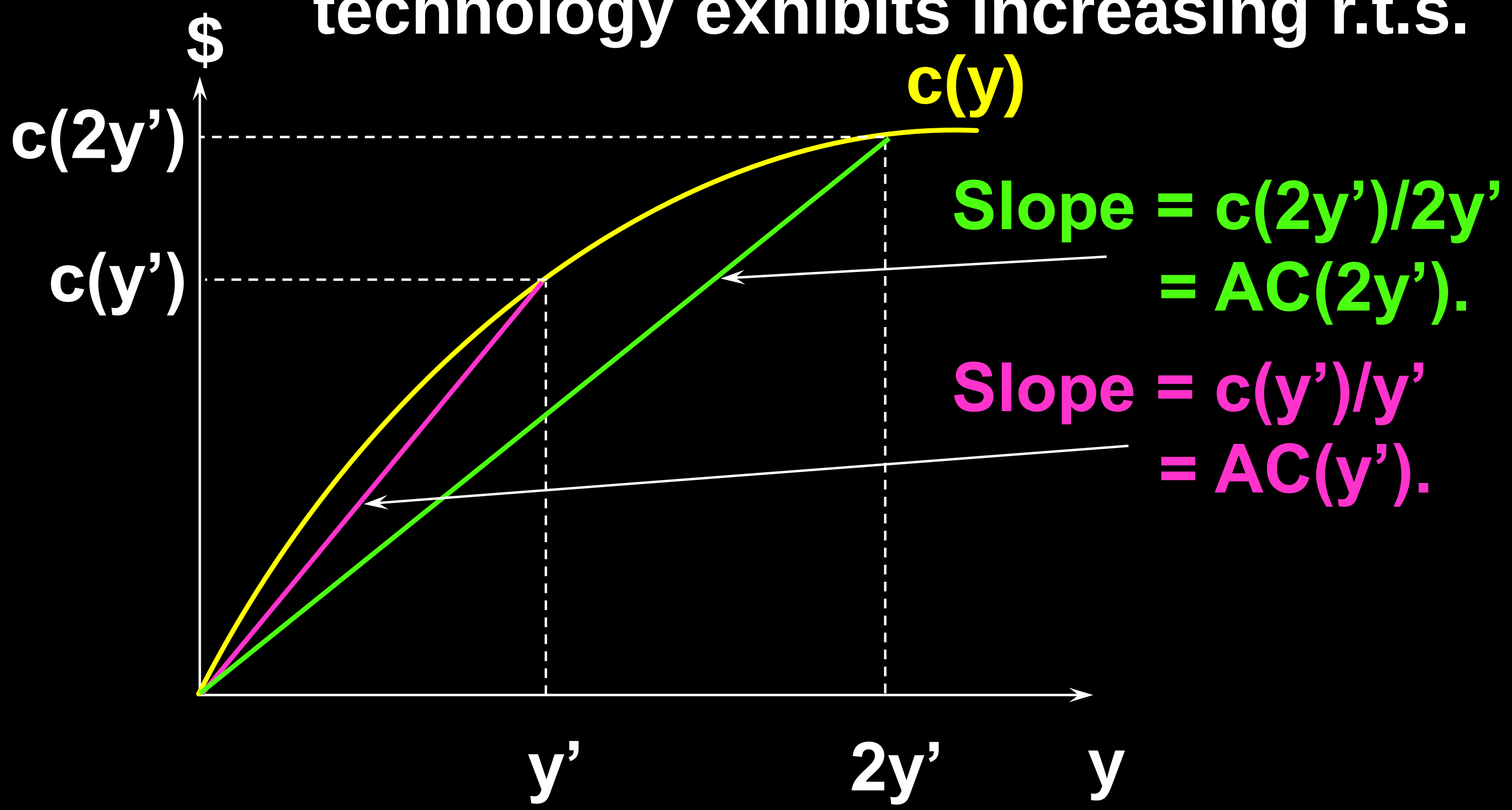
Returns-to-Scale and Total Costs

Av. cost increases with y if the firm's technology exhibits decreasing r.t.s.



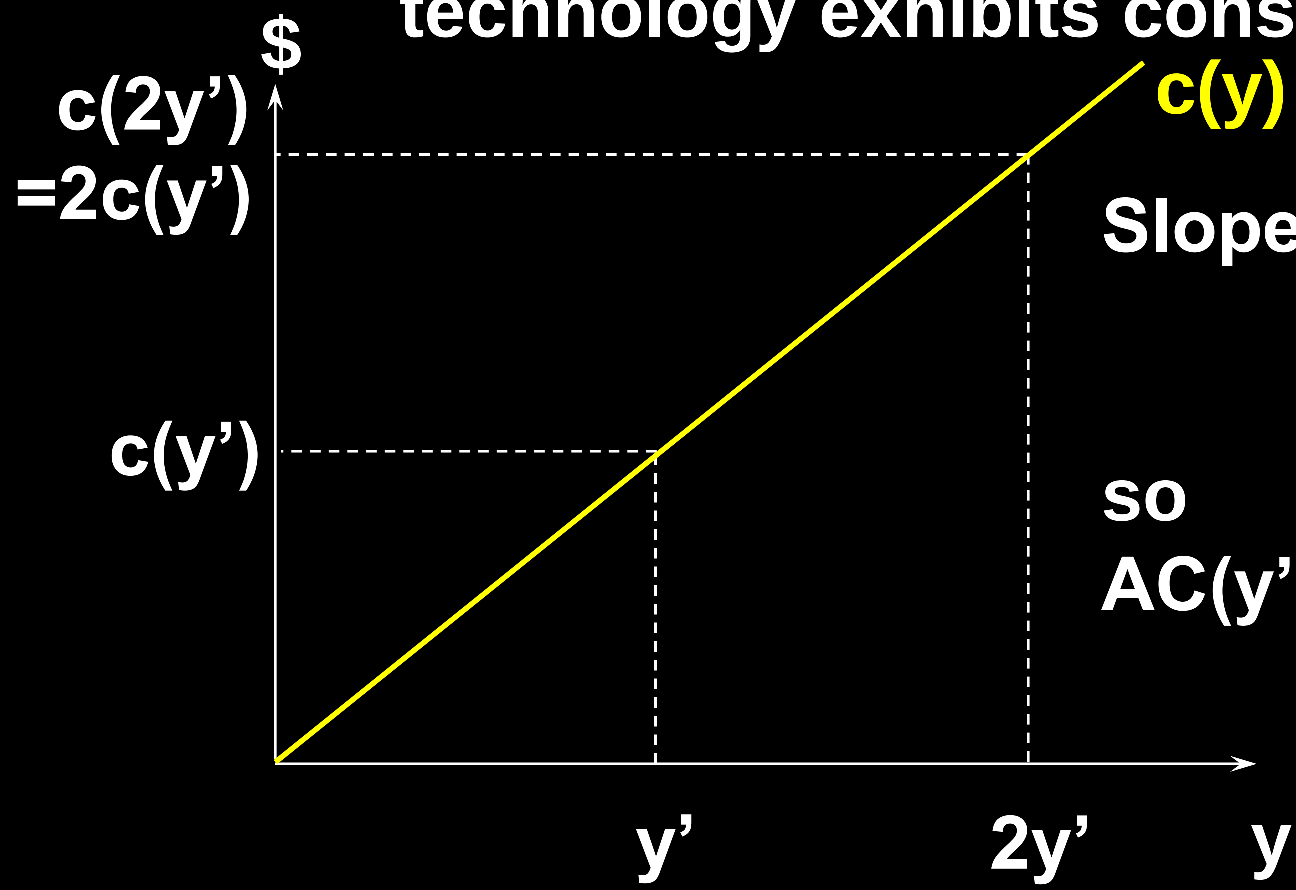
Returns-to-Scale and Total Costs

Av. cost decreases with y if the firm's technology exhibits increasing r.t.s.



Returns-to-Scale and Total Costs

Av. cost is constant when the firm's technology exhibits constant r.t.s.



Slope = c(2y')/2y'
= 2c(y')/2y'
= c(y')/y'

so

AC(y') = AC(2y').

Short-Run & Long-Run Total Costs

- In the long-run a firm can vary all of its input levels.
- In the short run, suppose that input 2 level is fixed at x_2' units.
- How does the short-run total cost of producing y output units compare to the long-run total cost of producing y units of output?

Short-Run & Long-Run Total Costs

- The long-run cost-minimization problem is $$\min_{x_1,x_2 \geq 0} \; w_1x_1 + w_2x_2$$ subject to $f(x_1, x_2) = y.$
- The short-run cost-minimization problem is $$\min_{x_1 \geq 0} \; w_1x_1 + w_2x_2'$$ subject to $f(x_1, x_2') = y.$

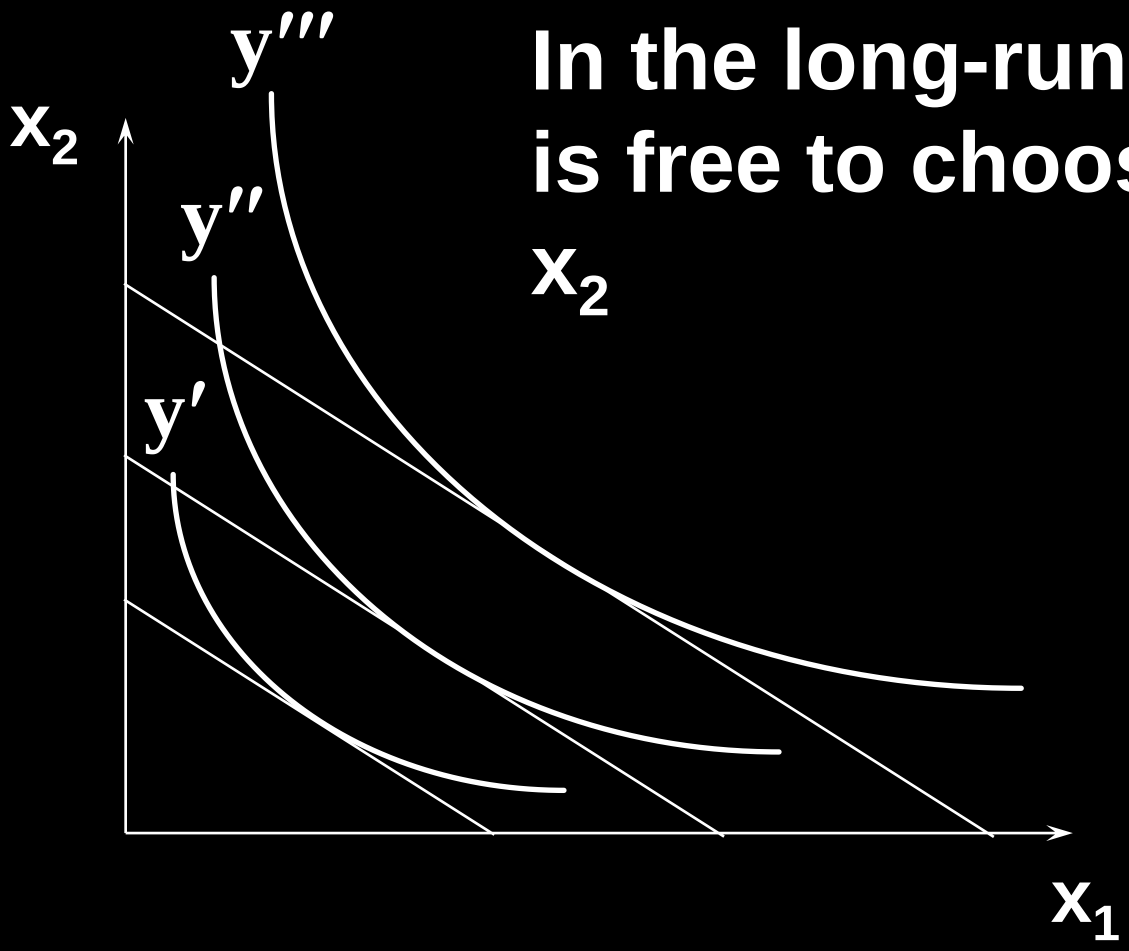
Short-Run & Long-Run Total Costs

- The short-run cost-min. problem can be viewed as the long-run problem subject to the extra constraint that $x_2 = x_2'$.
- If the long-run choice for x_2 happens to be x_2', then the extra constraint $x_2 = x_2'$ won't have any effect, and so the long-run and short-run total costs are the same.

Short-Run & Long-Run Total Costs

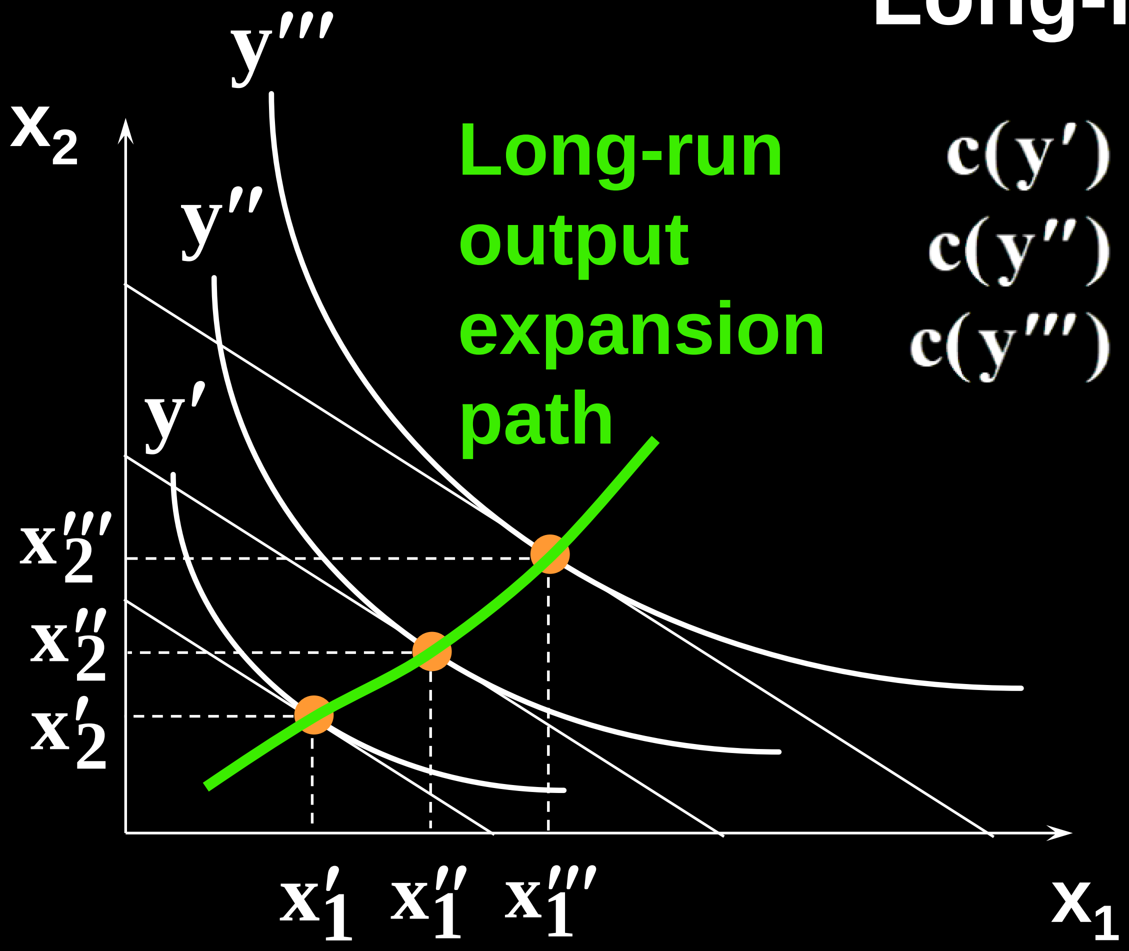
But, more likely, the long-run choice for $x_2 \neq x_2'$. Then the extra constraint $x_2 = x_2'$ prevents the firm in the short-run from achieving its long-run cost minimization, causing the short-run total cost to exceed the long-run total cost.

Short-Run & Long-Run Total Costs



In the long-run when the firm is free to choose both x_1 and x_2

Short-Run & Long-Run Total Costs



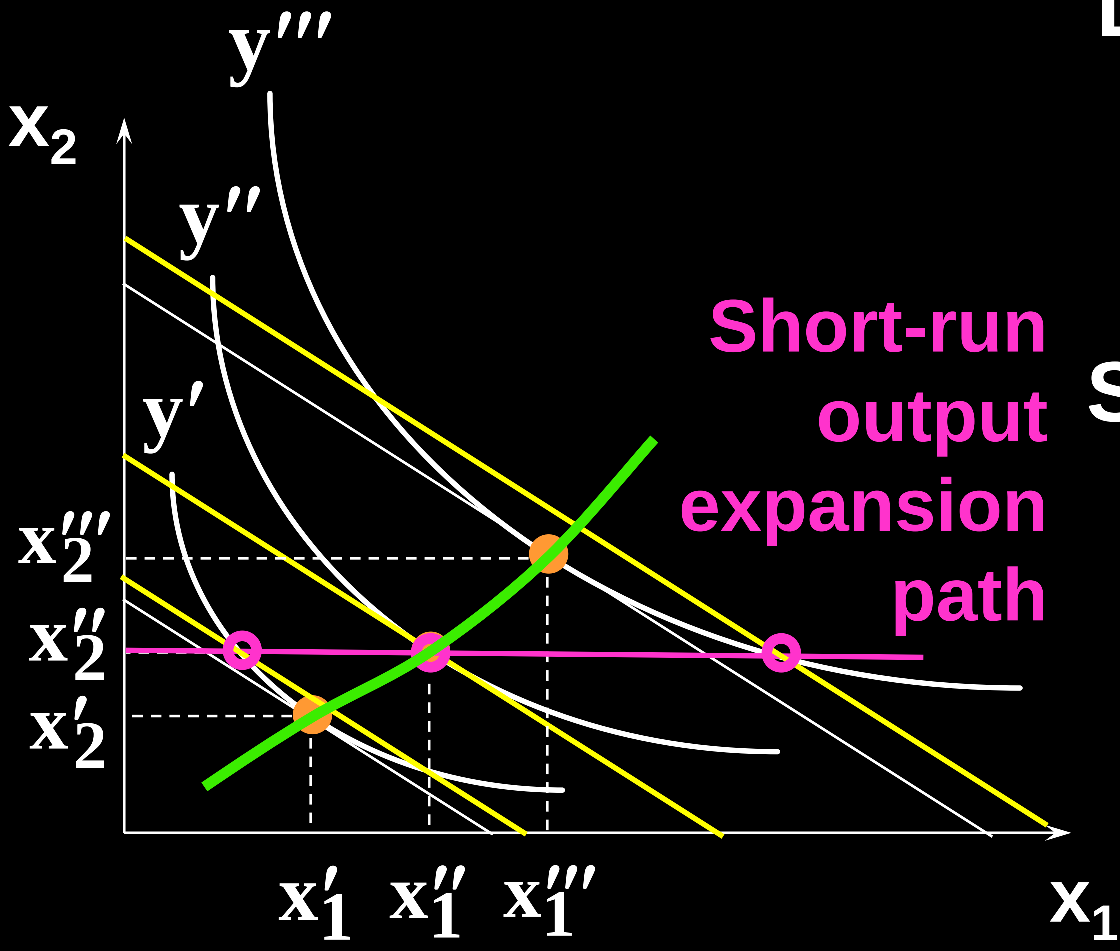
Long-run costs are:

$$c(y') = w_1 x_1' + w_2 x_2'$$
$$c(y'') = w_1 x_1'' + w_2 x_2''$$
$$c(y''') = w_1 x_1''' + w_2 x_2'''$$

Short-Run & Long-Run Total Costs

- **Now suppose the firm becomes subject to the short-run constraint that $x_2 = x_2$”.**

Short-Run & Long-Run Total Costs



Long-run costs are:

$$c(y') = w_1 x_1' + w_2 x_2'$$
$$c(y'') = w_1 x_1'' + w_2 x_2''$$
$$c(y''') = w_1 x_1''' + w_2 x_2'''$$

Short-run costs are:

$$c_s(y') > c(y')$$
$$c_s(y'') = c(y'')$$
$$c_s(y''') > c(y''')$$

Short-Run & Long-Run Total Costs

Short-run total cost exceeds long-run total cost except for the output level where the short-run input level restriction is at the long-run optimal input level choice.

Short-Run & Long-Run Total Costs

A short-run total cost curve typically has one point in common with the long-run total cost curve, and is elsewhere higher than the long-run total cost curve.