1.

(6) $(\exists x)(P(x)\to Q(x))$

$=(\exists x)(\neg P(x)\vee Q(x))$

$=\neg(\forall x)(P(x)\wedge\neg Q(x))$

$=\neg(\forall x)P(x)\vee\neg(\forall x)\neg Q(x)$

$=\neg(\forall x)P(x)\vee(\exists x)Q(x)$

$=(\forall x)P(x)\to(\exists x)Q(x)$

(7) $(\exists x)P(x)\to(\forall x)Q(x)$

$=\neg(\exists x)P(x)\vee(\forall x)Q(x)$

$=(\forall x)\neg P(x)\vee(\forall x)Q(x)$

$\Rightarrow(\forall x)(\neg P(x)\vee Q(x))$

$=(\forall x)(P(x)\to Q(x))$

(8) $(\exists x)P(x)\wedge(\forall x)Q(x)$

$=(\exists x)P(x)\wedge(\forall y)Q(y)$

$=(\exists x)(P(x)\wedge(\forall y)Q(y))$

$\Rightarrow(\exists x)(P(x)\wedge Q(x))$

(9) $((\forall x)P(x)\wedge(\forall x)Q(x)\wedge(\exists x)R(x))$

$\vee((\forall x)P(x)\wedge(\forall x)Q(x)\wedge(\exists x)S(x))$

$=((\forall x)P(x)\wedge(\forall x)Q(x)\wedge(\exists z)R(z))$

$\vee((\forall x)P(x)\wedge(\forall x)Q(x)\wedge(\exists u)S(u))$

$=((\forall x)(P(x)\wedge Q(x))\wedge(\exists z)R(z))$

$\vee((\forall x)(P(x)\wedge Q(x))\wedge(\exists u)S(u))$

$=(\forall x)(P(x)\wedge Q(x))\wedge((\exists z)R(z)\vee(\exists u)S(u))$

$=(\forall x)(P(x)\wedge Q(x))\wedge(\exists x)(R(x)\vee S(x))$

2.

(3) 普遍有效

证：$(\exists x)P(x) \rightarrow (\forall x)Q(x)$

$= \neg(\exists x)P(x) \vee (\forall x)Q(x)$

$= (\forall x)\neg P(x) \vee (\forall x)Q(x)$

$\Rightarrow (\forall x)(\neg P(x) \vee Q(x))$

$= (\forall x)(P(x) \rightarrow Q(x))$

因此 $((\exists x)P(x) \rightarrow (\forall x)Q(x))$

$\rightarrow ((\forall x)(P(x) \rightarrow Q(x)))$

为普遍有效的

(7) 反例：

在{1, 2}域上：

$P(1) = T$, $P(2) = F$

$Q(1) = F$, $Q(2) = T$

则 $(\exists x)P(x) \wedge (\exists x)Q(x) = T$

$(\exists x)(P(x) \wedge Q(x)) = F$

因此公式不是普遍有效的

(8) 反例：

在{1, 2}域上

$P(1, 1) = T$ $P(1, 2) = F$

$P(2, 1) = F$ $P(2, 2) = T$

则 $(\forall x)(\exists y)P(x, y) = T$

$(\exists y)(\forall x)P(x, y) = F$

因此公式不是普遍有效的

4.

(2) $(\forall x)(\forall y)(\forall z)\big(P(x,y,z)\land((\exists u)Q(x,u)\to(\exists w)Q(y,w))\big)$

$=(\forall x)(\forall y)(\forall z)\big(P(x,y,z)\land((\forall u)\neg Q(x,u)\lor(\exists w)Q(y,w))\big)$

$=(\forall x)(\forall y)(\forall z)(\forall u)(\exists w)\big(P(x,y,z)\land(\neg Q(x,u)\lor Q(y,w))\big)$

(7) $(\exists x)(\exists y)P(x,y)\to(\exists y)(\exists x)P(x,y))$

$=(\forall x)(\forall y)\neg P(x,y)\lor(\exists x)(\exists y)P(x,y))$

$=(\forall x)(\forall y)\neg P(x,y))\lor(\exists z)(\exists u)P(z,u)$

$=(\exists z)(\exists u)(\forall x)(\forall y)(\neg P(x,y))\lor P(z,u))$

(9) $(\forall x)\big(P(x)\to(\exists y)Q(x,y)\big)\lor(\forall z)R(z)$

skolem范式：$(\forall x)(\forall z)\big(\neg P(x)\lor Q(x,f(x))\lor R(z)\big)$

5.

(3)

① 推理规则法：

前提 $(\forall x)(P(x) \vee Q(x))$

$(\forall x)(Q(x) \to \neg R(x))$

$(\forall x) R(x)$

结论：$(\forall x) P(x)$

(1) $(\forall x) R(x)$ 前提

(2) $R(x)$ 全称量词消去

(3) $(\forall x)(Q(x) \to \neg R(x))$ 前提

(4) $Q(x) \to \neg R(x)$ 全称量词消去

(5) $R(x) \to \neg Q(x)$ (4)置换

(6) $\neg Q(x)$ (2)(5)假言

(7) $(\forall x)(P(x) \vee Q(x))$ 前提

(8) $P(x) \vee Q(x)$ 全称量词消去

(9) $P(x)$ (6)(8)

(10) $(\forall x) P(x)$ 全称量词引入

② 归结法：

$G = (\forall x)(P(x) \vee Q(x)) \wedge (\forall x)(Q(x) \to \neg R(x)) \wedge (\forall x) R(x) \wedge \neg(\forall x) P(x)$

子句集：$\{P(x) \vee Q(x),\ \neg Q(x) \vee \neg R(x),\ R(x),\ \neg P(a)\}$

(1) $P(x) \vee Q(x)$

(2) $\neg Q(x) \vee \neg R(x)$

(3) $P(x) \vee \neg R(x)$ (1)(2)归结

(4) $R(x)$

(5) $P(x)$ (3)(4)归结

(6) $\neg P(a)$

(7) $\square$ (5)(6)归结

(4)

$P(x)$: x是学生, $Q(x)$: x是本科生, $R(x)$: x是研究生, $S(x)$: x是高材生

① 推理规则法:

前提: $(\forall x)(P(x) \rightarrow (Q(x) \overline{\vee} R(x)))$

$(\exists x)(P(x) \land R(x))$

$\neg R(John)$, $S(John)$

结论: $P(John) \rightarrow Q(John)$

(1)	$\neg R(John)$	前提
(2)	$P(John)$	额外前提引入
(3)	$(\forall x)(P(x) \rightarrow (Q(x) \overline{\vee} R(x)))$	前提
(4)	$P(John) \rightarrow (Q(John) \overline{\vee} R(John))$	(3) 全称量词消去
(5)	$Q(John) \overline{\vee} R(John)$	(2)(4) 分离
(6)	$Q(John)$	(3)(7)
(7)	$P(John) \rightarrow Q(John)$	条件证明规则

② 归结法:

$G = (\forall x)(P(x) \rightarrow (Q(x) \overline{\vee} R(x))) \land (\exists x)(P(x) \land R(x)) \land \neg R(John) \land S(John)$

$\land \neg (P(John) \rightarrow Q(John))$

子句集: $\{ \neg P(x) \vee (Q(x) \overline{\vee} R(x))$, $P(a)$, $R(a)$, $\neg R(John)$, $S(John)$,

$P(John)$, $\neg Q(John) \}$

(1)	$\neg P(x) \vee (Q(x) \overline{\vee} R(x))$	
(2)	$P(John)$	
(3)	$Q(John) \overline{\vee} R(John)$	(1)(2) 归结
(4)	$\neg Q(John)$	
(5)	$R(John)$	(3)(4) 归结
(6)	$\neg R(John)$	
(7)	$\square$	(5)(6) 归结