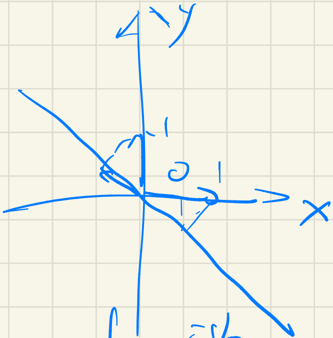
HW for Week 1

1.1

①



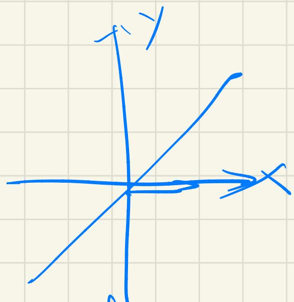
$\begin{bmatrix}1\\0\end{bmatrix} \xrightarrow{f} \begin{bmatrix}-1/2\\-1/2\end{bmatrix}$

$\begin{bmatrix}0\\1\end{bmatrix} \longrightarrow \begin{bmatrix}-1/2\\+1/2\end{bmatrix}$

$\begin{bmatrix}-1/2 & -1/2\\-1/2 & 1/2\end{bmatrix}$

Surjective

②



$\begin{bmatrix}1\\0\end{bmatrix} \xrightarrow{f} \begin{bmatrix}0\\1\end{bmatrix}$

$\begin{bmatrix}0\\1\end{bmatrix} \longrightarrow \begin{bmatrix}1\\0\end{bmatrix}$

$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$

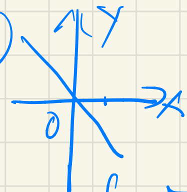
Bijective

③ not linear

④ $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ Bijective

⑤ not linear

⑥



$\begin{bmatrix}\pm1\\0\end{bmatrix} \xrightarrow{f} \pm\frac{\sqrt{2}}{2}$

$\begin{bmatrix}0\\\pm1\end{bmatrix} \xrightarrow{f} \pm\frac{\sqrt{2}}{2}$

$\begin{bmatrix}\frac{\sqrt{2}}{2}, & \frac{\sqrt{2}}{2}\end{bmatrix}$

Surjective

⑦ $\begin{bmatrix}2\\3\end{bmatrix}$ Bijective

1.2

① $\text{sum} = 1 + \cdots + 9 = 45$

$45/3 = 15$

$$M = \begin{bmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 \end{bmatrix} \begin{matrix} =15 \\ =15 \\ =15 \end{matrix}$$

$$\begin{matrix} \| & \| & \| \\ 15 & 15 & 15 \end{matrix}$$

$$\therefore M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}m_1\\m_4\\m_7\end{bmatrix} + \begin{bmatrix}m_2\\m_5\\m_8\end{bmatrix} + \begin{bmatrix}m_3\\m_6\\m_9\end{bmatrix}$$

$$= \begin{bmatrix}15\\15\\15\end{bmatrix}$$

② the sum of each row $= 45$

$$\therefore M\begin{bmatrix}1\\ \vdots \\1\end{bmatrix} = \underbrace{\begin{bmatrix}45\\ \vdots \\45\end{bmatrix}}_{\to 9 \text{ rows}}$$

1.3

① $\text{sum1} = \begin{bmatrix}0\\0\end{bmatrix}$

② $\text{sum2} = -\vec{x}_2 = -\begin{bmatrix}\sqrt{3}/2\\1/2\end{bmatrix}$

$$= \begin{bmatrix}-\sqrt{3}/2\\-1/2\end{bmatrix}$$

③ $\text{sum3} = \text{sum1} + 12(-\vec{x}_6)$

$$= \begin{bmatrix}0\\0\end{bmatrix} + 12\left(-\begin{bmatrix}0\\-1\end{bmatrix}\right)$$

$$= \begin{bmatrix}0\\12\end{bmatrix}$$

1.4

① $\vec{b} = \begin{bmatrix}1 & 0 & 1\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}2\\4\\5\end{bmatrix}$

② $\vec{b} = \begin{bmatrix}1 & 5\\2 & 4\\3 & 3\\4 & 2\\5 & 1\end{bmatrix}\begin{bmatrix}5\\4\end{bmatrix}$

③ $\vec{b} = \begin{bmatrix}1 & 2 & 1\\0 & 4 & 1\\1 & 1 & 1\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}$

④ $\vec{b} = \begin{bmatrix}0 & 0 & 0 & 0 & 5\\0 & 0 & 0 & 4 & 0\\0 & 0 & 3 & 0 & 0\\0 & 2 & 0 & 0 & 8\\1 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix}$

$$= \begin{bmatrix}5\\8\\9\\8\\5\end{bmatrix}$$

⑤ today   tomorrow

0.5 ✓ 0.5×0.8

0.5 ✗ $+ \; 0.5 \times 0.3 = 0.4 + 0.15$

$= 0.55$

$$\therefore \vec{b} = \begin{bmatrix}0.8 & 0.3\\0.2 & 0.7\end{bmatrix}\begin{bmatrix}0.5\\0.5\end{bmatrix}$$

$$= \begin{bmatrix}0.55\\0.45\end{bmatrix}$$

1.5

① original set: $\{\vec{a} + t(\vec{b} - \vec{a}) ; t \in R\}$

$\vec{a} + t(\vec{b} - \vec{a}) = (1-t)\vec{a} + t\vec{b}$

$\exists s \in R,\ s = 1-t \Rightarrow s\vec{a} + t\vec{b},\ s+t=1$

$\Rightarrow$ target set: $\{s\vec{a} + t\vec{b} ; s, t \in R$ and $s+t=1\}$

②: from ① $\Rightarrow \underbrace{\exists s, t \in R}_{\text{at least one pair}},\ \vec{p} = s\vec{a} + t\vec{b},\ s+t=1$

Then prove its uniqueness:

suppose: $\exists s', t' \in R,\ \vec{p} = s'\vec{a} + t'\vec{b},\ s'+t'=1$

$\Rightarrow s'\vec{a} + t'\vec{b} = s\vec{a} + t\vec{b}$

$s'\vec{a} + (1-s')\vec{b} = s\vec{a} + (1-s)\vec{b}$

$(s'-s)\vec{a} + (s-s')\vec{b} = \vec{0}$

since $\vec{a} \neq k\vec{b}$, then $\vec{a}, \vec{b} \neq \vec{0}$

so $s'-s=0,\ s-s'=0$

$\therefore s' = s$

$t' = t$

$\therefore$ done

③ suppose: $s < 0$

then: $t > 1$

$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$ $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

$\vec{p} = s\vec{a} + t\vec{b} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$

$\therefore p_1 = sa_1 + tb_1 = b_1 + [(t-1)b_1 + (1-t)a_1]$

if $a_1 > b_1$, $p_1 < b_1$ ✗

if $a_1 < b_1$, $p_1 > b_1$ ✗

$\therefore s, t \in [0,1] \Rightarrow$ done

④ $\{s\vec{a} + t\vec{b} + u\vec{c} : s, t, u \in R,$ $s+t+u=1\}$

⑤ $\{s\vec{a} + t\vec{b} + u\vec{c} : s, t, u \in R,\ s+t+u=1,$ $s, t, u \geq 0$ and $s, t, u \leq 1\}$

1.6

① $\vec{a} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$, $\vec{c} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

② from ① $\Downarrow$

equation for the plane: $ax + by + cz = t$

then $\begin{cases} 6a = t \\ 3b = t \\ 2c = t \end{cases} \Rightarrow$ let $a = 1$ (it's okay $a \neq 1$, I can get the same answer), so $\begin{cases} b = 2 \\ c = 3 \\ t = 6 \end{cases}$

$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + 2y + 3z = 6 \right\}$ is a plane through $\vec{a}, \vec{b}, \vec{c}$

③ $\vec{a} - \vec{b} = \begin{bmatrix} 6 \\ -3 \\ 0 \end{bmatrix}$ $\vec{b} - \vec{c} = \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}$ $\vec{c} - \vec{a} = \begin{bmatrix} -6 \\ 0 \\ 2 \end{bmatrix}$

let $\vec{t} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\vec{t} \cdot (\vec{a} - \vec{b}) = 6 \times 1 + (-3) \times 2 + 0 \times 3 = 0$

$\therefore \vec{t} \perp (\vec{a} - \vec{b})$

$\vec{t} \cdot (\vec{b} - \vec{c}) = 0 \times 1 + 2 \times 3 + 3 \times (-2) = 0$

$\therefore \vec{t} \perp (\vec{b} - \vec{c})$

$\vec{t} \cdot (\vec{c} - \vec{a}) = (-6) \times 1 + 0 \times 2 + 2 \times 3 = 0$

$\therefore \vec{t} \perp (\vec{c} - \vec{a})$