These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Five

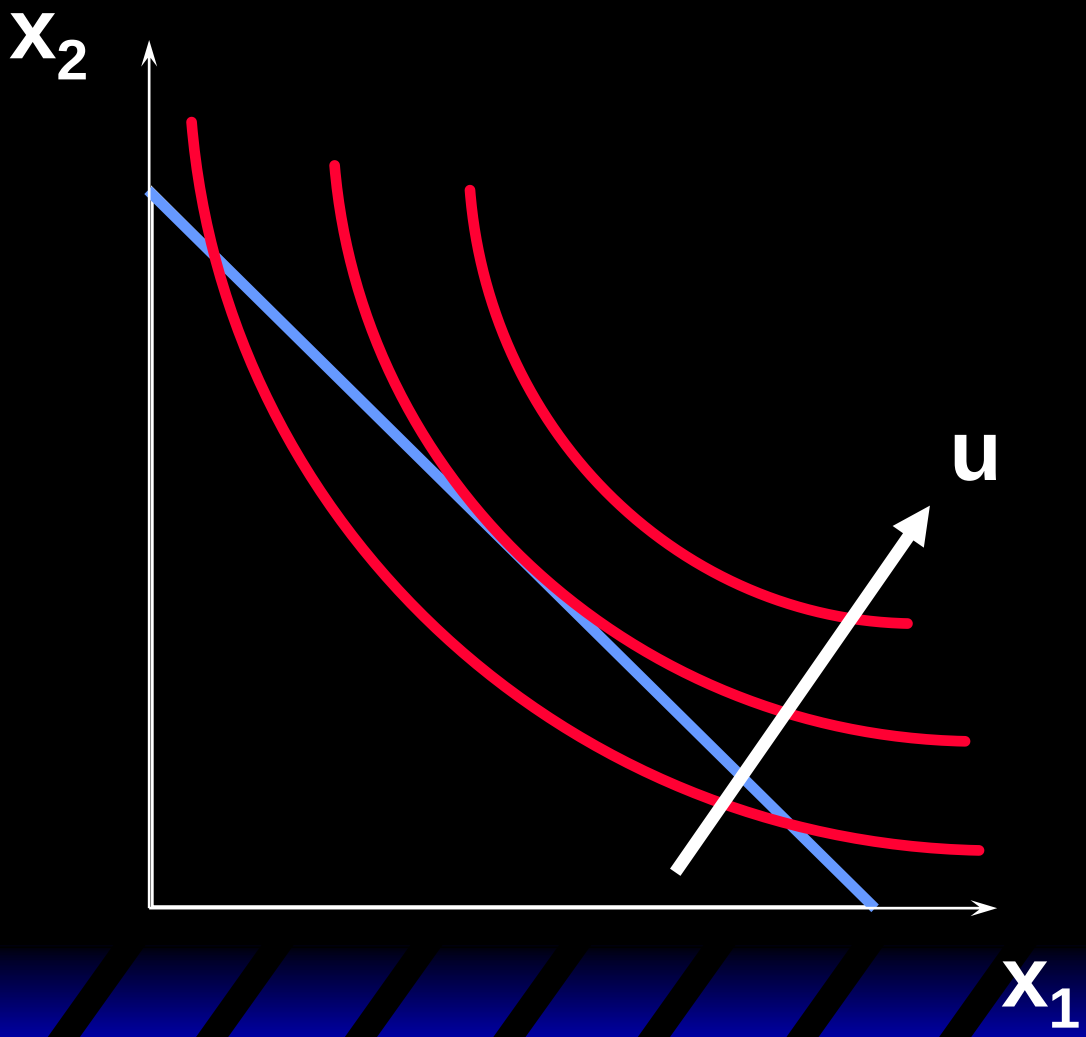
Choice
消费者最优选择

Utility Functions + Budget Constraint

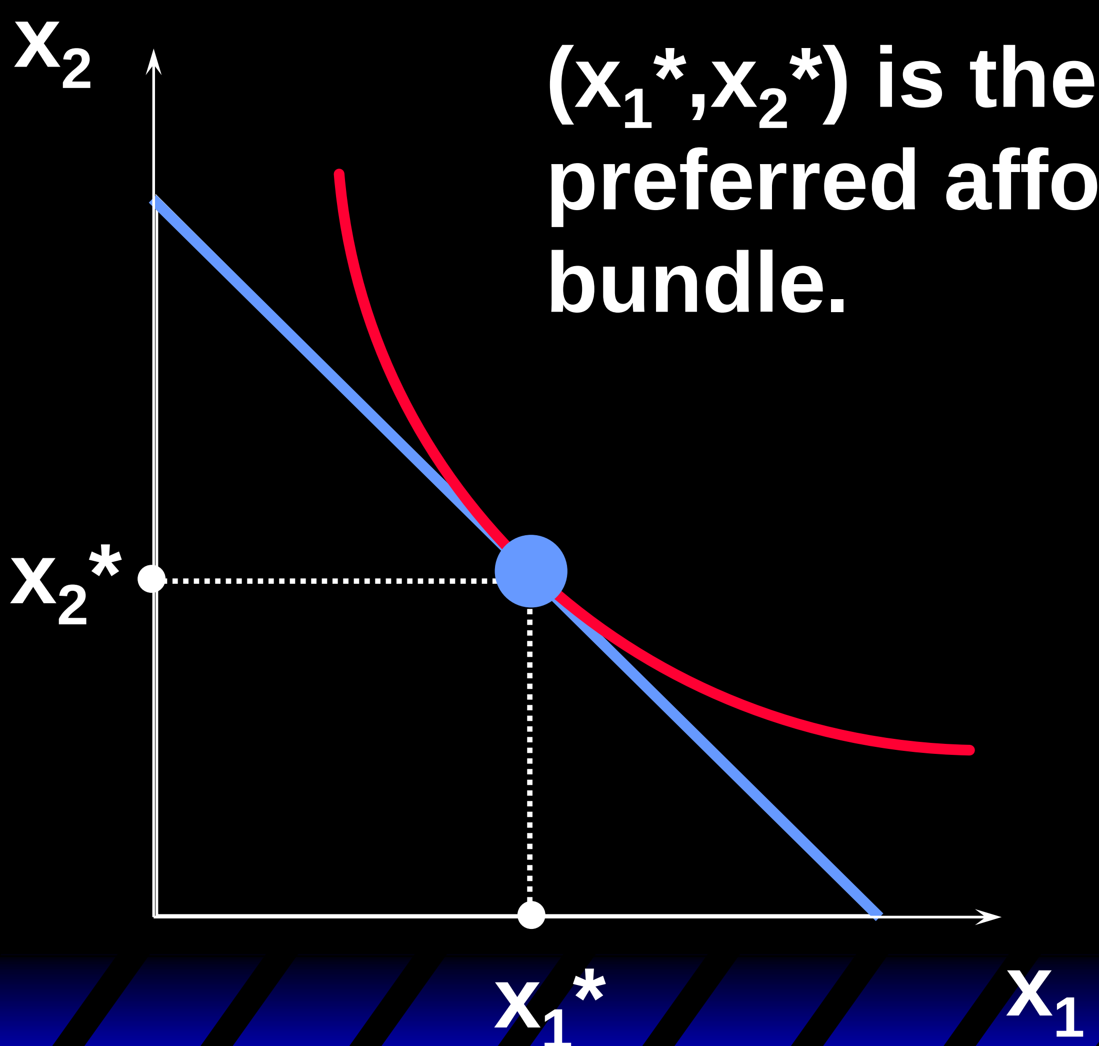
The consumer makes her choice as if she is solving a constraint maximization problem.

- **Objective function: Utility**
- **Constraint: Budget**

Rational Constrained Choice

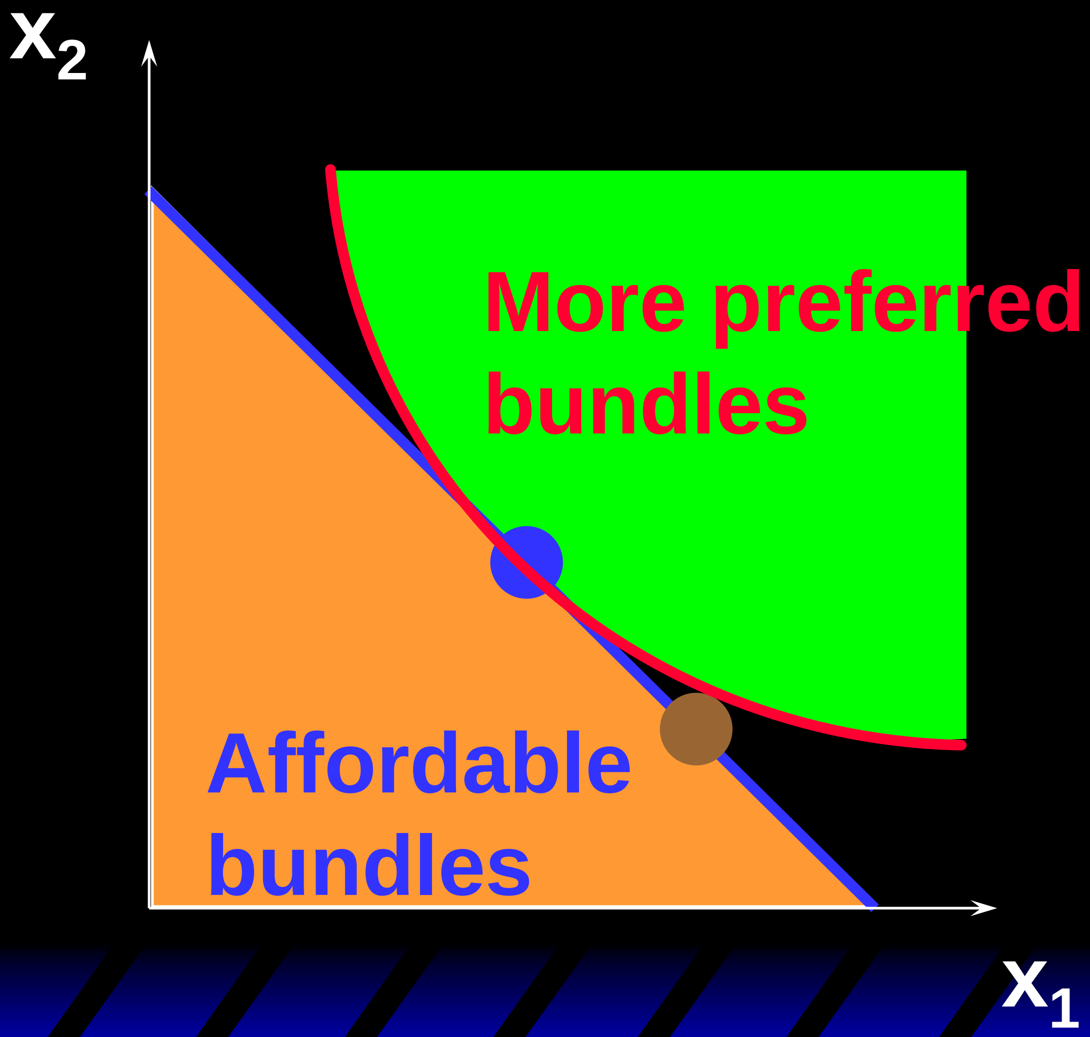


Rational Constrained Choice



(x_1^*,x_2^*) is the most preferred affordable bundle.

Rational Constrained Choice



Rational Constrained Choice

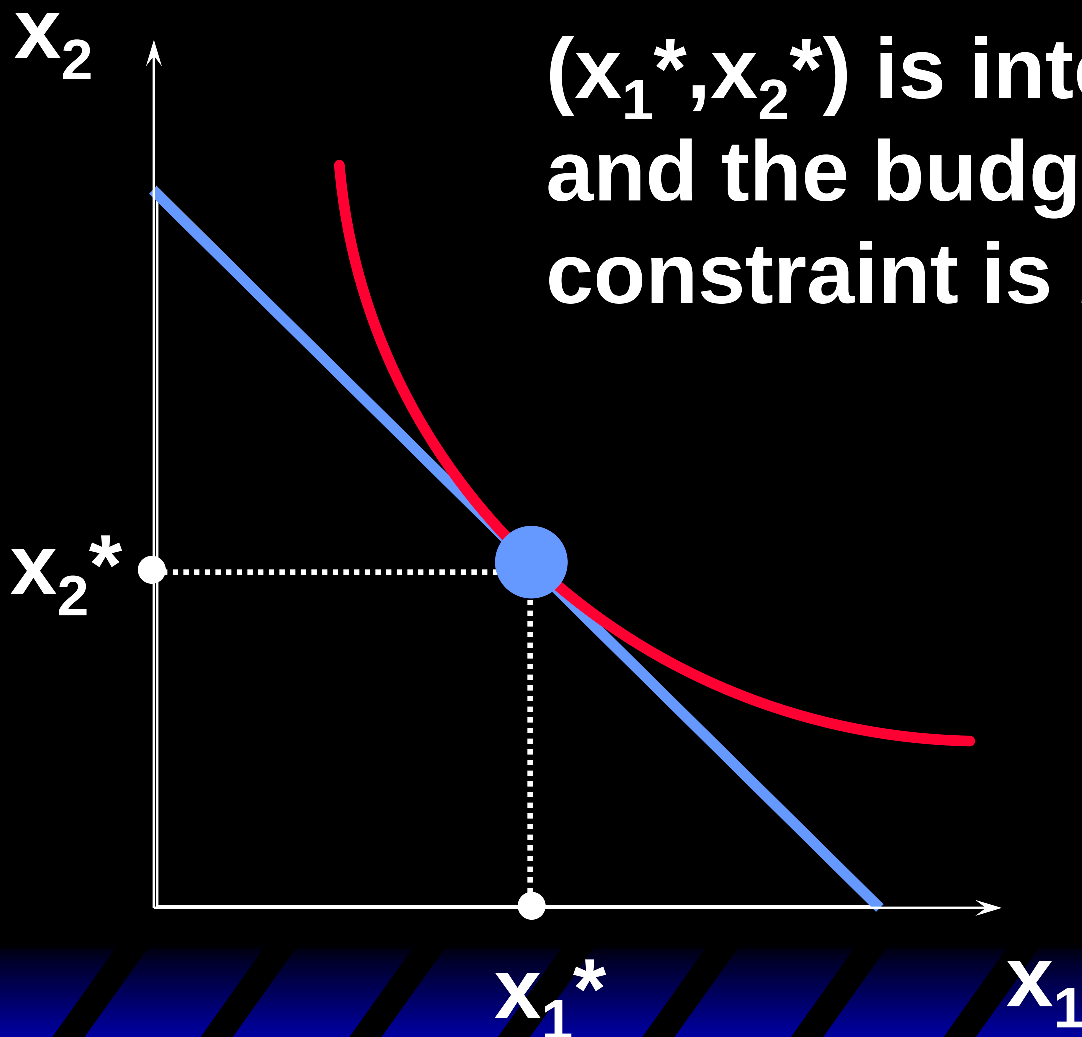
- The most preferred affordable bundle is called the consumer's **demand** at the given prices and budget.
- Demands will be denoted by $x_1^*(p_1,p_2,m)$ and $x_2^*(p_1,p_2,m)$.

Rational Constrained Choice

- When $x_1^* > 0$ and $x_2^* > 0$ the demanded bundle is **interior**. (Not on the axes.)

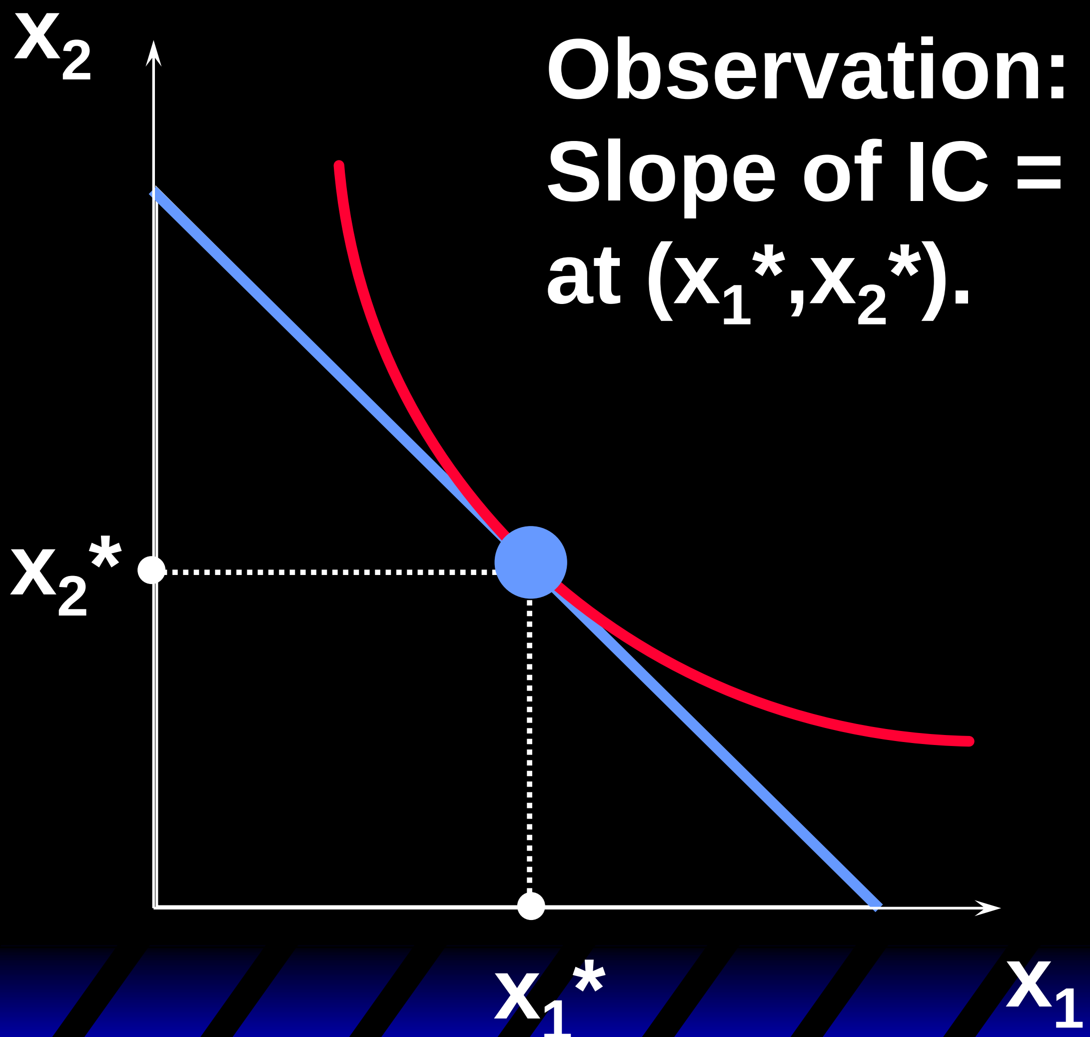
- If (x_1^*,x_2^*) is on the budget line, we say that the budget constraint is **binding**.
  - (x_1^*,x_2^*) exhausts the budget.

Rational Constrained Choice



(x_1^*, x_2^*) is interior, and the budget constraint is binding.

Rational Constrained Choice



Observation:
Slope of IC = Slope of BL at (x_1^*,x_2^*).

Rational Constrained Choice

(x_1^*, x_2^*) satisfies two conditions:

- (a) the budget is binding;

  $$p_1x_1^* + p_2x_2^* = m$$

- (b) Slope of IC = Slope of BL

  $$MRS = -p_1/p_2$$

Two equations and two unknowns.

Computing Demands - a Cobb-Douglas Example.

Suppose that the consumer has Cobb-Douglas preferences.

$$U(x_1, x_2) = x_1^a x_2^b$$

Then $$MU_1 = \frac{\partial U}{\partial x_1} = a x_1^{a-1} x_2^b$$

$$MU_2 = \frac{\partial U}{\partial x_2} = b x_1^a x_2^{b-1}$$

Computing Demands - a Cobb-Douglas Example.

So the MRS is

$$\mathrm{MRS} = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{a x_1^{a-1} x_2^{b}}{b x_1^{a} x_2^{b-1}} = -\frac{a x_2}{b x_1}.$$

Computing Demands - a Cobb-Douglas Example.

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At (x_1^*, x_2^*), MRS = $-p_1/p_2$ so

Computing Demands - a Cobb-Douglas Example.

So the MRS is

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{ax_1^{a-1}x_2^b}{bx_1^a x_2^{b-1}} = -\frac{ax_2}{bx_1}.$$

At (x_1^*, x_2^*), MRS = $-p_1/p_2$ so

$$-\frac{ax_2^*}{bx_1^*} = -\frac{p_1}{p_2} \quad \Rightarrow \quad x_2^* = \frac{bp_1}{ap_2}x_1^*. \qquad \text{(A)}$$

Computing Demands - a Cobb-Douglas Example.

Also, (x_1^*, x_2^*) exhausts the budget:

$$p_1x_1^* + p_2x_2^* = m. \qquad \textbf{(B)}$$

Now we have two equations and two unknows.

Computing Demands - a Cobb-Douglas Example.

With some algebra, we discovered that the most preferred affordable bundle for a consumer with Cobb-Douglas preferences

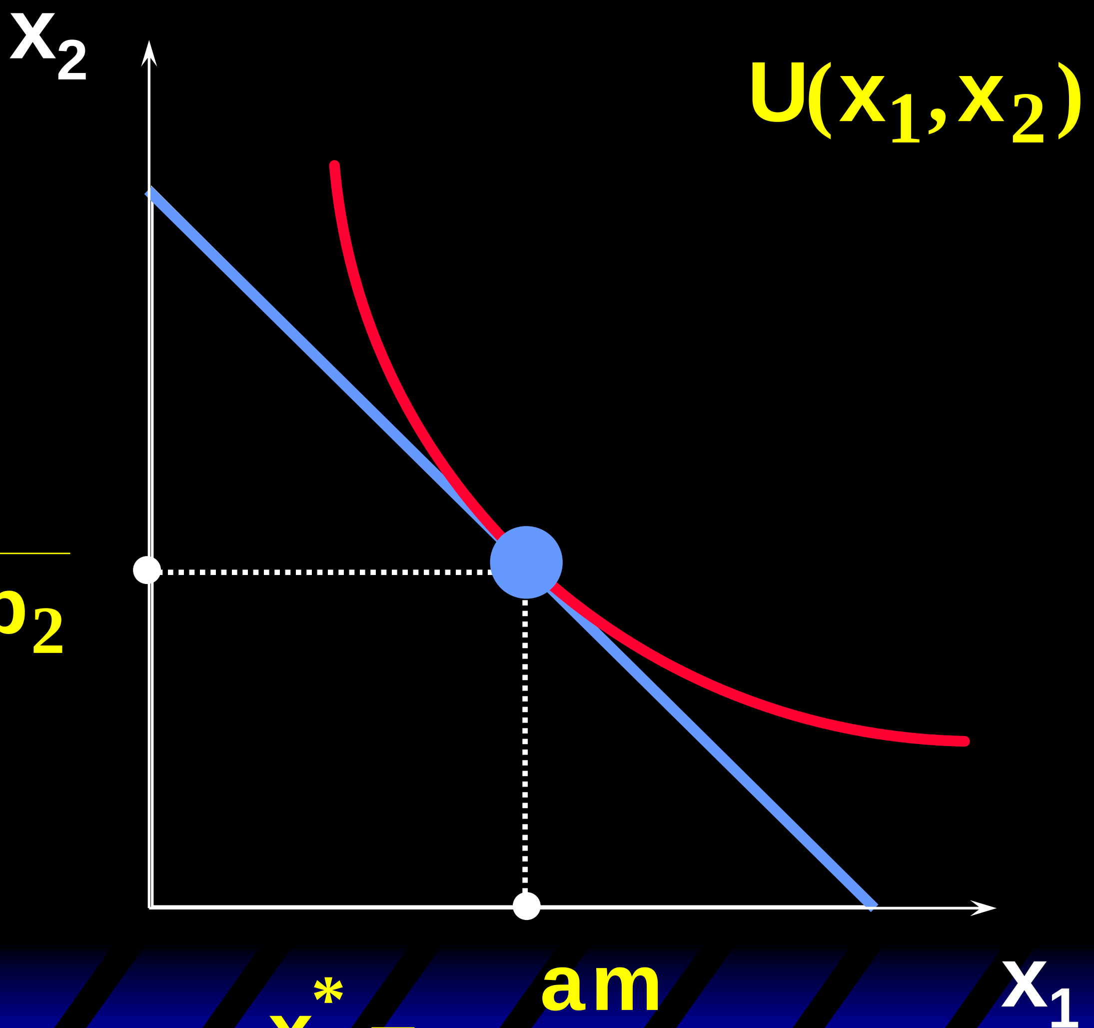
$$U(x_1, x_2) = x_1^a x_2^b$$

is

$$(x_1^*, x_2^*) = \left(\frac{am}{(a+b)p_1}, \frac{bm}{(a+b)p_2}\right).$$

Computing Demands - a Cobb-Douglas Example.

$$U(x_1, x_2) = x_1^a x_2^b$$



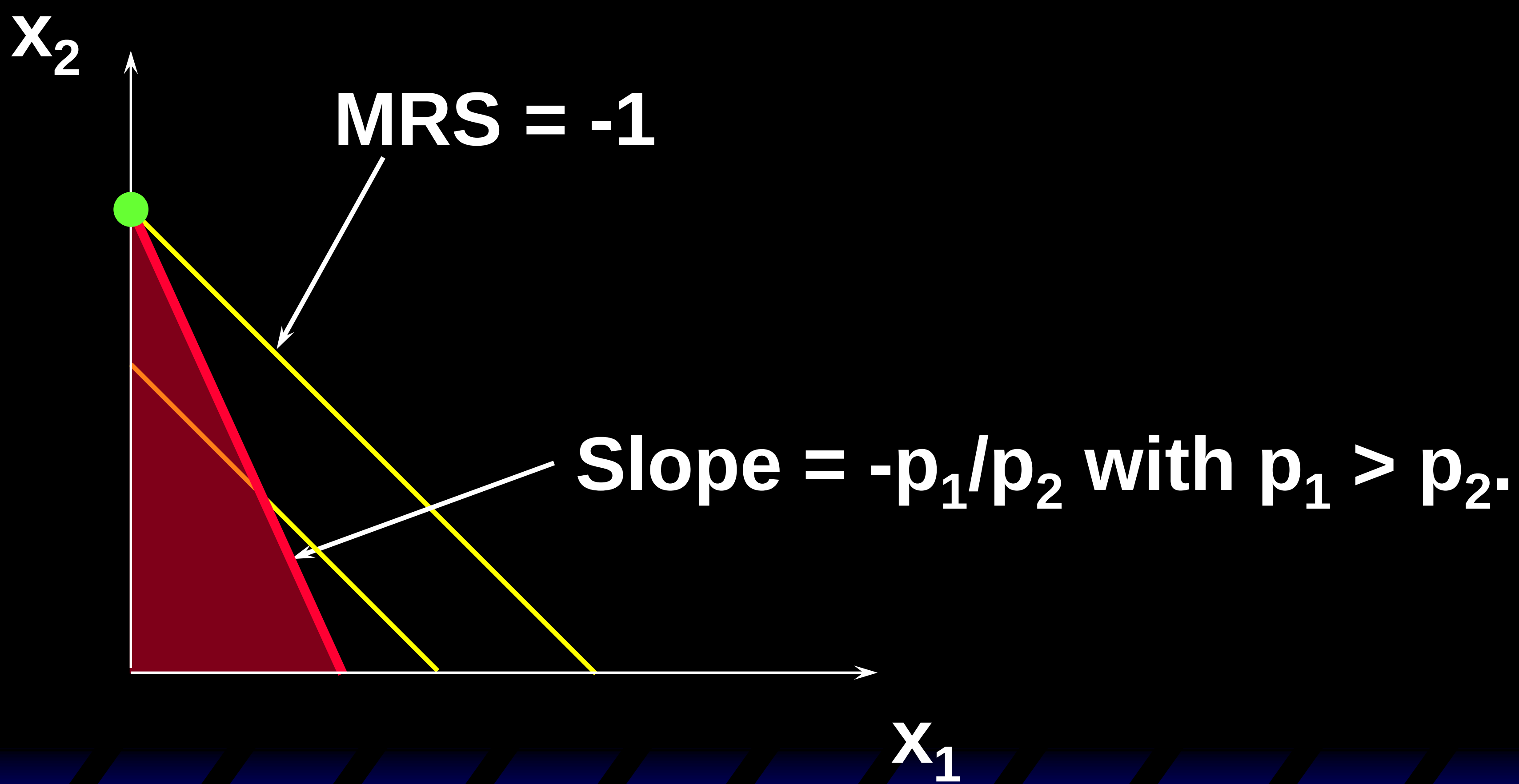
$$x_2^* = \frac{bm}{(a+b)p_2}$$

$$x_1^* = \frac{am}{(a+b)p_1}$$

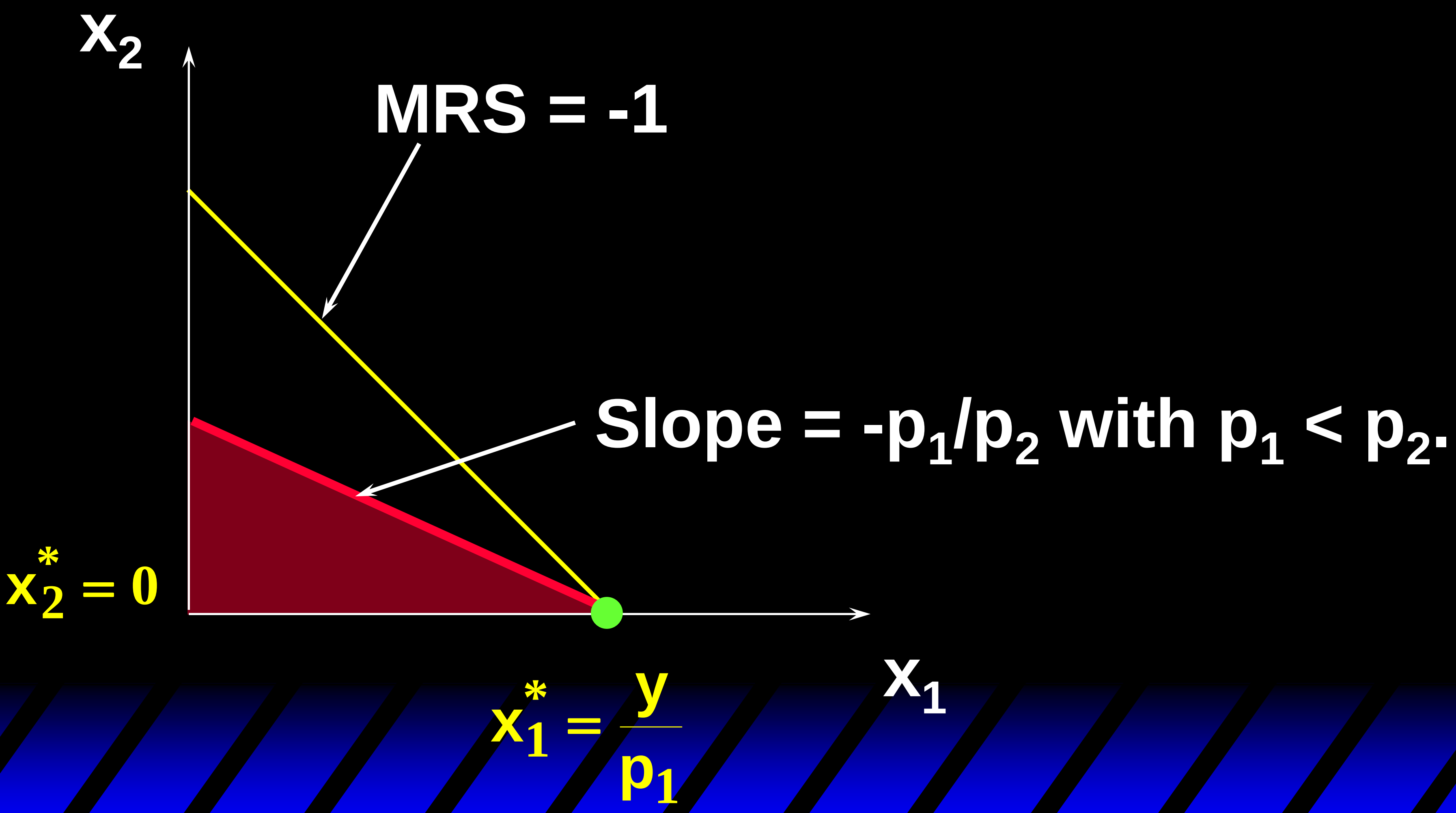
Rational Constrained Choice

- Interior solution implies MRS = $-p_1/p_2$
- What if we have a corner solution?
  - $x_1^* = 0$ or $x_2^* = 0$

Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case



Examples of Corner Solutions -- the Perfect Substitutes Case

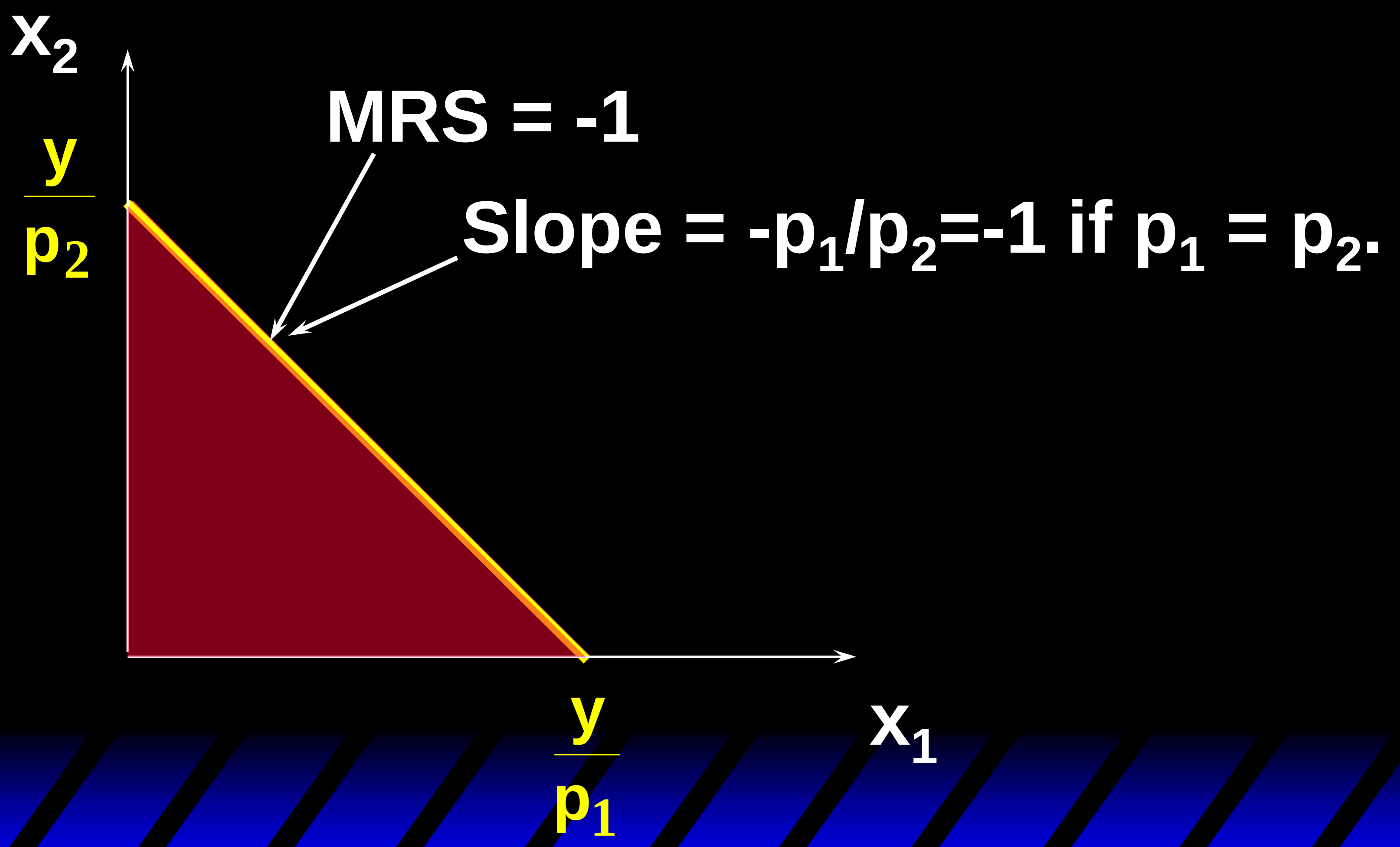
So when $U(x_1,x_2) = x_1 + x_2$, the most preferred affordable bundle is (x_1^*,x_2^*) where

$$(x_1^*, x_2^*) = \left(\frac{y}{p_1}, 0\right) \quad \text{if } p_1 < p_2$$

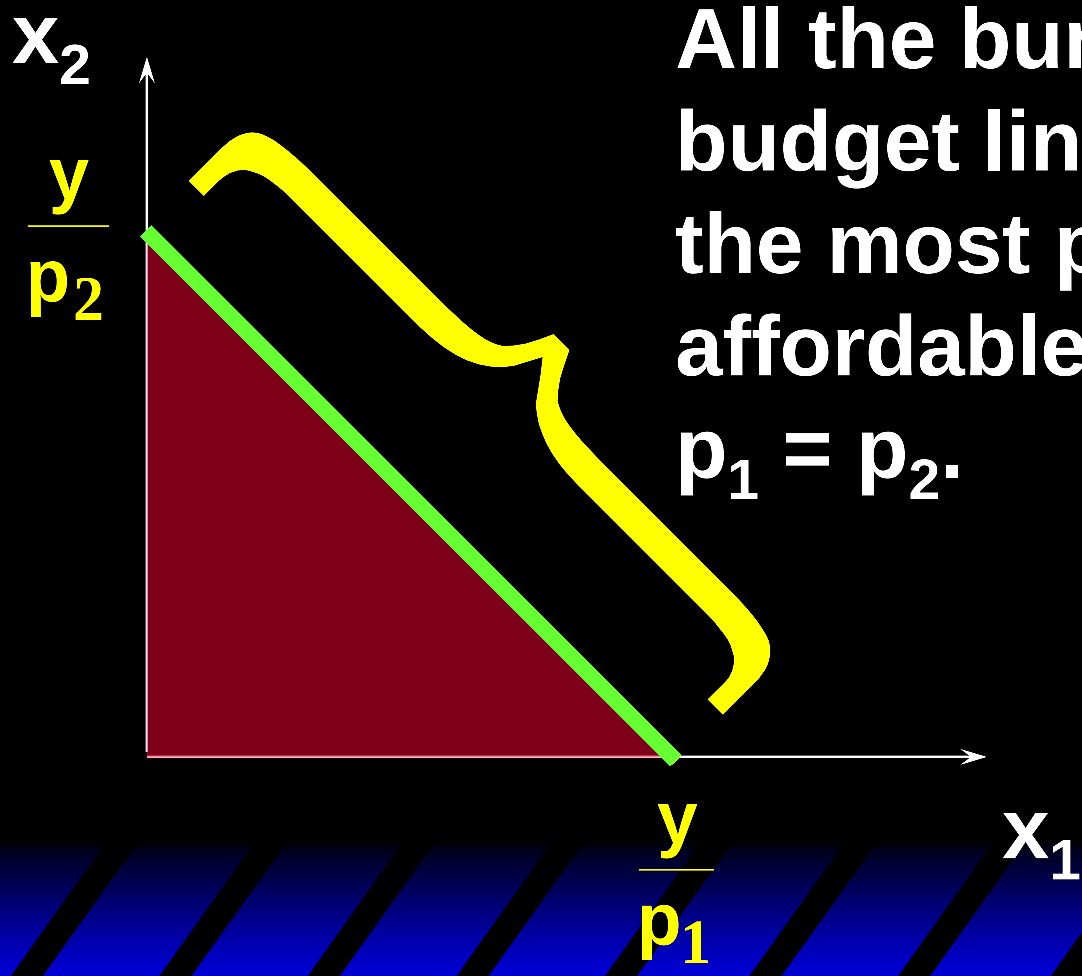
and

$$(x_1^*, x_2^*) = \left(0, \frac{y}{p_2}\right) \quad \text{if } p_1 > p_2.$$

Examples of Corner Solutions -- the Perfect Substitutes Case

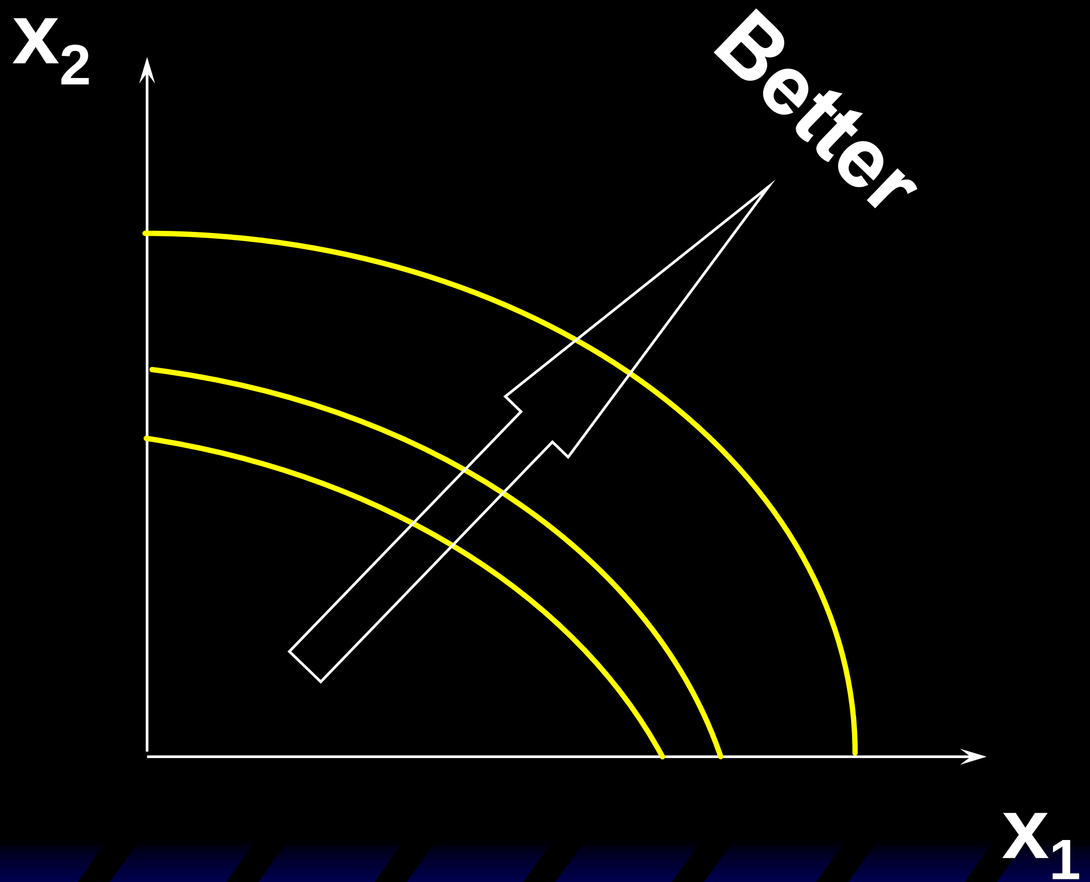


Examples of Corner Solutions -- the Perfect Substitutes Case



All the bundles on the budget line are equally the most preferred affordable bundle when $p_1 = p_2$.

Examples of Corner Solutions -- the Non-Convex Preferences Case



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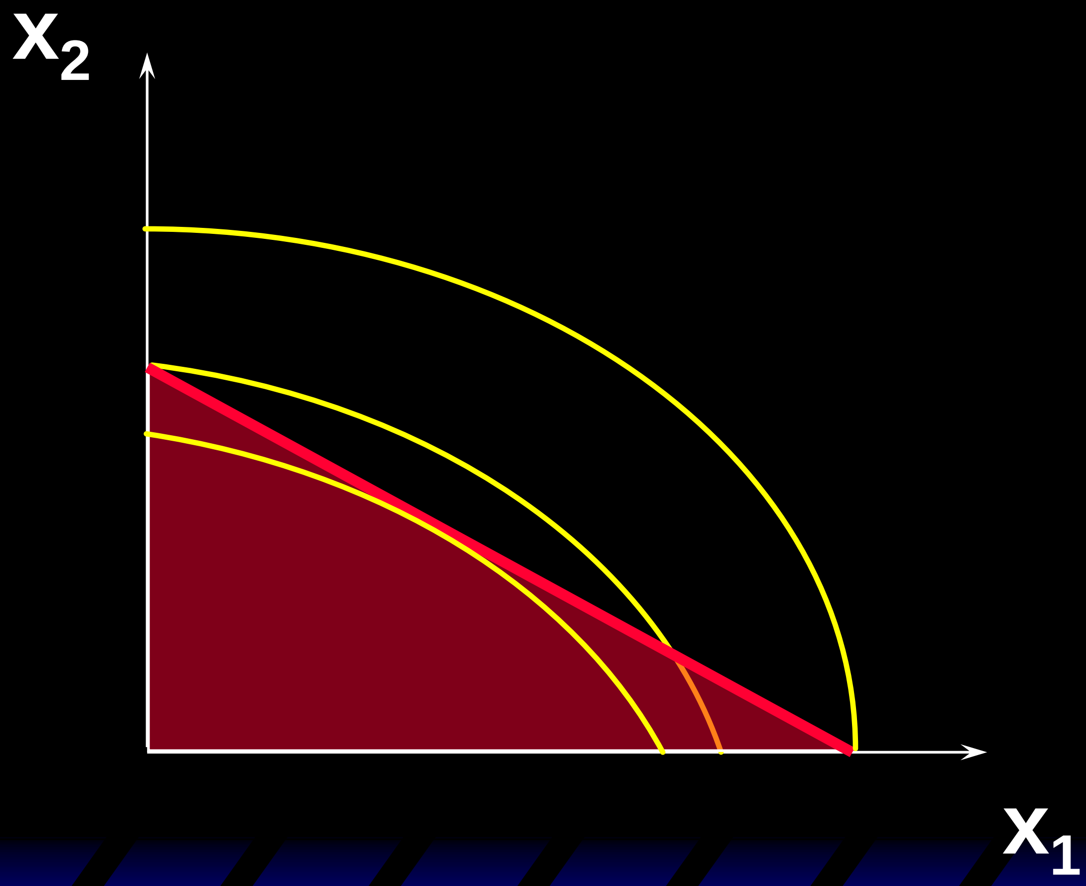

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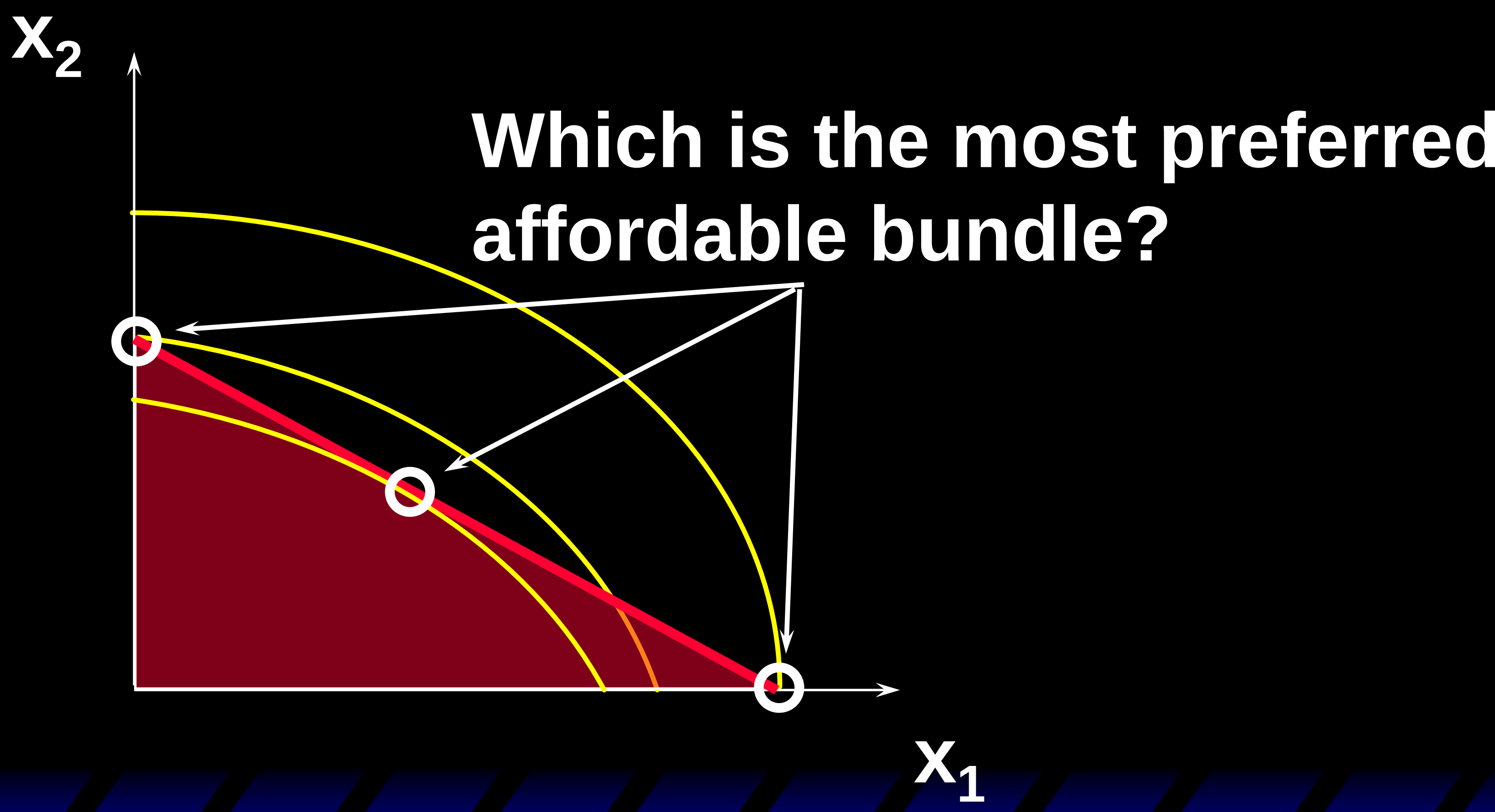

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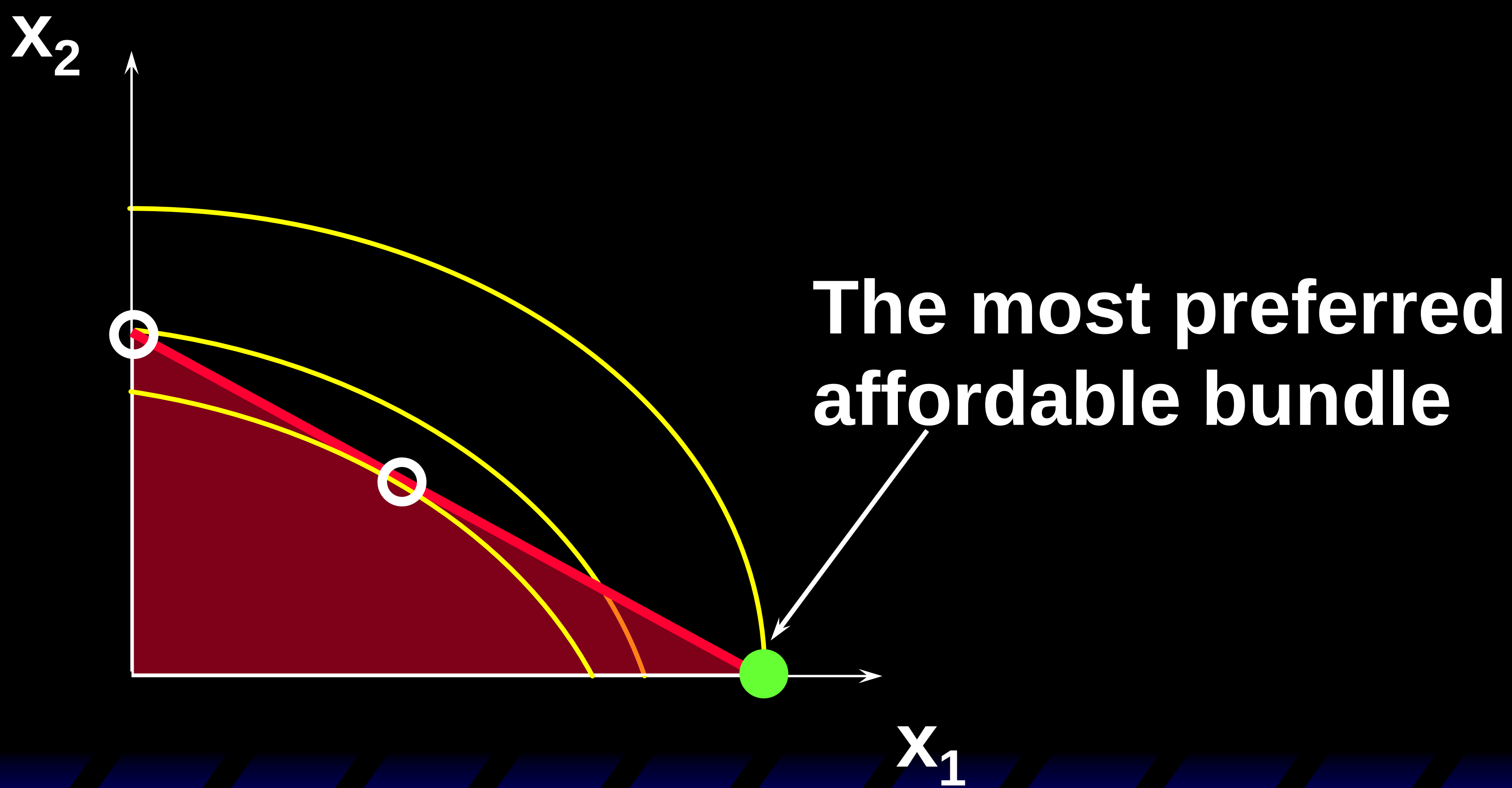

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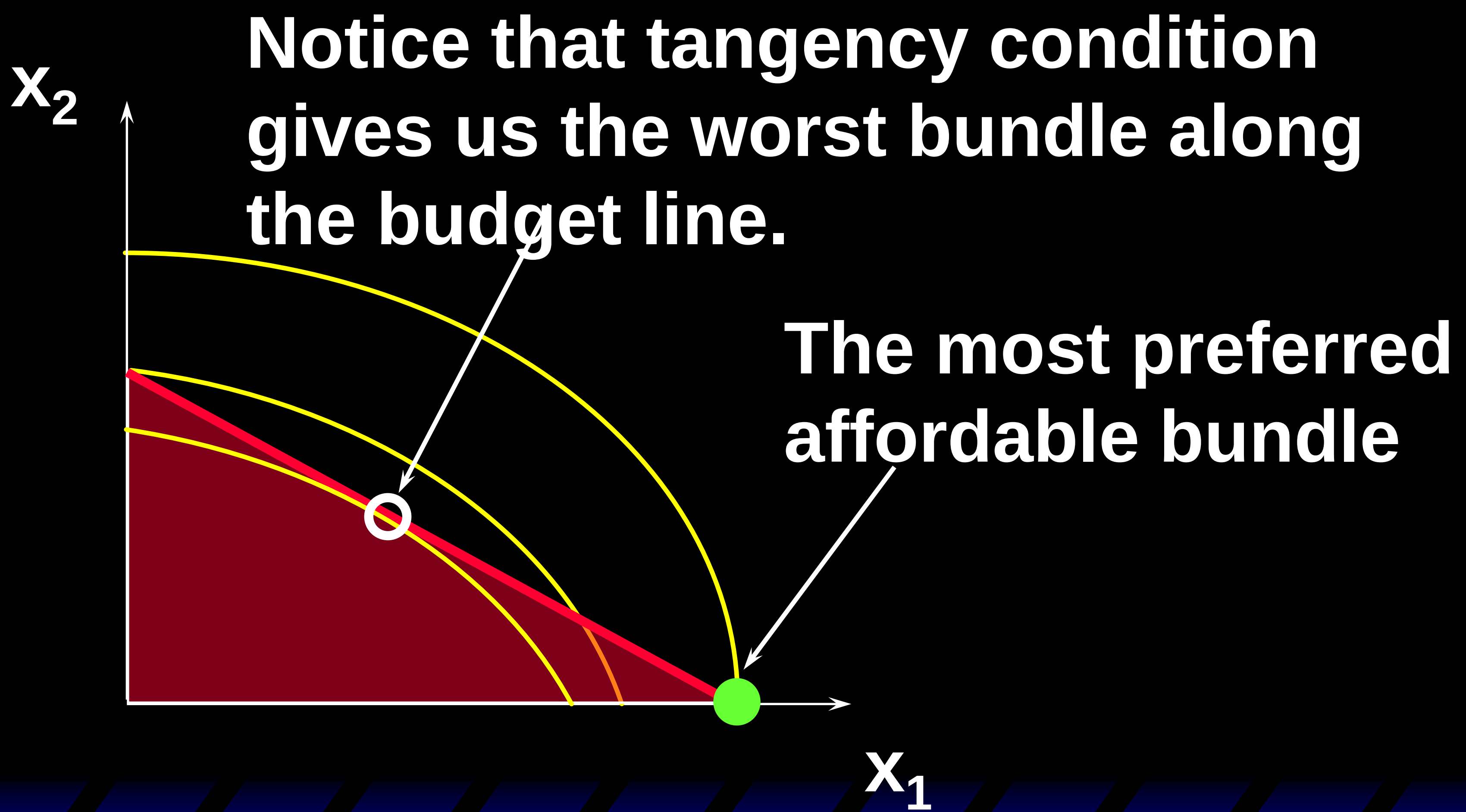
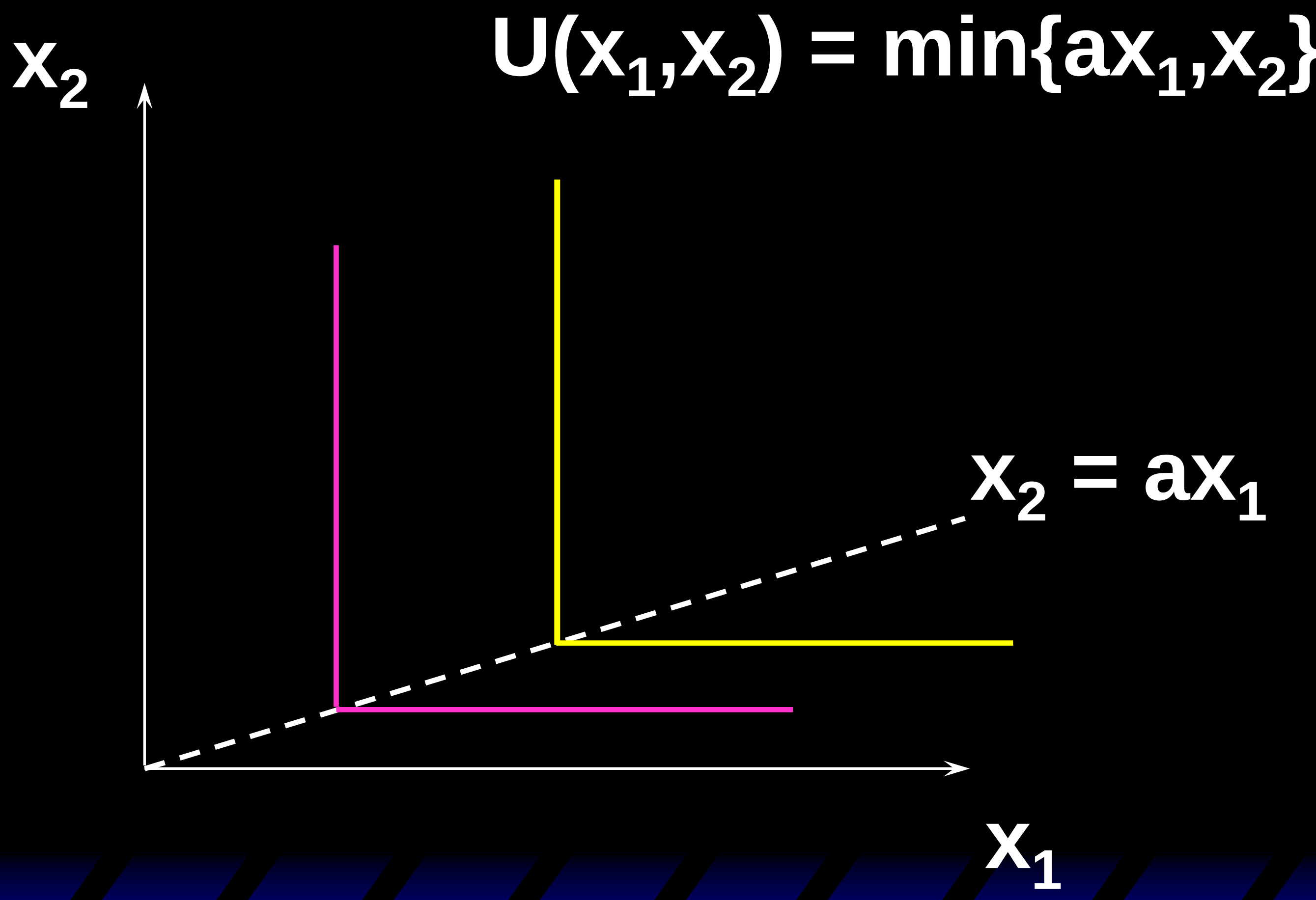


Examples of 'Kinky' Solutions -- the Perfect Complements Case



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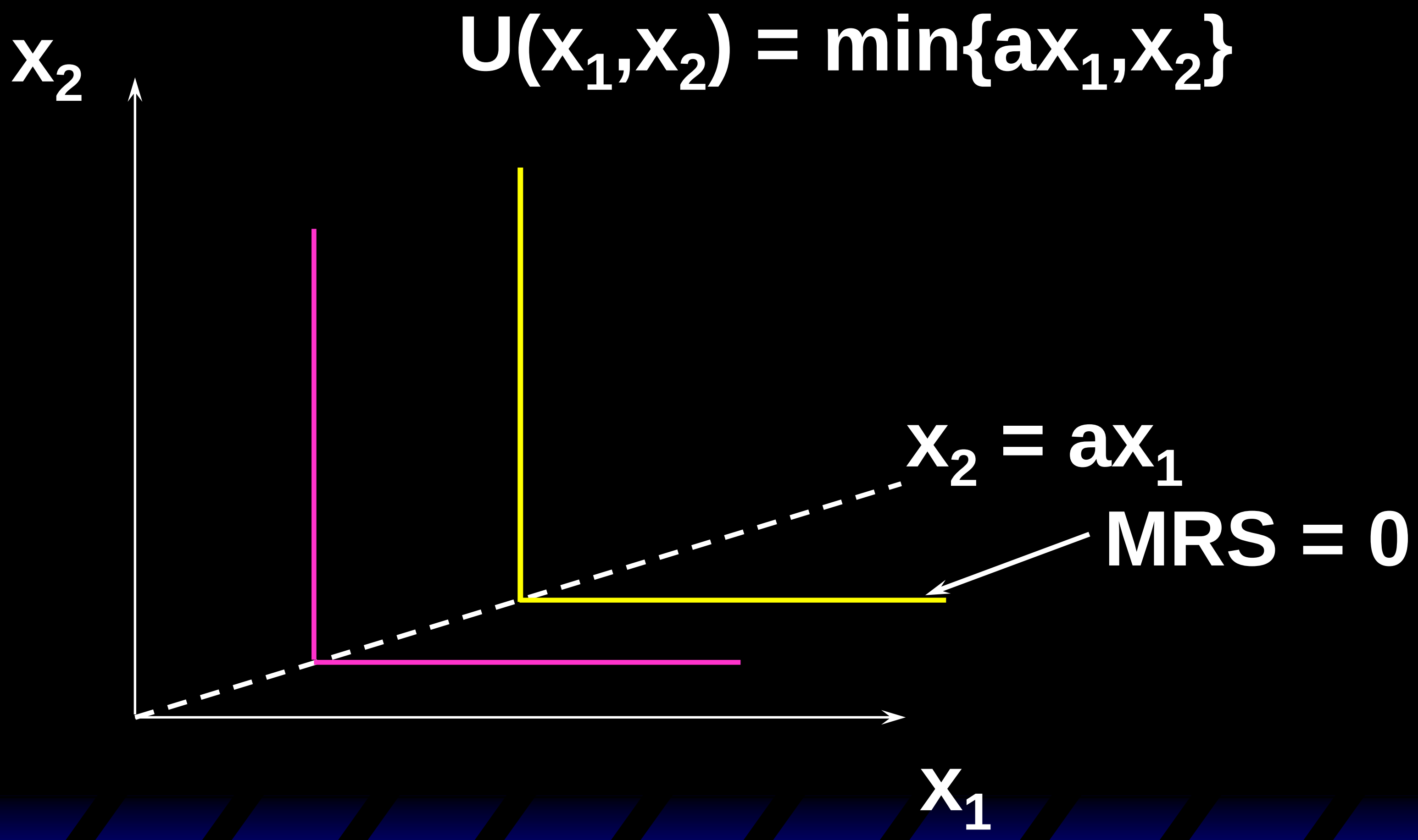

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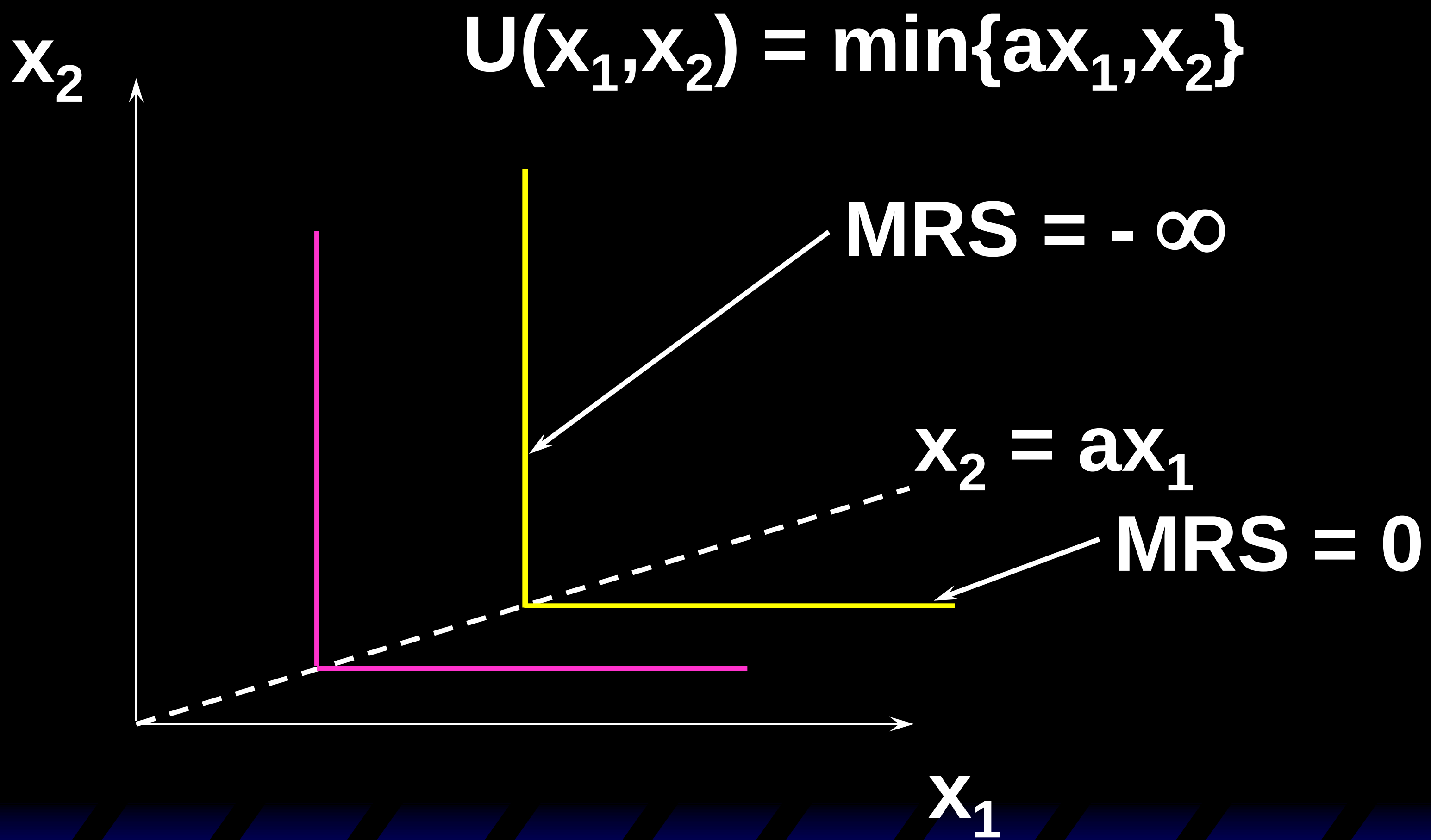

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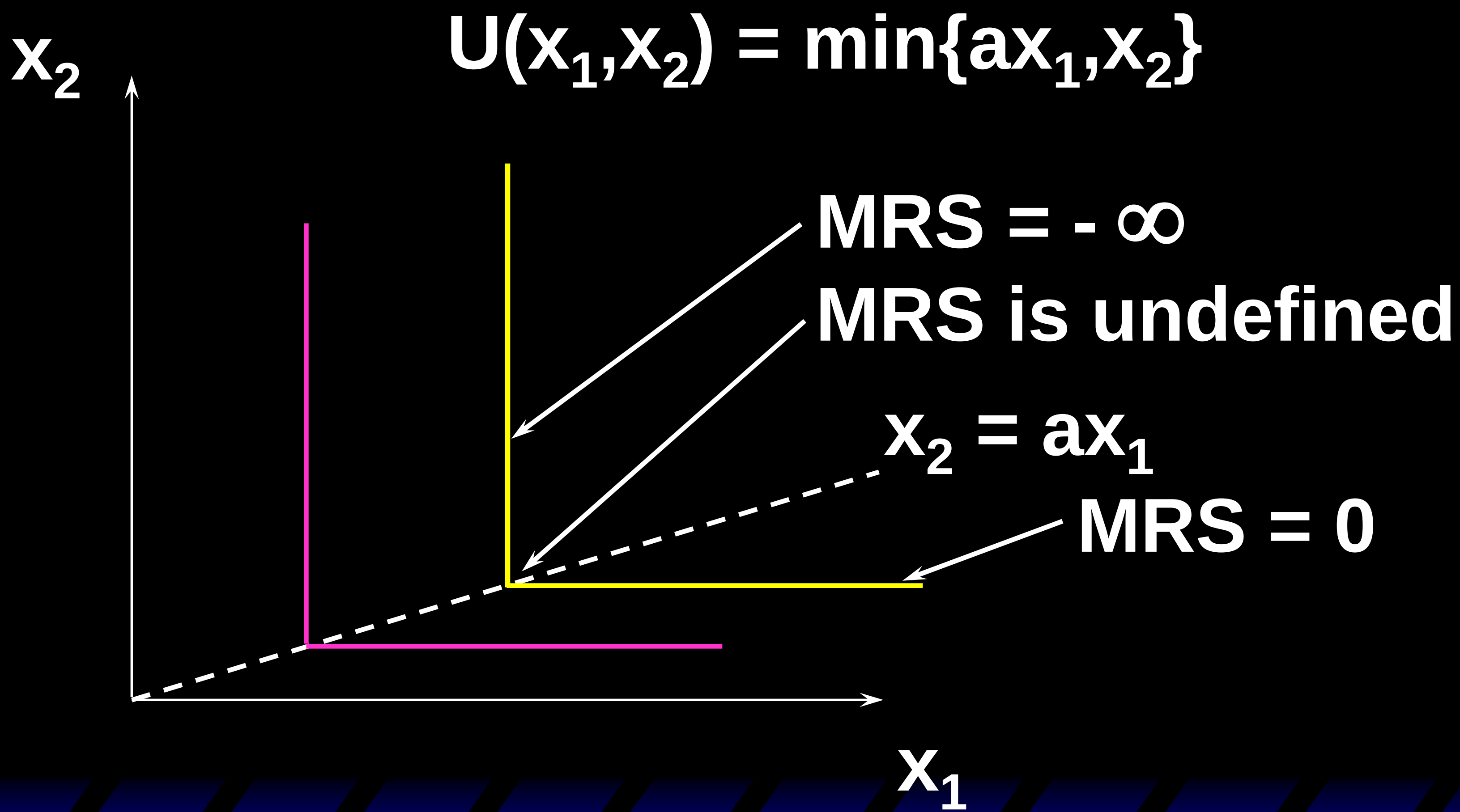

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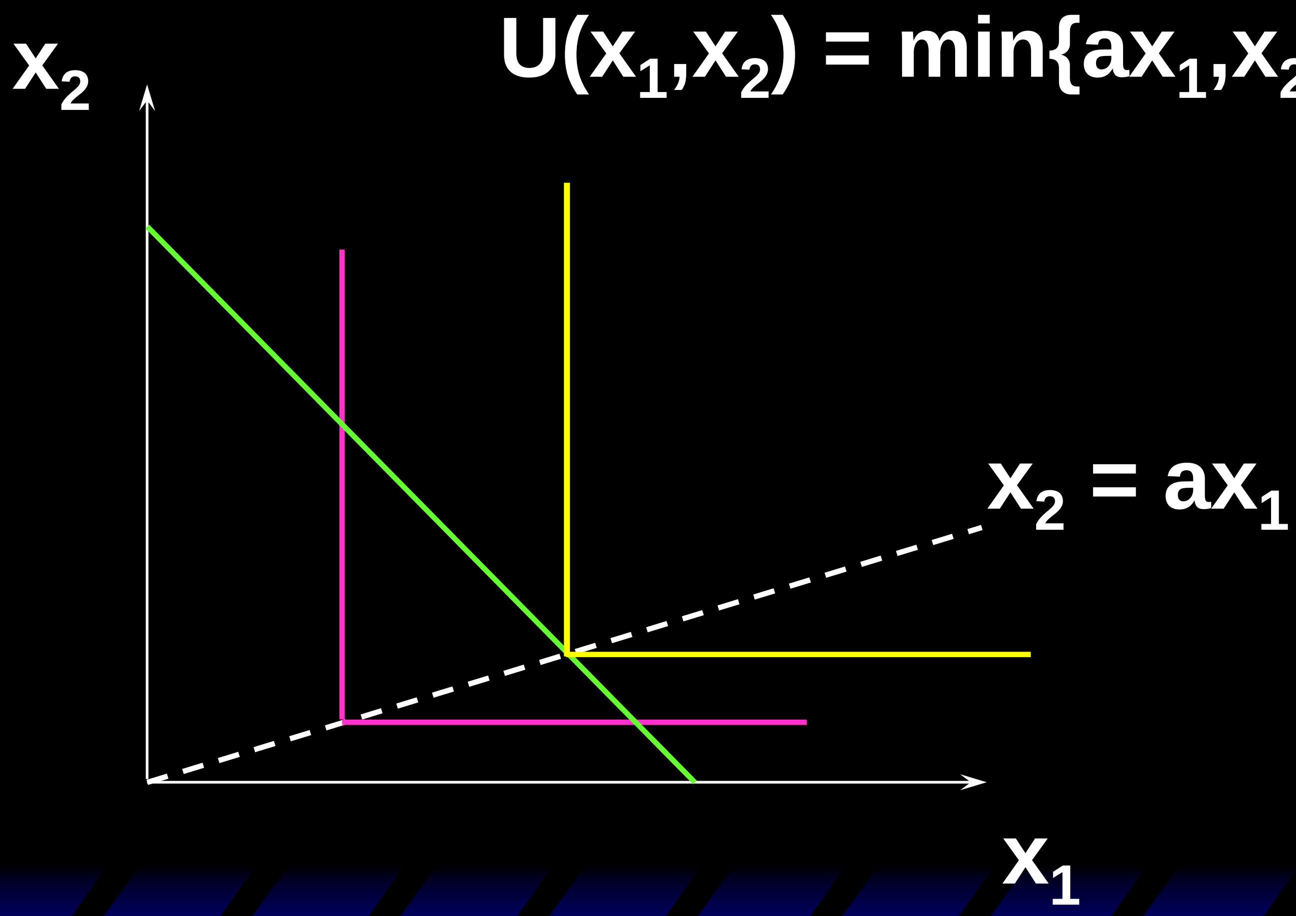

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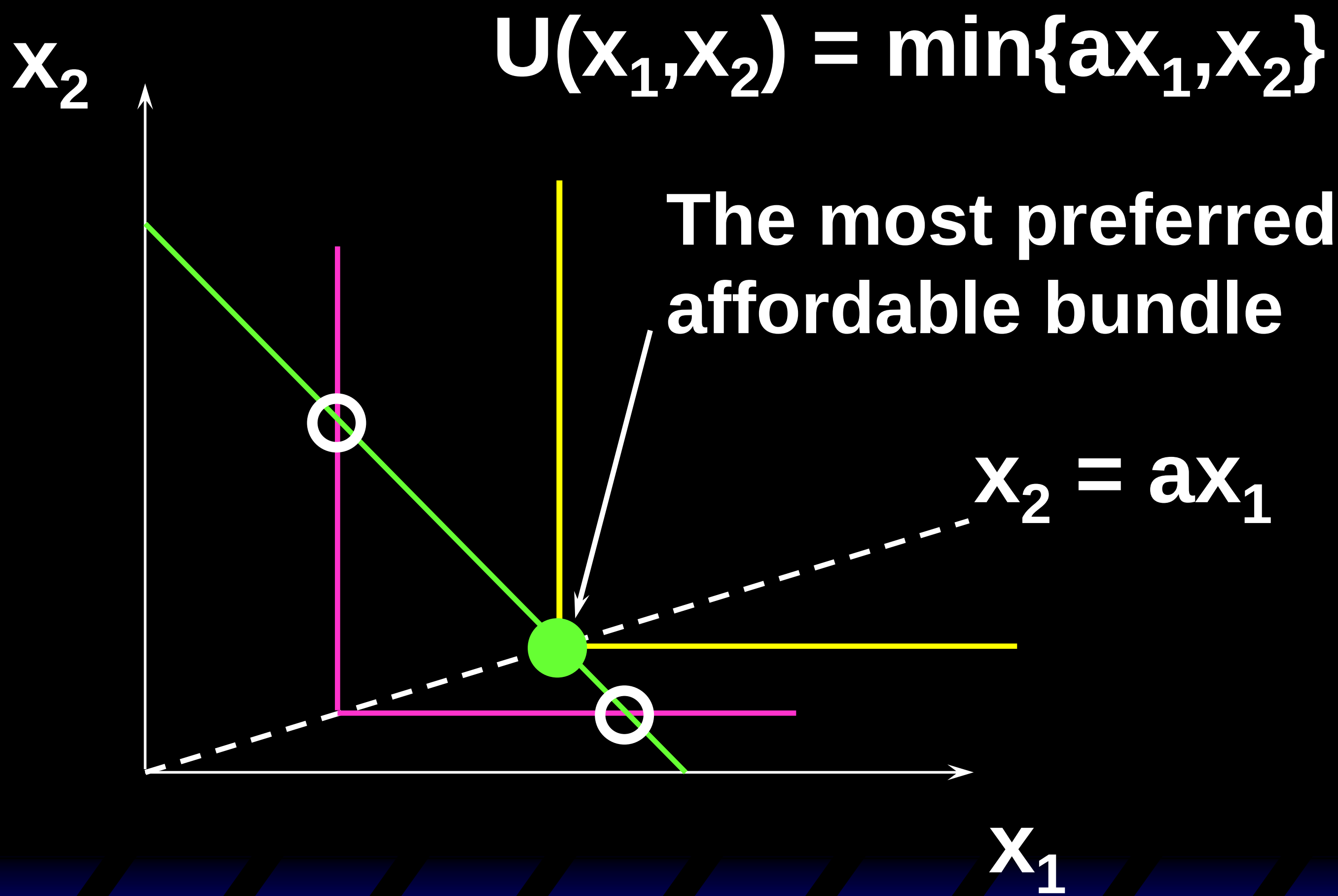
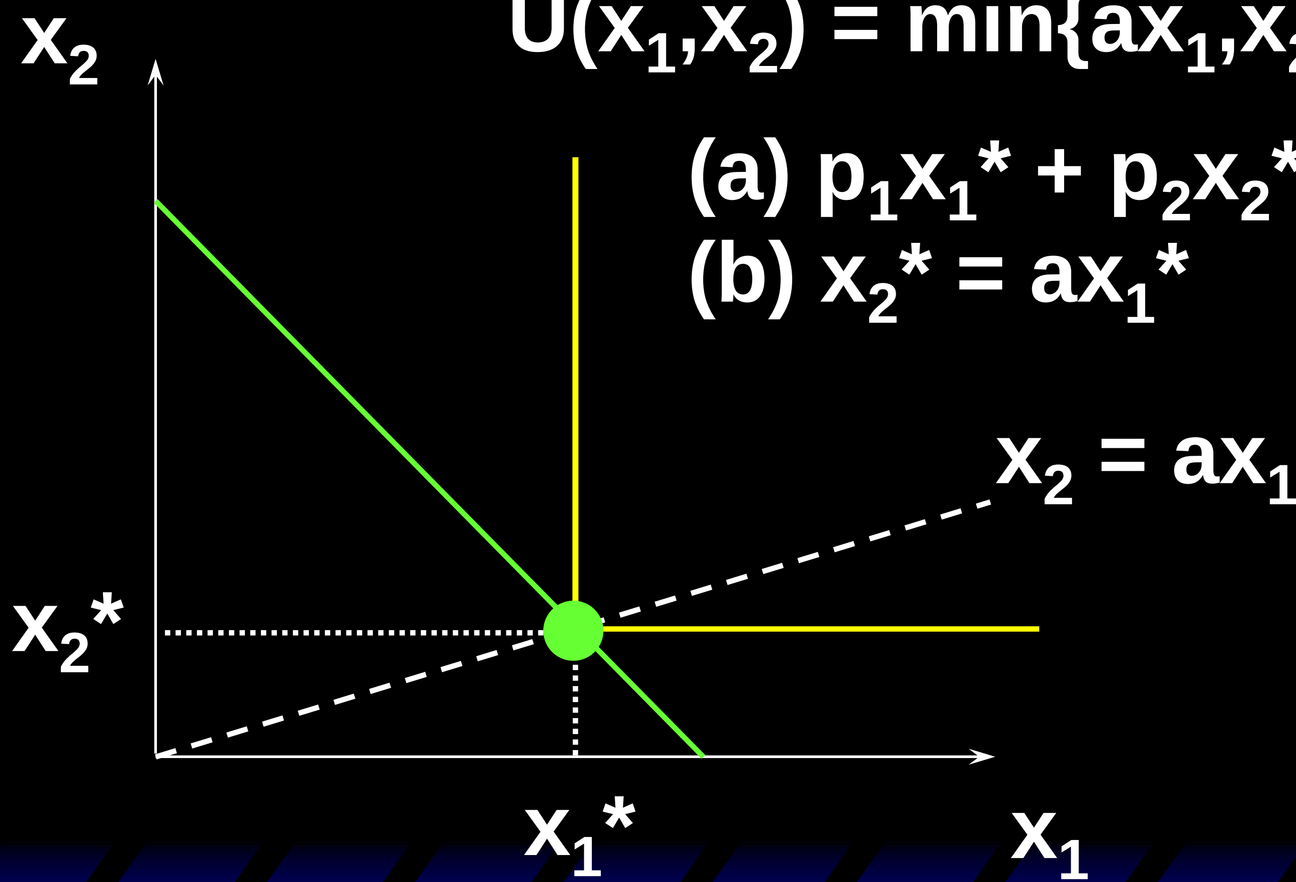


Examples of ‘Kinky’ Solutions -- the Perfect Complements Case

$U(x_1,x_2) = \min\{ax_1,x_2\}$

(a) $p_1x_1^* + p_2x_2^* = m$

(b) $x_2^* = ax_1^*$



Summary: Three Steps to Find the Optimal Choice of the Consumer

Step 1: Draw the budget set;

Step 2: Draw the indifference curves;

Step 3: Locate the point of optimal choice and calculate the solution.

If you know some more math, you may use Lagrangian.