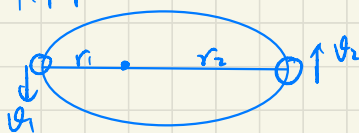


4.17



角动量守恒 $\Rightarrow r_1 \cdot v_1 = r_2 \cdot v_2$

能量守恒 $\Rightarrow E_{p1} + E_{k1} = E_{p2} + E_{k2} = E$

$$\begin{aligned} E_p &= -\frac{GMm}{r} \\ \Rightarrow \begin{cases} E_{p1} &= -\frac{GMm}{r_1} \\ E_{p2} &= -\frac{GMm}{r_2} \end{cases} \end{aligned}$$

$$E_{k1} = \frac{1}{2} m v_1^2$$

$$E_{k2} = \frac{1}{2} m v_2^2 = \left(\frac{r_1}{r_2}\right)^2 E_{k1}$$

$$-\frac{GMm}{r_1} + E_{k1} = -\frac{GMm}{r_2} + \left(\frac{r_1}{r_2}\right)^2 E_{k1}$$

$$\begin{aligned} E_{k1} &= \frac{(r_2 - r_1) GMm}{r_1 r_2} \cdot \frac{r_2^2}{r_2 - r_1^2} \\ &= \frac{r_2 GMm}{r_1 (r_1 + r_2)} \end{aligned}$$

$$\begin{aligned} \Rightarrow E &= E_{p1} + E_{k1} \\ &= GMm \cdot \left(-\frac{1}{r_1} + \frac{r_2}{r_1(r_1 + r_2)}\right) \\ &= -\frac{GMm}{r_1 + r_2} \end{aligned}$$

4.18

(1) 由 4.17 知轨道上能量 $E = -\frac{GMm}{r_1 + r_2}$

$$E_{\text{停}} = -\frac{GMm}{2(R+h_1)} = -1.28 \times 10^9 \text{ J}$$

$$E_{\text{登}} = -\frac{GMm}{2R+h_1+h_2} = -3.97 \times 10^9 \text{ J}$$

$$E_{\text{回}} = -\frac{GMm}{2(R+h_2)} = -2.35 \times 10^9 \text{ J}$$

$$\begin{aligned} \Delta E_1 &= E_{\text{登}} - E_{\text{停}} \\ &= 8.8 \times 10^9 \text{ J} \end{aligned}$$

$$\begin{aligned} \Delta E_2 &= E_{\text{回}} - E_{\text{登}} \\ &= 1.62 \times 10^9 \text{ J} \end{aligned}$$

(2) $E_{k\text{停}} = -E_{\text{停}} = 1.28 \times 10^9 \text{ J} = \frac{1}{2} m v_1^2$

$$E_{k\text{登}} = \frac{GMm \cdot (R+h_2)}{(R+h_1)(2R+h_1+h_2)} = 2.16 \times 10^9 \text{ J} = \frac{1}{2} m v_2^2$$

$$E_{k\text{回}} = \frac{GMm (R+h_1)}{(R+h_2)(2R+h_1+h_2)} = 7.31 \times 10^8 \text{ J} = \frac{1}{2} m v_3^2$$

$$E_{k\text{回}} = -E_{\text{回}} = 2.36 \times 10^9 \text{ J} = \frac{1}{2} m v_4^2$$

$$\Delta v_1 = v_2 - v_1 = 2.14 \times 10^3 \text{ m/s}$$

$$\Delta v_2 = v_4 - v_3 = 1.36 \times 10^3 \text{ m/s}$$

4.23

$$M \vec{v}_1 + m \vec{v}_2 = 0$$

$$\frac{m}{M} = \frac{4}{238-4} = \frac{4}{234} = \frac{2}{167}$$

$$E_{K1} = \frac{1}{2} M v_1^2$$

$$\frac{E_{K1}}{E_{K2}} = \frac{M}{m} \cdot \frac{m^2}{M^2} = \frac{m}{M} = \frac{2}{167}$$

$$E_{K2} = \frac{1}{2} m v_2^2$$

$$\Rightarrow E_{K2} = \frac{167}{169} E = 4.21 \text{ MeV}$$

4.25 \vec{v}_0

$$\boxed{m} \boxed{m} \boxed{m} \boxed{M}$$

$$\mu = \frac{Mm}{M+m}$$

$$E_{K0} = \frac{1}{2} \mu v_0^2$$

当最大压缩长度时,

$$0 + \frac{1}{2} k x_m^2 = \frac{1}{2} \mu v_0^2 + 0$$

$$\Rightarrow x_m = \sqrt{\frac{\mu v_0^2}{k}} = v_0 \sqrt{\frac{Mm}{k(M+m)}}$$

4.27

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

$$\begin{cases} p_1 - p_2 = 0 \\ h_1 - h_2 = 5 \text{ m} \\ v_1 = 0 \end{cases} \Rightarrow v_2 = 9.9 \text{ m/s}$$

$$\frac{\Delta V}{\Delta t} = v_2 \cdot \pi \cdot r^2 = 1.24 \text{ m}^3/\text{s} \\ \approx 4.46 \times 10^3 \text{ m}^3/\text{h}$$

4.28

$$p_1 - p_2 = \frac{1}{2} \rho v^2$$

$$v = \sqrt{\frac{2(p_1 - p_2)}{\rho}} = 63.6 \text{ m/s}$$

5.2

$$(1) \omega_0 = \frac{2\pi \times 200}{60} = \frac{20}{3} \pi \text{ rad/s}$$

$$\omega_1 = \frac{2\pi \times 3000}{60} = 100\pi \text{ rad/s}$$

$$\alpha = \frac{\omega_1 - \omega_0}{\Delta t} = \frac{40}{3} \pi \text{ rad/s}^2$$

(2)

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$= \frac{1120}{3} \pi \text{ rad}$$

$$n = \frac{\theta}{2\pi} = \frac{560}{3} \text{ (圈)}$$

5.3

$$\omega_1 = \frac{365 \times 2\pi}{T_1}$$

$$T_2 - T_1 = 1.14 \text{ s} = \Delta T$$

$$\omega_2 = \frac{365 \times 2\pi}{T_2}$$

$$\alpha = \frac{\omega_2 - \omega_1}{\Delta t} = 365 \times 2\pi \times \frac{\frac{1}{T_2} - \frac{1}{T_1}}{87T_1} = 365 \times 2\pi \times \frac{-\Delta T}{87T_1^2} \approx -9.6 \times 10^{-22} \text{ rad/s}^2$$

$\Delta t = 87T_1$

5.4

$$\lambda = 40^\circ$$

$$g = R \cos \lambda \cdot \omega \approx 462.966 \text{ m/s}$$

方向与地轴垂直, 指向东

$$\alpha = \omega^2 R \cos \lambda = 3.370 \times 10^{-2} \text{ m/s}^2$$

方向指向地轴