HW4

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1. $\Pi(s) = 8s - c(s) = 2s^2 - 8s + 18 \leq \Pi(2) = 10$, so he'll fix 2 cars per week.

2. (a) $\mathrm{MC}(y) = 2y$, $\mathrm{AVC}(y) = y$, $\mathrm{ATC}(y) = y + \frac{10}{y}$.

   (b) $p_{min} = \mathrm{ATC}(y)_{min} = 2\sqrt{10}$.

3. The expected cost of each bird on the fine is $\$1,000 \times 0.1 = \$100$.
   So the expected cost per bird is $\$40 \times 2 + \$100 = \$180$.
   Since the market is competitive, the price of bird is \$180 .

4. Average cost for each firm is $y + \frac{4}{y}$.
   In long run, the price should be equal to the minimum AC, which is 4 .
   Assume there's N firms in the market, then the equilibrium quantity is $2N$.
   According to the demand curve, $2N = D(4) = 50 - 4 = 46$, so $N = 23$.

5.1 a) $s = 4T + 0 = 80 \Rightarrow T = 20$.

b) Cost of teaching to raise one point: $\frac{\omega_T}{4}$, cost of encouragement to raise one point: ω_e .
So if $\omega_T > 4\omega_e$, encouragement is cheaper, otherwise teaching is cheaper.

c)

$$c(\omega_T, \omega_e; s) = \begin{cases} \omega_e \cdot s & \text{if } \omega_T > 4\omega_e \\ \omega_T \cdot \frac{s}{4} & \text{if } \omega_T \leq 4\omega_e \end{cases}$$

d)

$$e(\omega_T, \omega_e; s) = \begin{cases} s & \text{if } \omega_T > 4\omega_e \\ 0 & \text{if } \omega_T \leq 4\omega_e \end{cases}$$

5.2 e) $s_i = 2\sqrt{T} + \sqrt{e_i} = 2\sqrt{T} + \sqrt{h - T}$
$\Rightarrow \frac{\partial s_i}{\partial T} = \frac{1}{\sqrt{T}} - \frac{1}{2\sqrt{h-T}} = 0$
$\Rightarrow T = \frac{4}{5}h$, $e_i = \frac{1}{5}h$.

f)

$$\begin{aligned} S &= \sum_{i=1}^{n} s_i \\ &= 2n\sqrt{T} + \sum_{i=1}^{n} \sqrt{e_i} \\ &= 2n\sqrt{T} + \sqrt{nh - nT} \quad \text{consider all students are equivalent} \end{aligned}$$

$\Rightarrow \frac{\partial S}{\partial T} = \frac{n}{\sqrt{T}} - \frac{n}{2\sqrt{nh-nT}} = 0$
$\Rightarrow T = \frac{4n}{4n+1}h$, $e_i = \frac{1}{4n+1}h$.

g) The more students, the less time spent on encouragement.

5.3 h) The larger A_i , the more time spent on this student's encouragement. If his compensation is determined by the top-performing students in the class, he'll spend more time on the top-performing student's encouragement.
If it's determined by the lowest score, he'll spend more time on the bottom-performing student's encouragement.

6 (a) y is the quantity of good the monopolist produce, q_l is the quantity of labor the monopolist use, ω is the labor's wage, and suppose one unit of labor produce k units of production .

Assume the demand curve is $D(p) = D_0 - p$.

$\text{MC}(y) = \frac{k}{\omega}$

(b) $\text{MR}(y) = D_0 - 2y$.

(c) $y^* = \frac{D_0 - \frac{k}{\omega}}{2}$, $p^* = \frac{D_0 + \frac{k}{\omega}}{2}$.

(d) $\omega = n \times kp^* = \frac{kD_0 + \frac{k^2}{\omega}}{2}$.