These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Twenty-Two

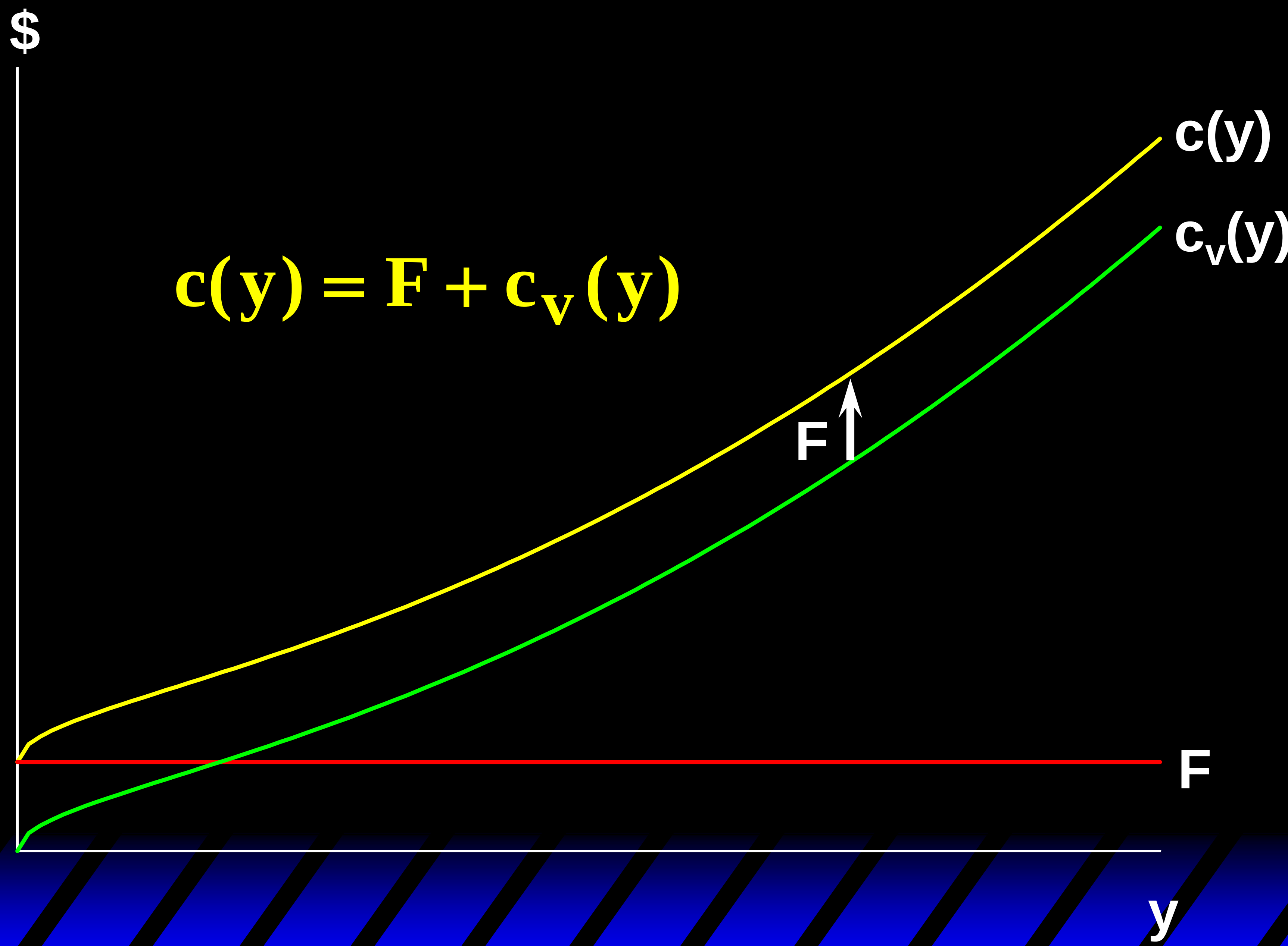
Cost Curves

Fixed, Variable & Total Cost Functions

- Fix a time horizon: long run or short run
- F: fixed cost + quasi-fixed cost
  - F does not vary with the firm's output level.
- $c_v(y)$: variable cost function.
- Then the total cost function is (if y>0)

$$c(y) = F + c_v(y).$$

$$c(y) = F + c_v(y)$$

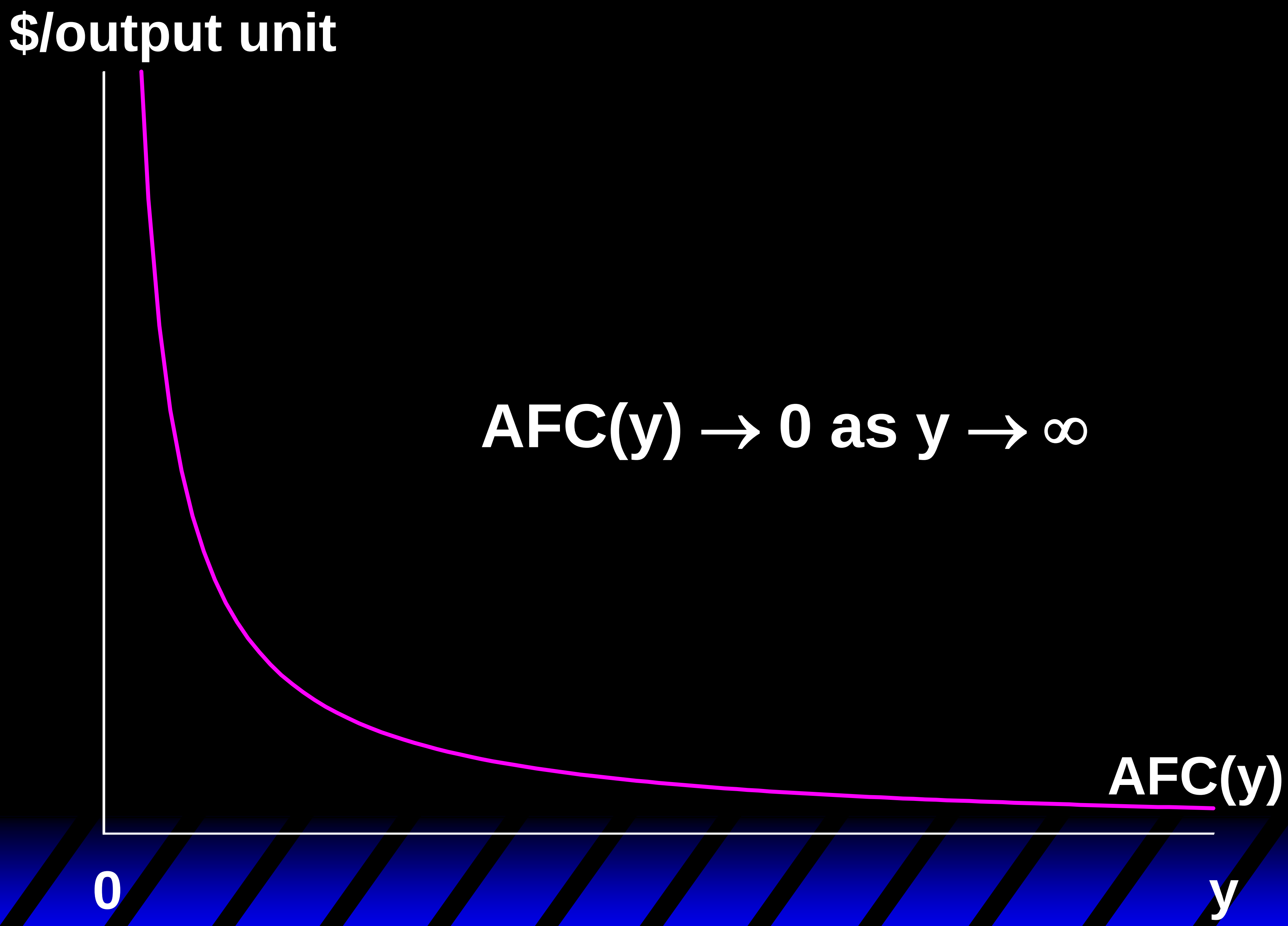


Av. Fixed, Av. Variable & Av. Total Cost Curves

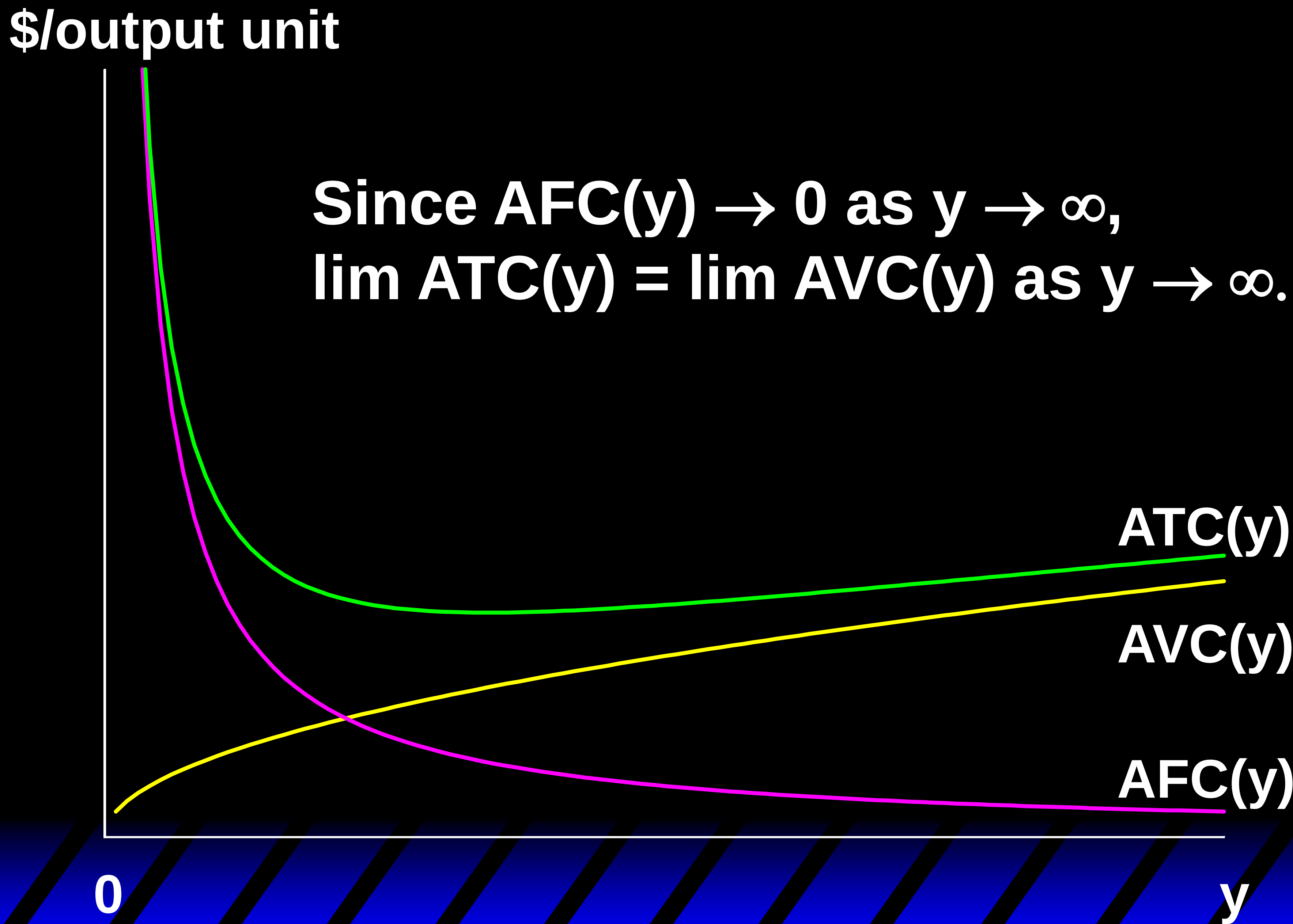
For y > 0, the firm's average total cost function is

$$\mathbf{AC(y) = \frac{F}{y} + \frac{c_v(y)}{y}}$$

$$\mathbf{= AFC(y) + AVC(y).}$$


$/output unit
AFC(y) → 0 as y → ∞
AFC(y)
0
y

Since AFC(y) $\to$ 0 as y $\to \infty$,
lim ATC(y) = lim AVC(y) as y $\to \infty$.



Marginal Cost Function

Since the firm's total cost function is:

$$c(y) = F + c_v(y)$$

The marginal cost is:

$$MC(y) = \frac{\partial c_v(y)}{\partial y} = \frac{\partial c(y)}{\partial y}.$$

Marginal and Variable Cost Functions

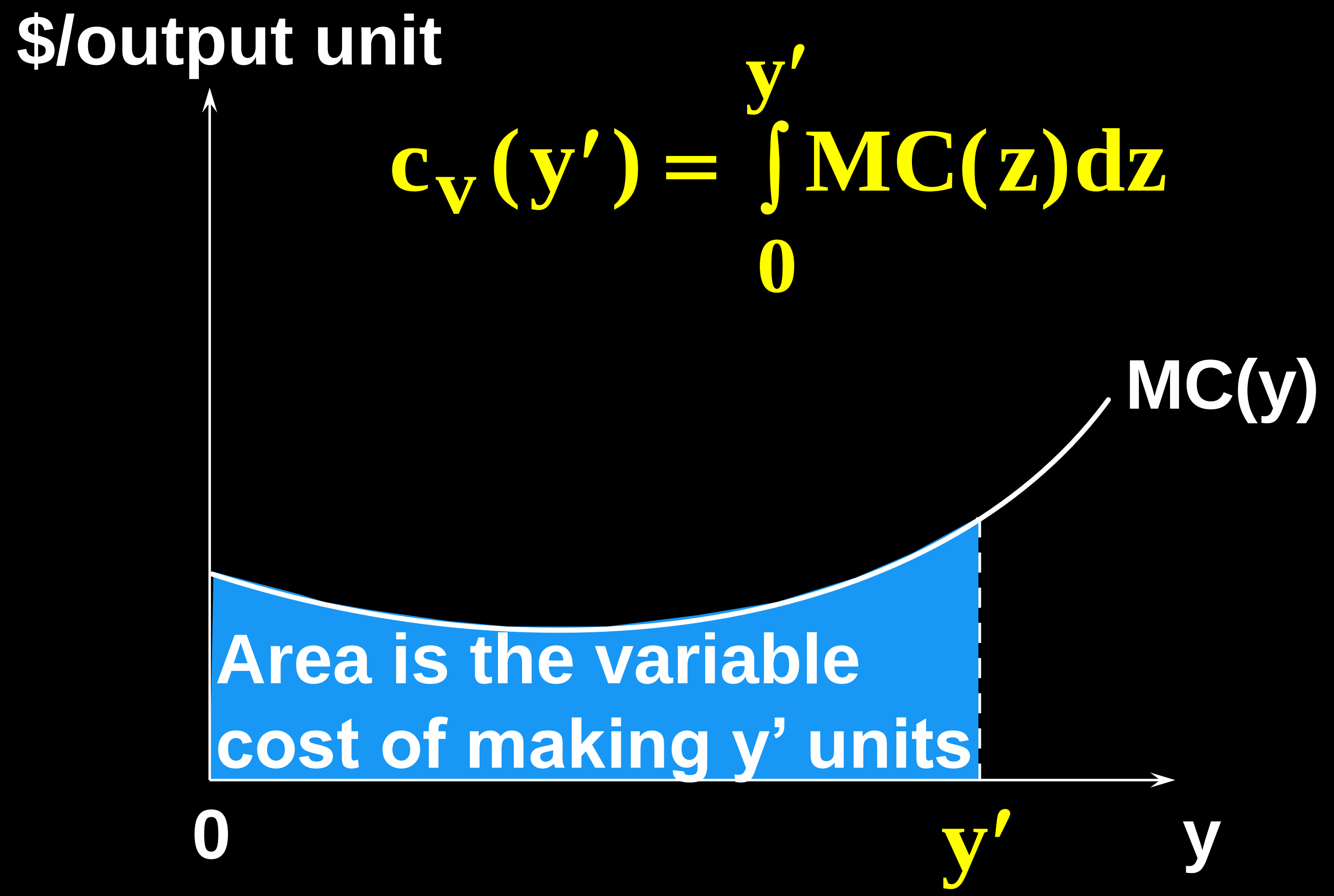
Note:

$$MC(y) = \frac{\partial c_v(y)}{\partial y}$$

$$\Rightarrow \quad c_v(y) = \int_0^y MC(z)\,dz.$$

Marginal and Variable Cost Functions

$$c_v(y') = \int_0^{y'} MC(z)\,dz$$



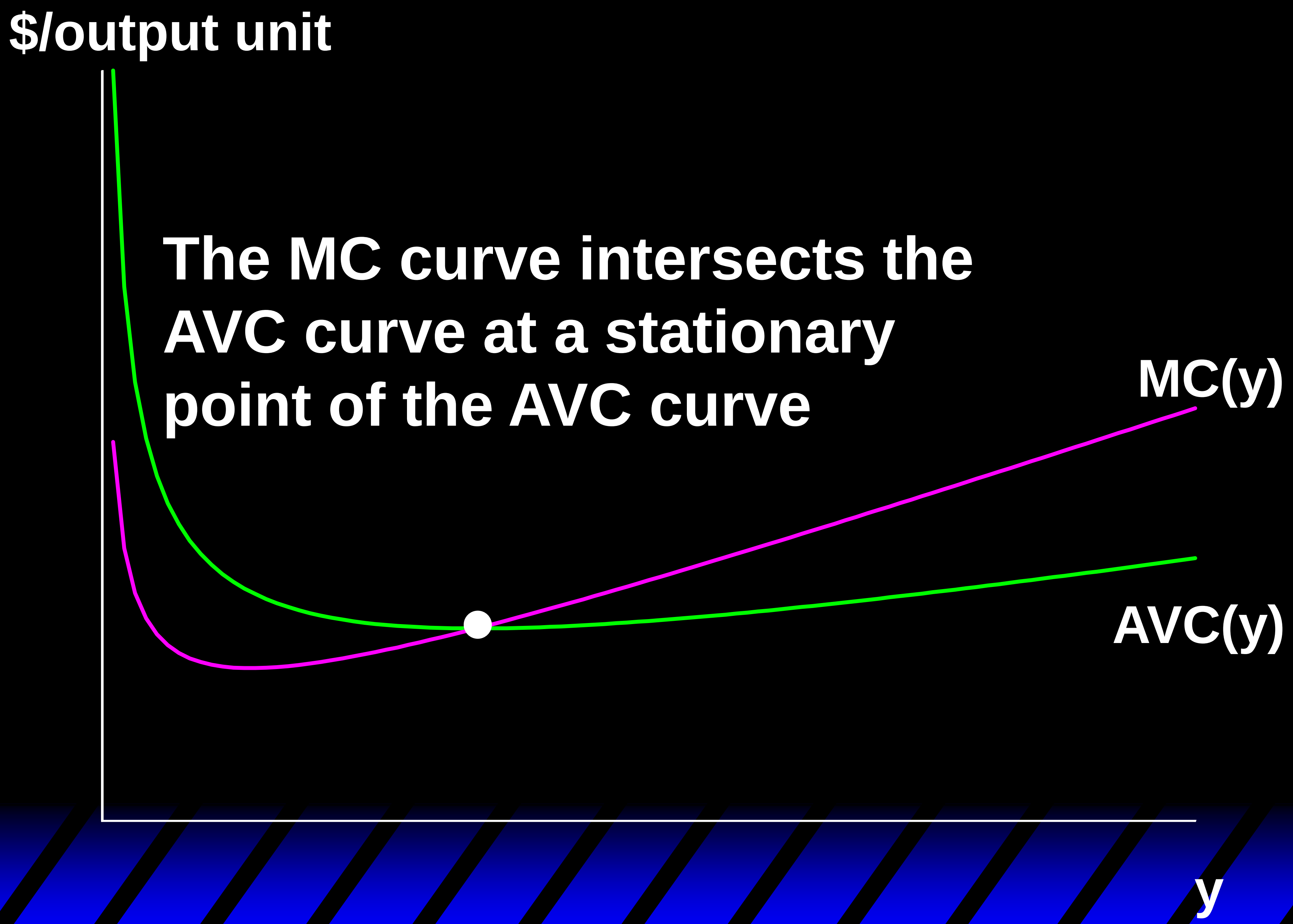
How is marginal cost related to average variable cost?

Since $$\mathbf{AVC(y) = \frac{c_v(y)}{y},}$$

$$\frac{\partial\, \mathbf{AVC(y)}}{\partial\, \mathbf{y}} = \frac{\mathbf{y \times MC(y) - 1 \times c_v(y)}}{\mathbf{y^2}}.$$

Therefore,

$$\frac{\partial\, \mathbf{AVC(y)}}{\partial\, \mathbf{y}} \begin{matrix} > \\ = \\ < \end{matrix} \mathbf{0} \quad \text{iff} \quad \mathbf{MC(y)} \begin{matrix} > \\ = \\ < \end{matrix} \frac{\mathbf{c_v(y)}}{\mathbf{y}} = \mathbf{AVC(y)}.$$


$/output unit
The MC curve intersects the AVC curve at a stationary point of the AVC curve
MC(y)
AVC(y)
y

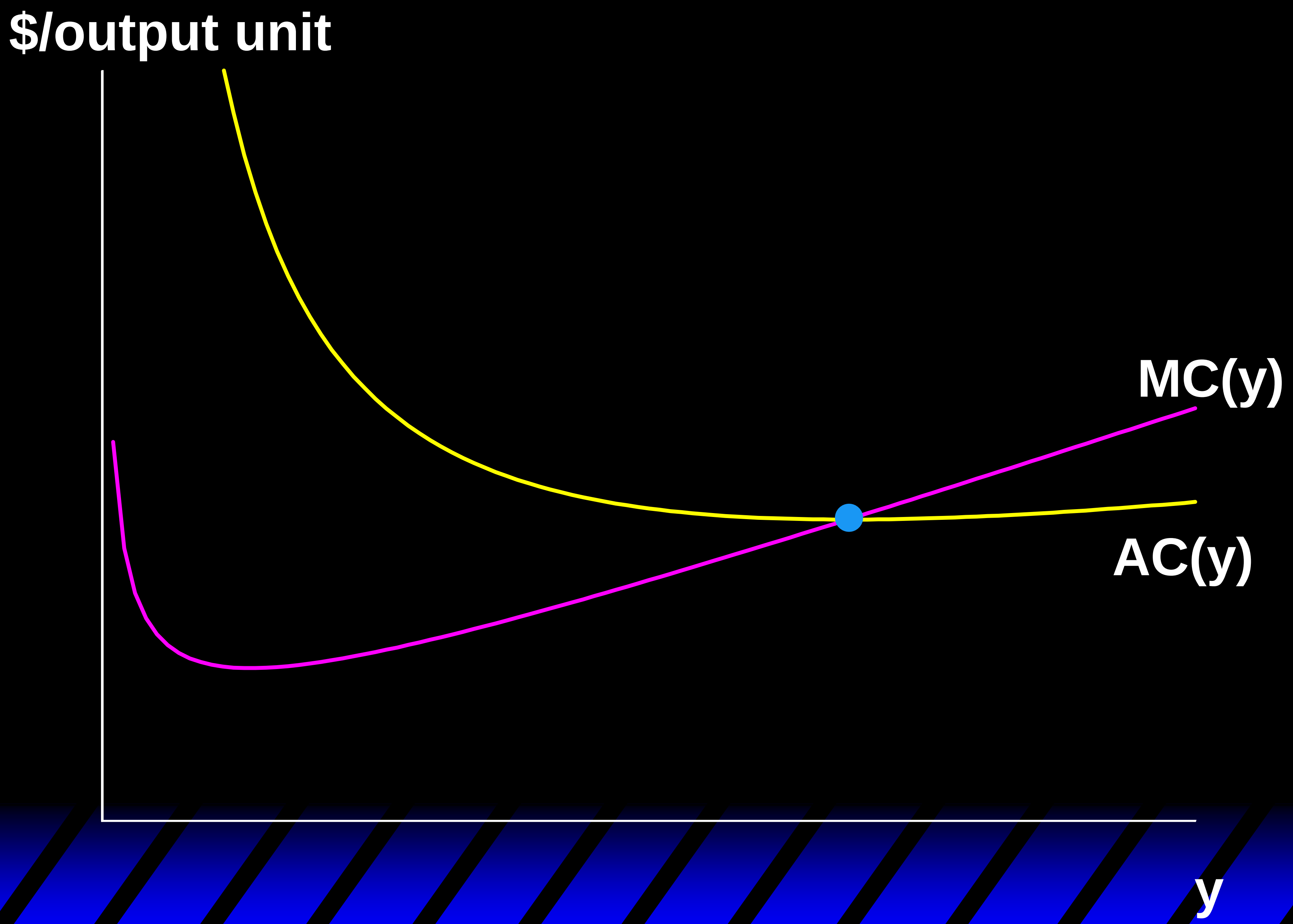
MC and ATC are similarly related

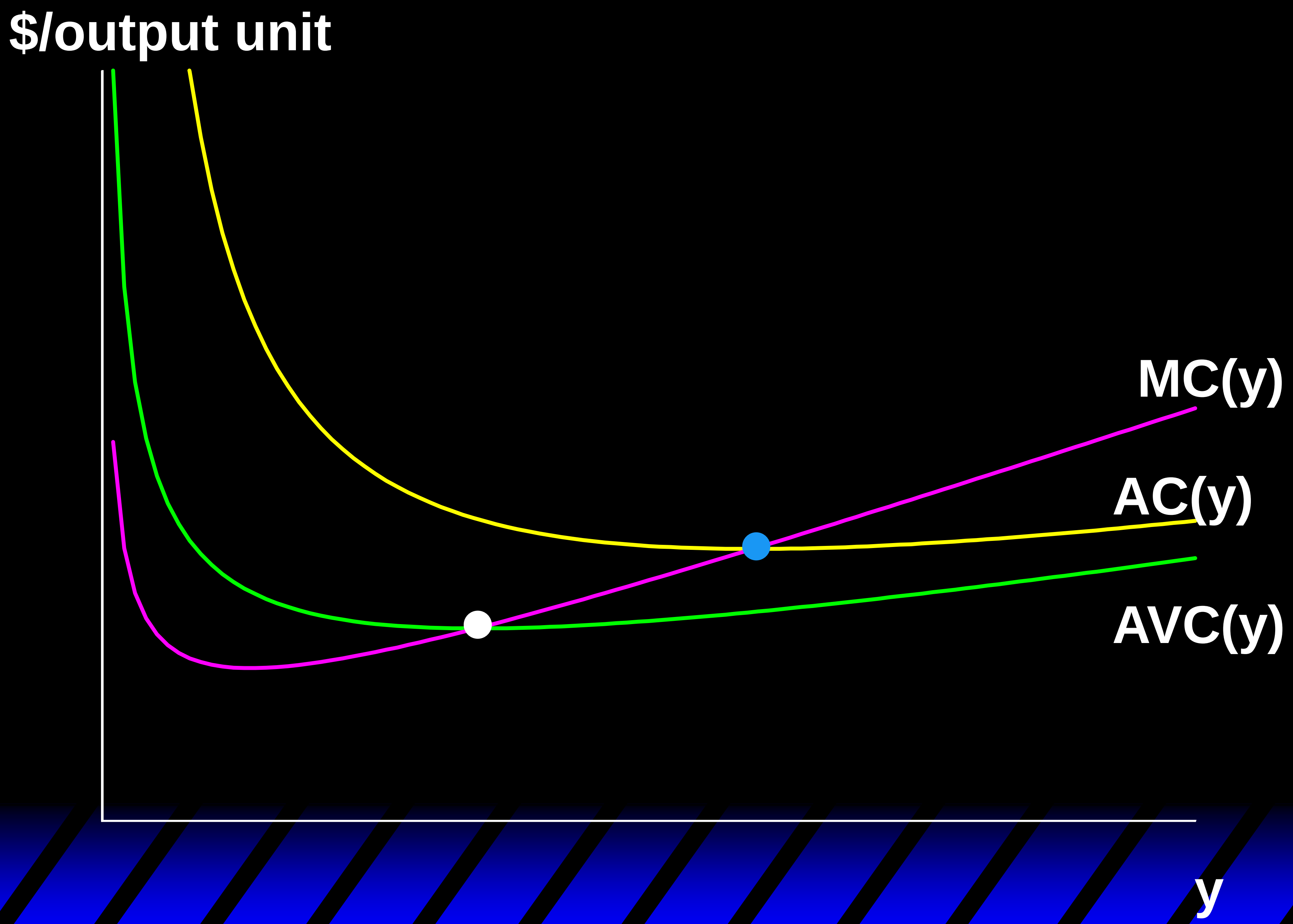
Similarly, since $$AC(y) = \frac{c(y)}{y},$$

$$\frac{\partial\ AC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c(y)}{y^2}.$$

Therefore,

$$\frac{\partial\ AC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} \frac{c(y)}{y} = AC(y).$$


$/output unit
MC(y)
AC(y)
y



$/output unit
MC(y)
AC(y)
AVC(y)
y

Short-Run & Long-Run Total Cost Curves

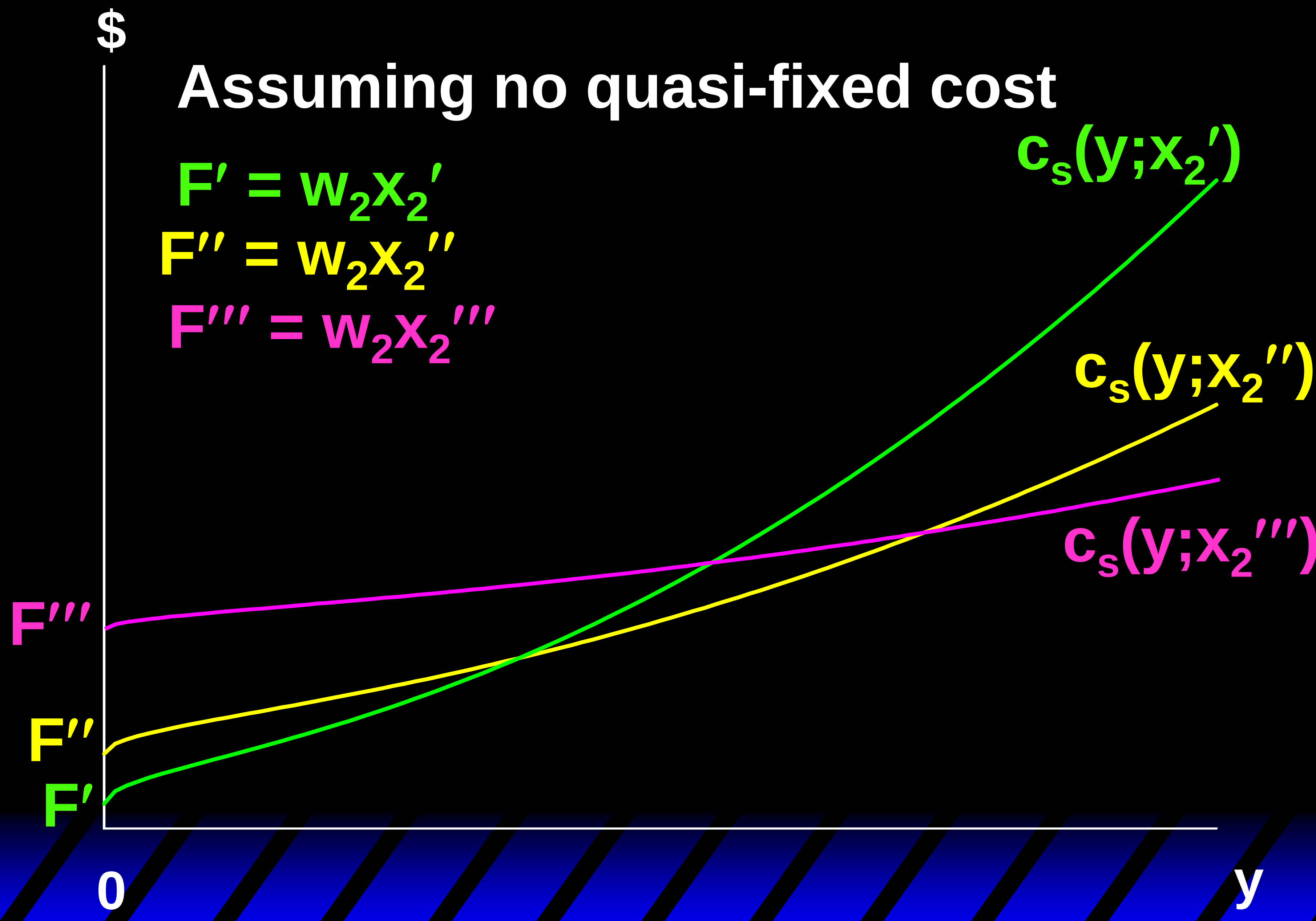
Suppose x_2 is a fixed input in the short-run, and can only be fixed at the following three levels:

$x_2 = x_2'$

or $x_2 = x_2''$ $x_2' < x_2'' < x_2'''$.

or $x_2 = x_2'''$.

Assuming no quasi-fixed cost



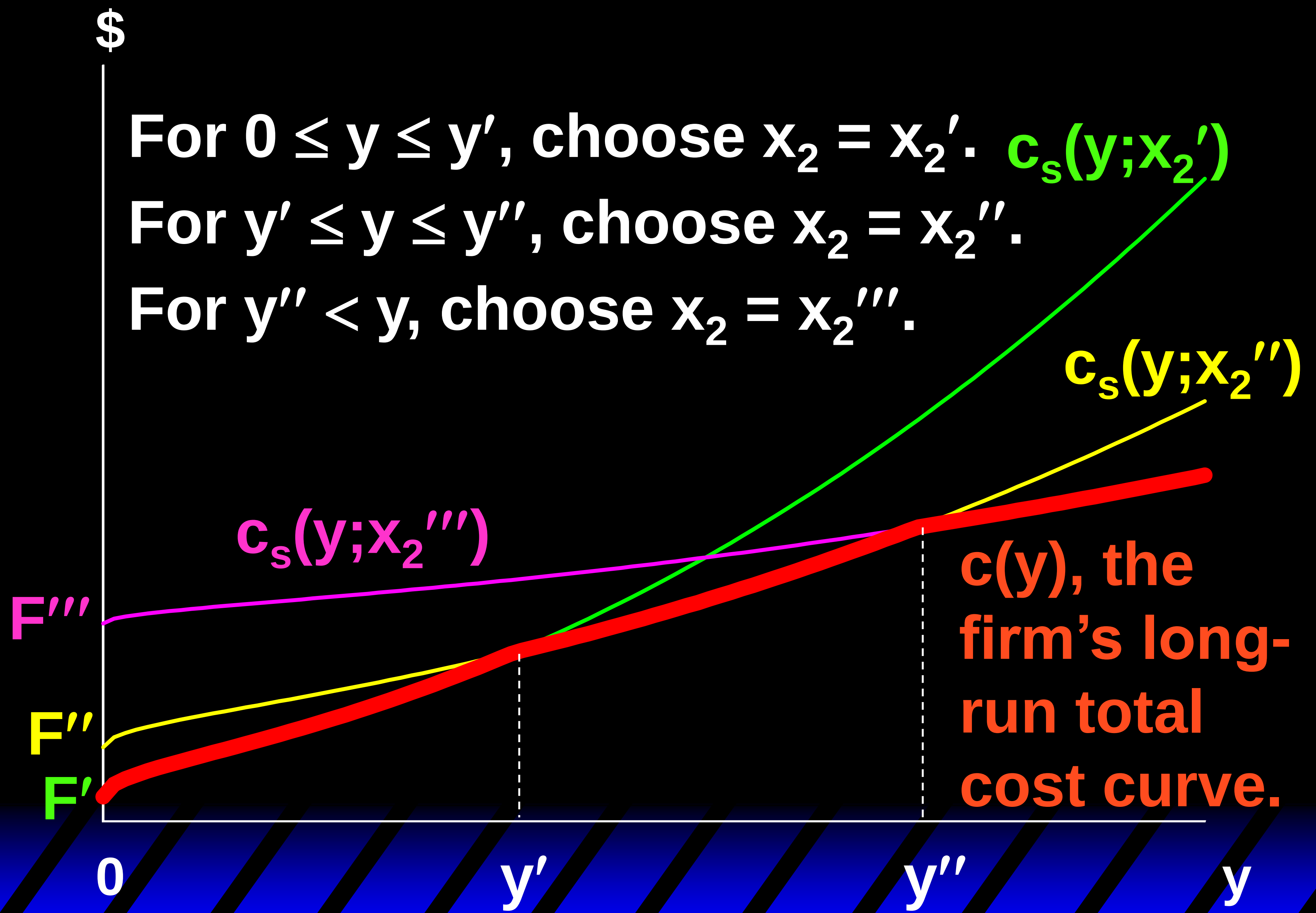
Short-Run & Long-Run Total Cost Curves

- The firm has three short-run total cost curves.
- In the long-run, how does the firm optimally choose x_2?

For $0 \leq y \leq y'$, choose $x_2 = x_2'$.

For $y' \leq y \leq y''$, choose $x_2 = x_2''$.

For $y'' < y$, choose $x_2 = x_2'''$.

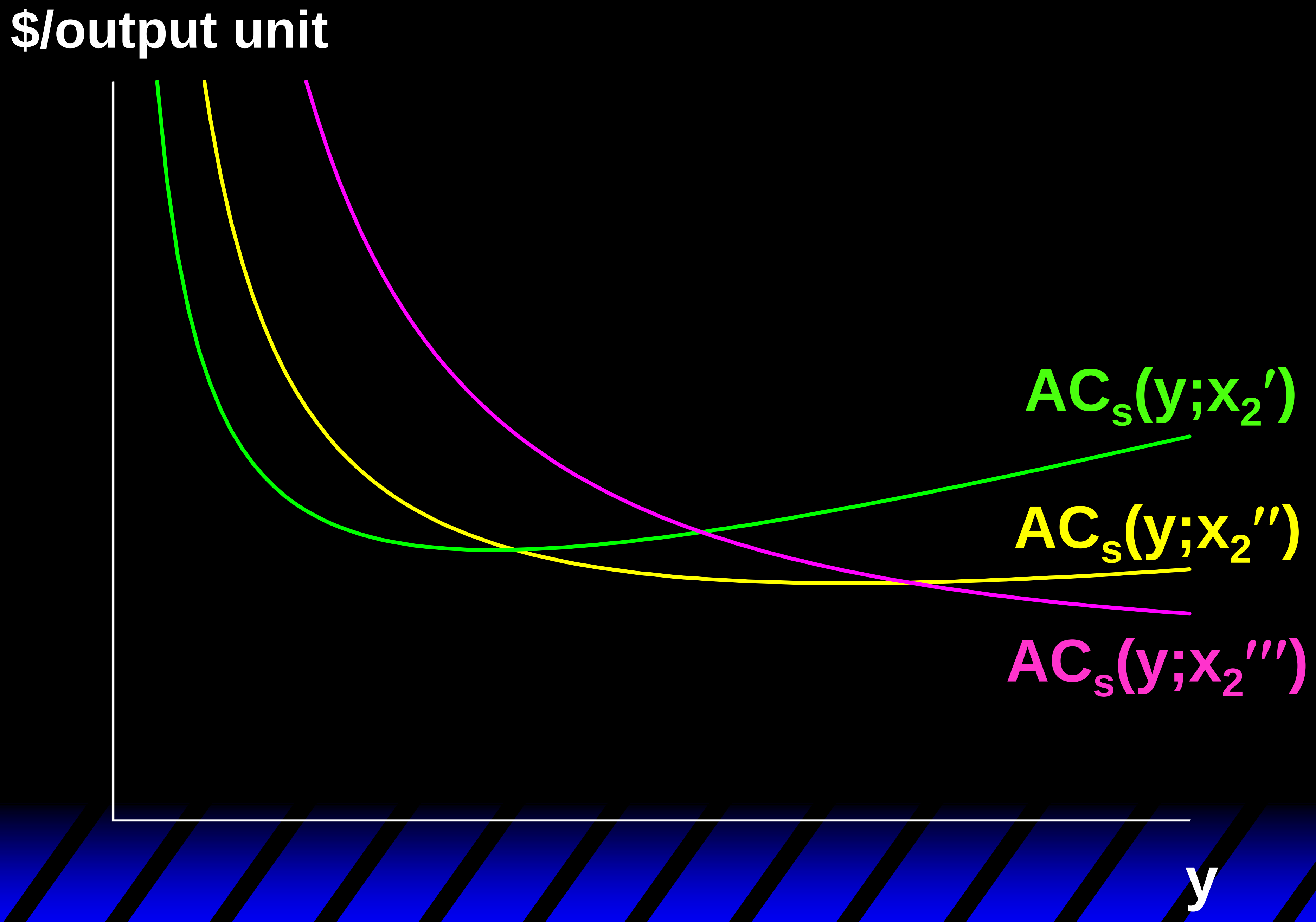


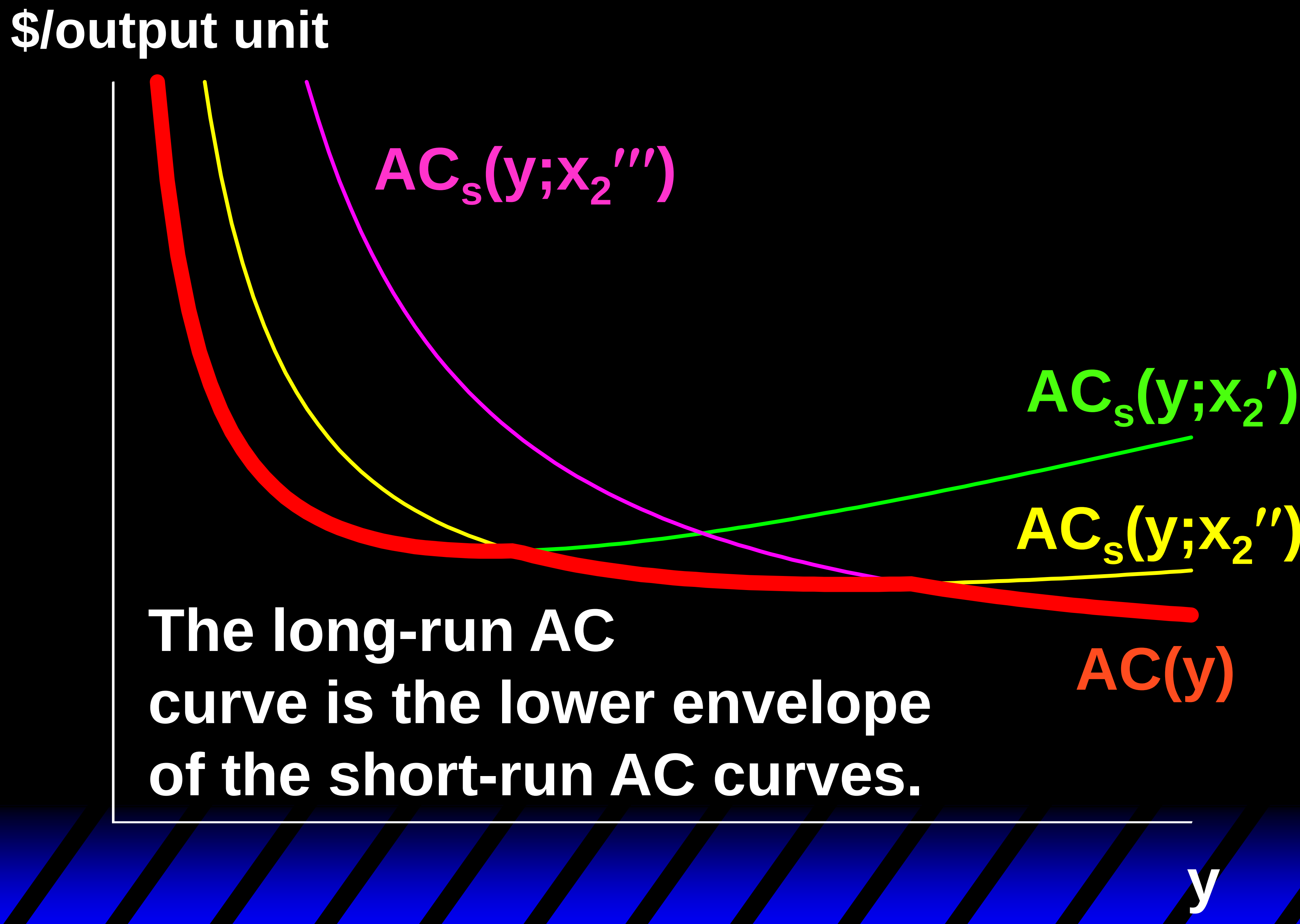
Short-Run & Long-Run Total Cost Curves

- The long-run total cost curve is the **lower envelope** of the short-run total cost curves.

Short-Run & Long-Run Average Total Cost Curves

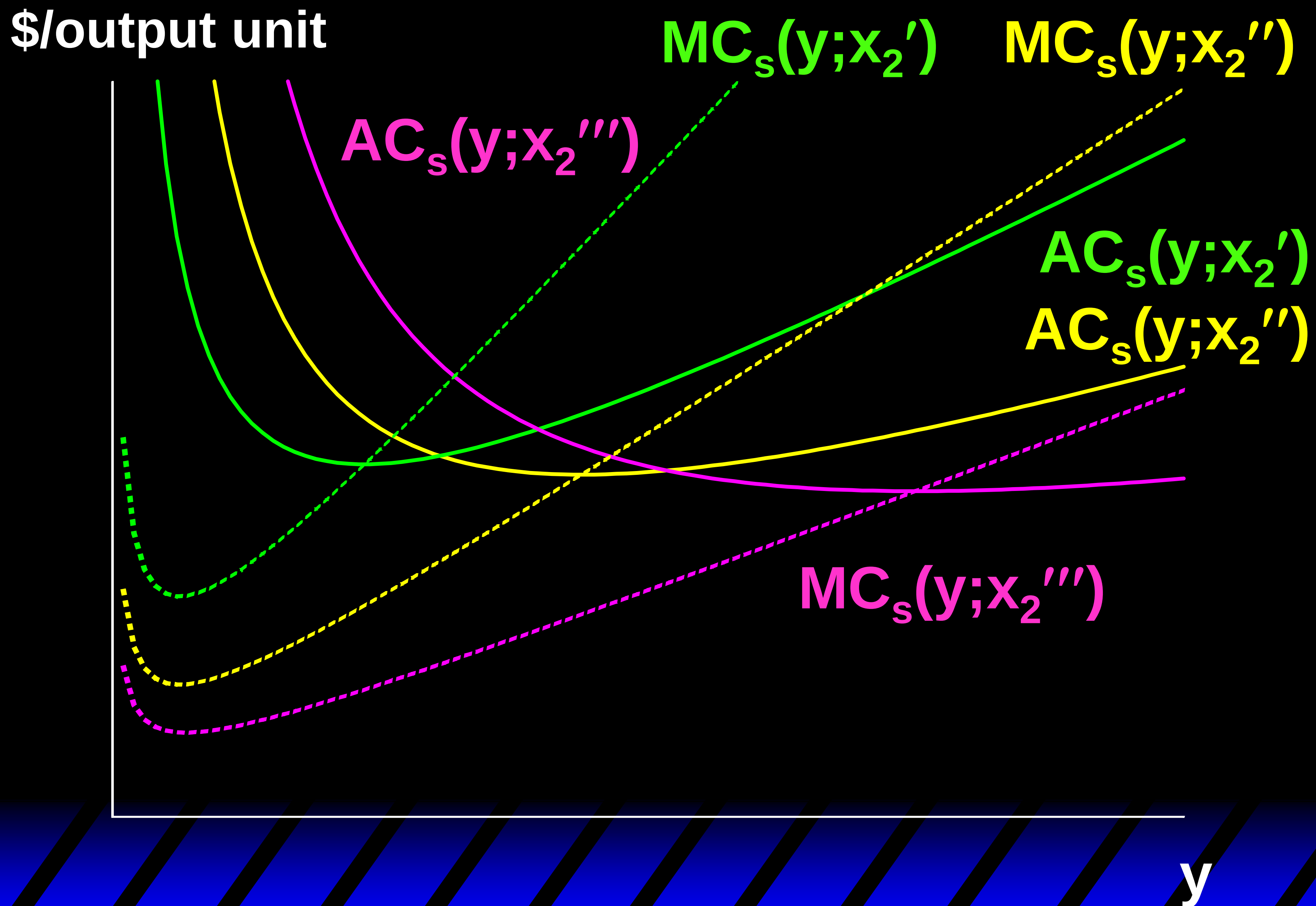
- The same is true with AC curves:
  - The long-run AC curve must be the lower envelope of all of the firm's short-run AC curves.


$/output unit
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$AC_s(y;x_2''')$
y

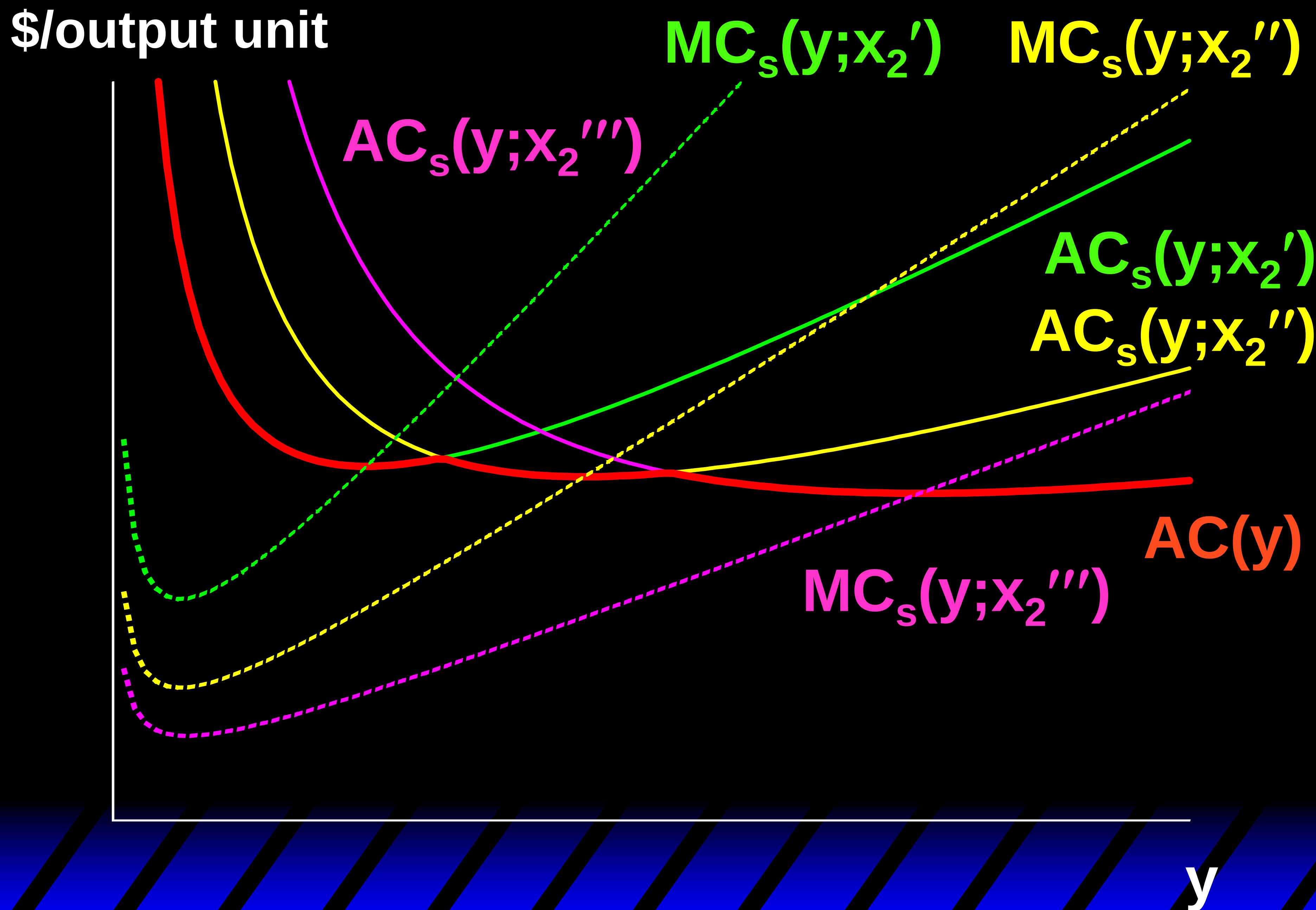

$/output unit
ACs(y;x2‴)
ACs(y;x2′)
ACs(y;x2″)
The long-run AC
curve is the lower envelope
of the short-run AC curves.
AC(y)
y

How about the Long-Run Marginal Cost Curves?

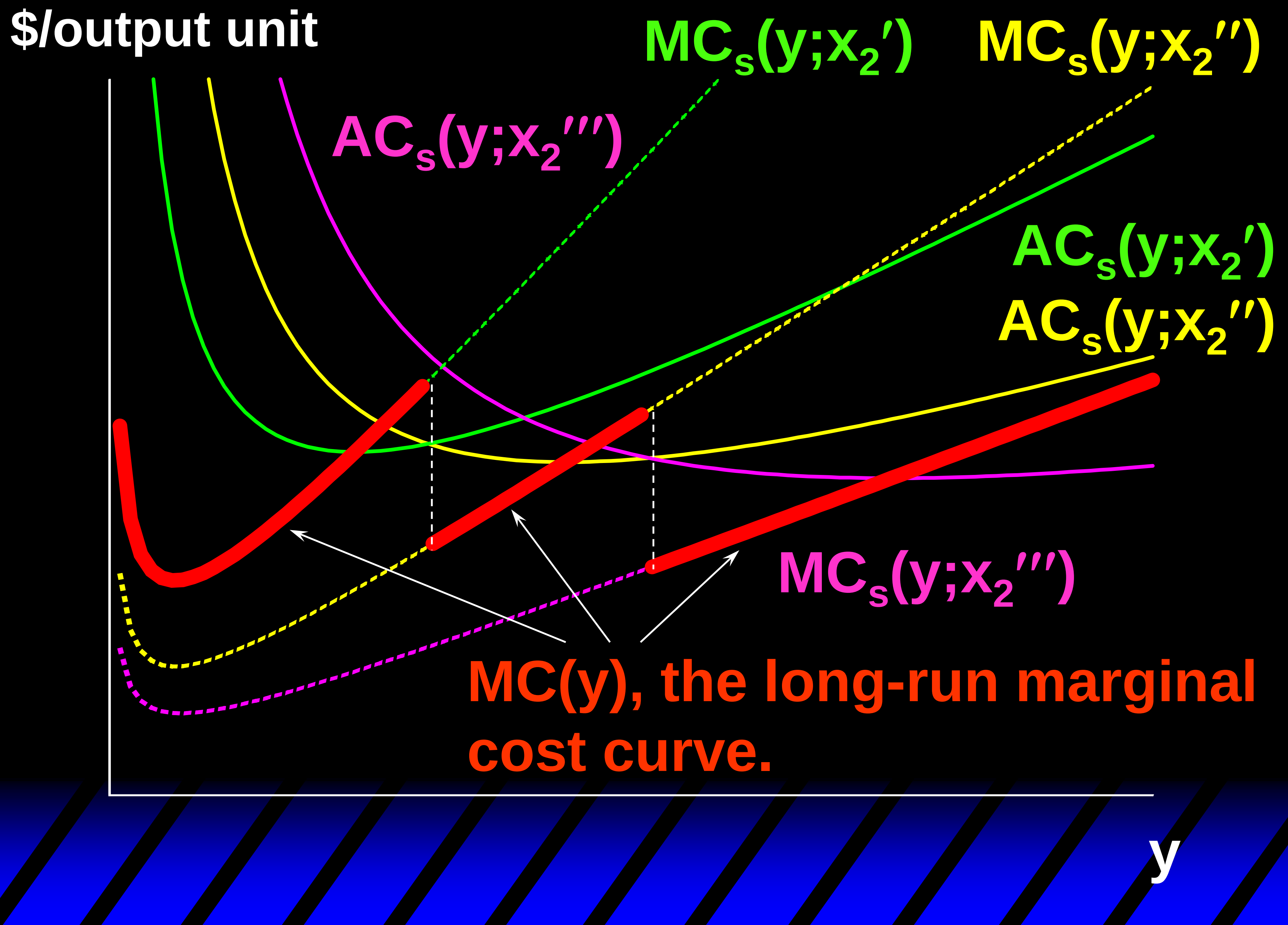
- What is the definition of MC(y)?
- We need to ask: Which short run technology is chosen to produce y?
- Here is an example:



$/output unit
MCs(y;x2′)
MCs(y;x2″)
ACs(y;x2‴)
ACs(y;x2′)
ACs(y;x2″)
MCs(y;x2‴)
y



$/output unit
MCs(y;x2′)
MCs(y;x2″)
ACs(y;x2‴)
ACs(y;x2′)
ACs(y;x2″)
AC(y)
MCs(y;x2‴)
y

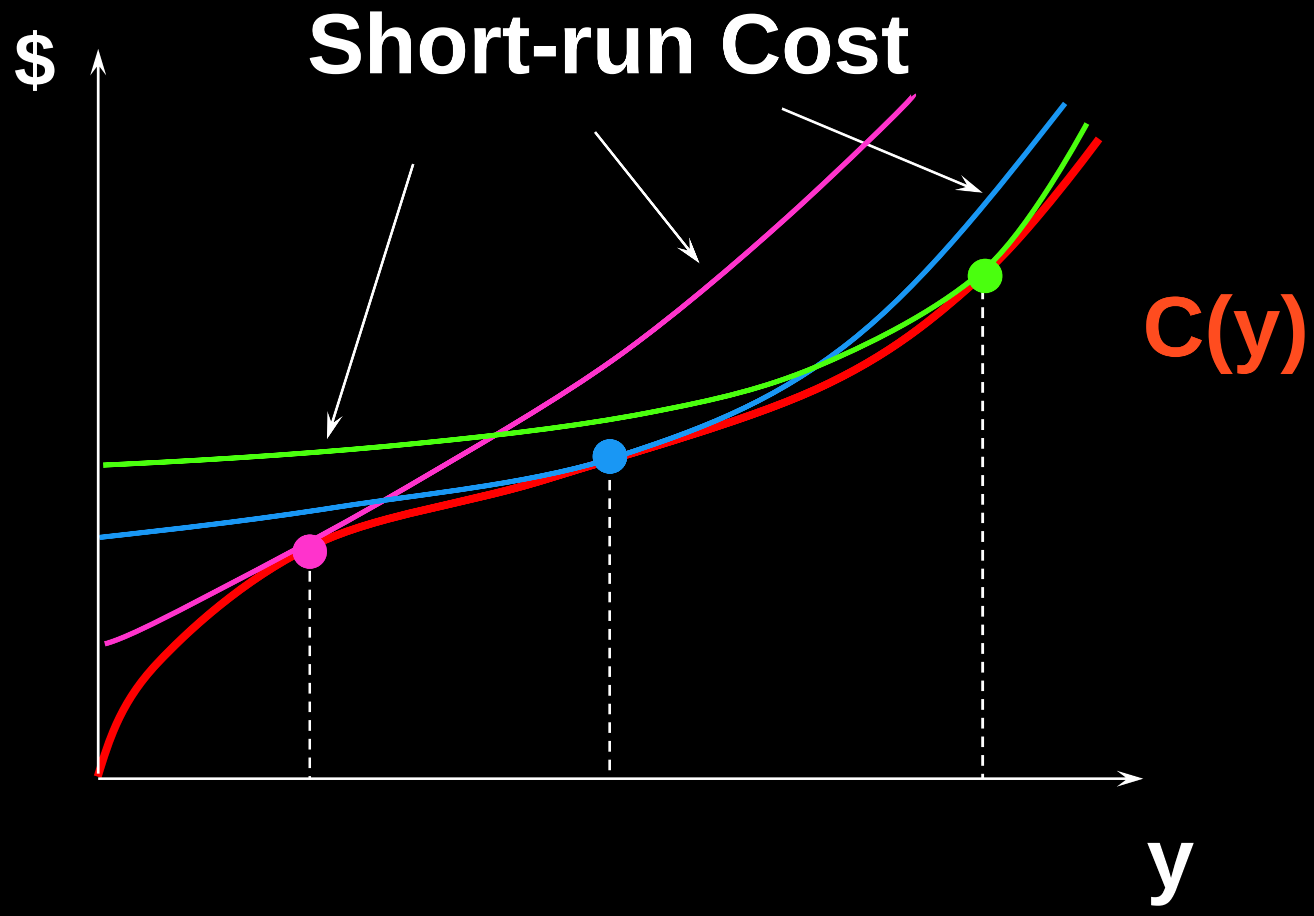


\$/output unit
$MC_s(y;x_2')$
$MC_s(y;x_2'')$
$AC_s(y;x_2''')$
$AC_s(y;x_2')$
$AC_s(y;x_2'')$
$MC_s(y;x_2''')$
MC(y), the long-run marginal cost curve.
y

General Picture

- In general, there is a continuum of short-run total cost curves
  - Each corresponds to a level of x_2
  - The long-run total cost curve is the lower envelope of the short-run ones.

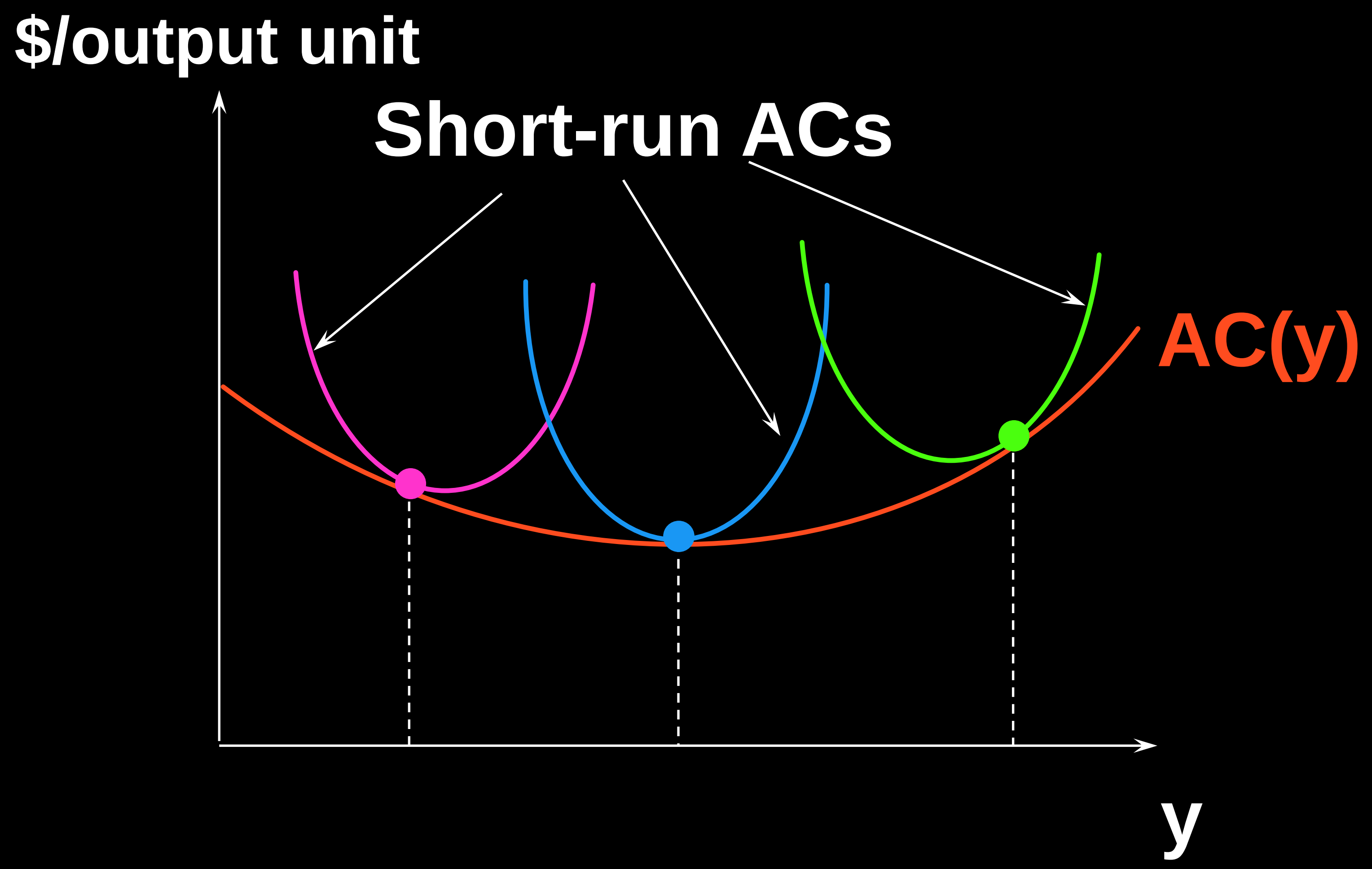
Short-Run & Long-Run Cost Curves



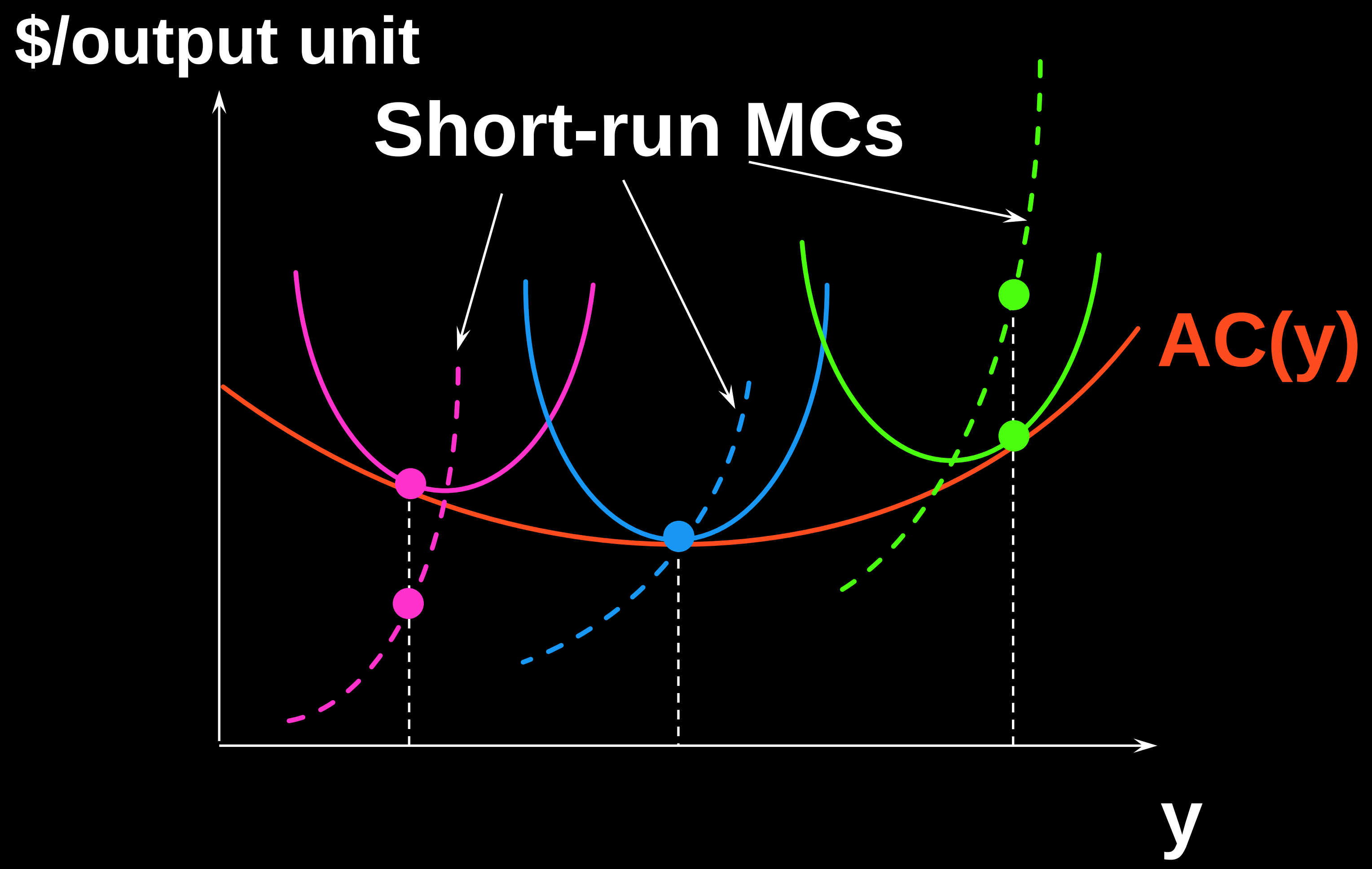
An Observation

If $c(y') = c_s(y';x_2')$ at some y' and x_2', then $MC(y') = MC_s(y';x_2')$

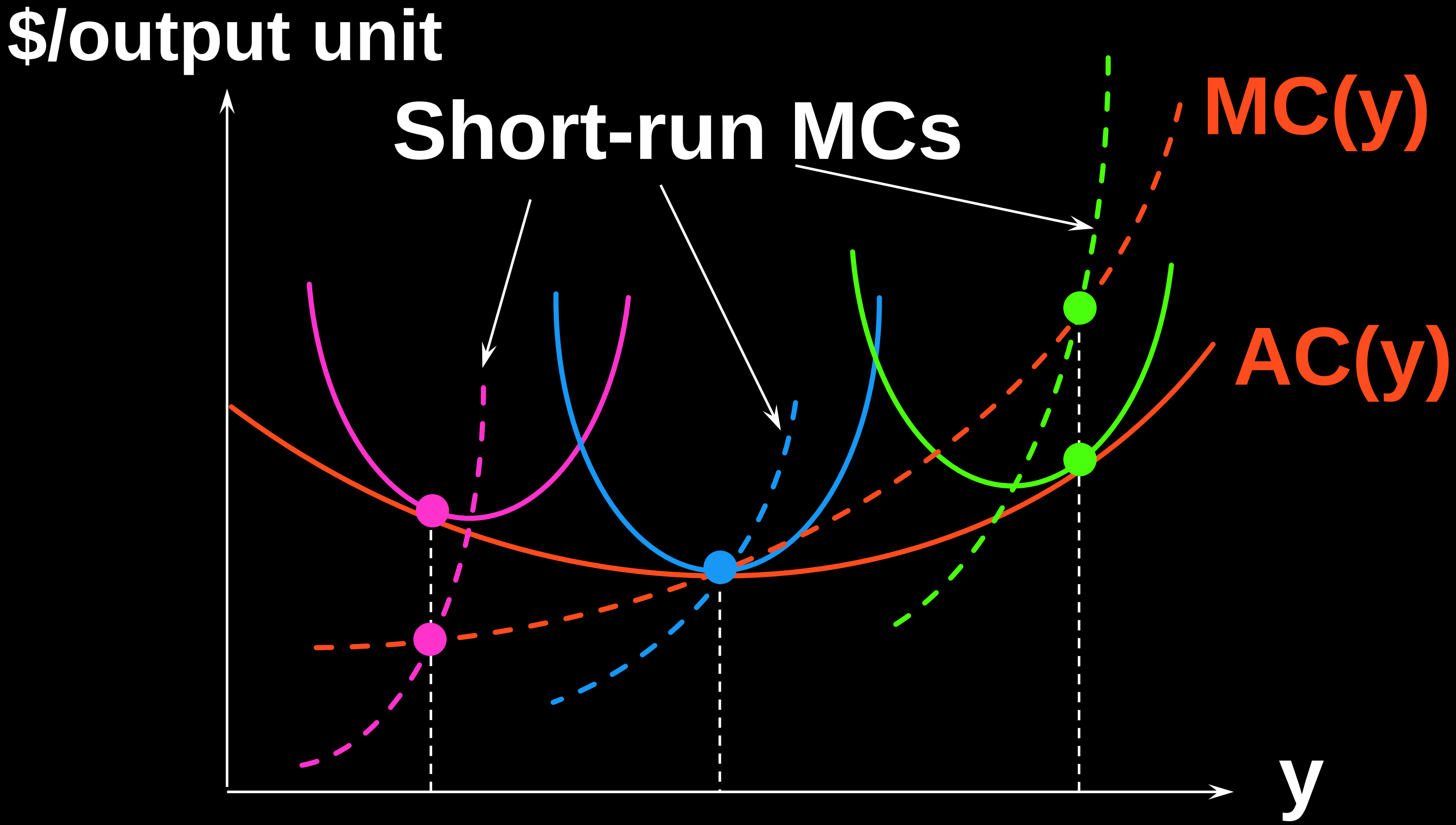
Short-Run & Long-Run AC Curves



Short-Run & Long-Run Marginal Cost Curves



Short-Run & Long-Run Marginal Cost Curves



At each output level y, MC(y) = $MC_s(y;x_2)$ where x_2 is optimally chosen for y.

Summary: Key Relationships

1. **AC and AVC;**
2. **MC and AVC (AC);**
3. **Long-run C (AC) with short-run C (AC);**
4. **Long-run MC and short-run MC’s.**