week 1

1.1

$$h_{max} = 2\times10^{5}\ m, \quad g_m = 1.8\ m/s^2$$

$$\parallel$$

$$\frac{v_0^2}{2g_m}$$

$$\Rightarrow v_0 = 600\sqrt{2}\ m/s$$

1.3

$$v_1 = 90\,km/h \quad v_2 = 65\,km/h \quad t_0 = 0.7\,s$$

$$a = 7.5\,m/s^2$$

$$d = 80\,m$$

$$d_t = d - (v_1+v_2)t_0 - \frac{(v_1+v_2)^2}{2\cdot(2a)} < 0$$

∴ 两车会相撞

$$d - (v_1+v_2)t_0 = \left(v_1 t - \tfrac{1}{2}at^2\right) + \left(v_2 t - \tfrac{1}{2}at^2\right)$$

$$\Rightarrow t \approx 1.6\,s$$

$$v_{末} = v_1 - at = (25 - 7.5\times1.6)\,m/s$$

$$= 13\,m/s$$

1.5

$$h_{max} = \frac{v_0^2}{2g} \approx 34.5\,m$$

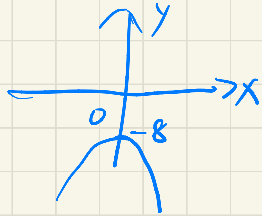
$$t = \frac{2v_0}{g} \approx 5.3\,s$$

$$V = qt \approx 24.7\,L$$

1.7

(1) $\begin{cases} x = 2t \\ y = 4t^2 - 8 \end{cases} \Rightarrow t = \frac{1}{2}x$

$\Rightarrow y = x^2 - 8$



(2) $x'(t) = 2 \quad y'(t) = 8t$

$x''(t) = 0 \quad y''(t) = 8$

$t_1 = 1s: \begin{cases} \vec{r} = (2, -4) \\ \vec{v} = (2, 8) \\ \vec{a} = (0, 8) \end{cases}$ $\quad t_2 = 2s: \begin{cases} \vec{r} = (4, 8) \\ \vec{v} = (2, 16) \\ \vec{a} = (0, 8) \end{cases}$

1.10

轨道方程: $y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta} \quad \theta \in [0, \frac{\pi}{2})$

代入 $x = L$

$y_L(\theta) = L\tan\theta - \frac{gL^2}{2v^2\cos^2\theta}$

$\cos^2\theta = \frac{1}{1+\tan^2\theta}$

$y_L'(\theta) = \frac{L}{\cos^2\theta} - \frac{gL^2}{2v^2} \cdot (-2) \frac{-\sin\theta}{\cos^3\theta}$

$= \frac{L}{\cos^2\theta} - \frac{gL^2}{v^2} \cdot \frac{\sin\theta}{\cos^3\theta} = 0$

$\Rightarrow \tan\theta = \frac{v^2}{gL} = \frac{125}{98}$

$y_L(\theta)_{max} = L\tan\theta - \frac{gL^2}{2v^2} \cdot (1 + \tan^2\theta)$

$\approx 12.2m < h \Rightarrow$ 扔不过, 最高12.2m

1.12

由于两枚炮弹均为抛物体，初速度等大反向

若能相碰，则必在位置中点

$$\begin{cases} y_中 = h - v_0 \sin\theta \, t_中 - \frac{1}{2} g t_中^2 > 0 \\ t_中 = \frac{s}{2} \div v_0 \cos\theta \quad (\tan\theta = \frac{h}{s} = \frac{1}{4}) \end{cases}$$

$t_中 = \frac{25\sqrt{17}}{v_0}$

$\Rightarrow v_0 > 45.6 m/s$