5.6

(1) $J = \frac{2}{3} m R^2 = \frac{2}{3} \times 60 \times 12 \times 1.67 \times 10^{-27} \times (71 \times 10^{-9})^2$

$= 1.01 \times 10^{-39}\ kg \cdot m^2$

(2) $E_K = \frac{1}{2} J \omega^2$

$\Rightarrow \omega = \sqrt{\frac{2E_K}{J}}$

$f = \frac{1}{2\pi} \omega = 5.58 \times 10^8\ Hz$

5.10

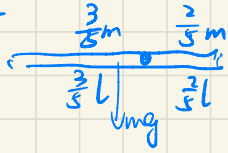
面密度 $\sigma = \frac{m}{\pi(R^2 - \frac{1}{4}R^2)} = \frac{4m}{3\pi R^2}$

$J = \frac{1}{2}(\sigma \pi R^2) \cdot R^2 - \left[\frac{1}{2}(\sigma \pi (\frac{R}{2})^2) \cdot (\frac{R}{2})^2 + \sigma \pi \frac{R^2}{4} \cdot \frac{R^2}{4}\right]$

$= \frac{\pi}{2} \sigma R^4 \cdot \frac{13}{16} = \frac{13\pi}{32} R^4 \cdot \frac{4m}{3\pi R^2}$

$= \frac{13}{24} m R^2$

5.12



$J = \frac{1}{3} \cdot \frac{3}{5} m \cdot (\frac{3}{5} l)^2 + \frac{1}{3} \cdot \frac{2}{5} m \cdot (\frac{2}{5} l)^2$

$= \frac{35}{375} m l^2 = \frac{7}{75} m l^2$

$M = mg \times \frac{1}{10} l = \frac{1}{10} mgl = J\alpha$

$\Rightarrow \alpha = \frac{15}{14} \frac{g}{l} = 10.5\ rad/s^2$

$E_K = \frac{1}{2} J \omega^2 = mg \cdot \frac{1}{10} l$

$\Rightarrow \omega = \sqrt{\frac{15g}{7l}} = 4.58\ rad/s$

5.14

面密度 $\sigma = \frac{m}{\pi R^2}$

$ds = 2\pi r dr$

$M_f = \int \sigma ds \cdot g \cdot \mu_k \cdot r = \int_0^R \mu_k \sigma g r \cdot 2\pi r dr = \frac{2}{3}\mu_k \sigma g \cdot \pi r^3 \Big|_0^R$

$= \frac{2}{3}\mu_k \sigma g \pi R^3 = \frac{2}{3}\mu_k mgR$

$\begin{cases} M_f = J\alpha = \frac{2}{3}\mu_k mgR \\ J = \frac{1}{2}mR^2 \end{cases} \Rightarrow \alpha = \frac{4}{3}\mu_k g/R$

$t = \frac{\omega}{\alpha} = \frac{3\omega R}{4\mu_k g}$

$M_F = M_f$

$W_F = M_F \cdot \omega t = \frac{2}{3}\mu_k mgR \cdot \omega \cdot \frac{3\omega R}{4\mu_k g}$

$= \frac{1}{2} m \omega^2 R^2$

$E_k = \frac{1}{2} J \omega^2 = \frac{1}{4} m \omega^2 R^2$

5.16

(1) 角动量 $l = \frac{3}{4} L m v_0 = J \cdot \omega$

$\Rightarrow \omega = \frac{Lmv_0}{J} = \frac{\frac{3}{4}Lmv_0}{\frac{1}{3}ML^2 + \frac{3}{16}mL^2} = \frac{\frac{3}{4}mv_0}{\frac{1}{3}mL + \frac{3}{16}mL} = 8.88\ rad/s$

(2)

$\frac{1}{2} J \omega^2 = \frac{3}{4}L \cdot (1-\cos\theta) mg + \frac{L}{2}(1-\cos\theta) \cdot Mg$

$\Rightarrow \cos\theta = -0.074$

$\Rightarrow \theta = 94^\circ$

5.17

$J_0 = J + 0 = 1200\ kg \cdot m^2$

$J_1 = J + M \cdot r^2 = 1520\ kg \cdot m^2$

$J_0 \omega = \frac{1}{2} J_1$

$\Rightarrow \omega' = \frac{J_0}{J_1}\omega = \frac{J_0}{J_1}\frac{2\pi}{t} = 0.50\ rad/s$

5.22

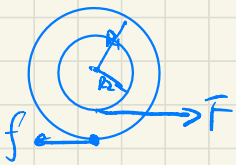
$$E_K = \frac{1}{2}Jw^2 = \frac{1}{2}\cdot 0.33MR^2\cdot\frac{4\pi^2}{T^2}$$
$$= 0.66\pi^2\cdot\frac{MR^2}{T^2}$$
$$= 2.13\times10^{29}\ J$$

$$W = -0.66\pi^2 MR^2\left(\frac{1}{T^2}-\frac{1}{(T+\Delta T)^2}\right)\div\Delta t$$
$$= -2.5\times10^{12}\ J/s$$

相当于我国4年发电量的11倍

$$M = J\cdot\alpha$$
$$= -0.33MR^2\cdot 2\pi\left(\frac{1}{T}-\frac{1}{T+\Delta T}\right)\div\Delta t$$
$$\approx -3.5\times10^{16}\ N\cdot m$$

5.25



$$\begin{cases} M = fR_1 - FR_2 = J\alpha \\ f - F = Ma_c = M\cdot\alpha R_1 \end{cases}$$
$$\Rightarrow \begin{cases} \alpha = 0.62\ rad/s^2 \\ f = 1384.62N \end{cases}$$

⇒ 向下方向滚动，角加速度为 0.62 rad/s

$$a_c = \alpha R_1 = 0.62\ m/s^2$$

5-26

(1) 

$$\begin{cases} mg - T = ma \\ RT = J\alpha = \frac{1}{3}mR^2\alpha \\ \alpha R = a \end{cases} \Rightarrow a = \frac{2}{5}g$$

$$\Rightarrow T = \frac{1}{3}mg$$

(2) $T' = mg$

$$\alpha = \frac{RT'}{J} = \frac{Rmg}{\frac{1}{3}mR^2} = 2\frac{g}{R}$$

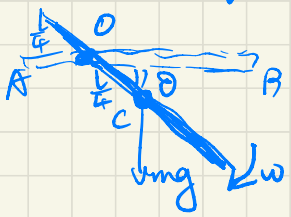
(3) $a' = \frac{2mg - mg}{m} = g$ 竖直向上

$$R \cdot 2mg = J\alpha' \Rightarrow \alpha' = 4\frac{g}{R}$$

$$a_{相对} = \alpha' R = 4g$$

$$a_{绝} = a_{相对} + a' = 5g$$

week6-1 PJ0例题



$$\begin{cases} M = mg\frac{l}{4}\cos\theta \\ \quad = J\cdot\alpha \\ J = \frac{1}{3}\cdot\frac{m}{4}\cdot(\frac{l}{4})^2 + \frac{1}{3}\cdot\frac{3m}{4}\cdot(\frac{3l}{4})^2 \\ \quad = \frac{7}{48}ml^2 \end{cases}$$

$$\Rightarrow \alpha = \frac{12}{7}\cos\theta\frac{g}{l}$$

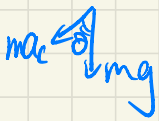
$$v_c = w\cdot\frac{l}{4} = \frac{1}{2}\sqrt{\frac{6gl\sin\theta}{7}}$$

$$a_{c//} = \alpha\cdot\frac{l}{4} = \frac{3}{7}\cos\theta g$$

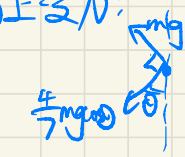
$$a_{c\perp} = \omega^2\frac{l}{4} = \frac{6gl\sin\theta}{7}$$

$$a_c = \sqrt{a_{c//}^2 + a_{c\perp}^2} = \frac{g}{7}\sqrt{9\cos^2\theta + 36\sin^2\theta}$$

$$= \frac{g}{7}\sqrt{9+27\sin^2\theta}$$



车轴上受力：



$$+ a_{c\perp}\cdot m = \frac{13}{7}g\sin\theta$$

$$F_{车轴} = \frac{mg}{7}\sqrt{16\cos^2\theta + 169\sin^2\theta}$$