

22.

$$R_1 \circ R_2 = \{ \langle c, d \rangle \}$$

$$R_2 \circ R_1 = \{ \langle a, d \rangle, \langle a, c \rangle \}$$

$$R_1^2 = \{ \langle a, a \rangle, \langle a, b \rangle, \langle a, d \rangle \}$$

$$R_2^2 = \{ \langle b, b \rangle, \langle c, c \rangle, \langle c, d \rangle \}$$

23.

$$R_1 = \{ \langle a, b \rangle, \langle b, c \rangle, \langle c, a \rangle \}$$

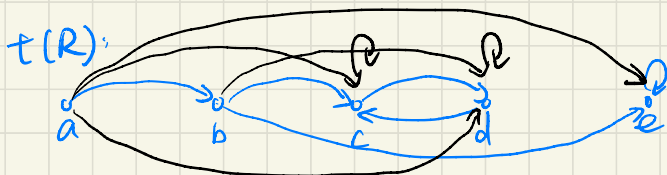
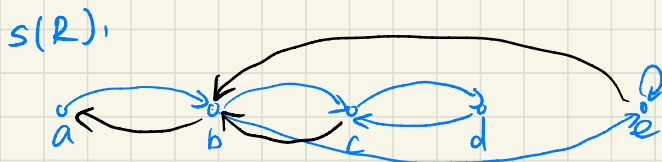
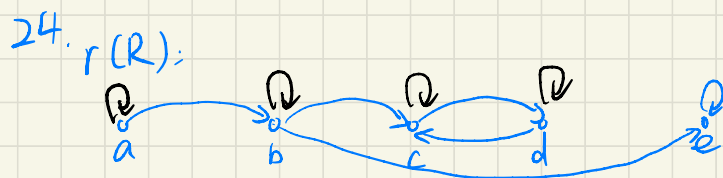
$$R_2 = \{ \langle a, c \rangle, \langle b, a \rangle, \langle c, b \rangle \}$$

$$M(R_1) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$M(R_2) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$M(R_1^2) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = M(R_2)$$

$$M(R_2^2) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = M(R_1)$$



27.

(1) 矩阵运算:

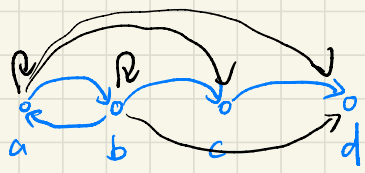
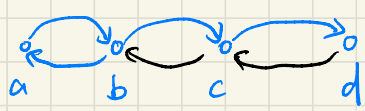
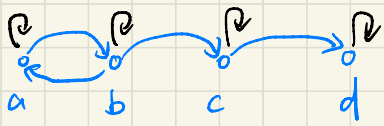
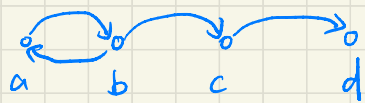
$$M(R) = \begin{pmatrix} 1 & & & \\ 1 & & 1 & \\ & & & 1 \end{pmatrix}$$

$$M(R^2) = \begin{pmatrix} 1 & & & \\ 1 & 1 & & \\ & 1 & 1 & \\ & & 1 & 1 \end{pmatrix}$$

$$M(R^3) = \begin{pmatrix} 1 & & & \\ 1 & & 1 & \\ & 1 & & 1 \\ & & 1 & 1 \end{pmatrix}$$

$$\begin{aligned} M(t(R)) &= M(R) + M(R^2) + M(R^3) + M(R^4) \\ &= \begin{pmatrix} 1 & & & \\ 1 & & 1 & \\ & & & 1 \end{pmatrix} + \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & 1 \end{pmatrix} \\ &\quad + \begin{pmatrix} 1 & & & \\ 1 & & 1 & \\ & 1 & & 1 \end{pmatrix} + \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & & & \\ 1 & 1 & 1 & 1 \\ & 1 & 1 & 1 \\ & & 1 & 1 \end{pmatrix} \end{aligned}$$

关系图:



(2) $M(R) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \xrightarrow{\begin{smallmatrix} \text{col 1} \\ [1,2] \\ [2,1] \end{smallmatrix}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \xrightarrow{\begin{smallmatrix} \text{col 2} \\ [1,2] \\ [2,2] \end{smallmatrix}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \xrightarrow{\begin{smallmatrix} \text{col 3} \\ [1,3] \\ [2,3] \end{smallmatrix}} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

$M(t(R)) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \xrightarrow{\begin{smallmatrix} \text{col 4} \\ [1,4] \\ [2,4] \\ [3,4] \end{smallmatrix}}$