Chapter Thirty-Four

Welfare

In This Chapter

- **We focus on the topics of social welfare function and fairness**
  - **Let's skip the topic of Arrow's impossibility theorem (Ch. 34.1)**

Social Welfare Functions

- $u_i(x)$ is individual i's utility from social alternative x.

- Utilitarian: $W = \sum_{i=1}^{n} u_i(x).$

- Weighted-sum: $W = \sum_{i=1}^{n} a_i u_i(x)$ with each $a_i > 0.$

Social Welfare Functions

- **A more general form:**

$$W = f(u_1(x), u_2(x), \ldots, u_n(x))$$

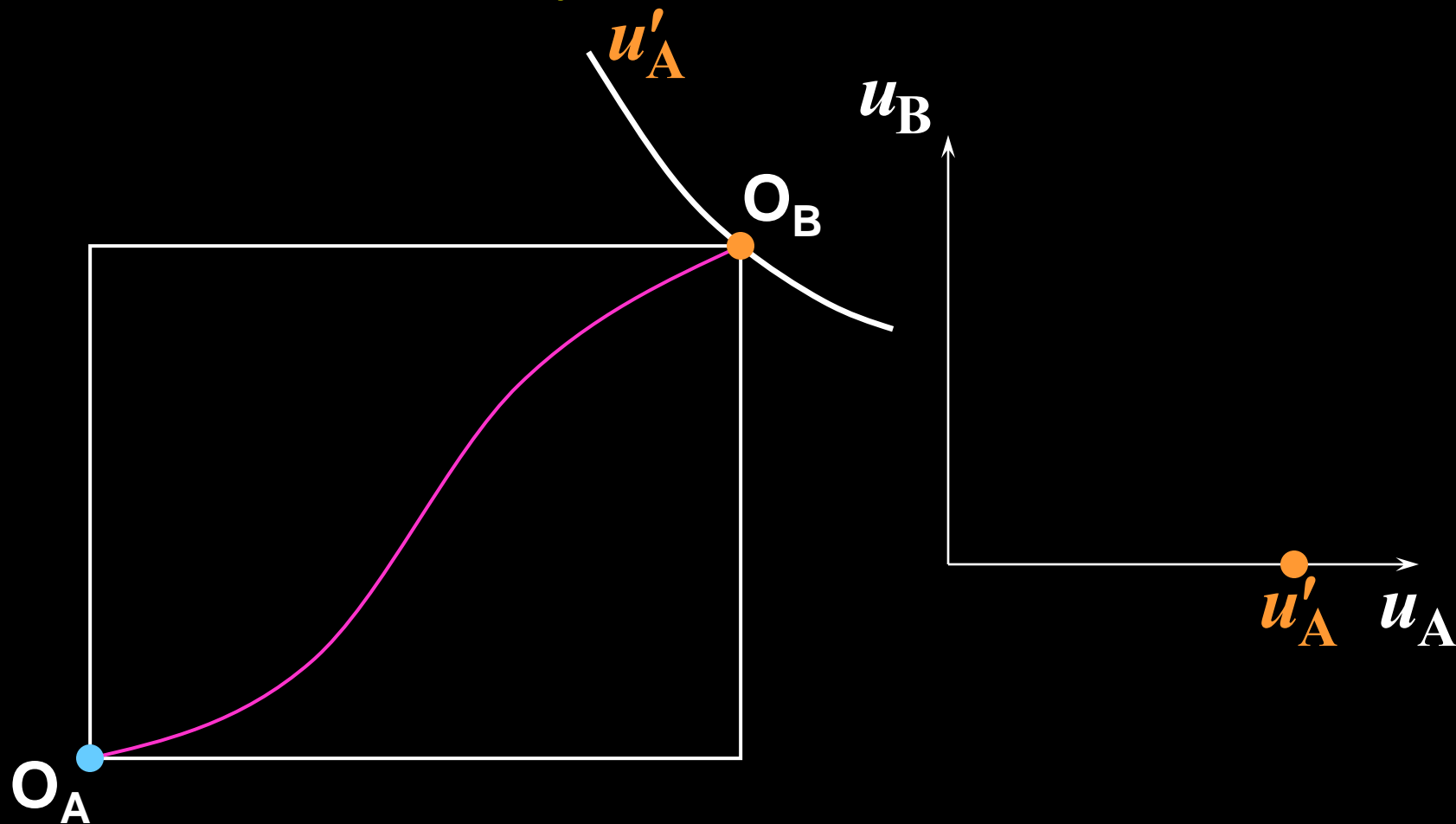where f is increasing in every argument.

Social Optima & Efficiency

- **A social alternative that maximizes a social welfare function must be Pareto optimal.**

  - **If not, then somebody's utility can be increased without reducing anyone else's utility, which means the value of the social welfare function can be further improved.**
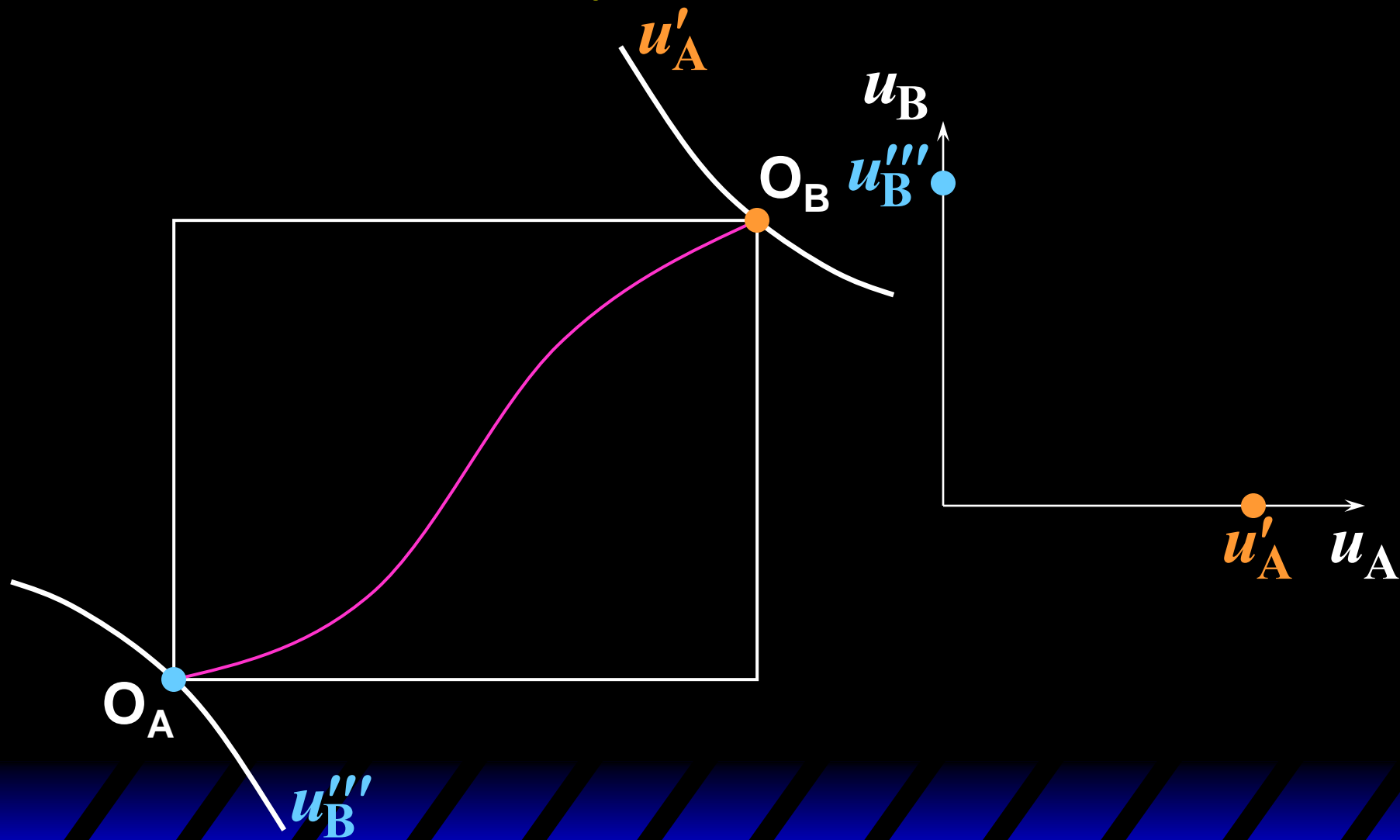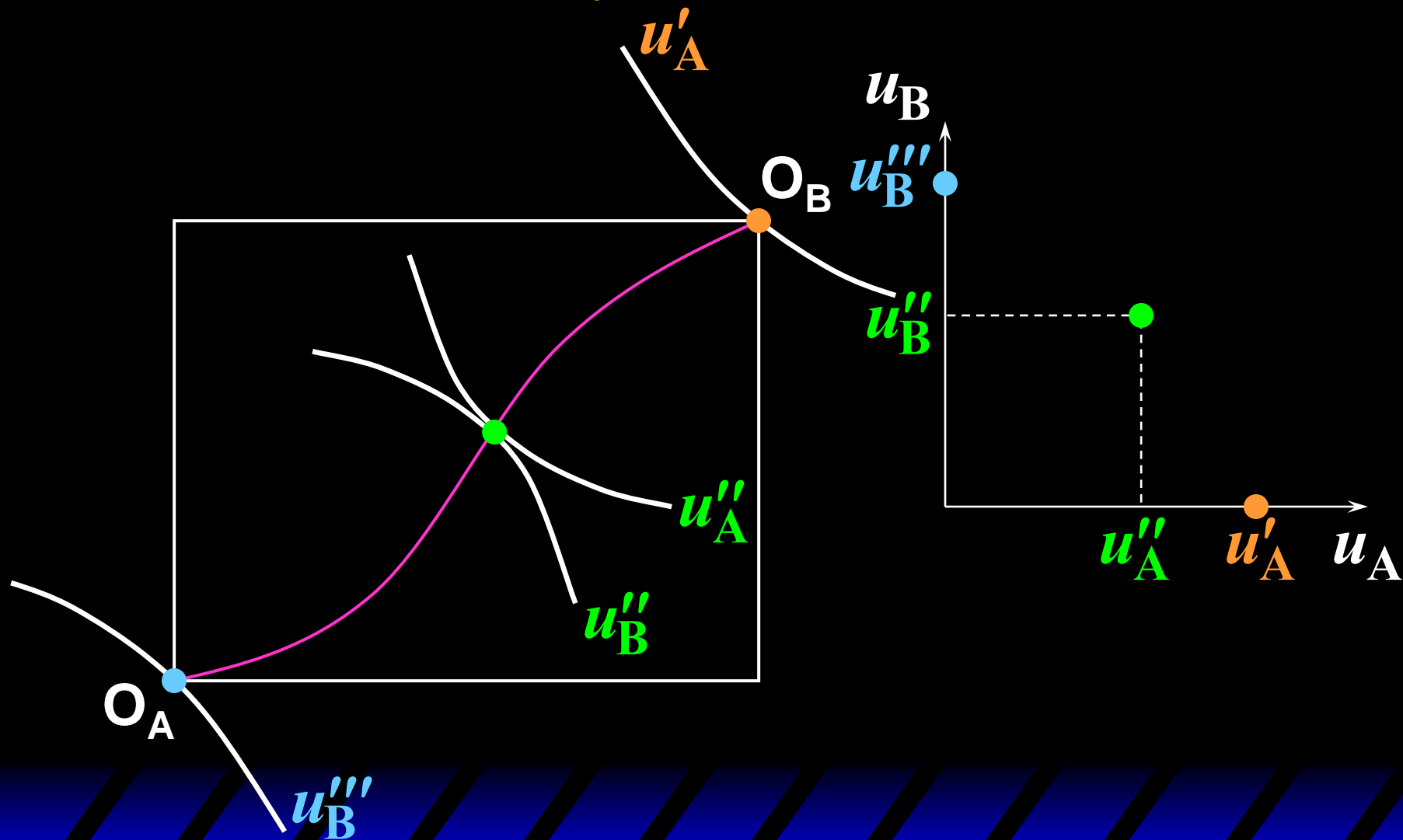
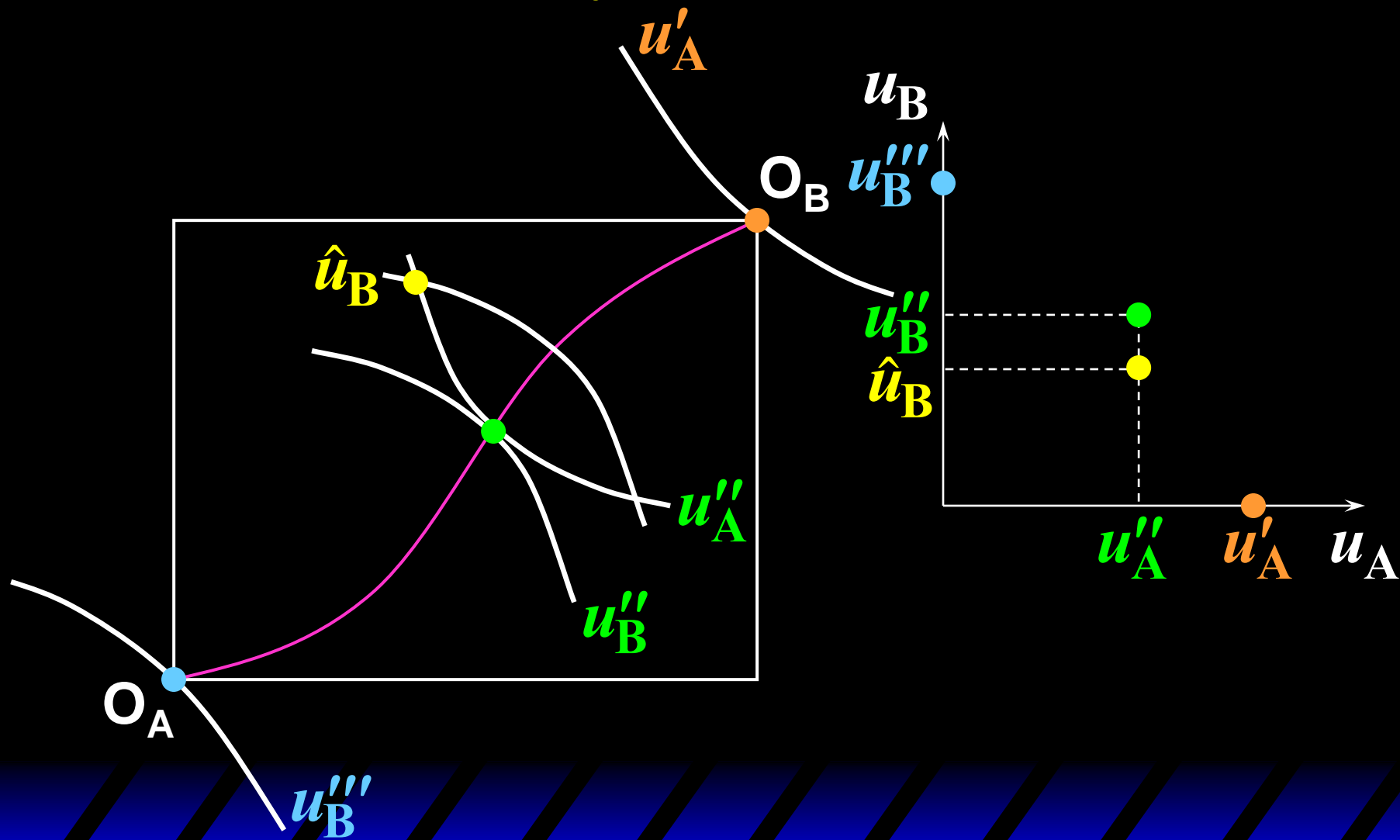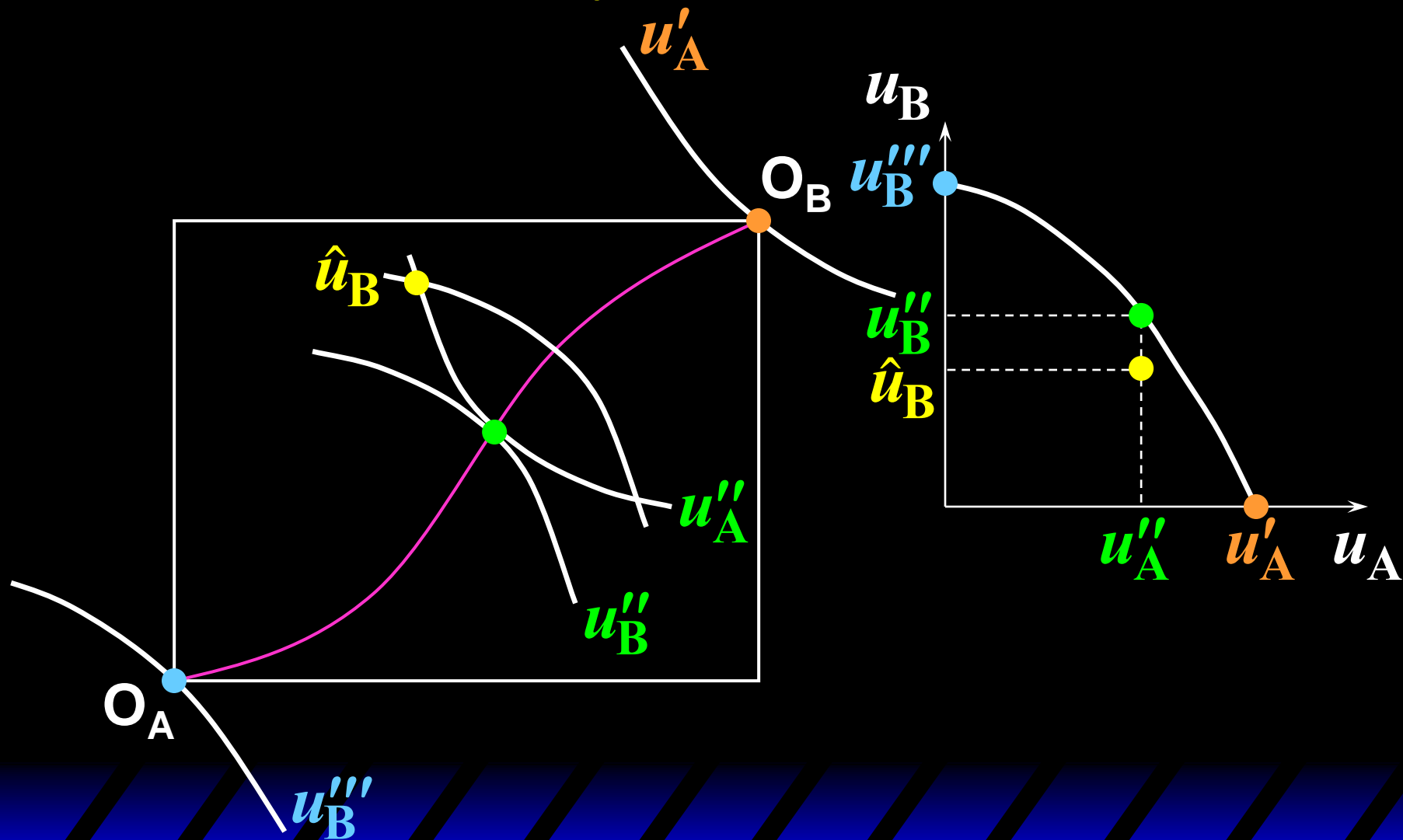# Utility Possibilities
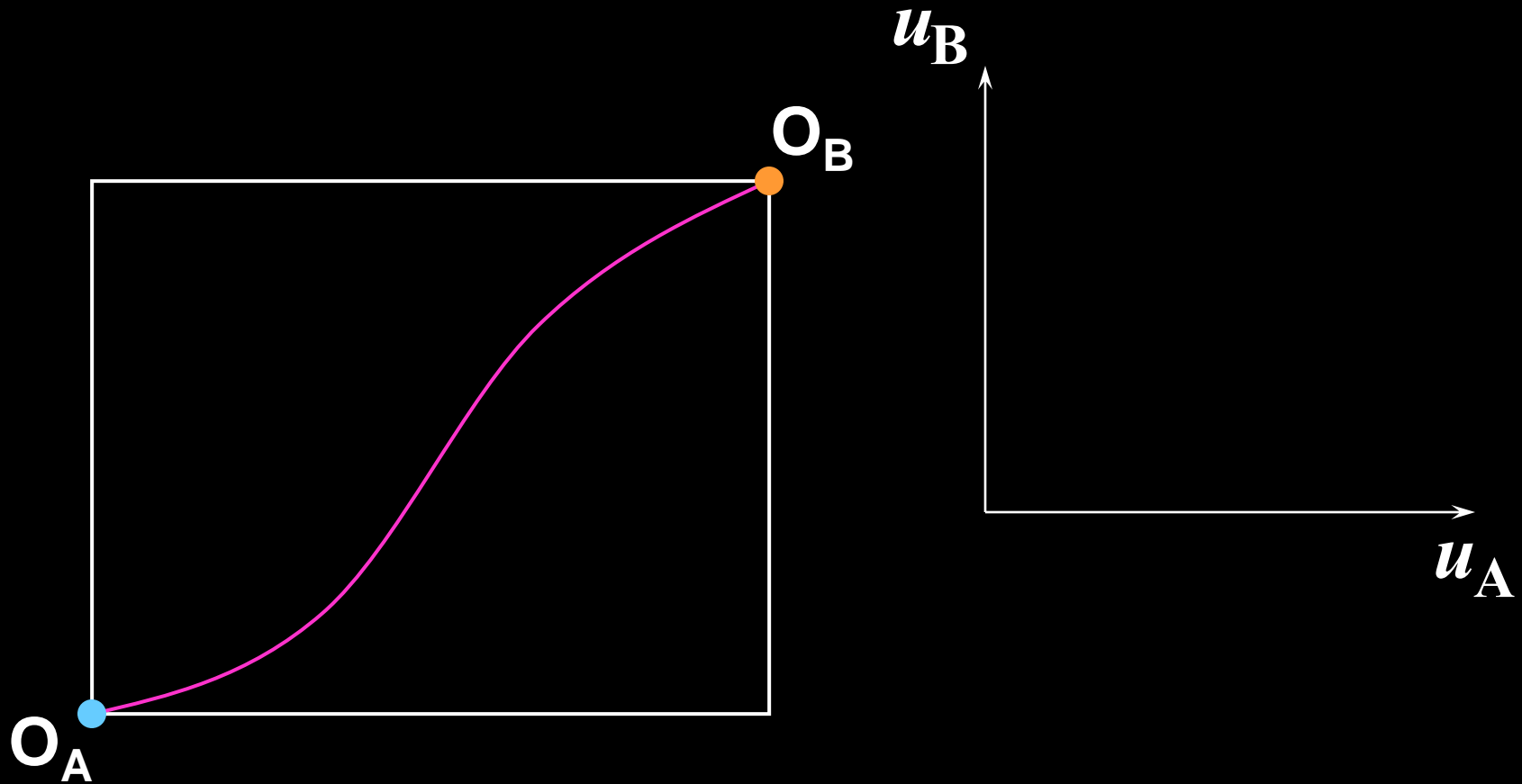
Utility Possibilities

Utility Possibilities
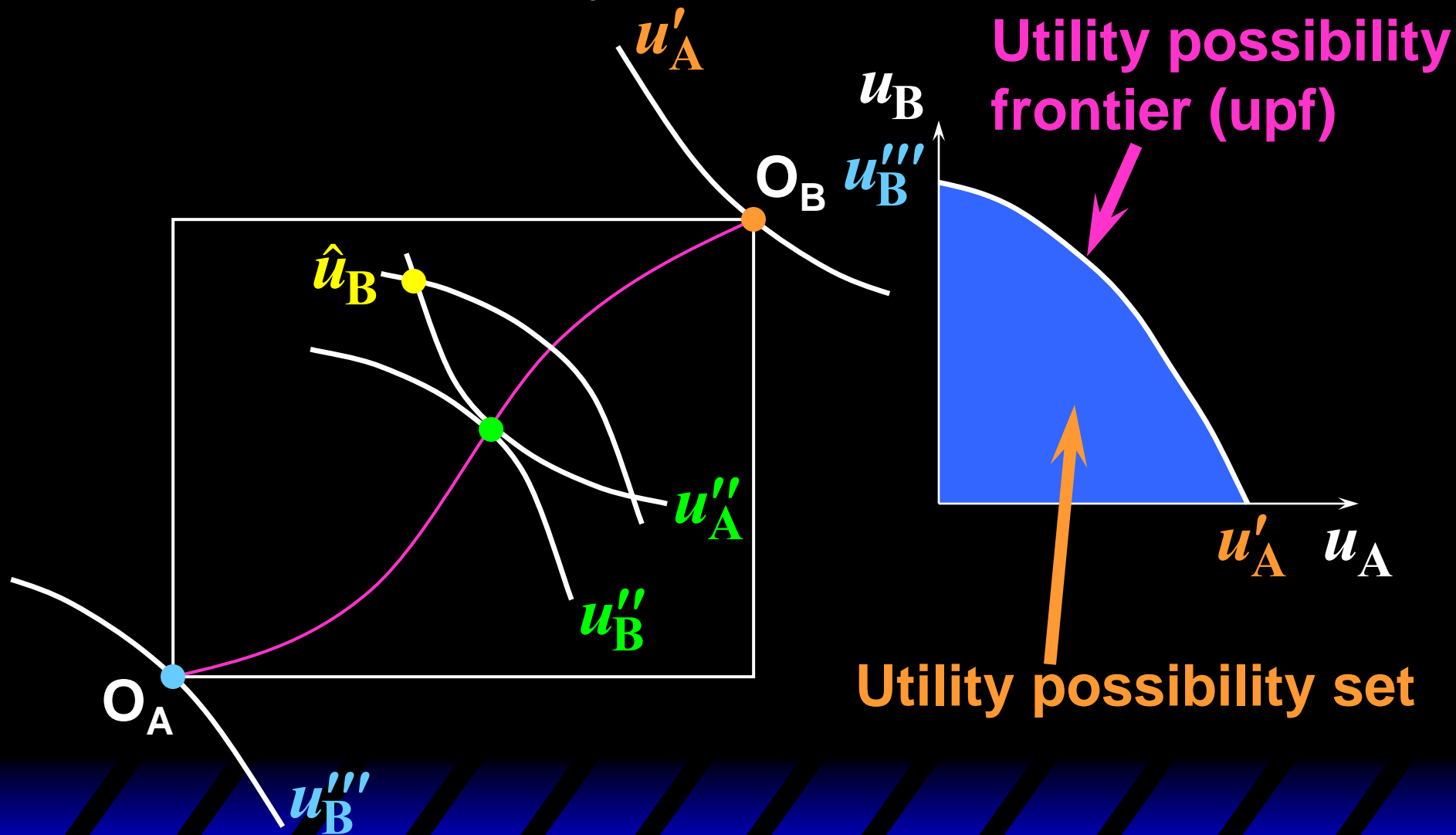
Utility Possibilities

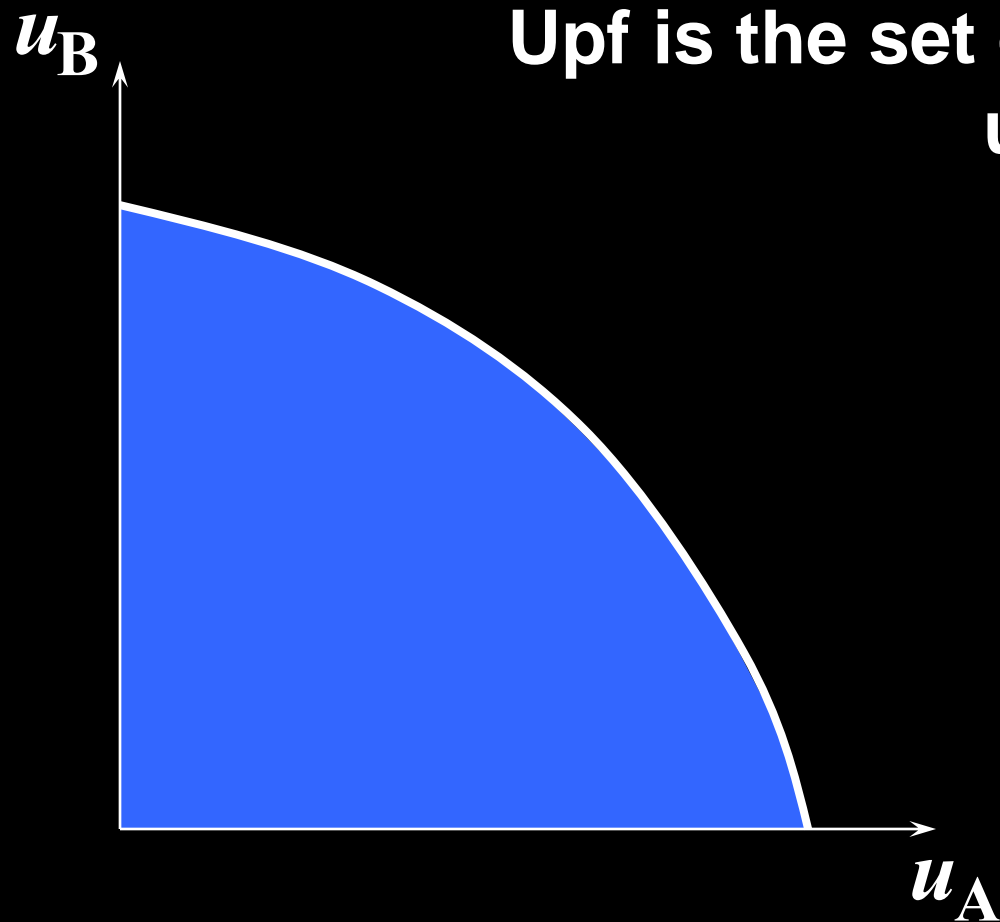# Utility Possibilities

Utility Possibilities

Utility Possibilities

Social Optima & Efficiency

Upf is the set of efficient utility pairs.

Social Optima & Efficiency

Upf is the set of efficient utility pairs.

Social indifference curves

Social Optima & Efficiency

u_B

Upf is the set of efficient utility pairs.

Higher social welfare

Social indifference curves

u_A

Social Optima & Efficiency

u_B

Upf is the set of efficient utility pairs.

Social optimum is efficient.

Social indifference curves

u_A
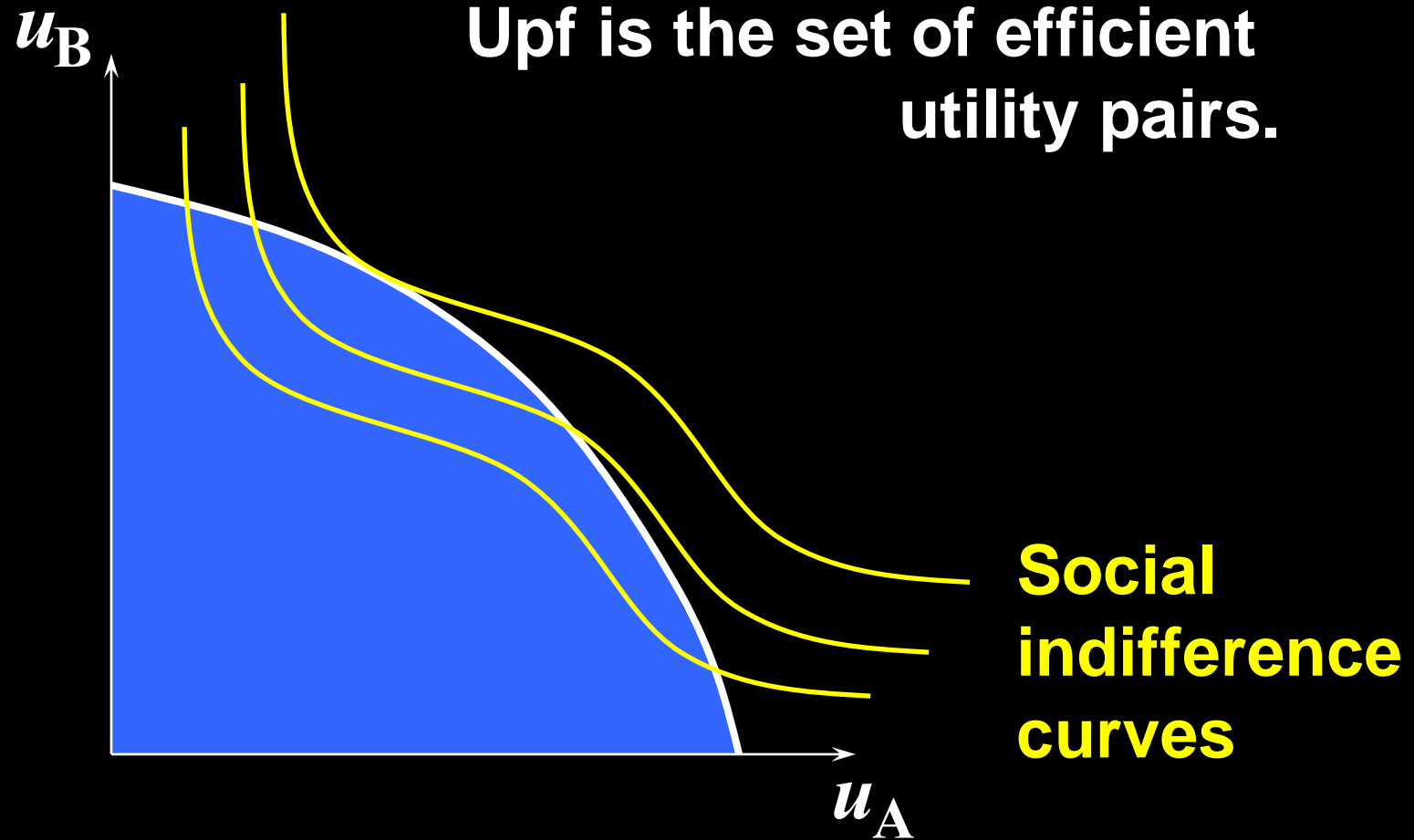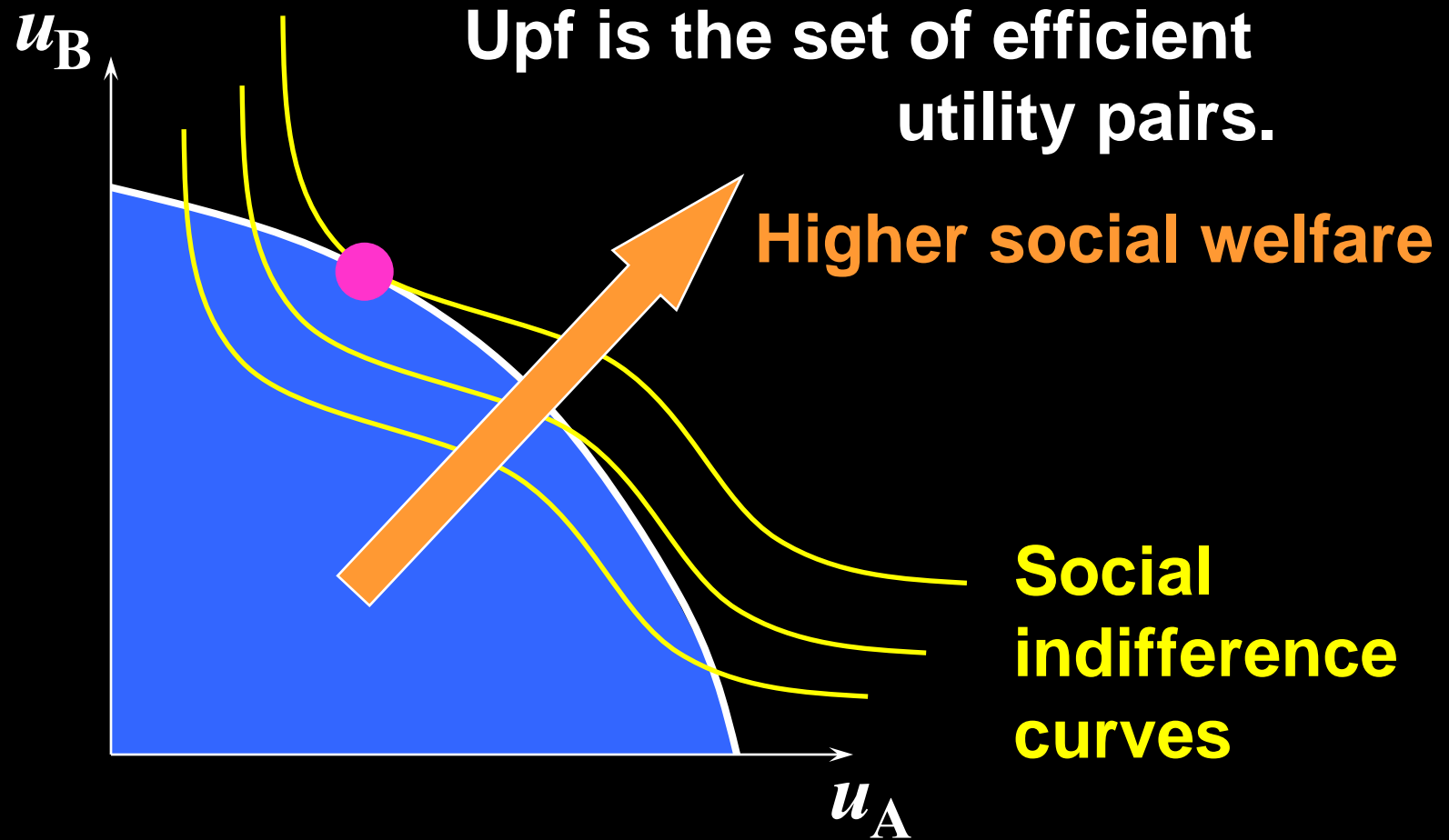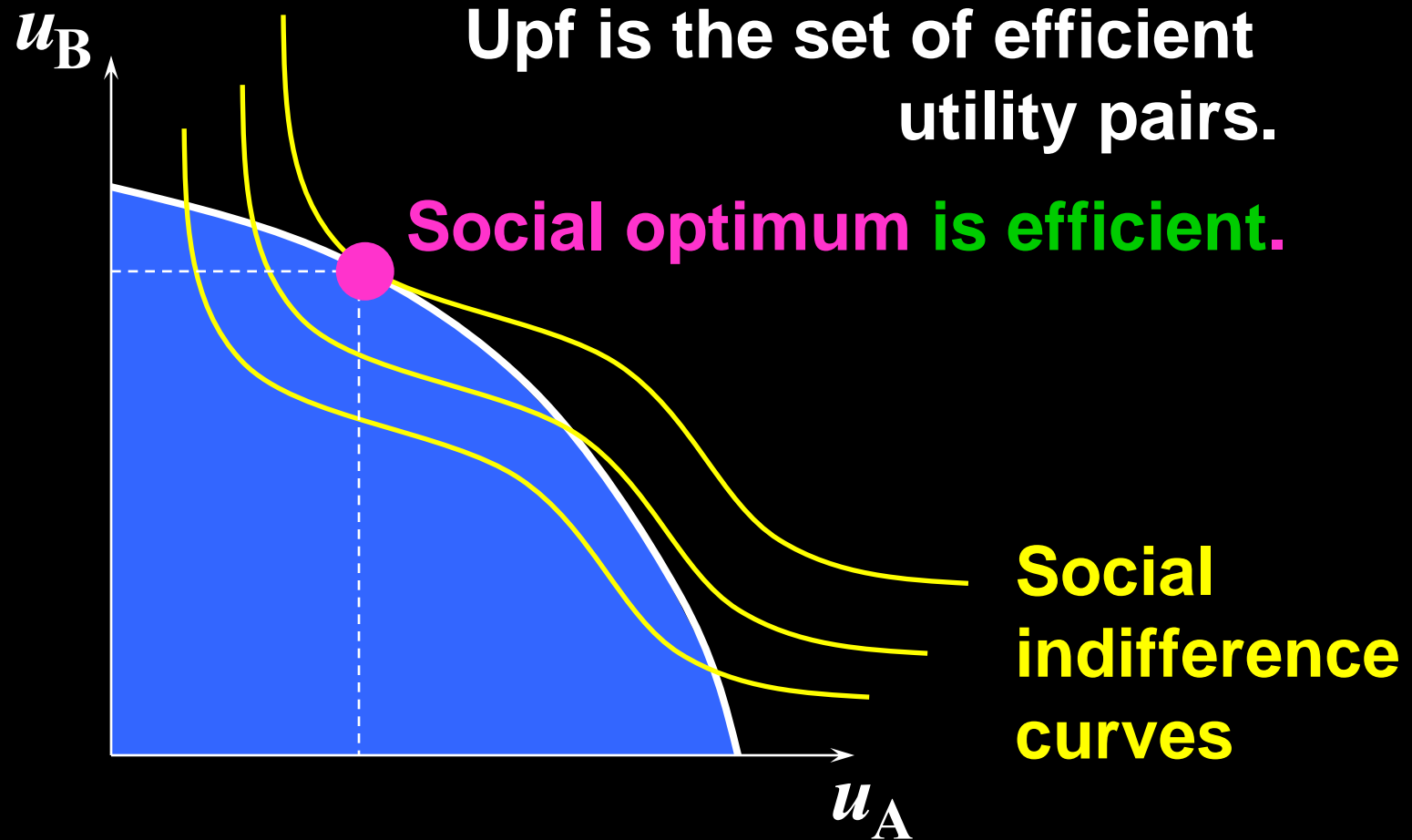
Fair Allocations

- **Some Pareto efficient allocations seem to be "unfair".**
  - **E.g. one consumer getting everything**
- **What does it mean by "fairness" or "equity"?**
  - **There are many different notions**
  - **We will see one based on the idea of no envy**

Fair Allocations

- **If consumer A prefers B's consumption bundle to his own, then agent A envies agent B.**

- **An allocation is equitable, if there is no envy under the allocation.**

- **An allocation is fair, if it is both Pareto efficient and equitable.**

Fair Allocations

- **Observation: If all consumers start with the same endowment bundle, then a general equilibrium must be fair.**

- **Pareto efficiency is due to First Welfare Theorem.**

- **Why is there no envy?**

Fair Allocations

- **Why is there no envy?**
  - **Suppose some consumer A envies some B, i.e. A strictly prefers x_B to x_A.**
  - **But B consuming x_B means x_B is affordable to B. Then it is also affordable to A.**
  - **So A is not choosing her most preferred affordable bundle.**
  - **This contradicts to the definition of an equilibrium.**

Fair Allocations

ω_1

O_B

ω_2
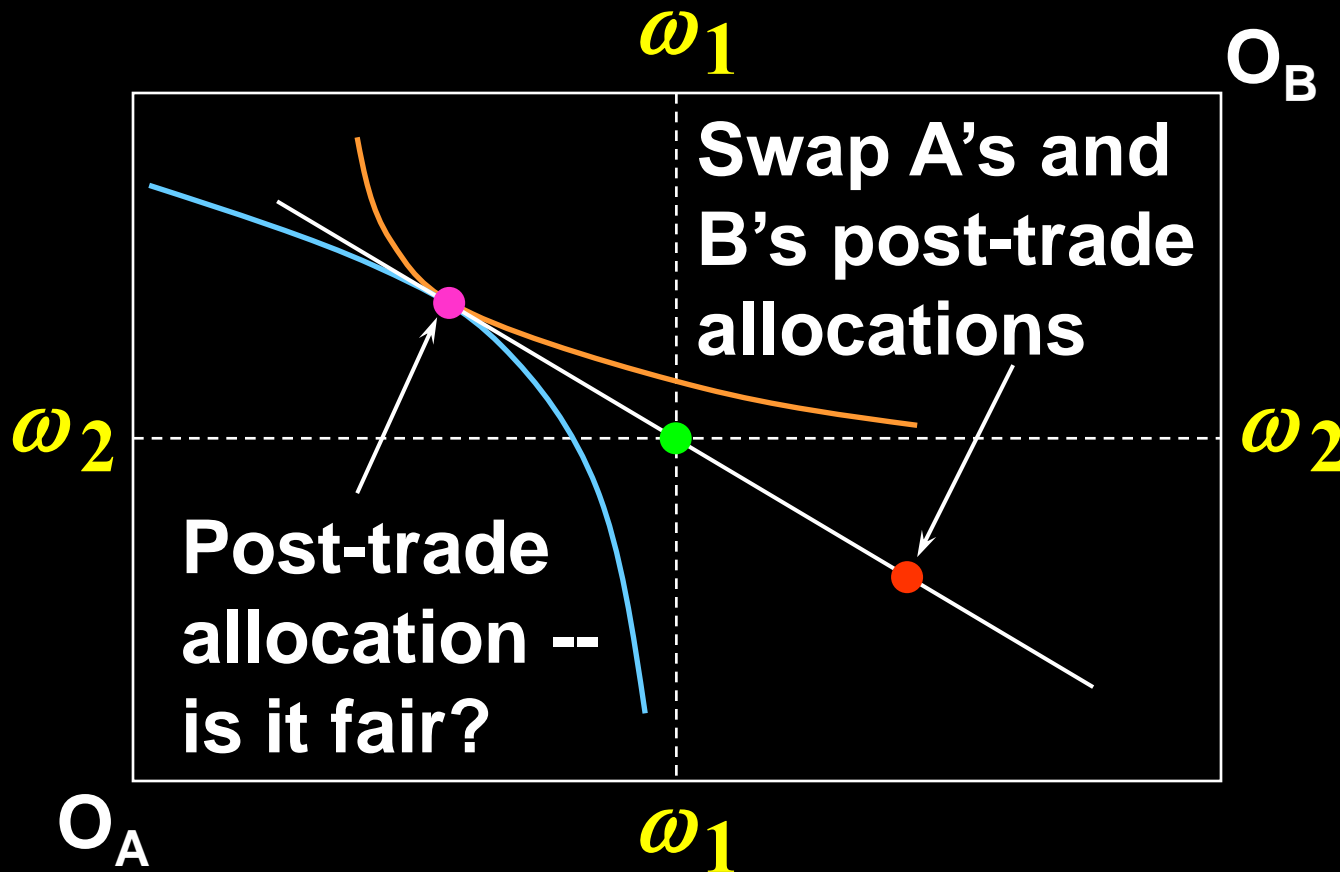
ω_2

O_A

ω_1
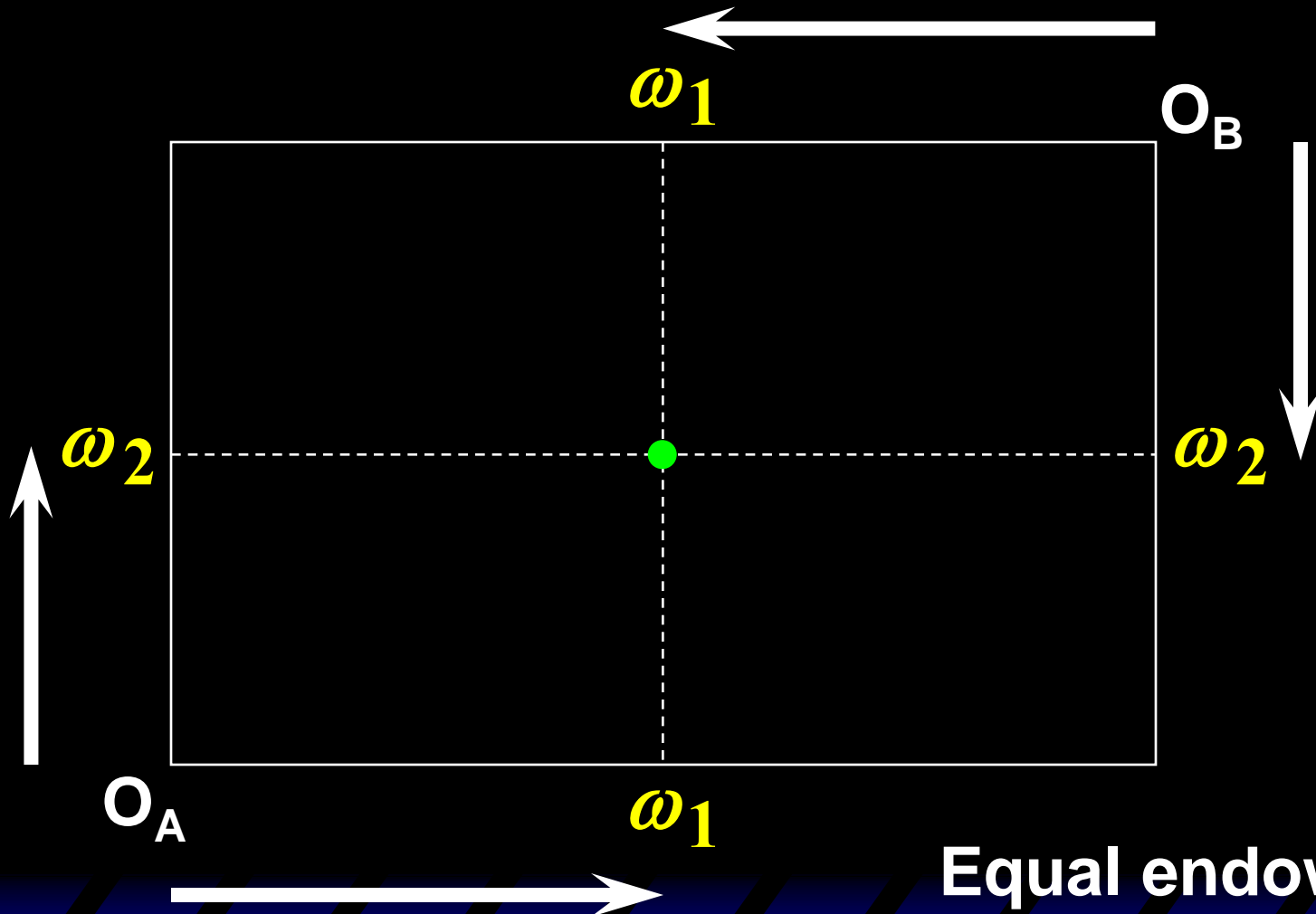
Equal endowments.

Fair Allocations
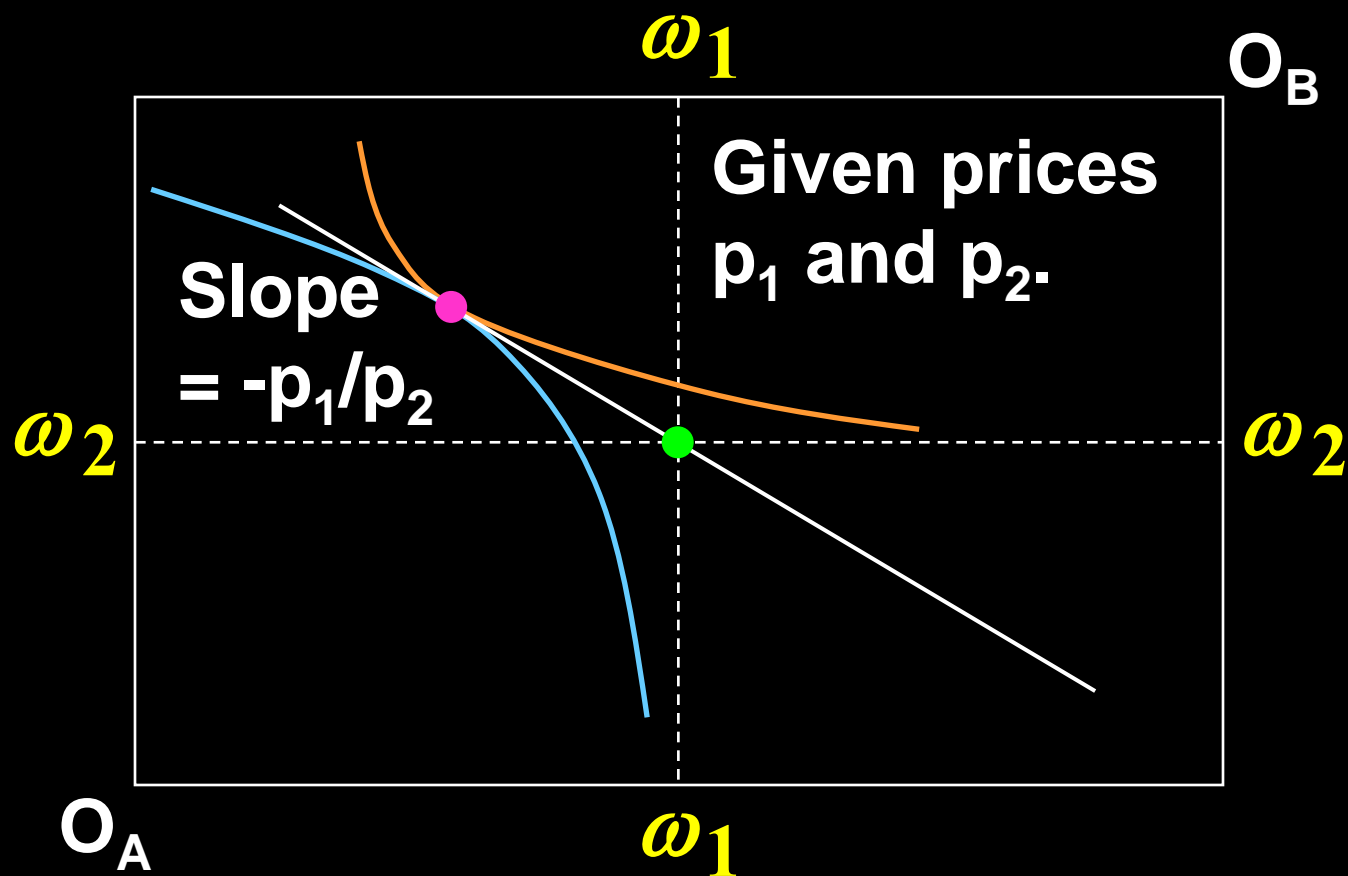
Fair Allocations

A does not envy B's allocation.
B does not envy A's allocation.

Summary

- **Social welfare function**
- **Fairness**