HW3

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1. net surplus $= \int_0^3 (150-30q)\mathrm{d}q - 3\times 60 = 135$.
2. $(1,2)$: $x_1 = 10$, $x_2 = 0$, $U = 10$.
   $(4,2)$: $x_1 = 0$, $x_2 = 5$, $U = 5$.
   $\Rightarrow$ CV $= 20 - 10 = 10$, EV $= 10 - 5 = 5$.
3. (a) $D(50) = 50$.
   (b) $\int_0^{50} D(p)\mathrm{d}p = \int_0^{50}(100-p)\mathrm{d}p = 3750$.
   (c) \$2500 .
   (d) $3750 - 2500 = 1250$.
4. (a) $\epsilon = \frac{\frac{\mathrm{d}D}{D}}{\frac{\mathrm{d}p}{p}} = \frac{\mathrm{d}D}{\mathrm{d}p}\frac{p}{D} = \frac{p}{p-40}$.
   (b) $\epsilon = \frac{\mathrm{d}D}{\mathrm{d}p}\frac{p}{D} = -60p^{-4}\frac{p}{20p^{-3}} = -3$.
   (c) $\epsilon = \frac{\mathrm{d}D}{\mathrm{d}p}\frac{p}{D} = -2(p+4)^{-3}\frac{p}{(p+4)^{-2}} = \frac{-2p}{p+4}$.
5. (a) $P(q) = 10 - q$.
   (b) total revenue $= 10q - q^2$, average revenue $= 10 - q$, marginal revenue $= 10 - 2q$.
   (c) $q = 5 \Rightarrow p = 5$.
   (d) $q = 5$, $\epsilon = \frac{\mathrm{d}q}{\mathrm{d}p}\frac{p}{q} = -\frac{p}{10-p} = -1$
6. (a) equilibrium price $= 20$, equilibrium quantity $= 60$.
   (b) $p_D = p_S + 10$
   $D = 100 - 2(p_S + 10) = S = 3p_S$
   (c) $\Rightarrow p_S = 16$, $p_D = 26$, $q = 48$.
7. (a) equilibrium price $= 20$, equilibrium quantity $= 100$.
   (b) $p_D + 10 = p_S \Rightarrow p_D = 15$, $p_S = 25$, $q = 125$.
8. (a)

$$\begin{cases} \text{decreasing returns to scale} & \text{if } a+b>1 \\ \text{constant returns to scale} & \text{if } a+b=1 \\ \text{increasing returns to scale} & \text{if } a+b<1 \end{cases} \tag{1}$$

   (b) $\frac{\partial^2 f}{\mathrm{d}x_1^2} = Ca(a-1)x_1^{a-2}x_2^b < 0 \Rightarrow a < 1$.
   (c) $\frac{\partial^2 x_2}{\partial x_1^2} = (1+\frac{a}{b})\frac{af^{\frac{1}{b}}}{bC^{\frac{1}{b}}}x_1^{-2-\frac{a}{b}} > 0 \Rightarrow a,b,c > 0$.
9. (a) $\Pi(x) = 400\sqrt{x} - 50x$.
   (b) $x^* = 16$, $\max(\Pi) = \Pi(16) = 800$.
   (c) $\Pi^{'}(x) = 320\sqrt{x} - 40x \Rightarrow x^{'*} = 16$, $\max(\Pi^{'}) = 640$.
   (d) $\Pi_{\text{after-tax}} = 200\sqrt{x} - 25x \Rightarrow x^*_{\text{after-tax}} = 16$, $\max(\Pi_{\text{after-tax}}) = 400$.
10. (a) $4\times(\frac{1}{2}x_1^{-\frac{1}{2}}x_2^{\frac{1}{4}}) = \omega_1$, $4\times(\frac{1}{4}x_1^{\frac{1}{2}}x_2^{-\frac{3}{4}}) = \omega_2$.
    (b) $\Rightarrow x_1 = \frac{8}{\omega_1^3\omega_2}$, $x_2 = \frac{4}{\omega_1^2\omega_2^2}$.