These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Four

Utility

效用

Utility Functions

A utility function U(x) represents a preference relation $\succsim$ if and only if:

$x' \succ x''$ $\Leftrightarrow$ $U(x') > U(x'')$

$x' \prec x''$ $\Leftrightarrow$ $U(x') < U(x'')$

$x' \sim x''$ $\Leftrightarrow$ $U(x') = U(x'')$.

Utility Functions & Indiff. Curves

All bundles in an indifference curve have the same utility level.

U(x1, x2)=Constant is the equation of an indifference curve.

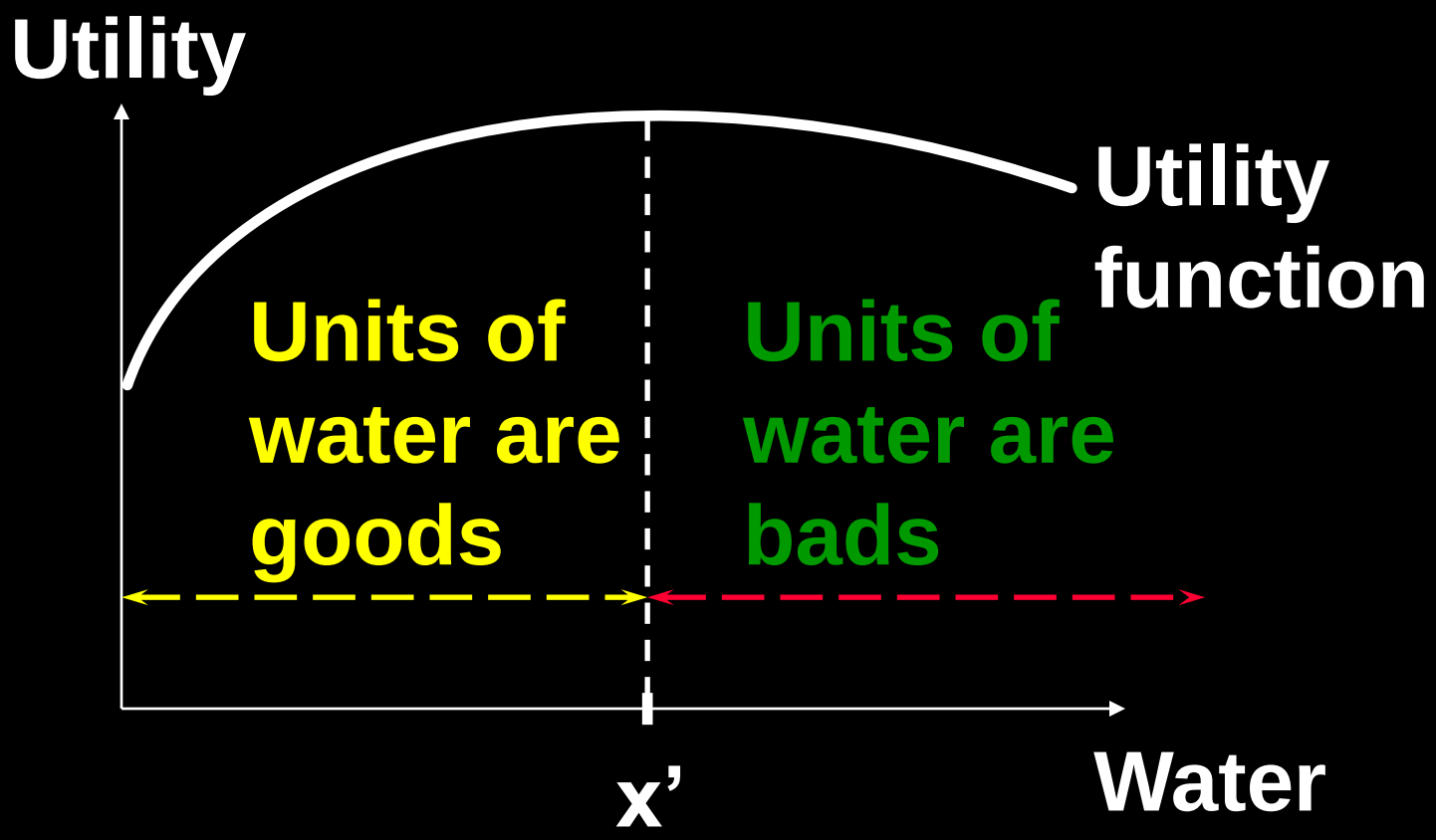
Goods, Bads and Neutrals

A good is a commodity which increases utility (gives a more preferred bundle).

A bad is a commodity which decreases utility (gives a less preferred bundle).

A neutral is a commodity which does not change utility (gives an equally preferred bundle).

Goods, Bads and Neutrals



Around x' units, a little extra water is a neutral.

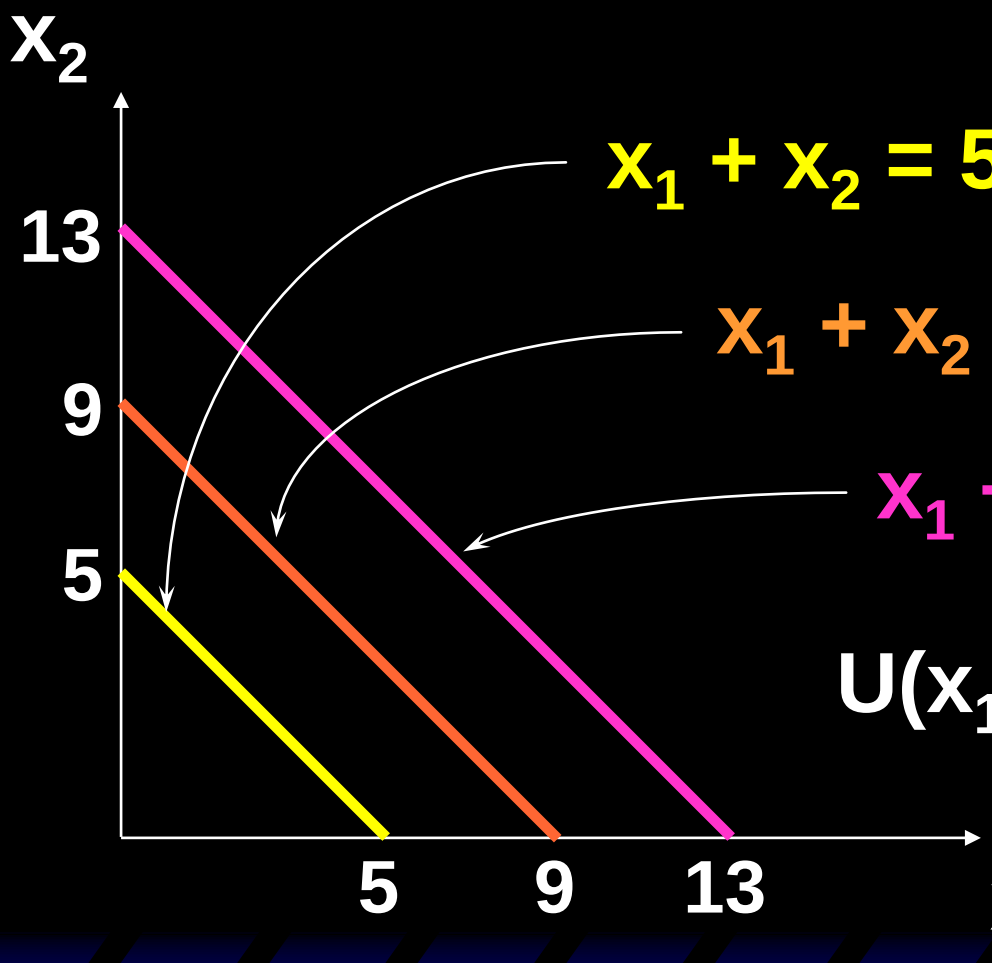
Some Utility Functions and Their Indifference Curves

Consider

$$U(x_1,x_2) = x_1 + x_2.$$

What do the indifference curves for this "perfect substitution" utility function look like?

Perfect Substitution Indifference Curves



$U(x_1,x_2) = x_1 + x_2$.

All are linear and parallel.

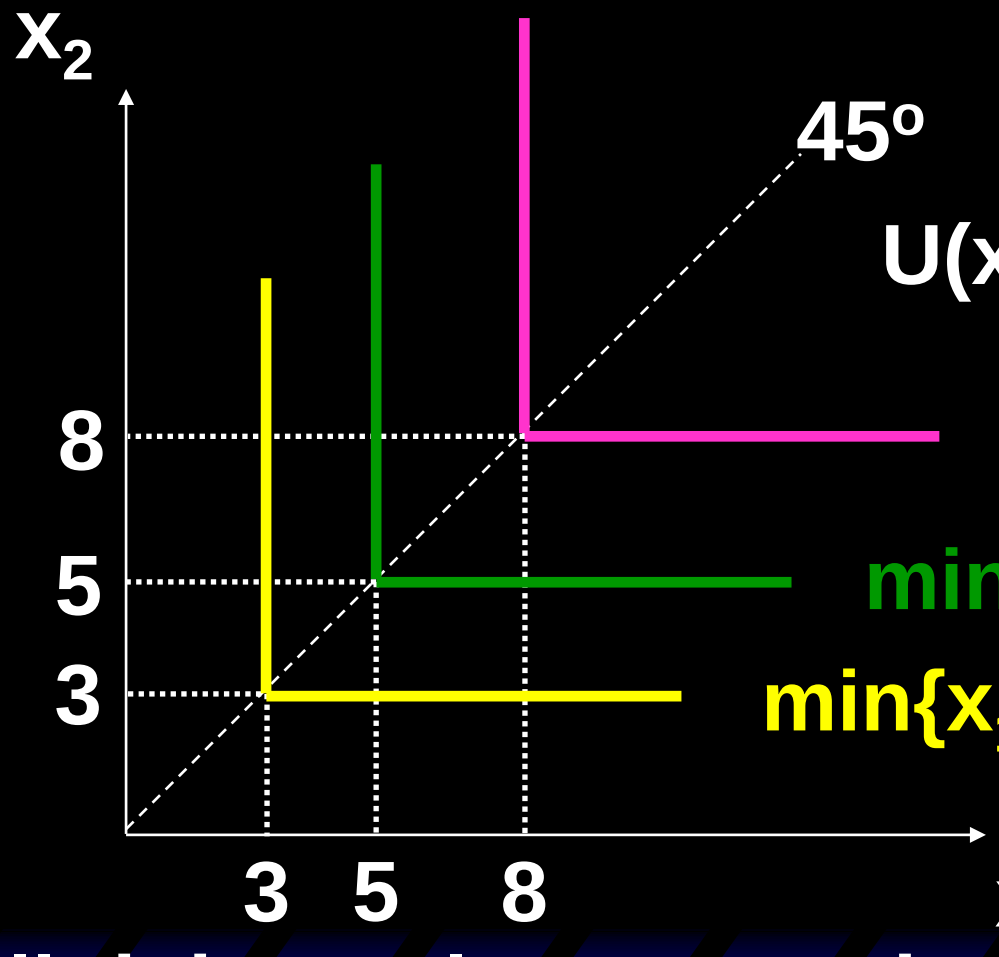
Some Other Utility Functions and Their Indifference Curves

Consider

$$U(x_1,x_2) = \min\{x_1,x_2\}.$$

What do the indifference curves for this "perfect complementarity" utility function look like?

Perfect Complementarity Indifference Curves



$U(x_1,x_2) = \min\{x_1,x_2\}$

$\min\{x_1,x_2\} = 8$

$\min\{x_1,x_2\} = 5$

$\min\{x_1,x_2\} = 3$

All right angles are on the 45° line.

Some Other Utility Functions and Their Indifference Curves

A utility function of the form

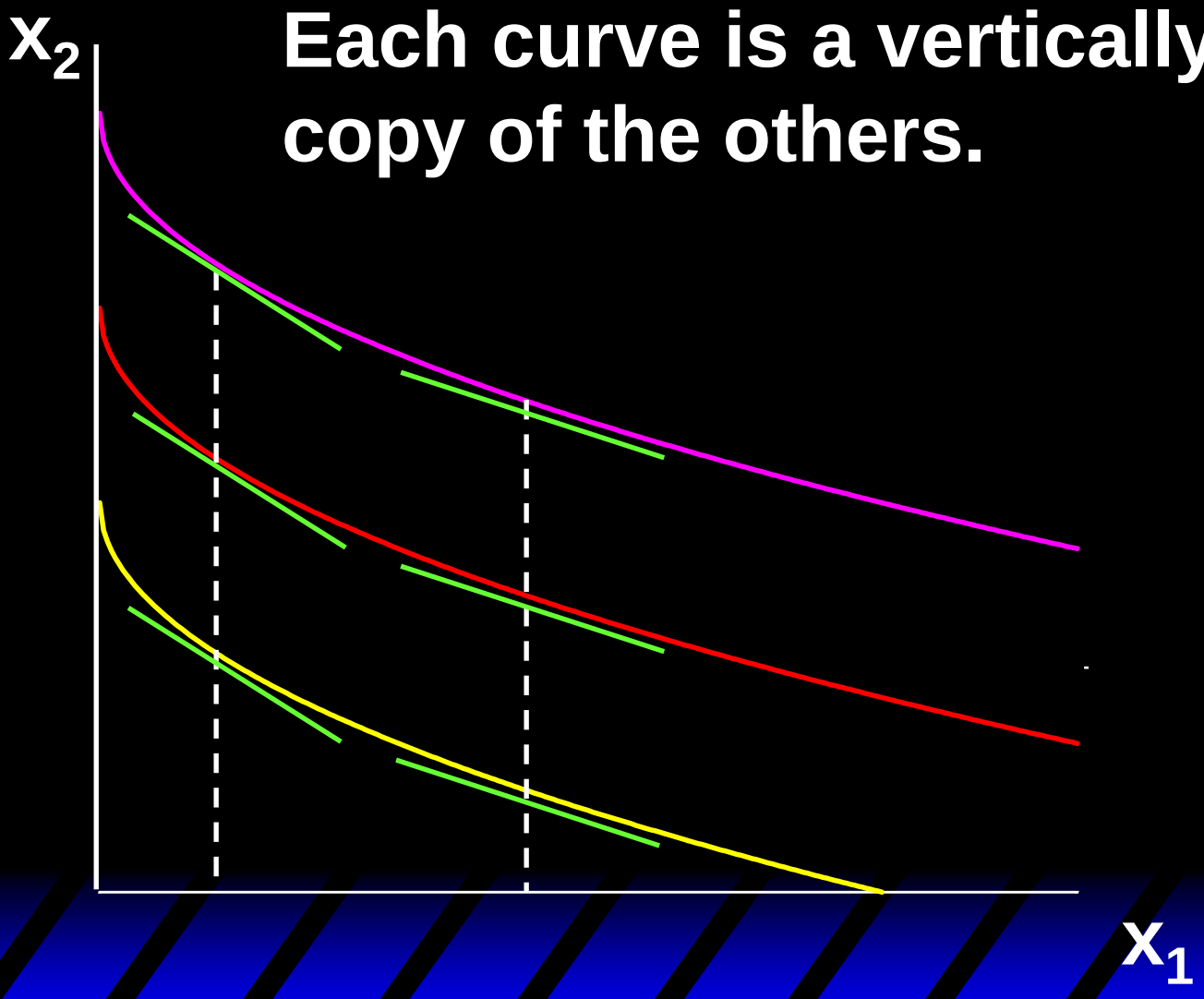
$$U(x_1,x_2) = f(x_1) + x_2$$

is linear in just x_2 and is called quasi-linear.

E.g. $U(x_1,x_2) = 2x_1^{1/2} + x_2$.

Quasi-linear Indifference Curves

Each curve is a vertically shifted copy of the others.



Some Other Utility Functions and Their Indifference Curves

Any utility function of the form

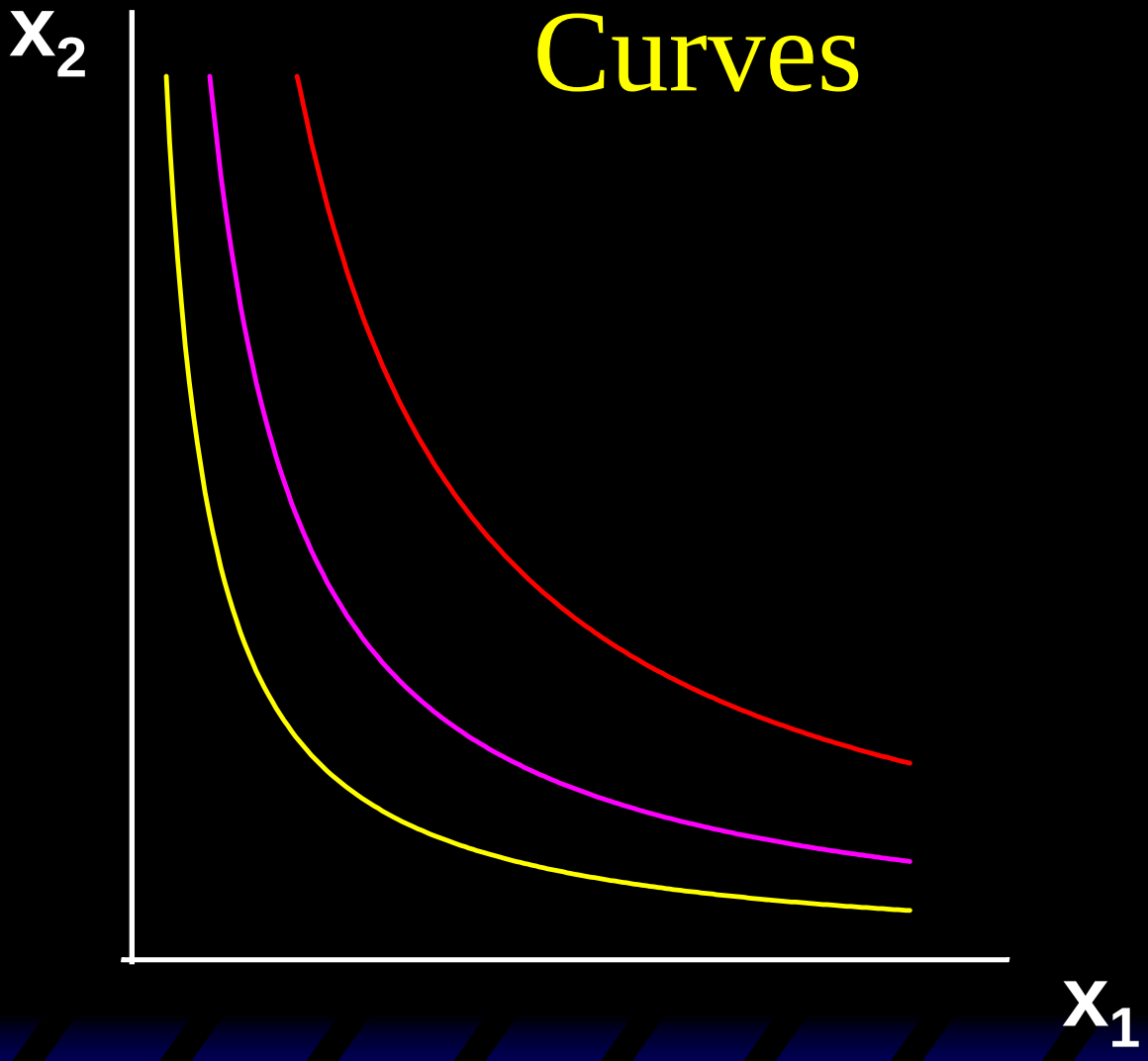
$$U(x_1,x_2) = x_1^{a} x_2^{b}$$

with a > 0 and b > 0 is called a Cobb-Douglas utility function.

E.g. $U(x_1,x_2) = x_1^{1/2} x_2^{1/2}$ **(a = b = 1/2)**

$U(x_1,x_2) = x_1 x_2^{3}$ **(a = 1, b = 3)**

Cobb-Douglas Indifference Curves



Marginal Utilities

Marginal means "incremental".

The marginal utility of commodity i is the rate-of-change of total utility as the quantity of commodity i consumed changes; *i.e.*

$$MU_i = \frac{\partial U}{\partial x_i}$$

Marginal Utilities and MRS

The general equation for an indifference curve is

$$U(x_1, x_2) \equiv k, \text{ a constant.}$$

Totally differentiating this identity gives

$$\frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0$$

Marginal Utilities and MRS

$$\frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0$$

rearranged is

$$\frac{dx_2}{dx_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}.$$

This is the MRS.

MRS for Quasi-linear Utility Functions

A quasi-linear utility function is of the form $U(x_1,x_2) = f(x_1) + x_2$.

$$\frac{\partial U}{\partial x_1} = f'(x_1) \qquad \frac{\partial U}{\partial x_2} = 1$$

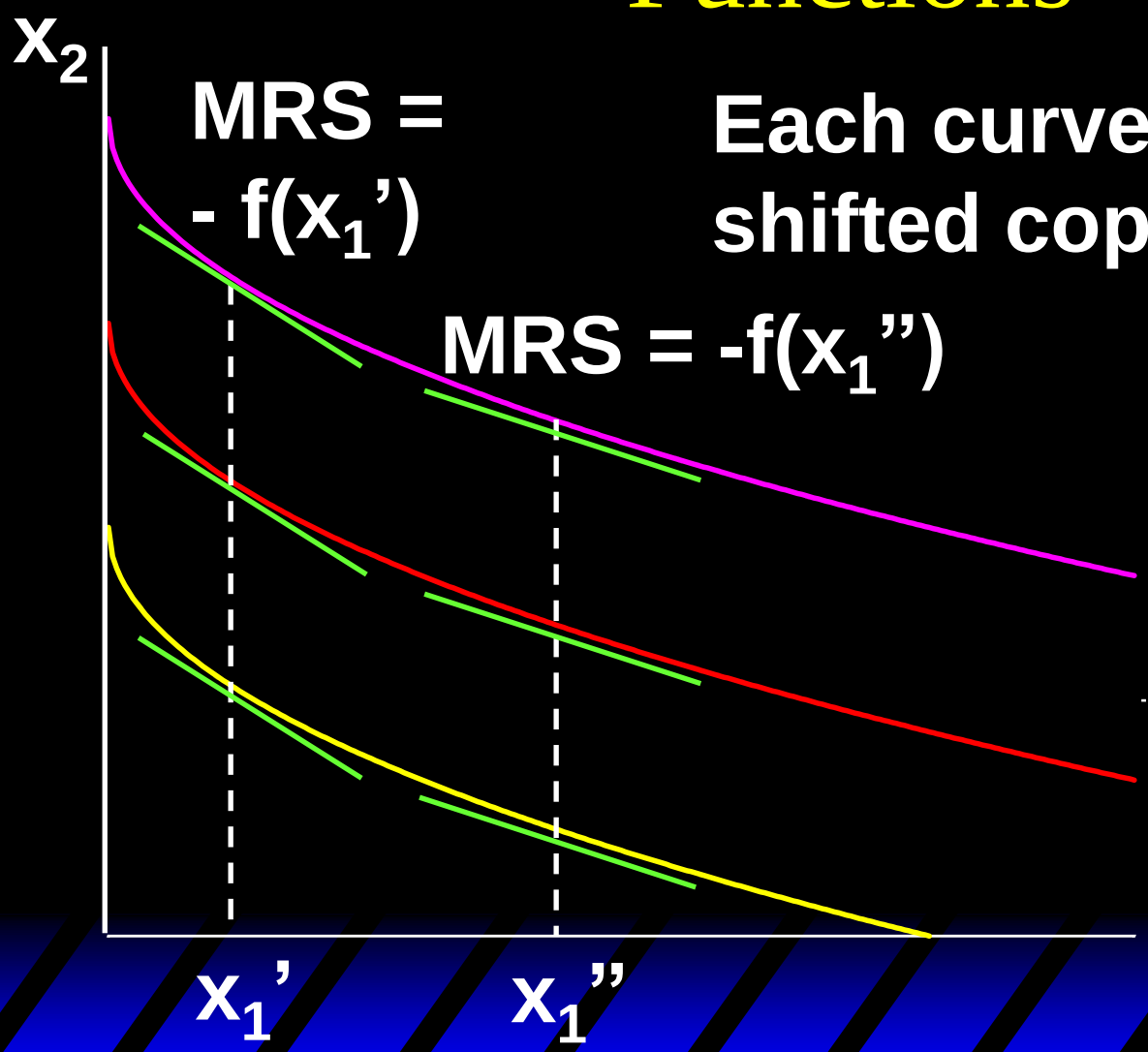
so $$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = -f'(x_1).$$

MRS for Quasi-linear Utility Functions

MRS = - $f'(x_1)$ does not depend upon x_2 .

So the slope of indifference curves for a quasi-linear utility function is constant along vertical lines.

MRS for Quasi-linear Utility Functions



Each curve is a vertically shifted copy of the others.

Ordinal vs Cardinal

Utility is usually an ordinal (i.e. ordering) concept.

***E.g*. if U(x) = 6 and U(y) = 2 then bundle x is strictly preferred to bundle y. But x is not preferred three times as much as is y.**

Strictly Increasing Transformation

If

- **U is a utility function that represents a preference relation $\succeq$ and**
- **f is a strictly increasing function,**

then V = f(U) is also a utility function representing $\succeq$.

MRS

Moreover, note that

$$MRS = -\frac{\partial V / \partial x_1}{\partial V / \partial x_2} = -\frac{f'(U) \times \partial U / \partial x_1}{f'(U) \times \partial U / \partial x_2}$$

$$= -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}.$$

So MRS is unchanged by a strictly increasing transformation.

Key Concepts

Utility

- **Its relation to preferences, indifference curves, and MRS**
- **Some commonly used utility functional forms**