1.14

$R = 6.4 \times 10^6\ m$

$\theta = 39.9^\circ$

$\omega = \frac{2\pi}{T} = \frac{\pi}{12}\ h^{-1}$

$v = \omega \cdot R\cos\theta = 357\ m/s$

$a = \omega^2 R\cos\theta = 2.6 \times 10^{-2}\ m/s^2$

1.18

(1) $t = \int_{R_1}^{R_2} \frac{2\pi n r}{v}\, dr = \frac{\pi n}{v}(R_2^2 - R_1^2)$

$\approx 4166\ s$

$\approx 69.4\ min$

(2)

$\omega = \frac{v}{r} = 26\ rad/s$

$\alpha = \dot{\omega} = -\frac{v}{r^2} \cdot \dot{r}$

$= -\frac{v}{r^2} \cdot \frac{v}{2\pi n r}$

$= -\frac{v^2}{2\pi n r^3}$

$\approx -3.3 \times 10^{-3}\ rad/s^2$

1.21

以电梯为参考系

$$a' = g - a = 8.6 m/s^2$$

$$t = \sqrt{\frac{2h}{a'}} = 0.59 s$$

$$s = \frac{1}{2} a t_0^2 + \left((a t_0) t + \frac{1}{2} a' t^2 \right)$$

$$\approx 2.0 m$$

1.24

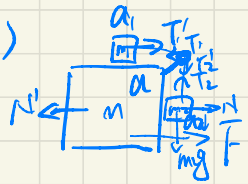
$$\Delta v = 2 \cdot \omega_{地} r_{地}$$

$$= \frac{4\pi}{T_{地}} r_{地}$$

$$= 933 m/s$$

2·8

(1)



$$\begin{cases} a = \dfrac{N}{m_2} = \dfrac{F - N' - T_1}{M} \\ N = N' \\ T_1' = T_1 = m_1 a_1 = T_2 = m_2 g - m_2 a_2 \\ a_1 = a + a_2 \end{cases} \Rightarrow$$

$$\Rightarrow a = \frac{F - m_2 a - \frac{m_1 m_2}{m_1 + m_2}(g + a)}{M}$$

$$\Rightarrow a = \frac{F - \frac{m_1 m_2}{m_1 + m_2} g}{M + m_2 + \frac{m_1 m_2}{m_1 + m_2}}$$

$$a_2 = \frac{m_2 g - m_1 a}{m_1 + m_2}$$

$$T_1 = \frac{m_1 m_2}{m_1 + m_2}(g + a)$$

(2)

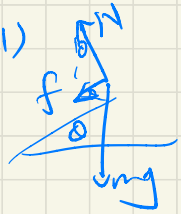
$$a_2 = 0$$

$$\Rightarrow a = \frac{m_2}{m_1} g = \frac{F - \frac{m_1 m_2}{m_1 + m_2} g}{M + m_2 + \frac{m_1 m_2}{m_1 + m_2}}$$

$$\Rightarrow F = \frac{m_2 g}{m_1}(M + m_1 + m_2)$$

2.11

(1)



$$\begin{cases} y: N\cos\theta - f\sin\theta = mg \\ x: f\cos\theta + N\sin\theta = m\dfrac{v^2}{R} \end{cases}$$

$$\Rightarrow N = \frac{mg}{\cos\theta} + f\tan\theta$$

$$= m\frac{v^2}{R\sin\theta} - \frac{f}{\tan\theta}$$

$$\Rightarrow f = m\frac{v^2}{R}\cos\theta - mg\sin\theta$$

$$= 635\ N$$

$$F_{向} = m\frac{v^2}{R} = 1.88\times10^3\ N$$

(2)

$$f_{max} = \mu_s N = \frac{\mu_s mg}{\cos\theta} + \mu_s f_{max}\tan\theta$$

$$\Rightarrow f_{max} = \frac{\frac{\mu_s mg}{\cos\theta}}{1-\mu_s\tan\theta} = \frac{\mu_s mg}{\cos\theta - \mu_s\sin\theta} = m\frac{v_m^2}{R}\cos\theta - mg\sin\theta$$

$$\Rightarrow mg\left(\frac{\mu_s}{\cos\theta - \mu_s\sin\theta} + \sin\theta\right) = \frac{m\cos\theta}{R}v_m^2$$

$$v_m^2 = \frac{gR}{\cos\theta}\left(\frac{\mu_s}{\cos\theta - \mu_s\sin\theta} + \sin\theta\right)$$

$$v_m \approx 66.0\ m/s$$

2.14

(1) 取表面上 Δm：

$$\frac{GM\Delta m}{R^2} = \Delta m \frac{4\pi^2}{T_{min}^2} R$$

$$\Rightarrow \begin{cases} T_{min}^2 = \dfrac{4\pi^2 R^3}{GM} \\ \rho = \dfrac{M}{\frac{4}{3}\pi R^3} \end{cases}$$

$$\Rightarrow T_{min} = \sqrt{\frac{3\pi}{G\rho}}$$

(2) $T_{min} = 6.9 \times 10^3\ s$

(3) $$T \geq \sqrt{\frac{4\pi^2 R^3}{GM}}$$

$$\Rightarrow M \geq \frac{4\pi^2 R^3}{GT^2} = 2.31 \times 10^{29}\ kg$$

$$= 0.12 \text{ 太阳质量}$$

2.15

取半径为 r 的球壳，对内压强 $dp = \dfrac{G(\rho \cdot \frac{4}{3}\pi r^3)\,dm}{r^2} \div 4\pi r^2$

$$dm = \rho \cdot (4\pi r^2 dr)$$

$$\Rightarrow dp = G\rho^2 \frac{4}{3}\pi r\, dr$$

$$p = \frac{4\pi}{3} G\rho^2 \int_0^R r\, dr$$

$$= \frac{2\pi}{3} G\rho^2 R^2$$

2.19

(1) $\omega = \frac{5\times10^4\times2\pi}{60}$ rad/s

$= \frac{5\times10^3}{3}\pi$ rad/s

$a_口 = \omega^2 r_口 = (\frac{5\times10^3}{3}\pi)^2 \times 2\times10^{-2}$ m/s²

$= 5.5\times10^5$ m/s²

$\approx 5.5\times10^4$ g

$a_底 = \omega^2 r_底 = 5a_口 \approx 2.8\times10^5$ g

(2) $F_压 = \int_{r_口}^{r_底} \omega^2 r \rho s dr$

$= \frac{1}{2}\omega^2\rho s(r_底^2 - r_口^2)$

$= \frac{m\omega^2}{2}(r_口 + r_底)$

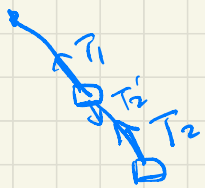
$\approx 2.0\times10^4$ N

相当于2t物体受的重力

(3) $F = m_0\omega^2 r_底$

$\approx 4.6\times10^{-6}$ N

2.22



$$T_2 = m_2\omega^2(L_1+L_2)$$

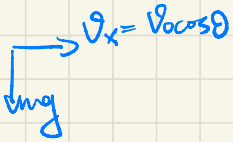
$$\begin{cases} T_2' = T_2 \\ T_1 - T_2' = m_1\omega^2 L_1 \end{cases}$$

$$\Rightarrow T_1 = m_1\omega^2 L_1 + m_2\omega^2(L_1+L_2)$$

附加题1：

$$mg\cos\theta = m\frac{v_0^2}{\rho_{初}}$$

$$\Rightarrow \rho_{初} = \frac{v_0^2}{g\cos\theta}$$



$$mg = m\frac{v_x^2}{\rho_{高}}$$

$$\Rightarrow \rho_{高} = \frac{v_0^2\cos^2\theta}{g}$$

附加题2：

都不一定是圆

e.g 阿基米德螺线