## 2021F Final

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matrix A = \begin{bmatrix} -2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & 1 & -2 \end{bmatrix}. (Note that the point
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Stepl. Find ATA Method 1.

$$A^{T}A = \left(g_{ij} = (i-t) \cdot col, j-ts \cdot col\right) = \left(\begin{matrix} 6 & -3 & -3 \\ -3 & 6 & -3 \end{matrix}\right)$$

$$\left(\begin{matrix} -3 & -3 & -3 \\ -3 & -3 & 6 \end{matrix}\right)$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix} = \mathcal{D}A$$

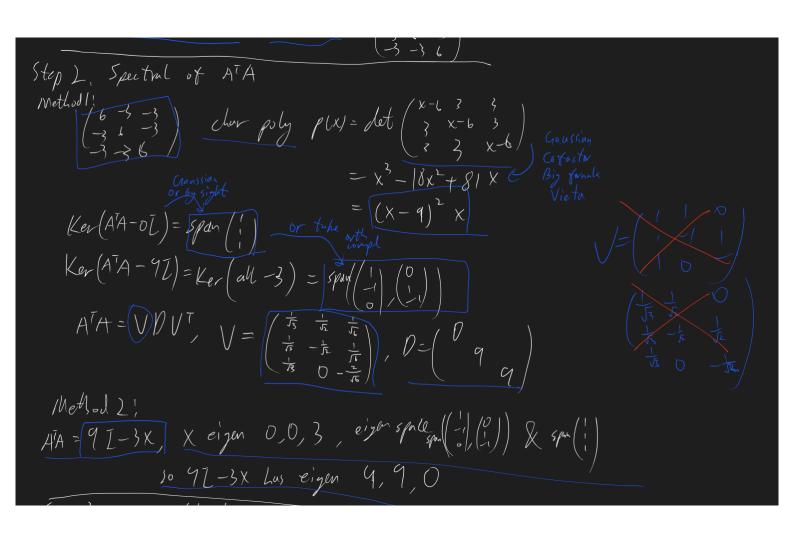
$$A^{T}A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

$$A = ('-1)/(-2-1) = pA'$$

$$A'A = (A')''D'DA' = (A')^{2} = (X-3)^{2} \text{ where } X = (all 1)$$

$$= X^{2} - 6X + 97 = 97 - 3y - (6-3-5)$$

$$= X^{2} - 6X + 7\overline{L} = 7\overline{L} - 3X = \begin{pmatrix} 6 - 3 - 3 \\ -3 & 1 \\ -3 & -3 \end{pmatrix}$$



Step 3. 
$$A^{2}A = \begin{pmatrix} \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac$$

## Takenung! It SVD problem, you can simplify F

(b) (3 points) Find two mutually orthogonal lines of best fit.

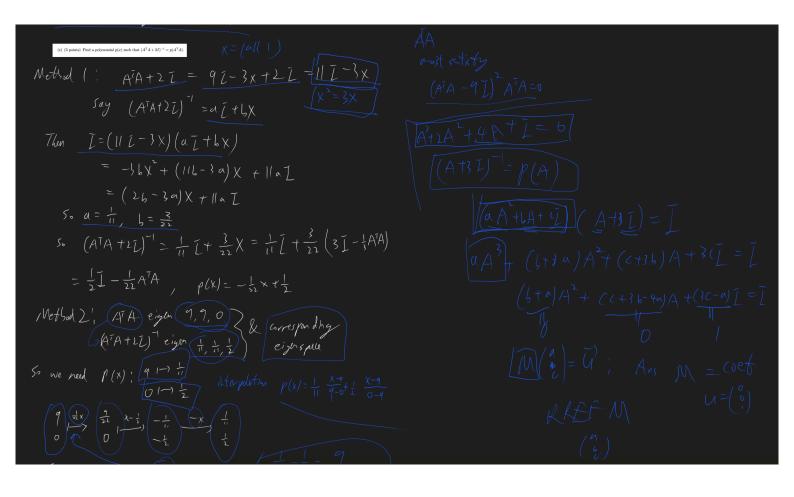
direction of lines are U' for largest  $\sigma$ but  $\sigma_i = \sigma_s$  both (ergest so Ans = lines with directions  $U_i$ ,  $U_i$ ) = span  $(U_i)$ , span  $(U_i)$ 

(c) (2 points) Find the plane of best fit.

span (vi, vi)

(d) (2 points) Find the maximum and minimum Rayleigh quotient  $\frac{v^TSv}{v^Tv}$  for  $S=(A^TA)^2+2A^TA+3I.$ 

ATA hus eigen 9, 9, 07 5 hos eigen 102, 102, 3



Method 2! ATA eigen 9,9,0 & corresponding eigenspale

So we need 
$$P(x)$$
:  $\frac{1}{9}$   $\frac{1}{11}$ ,  $\frac{1}{11}$ ,  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{11}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{11}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{11}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{11}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{9}$   $\frac{1}{11}$   $\frac{1}{9}$   $\frac{$ 

- (a) (2 points) Find a  $3 \times 2$  matrix A and a vector  $\boldsymbol{u}$  such that a, b is a possible solution to the problem above if and only if  $A\begin{bmatrix} a \\ b \end{bmatrix} = \boldsymbol{u}$ .

$$\begin{cases} 4+6=1\\ 24-6=1\\ -24+46=1 \end{cases}$$

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}, \quad \overrightarrow{U} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(b) (2 points) Show that  $A \begin{bmatrix} a \\ b \end{bmatrix} = \boldsymbol{u}$  above has no solution.

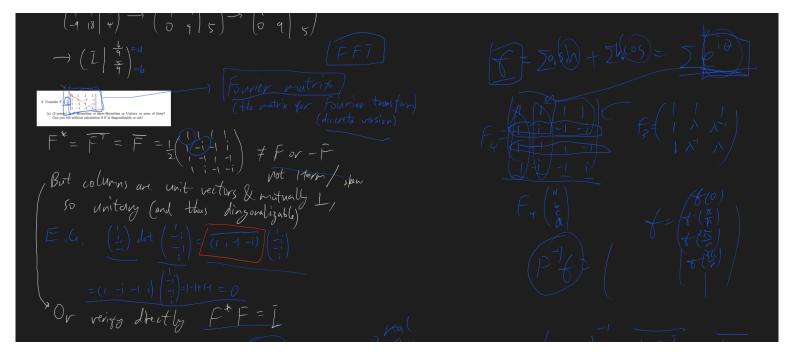
$$\begin{pmatrix} 2 & -1 & | & 1 \\ 2 & 2 & | & | & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & | & 1 \\ 0 & -3 & | & -1 \\ 0 & 6 & | & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & | & 1 \\ 0 & -3 & | & -1 \\ 0 & 0 & | & 1 \end{pmatrix}$$
 So he solution

$$\begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 3 & -3 \\ 3 & \frac{2}{3} \end{pmatrix}$$

(d) (2 points) Find the least square solution to  $A\begin{bmatrix} a \\ b \end{bmatrix} = u.$ 

$$A^{T}A = R^{T}R = \begin{pmatrix} 3 \\ -3 \end{pmatrix} = \begin{pmatrix} 9 & -9 \\ -9 & 18 \end{pmatrix}$$
 $A^{T}u^{2} = \begin{pmatrix} 4 \end{pmatrix}$ 

$$\begin{pmatrix} 9 & -9 & 1 \\ -9 & 18 & 4 \end{pmatrix} \longrightarrow \begin{pmatrix} 9 & -9 & 1 \\ 0 & 9 & 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 9 & 0 & 6 \\ 0 & 9 & 5 \end{pmatrix}$$



Mothod [ F is "scertly block diagonal"

(ast eigen veitors [ F ] =  $\frac{1}{2}F$  =  $\frac{1}{2}F$ Nothod [ F is "scertly block diagonal"

(and red portion has eigenvitar [ F ] for -1

(Nothod F is a sump motion F one F = F |

(ast eigen veitors)

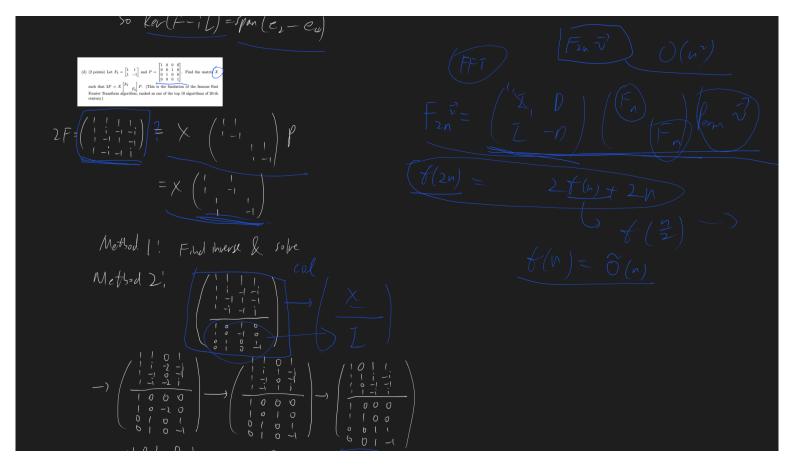
(b) F is eigenveitor F is a sump motion F or F and F is a sump motion F one F = F |

(ast eigen veitors)

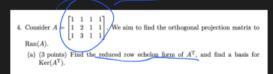
(b) F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F | F |

(c) (3 points) Find all eigenvalues of F and their algebraic multiplicity, and find a basis for the eigenspaces of non-real eigenvalues.

Method 1; Brute forle Method 2! 76 F eigenvalue 1, 12, 13, 14 Then P' eigenvalues () () 50 /1, /2, /3, /4 t1 t1 t1 (ti) tr(F) = |t|So eigenvalues of Fare 1,1,1,1 Leignvector of Ffori = eigenvector of P for i So Ker(F-i] = span(e, -eu)



Mothod 
$$\begin{cases} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 &$$





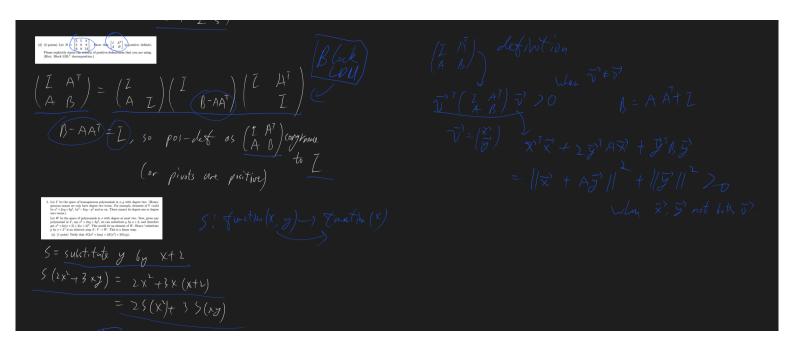
$$A^{T} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

So 
$$\operatorname{Ker}(A^{T}) = \operatorname{Span}(-2)$$

(b) (2 points) Find the orthogonal projection matrix 
$$P_1$$
 to  $\ker(A^T)$ .

(c) (2 points) Find the orthogonal projection matrix 
$$P_2$$
 to Ran(A)

$$\int_{2}^{2} = \int_{1}^{2} - \int_{1}^{2} = \int_{1}^{2} \left( \frac{5}{2} - \frac{2}{2} - \frac{1}{2} \right)$$



(c) (2 points) Find all possible  $p \in V$  such that  $S(p) = 3x^2 + 6x + 4$ .

$$A = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \text{find } X,$$

$$\begin{cases} 6 \\ 2 \\ 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \\ 7 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} \\ 7 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ 9 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

 $\mathcal{G}=X$  (d) (2 points) Pick basis  $\frac{L^2, x(y-x), (y-x)^2}{L^2}$  for V and basis  $x^2, x, 1$  for W, find the matrix B for S under these basis.

