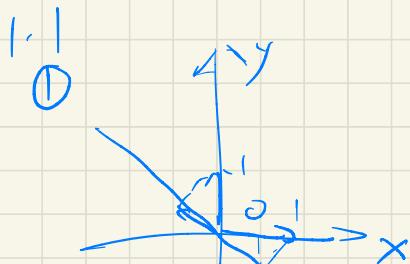


HW for Week 1

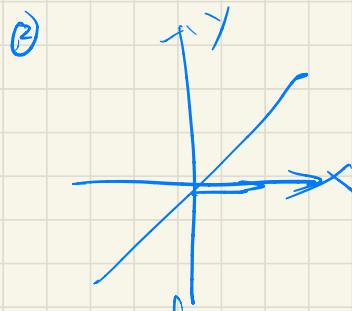


$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{f} \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\begin{bmatrix} -1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

Surjective



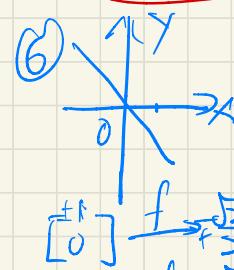
$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{f} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Bijective

- ③ not linear
- ④ $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ Bijective
- ⑤ not linear



$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{f} \begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{f} \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

Surjective

⑦ $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ Bijective

1.2

$$\textcircled{1} \text{ sum} = 1 + \dots + 9 = 45$$

$$45/3 = 15$$

$$M = \begin{bmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 \end{bmatrix} = \begin{bmatrix} 15 & 15 & 15 \\ 15 & 15 & 15 \\ 15 & 15 & 15 \end{bmatrix}$$

$$\therefore M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} m_1 \\ m_4 \\ m_7 \end{bmatrix} + \begin{bmatrix} m_2 \\ m_5 \\ m_8 \end{bmatrix} + \begin{bmatrix} m_3 \\ m_6 \\ m_9 \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 15 \\ 15 \end{bmatrix}$$

\textcircled{2} the sum of each row = 45

$$\therefore M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 45 \\ 45 \\ 45 \end{bmatrix}$$

\(\Rightarrow 9\) rows

1.3

$$\textcircled{1} \text{ sum1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\textcircled{2} \text{ sum2} = -x_2 = -\begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} -\sqrt{3}/2 \\ -1/2 \end{bmatrix}$$

$$\textcircled{3} \text{ sum3} = \text{sum1} + 12(-\vec{x}_6)$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 12 \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 12 \end{bmatrix}$$

1.4

$$\textcircled{1} \vec{b} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$$

$$\textcircled{2} \vec{b} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 4 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix}$$

$$\textcircled{3} \vec{b} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\textcircled{4} \vec{b} = \begin{bmatrix} 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 2 & 0 & 0 & 8 \\ 1 & 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 8 \\ 9 \\ 8 \\ 5 \end{bmatrix}$$

$$\textcircled{5} \text{ today tomorrow}$$

$$0.5 \times 0.8 + 0.5 \times 0.3 = 0.4 + 0.15 = 0.55$$

$$\vec{b} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.55 \\ 0.45 \end{bmatrix}$$

1.5

① original set: $\{\vec{a} + t(\vec{b} - \vec{a}); t \in \mathbb{R}\}$

$$\vec{a} + t(\vec{b} - \vec{a}) = (1-t)\vec{a} + t\vec{b}$$

$$\exists s \in \mathbb{R}, s=1-t \Rightarrow s\vec{a} + t\vec{b}, s+t=1$$

\Rightarrow target set: $\{s\vec{a} + t\vec{b}; s, t \in \mathbb{R} \text{ and } s+t=1\}$

②: from ① $\Rightarrow \exists \underline{s, t \in \mathbb{R}}, \vec{p} = s\vec{a} + t\vec{b}, s+t=1$
 ↓
 at least one pair

Then prove its uniqueness:

suppose: $\exists s', t' \in \mathbb{R}, \vec{p} = s'\vec{a} + t'\vec{b}, s'+t'=1$

$$\Rightarrow s'\vec{a} + t'\vec{b} = s\vec{a} + t\vec{b}$$

$$s'\vec{a} + (1-s')\vec{b} = s\vec{a} + (1-s)\vec{b}$$

$$(s'-s)\vec{a} + (s-s')\vec{b} = \vec{0}$$

since $\vec{a} \neq k\vec{b}$, then $\vec{a}, \vec{b} \neq \vec{0}$

$$\text{so } s'-s=0, s-s'=0$$

$$\therefore s' = s$$

$$t' = t$$

∴ done

③ suppose: $s < 0$

then: $t > 1$

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}, \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$\vec{p} = s\vec{a} + t\vec{b} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$$

$$\therefore p_1 = s a_1 + t b_1 = b_1 + [(t-1)b_1 + (1-t)a_1]$$

if $a_1 > b_1$, $p_1 < b_1 \times$

if $a_1 < b_1$, $p_1 > b_1 \times$

∴ $s, t \in [0, 1] \Rightarrow$ done

④ $\{s\vec{a} + t\vec{b} + u\vec{c}; s, t, u \in \mathbb{R}, s+t+u=1\}$

⑤ $\{s\vec{a} + t\vec{b} + u\vec{c}; s, t, u \in \mathbb{R}, s+t+u=1, s, t, u \geq 0 \text{ and } s+t+u=1\}$

1. b

$$\textcircled{1} \quad \vec{a} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}, \vec{b} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \vec{c} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

\textcircled{2} from \textcircled{1} \quad \Downarrow

equation for the plane: $ax + by + cz = t$

then $\begin{cases} 6a = t \\ 3b = t \\ 2c = t \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = 2 \\ c = 3 \end{cases}$ let $a = 1$ (it's okay at),
I can get the same answer)

$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + 2y + 3z = 6 \right\}$ is a plane through $\vec{a}, \vec{b}, \vec{c}$

$$\textcircled{3} \quad \vec{a} - \vec{b} = \begin{bmatrix} 6 \\ -3 \\ 0 \end{bmatrix} \quad \vec{b} - \vec{c} = \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix} \quad \vec{c} - \vec{a} = \begin{bmatrix} -6 \\ 0 \\ 2 \end{bmatrix}$$

let $\vec{t} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$\vec{t} \cdot (\vec{a} - \vec{b}) = 6 \times 1 + (-3) \times 2 + 0 \times 3 = 0$$

$$\therefore \vec{t} \perp (\vec{a} - \vec{b})$$

$$\vec{t} \cdot (\vec{b} - \vec{c}) = 0 \times 1 + 2 \times 3 + 3 \times (-2) = 0$$

$$\therefore \vec{t} \perp (\vec{b} - \vec{c})$$

$$\vec{t} \cdot (\vec{c} - \vec{a}) = (-6) \times 1 + 0 \times 2 + 2 \times 3 = 0$$

$$\therefore \vec{t} \perp (\vec{c} - \vec{a})$$