Suggested Solutions to Problem Set 2

Intermediate Microeconomics

Fall, 2023

1. A consumer's preferences are represented by the utility function $u(x_1, x_2) = x_1^3 x_2^5$. The prices of x_1 and x_2 are p_1 and p_2. Please solve the following problems.

   (a) The slope of the consumer's indifference curve at the point (x_1, x_2).

      Answer: $-3x_2/5x_1$

   (b) If this consumer's budget line is tangent to his indifference curve at (x_1, x_2). Please solve the expenditure share of goods 1.

      Answer: 3/8.

   (c) From the example above, please derive the expenditure share of goods 1 for a consumer with preferences represented by the utility function $u(x_1, x_2) = cx_1^a x_2^b$, where a, b, c are parameters.

      Answer: $\frac{a}{a+b}$.

2. A consumer's utility function is $u(x_1, x_2) = x_1 + 100 * x_2 - x_2^2$ and his/her budget constraint is $x_1 + 4x_2 = 500$.

   (a) The optimal consumption amount of goods 1 and 2.

      Answer: 308 units of goods 1 and 48 units of goods 2.

   (b) If the budget constraint changes to $x_1 + 4x_2 = 1000$, what about the changes of goods consumption?

      Answer: The consumption of goods 1 increases with 500 units while goods 2 does not change.

   (c) If the budget constraint changes to $x_1 + 4x_2 = a$, please show the condition for a that the consumer consumes both goods.

      Answer: $a > 192$.

3. Li Hua is an enthusiastic fan of Popmarts. When the prices of all other goods are fixed at current levels, Li Hua's demand function for highquality Popmart blind boxes is $q = 0.02m - 3p$, where m is his income, p is the price of a Popmart blind box, and q is the number of blind boxes that he wants to buy. Li Hua's income is 7,500 Yuan, and the price of a Popmart blind box is 30 Yuan.

   (a) How many blind boxes will Li Hua buy?

      Answer: 60.

(b) If the price of a blind box rose to 40 Yuan, how much income would Li Hua have to have in order to be exactly able to afford the amount of blind boxes and the amount of other goods that he bought before the price change? At this income, and a price of 40 Yuan, how many boxes would Li Hua buy?

Answer: 8,100 Yuan. 42 boxes.

(c) At his original income of 7,500 and a price of 40, how many blind boxes would Li Hua demand?

Answer: 30 boxes.

(d) From the analysis above, please gives the impact of substitution effect and income effect when the price changes from 30 Yuan to 40 Yuan.

Answer: The substitution effect reduced his demand by 18 boxes and the income effect reduced his demand by 12 boxes.

4. Suppose a hotel manager have a brand new empty building. There are 150 rooms in total. The manager need to buy mattresses for every rooms but with different qualities. That is, he can choose to make any fraction of mattresses with either soft ones or hard ones. The price is 1000 Yuan for a hard mattress and 2000 Yuan for a soft one. Because the rooms with soft mattress can be rent with a higher price, the manager always trys to buy as much soft ones as possible. However, he has only 200,000 Yuan to buy mattresses, which obviously is not enough money to give every room with a soft mattress.

(a) Please draw a graph to show his choice of mattresses. Please list the equations and its meanings for the lines and answer the number of hard mattresses he will buy.

Answer: On the graph below, use red line to show the locus of combinations of soft and hard mattresses that the manager can just afford to purchase with 200,000 Yuan. Use blue line to show the locus of combinations of mattresses that are sufficient to give every room one. Locate the combination of soft and hard mattresses that he will choose and label it A.

The equations are the budget constraint

$$2000m_1 + 1000m_2 = 200000$$

and the demand function:
$$m_1 + m_2 = 150$$

Thus, the number of hard mattresses is 100.

(b) Suppose the price of hard mattress fell to 500 Yuan while the price of soft mattress remained at 2000 Yuan. On the graph that you drew above, the combinations that he can afford with 200,000 Yuan at these prices. How many hard mattresses will he buy now? Is it Giffen goods for him?

Soft mattress
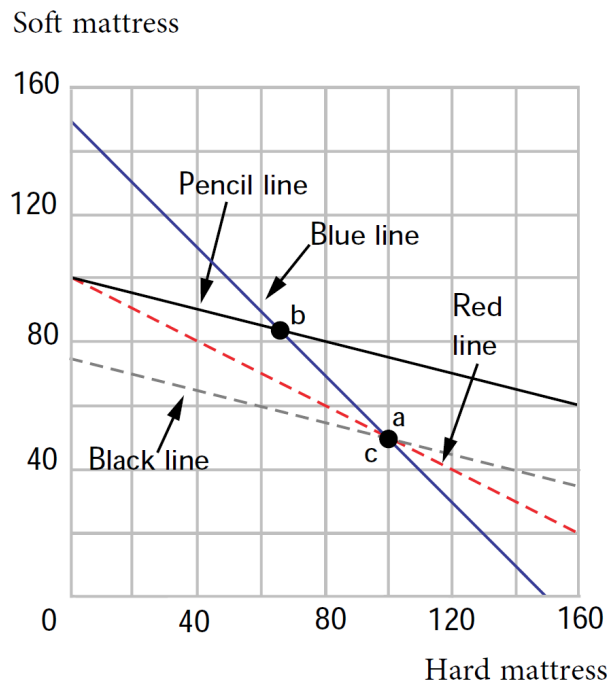
Hard mattress

Answer: The graph change is shown above. He will buy 67 hard mattresses. Yes, it is Giffen goods.

5. Xavier and Magneto work in a gas station where the salaries are given by the working hours in a week. Xavier gets 40 Yuan an hour for the first 40 hours that she works and 60 Yuan an hour for every hour beyond 40 hours a week. Magneto gets 50 Yuan an hour no matter how many hours he works. Each has 80 hours a week to allocate between work and leisure and neither has any income from sources other than labour. Each has a utility function $U = cr$, where c is consumption and r is leisure. Each can choose the number of hours to work. Please answer the following questions.

(a) How many hours will Magneto choose to work?

Answer: 40.

(b) List Xavier's budget constraint.

Answer: Xavier gets the budget constraint of $c + 40r = 3200$ when $r \geq 40$ and $c + 60r = 4000$ when $r < 40$.

(c) Will Xavier choose to work over 40 hours? What is the best choice for Xavier?

Answer: Yes. $(c, r) = (200, 33.3)$

(d) Suppose the jobs are equally agreeable in all other respects. Who has the better job?

Answer: Magneto.

6. Michael's utility function is $U(C, R) = C - (10 - R)^2$, where R is the amount of leisure he has per day. He has 16 hours a day to divide between work and leisure. He has an income

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of $30 a day from nonlabor sources. The price of consumption goods is $1 per unit.

(a) If Michael can work as many hours a day as he likes but gets zero wages for his labor, how many hours of leisure will he choose?

Answer: 10.

(b) If Michael can work as many hours a day as he wishes for a wage rate of $10 an hour, how many hours will he choose to work?

Answer: 11.

(c) Suppose that Michael has to pay an income tax of 20 percent on all of his income, and suppose that his before-tax wage remained at $10 an hour. How many hours would he choose to work?

Answer: 10.

7. Jack consumes (c_1, c_2) and earns (m_1, m_2) in periods 1 and 2 respectively. Suppose the interest rate is r.

(a) Write down Jack's intertemporal budget constraint in present value terms.

Answer:
$$c_1 + \frac{c_2}{(1+r)} = m_1 + \frac{m_2}{(1+r)}$$

(b) If Jack does not consume anything in period 1, what is the most he can consume in period 2? Is this the future value or present value of his endowment?

Answer: $m_1(1+r) + m_2$. Future value.

(c) If Jack does not consume anything in period 2, what is the most he can consume in period 1? Is this the future value or present value of his endowment?

Answer: $m_1 + \frac{m_2}{(1+r)}$. Present value.

(d) What is the slope of Jack's budget line?

Answer: $-(1+r)$.

8. In an isolated mountain village, the villagers grow wheat. They harvest 1,000 kilos of wheat this year and are definitely sure that they can only harvest 150 kilos next year. Suppose There is no trade with the outside world. wheat can be stored from one year to the next, but 25% of the stored will be lost. The villagers have utility functions as $U(c_1, c_2) = c_1 c_2$ where c_1 is consumption this year, and c_2 is consumption next year.

(a) How much corn will the villagers consume this year and next year?

Answer: 600 kilos and 450 kilos.

(b) Suppose that a road is built to the village so that now the village is able to trade with the rest of the world. Now the villagers are able to buy and sell wheat at the world

price, which is $1 per kilo. They are also able to borrow and lend money at an interest rate of 10%. Solve for the amount they would now consume in each period.

Answer: 568 kilos in the first period and 624 kilos in the second period.

9. Mr. Hacker favours milk tea. He has quasilinear preferences and his inverse demand function for a cup of milk tea is $P(m) = 40 - 2m$, where m is the number of cups of milk tea he drink a month. He is currently consuming Heytea (a kind of milk tea) at the price of 20 Yuan.

   (a) How many cups of Heytea does he consume in a month? What is his level of consumer's surplus?

   Answer: 10 cups. 100 Yuan.

   (b) Another milk tea shop called Coco comes. They offer milk tea at the price of 10 Yuan. Suppose he changes to buy milk tea at Coco. What is Mr. Hacker's change in consumer's surplus?

   Answer: 125 Yuan more.

10. Oliver consumes wine and other drinks. Suppose he has 1000 Yuan a month in total to spend for drinks. His utility function for wine x and other drinks y is given by

$$u(x,y) = 60x - \frac{x^2}{2} + y.$$

   (a) What kind of utility function does Oliver have? What is his inverse demand function for wine?

   Answer: Quasilinear. $p = 60 - x$.

   (b) If the price of wine is 30 Yuan, how many bottles of wine will he consume?

   Answer: 30.

   (c) If the price of wine changes to 50 Yuan, how many bottles of wine will he consume?

   Answer: 10.

   (d) What is the change in consumer's surplus when the price of wine changes from 30 Yuan to 50 Yuan?

   Answer: Decrease 400 Yuan.