These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Thirty-Two

Exchange

In Chapter 32 and 33

- **We try to get some flavor of the theory of general equilibrium (一般均衡), which is a typically an advanced topic.**
  - **A general equilibrium model puts together many consumers and firms trading many goods, and prices for all goods are determined endogenously by a set of market clearing conditions.**

In Chapter 32 and 33

- **Chapter 32**
  - **2 consumers, no firm, 2 goods**
- **Chapter 33**
  - **1 consumer, 1 firm, 2 goods**

The Exchange Economy

- Two consumers, A and B.
- Their endowments of goods 1 and 2 are $\omega^A = (\omega_1^A, \omega_2^A)$ and $\omega^B = (\omega_1^B, \omega_2^B)$.
- E.g. $\omega^A = (6,4)$ and $\omega^B = (2,2)$.
- The total quantities available

are $\omega_1^A + \omega_1^B = 6 + 2 = 8$ units of good 1

and $\omega_2^A + \omega_2^B = 4 + 2 = 6$ units of good 2.

Exchange

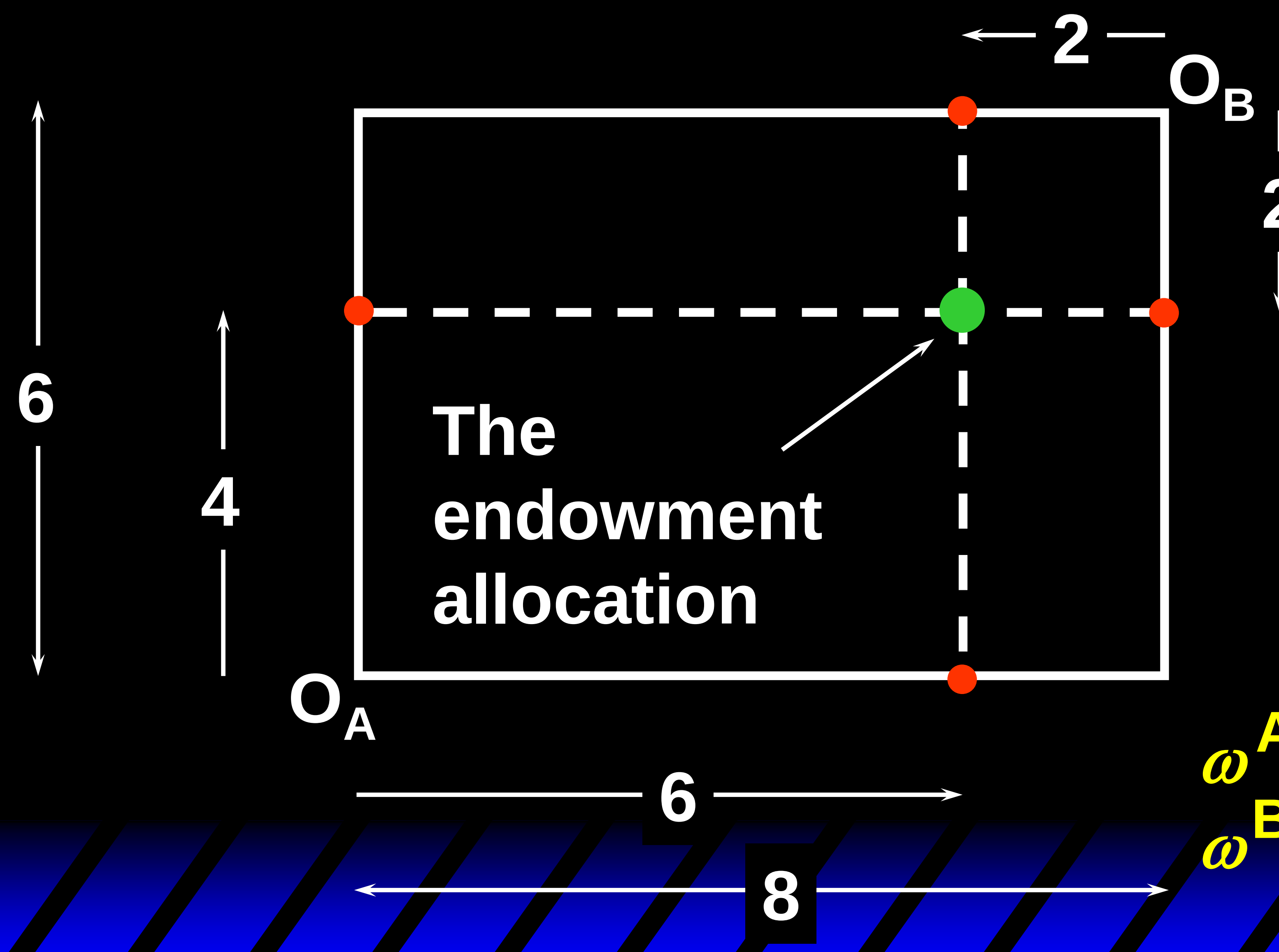
- A diagram, called an **Edgeworth box**, shows all possible allocations of the available quantities of goods 1 and 2 between the two consumers.

Starting an Edgeworth Box

Height = $\omega_2^A + \omega_2^B = 4 + 2 = 6$

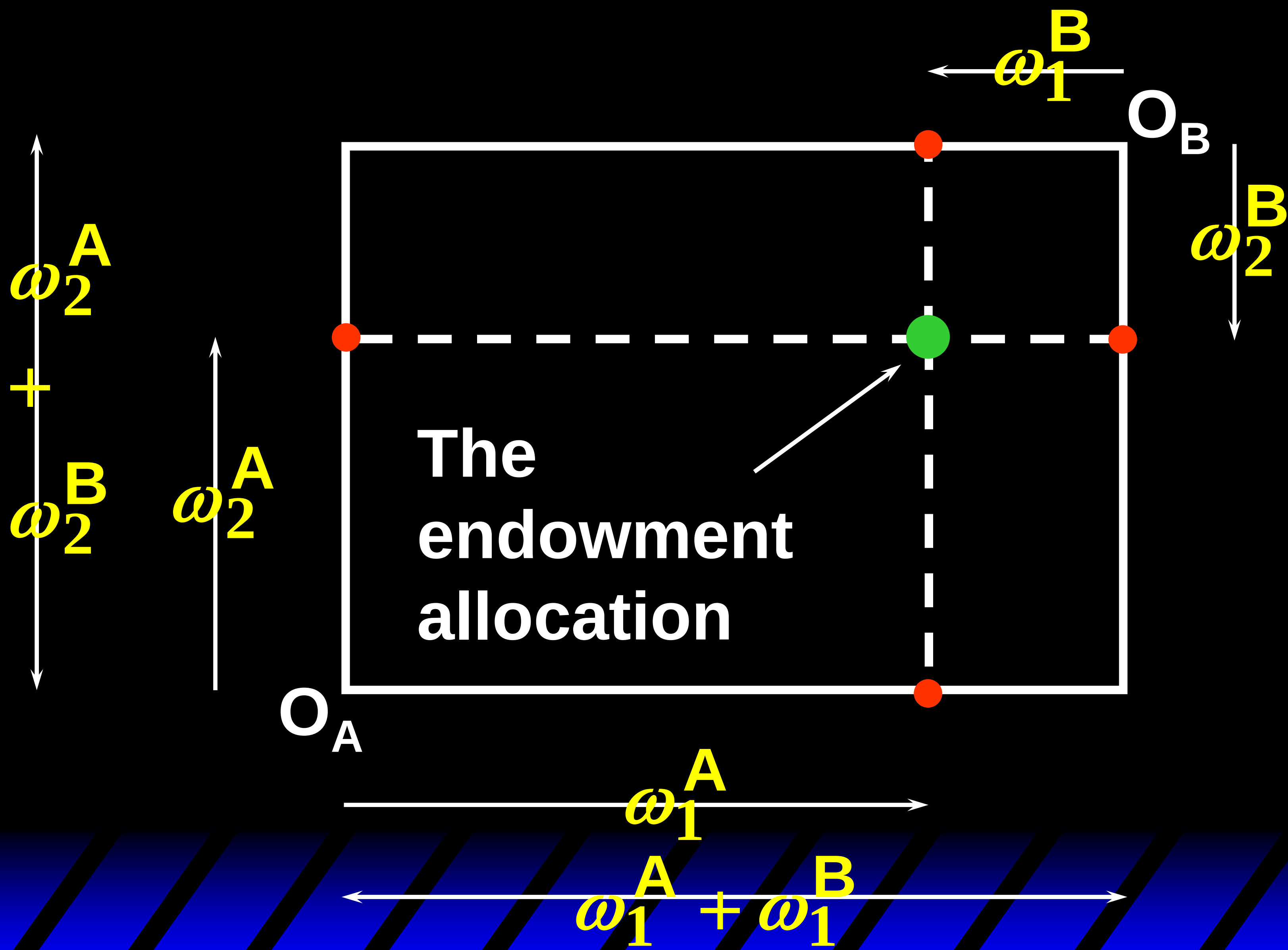
Width = $\omega_1^A + \omega_1^B = 6 + 2 = 8$

The Endowment Allocation



$$\omega^A = (6,4)$$
$$\omega^B = (2,2)$$

The Endowment Allocation



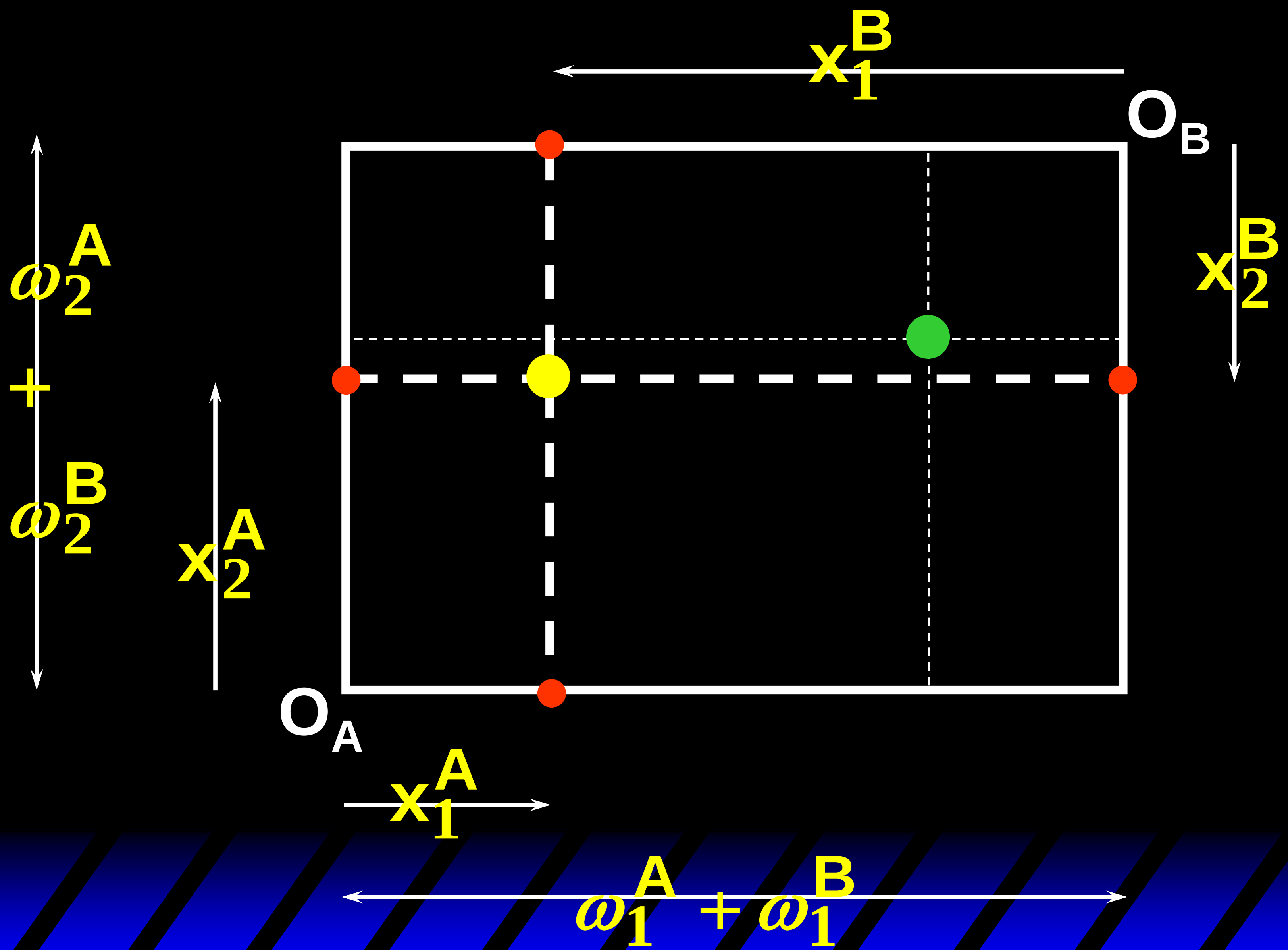
Other Feasible Allocations

- (x_1^A, x_2^A) denotes an allocation to consumer A.
- (x_1^B, x_2^B) denotes an allocation to consumer B.
- An allocation is feasible if and only if

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$

and $$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

Feasible Reallocations

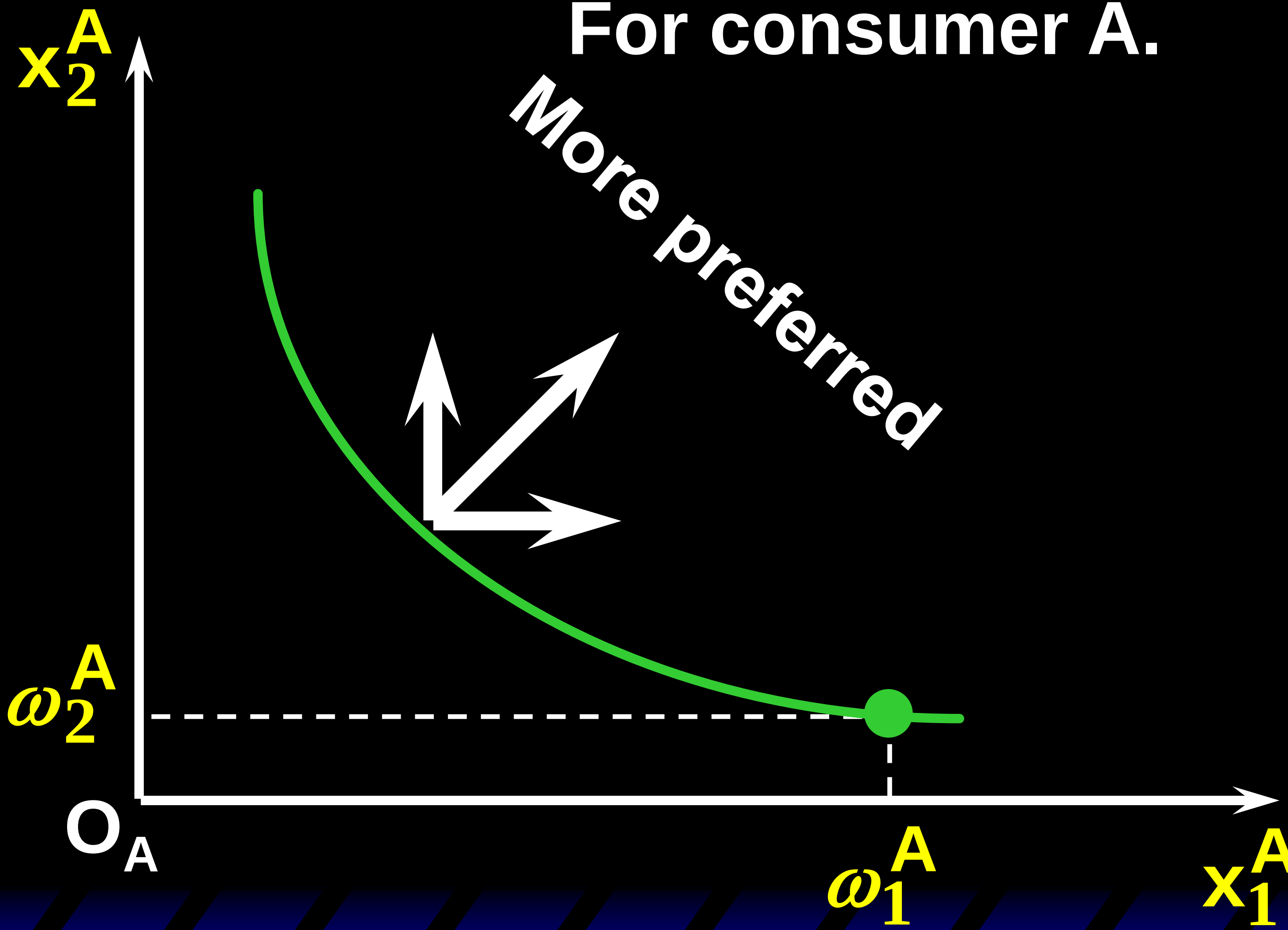


Feasible Reallocations

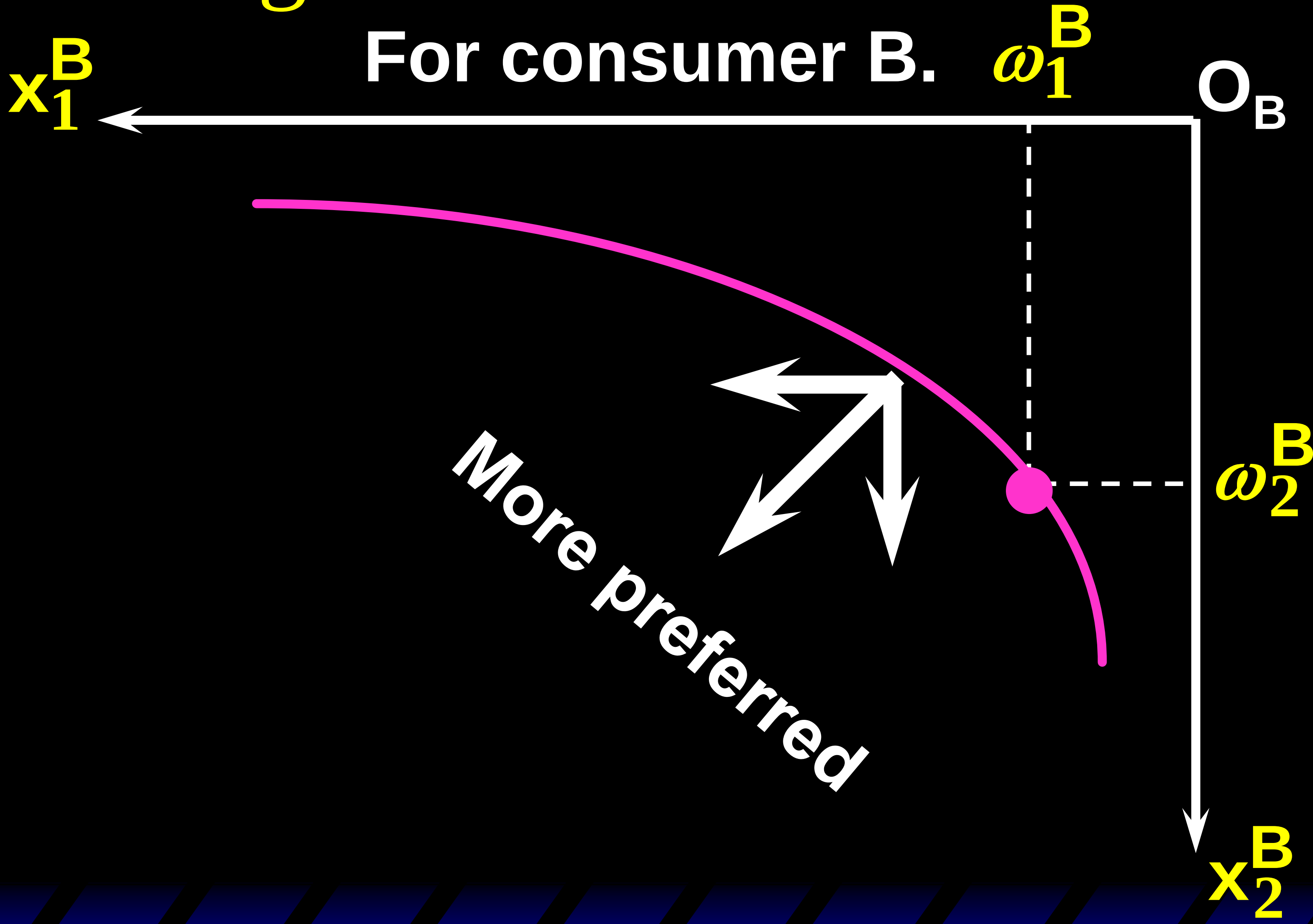
- **Each point in the box, including the boundary, represents a feasible allocation.**

Adding Preferences to the Box

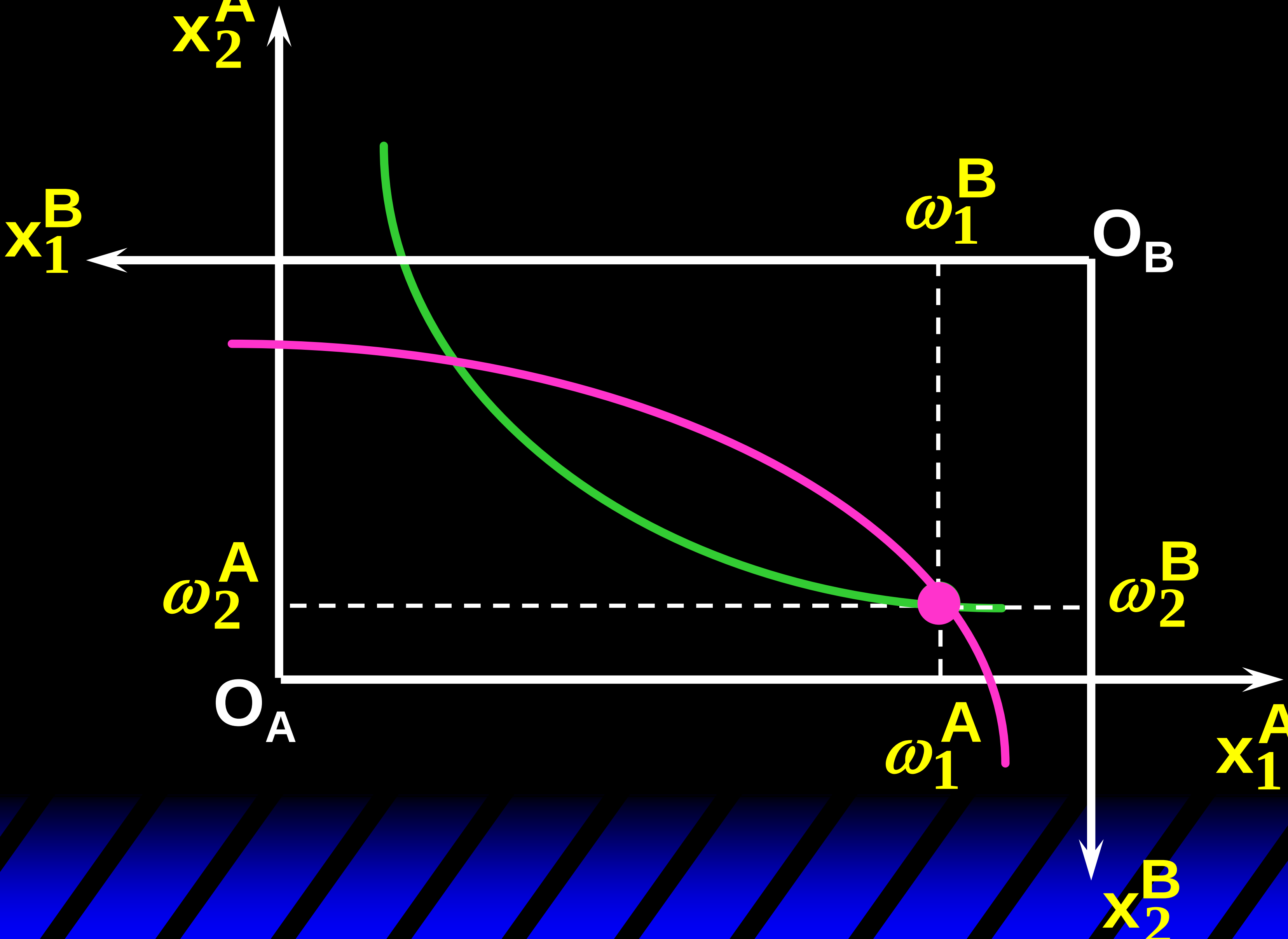
For consumer A.



Adding Preferences to the Box



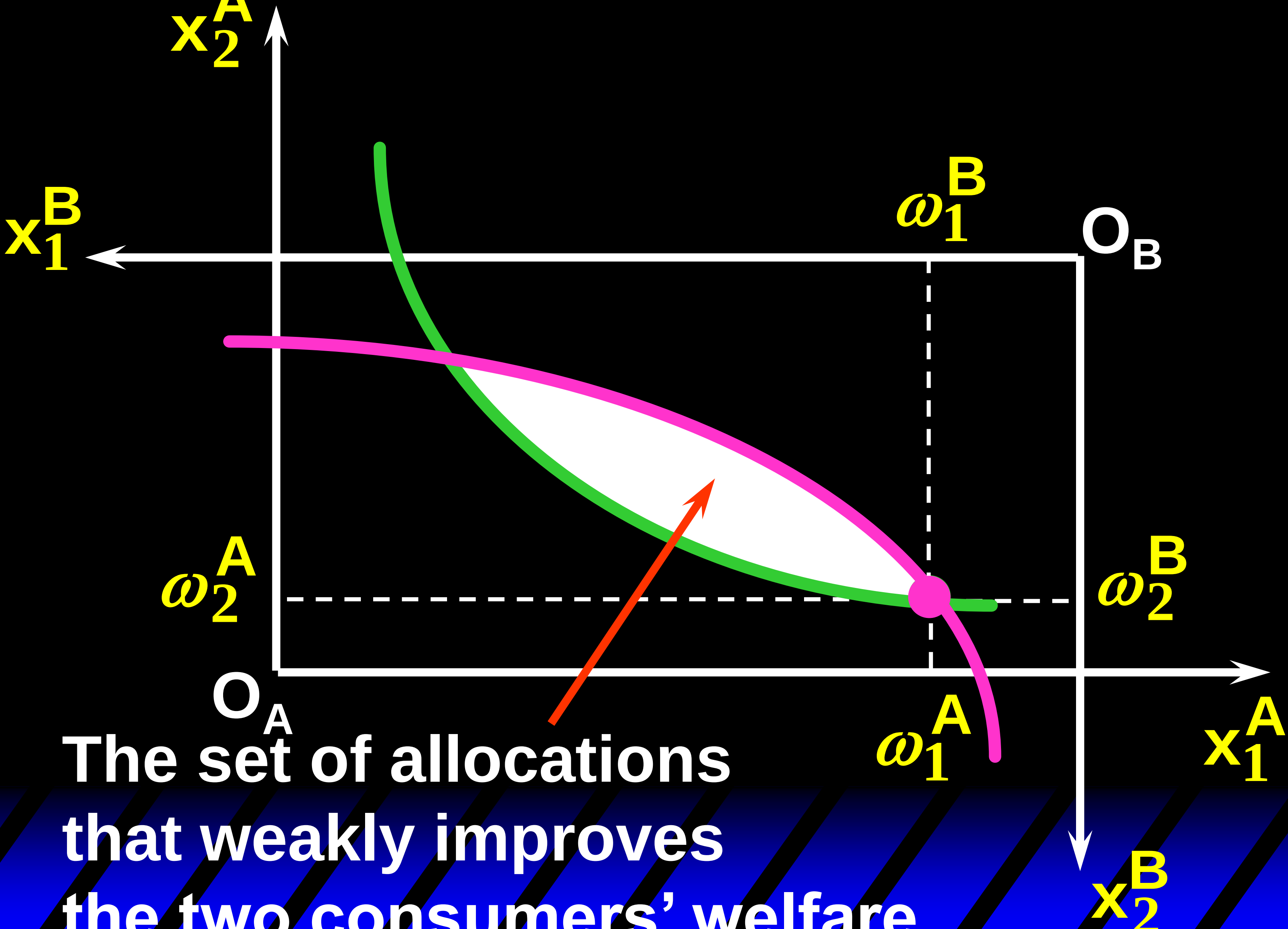
Edgeworth's Box



Assumption of Voluntary Trade

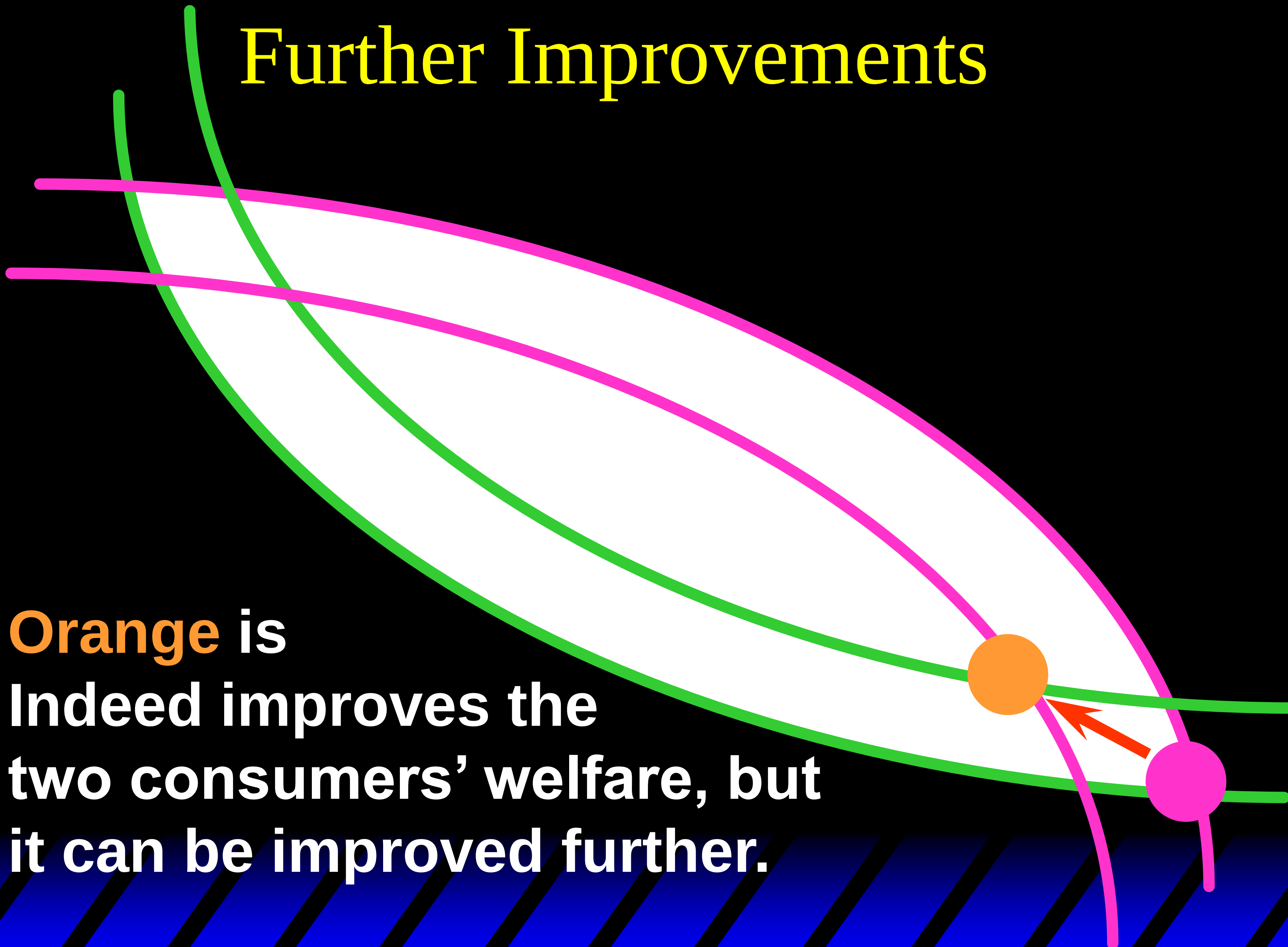
- If we assume that trade is voluntary, i.e. each consumer has the right to refuse to trade, we should expect the outcomes cannot make any consumer worse off.
- Can the model make a sharper prediction?

Welfare Improvements



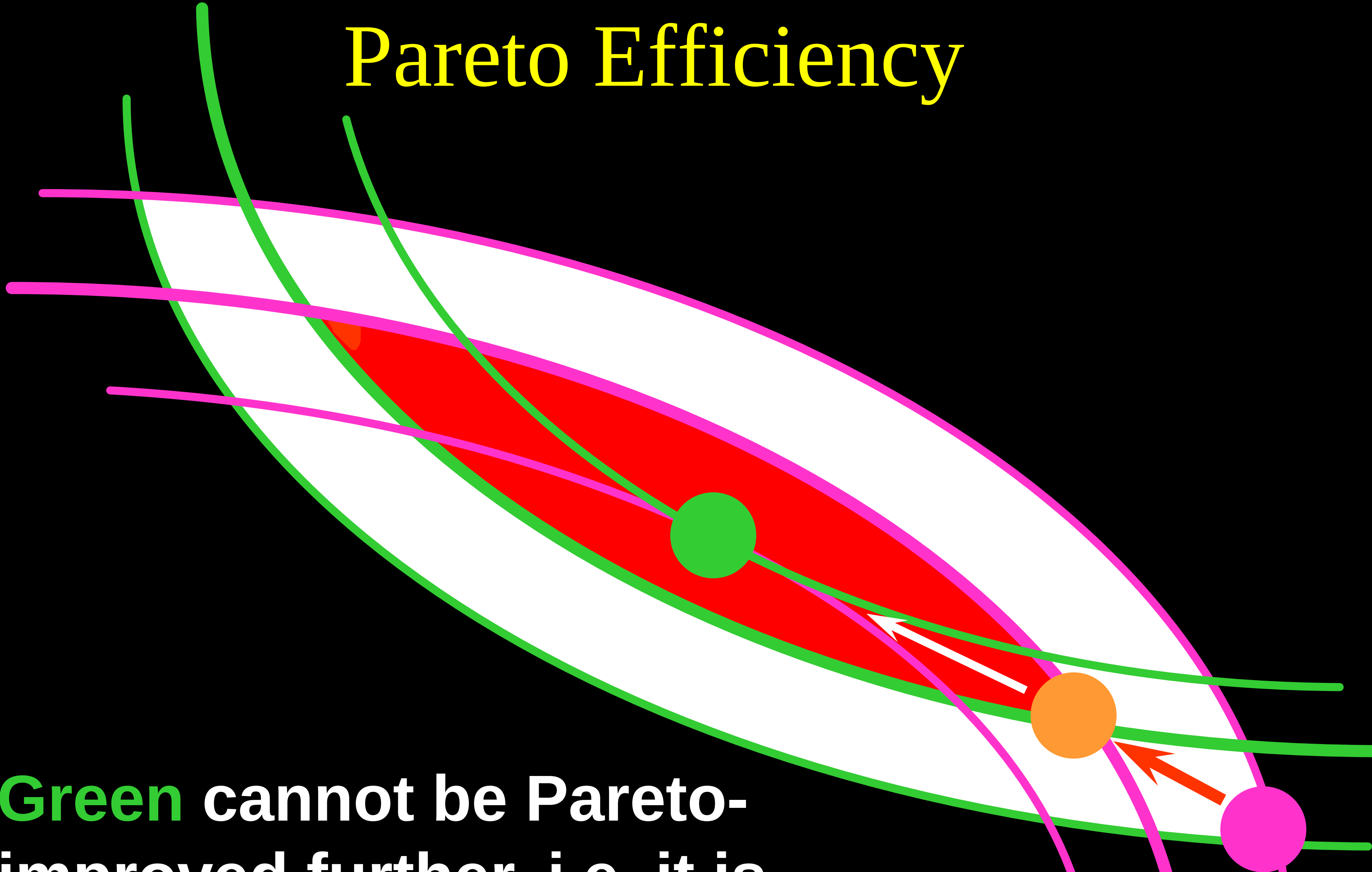
The set of allocations that weakly improves the two consumers' welfare

Further Improvements



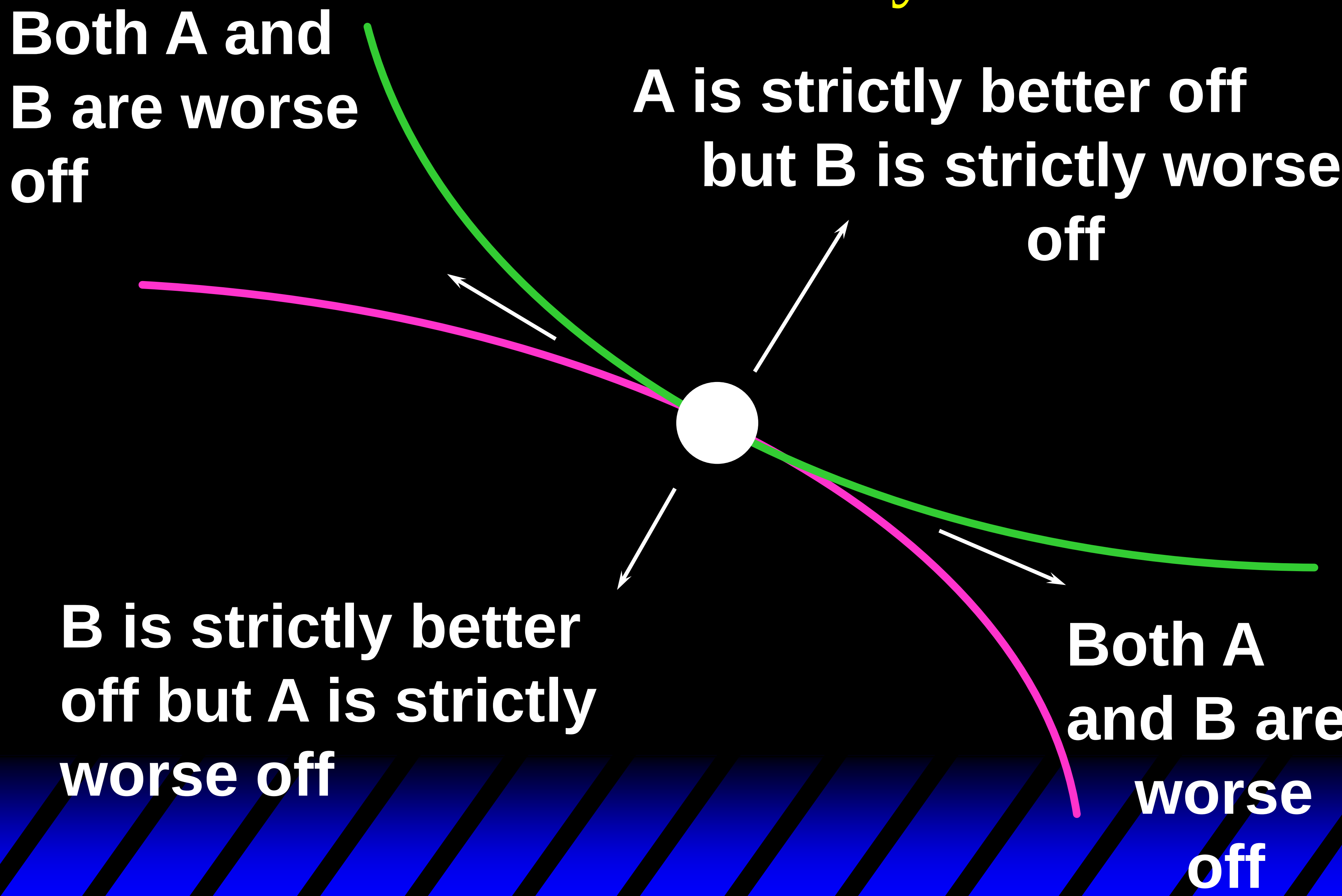
Orange is Indeed improves the two consumers' welfare, but it can be improved further.

Pareto Efficiency

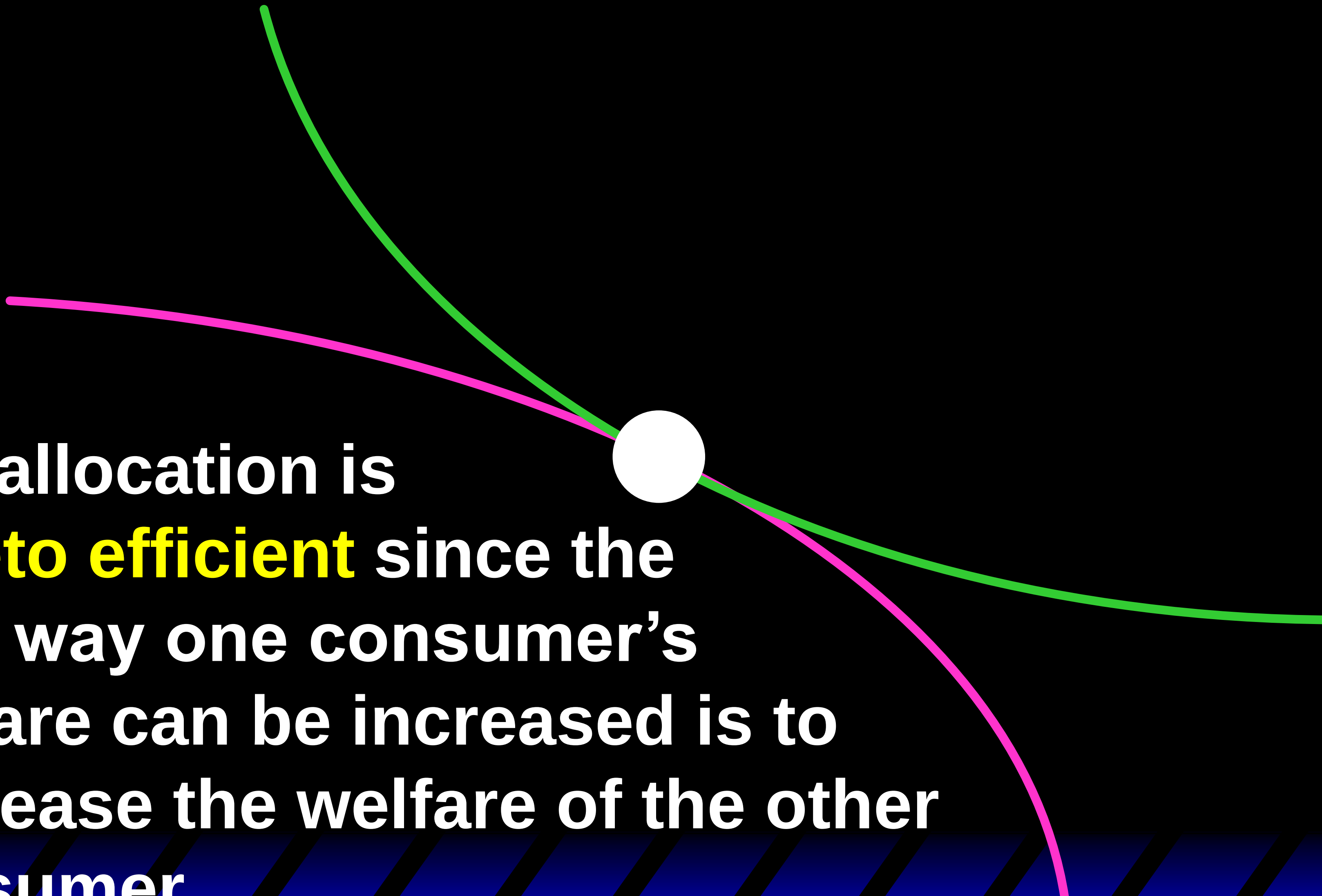


Green cannot be Pareto-improved further, i.e. it is Pareto efficient.

Pareto Efficiency



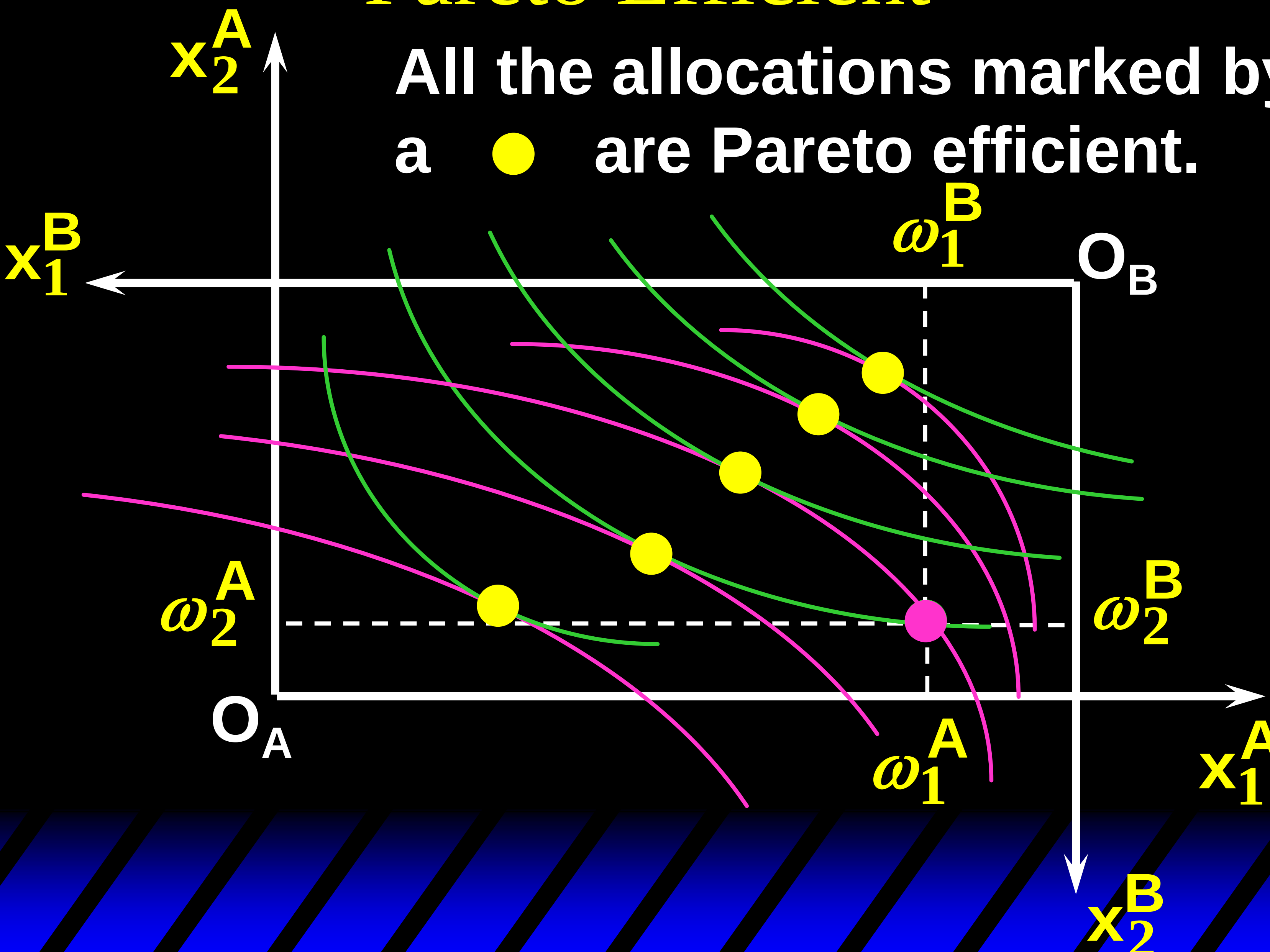
Pareto Efficient



The allocation is Pareto efficient since the only way one consumer's welfare can be increased is to decrease the welfare of the other consumer.

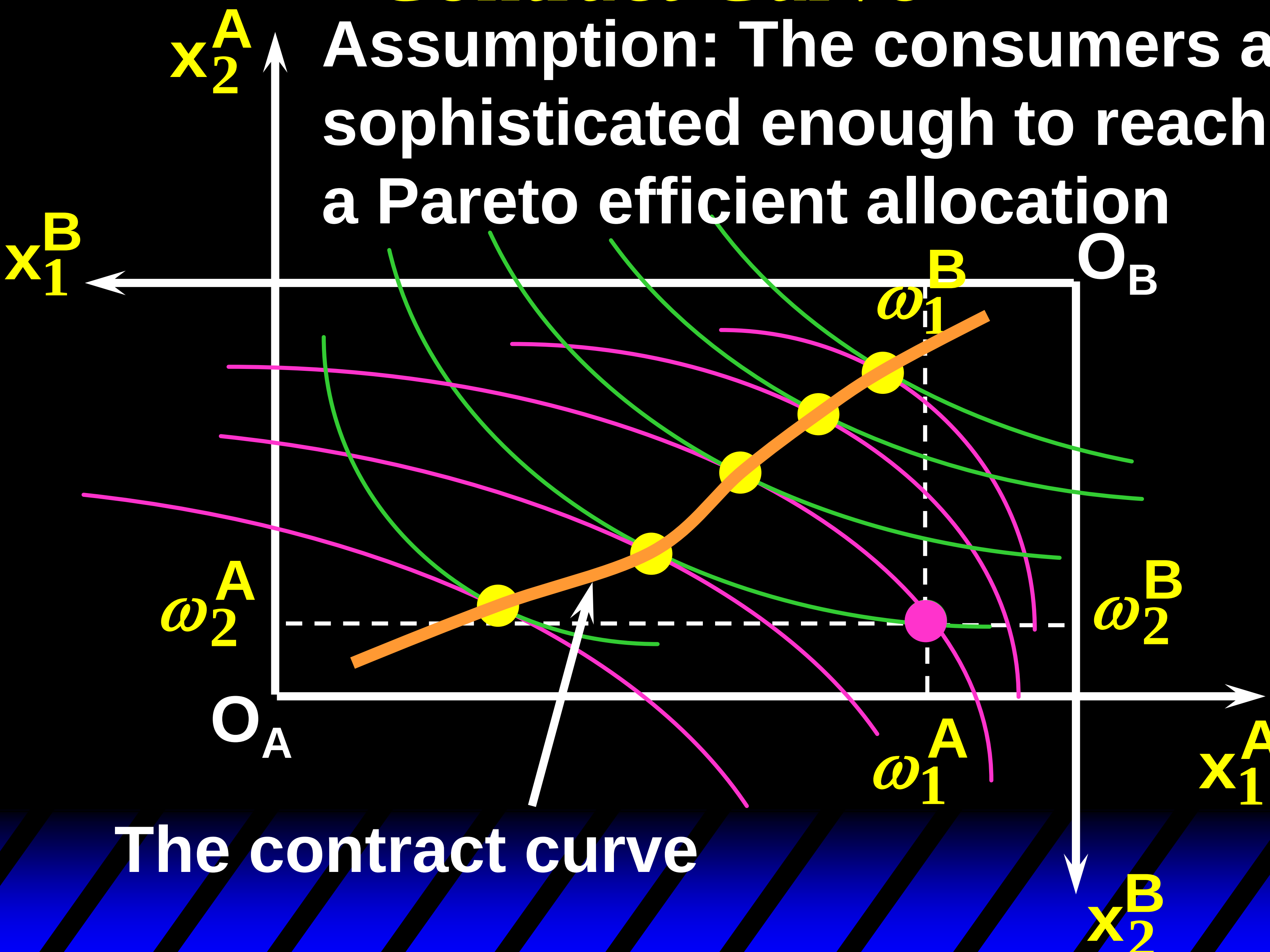
Pareto Efficient

All the allocations marked by a ● are Pareto efficient.



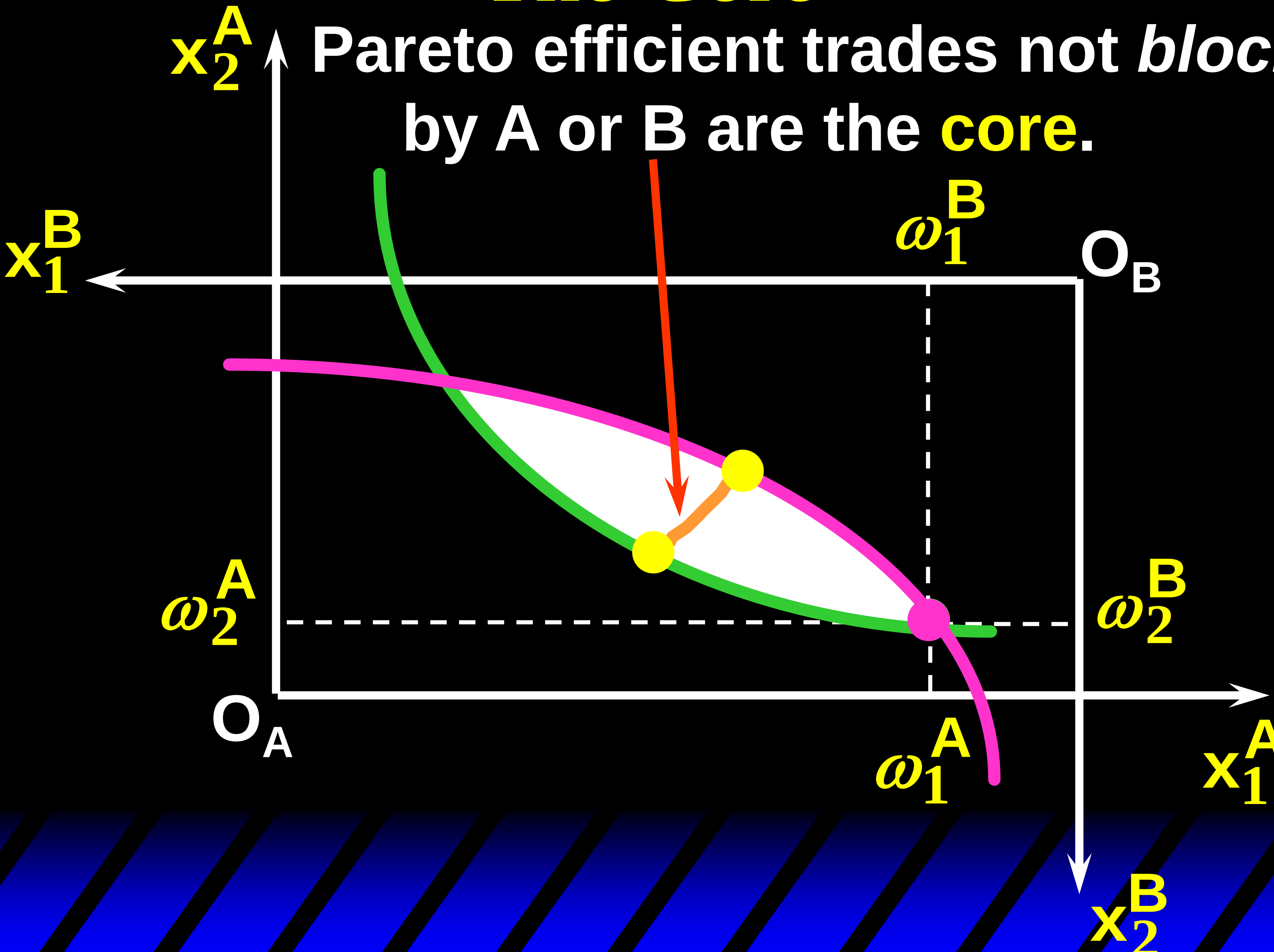
Contract Curve

Assumption: The consumers are sophisticated enough to reach a Pareto efficient allocation



The Core

Pareto efficient trades not *blocked* by A or B are the core.



The Core

- **The core is the set of all Pareto efficient allocations that are welfare-improving for both consumers relative to their own endowments.**
- **We usually expect voluntary trade to achieve a core allocation.**

Further Refinement of the Solution Concept

- Can the model make a even sharper prediction?
- That depends on how trade is conducted.
  - Competitive market (general equilibrium theory)
  - Bargaining game
  - Other assumptions…

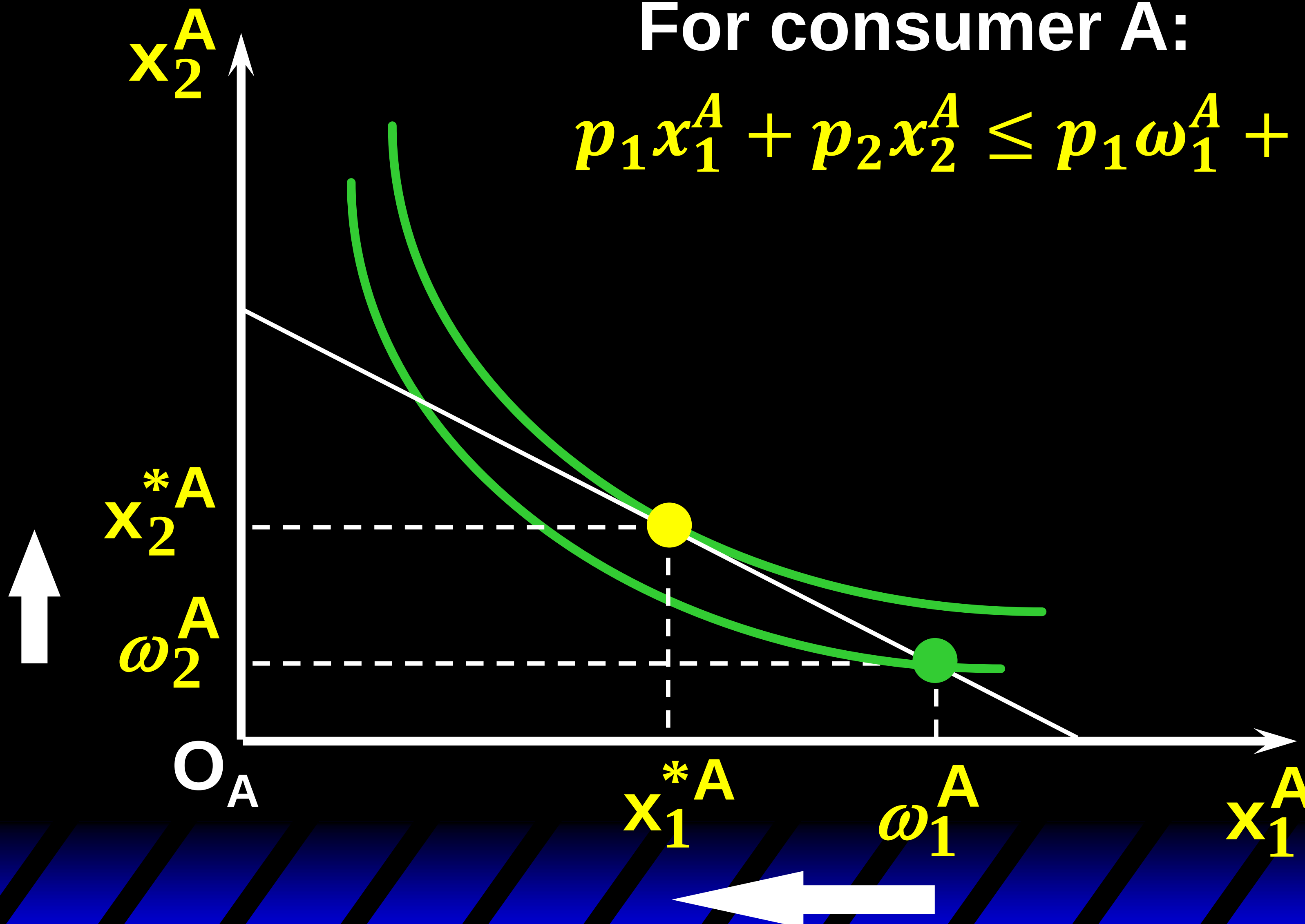
Trade in Competitive Markets

- Under the assumption of competitive markets, each consumer is a price-taker.
- Typically, the price taking assumption only makes sense when there are many participants.
  - We should interpret the 2-consumer Edgeworth box model as a simplistic illustration of the general equilibrium theory with many market participants.

Trade in Competitive Markets

For consumer A:

$$p_1 x_1^A + p_2 x_2^A \leq p_1 \omega_1^A + p_2 \omega_2^A$$



Trade in Competitive Markets

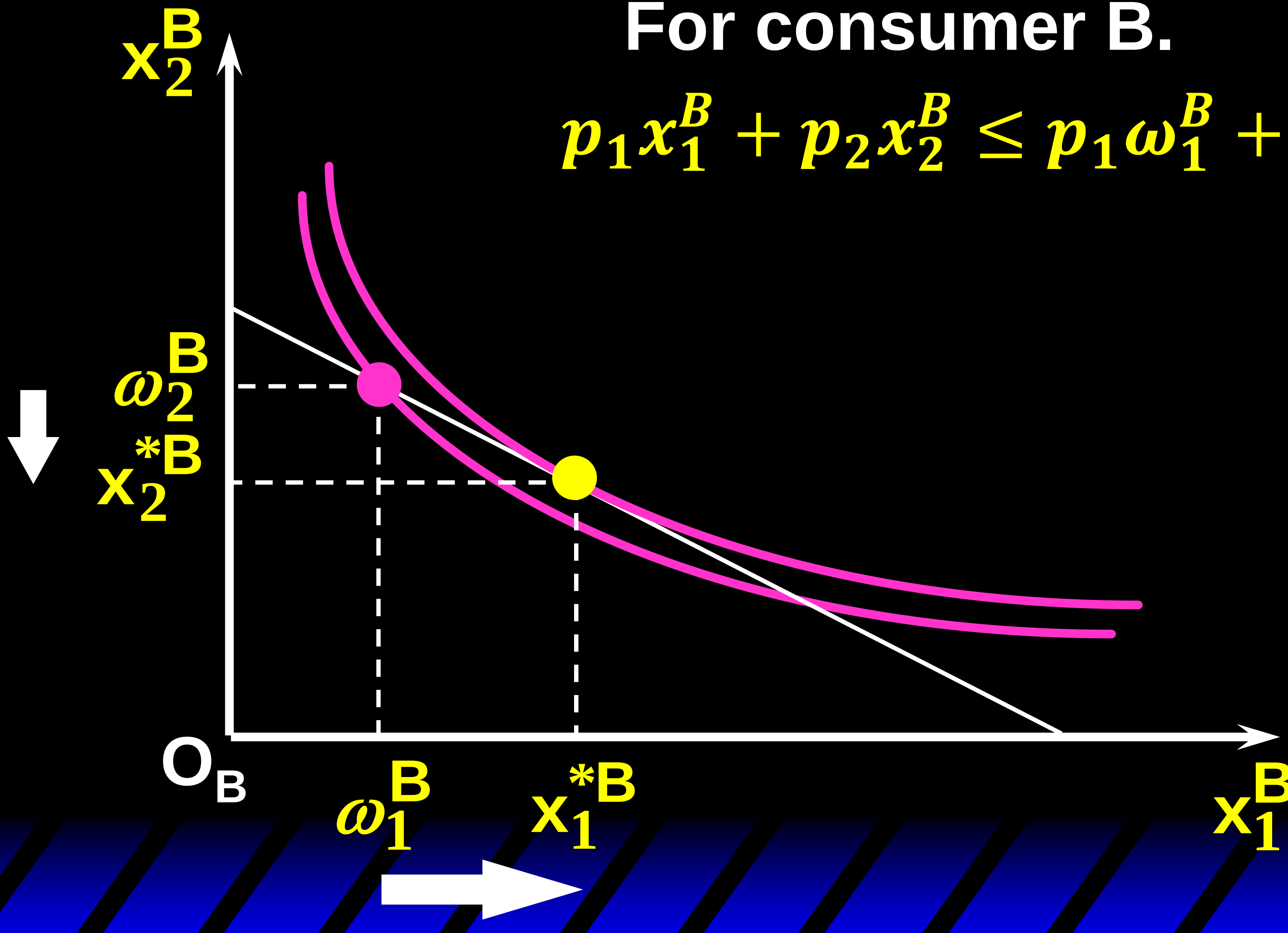
- So given p_1 and p_2, consumer A's net demands (a.k.a. excess demand) for goods 1 and 2 are

$$x_1^{*A} - \omega_1^A \quad \text{and} \quad x_2^{*A} - \omega_2^A.$$

Trade in Competitive Markets

For consumer B.

$$p_1x_1^B + p_2x_2^B \le p_1\omega_1^B + p_2\omega_2^B$$



Trade in Competitive Markets

- So given p_1 and p_2, consumer B's net demands for goods 1 and 2 are

$$x_1^{*B} - \omega_1^B \quad \text{and} \quad x_2^{*B} - \omega_2^B.$$

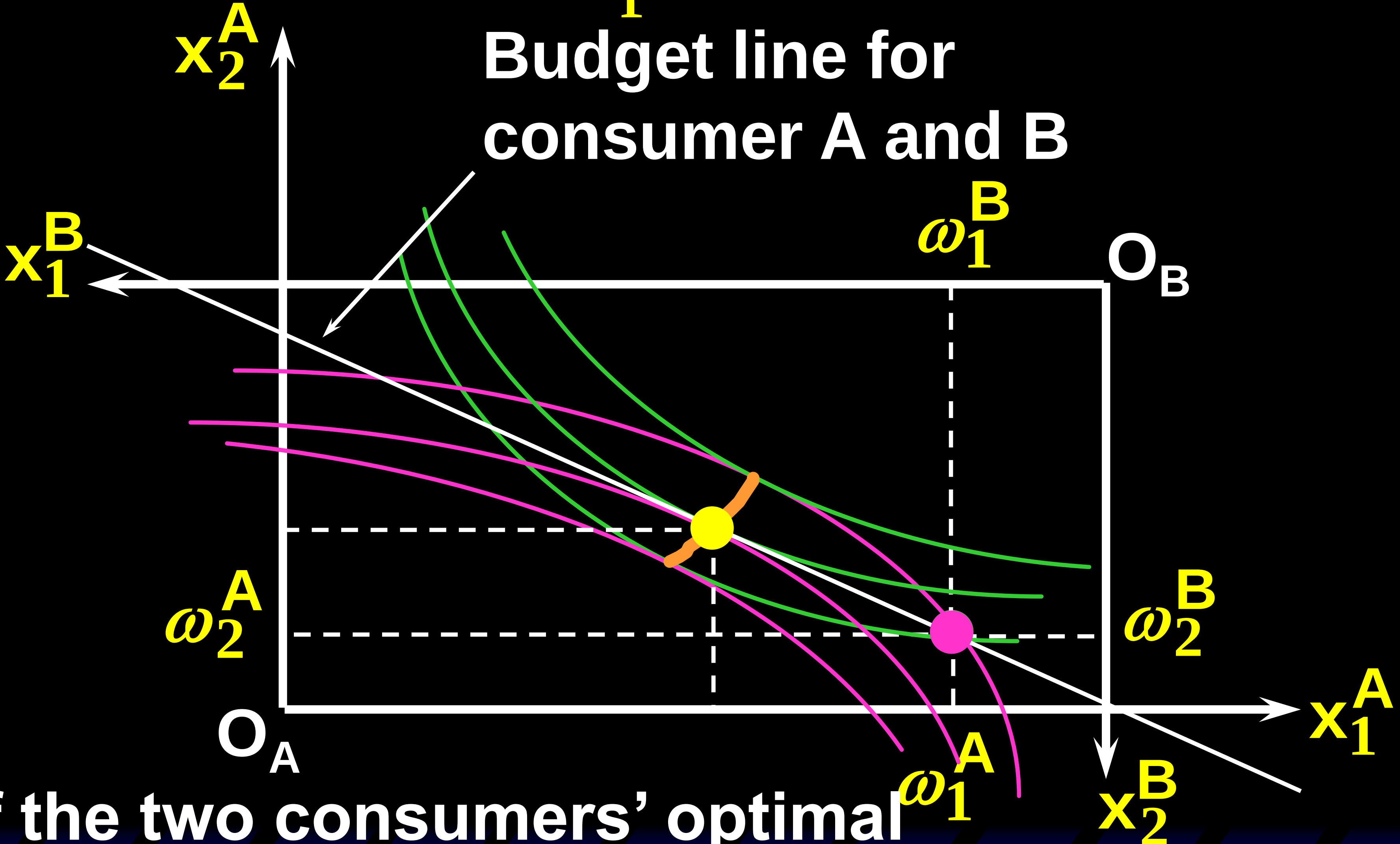
Trade in Competitive Markets

- A **general equilibrium** occurs when prices p_1 and p_2 cause both the markets for goods 1 and 2 to clear; i.e.

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$
$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B.$$

- The above is gross demand = gross supply.
- Equivalently, we can rewrite it as the sum of net demand = 0.

Trade in Competitive Markets



If the two consumers' optimal choice coincide, then the markets clear.

Equilibrium

- **If prices p_1 and p_2 clear both markets, then this price vector together with the resulted allocation is called a Walrasian equilibrium (a.k.a. competitive/market/price equilibrium).**
- **It has been shown that Walrasian equilibrium exists under relatively mild conditions (including convexity of preferences.)**

First Welfare Theorem

- **Note that trading in competitive markets achieves a Pareto efficient allocation.**
- **This observation holds in general under very mild assumption (monotonic preferences), which is known as First Welfare Theorem, a.k.a. First Fundamental Theorem of Welfare Economics.**

Walras' Law

- **Assuming both consumers' preferences are monotonic, and so their budget constraint is binding, i.e.**
- **For consumer A:**

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

For consumer B:

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

Walras' Law

$$p_1x_1^{*A} + p_2x_2^{*A} = p_1\omega_1^A + p_2\omega_2^A$$

$$p_1x_1^{*B} + p_2x_2^{*B} = p_1\omega_1^B + p_2\omega_2^B$$

Summing gives

$$p_1(x_1^{*A} + x_1^{*B}) + p_2(x_2^{*A} + x_2^{*B})$$

$$= p_1(\omega_1^A + \omega_1^B) + p_2(\omega_2^B + \omega_2^B).$$

Walras' Law

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$
$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B)$$
$$= 0.$$

This says that the summed market value of excess demands is zero for any positive prices p_1 and p_2 -- this is Walras' Law.

Implications of Walras' Law

Suppose the market for good A is in equilibrium; that is,

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B = 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies market for good B also clears:

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B = 0.$$

Implications of Walras' Law

- **If one market clears, then then other market must also clears.**
- **More generally, if there are n goods, and n-1 markets clear, then the remaining market must also clears.**

Implications of Walras' Law

What if, for some positive prices p_1 and p_2, there is an excess quantity supplied of good 1? That is,

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B < 0.$$

Then

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies market for good B is in shortage:

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B > 0.$$

Second Welfare Theorem

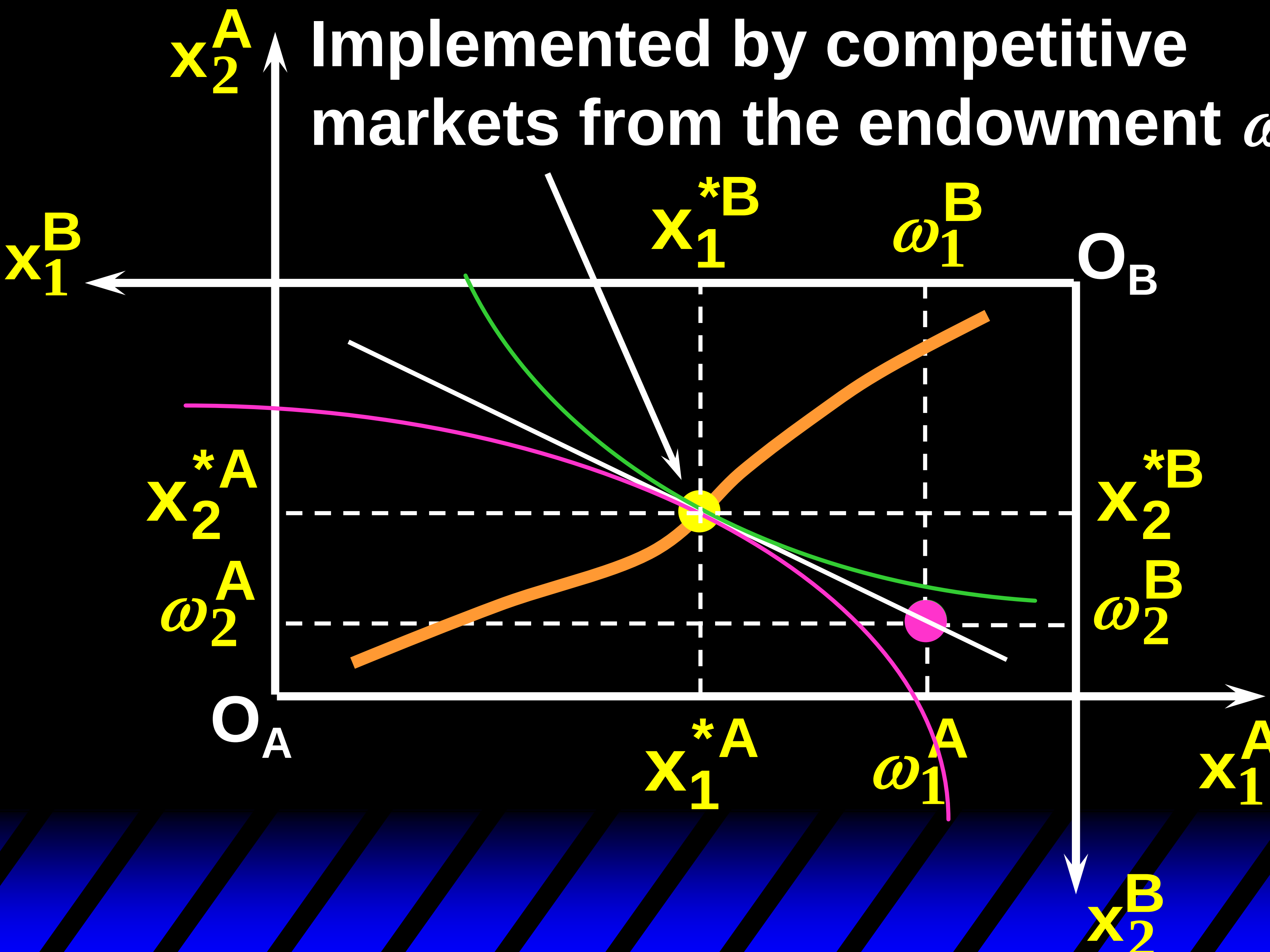
- **First Welfare Theorem states that the competitive markets always implement one of the Pareto efficient allocations.**
- **What if the implemented allocation is not desirable in other aspects, e.g. "fairness"?**

Second Welfare Theorem

- **What if the government want to implement another Pareto efficient allocation?**
- **Second welfare theorem states that competitive markets can implement any Pareto efficient allocation, as long as the government can redistribute the consumers' endowment or monetary income.**
  - **It holds under mild assumptions, including convexity of preferences.**

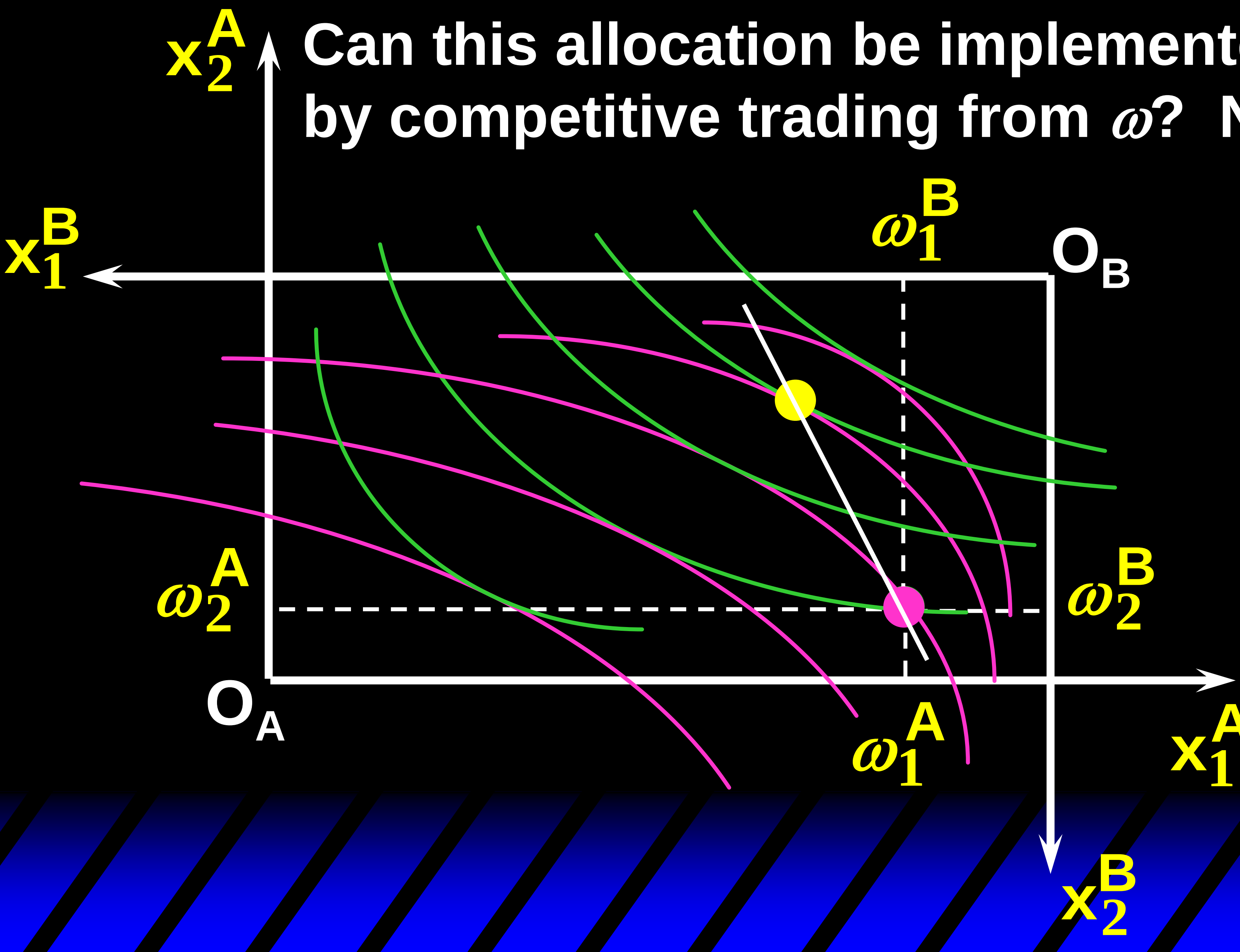
Second Welfare Theorem

Implemented by competitive markets from the endowment ω.



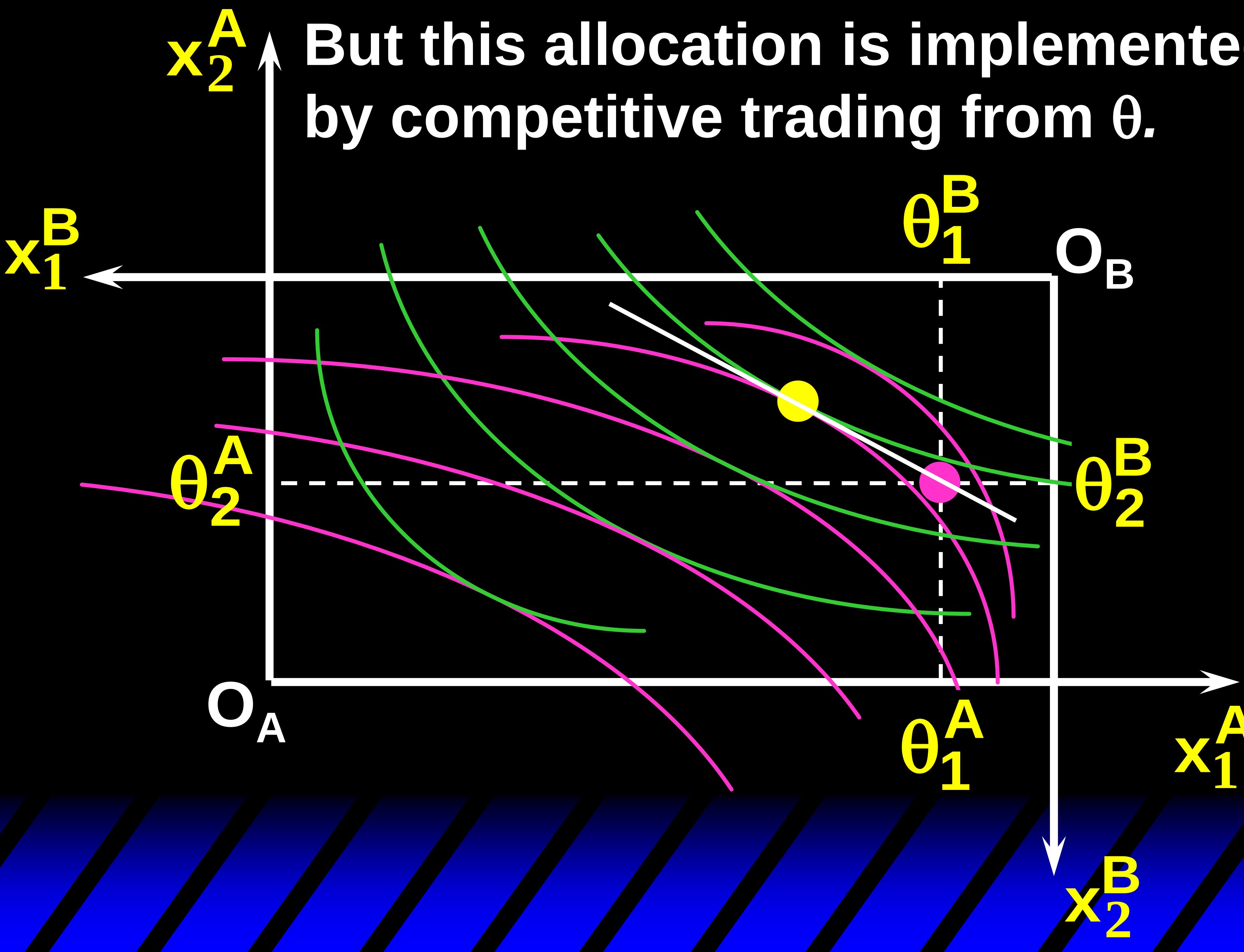
Second Fundamental Theorem

Can this allocation be implemented by competitive trading from ω? No.



Second Fundamental Theorem

But this allocation is implemented by competitive trading from θ.



Summary

- Key concepts:
  – Pareto efficiency
  – Contract curve
  – Core
  – Walrasian equilibrium
  – Walras' Law
  – First and Second Welfare Theorems
- Key methodology: The Edgeworth box