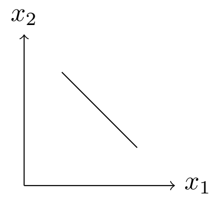
HW1

Li Haolun 2022011545

October 15, 2023

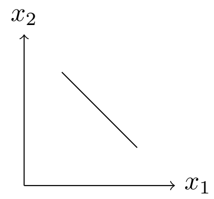
1.



2. *Proof.* Since the goods for 2 curves are the same, the relative price of the goods is the same. Therefore, the slope of the 2 curves are the same, which means the 2 curves are parallel. □

3.

$$\begin{aligned} U &= 3(x_1^2 + 2x_1x_2 + x_2^2) + 10 \\ &= 3(x_1 + x_2)^2 + 10 \\ \Rightarrow \sqrt{\frac{U-10}{3}} &= x_1 + x_2 \end{aligned}$$



It is the preference of substitutes.

4. (1) *Proof.* For perfect substitutes, a consumer is willing to substitute one good for the other at a constant rate. So,

$$\begin{aligned} \frac{\partial x_2}{\partial x_1} &= \frac{\partial U}{\partial x_1} \Big/ \frac{\partial U}{\partial x_2} \\ &= \frac{\frac{1}{\rho}(\alpha_1 x_1^\rho + \alpha_2 x_2^\rho)^{\frac{1}{\rho}-1}\alpha_1 \rho x_1^{\rho-1}}{\frac{1}{\rho}(\alpha_1 x_1^\rho + \alpha_2 x_2^\rho)^{\frac{1}{\rho}-1}\alpha_2 \rho x_2^{\rho-1}} \\ &= \frac{\alpha_1}{\alpha_2}(\frac{x_1}{x_2})^{\rho-1} \\ &= \text{constant rate} \end{aligned}$$

□

It's clear that $\frac{\alpha_1}{\alpha_2}(\frac{x_1}{x_2})^{\rho-1}$ is constant if and only if $\rho = 1$.

(2) *Proof.* For perfect complements, a consumer always consumes commodities 1 and 2 in fixed proportion. So,

$$\begin{aligned} \frac{\partial x_2}{\partial x_1} &= \frac{\partial U}{\partial x_1} \Big/ \frac{\partial U}{\partial x_2} \\ &= \frac{\alpha_1}{\alpha_2}(\frac{x_1}{x_2})^{\rho-1} \\ &= 0 \text{ or } \infty \end{aligned}$$

$\square$

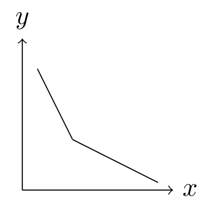
It's clear that $\frac{\alpha_1}{\alpha_2}(\frac{x_1}{x_2})^{\rho-1} = 0$ or ∞ if and only if $\rho = -\infty$.

(3) For Cobb-Douglas,

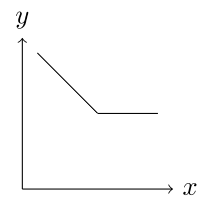
$$\begin{aligned} \frac{\partial x_2}{\partial x_1} &= \frac{\partial U}{\partial x_1} / \frac{\partial U}{\partial x_2} \\ &= \frac{\alpha_1}{\alpha_2}(\frac{x_1}{x_2})^{\rho-1} \\ &\propto \frac{x_2}{x_1} \end{aligned}$$

So, $\rho = 0$.

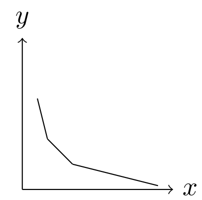
5. (1)



(2)



(3)



6.

$$\begin{aligned} 25x + 15y &= C \\ U &= \min(x, y^2) \\ \Rightarrow U &= \min(\frac{C - 15y}{25}, y^2) \end{aligned}$$

So, when $y = 7$, we have

$$\begin{aligned} \frac{C - 15y}{25} &= y^2 \\ \Rightarrow C &= 1330 \end{aligned}$$

7.

$$\begin{aligned} U &= x_1 + \sqrt{x_2 + x_3} \\ x_1 + 2x_2 + 3x_3 &= 100 \\ \Rightarrow U &= 100 - 2x_2 - 3x_3 + \sqrt{x_2 + x_3} \end{aligned}$$

For $x_2, x_3 > 0$:

$$\begin{aligned}
\frac{\partial U}{\partial x_2} &= \frac{1}{2\sqrt{x_2+x_3}} - 2 \\
&= 0 \\
\Rightarrow x_2 &= \frac{1}{16} - x_3 \\
\frac{\partial U}{\partial x_3} &= \frac{1}{2\sqrt{x_2+x_3}} - 3 \\
&= 0 \\
\Rightarrow x_3 &= \frac{1}{36} - x_2
\end{aligned}$$

There's no solution for $x_2, x_3 > 0$.

Then consider $x_2 = 0$:

$$\begin{aligned}
U &= 100 - 3x_3 + \sqrt{x_3} \\
\frac{\partial U}{\partial x_3} &= \frac{1}{2\sqrt{x_3}} - 3 \\
&= 0 \\
\Rightarrow x_3 &= \frac{1}{36} \\
x_1 &= \frac{1199}{12} \\
U &= 100\frac{1}{12}
\end{aligned}$$

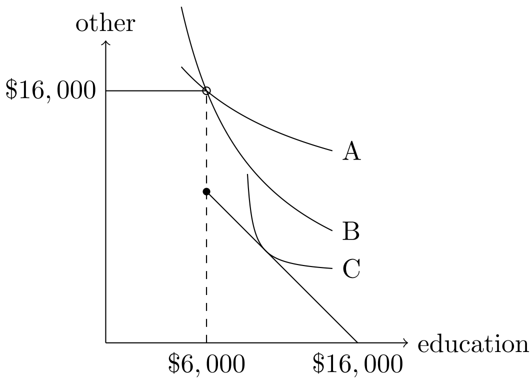
$x_3 = 0$:

$$\begin{aligned}
\Rightarrow x_2 &= \frac{1}{16} \\
x_1 &= \frac{799}{8} \\
U &= 100\frac{1}{8}
\end{aligned}$$

So, the optimal choice should be $x_1 = \frac{799}{8}, x_2 = \frac{1}{16}, x_3 = 0$.

But x_i are integer, so the optimal choice is $x_1 = 100, x_2 = 0, x_3 = 0$. And $U = 100$.

8.



9. (1)

$$\begin{aligned}
x + y &= 600 \\
U &= x^{0.1}y^{0.9} \\
-\frac{\partial y}{\partial x} &= -\frac{\partial U}{\partial x} \Big/ \frac{\partial U}{\partial y} \\
&= -\frac{0.1x^{-0.9}y^{0.9}}{0.9x^{0.1}y^{-0.1}} \\
&= -\frac{y}{9x} \\
&= -1 \\
\Rightarrow y &= 9x \\
\Rightarrow x &= 60 \\
y &= 540
\end{aligned}$$

(2)

$$\begin{aligned}
x + y &= 600 + 100 = 700 \\
y &= 9x \\
\Rightarrow x &= 70 \\
y &= 630
\end{aligned}$$

(3)

$$\begin{aligned}
\frac{1}{2}x + y &= 600 \\
\frac{\partial y}{\partial x} &= -\frac{y}{9x} \\
&= -\frac{1}{2} \\
\Rightarrow 2y &= 9x \\
\Rightarrow x &= 120 \\
y &= 540
\end{aligned}$$

(4)

$$\begin{cases} y = 600 & (0 \leq x < 100) \\ (x - 100) + y = 600 & (x \geq 100) \end{cases}$$

$$\begin{aligned}
\frac{\partial y}{\partial x} &= -\frac{y}{9x} \\
&= -1 (x \leq 100) \\
\Rightarrow y &= 9x (x \geq 100) \\
\Rightarrow x &= 100 \\
y &= 600
\end{aligned}$$

If there's a black market,

$$\begin{cases} 0.8x + y = 680 & (0 \geq x < 100) \\ (x - 100) + y = 600 & (x \geq 100) \end{cases}$$

$$\begin{cases} y = 7.2x & (0 \geq x < 100) \\ y = 9x & (x \geq 100) \end{cases}$$

$$\begin{aligned}
\Rightarrow x &= 85 \\
y &= 612
\end{aligned}$$

10. (1)

$$C + 8R = 160$$

(2)

$$
\begin{aligned}
-\frac{\partial C}{\partial R} &= -\frac{\partial U}{\partial R} / \frac{\partial U}{\partial C} \\
&= -\frac{C}{R} \\
&= -8 \\
\Rightarrow C &= 8R \\
\Rightarrow R &= 10 \\
C &= 80
\end{aligned}
$$

(3)

$$
\begin{aligned}
C + 12R &= 232 \\
C &= 12R \\
\Rightarrow R &= \frac{29}{3} \\
C &= 116
\end{aligned}
$$

(4)

$$
\begin{cases}
C + 4R = 112 \quad (R < 12) \\
C + 8R = 160 \quad (R \geq 12)
\end{cases}
$$

$$
\begin{aligned}
C &= 8R \\
\Rightarrow R &= 12 \\
C &= 96
\end{aligned}
$$