These slides are by courtesy of Prof. 李稻葵 and Prof. 郑捷.

Chapter Fifteen

Market Demand

From Individual to Market Demand Functions

- Think of an economy containing n consumers, denoted by i = 1, … ,n.
- Consumer i's demand function for commodity j is

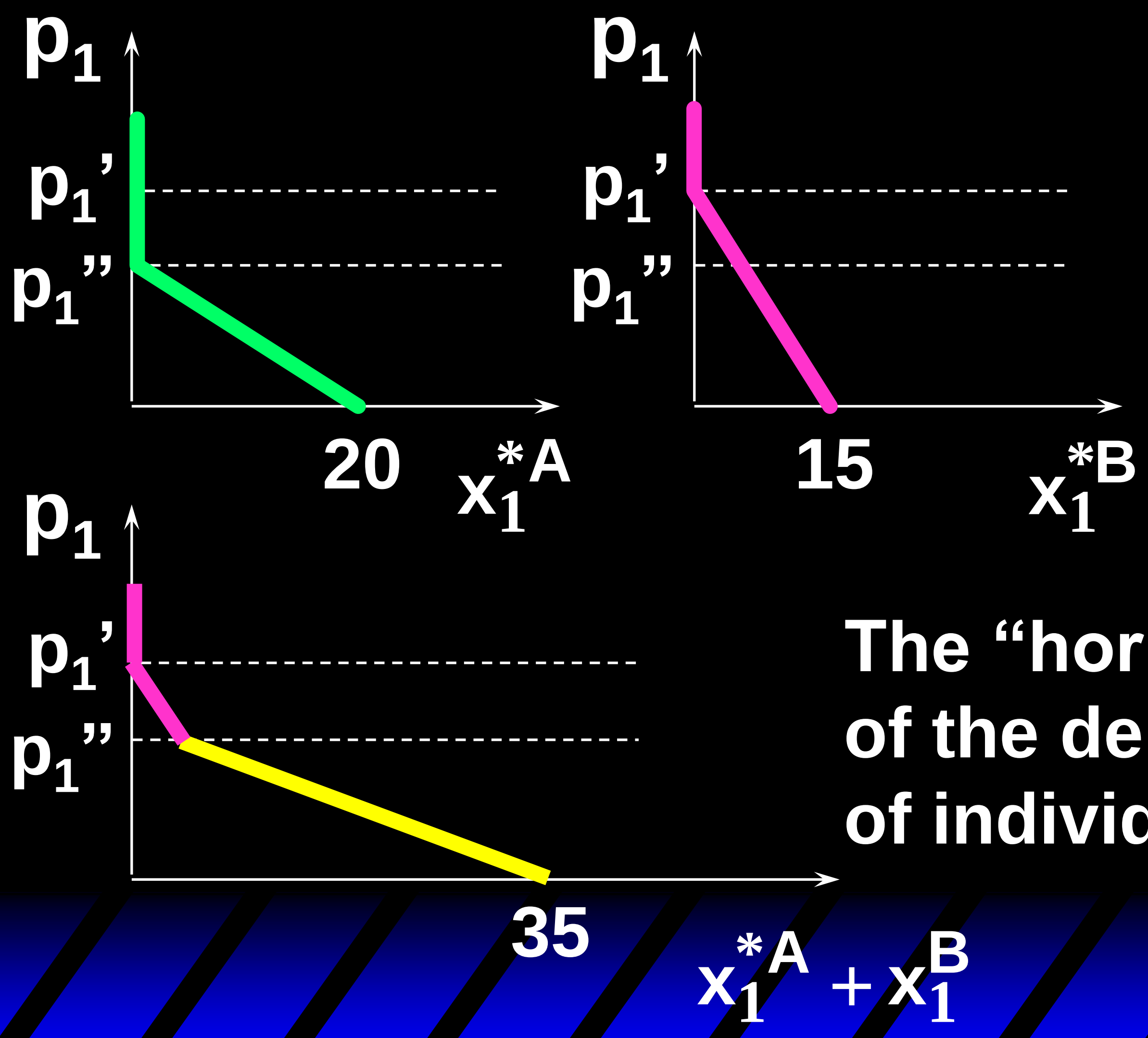
$$x_j^{*i}(p_1, p_2, m^i)$$

From Individual to Market Demand Functions

- **The market demand function for commodity j is**

$$X_j(p_1, p_2, m^1, \cdots, m^n) = \sum_{i=1}^{n} x_j^{*i}(p_1, p_2, m^i).$$

From Individual to Market Demand Functions



The "horizontal sum" of the demand curves of individuals A and B.

Elasticity

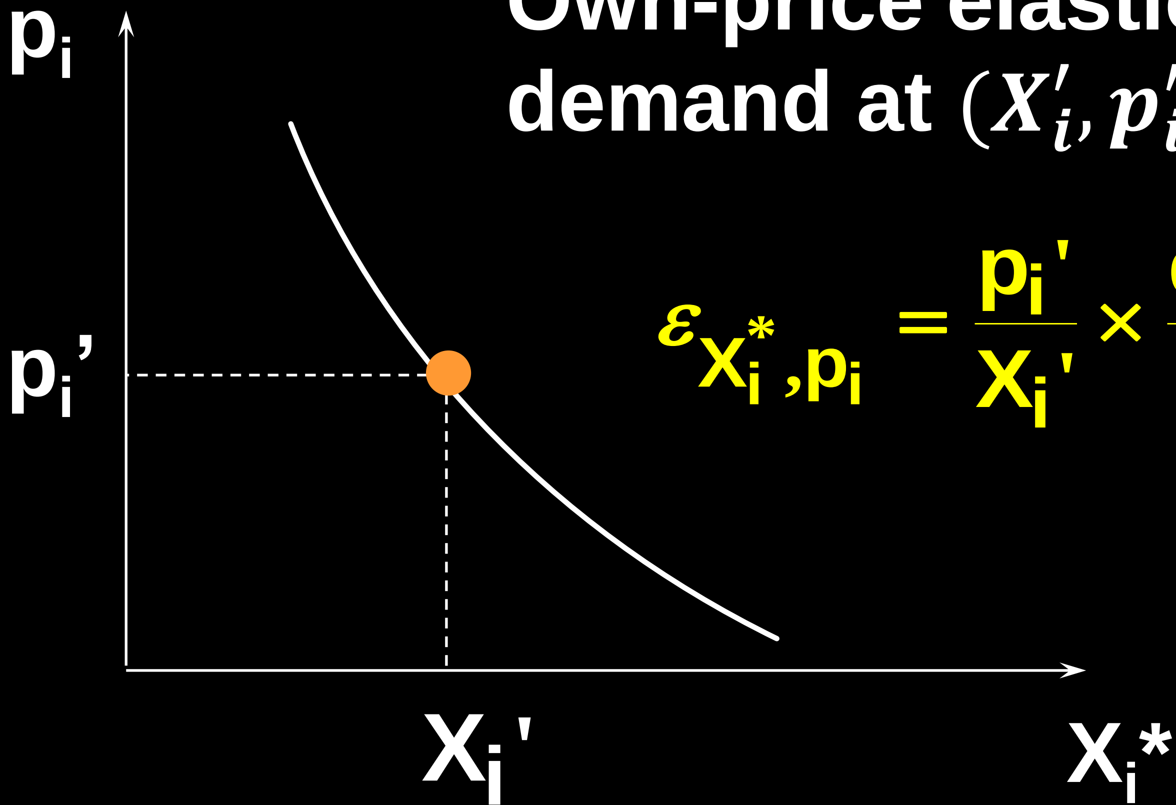
- Elasticity measures the "sensitivity" of one variable with respect to another.
- The elasticity of variable X with respect to variable Y is

$$\varepsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y}.$$

Slope Vs. Elasticity

- **Q: Why not just use the slope of a demand curve to measure the sensitivity of quantity demanded to a change in a commodity's own price?**

Own-Price Elasticity



Own-price elasticity of demand at (X_i', p_i')

$$\varepsilon_{X_i^*, p_i} = \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}(p_i')$$

Own-Price Elasticity

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

E.g. Suppose $p_i = a - bX_i$.
Then $X_i = (a-p_i)/b$ and

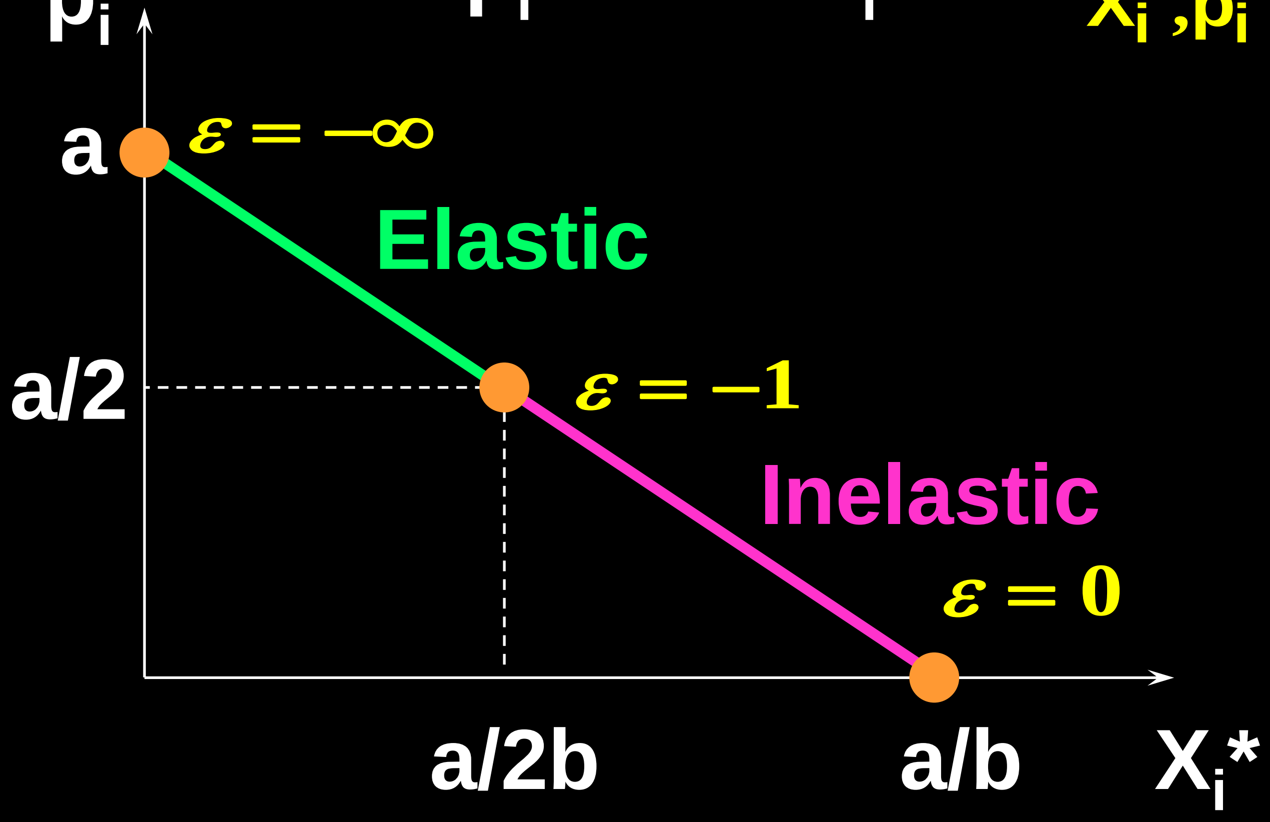
$$\frac{dX_i^*}{dp_i} = -\frac{1}{b}.$$ Therefore,

$$\varepsilon_{X_i^*, p_i} = \frac{p_i}{(a - p_i)/b} \times \left(-\frac{1}{b}\right) = -\frac{p_i}{a - p_i}.$$

Own-Price Elasticity

$p_i = a - bX_i^*$

$$\varepsilon_{X_i^*,p_i} = -\frac{p_i}{a-p_i}$$



Own-Price Elasticity

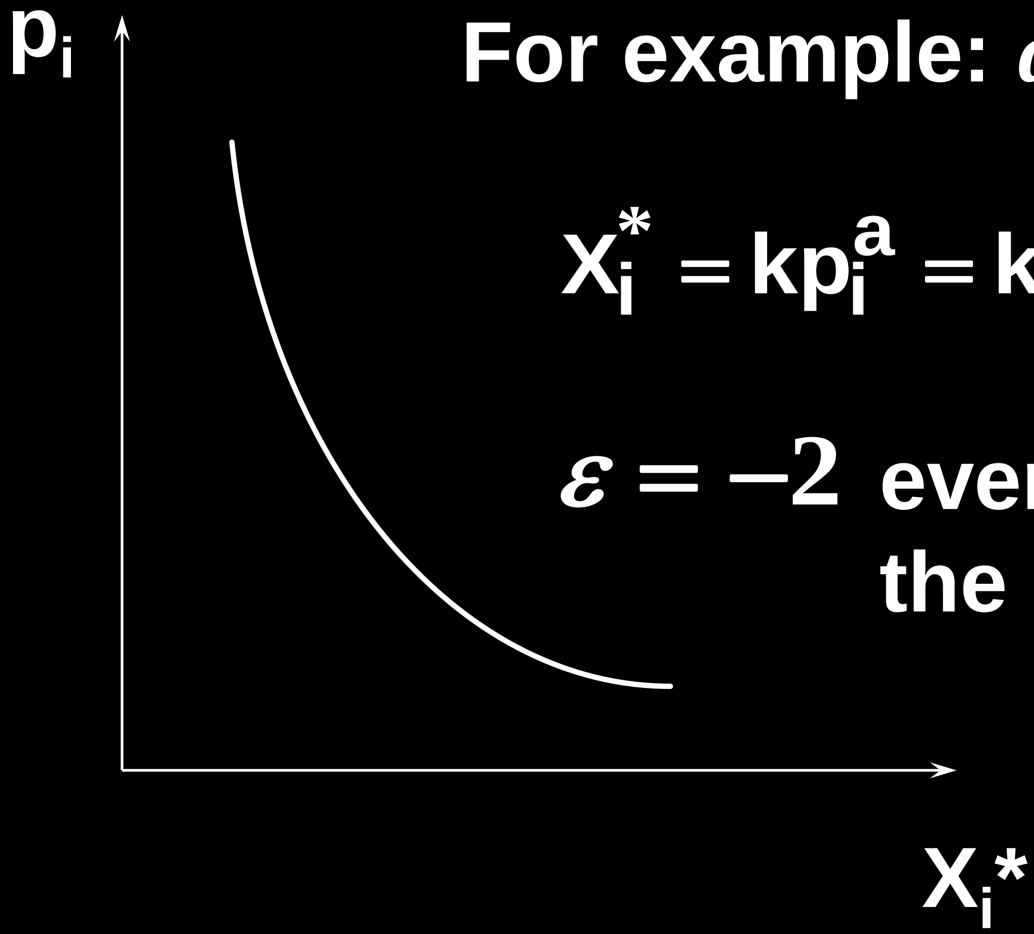
$$\varepsilon_{x_i^*, p_i} = \frac{p_i}{x_i^*} \times \frac{dx_i^*}{dp_i}$$

E.g. $x_i^* = kp_i^a$. Then $\dfrac{dx_i^*}{dp_i} = kap_i^{a-1}$

so

$$\varepsilon_{x_i^*, p_i} = \frac{p_i}{kp_i^a} \times kap_i^{a-1} = a\frac{p_i^a}{p_i^a} = a.$$

Own-Price Elasticity



For example: $a = -2$

$$X_i^* = kp_i^a = kp_i^{-2} = \frac{k}{p_i^2}$$

$\varepsilon = -2$ everywhere along the demand curve.

Revenue and Own-Price Elasticity of Demand

Monopoly's revenue is $R(p) = p \times X^*(p).$

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

$$= X^*(p)[1+\varepsilon].$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

so if $\varepsilon = -1$ then $\frac{dR}{dp} = 0$

and a small change in price at a point where $\varepsilon = -1$ does not change sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

if $\varepsilon > -1$ then $\frac{dR}{dp} > 0$

and a price increase raises sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

And if $\varepsilon < -1$ then $\frac{dR}{dp} < 0$

and a price increase reduces sellers' revenue.

Revenue and Own-Price Elasticity of Demand

- **We may also express revenue as a function of q, the quantity sold:**

$$R(q) = p(q) \times q$$

p(q): inverse demand

- **A seller's marginal revenue is the rate at which revenue changes with q.**

$$MR(q) = \frac{dR(q)}{dq}.$$

Revenue and Own-Price Elasticity of Demand

p(q): inverse demand function; i.e. the price at which the seller can sell q units. Then

$$R(q) = p(q) \times q$$

so

$$MR(q) = \frac{dR(q)}{dq} = \frac{dp(q)}{dq} q + p(q)$$

$$= p(q)\left[1 + \frac{q}{p(q)} \frac{dp(q)}{dq}\right].$$

Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{q}{p(q)}\frac{dp(q)}{dq}\right].$$

and $$\varepsilon = \frac{dq}{dp} \times \frac{p}{q}$$

so $$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right].$$

Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$

If $\varepsilon = -1$ then $MR(q) = 0$.

If $-1 < \varepsilon \leq 0$ then $MR(q) < 0$.

If $\varepsilon < -1$ then $MR(q) > 0$.

Revenue and Own-Price Elasticity of Demand

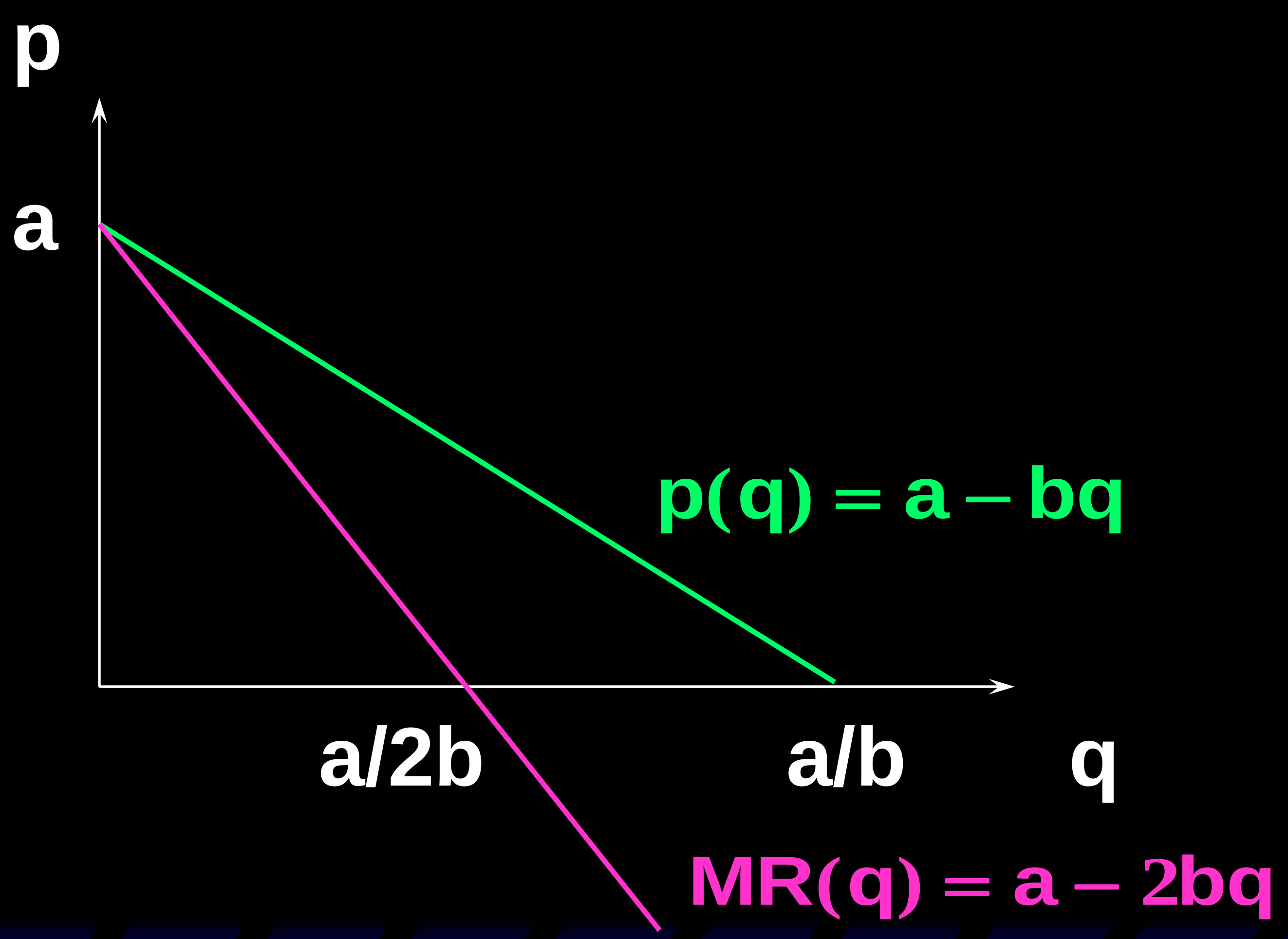
An example with linear inverse demand.

$$p(q) = a - bq.$$

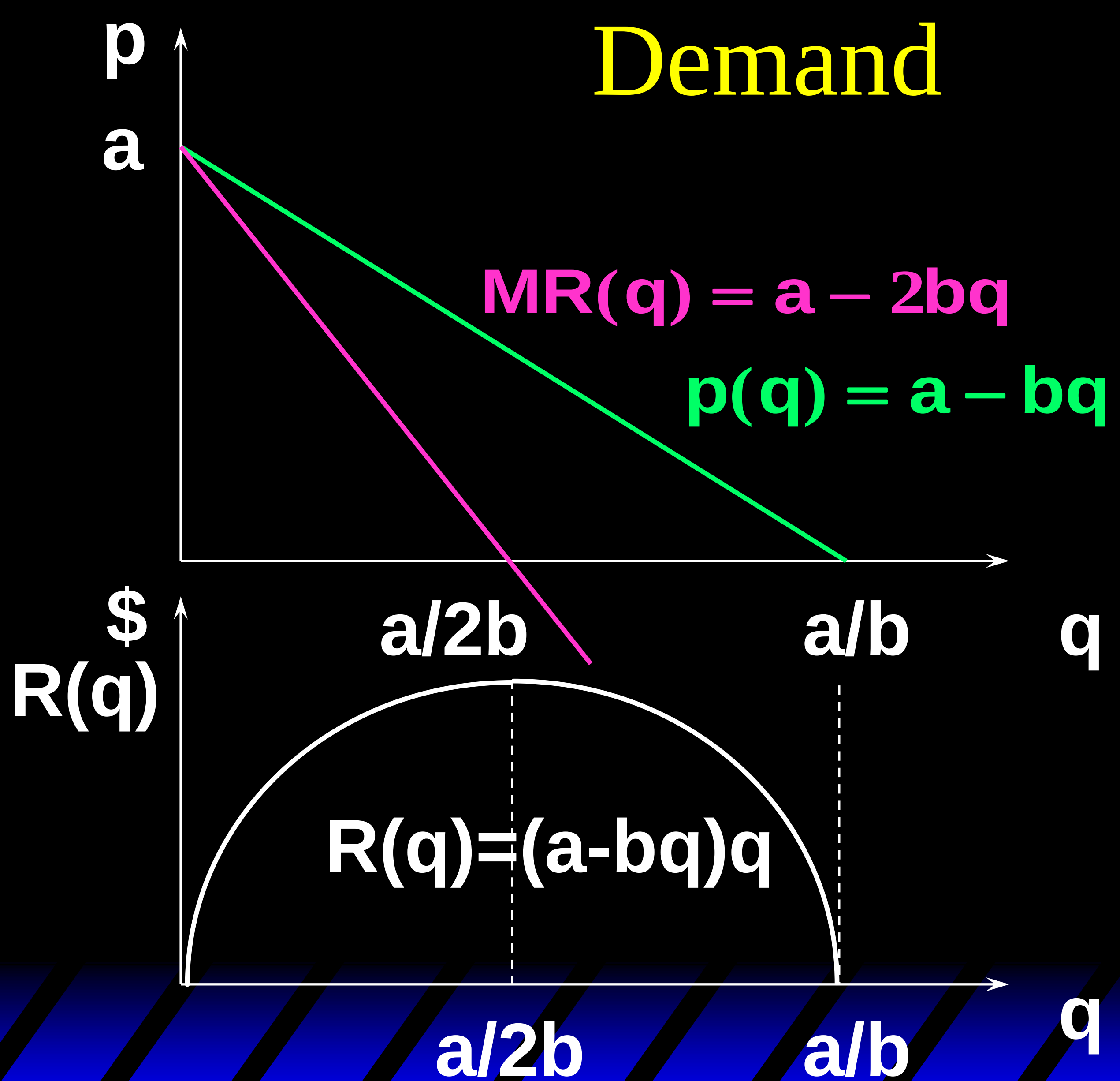
Then $R(q) = p(q)q = (a - bq)q$

and $MR(q) = a - 2bq.$

Revenue and Own-Price Elasticity of Demand



Revenue and Own-Price Elasticity of Demand



Summary: The Key Concept

- Market demand: Sum of individual demands
- Price elasticity of demand, and its relationship with:
  - Slope of demand function;
  - A monopolist’s marginal revenue (with respect to quantity).