Introduction to mathematical programming

Julien Meier

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Damn, it is about mathematics?!

Nope, it is not - at least not really if you're not delving into the theory.

This will be more about modelling of problems from practise and the theoretical basics.

What can we model?

A lot!

Let's look at some examples...

Example: Maximum flow ("easy" problem)

Problem: Maximum flow

Goal: How much flow can you push from node s to node t while complying with capacity constraints.

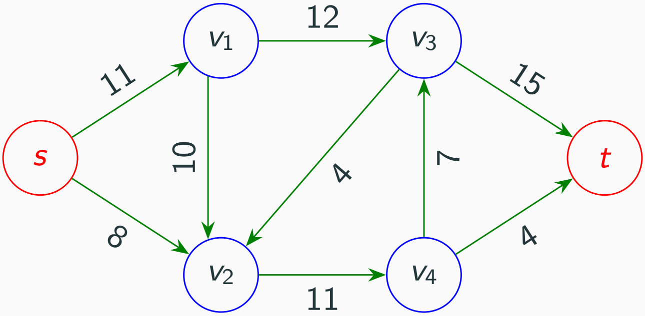

Figure 1: Example for maximum flow. Upper bound is 19 units due to the capacity constraints of the edges going into t.

Example: Minimum Vertex Cover ($\mathcal{NP}$-hard)

Problem: Minimum Vertex Cover

Goal: Find a subset of vertices that cover each edge while minimizing costs.

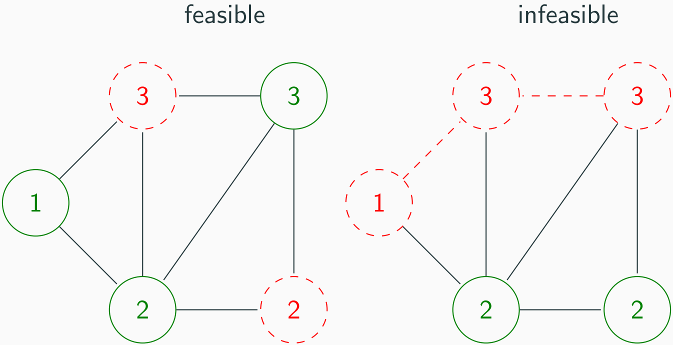

Figure 2: Example for minimum vertex cover. Numbers in nodes are associated costs. Dashed edges are not covered which renders the instance infeasible. All dashed nodes are not included in the subset.

Short "reminder" on what's $\mathcal{NP}$-hard

A problem being $\mathcal{NP}$-hard means

"probably not solvable deterministically in sub-exponential time"

$\rightarrow$ difficult problem...

More precisely, a problem B is $\mathcal{NP}$-hard if you can reduce another $\mathcal{NP}$-hard problem A to it. Basic idea: Solve A by solving B.

Reduction: $A \rightarrow B$ (formulate problem A as problem B, solve it, then derive knowledge from that)

Example: Graph coloring ($\mathcal{NP}$-hard)

Problem: Graph coloring

Goal: Color each node with adjacent nodes colored distinctly.

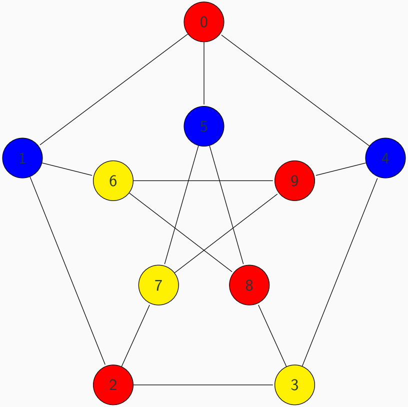

Figure 3: Example for graph coloring. Adjacent nodes are colored distinctly.

Example: Maximum Independent Set ($\mathcal{NP}$-hard)

Problem: Maximum Independent Set

Goal: Find a maximum subset of vertices that are not adjacent.

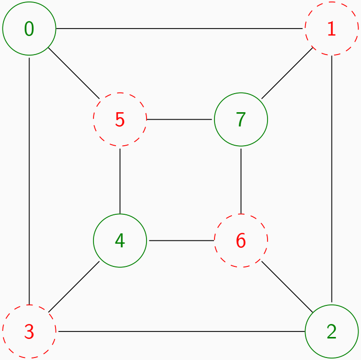

Figure 4: Example for maximum independent set. Dashed edges are excluded from the set. Note that none of the vertices included are adjacent.