#### 10.3 Deterministic RBGs Based on Number Theoretic Problems

#### 10.3.1 Discussion

A DRBG can be designed to take advantage of number theoretic problems (e.g., the discrete logarithm problem). If done correctly, such a generator's properties of randomness and/or unpredictability will be assured by the difficulty of finding a solution to that problem. Section 10.3.2 specifies a DRBG based on elliptic curves; Section 10.3.3 specifies a DRBG based on the RSA integer factorization problem.

## 10.3.2 Dual Elliptic Curve Deterministic RBG (Dual\_EC\_DRBG)

#### 10.3.2.1 Discussion

**Dual\_EC\_DRBG** (...) is based on the following hard problem, sometimes known as the "elliptic curve logarithm problem": given points P and Q on an elliptic curve of order n, find a such that Q = aP.

**Dual\_EC\_DRBG** (...) uses a seed m bits in length to initiate the generation of m-bit pseudorandom strings by performing scalar multiplications on two random points in an elliptic curve group, where the curve is defined over a field approximately  $2^m$  in size. For efficiency, m should be kept as small as possible, subject to the security strength required by the application. For all the NIST curves given in this Standard,  $m \ge 163$ . Figure 18 depicts the **Dual EC DRBG** (...).

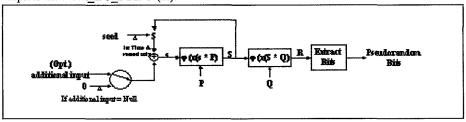


Figure 18: Dual\_EC\_DRBG (...)

The instantiation of this DRBG requires the selection of an appropriate elliptic curve and curve points specified in Annex E.4 for the desired security strength. The seed used to determine the initial value (S) of the DRBG shall have entropy that is at least the maximum of 128 and the desired security strength (i.e., entropy  $\geq$  max (128, strength). Its length shall be m bits. Further requirements for the seed are provided in Section 8.4. When optional additional input (additional\_input) is used, the value of additional\_input is arbitrary, in conformance with Section 9.8.3, but it will be hashed to an m-bit string. Figure 19 depicts the insertion of test input for the seed and the additional\_input. The tests shall be run on the output of the generator. Validation and Operational testing are discussed in Section 11. Detected errors shall result in a transition to the error state.

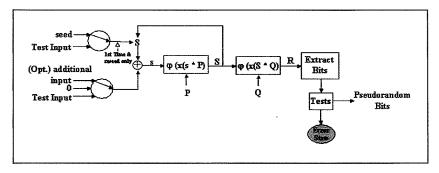


Figure 19: Dual\_EC\_DRBG (...) with Tests

10.3.2.2 Interaction with Dual\_EC\_DRBG (...)

10.3.2.2.1 Instantiating Dual\_EC\_DRBG (...)

Prior to the first request for pseudorandom bits, **Dual\_EC\_DRBG** (...) shall be instantiated using the following call:

status = Instantiate\_Hash\_DRBG ([usage\_class, ] requested\_strength, prediction\_resistance\_flag [, integer ([requested\_curve\_type] [, max\_ctr])])

as described in Section 9.6.1, with the addition of the optional requested\_curve\_type and max\_ctr parameters. requested\_curve\_type is used to specify a class of elliptic curves from which the instantiated elliptic curve is to be selected. max\_ctr indicates the maximum number of steps that may be taken along the curve before the DRBG must be seeded.

10,3.2.2.2 Reseeding a Dual\_EC\_DRBG (...) Instantiation

When a DRBG instantiation requires explicit reseeding (see Section 9.7), the DRBG shall be re-instantiated (i.e., reseeded) using the following call:

status = Reseed\_ Dual\_EC\_DRBG\_Instantiation ([usage\_class, ]

as described in Section 9.7.2.

10.3.2.2.3 Generating Pseudorandom Bits Using Dual\_EC\_DRBG (...)

An application shall request the generation of pseudorandom bits by **Dual\_EC\_DRBG** (...) using the following call:

(status, pseudorandom\_bits) = Dual\_EC\_DRBG ([usage\_class, ] requested\_no\_of\_bits, requested\_strength, additional\_input\_flag)

as described in Section 9.8.2.

10.3.2.2.4 Inserting Additional Entropy into the State Using Dual\_EC\_DRBG (...)

Additional entropy may be inserted into the state of the **Dual\_EC\_DRBG** (...) between requests for pseudorandom bits as follows:

(status) = Add\_Entropy\_to\_Dual\_EC\_DRBG ([usage\_class]) as described in Section 9.9.

### 10.3.2.3 Specifications

### 10.3.2.3.1 General

The instantiation of **Dual\_EC\_DRBG** (...) consists of selecting an appropriate elliptic curve and point pairing from Annex E.4 and obtaining a *seed* that is used to determine an initial value (S) for the DRBG that is one element of the initial *state*. The state consists of:

- (Optional) The usage\_class of the DRBG instantiation; if the DRBG is used for multiple usage\_classes, requiring multiple instantiations, then the usage\_class shall be indicated, and the implementation shall accommodate multiple states simultaneously; if the DRBG will be used for only one usage\_class, then the usage class may be omitted,
- A counter (ctr) that indicates the number of requests to Dual\_EC\_DRBG (...)
  during the current instance,
- 3. An optional *max\_counter* may be provided, which will be checked for automatic reseeding of the **Dual EC DRBG** (...),
- 4. A value (S) that is updated during each request for pseudorandom bits,
- 5. The elliptic curve domain parameters (*curve\_type*, *m*, [*p*], *a*, *b*, *n*), where curve\_type indicates a prime field F<sub>p</sub>, or a pseudorandom or Koblitz curve over the field is F<sub>2</sub><sup>m</sup>; *a* and *b* are two field elements that define the equation of the curve, and *n* is the order of the point *P*; one of the binary curve types may be requested at initialization; otherwise, the default *curve\_type* 0, indicating mod *p*, will be used,
- Two points P and Q on the curve; the generating point of the curve will be used as P.
- 7. The security *strength* provided by the instance of the DRBG; the curve will be selected to provide a maximum of *requested strength* bits of security,
- A prediction\_resistance\_flag that indicates whether or not prediction resistance is required by the DRBG, and
- 9. (Optional) A record of the seeding material in the form of a one-way function that is performed on the *seed* for later comparison with a new *seed* when the DRBG is reseeded; this value **shall** be present if the DRBG will potentially be reseeded; it **may** be omitted if the DRBG will not be reseeded.

The variables used in the description of Dual\_EC\_DRBG (...) are:

	a, b	Two field elements that define the equation of the curve.		
	additional_input	Optional additional input. A bitstring returned by		
		Get_additional_input(), a function that prompts the user to		
		supply an input. It will be hashed and truncated to $m$ bits.		
	additional_input_flag	A flag that indicates whether or not additional input may be		
		used, with values as follows:		
		0 = None requested, return $0$ .		
		1 = Request <i>additional_input</i> , but return 0 if no input is available.		
	В	The output block length of the hash function.		
	ctr	A count of the number of iterations of the of		
		Dual_EC_DRBG () since the last reseeding.		
	curve_type	Either 0,1,2 indicating a curve over a prime field, a random		
	•	binary curve, or a Koblitz curve, respectively.		

An elliptic curve defined over  $F_p$  or  $F_2^m$ Е The cofactor of the curve: 1 for all prime field curves, 2 or 4 for the binary curves. Comment: This value will be implicit from the *curve\_type* and *a*. A generating point of prime order n on the curve. GGet entropy (min entropy, min length, max length) A function that acquires a string of bits from an Approved entropy source. The parameters indicate the minimum entropy to be provided in the returned bit string, and the limits between which the length of that string must lie (i.e., min length and max length). Dual EC DRBG (....) will always specify (min length - max length) = m. A temporary value that is used as a loop counter. i Length in bits of the internal state S; the curve is defined over a field with approximately 2<sup>m</sup> elements. The maximum of the values A and B. max(A, B)The maximum number of steps taken along the curve before max ctr the DRBG must be reseeded. When counter reaches 100,000, a new seeding is recommended (see Annex D.3.2). min entropy A value used in the request to Get entropy (...) to indicate the minimum entropy to be provided. Comment: In fact, the value of strength is used in this determination, and strength is always at least requested strength. The order of the point P on the curve. old\_transformed\_ seed A record of the seed\_material used in this previous instance of the DRBG. order P The order of the point P. Note: For the NIST approved curves, the order of Q equals the order of P. The modulus when *curve* type = 0 (prime field); an *m*-bit p P, QRandom points on the elliptic curve E, such that each generates a large cyclic subgroup on E. The generating point G will be used as P. prediction\_resistance\_flag An indication of whether or not prediction resistance is to be provided by the DRBG. pseudorandom bits The pseudorandom bits produced by the DRBG. A value from which pseudorandom bits are extracted. The curve type can be specified as input to requested curve type Initialize\_Dual\_EC\_DRBG (...); if none is requested the default value of 0 is assigned. The number of pseudorandom bits to be returned on a call to requested no of bits Dual EC DRBG (...). requested stength The security *strength* of the bits requested from the DRBG. A temporary value.

A value that is initially determined by a *seed*, but assumes new values during each request of pseudorandom bits from the DRBG.

seed\_material

The seed used to derive the initial value of S.

seedlen state The length of the seed material.

The state of the DRBG that is carried between calls to the generator. In the following specifications, the entire state is

([usage\_class,] counter, max\_counter, S, curve\_type, [p], a, b, n, P, Q, strength, prediction\_resistance\_flag[, transformed\_seed]). A particular element of the state is

specified as state.element, e.g., state.S.

status

The *status* returned from a function call, where *status* = "Success" or an indication of a failure. Failure messages are:

- 1. Invalid requested strength.
- 2. Failure indication returned by the entropy source.
- 3. State not available for the indicated usage class.
- 4. Entropy source failure.
- 5. Invalid additional\_input\_flag value.
- 6. Failure from request for additional input.

strength

The maximum strength of an instance of the DRBG (i.e., 80, 112, 128, 192 or 256).

A temporary value.

temp temp\_input

A temporary value.

transformed seed

A record of the *seed\_material* used in the current instance of the DRBG.

# Truncate (bits, in len, out len)

A function that inputs a bit string of in len bits,

returning a string consisting of the leftmost *out\_len* bits of input. If *in\_len < out\_len*, the input string is padded on the right with (*out\_len - in\_len*) zeroes, and the result is

returned.

usage class

The usage\_class of a DRBG instance. This optional integer parameter may be used to differentiate instantiations of the **Dual\_EC\_DRBG** (...), e.g., when there are multiple purposes being serviced that require differing strengths.

x(A)

The x-coordinate of the point A on the curve E.

φ

A mapping from field elements to non-negative integers, which takes the bit vector representation of a field element and interprets it as the binary expansion of an integer. Section 10.3.2.2.5 includes details of this mapping. Scalar multiplication of a point on the curve.

10.3.2.2.2 Instantiation of Dual\_EC\_DRBG (...)

The following process or its equivalent shall be used to instantiate the Dual\_EC\_DRBG (...) process. Let Hash (...) be an Approved hash function for the security strengths to be supported. If the DRBG will be used for multiple security strengths, and only a single hash

function will be available, that hash function **shall** be suitable for all supported security strengths (see SP 800-57).

### Instantiate Dual EC DRBG (...):

**Input:** integer ([usage\_class,] requested\_strength, prediction\_resistance\_flag [, integer ([requested\_curve\_type] [, max\_ctr])]

Output: string status.

### Process:

1. If (requested strength > 256), then Return ("Invalid requested strength").

Comment: Determine *m* appropriate for the requested strength: this will depend on *curve type*.

2. If (requested curve type = 0), then

Comment: choose one of the prime field curves:

If (requested strength  $\leq 96$ ), then  $\{strength = 96, m = 192\}$ 

Else if  $(requested\_strength \le 112)$ , then  $\{strength = 112, m = 224\}$ 

Else if  $(requested\_strength \le 128)$ , then  $\{strength = 128, m = 256\}$ 

Else if  $(requested\_strength \le 192)$ , then  $\{strength = 192, m = 384\}$ Else if  $(requested\_strength \le 256)$ , then  $\{strength = 256, m = 521\}$ 

Comment: There is no NIST curve with m = 512.

3. If (requested curve type  $\neq$  0), then

Comment: choose one of the binary curves.

If (requested strength  $\leq 80$ ), then {strength = 80, m = 163}

Else if  $(requested\_strength \le 112)$ , then  $\{strength = 112, m = 233\}$ 

Else if (requested strength  $\leq$  128), then {strength = 128, m = 283}

Else if (requested\_strength  $\leq$  192), then {strength = 192, m = 409}

Else if (requested\_strength  $\leq$  256), then {strength = 256, m = 571.

4. Choose a suitable elliptic curve E defined over Fp based on requested curve type, where p is an m-bit prime, or  $F_2^m$  from Annex E.4.

5. Set the point P to the generator G and determine the  $order\_of\_P$ .

6. Select the appropriate Q from Annex E.4.

Comment: Request *seed\_material* of length *m* and the appropriate amount of *entropy*.

7.  $min_{entropy} = max(128, strength)$ .

8. (status, seed\_material) = Get\_entropy (min\_entropy, m, m).

 If (status = "Failure"), then Return ("Failure indication returned by the entropy source").

10. seedlen = ||seed material||.

11. (Optional) Get additional input and combine with the seed material.

11.1 (status, additional\_input) = Get\_additional\_input ().

11.2 If (status = "Failure"), then **Return** ("Failure from request for additional input").

11.3 seed\_material = seed\_material || additional\_input.

Comment [ebb1]: Page: 128
Note that we don't have an official strength of 96 bits. Should we check for a requested strength of 80 bits?

Comment [ebb2]: Page: 128 Don't see that this is explicitly used.

Comment [ebb3]: Page: 128
The same curve type as P? Any other restrictions?

Comment [ebb4]: Page: 129
Note that min\_length is not requested\_strength

Comment: Perform a one-way function on the *seed* values for later comparison

- 12. (Optional) transformed\_seed = |Hash (seed\_material).
- 13. ctr = 0.
- 14. S = Hash\_df (seed\_material, seedlen). Comment: See Section 9.6.3.2.
- 15. If max ctr not present as an input parameter, then max ctr = 0.

Comment: Setting max\_counter = 0 means that there is no maximum.

- 16. state = {[usage\_class, ] ctr, max\_ctr, S, curve\_type, m, [p], a, b, n, P, Q, strength, prediction\_resistance\_flag[, transformed\_seed]}.
- 17. Return ("Success").

## 10.3.2.2.3 Reseeding of a Dual\_EC\_DRBG (...) Instantiation

The following process or its equivalent shall be used to reseed the Dual\_EC\_DRBG (...) process, after it has been instantiated. Let Hash (...) be an Approved hash function for the security strengths to be supported.

# Reseed\_Dual\_EC\_DRBG\_Instantiation (...):

Input: integer [usage class].

Output: string status.

### **Process:**

- 1. If a *state* is not available for the indicated *usage\_class*, **Return** ("State not available for the indicated *usage\_class*").
- Get the appropriate state values for the indicated usage\_class, e.g., S = state.S, m = state.m, strength = state.strength, old\_transformed\_seed = state.transformed\_seed.
- 3. Perform steps 7-13 of Instantiate\_Dual\_EC\_DRBG (...).
  - 3.1  $min\ entropy = max\ (128, strength)$ .
  - 3.2 (status, seed\_material) = Get\_entropy (min\_entropy, m, m).
  - 3.3 If (*status* = "Failure"), then **Return** ("Failure indication returned by the entropy source").
  - 3.4 seedlen = || seed material ||.
  - 3.5 (Optional) Get additional input and combine with the seed material.
    - 3.5.1 (status, additional input) = Get\_additional\_input().
    - 3.5.2 If (status = "Failure"), then Return ("Failure from request for additional input").
    - 3.5.3 seed\_material = seed\_material || additional\_input.
  - 3.6 transformed seed = Hash (seed material).
  - 3.7 ctr = 0.
- If (transformed\_seed = old\_transformed\_seed), then Return ("Entropy source failure").
- 5.  $temp = Hash_df((S || seed_material), B)$ .
- 6. S = Truncate (temp, B, m).
- 7. Update the changed values in the state.
  - 7.1 state.S = S.
  - 7.2 state.transformed seed = transformed seed
  - 7.3 state.ctr = ctr.

Comment [ebb5]: Page: 129
Should a hash derivation function be used, or can we devise an EC derivation function?

8. Return ("Success").

# 10.3.2.2.4 Generating Pseudorandom Bits Using Dual\_EC\_DRBG (...)

The following process or its equivalent shall be used to generate pseudorandom bits. **Dual EC DRBG (...)**:

Input: integer ([usage\_class, ] requested\_no\_of\_bits, requested\_strength, additional\_input\_flag).

Output: string status, bitstring pseudorandom\_bits.

### Process:

- 1. If a *state* is not available for the indicated *usage\_class*, **Return** ("State not available for the indicated *usage\_class*", Null).
- Get the appropriate state values for the indicated usage\_class, e.g., S = state.S,
  m = state.m, strength = state.strength, P = state.P, Q = state.Q, ctr = state.ctr,
  max\_ctr = state.max\_ctr, prediction\_resistance\_flag =
  state.prediction\_resistance\_flag.
- 3. If (requested\_strength > strength), then Return ("Incorrect requested strength", Null).
- 4. If ((additional\_input\_flag < 0) or (additional\_input\_flag > 1)), then **Return** ("Invalid additional\_input\_flag value", Null).
- 5. If (additional\_input\_flag = 0), then additional\_input = 0 Comment: m zeroes Else do
  - 5.1 (status, temp\_input) = Get\_additional\_input().
  - 5.2 If (status = "Failure"), then **Return** ("Failure from request for additional input", Null).
  - 5.3 temp input = Hash (temp\_input).
  - 5.4 additional input = Truncate (temp\_input, B, m).

Comment: Determine whether reseeding is required.

- 6. temp = the Null string.
- 7. i = 0.
- 8. If  $((max_ctr > 0))$  and  $(ctr = max_ctr)$ , then
  - 8.1 status = Reseed Dual EC DRBG ([usage class]).
  - 8.2 If (status ≠ "Success"), then Return (status, Null).
- 9.  $s = S \oplus additional\_input$ . Comment: s is to be interpreted as an mbit unsigned integer. To be precise, s

bit unsigned integer. To be precise, s should be reduced mod n; the scalar \*

will affect this.

10.  $S = \varphi(x(s * P))$ . Comment: S is an m-bit number.

11.  $R = \varphi(x(S * Q))$ . Comment: R is an m-bit number. See

footnote 1.

<sup>&</sup>lt;sup>1</sup> The precise definition of  $\varphi(x)$  used in steps 12 and 13 depends on the field representation of the curve points. In keeping with the convention of FIPS 186-2, the following elements will be associated with each other:

B:  $|c_{m-1}|c_{m-2}|\dots|c_1|c_0|$ , a bitstring, with  $c_{m-1}$  being leftmost

```
12. temp = temp || R.
13. i = i + 1.
```

14. ctr = ctr + 1.

15. If (||temp|| < requested\_no\_of\_bits), then go to step 7.

16. pseudorandom bits = Truncate (temp,  $i \times B$ , requested no of bits).

17. If (prediction resistance flag = 1), then

17.1 status = Reseed Dual EC DRBG ([usage class]).

17.2 If (status ≠ "Success"), then Return (status, Null).

Else Update the changed values in the state.

17.3 state.S = S.

17.4 state.ctr = ctr.

18. Return ("Success", pseudorandom\_bits).

10.3.2.2.5 Adding Additional Entropy to Dual\_EC\_DRBG (...)

The **Dual\_EC\_DRBG** (...) may be reseeded at any time. There is also the *additional\_input* parameter that allows a bitstring to be added to the current state (*seed*) whenever **Dual\_EC\_DRBG** (...) is invoked.

# Add Entropy to Dual EC DRBG (...):

Input: integer ([usage class,] always update flag).

Output: string status.

#### Process:

 If a state for the indicated usage\_class is not available, then Return ("State not available for the indicated usage class", Null).

Get the appropriate state values for the indicated usage\_class, e.g., S = state.S,
m = state.m, strength = state.strength, P = state.P, Q = state.Q, ctr = state.ctr,
max\_ctr = state.max\_ctr, prediction\_resistance\_flag =
state.prediction\_resistance\_flag.

3.  $(status, additional \ entropy) = Get \ entropy (1, 1, inlen).$ 

4. If (*status* = "Failure"), then **Return** ("Failure from request for additional entropy").

If ((additional\_entropy = Null) and (always\_update\_flag = 0)), then Return
("No update performed").

6. Perform steps 5.3-17 of Dual\_EC\_DRBG (...).

6.1 temp\_input = **Hash** (temp\_input).

6.2 additional input = Truncate (temp input, B, m).

Comment: Determine whether reseeding is required.

```
\begin{array}{lll} Z: & c_{m-1}2^{m-1} + \ldots + c_22^2 + c_12^1 + c_0 & \in Z \; ; \\ Fa: & c_{m-1}2^{m-1} + \ldots + c_22^2 + c_12^1 + c_0 \; \bmod p \; \in \mathrm{GF}(p) \; \; ; \\ Fb: & c_{m-1}t^{m-1} \oplus \ldots \oplus c_2t^2 \oplus c_1t \oplus c_0 \; \in \mathrm{GF}(2^m) \; , \; \text{when a polynomial basis is used;} \\ Fc: & c_{m-1}\beta \; \oplus \; c_{m-2}\beta^2 \oplus c_{m-3}\beta^2 \oplus \ldots \oplus \; c_0\beta^{2^{m-1}} \in \mathrm{GF}(2^m) \; , \; \text{when a normal basis is used.} \end{array}
```

Thus, any field element x of the form Fa, Fb or Fc will be converted to the integer Z or bitstring B, and vice versa, as appropriate.

Comment [ebb6]: Page: 132
This demands input each time unless the additional input\_flag = 0. Is this what is wanted?

```
6.3 temp = the Null string.
   6.4 i = 0.
   6.5 If ((max \ ctr > 0)) and (ctr = max \ ctr)), then
         6.5.1 status = Reseed_Dual_EC_DRBG ([usage class]).
         6.5.2 If (status \neq "Success"), then Return (status, Null).
                                              Comment: s is to be interpreted as an
   6.6. s = S \oplus additional input.
                                             m-bit unsigned integer. To be precise,
                                             s should be reduced mod n; the scalar
                                              * will affect this.
   6.7 S = \varphi(x(s * P)).
                                              Comment: S is an m-bit number.
   6.8 R = \varphi(x(S * Q)).
                                             Comment: R is an m-bit number. See
                                              footnote.
   6.9 temp = temp \parallel R.
   6.10 i = i + 1.
   6.11 ctr = ctr + 1.
   6.12 If (||temp|| < requested_no_of_bits), then go to step 3.
   6.13 pseudorandom bits = Truncate (temp, i \times B, requested no of bits).
   6.14 If (prediction resistance flag = 1), then
             6.14.1 status = Reseed Dual EC DRBG ([usage class]).
             6.14.2 If (status ≠ "Success"), then Return (status, Null).
         Else Update the changed values in the state.
             6.14.3 state.S = S.
             6.14.4 state.ctr = ctr.
7. Return ("Success").
```

Comment [ebb7]: Page: 133
This demands input each time unless the additional\_input\_flag = 0. Is this what is wanted?

# 10.3.2.3.7 Implementation Considerations

[To be inserted]

# 10.3.2.3 Generator Strength and Attributes

The particular curve used is based on *strength*, which is selected from one of five security levels and is always at least *requested\_strength*. The curves and associated security levels are those given in Section 1.2 of FIPS 186-2; they are meant to correspond to the strengths of various standard symmetric encryption algorithms.

There are three curves associated with each security level, one defined over a prime field GF(p) and two over a binary field  $GF(2^m)$ , where  $2^m \approx p$ . The mod p curves, assigned curve\_type 0, are used by default. Any of these three curves may be used for the security level.

Initial seeding is accomplished with a call to **Get\_entropy(...)**, which returns a bitstring of a specified length and entropy. The **Dual\_EC\_DRBG (...)** specifies **max** (128, *strength*) bits of entropy.

## 10.3.2.4 Reseeding and Rekeying

The reseeding process is covered in 10.3.2.2.3. Reseeding may be performed "automatically", by using a nonzero value for <code>max\_ctr</code>. Alternatively, or in addition, a call to <code>Reseed\_Dual\_EC\_DRBG\_Instantiation(...)</code> can be made at any time.

The <code>Dual\_EC\_DRBG</code> (...) is not keyed per se; however, the <code>additional\_input</code> feature may be used to effect keying, if desired.