

$$T: a \longleftarrow a + b, \quad b \longleftarrow a$$

$$T_{pq}: a \longleftarrow bq + aq + ap, \quad b \longleftarrow bp + aq$$

$$T_{pq}(a, b) = ((bq + aq + ap), (bp + aq))$$

$$T_{pq}(T_{pq}(a, b))$$

$$= T_{pq}((bq + aq + ap), (bp + aq))$$

$$= (((bp + aq)q + (bq + aq + ap)q + (bq + aq + ap)p), \\ ((bp + aq)p + (bq + aq + ap)q))$$

$$= ((bpq + aq^2 + bq^2 + aq^2 + apq + bpq + apq + ap^2), \\ (bp^2 + apq + bq^2 + aq^2 + apq))$$

$$= ((2bpq + 2aq^2 + 2apq + bq^2 + ap^2), \\ (2apq + aq^2 + bq^2 + bp^2))$$

$$= ((b(2pq + q^2) + a(2pq + q^2) + a(p^2 + q^2)), \\ (b(q^2 + p^2) + a(2pq + q^2))) \quad \textcircled{1}$$

$$\text{If } p' = p^2 + q^2 \quad \& \quad q' = q^2 + 2pq$$

$$\text{then } T_{pq}(T_{pq}(a, b)) = ((bq' + aq' + ap'), (bp' + aq'))$$

$$\text{and } T_{p'q'}(a, b) = ((bq' + aq' + ap'), (bp' + aq')) \\ = \textcircled{1}$$