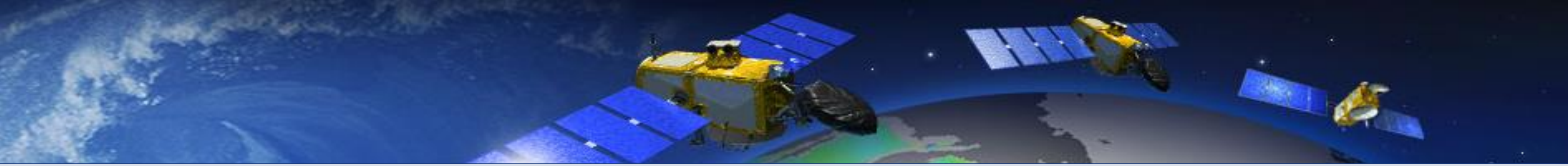


Orbit Mechanics Tutorial

Hao Ye

yehao0809@gmail.com

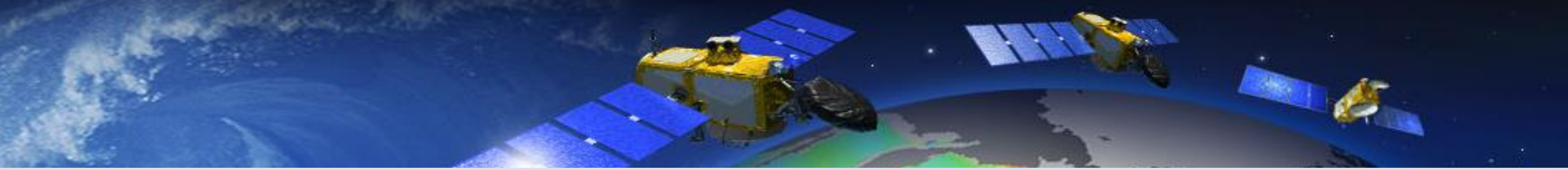
Directed by Prof. Urs Hugentobler

Three tutorials in total

1st --Keplerian Orbits (analytical way)

2rd --Numerical Integration of Satellite Orbits (numerical way)

3rd --Orbits Integration with Different Force Models(software)



Six Keplerian orbital elements

--- semimajor axis α

--- numerical eccentricity e

Shape of orbit

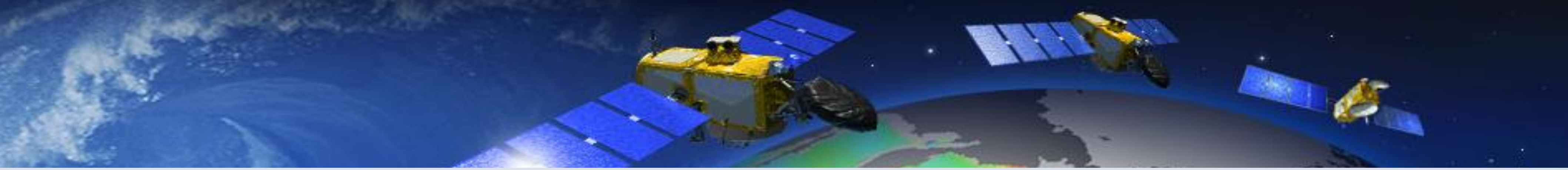
--- Inclination i

--- Right ascension Ω of the ascending node

Orientation of orbital plane

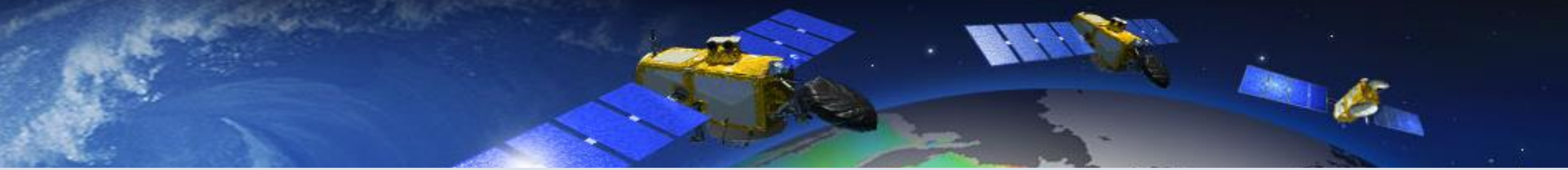
--- Argument of perigee ω

--- Perigee passing time T_0



Six Keplerian orbital elements

Satellite	a[km]	e	i [deg]	Ω [deg]	ω [deg]
GOCE	6629	0.004	96.6	257.7	144.2
GPS	26560	0.01	55	60	0
MOLNIYA	26554	0.7	63	245	270
GEO	Geostationary	0	0	0	0
MICHIBIKI	Geosynchronous	0.075	41	195	270



Main tasks

1. Keplerian elements $\Longrightarrow$ Orbits in 2D plane

- 2D orbit
- Mean anomaly, eccentric anomaly and true anomaly

2. 2D plane $\Longrightarrow$ 3D trajectory in space-fixed system

3. Orbit in space-fixed system $\Longrightarrow$ Orbit in earth-fixed system

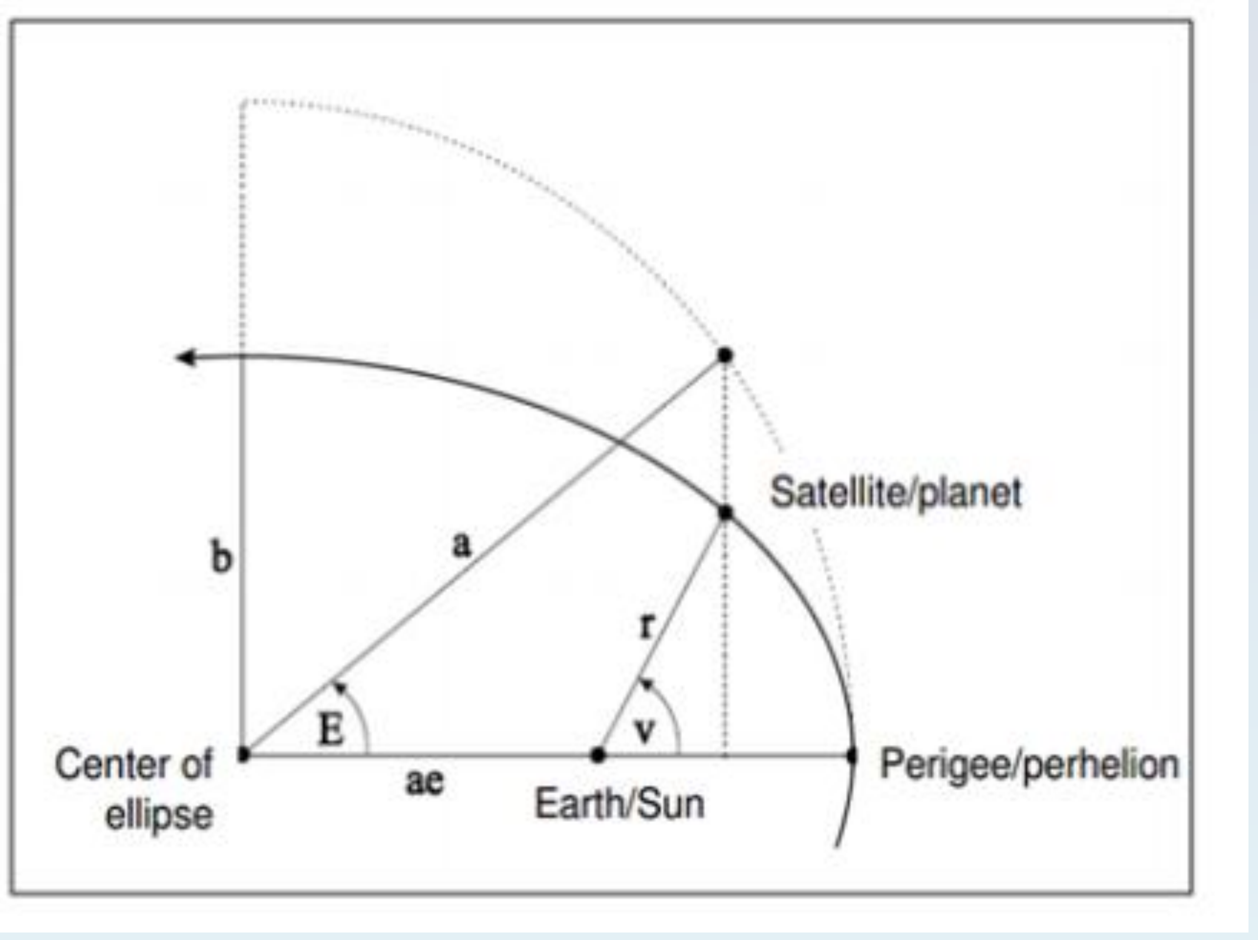
- 3D orbit
- Tracks on the ground

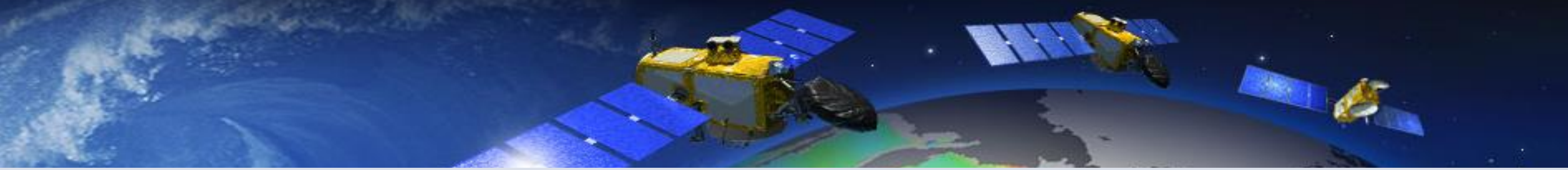
4. Orbit in earth-fixed system $\Longrightarrow$ Orbit in topo-centric system

- Azimuth and elevation orbit
- Judge visibility

Task I

Keplerian elements $\Longrightarrow$ Orbits in 2D plane





Task I

Keplerian elements $\Longrightarrow$ Orbits in 2D plane

$$n = \sqrt{\frac{GM}{a^3}} \qquad M(\mathrm{t}) = \mathrm{n} \cdot (\mathrm{t} - \mathrm{T_0})$$

Mean anomaly

$\Downarrow$

Kepler's equation: $M = E - e\sin E \quad \Delta E_i < 10^{-6}$

Eccentric anomaly

$\Downarrow$

$$r = a(1 - e\cos\mathrm{E}) \qquad \tan\frac{v}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

Radius

True anomaly

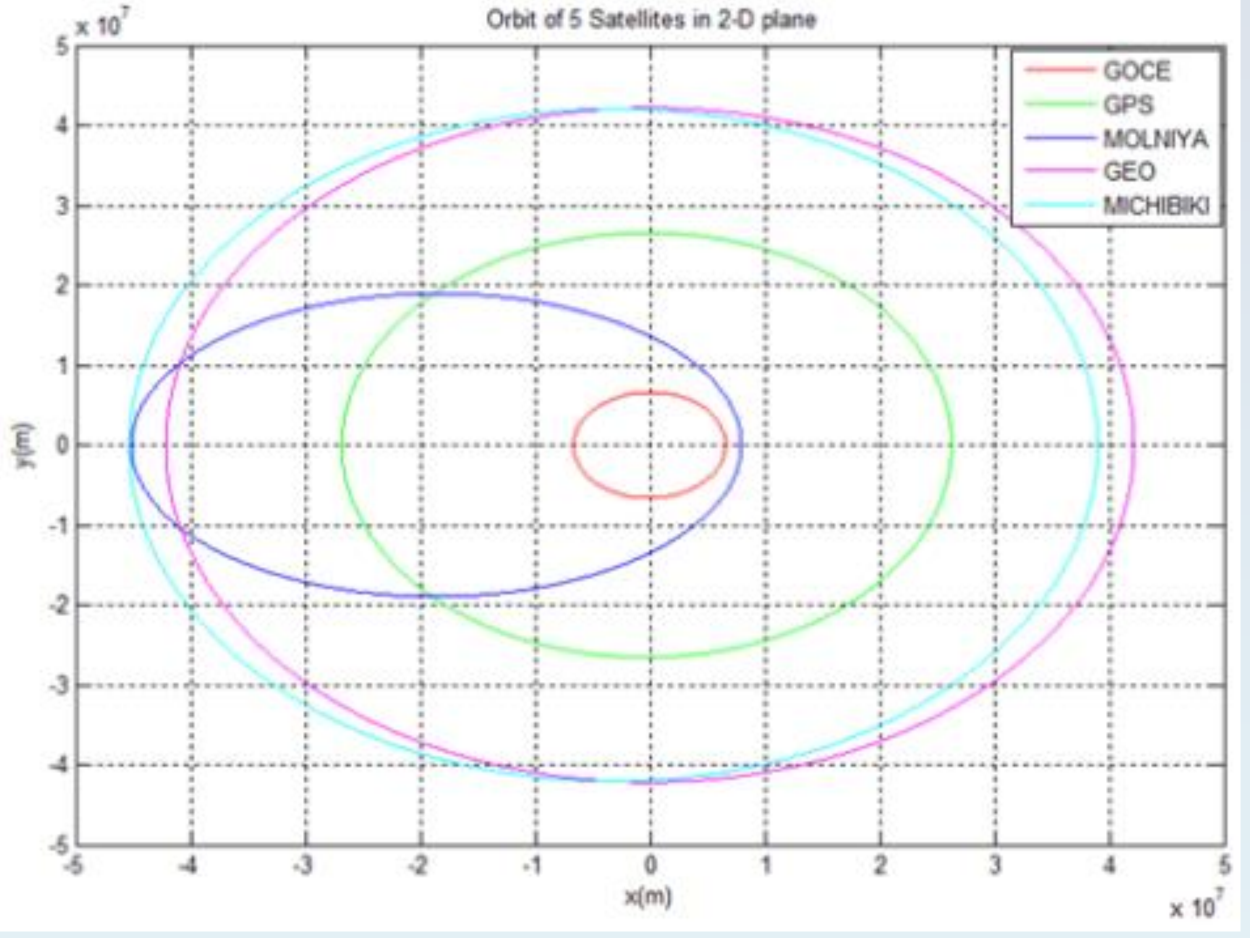
$\Downarrow$

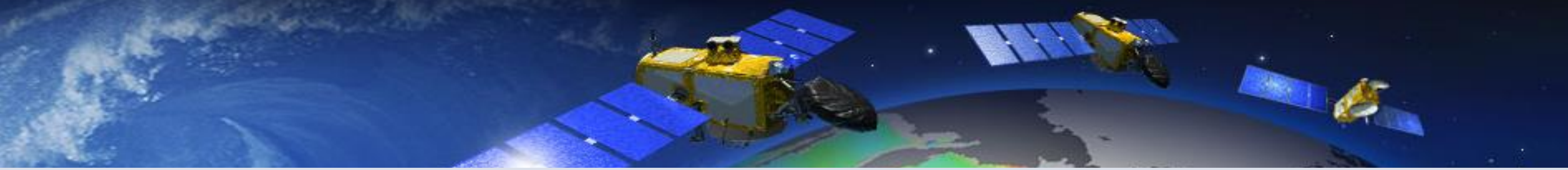
$$x = r\cos v \qquad y = r\sin v$$

2D coordinates

Task I

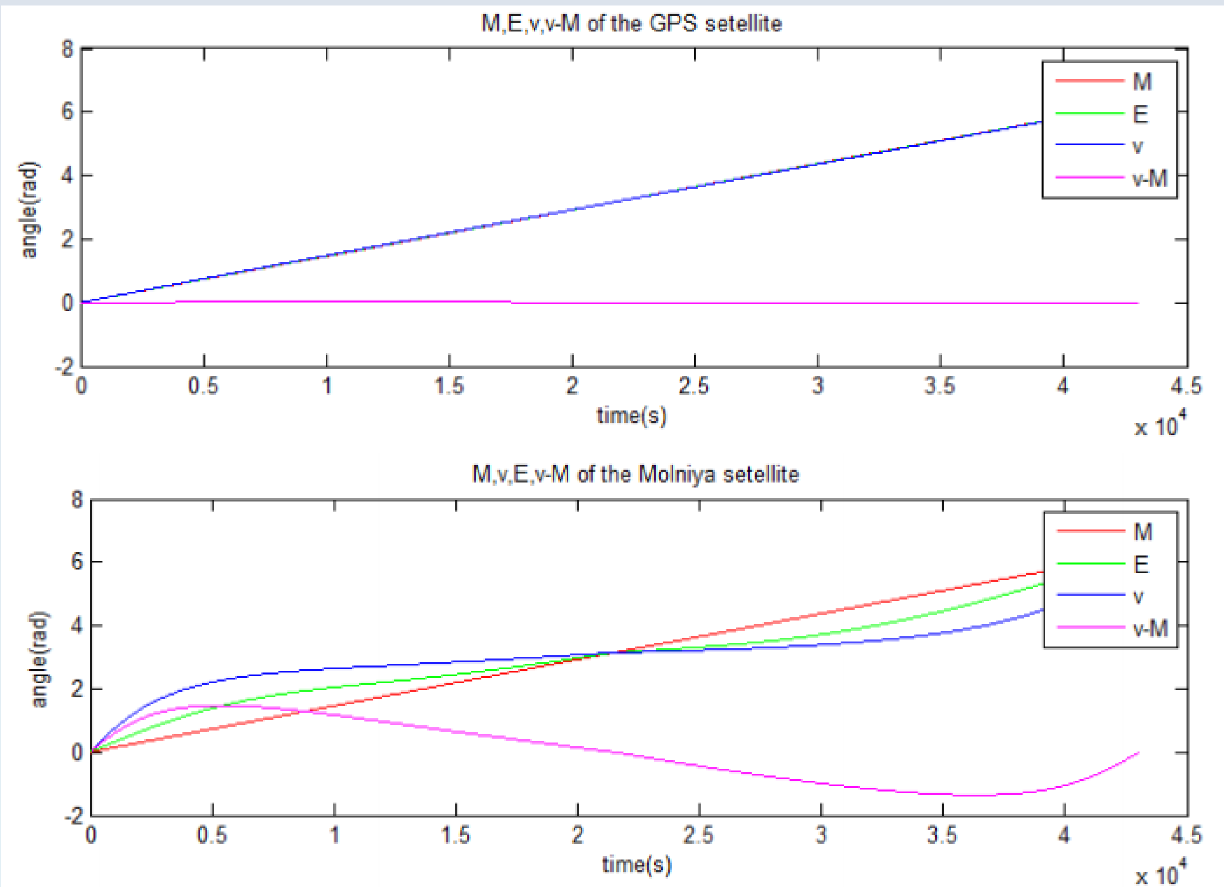
Keplerian elements ⟹ Orbits in 2D plane





Task I

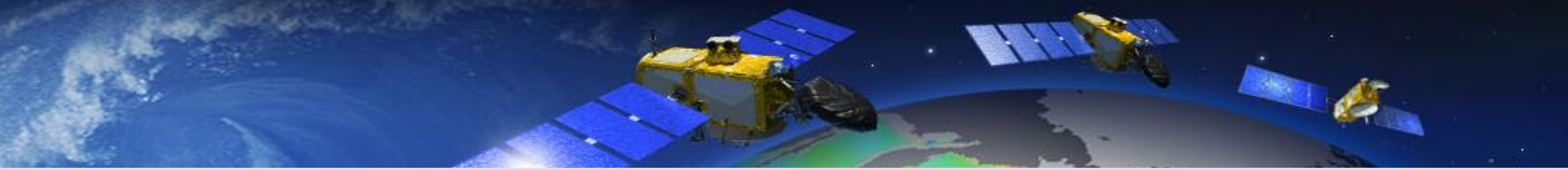
Keplerian elements $\Longrightarrow$ Orbits in 2D plane



Mean anomaly

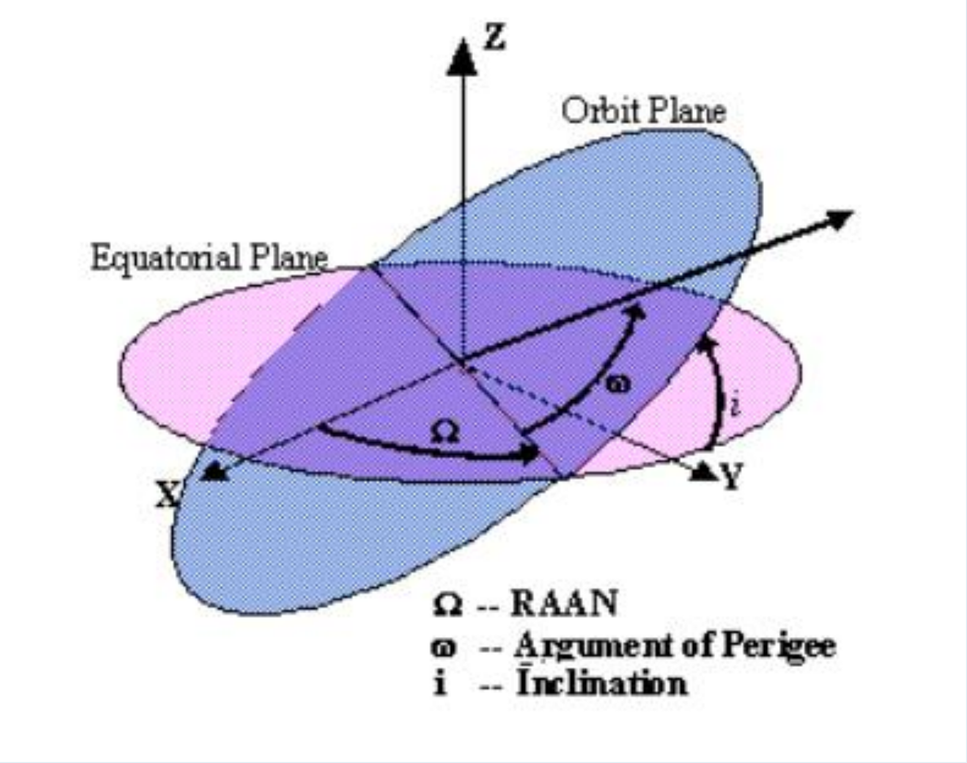
Eccentric anomaly

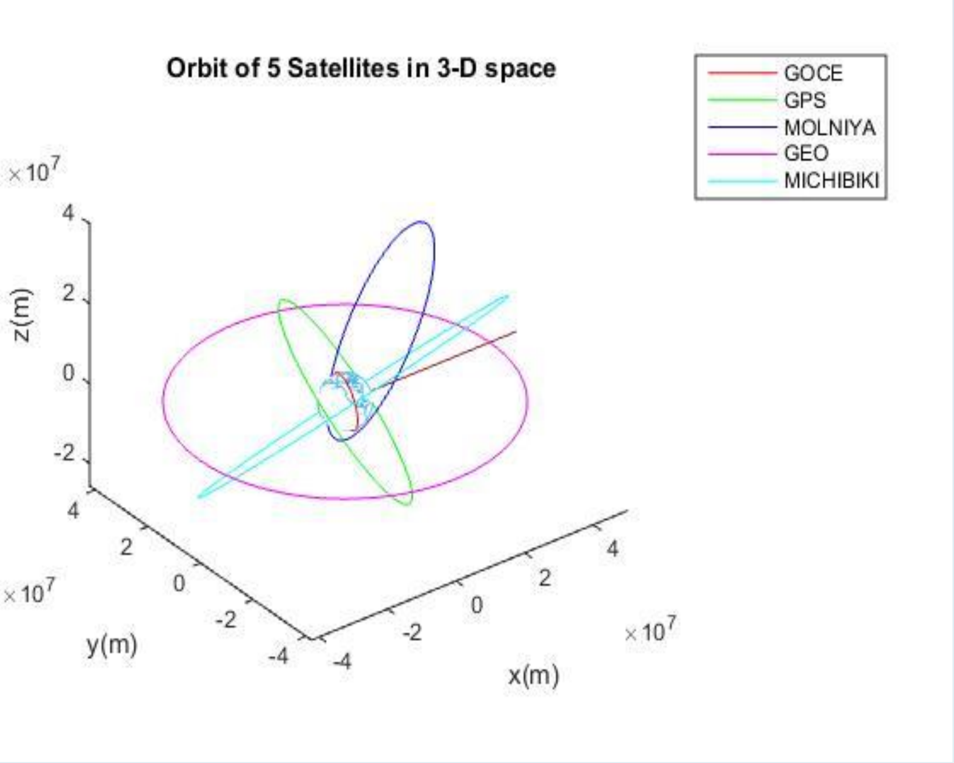
True anomaly

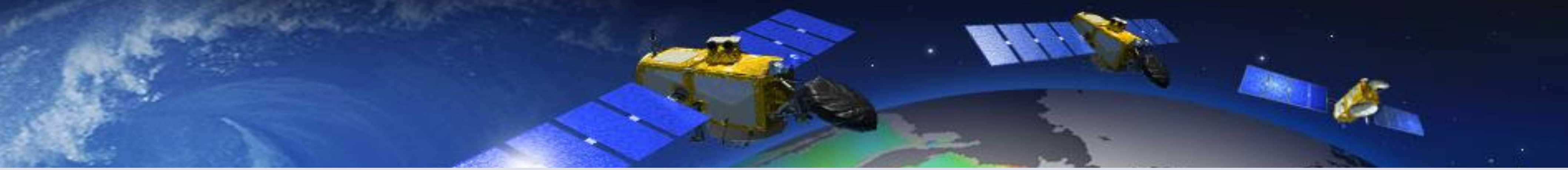


Task II

2D plane ⟹ 3D plane in space-fixed system







Task II

2D plane $\Longrightarrow$ 3D plane in space-fixed system

$$\vec{r}_b = r(\cos\nu, \sin\nu, 0)$$

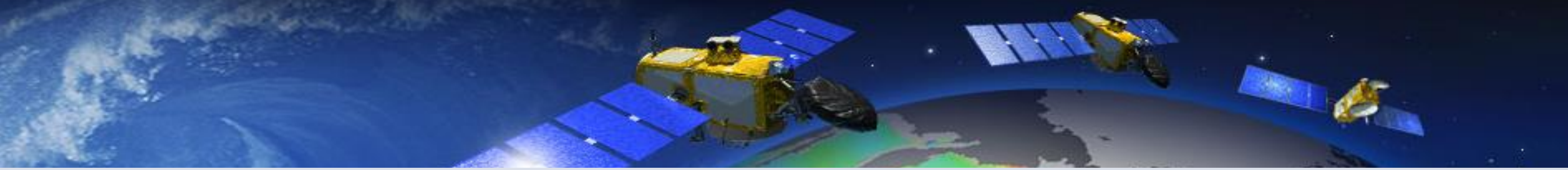
$$\dot{\vec{r}}_b = \sqrt{\frac{GM}{a(1-e^2)}}(-\sin\nu, e+\cos\nu, 0)$$

rotation $\Longrightarrow$

$$\vec{r} = R_3(-\Omega)R_1(-i)R_3(-\omega)\vec{r}_b$$

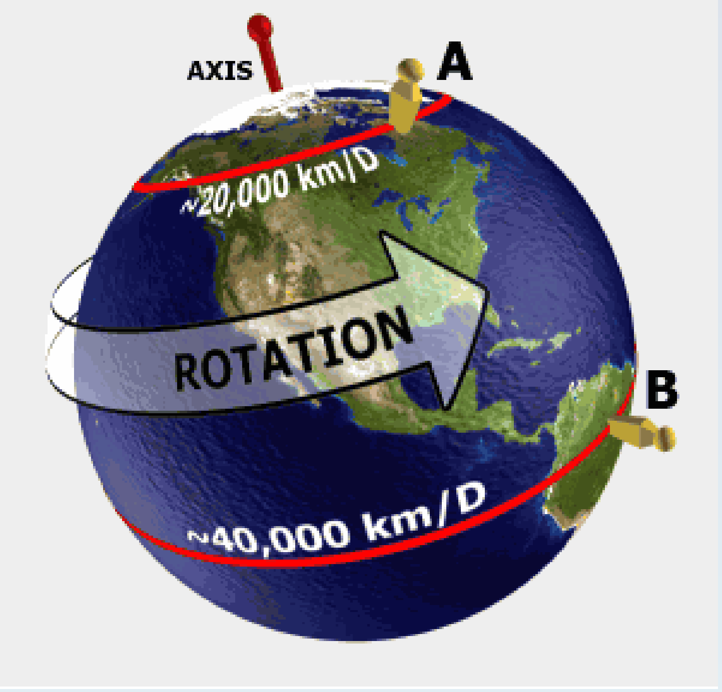
$$\dot{\vec{r}} = R_3(-\Omega)R_1(-i)R_3(-\omega)\dot{\vec{r}}_b$$

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \quad R_2(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_3(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Task III

Space-fixed system $\Longrightarrow$ Earth-fixed system

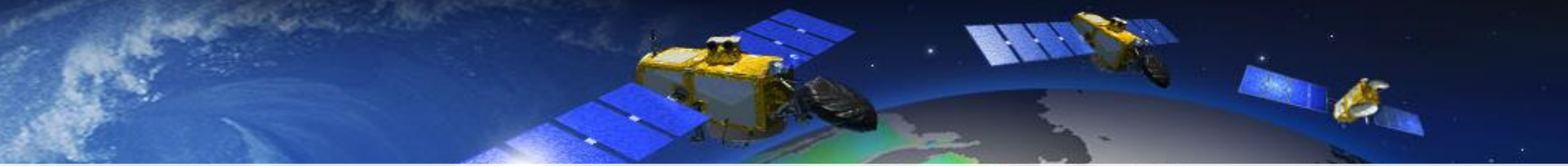


Rotating rate: $\dot{\Omega}_E = \dfrac{2\pi}{86164s}$

Rotating angle: $\theta_0(\mathrm{t}) = \dot{\Omega}_E t$

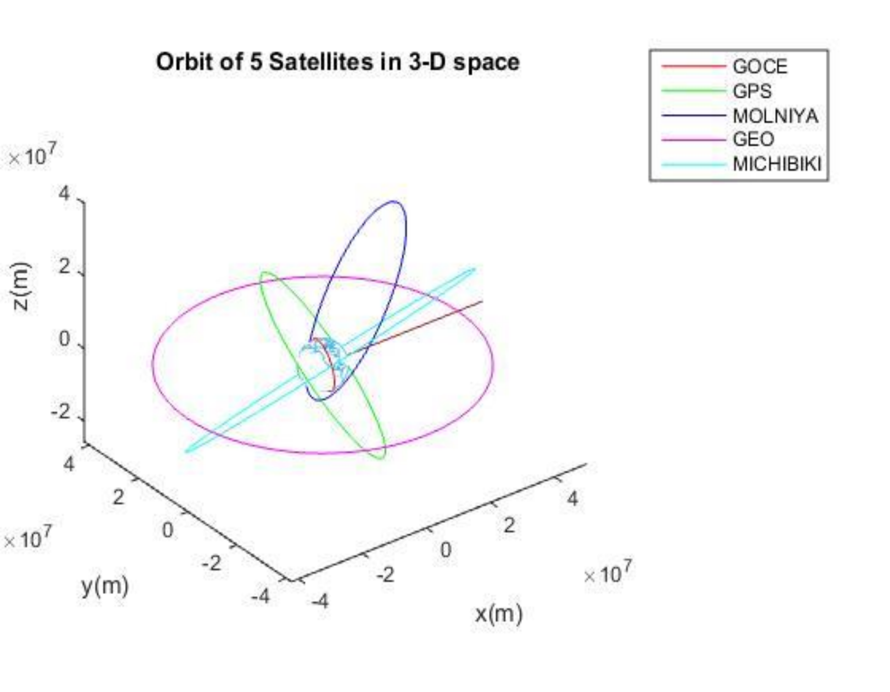
$$\boxed{\vec{r}_{Earth-fixed}(\mathrm{t}) = \mathrm{R}_3(\theta_0(\mathrm{t})) \cdot \vec{r}_{Space-fixed}(\mathrm{t})}$$

$$R_3(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

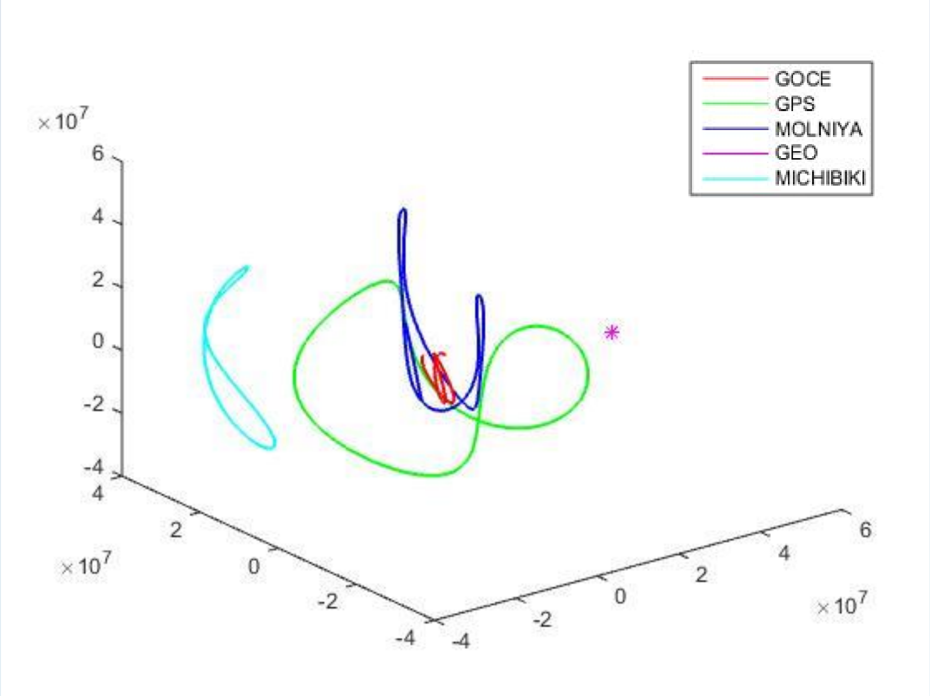


Task III

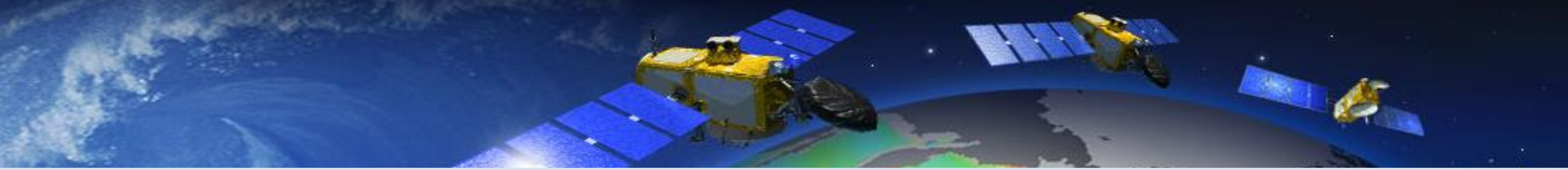
Space-fixed system ⟹ Earth-fixed system



Space-fixed system



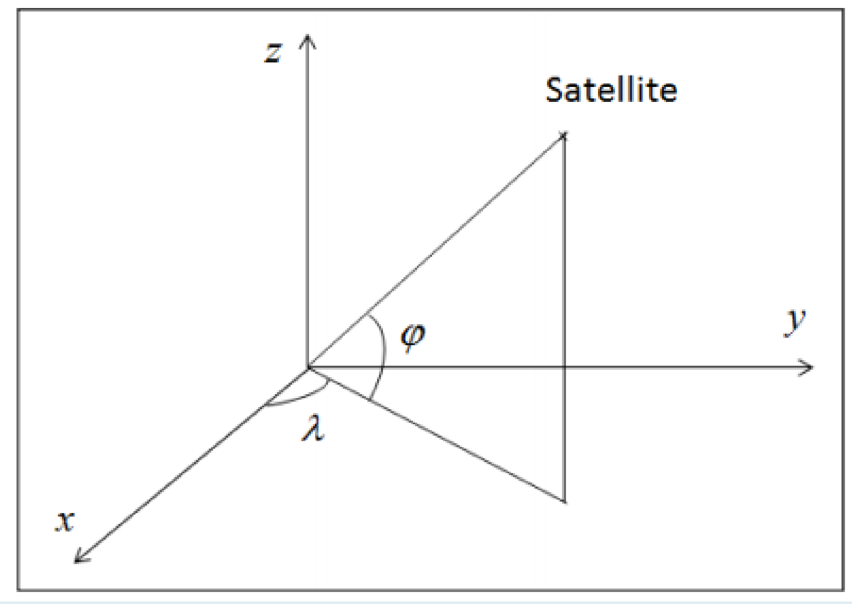
Earth-fixed system



Task III

Space-fixed system $\Longrightarrow$ Earth-fixed system

Ground-tracks on the Earth surface



Latitude

$$\tan\lambda = \frac{y_{Earth-fixed}}{x_{Earth-fixed}}$$

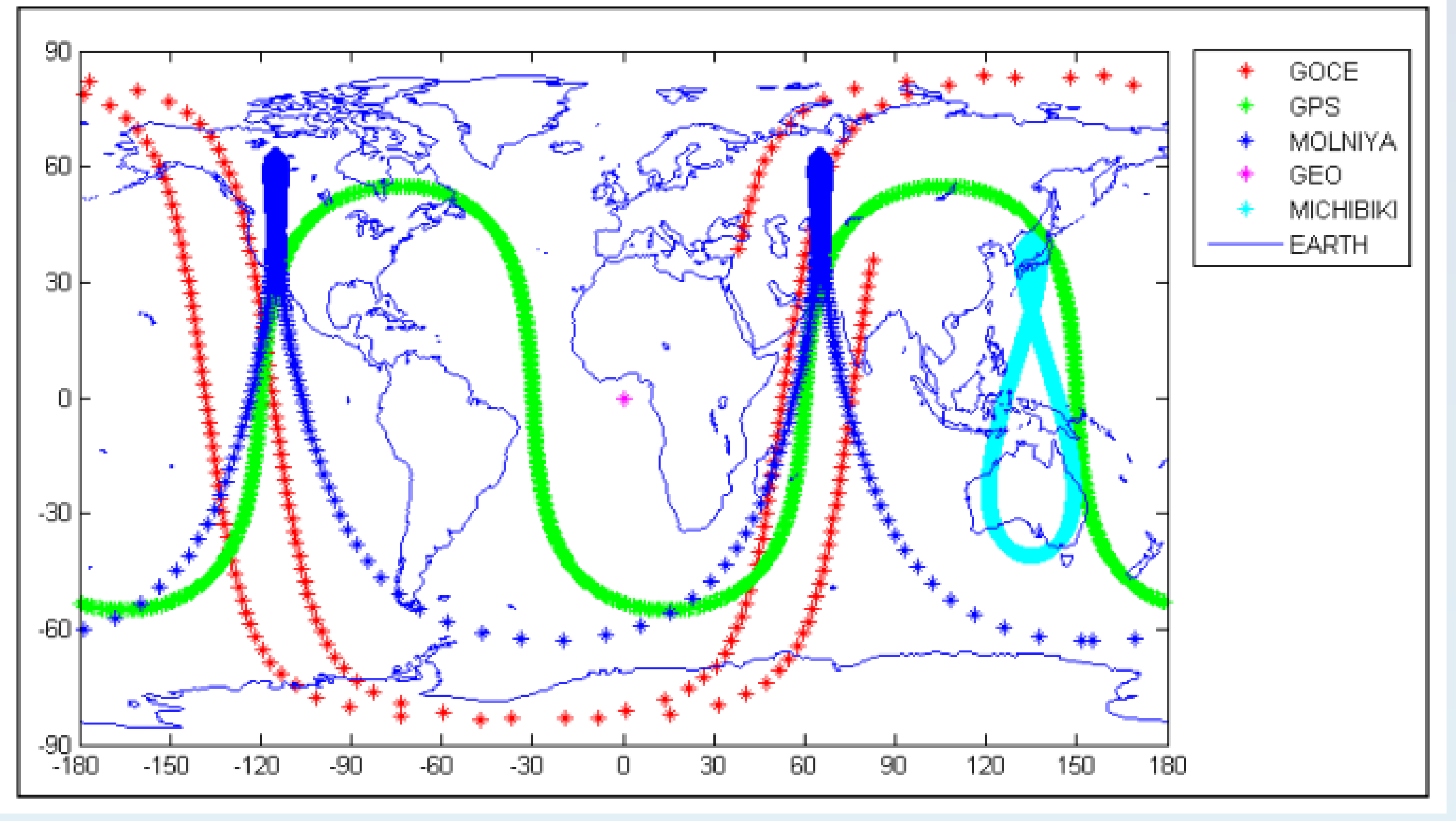
Longitude

$$\tan\psi = \frac{z_{Earth-fixed}}{\sqrt{x^2_{Earth-fixed} + y^2_{Earth-fixed}}}$$

Task III

Space-fixed system ⟹ Earth-fixed system

Ground-tracks on the Earth surface



Task IV

Earth-fixed system $\Longrightarrow$ Topocentric system

Wettzell

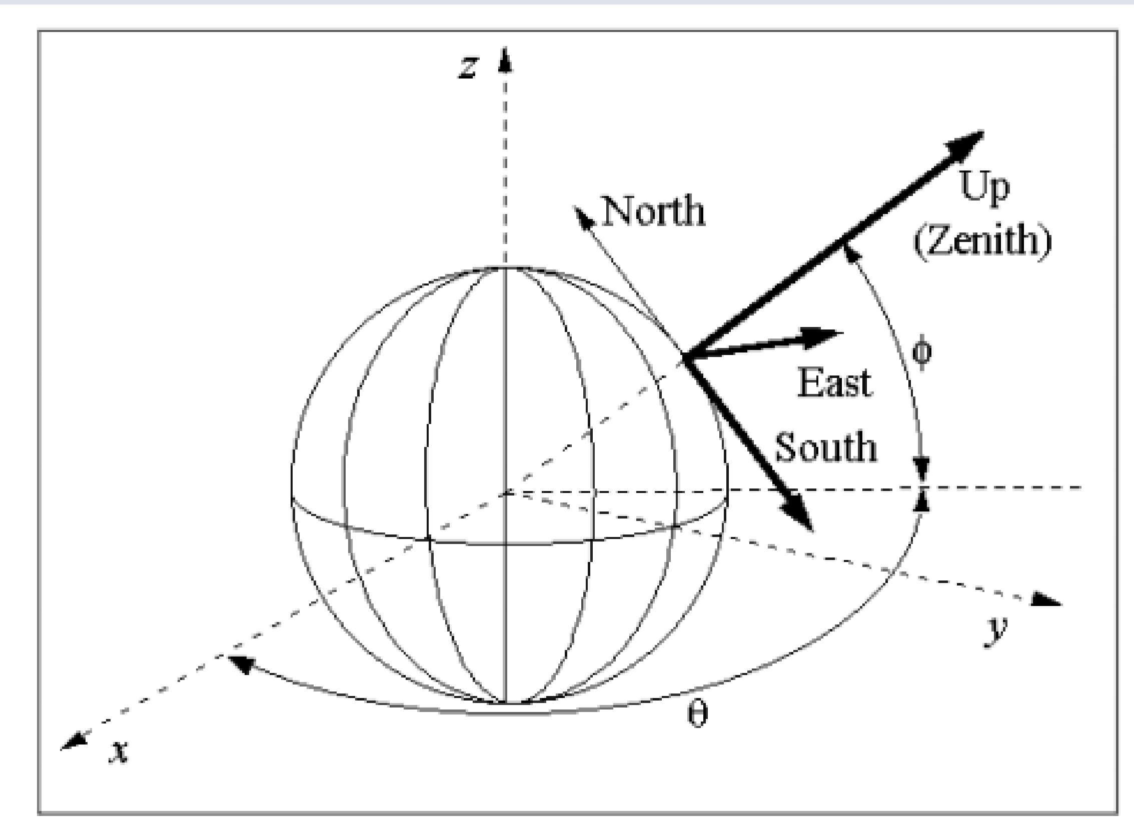
$(4075.53022, 931.78130, 4801.61819)^T$

Key point:

You should move the center of the coordinate system to the Wettzell

Task IV

Earth-fixed system $\Longrightarrow$ Topocentric system



Transformation

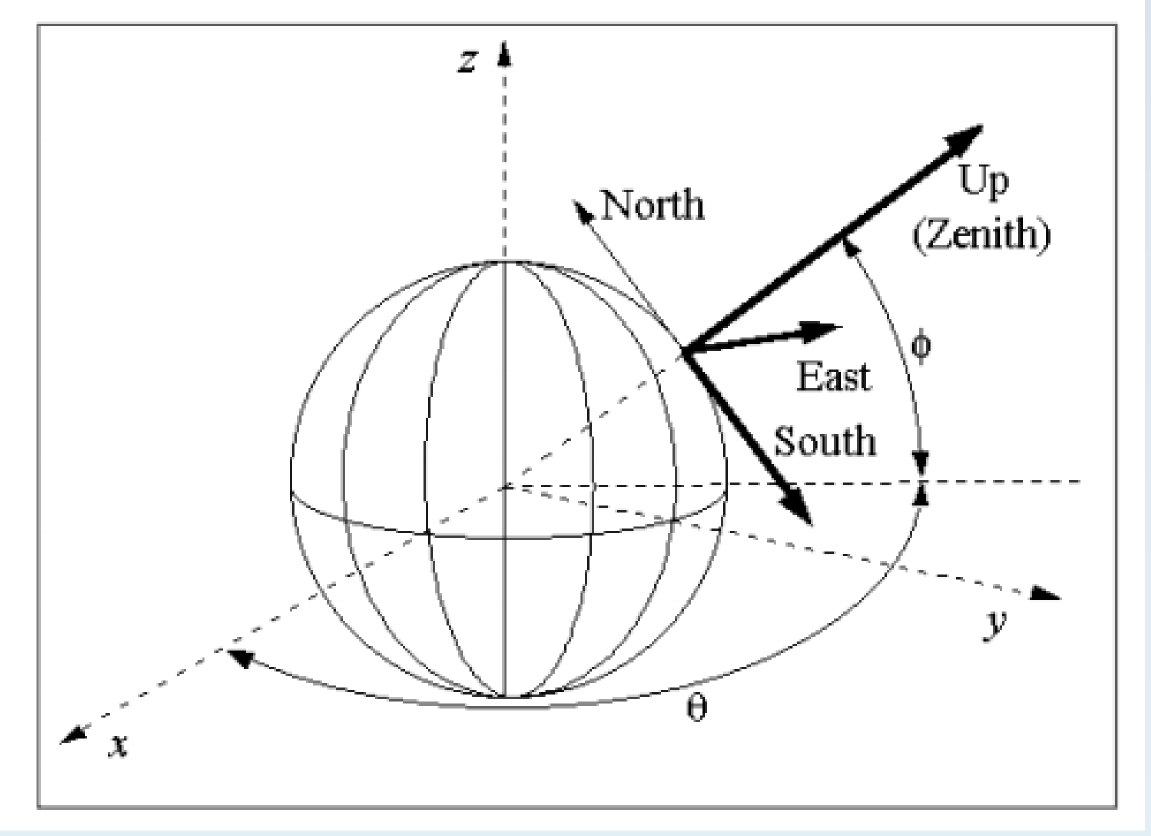
$$\boxed{\boldsymbol{d} = \boldsymbol{R} - \boldsymbol{r}}$$

Rotation

$$\boxed{\boldsymbol{d_{hori}} = Q_1 R_2(90 - \varphi) R_3(\lambda) \boldsymbol{d}}$$

Task IV

Earth-fixed system ⟹ Topocentric system



Transformation

$$\boxed{\boldsymbol{d} = \boldsymbol{R} - \boldsymbol{r}}$$

Rotation

$$\boxed{\boldsymbol{d_{hori}} = Q_1 R_2(90 - \varphi) R_3(\lambda) \boldsymbol{d}}$$

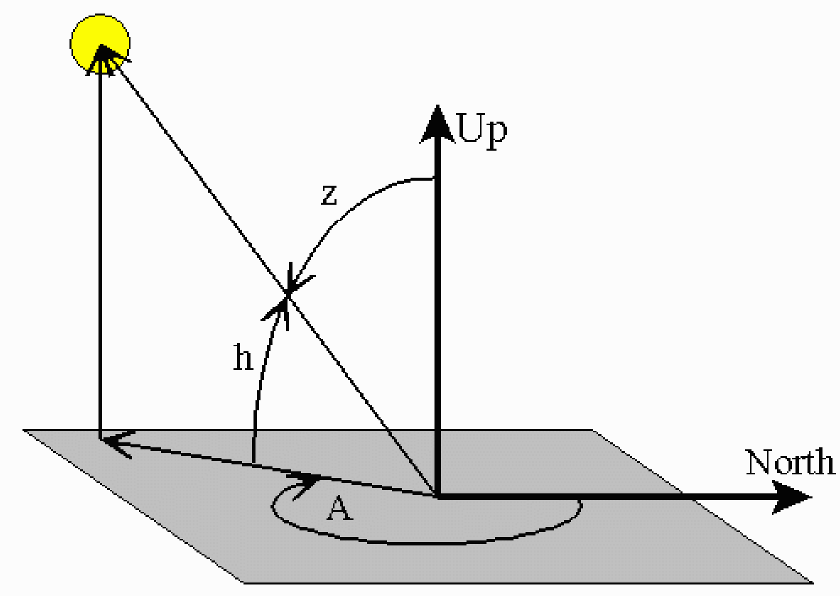
Notice:

$$Q_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Left hand --->right hand

Task IV

Earth-fixed system $\Longrightarrow$ Topocentric system



h = elevation angle, measured up from horizon

z = zenith angle, measured from vertical

A = Azimuth angle, measured clockwise from North

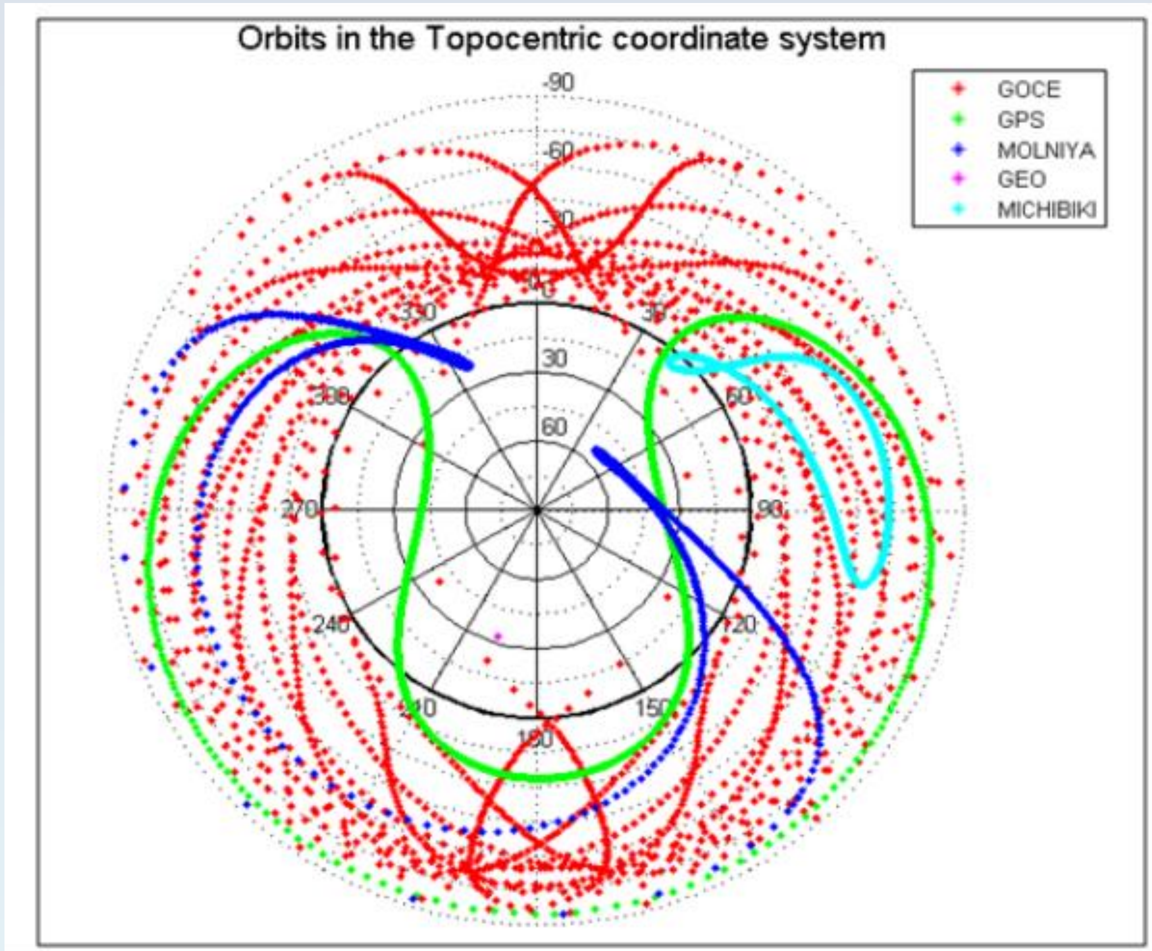
$$\tan A = \frac{y}{x}$$

$$\cos z = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$E = \frac{\pi}{2} - z$$

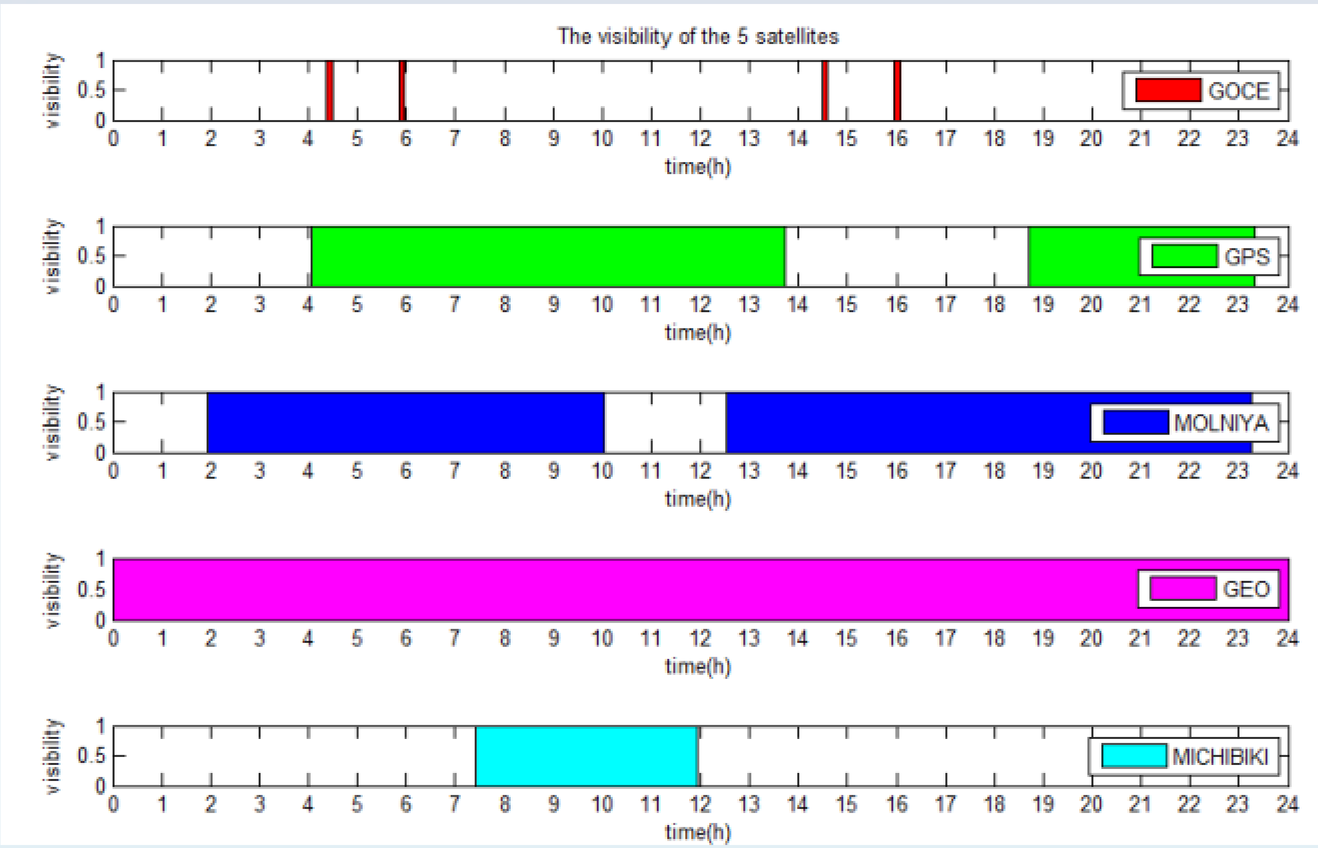
Task IV

Earth-fixed system ⟹ Topocentric system



Task IV

Earth-fixed system ⟹ Topocentric system



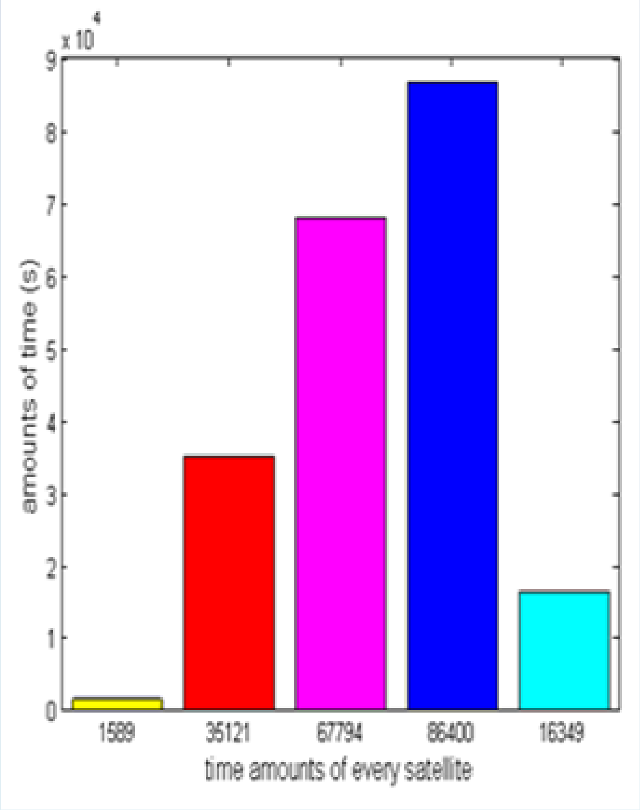
Judge the Visibility

Rule:

Elevation angle>0---->visible

Task IV

Earth-fixed system ⟹ Topocentric system

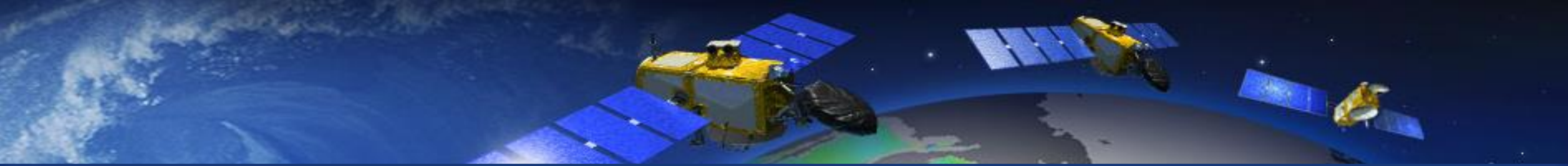


Satellites	GOCE	GPS	MOLNIYA	GEO	MICHIBIKI
Visibility time/s	1589	35121	67794	86400	16349
Percentage/%	1.84	40.65	78.47	100	18.96

Or some other formats you like……

Tips in programming

- Avoid using loop
- Avoid putting all the code in one function and trying to design the structure of your task
- Check your input parameters and typing errors
- Try to use the debugging of MATLAB and be patient
- No hesitation to get help from you tutor



Thank you for your attention !