Homework 5 — due March 28 @ 11:5pm

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Assumptions & previous knowledge:

$\underline{X}_1, \underline{X}_2 \ldots \underline{X}_n \sim$ some p variate dist.
$\underline{Y}_1, \underline{Y}_2 \quad \underline{Y}_m \sim$ some p variate dist.
} these two dist are ind.

$\underline{\lambda} = (\lambda_1 \cdots \lambda_p)$

Box Cox Transformation:

$U_i^{(\lambda)} = \psi(X_i, \lambda)$

$\psi(\underline{w}; \underline{\lambda}) \equiv (\psi(w_1; \lambda_1), \psi(w_2, \lambda_2) \ldots \psi(w_p, \lambda_p))$

$V_i^{(\lambda)} = \psi(Y_i, \lambda)$

$$\psi(w_j; \lambda_j) = \begin{cases} \frac{w_j^{\lambda_j} - 1}{\lambda_j} & \lambda_j \neq 0 \\ \log w_j & \lambda_j = 0 \end{cases}$$

- $U_1^{(\lambda)}, U_2^{(\lambda)}, \ldots U_n^{(\lambda)} \sim N_p(\mu(\lambda), \Sigma_{(\lambda)})$

&

- $V_1^{(\lambda)}, V_2^{(\lambda)} \ldots V_m^{(\lambda)} \sim N_p(\nu(\lambda), \Sigma_{(\lambda)})$

J_x = Jacobian matrix of $U_i^{(\lambda)} = \psi(X_i, \lambda)$
J_y = " " of $V_j^{(\lambda)} = \psi(Y_i, \lambda)$

$$J_x = \prod_{i=1}^{n} \prod_{k=1}^{p} X_{ik}^{\lambda_k - 1} \qquad J_y = \prod_{j=1}^{m} \prod_{k=1}^{p} Y_{jk}^{\lambda_j - 1}$$

Define the Likelihood:

$$L(\underline{\lambda},\underline{\mu},\underline{v},\Sigma;\underline{X},\underline{Y}) = f(\underline{X},\underline{Y})$$ ← here we know that $\underline{X}$ & $\underline{Y}$ are from ind. dist.

$$= f(\underline{X})\cdot f(\underline{Y})$$ ← We don't know $\underline{X}$ & $\underline{Y}$'s dist, but we know $\underline{U}, \underline{V}$

$$= f(\underline{U})\cdot|J_X|\cdot f(\underline{V})\cdot|J_Y|$$ ← we know that $\underline{U}_1, \underline{U}_2 \dots \underline{U}_n$ are ind. obs.

$$= \prod_{i=1}^{n} f(\underline{U}_i)\cdot|J_{x_i}| \cdot \prod_{j=1}^{m} f(\underline{V}_j)\cdot|J_{y_j}|$$ ← here U_i & V_i follow a $MVN(\underline{\mu},\Sigma)$ & $MVN(\underline{V},\Sigma)$, respectively

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Here I will solve for $\prod_{i=1}^{n} f(U_i)\cdot|J_i|$, it will be similar for V_j & $|J_j|$

$$= \prod_{i=1}^{n} f(U_i)\cdot|J_i| = \prod_{i=1}^{n}\left(\frac{1}{(2\pi)^{p/2}\cdot|\Sigma|^{1/2}}\exp\left\{-\frac{1}{2}(\underline{U}_i-\underline{\mu})^T\Sigma^{-1}(\underline{U}_i-\underline{\mu})\right\}\cdot|J_i|\right.$$

Distribute the product:

$$= \frac{1}{(2\pi)^{np/2}\cdot|\Sigma|^{n/2}}\exp\left\{-\frac{1}{2}\sum_{i=1}^{n}(\underline{U}_i-\underline{\mu})^T\Sigma^{-1}(\underline{U}_i-\underline{\mu})\right\}\cdot\prod_{i=1}^{n}|J_{x_i}|$$

$$= \frac{1}{(2\pi)^{np/2}\cdot|\Sigma|^{n/2}}\exp\left\{-\frac{1}{2}\sum_{i=1}^{n}(\underline{U}_i-\underline{\mu})^T\Sigma^{-1}(\underline{U}_i-\underline{\mu})\right\}\cdot\prod_{i=1}^{n}(x_i^{\lambda-1}),$$

Plug everything in before taking the log...

$$L(\underline{\lambda},\underline{\mu},\underline{v},\Sigma;\underline{X},\underline{Y}) = \frac{1}{(2\pi)^{\left(\frac{m+n}{2}\right)\cdot p}|\Sigma|^{\frac{n+m}{2}}}\exp\left\{-\frac{1}{2}\sum_{i=1}^{n}(\underline{U}_i-\underline{\mu})^T\Sigma^{-1}(\underline{U}_i-\underline{\mu}) - \frac{1}{2}\sum_{j=1}^{m}(\underline{V}_j-\underline{v})^T\Sigma^{-1}(\underline{V}_j\right.$$

$$\times\prod_{i=1}^{n}\prod_{k=1}^{p} x_{ik}^{\lambda_k-1}\times\prod_{j=1}^{m}\prod_{k=1}^{p} y_{jk}^{\lambda_k-1}$$

When we take natural log...

$$= \left(-\frac{m+n}{2}\right)\cdot p\ln(2\pi) + \left(\frac{n+m}{2}\right)\ln(|\Sigma|) + -\frac{1}{2}\sum_{i=1}^{n}(\underline{U}_i-\underline{\mu})^T\Sigma^{-1}(\underline{U}_i-\underline{\mu}) - \frac{1}{2}\sum_{j=1}^{m}(\underline{V}_j-\underline{v})^T$$

$$\Sigma^{-1}\cdot(\underline{V}_j-\underline{v}) + \sum_{k=1}^{p}(\lambda_k-1)\left[\sum_{i=1}^{n}\ln x_{ki} + \sum_{j=1}^{m}\ln y_{kj}\right]$$

$$\ell(\underline{\lambda}, \underline{\mu}, \underline{\nu}, \Sigma ; \underline{X}, \underline{Y}) = \overbrace{-\frac{(m+n)p}{2}\ln(2\pi)}^{\text{Constant}} + \left(\frac{n+m}{2}\right)\ln|\Sigma| - \frac{1}{2}\sum_{i=1}^{n}(\underline{U}_i - \underline{\mu})^T \Sigma^{-1} \cdot (\underline{U}_i - \underline{\mu}) -$$

$$-\frac{1}{2}\sum_{j=1}^{m}(\underline{V}_j - \underline{\nu})^T \cdot \Sigma^{-1} \cdot (\underline{V}_j - \underline{\nu}) + (\underline{\lambda} - 1)\sum_{j=1}^{n} \ln(x_i)$$

$$+ (\lambda_1 - 1)\left[\sum_{i=1}^{n} \ln x_{1i} + \sum_{j=1}^{m} \ln y_{1j}\right] + (\lambda_2 - 1)\left[\sum_{i=1}^{n} \ln x_{2i} + \sum_{j=1}^{m} \ln y_{2j}\right]$$

$$+ \dots + (\lambda_p - 1)\left[\sum_{i=1}^{n} \ln x_{pi} + \sum_{j=1}^{m} \ln y_{pj}\right]$$

First find the MLE for $\underline{\mu}, \underline{\nu}, \underline{\Sigma}$

$$\frac{d}{d\underline{\mu}}\left(\ell(\underline{\lambda}, \underline{\mu}, \underline{\nu}, \Sigma ; \underline{X}, \underline{Y})\right) = 0 + 0 + -\frac{1}{2}\cdot \Sigma^{-1}\sum_{i=1}^{n} -2(\underline{U}_i - \underline{\mu}) + 0 + 0 + 0 + 0 + 0$$

$$0 = \Sigma^{-1}\sum_{i=1}^{n}(\underline{U}_i - \hat{\underline{\mu}})$$

$$0 = \sum_{i=1}^{n} \underline{U}_i - n\hat{\underline{\mu}}$$

$$n\hat{\underline{\mu}} = \sum_{i=1}^{n} \underline{U}_i$$

$$\boxed{\hat{\underline{\mu}} = \frac{1}{n}\sum_{i=1}^{n} \underline{U}_i} \quad \text{Similarly} \dots \quad \boxed{\hat{\underline{\nu}} = \frac{1}{m}\sum_{j=1}^{m} \underline{V}_j}$$

$$\frac{d}{d\Sigma}\left(\ell(\underline{\lambda}, \underline{\mu}, \underline{\nu}, \Sigma ; \underline{X}, \underline{Y})\right) = 0 + -\frac{n}{2}\Sigma^{-1} + -\frac{1}{2}\sum_{i=1}^{n}\left(-\Sigma^{-1}\cdot(\underline{U}_i - \underline{\mu})\cdot(\underline{U}_i - \underline{\mu})^T \cdot \Sigma^{-1}\right) + 0$$

$$+ 0 + -\frac{m}{2}\Sigma^{-1} + -\frac{1}{2}\sum_{j=1}^{m}\left(-\Sigma^{-1}\cdot(\underline{V}_j - \underline{\nu})\cdot(\underline{V}_j - \underline{\nu})^T \cdot \Sigma^{-1}\right) + 0$$

$$= -\frac{n}{2}\Sigma^{-1} + -\frac{m}{2}\Sigma^{-1} + +\frac{1}{2}\cdot +\Sigma^{-1} \cdot \Sigma^{-1}\sum_{i=1}^{n}(\underline{U}_i - \underline{\mu})(\underline{U}_i - \underline{\mu})^T + +\frac{1}{2}\cdot \Sigma^{-1}\cdot\Sigma^{-1}\sum_{j=1}^{m}(\underline{V}_j - \underline{\nu})(\underline{V}_j$$

$$0 = \hat{\Sigma}^{-1}\left(-\frac{n}{2} + -\frac{m}{2} + \frac{1}{2}\cdot\hat{\Sigma}^{-1}\sum_{i=1}^{n}(\underline{U}_i - \underline{\mu})(\underline{U}_i - \underline{\mu})^T + \frac{1}{2}\cdot\hat{\Sigma}^{-1}\sum_{j=1}^{m}(\underline{V}_j - \underline{\nu})(\underline{V}_j - \underline{\nu})^T\right)$$

$$0 = -\frac{(n+m)}{2} + \frac{1}{2}\cdot\hat{\Sigma}^{-1}\sum_{i=1}^{n}(\underline{U}_i - \underline{\mu})(\underline{U}_i - \underline{\mu})^T + \frac{1}{2}\cdot\hat{\Sigma}^{-1}\sum_{j=1}^{m}(\underline{V}_j - \underline{\nu})(\underline{V}_j - \underline{\nu})^T$$

$$0 = -(n+m) + \hat{\Sigma}^{-1}\sum_{i=1}^{n}(\underline{U}_i - \underline{\mu})(\underline{U}_i - \underline{\mu})^T + \hat{\Sigma}^{-1}\sum_{j=1}^{m}(\underline{V}_j - \underline{\nu})(\underline{V}_j - \underline{\nu})^T$$

$$\hat{\Sigma}\cdot n+m = \hat{\Sigma}^{-1}\left(\sum_{i=1}^{n}(\underline{U}_i - \underline{\mu})(\underline{U}_i - \underline{\mu})^T + \sum_{j=1}^{m}(\underline{V}_j - \underline{\nu})(\underline{V}_j - \underline{\nu})^T\right)\cdot\hat{\Sigma}$$

$$\boxed{\hat{\Sigma} = \frac{1}{n+m}\left(\sum_{i=1}^{n}(\underline{U}_i - \underline{\mu})(\underline{U}_i - \underline{\mu})^T + \sum_{j=1}^{m}(\underline{V}_j - \underline{\nu})(\underline{V}_j - \underline{\nu})^T\right)}$$

The maximized log likelihood function for μ, ν, Σ is...

$$l(\lambda; X, Y) = l(\lambda_1, \lambda_2 \ldots \lambda_p) = -\frac{n}{2} \ln|\hat{\Sigma}_x| + \frac{m}{2} \ln|\hat{\Sigma}_y| + (\lambda_1 - 1)\left(\sum_{i=1}^{n} \ln(x_{1i}) + \sum_{j=1}^{m} \ln y_{1j}\right)$$

$$+ (\lambda_2 - 1)\left(\sum_{i=1}^{n} \ln(x_{2i}) + \sum_{j=1}^{m} \ln(y_{2j})\right) + \cdots + (\lambda_p - 1)\left(\sum_{i=1}^{n} \ln(x_{pi}) + \sum_{j=1}^{m} \ln(y_{pj})\right)$$

The maximize log likelihood function occurs at the value of lambda that maximizes the log likelihood.

Stat501_Homework5

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Question 1:

Part b.)

i.) Because the variables in each of the two populations may not be multivariate normally distributed, we will find the λ which transforms the data such that they are so. For a grid of λ values, where each component λ_j takes values in {0, 1/4, 1/3, 1/2, 1, 2, 3, 4}, find the λ which maximizes the joint likelihood of λ (from among the grid) give the observations. [10 points]

```
library(car)
# Read in the data
colleges <- read.table('~/Desktop/stat_501/Colleges.txt', sep = '\t', header = T)

# Box Cox Function
box_cox <- function(w, lambda, eps = 1e-03)
  { if (abs(lambda) < eps)
    log(w)
  else
    ((w^lambda) - 1)/lambda
}

# Log Likelihood function
llhd <- function(lambda, x, y)
  {
  # Calculate means mu and v:
  mu <- mean(box_cox(x, lambda))
  v <- mean(box_cox(y, lambda))
  sigma_x <- var(box_cox(x, lambda))
  sigma_y <- var(box_cox(y, lambda))

  length(x)/2 * log(sigma_x) + length(y)/2 * log(sigma_y) + ((lambda - 1) * (sum(log(x)) + sum(log(y)))
}


library(dplyr)
X_df <- dplyr::filter(colleges, colleges$School_Type == 'Lib Arts') %>% select(SAT, Acceptance, X..Stude
Y_df <- dplyr::filter(colleges, colleges$School_Type == 'Univ') %>% select(SAT, Acceptance, X..Student,
```

```r
final_grid <- data.frame(lamda_values = c(0, 1/2, 1/3, 1/4, 1, 2, 3, 4),
                         SAT = c(llhd(lambda = 0, x = X_df$SAT, y = Y_df$SAT), llhd(lambda = 1/4, x = X_
                                 llhd(lambda= 1/3, x = X_df$SAT, y = Y_df$SAT), llhd(lambda = 1/2, x = X
                                 llhd(lambda = 1, x = X_df$SAT, y = Y_df$SAT), llhd(lambda = 2, x = X_d
                                 llhd(lambda = 3, x = X_df$SAT, y = Y_df$SAT), llhd(lambda = 4, x = X_d

                         Acceptance = c(llhd(lambda = 0, x = X_df$Acceptance, y = Y_df$Acceptance), llh

                         X..Student = c(llhd(lambda = 0, x = X_df$X..Student, y = Y_df$X..Student), llh

                         Top.10. = c(llhd(lambda = 0, x = X_df$Top.10., y = Y_df$Top.10.), llhd(lambda
                         X.PhD = c(llhd(lambda = 0, x = X_df$X.PhD, y = Y_df$X.PhD), llhd(lambda = 1/4,
                                 llhd(lambda= 1/3, x = X_df$X.PhD, y = Y_df$X.PhD), llhd(lambda = 1/2,
                                 llhd(lambda = 1, x = X_df$X.PhD, y = Y_df$X.PhD), llhd(lambda = 2, x =
                                 llhd(lambda = 3, x = X_df$X.PhD, y = Y_df$X.PhD), llhd(lambda = 4, x =

                         Grad. = c(llhd(lambda = 0, x = X_df$Grad., y = Y_df$Grad.), llhd(lambda = 1/4,
                                 llhd(lambda= 1/3, x = X_df$Grad., y = Y_df$Grad.), llhd(lambda = 1/2,
                                 llhd(lambda = 1, x = X_df$Grad., y = Y_df$Grad.), llhd(lambda = 2, x =
                                 llhd(lambda = 3, x = X_df$Grad., y = Y_df$Grad.), llhd(lambda = 4, x =
                         )

print(final_grid)
```

```
##   lamda_values       SAT Acceptance X..Student    Top.10.     X.PhD      Grad.
## 1    0.0000000 -510.7680 -230.46020 -580.85791 -304.48270 -343.4347 -340.79900
## 2    0.5000000 -332.3428 -141.12268 -324.20421 -198.01712 -231.9765 -230.70999
## 3    0.3333333 -272.8660 -111.26682 -238.56475 -162.50616 -194.8109 -194.00698
## 4    0.2500000 -153.9098  -51.44105  -67.14815  -91.45079 -120.4604 -120.59105
## 5    1.0000000  202.9797  128.92997  448.24228  121.97942  102.7428   99.73459
## 6    2.0000000  916.8519  493.41091 1484.00564  549.98067  549.8008  540.72272
## 7    3.0000000 1630.8471  862.15749 2524.92693  979.37929  997.6497  982.13307
## 8    4.0000000 2344.9632 1234.28314 3569.00345 1410.00438 1446.1830 1423.93455
```

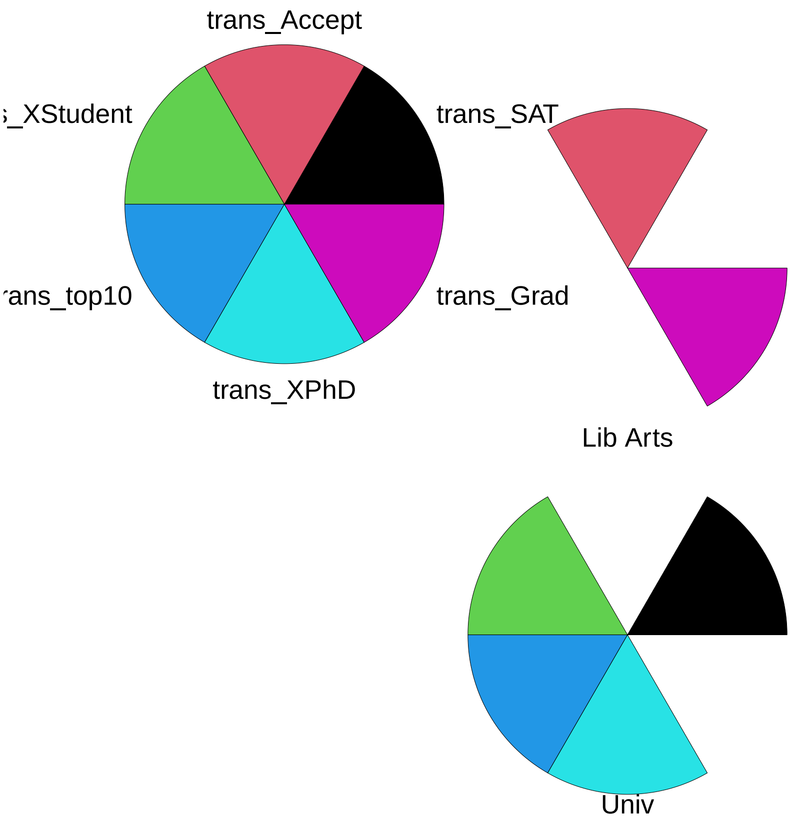
Printed above is my final grid that contains the calculated log likelihood function for each lambda value. I am not sure if I did this correctly, but I wasn't sure how to optimize the log liklihood function so that I am trying every different value (0,1/4,1/3,1/2,1,2,3,4) for each different position in the λ vector.

Based on what I have here it looks like the MLE is $\hat{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6) = (4, 4, 4, 4, 4, 4)$

ii.) With the transformed data, compare the mean values of the (transformed) SAT, % acceptance, cost per student, per cent of students in top 10 per cent of HS graduating class, per cent faculty with Ph.D.s and graduation rate, for the liberal arts vis-a-vis public universities? Are any of these means equal? [10 points]

```
## [1] "Mean Values:"

##    Group.1 trans_SAT trans_Accept trans_XStudent trans_top10 trans_XPhD
## 1 Lib Arts  68.88795    10.591380       292.1179    14.34778   16.77421
## 2     Univ  69.27830     9.633949       383.4811    16.01783   17.24891
##   trans_Grad
## 1   16.33163
## 2   16.17709
```



The means for each variable are very close. For most of the variables there is only a 1-2 value difference. For the graduationg rate both are ~16, so they are almost equal. The biggest difference in means between school types is found in the cost per student variable.

This star plot is showing which school type has the highest in each category after transforming the data with the box-cox transformation. Public University schools have the highest means in SAT, cost per student, per cent of students in top 10 per cent of HS graduating class, per cent faculty with Ph.D.s categories. Liberal Arts schools have the highest means in acceptance rate and graduation rate.

iii.) Setting the False Discovery Rate at q = 0.05,which of the six variables have a significant difference between the liberal arts colleges and public universities. Interpret the results. 10 points]

```
# Add a group varibale
colleges2 <- transformed_dta  %>% mutate(group = ifelse(transformed_dta$colleges.School_Type == "Lib Ar
colleges2$colleges.School_Type <- as.factor(colleges2$colleges.School_Type)

fit.lm <- lm(group ~ trans_SAT + trans_Accept + trans_XStudent + trans_top10 + trans_XPhD + trans_Grad
summary(fit.lm)
```

```
##
## Call:
## lm(formula = group ~ trans_SAT + trans_Accept + trans_XStudent +
##     trans_top10 + trans_XPhD + trans_Grad, data = colleges2)
##
```

```
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.60926 -0.18948 -0.04881  0.22289  0.64858
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)     7.8694858  3.2264841   2.439 0.018929 *
## trans_SAT      -0.1731437  0.0471551  -3.672 0.000661 ***
## trans_Accept   -0.0095062  0.0370656  -0.256 0.798811
## trans_XStudent  0.0048825  0.0008911   5.479 2.08e-06 ***
## trans_top10     0.1329257  0.0453527   2.931 0.005393 **
## trans_XPhD     -0.0019725  0.0619576  -0.032 0.974750
## trans_Grad      0.0648462  0.0866561   0.748 0.458342
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3416 on 43 degrees of freedom
## Multiple R-squared:  0.5986, Adjusted R-squared:  0.5426
## F-statistic: 10.69 on 6 and 43 DF,  p-value: 3.011e-07
```

Here we see that the transformed SAT and cost per student have a significant effect between the different types of colleges at the 0.001 significance level. The percent of students in the top 10% of HS graduating class has a signifcant effect between the different types of colleges at the 0.01 level. These are the only variables that seem to have significance.

Question 2:

```r
library(sas7bdat)
library(car)
#source('~/Desktop/stat_501/manova.R')
psych_sas7bdat <- read.sas7bdat('~/Desktop/stat_501/hw5/psych.sas7bdat', debug=FALSE)
psych_sas7bdat$PROG <- as.factor(psych_sas7bdat$PROG)
```

Part a.) Fit a linear model to the above and all the variables. Ignore interactions for now. Assume that the first level in the categorical variable has no additional effect (i.e. $\tau_1 = 0$) in the contrast. Summarize the results. [10 points]

```r
# Assume that the first level in PROG has no additional effect in the contrast.
psych_sas7bdat$PROG <- C(object = psych_sas7bdat$PROG, contr = contr.treatment(n = 3,  base = 2))
psych_lm_a <- lm(cbind(LOCUS_OF_CONTROL, SELF_CONCEPT, MOTIVATION) ~ READ + WRITE + SCIENCE + PROG, data
psych_lm_a
```

```
##
## Call:
## lm(formula = cbind(LOCUS_OF_CONTROL, SELF_CONCEPT, MOTIVATION) ~
##     READ + WRITE + SCIENCE + PROG, data = psych_sas7bdat)
##
## Coefficients:
```

```
##               LOCUS_OF_CONTROL  SELF_CONCEPT  MOTIVATION
## (Intercept)  -1.496970         -0.095858     -0.950513
## READ          0.012505          0.001308      0.009674
## WRITE         0.012145         -0.004293      0.017535
## SCIENCE       0.005761          0.005306     -0.009001
## PROG1        -0.127795         -0.276483     -0.360329
## PROG3         0.123875          0.146876      0.259367
```

```r
psych_lm_a_manova <- Manova(psych_lm_a)
summary(psych_lm_a_manova)
```

```
##
## Type II MANOVA Tests:
##
## Sum of squares and products for error:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL        218.85624     34.14870   35.93761
## SELF_CONCEPT             34.14870    282.04029   77.83401
## MOTIVATION               35.93761     77.83401  344.36143
##
## ------------------------------------------
##
## Term: READ
##
## Sum of squares and products for the hypothesis:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL        4.1681596   0.43586639  3.2244794
## SELF_CONCEPT            0.4358664   0.04557875  0.3371853
## MOTIVATION              3.2244794   0.33718531  2.4944504
##
## Multivariate Tests: READ
##                  Df test stat approx F num Df den Df    Pr(>F)
## Pillai            1 0.0235748 4.764416      3    592 0.0027266 **
## Wilks             1 0.9764252 4.764416      3    592 0.0027266 **
## Hotelling-Lawley  1 0.0241440 4.764416      3    592 0.0027266 **
## Roy               1 0.0241440 4.764416      3    592 0.0027266 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## ------------------------------------------
##
## Term: WRITE
##
## Sum of squares and products for the hypothesis:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL         4.725243   -1.6704333   6.822473
## SELF_CONCEPT            -1.670433    0.5905193  -2.411831
## MOTIVATION               6.822473   -2.4118306   9.850527
##
## Multivariate Tests: WRITE
##                  Df test stat approx F num Df den Df     Pr(>F)
## Pillai            1 0.0526060 10.95734      3    592 5.1862e-07 ***
## Wilks             1 0.9473940 10.95734      3    592 5.1862e-07 ***
## Hotelling-Lawley  1 0.0555271 10.95734      3    592 5.1862e-07 ***
```

```
## Roy               1 0.0555271 10.95734      3    592 5.1862e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## ------------------------------------------
##
## Term: SCIENCE
##
## Sum of squares and products for the hypothesis:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL        0.9224864    0.8495491  -1.441248
## SELF_CONCEPT            0.8495491    0.7823788  -1.327294
## MOTIVATION             -1.4412481   -1.3272945   2.251736
##
## Multivariate Tests: SCIENCE
##                  Df test stat approx F num Df den Df   Pr(>F)
## Pillai            1 0.0165945 3.329911      3    592 0.019305 *
## Wilks             1 0.9834055 3.329911      3    592 0.019305 *
## Hotelling-Lawley  1 0.0168745 3.329911      3    592 0.019305 *
## Roy               1 0.0168745 3.329911      3    592 0.019305 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## ------------------------------------------
##
## Term: PROG
##
## Sum of squares and products for the hypothesis:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL         5.029620     8.290863   12.25844
## SELF_CONCEPT             8.290863    14.218385   20.61640
## MOTIVATION              12.258441    20.616397   30.18084
##
## Multivariate Tests: PROG
##                  Df test stat approx F num Df den Df     Pr(>F)
## Pillai            2 0.1086487 11.35496      6   1186 2.2795e-12 ***
## Wilks             2 0.8914383 11.67076      6   1184 9.8057e-13 ***
## Hotelling-Lawley  2 0.1216850 11.98597      6   1182 4.2255e-13 ***
## Roy               2 0.1208775 23.89346      3    593 1.3102e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Based on the Manova summary, all of our predictor variables have a significant effect on predicting locus of control, self-concept and motivation of high school students at the 0.05 level.The PROG and WRTIE variables seem to have stronger effect at the 0.0001 level, while the SCIENCE variable has the weakest effect at the 0.05 level, although still significant.

Part b.) Refit the model but after dropping the dependent variables on the test scores of writing and science. Summarize the results. [8 points]

```
psych_lm_b <- lm(cbind(LOCUS_OF_CONTROL, SELF_CONCEPT, MOTIVATION) ~ READ + PROG, data = psych_sas7bdat)
```

```r
psych_lm_b_manova <- Manova(psych_lm_b)
summary(psych_lm_b_manova)
```

```
##
## Type II MANOVA Tests:
##
## Sum of squares and products for error:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL        225.90750     33.57873   41.58308
## SELF_CONCEPT             33.57873    283.15145   74.86532
## MOTIVATION               41.58308     74.86532  354.80754
##
## ------------------------------------------
##
## Term: READ
##
## Sum of squares and products for the hypothesis:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL        33.616976    3.1957148  20.255356
## SELF_CONCEPT             3.195715    0.3037927   1.925525
## MOTIVATION              20.255356    1.9255254  12.204532
##
## Multivariate Tests: READ
##                  Df test stat approx F num Df den Df     Pr(>F)
## Pillai            1 0.1439298 33.28946      3    594 < 2.22e-16 ***
## Wilks             1 0.8560702 33.28946      3    594 < 2.22e-16 ***
## Hotelling-Lawley  1 0.1681286 33.28946      3    594 < 2.22e-16 ***
## Roy               1 0.1681286 33.28946      3    594 < 2.22e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## ------------------------------------------
##
## Term: PROG
##
## Sum of squares and products for the hypothesis:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL         5.652949      8.49380   13.37923
## SELF_CONCEPT             8.493800     13.90261   20.98700
## MOTIVATION              13.379228     20.98700   32.35107
##
## Multivariate Tests: PROG
##                  Df test stat approx F num Df den Df     Pr(>F)
## Pillai            2 0.1121308 11.78009      6   1190 7.2868e-13 ***
## Wilks             2 0.8880617 12.10849      6   1188 3.0304e-13 ***
## Hotelling-Lawley  2 0.1258310 12.43630      6   1186 1.2626e-13 ***
## Roy               2 0.1240839 24.60996      3    595 5.0622e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

After refitting the linear model and removing the effect for the WRITE and SCIENCE variable, we see that both PROG and READ have a strong effect on the locus of control, self-concept and motivation at the

0.0001 significance level. By removing these variable, the READ effect became stronger at predicting locus of control, self-concept and motivation.

Part c.) Is there a significant evidence that the writing and science test scores are related to the psychological profiles? [2 points]

```
#LRT
library(stats)
anova(psych_lm_a, psych_lm_b, test = "Wilks")
```

```
## Analysis of Variance Table
##
## Model 1: cbind(LOCUS_OF_CONTROL, SELF_CONCEPT, MOTIVATION) ~ READ + WRITE +
##     SCIENCE + PROG
## Model 2: cbind(LOCUS_OF_CONTROL, SELF_CONCEPT, MOTIVATION) ~ READ + PROG
##   Res.Df Df Gen.var.   Wilks approx F num Df den Df    Pr(>F)
## 1    594     0.45201
## 2    596  2  0.46105 0.93285   6.9794      6   1184 2.618e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Here we see that the WRITE and SCIENCE terms are needed to significantly improve the model based on the 0.05 significance level because the P-value here is $2.618e-07$.Also the Wilks statistic is very large and close to 1. We will only reject the null if the wilk's statistic is small. Therefore we fail to reject the null hypothesis on the 0.05 significance level.

Part d.) From the model in your results in (c) above, test simultaneously for whether there is a difference in psychological profiles between Program 1 and 2 and between Program 2 and 3. [10 points]

```
# Decide which model from the LRT in partc
print("Here is my Beta Matrix:")
```

```
## [1] "Here is my Beta Matrix:"
```

```
print(coef(psych_lm_a))
```

```
##             LOCUS_OF_CONTROL SELF_CONCEPT   MOTIVATION
## (Intercept)     -1.496969664 -0.095857801 -0.950512536
## READ             0.012504619  0.001307614  0.009673547
## WRITE            0.012145048 -0.004293428  0.017535449
## SCIENCE          0.005761477  0.005305940 -0.009001453
## PROG1           -0.127795079 -0.276483394 -0.360329390
## PROG3            0.123875431  0.146875797  0.259366647
```

```
# For that model, SIMULtaneously test whether there is a difference in psychological profiles between 1
# Test by testing for a diff in beta coefficinets : C* Beta

C <- matrix(c(0,0,0,0,1,0,0,0,0,0,1,-1), ncol = 6,  by = T)
C
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    0    0    0    1    0
## [2,]    0    0    0    0    1   -1
```

```r
psych_lm_a_hyp <- linearHypothesis(model = psych_lm_a, hypothesis.matrix = C)
psych_lm_a_hyp
```

```
##
## Sum of squares and products for the hypothesis:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL         5.029620     8.290863   12.25844
## SELF_CONCEPT             8.290863    14.218385   20.61640
## MOTIVATION              12.258441    20.616397   30.18084
##
## Sum of squares and products for error:
##                  LOCUS_OF_CONTROL SELF_CONCEPT MOTIVATION
## LOCUS_OF_CONTROL        218.85624     34.14870   35.93761
## SELF_CONCEPT             34.14870    282.04029   77.83401
## MOTIVATION               35.93761     77.83401  344.36143
##
## Multivariate Tests:
##                  Df test stat approx F num Df den Df     Pr(>F)
## Pillai            2 0.1086487 11.35496      6   1186 2.2795e-12 ***
## Wilks             2 0.8914383 11.67076      6   1184 9.8057e-13 ***
## Hotelling-Lawley  2 0.1216850 11.98597      6   1182 4.2255e-13 ***
## Roy               2 0.1208775 23.89346      3    593 1.3102e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Here I have set up a linearhypothesis() function that is testing to see if there is a difference in psychological profiles (Locus, movitvation, self-concept) between the groups Prog 1&2 and 2&3 simultaneously. If I did this correctly, then there is a significant difference in profiles between the 3 programs based on the 0.001 significance level.

Part e.) Test the null hypothesis that the coefficient for the written test scores with locus of control as the outcome is equal to the corresponding coefficient with self concept as the outcome. [10 points]

```r
print("Here is my Beta Matrix:")
```

```
## [1] "Here is my Beta Matrix:"
```

```r
print(coef(psych_lm_a))
```

```
##             LOCUS_OF_CONTROL SELF_CONCEPT   MOTIVATION
## (Intercept)     -1.496969664 -0.095857801 -0.950512536
## READ             0.012504619  0.001307614  0.009673547
## WRITE            0.012145048 -0.004293428  0.017535449
## SCIENCE          0.005761477  0.005305940 -0.009001453
## PROG1           -0.127795079 -0.276483394 -0.360329390
## PROG3            0.123875431  0.146875797  0.259366647
```

```r
print("Here is my C Matrix:")
```

```
## [1] "Here is my C Matrix:"
```

```r
C_e <- matrix(c(0,0,1,0,0,0), nrow = 1, by = T)

print("Here is my M Matrix:")
```

```
## [1] "Here is my M Matrix:"
```

```r
M_e <- matrix(c(1,-1,0), nrow = 3, by = T)

part_e_answer <- linearHypothesis(model = psych_lm_a, hypothesis.matrix = C_e, P = M_e)
part_e_answer
```

```
##
##  Response transformation matrix:
##                   [,1]
## LOCUS_OF_CONTROL    1
## SELF_CONCEPT       -1
## MOTIVATION          0
##
## Sum of squares and products for the hypothesis:
##          [,1]
## [1,] 8.656629
##
## Sum of squares and products for error:
##          [,1]
## [1,] 432.5991
##
## Multivariate Tests:
##                  Df test stat approx F num Df den Df     Pr(>F)
## Pillai            1 0.0196182 11.88638      1    594 0.00060546 ***
## Wilks             1 0.9803818 11.88638      1    594 0.00060546 ***
## Hotelling-Lawley  1 0.0200107 11.88638      1    594 0.00060546 ***
## Roy               1 0.0200107 11.88638      1    594 0.00060546 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Here I have set up a linearhypothesis() function that is testing if the coefficient for the written test scores with locus of control as the outcome is equal to the corresponding coefficient with self concept as the outcome. Here we can see that there is a significant difference in these coefficients when tested at the significance level 0.001.

Part f.) Now, test the null hypothesis that the coefficient for science scores for locus of control is equal to the corresponding coefficient for science for the self concept variable, and that the coefficient for the written scores for locus of control is equal to the coefficient for the written scores for self concept. [10 points]

```
C_f <- matrix(c(0, 0, 1, 1, 0, 0), nrow = 1,  by = T)
M_f <- matrix(c(1,-1,0), nrow = 3, by = T)

final_answerf <- linearHypothesis(model = psych_lm_a, hypothesis.matrix = C_f, P = M_f)
final_answerf
```

```
##
##  Response transformation matrix:
##                  [,1]
## LOCUS_OF_CONTROL    1
## SELF_CONCEPT       -1
## MOTIVATION          0
##
## Sum of squares and products for the hypothesis:
##          [,1]
## [1,] 5.579309
##
## Sum of squares and products for error:
##          [,1]
## [1,] 432.5991
##
## Multivariate Tests:
##                  Df test stat approx F num Df den Df    Pr(>F)
## Pillai            1 0.0127330 7.660925      1    594 0.0058189 **
## Wilks             1 0.9872670 7.660925      1    594 0.0058189 **
## Hotelling-Lawley  1 0.0128972 7.660925      1    594 0.0058189 **
## Roy               1 0.0128972 7.660925      1    594 0.0058189 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Here I have set up a linearhypothesis() function that is testing the following: 1.) if the coefficient for science scores for locus of control is equal than the science score for selc concept 2.) if the coefficient for the written test scores with locus of control is equal to the coefficient for the written scores for self concept Here we can see that there is a significant difference in both of these coefficient comparisons at the significance level 0.01.

Part g.) Depending on the results from (c), fit a linear model with all interactions included. Interpret the results. [10 points]

```
psych_lm_g <- lm(cbind(LOCUS_OF_CONTROL, SELF_CONCEPT, MOTIVATION) ~ (READ + WRITE + SCIENCE + PROG)^4,
psych_lm_g
```

```
##
## Call:
```

```
## lm(formula = cbind(LOCUS_OF_CONTROL, SELF_CONCEPT, MOTIVATION) ~
##     (READ + WRITE + SCIENCE + PROG)^4, data = psych_sas7bdat)
##
## Coefficients:
##                            LOCUS_OF_CONTROL  SELF_CONCEPT  MOTIVATION
## (Intercept)                 1.499e+00         5.752e+00     1.885e+00
## READ                       -5.327e-02        -1.416e-01    -5.367e-03
## WRITE                      -7.331e-02        -1.211e-01    -8.957e-02
## SCIENCE                    -1.921e-02        -7.847e-02    -5.774e-02
## PROG1                       2.117e+00        -5.757e+00    -7.157e+00
## PROG3                      -3.913e+00        -1.466e+01    -1.063e+01
## READ:WRITE                  1.678e-03         2.751e-03     1.359e-03
## READ:SCIENCE                6.145e-04         2.035e-03     1.241e-04
## READ:PROG1                  2.440e-02         7.830e-02     8.935e-02
## READ:PROG3                  1.033e-01         2.688e-01     5.194e-02
## WRITE:SCIENCE               9.947e-04         1.610e-03     1.746e-03
## WRITE:PROG1                -2.540e-02         1.715e-01     1.347e-01
## WRITE:PROG3                 1.115e-01         3.244e-01     3.070e-01
## SCIENCE:PROG1              -1.798e-01         1.047e-01     1.333e-01
## SCIENCE:PROG3               5.088e-02         2.382e-01     2.383e-01
## READ:WRITE:SCIENCE         -1.946e-05        -3.785e-05    -1.913e-05
## READ:WRITE:PROG1           -7.181e-04        -2.794e-03    -1.636e-03
## READ:WRITE:PROG3           -2.376e-03        -5.758e-03    -3.116e-03
## READ:SCIENCE:PROG1          2.059e-03        -8.331e-04    -1.742e-03
## READ:SCIENCE:PROG3         -1.394e-03        -4.155e-03    -1.425e-03
## WRITE:SCIENCE:PROG1         3.192e-03        -3.481e-03    -2.355e-03
## WRITE:SCIENCE:PROG3        -1.630e-03        -5.053e-03    -5.995e-03
## READ:WRITE:SCIENCE:PROG1   -3.581e-05         4.475e-05     2.697e-05
## READ:WRITE:SCIENCE:PROG3    3.439e-05         8.617e-05     5.943e-05
```

```
summary(psych_lm_g)
```

```
## Response LOCUS_OF_CONTROL :
##
## Call:
## lm(formula = LOCUS_OF_CONTROL ~ (READ + WRITE + SCIENCE + PROG)^4,
##     data = psych_sas7bdat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.04523 -0.38841 -0.02312  0.37595  1.94187
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                1.499e+00  3.366e+00   0.445    0.656
## READ                      -5.327e-02  7.656e-02  -0.696    0.487
## WRITE                     -7.331e-02  7.714e-02  -0.950    0.342
## SCIENCE                   -1.921e-02  6.419e-02  -0.299    0.765
## PROG1                      2.117e+00  5.410e+00   0.391    0.696
## PROG3                     -3.913e+00  6.960e+00  -0.562    0.574
## READ:WRITE                 1.678e-03  1.592e-03   1.054    0.292
## READ:SCIENCE               6.145e-04  1.305e-03   0.471    0.638
## READ:PROG1                 2.440e-02  1.264e-01   0.193    0.847
## READ:PROG3                 1.033e-01  1.404e-01   0.735    0.462
```

```
## WRITE:SCIENCE             9.947e-04  1.375e-03   0.724    0.470
## WRITE:PROG1              -2.540e-02  1.198e-01  -0.212    0.832
## WRITE:PROG3               1.115e-01  1.511e-01   0.738    0.461
## SCIENCE:PROG1            -1.798e-01  1.151e-01  -1.562    0.119
## SCIENCE:PROG3             5.088e-02  1.408e-01   0.361    0.718
## READ:WRITE:SCIENCE       -1.946e-05  2.572e-05  -0.756    0.450
## READ:WRITE:PROG1         -7.181e-04  2.525e-03  -0.284    0.776
## READ:WRITE:PROG3         -2.376e-03  2.791e-03  -0.851    0.395
## READ:SCIENCE:PROG1        2.059e-03  2.345e-03   0.878    0.380
## READ:SCIENCE:PROG3       -1.394e-03  2.565e-03  -0.543    0.587
## WRITE:SCIENCE:PROG1       3.192e-03  2.326e-03   1.373    0.170
## WRITE:SCIENCE:PROG3      -1.630e-03  2.906e-03  -0.561    0.575
## READ:WRITE:SCIENCE:PROG1 -3.581e-05  4.290e-05  -0.835    0.404
## READ:WRITE:SCIENCE:PROG3  3.439e-05  4.901e-05   0.702    0.483
##
## Residual standard error: 0.609 on 576 degrees of freedom
## Multiple R-squared:  0.2062, Adjusted R-squared:  0.1745
## F-statistic: 6.504 on 23 and 576 DF,  p-value: < 2.2e-16
##
##
## Response SELF_CONCEPT :
##
## Call:
## lm(formula = SELF_CONCEPT ~ (READ + WRITE + SCIENCE + PROG)^4,
##     data = psych_sas7bdat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.34313 -0.43875 -0.00875  0.45591  2.20948
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)               5.752e+00  3.792e+00   1.517   0.1298
## READ                     -1.416e-01  8.625e-02  -1.642   0.1011
## WRITE                    -1.211e-01  8.690e-02  -1.394   0.1640
## SCIENCE                  -7.847e-02  7.231e-02  -1.085   0.2783
## PROG1                    -5.757e+00  6.094e+00  -0.945   0.3453
## PROG3                    -1.466e+01  7.840e+00  -1.870   0.0620 .
## READ:WRITE                2.751e-03  1.793e-03   1.535   0.1254
## READ:SCIENCE              2.035e-03  1.470e-03   1.385   0.1666
## READ:PROG1                7.830e-02  1.423e-01   0.550   0.5825
## READ:PROG3                2.688e-01  1.582e-01   1.699   0.0898 .
## WRITE:SCIENCE             1.610e-03  1.548e-03   1.040   0.2989
## WRITE:PROG1               1.715e-01  1.350e-01   1.271   0.2044
## WRITE:PROG3               3.244e-01  1.702e-01   1.906   0.0571 .
## SCIENCE:PROG1             1.047e-01  1.296e-01   0.807   0.4198
## SCIENCE:PROG3             2.382e-01  1.587e-01   1.502   0.1337
## READ:WRITE:SCIENCE       -3.785e-05  2.897e-05  -1.306   0.1920
## READ:WRITE:PROG1         -2.794e-03  2.844e-03  -0.982   0.3263
## READ:WRITE:PROG3         -5.758e-03  3.144e-03  -1.831   0.0676 .
## READ:SCIENCE:PROG1       -8.331e-04  2.641e-03  -0.315   0.7525
## READ:SCIENCE:PROG3       -4.155e-03  2.890e-03  -1.438   0.1511
## WRITE:SCIENCE:PROG1      -3.481e-03  2.620e-03  -1.329   0.1844
## WRITE:SCIENCE:PROG3      -5.053e-03  3.274e-03  -1.544   0.1232
```

```
## READ:WRITE:SCIENCE:PROG1  4.475e-05  4.833e-05   0.926   0.3549
## READ:WRITE:SCIENCE:PROG3  8.617e-05  5.521e-05   1.561   0.1191
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.686 on 576 degrees of freedom
## Multiple R-squared:  0.09076,    Adjusted R-squared:  0.05446
## F-statistic:   2.5 on 23 and 576 DF,  p-value: 0.0001512
##
##
## Response MOTIVATION :
##
## Call:
## lm(formula = MOTIVATION ~ (READ + WRITE + SCIENCE + PROG)^4,
##     data = psych_sas7bdat)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.31781 -0.50971 -0.00794  0.49580  2.24623
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)               1.885e+00  4.205e+00   0.448   0.6542
## READ                     -5.367e-03  9.565e-02  -0.056   0.9553
## WRITE                    -8.957e-02  9.637e-02  -0.929   0.3531
## SCIENCE                  -5.774e-02  8.020e-02  -0.720   0.4718
## PROG1                    -7.157e+00  6.759e+00  -1.059   0.2901
## PROG3                    -1.063e+01  8.695e+00  -1.223   0.2219
## READ:WRITE                1.359e-03  1.988e-03   0.684   0.4945
## READ:SCIENCE              1.241e-04  1.630e-03   0.076   0.9393
## READ:PROG1                8.935e-02  1.579e-01   0.566   0.5716
## READ:PROG3                5.194e-02  1.754e-01   0.296   0.7672
## WRITE:SCIENCE             1.746e-03  1.717e-03   1.017   0.3097
## WRITE:PROG1               1.347e-01  1.497e-01   0.899   0.3688
## WRITE:PROG3               3.070e-01  1.887e-01   1.627   0.1043
## SCIENCE:PROG1             1.333e-01  1.438e-01   0.927   0.3541
## SCIENCE:PROG3             2.383e-01  1.760e-01   1.354   0.1761
## READ:WRITE:SCIENCE       -1.913e-05  3.213e-05  -0.595   0.5520
## READ:WRITE:PROG1         -1.636e-03  3.155e-03  -0.519   0.6042
## READ:WRITE:PROG3         -3.116e-03  3.487e-03  -0.894   0.3719
## READ:SCIENCE:PROG1       -1.742e-03  2.929e-03  -0.595   0.5523
## READ:SCIENCE:PROG3       -1.425e-03  3.205e-03  -0.444   0.6569
## WRITE:SCIENCE:PROG1      -2.355e-03  2.905e-03  -0.811   0.4179
## WRITE:SCIENCE:PROG3      -5.995e-03  3.631e-03  -1.651   0.0993 .
## READ:WRITE:SCIENCE:PROG1  2.697e-05  5.360e-05   0.503   0.6150
## READ:WRITE:SCIENCE:PROG3  5.943e-05  6.123e-05   0.971   0.3321
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.7608 on 576 degrees of freedom
## Multiple R-squared:  0.177,  Adjusted R-squared:  0.1441
## F-statistic: 5.385 on 23 and 576 DF,  p-value: 4.564e-14
```

```r
anova(psych_lm_g)
```

```
## Analysis of Variance Table
##
##                           Df   Pillai approx F num Df den Df    Pr(>F)
## (Intercept)                1 0.025885    5.084      3    574   0.00176 **
## READ                       1 0.173281   40.104      3    574 < 2.2e-16 ***
## WRITE                      1 0.059076   12.013      3    574 1.229e-07 ***
## SCIENCE                    1 0.017404    3.389      3    574   0.01784 *
## PROG                       2 0.111886   11.358      6   1150 2.320e-12 ***
## READ:WRITE                 1 0.002922    0.561      3    574   0.64112
## READ:SCIENCE               1 0.003003    0.576      3    574   0.63076
## READ:PROG                  2 0.012924    1.247      6   1150   0.27970
## WRITE:SCIENCE              1 0.003174    0.609      3    574   0.60923
## WRITE:PROG                 2 0.015347    1.482      6   1150   0.18079
## SCIENCE:PROG               2 0.010378    1.000      6   1150   0.42393
## READ:WRITE:SCIENCE         1 0.001600    0.307      3    574   0.82062
## READ:WRITE:PROG            2 0.020323    1.968      6   1150   0.06740 .
## READ:SCIENCE:PROG          2 0.006213    0.597      6   1150   0.73271
## WRITE:SCIENCE:PROG         2 0.013614    1.314      6   1150   0.24791
## READ:WRITE:SCIENCE:PROG    2 0.008035    0.773      6   1150   0.59108
## Residuals                576
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Here I have printed the linear model fitted with all of the interaction terms, the summary of the linear model and the results of anova test. In referring to the anova resuls we see that the READ,WRITE & PROG terms have a significant effect at the 0.001 level. While SCIENCE is only significant at eh 0.05 level. We also see here that none of the interaction terms are found to be significant.