Kelsey Cameron
NID: ke110861
PID: k3593775
Assignment 3 Discrete Structures

1.

Problem 1

Let $A = \{4, \{4\}, \{2\}\}$. Indicate whether each of the following statements is true or false.

a) $1 \in A$
b) $1 \notin A$
c) $4 \in A$
d) $\{2\} \in A$
e) $\{2\} \subseteq A$
f) $\{\{4\}\} \subseteq A$
g) $\{4\} \in A$
h) $\{4\} \subseteq A$
i) $\{\{2\}\} \subseteq A$
j) $\{\{2\}\} \subset A$

a) False
b) True
c) True
d) True
e) False
f) True
g) True
h) True
i) True
j) True

2. If $A \subseteq B$ and $B \nsubseteq C$, then $A \nsubseteq C$

Let x ε A.

a)	1.	A ⊆ B	Premise
	2.	B ⊈ C	Premise
	3.	∀x (x ε A → x ε B)	Definition of Subsets (1)
	4.	x ε A → x ε B	Universal Instantiation (3)
	5.	∀x (x ε B→ ¬x ε C))	Definition of Subsets (2)
	6.	x ε B → ¬ x ε C	Universal Instantiation (5)
	7.	x ε A → ¬ x ε C	Law of Syllogism (4)(6)
	8.	∀x (x ε A → ¬ x ε C)	Universal Generalization (7)
	9.	A ⊈ C	Definition of Subsets (8)

Q.E.D
Therefore, the statement is true.

b)

Prove or Disprove:
A – B = ¬(B – A)

B: {1, 2, 3, 4, 5}
A: {1, 2, 3, 4, 5}

A – B = {}
B – A = {}
Since A and B both equal the null set, A – B = ¬(B- A) is not true.
Therefore, the statement is false.

c)

1.	$(A \cap B \cap C) \subseteq (A \cap B)$	Given
2.	$\forall x\ (x \varepsilon A \wedge x \varepsilon B \wedge x \varepsilon C \rightarrow x \varepsilon A \wedge x \varepsilon B)$	Definition of Subsets(1)
3.	$x \varepsilon A \wedge x \varepsilon B \wedge x \varepsilon C \rightarrow x \varepsilon A \wedge x \varepsilon B$	Universal Instantiation (3)
4.	$x \varepsilon A \wedge x \varepsilon B \rightarrow x \varepsilon A \wedge x \varepsilon B$	Conjunctive Simplification(4)
5.	Q.E.D.	

Therefore, the statement must be true.

d)

Prove or Disprove: $A \subseteq B \wedge A \subseteq C \rightarrow B \cap C \neq \emptyset$
B: {1, 3, 5, 7, 8}
C: {2, 4, 6, 8}
A: {8}

B ∩ C = {8}
Therefore, the statement above is false.

e) $(A \cup B) \subseteq A$

A: {1, 3, 5, 7}
B: {2, 4, 6, 8}

A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8}.
Since this is not a subset of A, the statement must be false.

Kelsey Cameron
NID: ke110861
PID: k3593775

f)

$(A \cap C)=(B \cap C) \Rightarrow (A=B)$

A: {1, 2, 3}
B: {1, 2, 3, 4}
C: {1, 2, 3}
A ∩ C = {1, 2, 3} and A ∩ B = {1, 2, 3}, but A does not equal B.

Therefore, the statement is false.

g) $A \cup (B - A)=(A \cup B)$

1.	$A \cup (B - A)=(A \cup B)$	Given
2.	∀x (x ε A v (∀x (x ε B ^ x ε A)) = A v B	Definition of Subsets (1)
3.	x ε A v (x ε B ^ x ε A)) = A v B	Universal Instantiation (2)
4.	x ε A v x ε B	Conjunctive Simplification (3)
5.	∀x (x ε A v x ε B)	Universal Generalization (4)
6.	A ∪ B	Definition of Union (5)

Q.E.D

Therefore, the statement is true.

h)

1.	$(A \subseteq B) \Leftrightarrow (\neg B \subseteq \neg A)$	Given
2.	∀x(x ε A → x ε B) ⇔ ∀x (¬ x ε B → ¬ x ε A)	Definition of Subsets (1)
3.	x ε A → x ε B ⇔ ¬ x ε B → ¬ x ε A	Universal Instantiation (2)
4.	¬ x ε B → ¬ x ε A ⇔ ¬ x ε B → ¬ x ε A	Definition of Contraposition(3)

Q.E.D

Therefore, the statement is true.

i) Let x ε A

1.	$A \subseteq (B - C) \Rightarrow (A \cap C=\varnothing)$	Given
2.	∀x (x ε A → (∀x(x ε B ^ ¬ x ε C)) → x ε A ^ x ε C = false	Definition of Subsets(1)
3.	x ε A → (x ε B ^ ¬ x ε C) → x ε A ^ x ε C = false	Universal Instantiation (2)
4.	¬x ε A v (x ε B ^ ¬ x ε C)	Definition of Implication (3)

Kelsey Cameron
NID: ke110861
PID: k3593775

5.	$(\neg x \in A \wedge x \in B) \vee (\neg x \in A \wedge \neg x \in C)$	Distribution (4)
6.	$\neg x \in A$	Absorption (5)
7.	$\neg x \in A \wedge x \in A$ = False	Complementation(6)

Q.E.D

Therefore, the statement is true.

j)
$(A - C = B - C) \Rightarrow (A = B)$

True prove it

A: {2, 8}

B: {3, 8}

C: {2, 3, 4, 5, 6}

The set A – C = {8}, and the set B – C also equals {8}.

However, {2, 8} does not equal {3, 8}, therefore the statement above is false.

k)
Prove or Disprove:
$A \subseteq B \wedge A \subseteq C \Rightarrow A = \varnothing \vee (B \cap C \neq \varnothing)$

A: {1, 2, 4, 6}
B: {1, 2, 4, 6, 8}
C: {1, 2, 4, 6, 7}

Since the set A is not null, and B ∩ C = {1, 2, 4, 6}, the above statement must be false.

3.
Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

Since A ∩ B = {3,6,9) that means they both have 3, 6, and 9.
A: {3, 6, 9}
B: {3, 6, 9}

Since B – A = {2, 10}
This means that B has the elements of 2 and 10 that A does not have.
B: {2, 3, 6, 9, 10}

Since A – B = {1,5,7,8}
A must have the elements 1,5,7 and 8 that B does not have.
A: {1, 3, 5, 6, 7, 8, 9}
B: {2, 3, 6, 9, 10}

Q.E.D.

4.
a)
P= {ϕ, 1, {1, 1}, {1, 2}, {1, 6}, {1, 9}, 2, {2, 1}, {2, 2}, {2, 6}, {2, 9}, 6, {6, 1}, {6, 2}, {6, 6}, {6, 9}, 9, {9, 1}, {9, 2}, {9, 6}, {9, 9}, {1, 2, 6}, {1, 2, 6, 9}, {2, 6, 9}}

b)
S x S = {{1, 1}, {1, 2}, {1, 6}, {1, 9}, {2, 1}, {2, 2}, {2, 6}, {2, 9}, {6, 1}, {6, 2}, {6, 6}, {6, 9}, {9, 1}, {9, 2}, {9, 6}, {9, 9}}

5.
a)
S = {x ε Z | 2^x and x $\geq$ 0}

b)
S = {2x + 1 | x ε Z}

c)
S = {4n + 42 | x ε Z and x $\geq$ 0}