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Problem 1

Let $A = \{1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18\}$

(a) How many subsets of A contain six elements?
(b) How many six-element subsets of A contain four even integers and two odd integers?
(c) How many six-element subsets of A contain at least four even integers?
(d) How many subsets of A contain only odd integers?

a) 12 elements, six chosen.
$_6A_{12}$ = 12! / 6! 6! = 665280 / 6*5*4*3*2 = 924 subsets

b) $(^6{}_4)\ (^6{}_2)$ = (6! / 4!2!) (6! / 4!2!) = (6! / 48) (6! / 48) = 15 * 15 = 225 subsets
c) $(^6{}_4)(^6{}_2)$+ $(^6{}_5)(^6{}_1)$ + $(^6{}_6)(^6{}_0)$ + = 225 + 36 + 1 = 262 subsets.
d) $(^6{}_1)$ + $(^6{}_2)$ + $(^6{}_3)$ + $(^6{}_4)$ + $(^6{}_5)$ + $(^6{}_6)$ = 924 subsets

Problem 2

Suppose the letters from the set $\{A, B, C, D, E\}$ are used to form strings of length 3.

(a) How many strings can be formed if we allow repetitions?
(b) How many strings can be formed if we do not allow repetitions?
(c) How many strings begin with A if we allow repetitions?
(d) How many strings begin with A if we do not allow repetitions?

a) 5^3 = 125 strings
b) 5! / (5-3)! = 120 / 2 = 60 strings
c) 5^2 = 25 strings
d) 4*3 = 12 strings

Problem 3

A multiple choice exam contains 10 questions. There are four possible answers for each question.

(a) How many ways can a student answer the questions on the exam if every question is answered?
(b) How many ways can a student answer the questions on the exam if the student can leave answers blank?
(c) Assuming that there is only one correct answer to each question, and assuming that a student does not leave any questions blank, how many of the possibilities from part (a) of this question will earn the student a 70% or higher on the exam if all questions are weighted equally.

a) 4^{10} = 1, 048, 576 ways.
b) 5^{10} = 9, 765, 625 ways.
c) $(^{10}{}_7)$ *3 * 3 * 3 + $(^{10}{}_8)$ * 3 * 3 + $(^{10}{}_9)$ * 3 + $(^{10}{}_{10})$ = 10*9*8 / 6 (27) + 10*9/2 (9) + 10*3 + 1 = 3240 + 405 + 31 = 3676 ways.

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Problem 4

In how many ways can 12 distinct racy romance novels be divided among four unique librarians if each librarian gets three of the books?

12 books, three books each, four people.

Combos to get three books for 12 people * combos to get 3 books for 9 people * combos to get 3 books for 6 people * combos to get 3 books for 3 people.

$\binom{12}{3}\binom{9}{3}\binom{6}{3}\binom{3}{3}$ = 369, 600 ways.

Problem 5

A committee of fifteen people, nine women and six men, is to be seated at a circular table with 15 seats. In how many ways can the seats be assigned so that no two men are seated next to one another? (Here, assume that we do not consider a seating arrangement to be unique if it is just a rotation of one that we have already considered. Also consider each man and woman a unique individual.)

So each combo will produce 15 of the same rotations, so you'll have to divide by 15 in the end. Group together one woman and one man, because if no two men sit next to each other, each man must sit next to a woman. This creates a new set because they have to sit together.
If we decide a man sits at the head of the table, there are 6 ways to do this. Then, if we continue looping around the table, we have 9 ways to put a woman next to them. Then, as we keep looping around the table, each man and woman possibility decrease by 1.
Since each of the rotations is the same thing, you have to divide 9! / 9 = 8!. Any of the spare 3 women can be placed in between each of men spots.

$\binom{9}{6}$6!8! = 2438553600 ways.

Problem 6

How many ways can Matt and Sean distribute eight chocolate donuts and seven jelly donuts among Yuksel, Travis and Ryan if each of these TAs wants at least one donut of each kind?

If we partition the donuts, the chocolate kind has seven places where a bar could go, and three people do divide it into.
$\binom{7}{2}$ = 7! / 2! 5! = 7*6 / 2 = 21 ways for chocolate
If we partition the jelly donuts, the jelly kind has six places where a bar could go and three people to divide into.

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$\binom{6}{2} = 6! / 2!4! = 6*5 / 2 = 15$ ways for jelly

To get combinations of both, 21 * 15 = 315 ways.