Homework 4
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1- Given $F_1 = \sum m(0,2,5,7,9)$ and $F_1 = \sum m(2,3,4,7,8)$ find the minterm expression for F_1+F_2. State a general rule for finding the expression for F_1+F_2 given the minterm expansions for F_1 and F_2. Prove your answer by using the general form of the minterm expansion.

F1 + F2 = Σm(0, 2, 3, 4, 5, 7, 9)
Rule: You can add the minterm expressions because each minterm is or'ed together, and since F1 and F2 are also or'ed together, by the associative property, you can combine everything and or it all together.

F1 =

0000	a'b'c'd' +
0010	a'b'cd' +
0100	a'bc'd' +
0111	a'bcd +
1001	ab'c'd

F2 =

0010	a'b'cd' +
0011	a'b'cd +
0100	a'bc'd' +
0111	a'bcd +
1000	ab'c'd'

Therefore, F2 = a'b'c'd' + a'b'cd' + a'bc'd' + a'bc'd' + a'bcd + ab'c'd' + ab'c'd
Which is Σm(0, 2, 3, 4, 5, 7, 9).

2- Given: F(a,b,c,d)=(a+b+c'+d')(a'+b'+c')(a+b+d)(a'+c)

(a) Express F as a minterm expansion. (use m-notation)
(b) Express F as a maxterm expansion. (use M-notation)
(c) Express F' as a minterm expansion. (use m-notation)
(d) Express F' as a maxterm expansion. (use M-notation)

First I will put all the variables in each term by multiplying by 1.
(a+b+c'+d')(a'+b'+c')(d+d')(a+b+d)(c+c')(a'+c)(b'+b)(d'+d) Identity

{(a+b+c'+d)(a'+b'+c'+d))(a'+b'+c' + d')(a+b+c+d)(a+b+c'+d) Distributive
(a' + b' + c + d') (a' + b' + c + d) (a'+b + c+d') (a'+b + c + d)}

Your Maxterm values:
1101, 0001, 0000, 1111, 1101, 0010, 0011, 0110, 0111

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Sorted Maxterm values:
0000, 0001, 0010, 0011, 0110, 0111, 1101, 1111

a)
$F = \Sigma\, m(4, 5, 8, 9, 10, 11, 12, 14)$

b)
All values not minterm
$F = \prod M(0, 1, 2, 3, 6, 7, 13, 15)$

c)
$F' = \Sigma m(0, 1, 2, 3, 6, 7, 13, 15)$

d)
$F' = \prod m(4, 5, 8, 9, 10, 11, 12, 14)$

3- **Find the minterm expansion of f(a,b,c,d)=a'(b'+d)+acd' and then design the result.**

a'(b' + d) + acd'	Given
a'b' + a'd + acd'	Distributive
a'b' (c+c')(d+d') + a'd(c+c')(b+b') + acd'(b+b')	Identity
a'b'c(d+d') + a'b'c'(d+d') + a'cd (b+b') + a'c'd(b+b') + abcd' + ab'cd'	Distributive

a'b'cd + a'b'cd' + a'b'c'd + a'b'c'd' + a'bcd + a'b'cd + a'bc'd + a'b'c'd + abcd' + ab'cd'

Thus you have the values
0011, 0010, 0001, 0000, 0111, 0011, 0101, 0001, 1110, 1010
Sorted values:
0000, 0001, 0010, 0011, 0101,0111,1010
$F = \Sigma m(0, 1, 2, 3, 5, 7, 12)$

DESIGN – See Attached Paper

4- **Design a combinational logic circuit which has one output Z and a 4-bit input ABCD representing a binary number. Z should be 1 if the input is at least 5, but is no greater than 11. Use one OR gate (three input), and three AND gates.**

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DESIGN – See Attached Paper

5- A half adder is a circuit that adds two bits to give a sum and a carry. Give the truth table for a half adder, and design the circuit using only two gates. Then design a circuit which will find the 2's complement of a 4-bit binary number. Use four half adders and any additional gates. (Hint: recall that one way to find the 2's complement of a binary number is to complement all bits, and then add 1)

AB	BC	Carry	Sum
00	00	0	0
01	01	1	0
10	10	1	0
11	11	1	0

DESIGN – See Attached Paper

6- Find the minimum sum of products for each function using a Karnaugh map.
(a) $f_1(a, b, c) = m_0 + m_2 + m_5 + m_6$
(b) $f_2(d, e, f) = \sum m(0, 1, 2, 4)$
(c) $f_3(x, y, z) = xz' + x'y' + x'y$
(d) $f_4(r, s, t) = M_0 \cdot M_5$

See Attached Paper for 6-12

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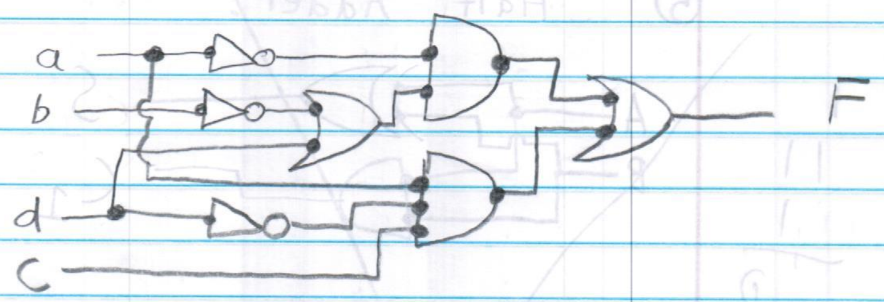
Page 1

Designs

(3)



(4)

ABCD	Z
0000	0
0001	0
0010	0
0011	0
0100	0
0101	1

$Z \geq 5$ ABCD 0101

$Z \leq 11$ 1011

5 through 11

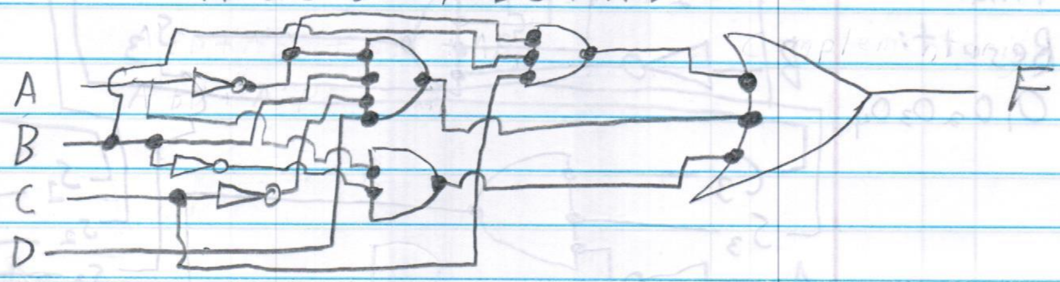
0101, 0110, 0111, 1000, 1001, 1010, 1011

$A'BC'D + A'BCD' + A'BCD + AB'C'D' + AB'C'D$
$AB'CD' + AB'CD$

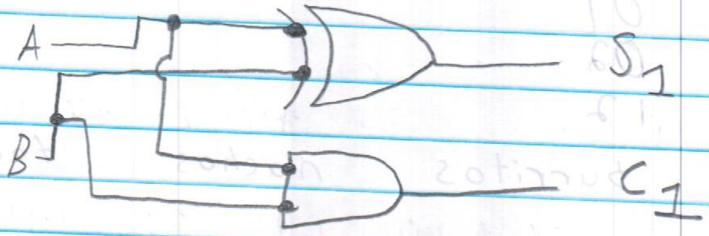
$A'B(C'D + CD' + CD) + AB'(C'D' + C'D + CD' + CD)$ Distributive

$A'B(C'D + C) + AB'$ Complement

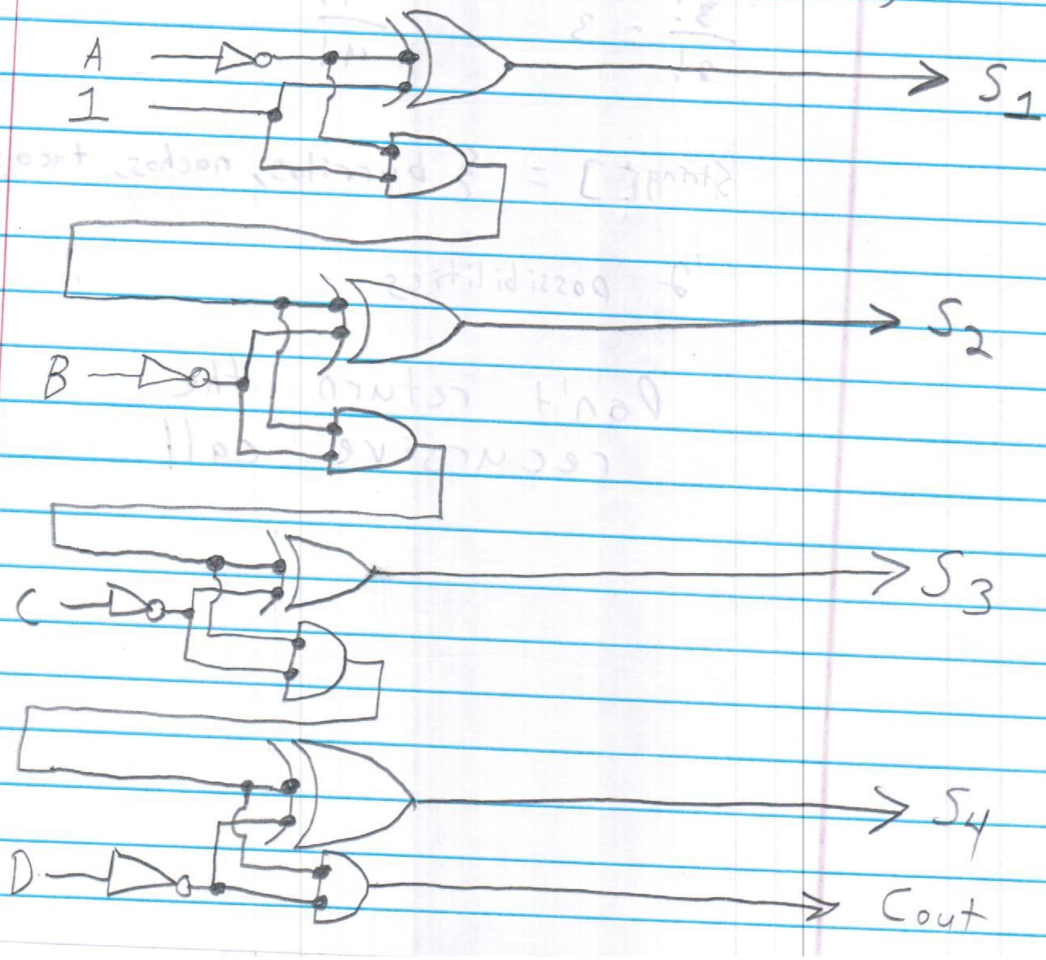
$A'BC'D + A'BC + AB'$



(5) Half Adder:



Find two's complement of 4 bit number. (Invert all bits, add 1)



⑥ ⓒ $f_3(x,y,z) = xz' + x'y' + x'y$

x	y	z	f_3
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

yz \ x	0	1
00	1	1
01	1	1
11	1	0
10	0	1

$xz'(y+y') + (x'y'+x'y)(z+z')$

$xyz' + xy'z' + x'y'z + x'y'z' + x'yz + x'yz'$

110 100 001 000 011 101

$$\boxed{y' + x'z + xz'}$$

6 d $f_4(r,s,t) = M_0 M_5$

$f_4(r,s,t) = m_1 + m_2 + m_3 + m_4$

RST	f_4
000	0
001	1
010	1
011	1
100	1
101	0
110	0
111	0

ST \ R	0	1
00	0	1
01	1	0
11	1	0
10	1	0

$$\boxed{R'T + R'S + S'T'R}$$

7) $F(A,B,C,D) = BC' + B'CD + ABC + ABCD + B'D'$

ABCD	F
0000	1
0001	0
0010	1
0011	1
0100	1
0101	1
0110	0
0111	0
1000	1
1001	0
1010	1
1011	1
1100	1
1101	1
1110	0
1111	1

$BC'(A+A')(D+D')$

$ABC'D + ABC'D' + A'BC'D + A'BC'D'$
$+ AB'CD + A'B'CD + ABCD +$
$AB'CD' + AB'C'D' +$
$A'B'CD' + A'B'C'D'$

$B'D'(A+A')(C+C')$

1101, 1100, 0101,
0100, 1011, 0011, 1111,
1010, 1000, 0010,
0000

7c

$F = \boxed{BC' + C'D' + A'B'C + ACD + AB'C}$

7) a)

CD \ AB	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	1	0	1	1
10	1	0	0	1

7 D

$$F = (B'+C)(C+D)(A+B+C')(A'+C'+D')(A'+B+C')$$

8 A

$$\Pi M(0,2,4,6,7,9,14) * \Pi D(10,11)$$

0000, 0010, 0100, 0110,
0111, 1001, 1110

10 - 1010
11 - 1011

ABCD	F
0000	1
0001	0
0010	1
0011	0
0100	1
0101	0
0110	1
0111	1
1000	0
1001	1
1010	X
1011	X
1100	0
1101	0
1110	1
1111	0

CD \ AB	00	01	11	10
00	1	1	0	0
01	0	0	0	1
11	0	1	0	X
10	1	1	1	X

$$\boxed{(A'+C'+D')(C+D')(A'+B+C)(A+B'+D)}$$

8 b $\Sigma m(1,3,7,8,15) + \Sigma d(5,12)$

0001, 0011, 0111, 1000, 1111

5 - 0101
12 - 1100

ABCD	F
0000	0
0001	1
0010	0
0011	1
0100	0
0101	X
0110	0
0111	1
1000	1
1001	0
1010	0
1011	0
1100	X
1101	0
1110	0
1111	1

CD \ AB	00	01	11	10
00	0	0	X	1
01	1	X	0	0
11	1	1	1	0
10	0	0	0	0

$\boxed{A'D + BCD + AC'D'}$

9) $F(a,b,c,d) = ab'd' + a'b' + a'cd + ac'd$

$a = c$ never occurs if $b = d = 1$

$ab'cd' + ab'c'd' + a'b'cd + a'b'cd' + a'b'c'd + a'b'c'd'$
$+ a'bcd + a'b'cd + abc'd + ab'c'd$

1010, 1000, 0011, 0010, 0001, 0000,
0111, 0011, 1101, 1001

abcd	F
0000	1
0001	1
0010	1
0011	1
0100	0
0101	0
0110	0
0111	~~1~~ 0
1000	1
1001	1
1010	1
1011	0
1100	0
1101	~~1~~ 0
1110	0
1111	0

cd \ ab	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	0
10	1	0	0	1

$\boxed{a'b' + b'c' + ab'd'}$

(10) $F = AB'D' + A'B + A'C + CD$

Minterms

$AB'CD' + AB'C'D' + A'BCD \quad A'BC'D$
$A'BCD' + A'BC'D' + A'BCD + A'BCD' +$
$A'BCD + A'B'CD + ABCD + A'BCD +$
$AB'CD + A'B'CD$

1010, 1000, 0111, 0101, 0110, 0100,
0111, 0110, 0111, 0011, 1111, 0111,
1011, 0011

ABCD	F
1111	1
0000	0
0001	0
0010	0
0011	1
0100	1
0101	1
0110	1
0111	1
1000	1
1001	0
1010	1
1011	1
1100	0

Since we want maxterms, use the 0's as 1's in k-map

CD \ AB	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	1	1	1	1
10	0	1	0	1

(10) (C)

$\boxed{F = CD + A'B + AB'D'}$

10

a) The K-Map for max-terms will be the exact opposite from the min terms.

CD \ AB	00	01	11	10
00	1	0	1	0
01	1	0	1	1
11	0	0	0	0
10	1	0	1	0

$$\boxed{F = (A' + B' + C')(A' + B' + D')(A + B + C')(A + C' + D)}$$

10 b) The K-map for maxterm for F' will just be the K-map for the min term.

(See previous page)

$$\boxed{F' = C'D' + AB' + A'BD}$$

11 $F = AB'C'D + A'B'D + A'CD + ABD + ABC$

$AB'C'D + A'B'C'D + A'B'CD + A'BCD + A'B'CD +$
$ABCD + ABC'D + ABCD + ABCD'$

1001, 0001, 0011, 0111, 0011, 1111,
1101, 1111, 1110

ABCD	F
0000	0
0001	1
0010	0
0011	1
0100	0
0101	0
0110	0
0111	1
1000	0
1001	0
1010	0
1011	0
1100	0
1101	1
1110	1
1111	1

CD \ AB	00	01	11	10
00	0	0	0	0
01	1	0	1	0
11	1	1	1	0
10	0	0	1	0

$\boxed{F = A'B'D + BCD + ABD + ABC}$