Problem Set One: Kelcey Davis

In [8]:
```
import astropy.units as u
import astropy.constants as c
from astropy.coordinates import SkyCoord
from astropy.time import Time
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import glob
%matplotlib inline
```

In [9]:
```
plt.rcParams['figure.figsize'] = (40, 10)
plt.rc('axes', labelsize=)
plt.rc('axes', labelweight='bold')
plt.rc('axes', titleweight='bold')
plt.rc('axes', titlesize='medium')
plt.rc('font', family='sans-serif')
```

Problem One

Angular Size: A second of arc (or arcsecond) is 1/60 of a minute of arc (or arcminute), which in turn is 1/60 of a degree of arc on the sky. So an arcsecond is 1/3600 of a degree. A radian, as you'll hopefully all remember, is the angle subtended at the center of a circle by a portion of the circle equal in length to the circle's radius. A steradian is a measure of area on the sky in units of square radians.

(a) From these definitions, write and evaluate an expression for the number of arcseconds in a radian. Give it to the nearest arcsecond.

(b) The Hubble Space Telescope can detect 30,000 galaxies brighter than m=32 in a 10 arcmin x 10 arcmin region of sky. If we surveyed the entire sky (not just one hemisphere), how many galaxies would Hubble detect?

1. a

Definitions we know:

1 arcsec = $\frac{1}{60}$ arcmin

1 arcmin = $\frac{1}{60}$ deg

1 arcsec = $\frac{1}{60 \times 60}$ deg

0 rad =180 deg

1 arcsec = ?? rad

We can just switch around our radians to degrees formula to get:

180 deg = π rad

and

1 arcsec = $\frac{1}{60 \times 60}$ deg

becomes:

1 arcsec = $\frac{1}{60 \times 60} \frac{\text{deg}}{180}$ rad

or

1 arcsec = $\frac{1}{60 \times 60 \times 180}$ rad

and

1 rad = $\frac{60 \times 60 \times 180}{1}$ arcsec

or

1 rad = 206264.8062470963E arcsec

rounding:

1 rad = 206265 arcsec ✓

1.b

First we need to know how many total arcseconds there are in the sky.

We know the sky has 360 degrees in one dimension.

In [10]:
```
print(f'There are {360*360} square degrees on the sky.')
```

```
There are 129600 square degrees on the sky.
```

There are 30,000 galaxies in 100 square arcminutes. 100 square arcminutes is:

$\frac{30000 \text{ galaxies}}{100 \text{ arcmin}^2}$

$1 \text{ Harcmin} = 0.1 \text{ deg}^2$

$100 \text{ arcmin}^2 = 0.027778 \text{ deg}^2$

By proportionality,

$\frac{30000 \text{ galaxies}}{0.027778 \text{ deg}^2} = \frac{x \text{ galaxies}}{129600 \text{ deg}^2}$

1.5 ✓

In [11]:
```
galaxies = 30000*(360*360)/(0.027778
print(f'This is roughly {galaxies:.1e} galaxies across the entire sky.')
```

```
This is roughly 1.4e+10 galaxies across the entire sky.
```

1 pt. for 1e10
2 pts for 30000
2 pts for 360*360

✓

Problem 2

Black Holes. The observation that all galaxies have supermassive black holes in their centers begs the question: how were they created? The essence theory is that small black holes were created in the early universe by the first stars (known as Population III stars)when they underwent collapse. These black holes then accreted matter and grew in mass. The oldest quasars we observe are at redshifts of about 6 billion, of little relevance by today. However, we have observed galaxies at redshifts greater than six, only one billion years after the Big Bang, which have black holes with masses in excess of 109 M⊙. Conventional methods of black hole growth would prevent a Population III black hole seed from growing so fast. Due to this difficulty, a number of theorists are now studying so-called 'direct collapse' black holes, which is when a large cloud of gas, with M, 105M⊙ collapses directly into a black hole. Calculate the radius of the event horizon in these two cases (Pop III 'seeds' and direct collapse), and discuss over-objects (for each case) in the physical world which is of a similar size

For pop III seeds:

$M = 10 \text{ Msun}$

For direct collapse:

$M = 10^5 \text{ Msun}$

The event horizon or schwarzschild radius is given by:

$R = \frac{2GM}{c^2}$

In [16]:
```
mass_kg = c.M_sun.to(u.kg)
print(f'The mass of the sun in kg is {mass_kg}.')
```

```
The mass of the sun in kg is 1.9884754153382105e+30.
```

In [21]:
```
print(f'The gravitational constant G is given by {c.G.decompose()}.')
```

```
The gravitational constant G is given by 6.6743e-11 m3 / (kg s2).
```

In [22]:
```
print(f'The speed of light c is given by {c.c.decompose()}.')
```

```
The speed of light c is given by 299792458.0 m / s.
```

For pop III seeds, a typical event horizon would be:

$R = \frac{2 \times 6.674e-11 \frac{m^3}{kg s^2} \times 1.9884754153382105e+30 \, kg}{(299792458.0 \frac{m}{s})^2}$

the units clearly cancel down to meters, and we get:

In [28]:
```
r = (2*6.674e-11*10*1.9884754153382105e+30)/(299792458**2)
print(f'This is an event horizon radius of {r:.1e} m.')
```

```
This is an event horizon radius of 3.0e+04 m.
```

This is an event horizon radius of 3.0e+04 m.

Now for the direct collapse, we have a radius of:

$R = \frac{2 \times 6.674e-11 \frac{m^3}{kg s^2} \times 1.9884754153382105e+30 \, kg}{(299792458.0 \frac{m}{s})^2}$

the units clearly cancel down to meters, and we get:

In [29]:
```
r = (2*6.674e-11*10^5*1.9884754153382105e+30)/(299792458**2)
print(f'This is an event horizon radius of {r:.1e} m.')
```

```
This is an event horizon radius of 3.0e+09 m.
```

This is an event horizon radius of 3.0e+09 m.

What objects are of similar size?

So the pop III stars, $3x10^4$ meters is 30 km.

30 Kilometers is about 20km from sea level, so this would be about the size of three Mt.Everests stacked on top of each other.

For a direct collapse black hole, we have a radius of $3x10^9$ m or $3x10^6$ km. 3 million km. 20057 orbits on a about 1.5 million km, so this is about twice the distance between us and 20057.

Problem 3

Galaxy Rotation. One piece of evidence against the extragalactic nature of 'spiral nebulae' came some from Edwards van Maanen, who in 1916 claimed to see the galaxy M101 rotating. The angular rotation rate he reported was 0.02 arcsec/year. The measured diameter of M101 was 180 kpc.

a. Using van Maanen's (flawed) observation and the measured diameter of M101, estimate the speed of a point at the edge of the galaxy and compare it to the characteristic orbital speed of the Milky Way, 220 km/s.

In [31]:
```
rot_reported = 0.02*arcsec/year
d_m101 = 180*kpc
```

$\omega_{rot} = ??$ angular speed ✓

1 rad = 206265 arcsec

$0.02 \text{arcsec} \times \frac{\text{rad}}{206265 \text{ arcsec}} = 9.69696e-05 \text{ rad}$

In [34]:
```
v_rot = 9.69696e05*d_m101/2
print(f'The rotational velocity would be {v_rot:.1e}[km/year].')
```

```
The rotational velocity would be 1.8e.00[km/year].
```

→ ~ 7,200 km/s [mpkm v Mkm]

b. The diameter of M101 is actually 25.4 kpc. Assuming it rotates at the same velocity as the Milky Way, estimate the angular rotation rate of stars at the edge of the galaxy.

$\omega = \frac{v}{r}$

$\frac{\omega}{\text{rad}} = \frac{\text{rad}}{\text{year}} \frac{\text{km}}{\text{km}}$

In [39]:
```
d = 25.4 kpc
v_mw = 220 * 3.2407e-17 kpc/s

omega = v_mw/(d.to)
print(f'The angular rotation rate is approximately {omega.to(rad/s)}')
```

```
The angular rotation rate is approximately 3.0e.12[rad/s]
```

Now long would it take for the galaxy to rotate through 1 arcsec?

$\frac{\omega}{\text{rad}} = \frac{\text{rad/sec}}{\text{sec}}$

$\frac{1}{\text{sec}} = \frac{\text{rad}}{\text{sec}} \times \frac{1}{\text{rad}} = \frac{1}{\text{rad}}$

In [79]:
```
omega_arcsec = omega*206265
print(f'The rotation rate in {omega_arcsec.to(arcsec/s)}.')
t = (1/omega_arcsec)
t = (t/3.1e.xx)/1000
print(f'This corresponds to the galaxy rotating through an arcsecond in {round(t, -1)} years.')
```

```
The rotation rate in 6.2e.16 [arcsec/s].
This corresponds to the galaxy rotating through an arcsecond in 790.0 years.
```

The maximum resolution observable from the ground today is roughly 0.5 arcsec. Could astronomers in the 1920s have detected the rotation of M101?

✓

Sadly no this would not be observable. They would need to observe for 350 years just for the rotation to match the available resolution.

Problem 4

When galaxies collide. Two galaxies collide, each of which has an apparent magnitude of mapp = 12. Assuming flux is conserved, what is the final magnitude of the merger product?

Apparent magnitudes:

$m_1 = -2.5 \log_{10}(F_1)$

$\frac{m_2}{\text{mag}} = \log_{10}(F)$

$F = 10^{\frac{m_2}{\text{mag}}}$

$F = F = 10^{\frac{m_1}{\text{mag}}} + 10^{\frac{m_2}{\text{mag}}}$

In [85]:
```
F = 2*10**(-12/2.5)
print(f'The combined flux is {F.:e}.')
```

```
The combined flux is 1.2e-05.
```

$m_{new} = -2.5 \log_{10}(1.2e-5)$

In [86]:
```
m = -2.5*np.log10(F)
print(f'The resulting combined magnitude will be {round(m,0)} magnitudes.')
```

```
The resulting combined magnitude will be 11.0 magnitudes.
```

Problem 5

Like the stars in the sky. The Andromeda galaxy has a distance of 0.8 Mpc and an apparent magnitude mapp = 3.5. Assuming each star has the same intrinsic brightness of the Sun (absolute magnitude of Msun = 4.74 (stars written as 5.48), estimate how many stars are in Andromeda.

In [96]:
```
d = 0.8e.Mpc
m_appr = 3.5 Andromeda
m_abs = 4.74 star
```

Absolute magnitude:

$M = m - 5 \log_{10}(\frac{d}{10pc})$ ✓

$M_{Andromeda} = 3.5 - 5 \log_{10}(\frac{800000}{10pc})$

In [98]:
```
M_and = 3.5-(5*np.log10(0.8e6/10))
print(f'The absolute Magnitude of Andromeda is {round(M_and, -1)}.')
```

```
The absolute Magnitude of Andromeda is -16.0.
```

In [99]:
```
Msun = 4.74
```

$M_* = M_\odot - 2.5 \log_{10} \frac{L_*}{L_\odot}$

$\frac{L_*}{L_\odot} = 10^{\frac{M_* - M_\odot}{-2.5}}$

so M_* for the andromeda galaxy and $M_\odot$ for the a single star.

4, 4

In [100]:
```
L_ratio = 10**((M_and-Msun)/(-2.5))
print(f'The luminosity ratio is {L_ratio:.1e}.')
```

```
The luminosity ratio is 2.5e+06.
```

✓

We can consider this ratio to be proportional to the number of stars that make up the galaxy. We can then conclude that:

In [101]:
```
print(f'The number of stars in the Andromeda galaxy is approximately {L_ratio:.1e}.')
```

```
The number of stars in the Andromeda galaxy is approximately 2.5e+06.
```

✓ ✓ ✓

Making sense?

Problem 6

Gravity. Show that the total, gravitational potential energy of a uniform density sphere is

$U = -\frac{3}{5} \frac{GM^2}{R}$

where M is the total mass of the sphere and R is its radius. Hint: The equation for the gravitational potential energy of a point mass, dm at radius r with filled mass enclosed within that shell:

$dU = -G \frac{M(r)\,dm}{r}$

$dU = -G \frac{M(r)\,\rho\,4\pi r^2\,dr}{r}$

```
$ let_ldu1 i=let_ldu1 G*Rho*(M_lmm[0]) dm_i $
```

$dU_G = G\rho r^4 dr$

density is uniform so ρ has no r dependence.

$\rho = \frac{M}{V}$

$V = \frac{4}{3}\pi R^3$

$M_{lmm} = \frac{4}{3}\pi r^3 \rho$

```
$ let_ldu1 i=let_ldu1 G*Rho*(((((4/3*pi*r3*rho Rho))))) dm_i $
$ let_ldu1 i=let_ldu1 G*Rho*(((((4/3*pi*r3*rho)) dm_i $
$ let_ldu1 i=let_ldu1 G*Rho*(((((4/3*pi*r3*rho)) dm_i $
$ let_ldu1 i=(G*Rho*(((((4/3*pi*r3*rho)) dm_i $
```

$dm_i = 4\pi r^2 \rho dr$

```
$ let_ldu1 i=(G*Rho*(((((4/3*pi*r3*rho)) 4/3pi*r2 rho dr $
$ let_ldu1 i=(G*Rho*(((((4/3*pi*r3*rho)) 4/3pi*r2 rho dr $
```

$U = -G\rho^2 \frac{4}{3}\pi \int_0^R r^4 dr$

uniform density

$\rho = \frac{M}{\frac{4}{3}\pi R^3}$

$U = -G\rho^2 \frac{4}{3}\pi \int_0^R r^4 dr$ $\left(\rho = \frac{M}{\frac{4}{3}\pi R^3}\right)$

$U = -G\rho^2 \frac{4}{3}\pi \frac{r^5}{5}\Big|_0^R$

$U = -\frac{3}{5} \frac{GM^2}{R}$

Problem 7

All together now. You observe a relaxed globular cluster of stars that contains 20,000 stars each with M = 0.5 Msl on average and an average velocity dispersion of σ v = 10 km s-1. The observed angular diameter of the star cluster is 10, arcmin.

a. What is the total mass of the cluster in units of $M_\odot$?

In [109]:
```
stars = 20000
Msun = 0.5msun
vel_disp = 10 xlm/s
d = 5kpcxlmm
```

In [109]:
```
Mclust = stars/Msun
print(f'The mass of the cluster is {Mclust.:e} solar masses.')
```

```
The mass of the cluster is 1.0e+04 solar masses.
```

✓

b. What are the expressions for the total kinetic (K) and potential (U) energies of the cluster?

$K = \frac{1}{2}mv^2$ (scalar dimensions)

$K = \frac{1}{2}(1.0e4 M_\odot)(10 \frac{km}{s})^2$

$U = -\frac{3}{5} \frac{GM^2}{R}$ ✓

$U = -\frac{\frac{3}{5} \times 6.67e-11 \frac{m^3}{kg\,s^2} \times (1.0e4 \times 1.989e30 \, kg)^2}{5 \times 3.086e19 \, m}$

$U = -\frac{\frac{3}{5} \times 6.67e-11 \times (1.0e4 \times 1.989e30)^2}{5 \times 3.086e19}$

c. What is the physical size of the cluster (hint: use the virial theorem)? Give your answer in pc.

$K.E = -\frac{1}{2}U$

$\frac{1}{2}(1.0e4 M_\odot)(10 \frac{km}{s})^2 = -\frac{1}{2} \left(-\frac{\frac{3}{5} \times 6.67e-11 \times (1.0e4 \times 1.989e30)^2}{5 \times 3.086e19}\right)$ 2/5 of 3/5

$(10 \frac{km}{s})^2 = \frac{\frac{3}{5} \times 6.67e-11 \times (1.0e4 \times 1.989e30)^2}{5 \times 3.086e19 \times 10 \frac{km}{s}}$

$(10 \frac{km}{s})^2 = \frac{\frac{3}{5} \times 6.67e-11 \times (1.0e4 \times 1.989e30)^2}{5 \times 3.086e19 \times 10 \frac{km}{s}}$

$R = \frac{\frac{3}{5} \times 6.67e-11 \times (1.0e4 \times 1.989e30)^2}{5 \times 3.086e19 \times 10 \frac{km}{s}}$ → 7.0 M Sun

d. How far away is the cluster (Hint: It is still considered a small angle)? Give your answer in kpc (kilo-parsec, 1 kpc = 1,000 pc).

Sin small angles,

$\tan\theta \approx \theta$ ✓

$\tan\theta \approx \theta$

$\theta = \frac{D}{d}$

in degrees,

$\theta = \frac{D}{d} \times \frac{180}{\pi}$

right idea

In [ ]:
```
D = 10/(180/pi) # diameter in deg
Ldiam =
```

In [ ]: