PHYS 4720/6720 - Galaxies and the ISM
Prof. Cara Battersby, Spring 2023

Problem Set #5

Due: Wednesday April 12th, 2023 at 12:30pm.

Reminder: Same instructions as previous PSs still apply.
4720: 3 questions, 38 pts total Written + Coding.
6720 and Honors: 4 questions, 50 pts total Written + Coding.

1. **(5 pts) Let me tell ya 'bout fish, Sir.** The Tully-Fisher relation can be expressed as:

$$M = -4.43 - 6.15 \log \Delta V \tag{1}$$

where ΔV is the velocity width of the rotational velocity profile of each galaxy in km/s, and M is the absolute magnitude of the galaxy. The figure below shows the velocity profiles of two spiral galaxies, NGC1241 and NGC5248.

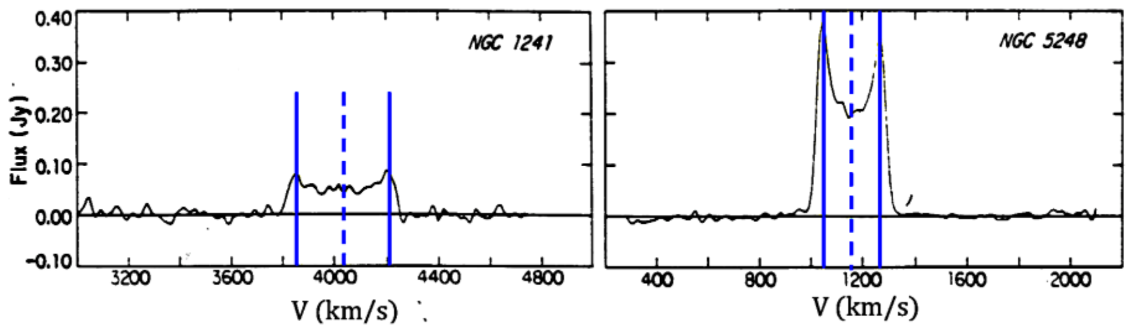

Figure 1: Velocity profiles observed for two spiral galaxies, NGC 1241 and NGC 5248.

(a) **(3 pts)** Assume that galaxies NGC 1241 and NGC 5248 have apparent magnitudes of $m = 14$ and $m = 12$, respectively. Calculate the distances to these two galaxies.

(b) **(2 pts)** From these and their recessional velocities, calculate the value of the Hubble constant. For simplicity, also assume that the galaxies are seen edge-on (inclination angle $i = 90$), so that the central velocity corresponds to the maximum rotational velocity.

2. **(8 pts) It's FUNdamental.** The Fundamental Plane relationship says that the velocity dispersion, σ, of an elliptical galaxy is related to its size r_e and surface brightness I_e by:

$$r_e \propto \sigma^{1.4} I_e^{-0.85} \tag{2}$$

(a) **(4 pts)** Derive the Fundamental Plane relationship using arguments similar to those used to derive the Tully-Fisher relationship and the Faber-Jackson relationship in class. You should get an answer that is close, but not exactly the same as the equation above (i.e. the exponents on σ and I_e will be slightly off).

(b) **(4 pts)** The reason your answer for part (a) differs from the actual relationship is because the Mass-to-Light ratio for elliptical galaxies is not constant. Instead it varies with the mass of the galaxy. Determine how the M/L ratio must depend on mass to obtain the observed Fundamental Plane relationship. In other words, assuming M/L $\propto$ M^x, solve for x.

3. **(25 pts) Stellar Populations.** For this problem, you will generate a model galaxy spectrum for two idealized cases. The first is a burst of star formation which has just occurred (e.g., t=0, no stars have evolved off of the main sequence), while the second will be that same population, only viewed one billion years later.

   (a) **(7 pts)** Assume that a gas cloud with a mass of 10^9 $M_\odot$ collapses instantaneously (yes, this is unphysical, but it's an idealized case, so let's go with it), and stars are formed with masses distributed according to a Salpeter initial mass function (power law index = -2.35), with a minimum mass of 0.08 $M_\odot$and a maximum of 100 $M_\odot$. Calculate the number of stars of each spectral type, using the following definitions: O=20-100 $M_\odot$; B=3-20 $M_\odot$; A=1.7-3 $M_\odot$; F=1.1-1.7 $M_\odot$; G=0.8-1.1 $M_\odot$; K=0.6-0.8 $M_\odot$; M=0.08-0.6 $M_\odot$. Write these values down in tabular form and show all work.

   (b) **(4 pts)** In this folder, there is a file "Kurucz_models.csv". These are theoretical stellar spectra models (from the paper Kurucz 1993). I have provided you in this file with the model stellar spectra for stars of type O5V, B5V, A5V, F5V, G5V, K5V and M5V. The first column is wavelength, and the subsequent columns are the spectra (in Jy) for each spectral type at a distance of 10 pc. Please put these spectra together on a single plot (with each spectra in a different color, ideally in some logical way); you'll want to make the y-axis in log.

   (c) **(6 pts)** Using the results from part a, now calculate the integrated spectrum of your entire stellar population. Assume that all O stars have the spectrum of an O5V, all B stars have B5V, etc. Plot this.

   (d) **(6 pts)** In part c, you plotted what the spectrum of the population would look like just after all of the stars were formed. In this part, you'll plot the integrated spectrum for the same population, only after a time of 1 Gyr. Think hard about which spectral types should be represented. Plot this model and add on top the same plot as part c, with a different line type. Be sure to add a legend to explain all of your lines. **Be sure to explain your method and reasoning in words, either in your written or coding submission.**

   (e) **(2 pts)** We made several assumptions in the problem which resulted in our final spectra not being all that accurate. Describe two such assumptions which are likely not well-justified.

Extra Question for Honors and 6720 Students

4. **(12 pts) Active Galactic Nuclei.** Galaxy interactions often result in gas being funneled to the center of a galaxy, which triggers a growth phase of the supermassive black hole. In this problem, you'll calculate the typical timescale over which this phase would be visible as an AGN. Suppose an elliptical galaxy undergoes a merger and the central black hole begins to accrete matter at the Eddington rate. If the total gas mass, M_{gas}, funneled to the black hole is 1% of the overall mass of the galaxy's bulge, M_{bulge}, **estimate the maximum lifetime of the resulting growth phase**.

   The measured stellar velocity dispersion of this elliptical galaxy is about 220 km/s. The publication McConnell & Ma 2013, ApJ, 764, 184M gives a comprehensive analysis of the scaling relationships between black holes and their host galaxies. The AGN luminosity is about 10^{13} $L_\odot$.

   (a) **(3 pts)** Using the appropriate scaling relationships given in McConnell & Ma, what is the estimated black hole mass of this Galaxy in units of $M_\odot$? Describe the scaling relationship that you chose and why (no need to make it overly complicated, just choose one that is reasonable).

   (b) **(3 pts)** Using the appropriate scaling relationships given in McConnell & Ma, what is the estimated bulge mass of this Galaxy in units of $M_\odot$? Describe the scaling relationship that you chose and why.

   (c) **(1 pts)** What is the total amount of gas mass funneled into the black hole in units of $M_\odot$?

   (d) **(2 pts)** The Eddington accretion rate is given by: $\dot{m} = \frac{L}{\epsilon c^2}$ where L is the AGN luminosity, ϵ is the efficiency (typically 0.1 is assumed), and c is the speed of light. What is the Eddington accretion rate (in solar masses per year) for this growing black hole?

   (e) **(2 pts)** Assuming a constant accretion rate with time, how long will it take for all the available gas to be accreted? Give your answer in Myr.

   (f) **(1 pts)** How does this compare with the timescale for a typical merger, which is about 1 Gyr? Could this explain why only $\sim$ 10% of galaxies are observed to host an AGN at any given time?