Problem set 5

In [1]:

```
import astropy.units as u
import astropy.constants as c
from astropy.coordinates import SkyCoord
from astropy.time import Time
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import glob
%matplotlib inline
```

In [2]:

```
plt.rcParams['figure.figsize'] = (10, 10)
plt.rc('axes', labelsize=14)
plt.rc('axes', labelweight='bold')
plt.rc('axes', titlesize=16)
plt.rc('axes', titleweight='bold')
plt.rc('font', family='sans-serif')
```

Problem 1

The Tully-Fisher relation can be expressed as:

$$M = -4.43 - 6.15 log \Delta V$$

where ΔV is the velocity width of the rotational velocity pro□le of each galaxy in km/s, and M is the absolute magnitude of the galaxy. The □gure below shows the velocity pro□les of two spiral galaxies, NGC1241 and NGC5248.

(a) (3 pts) Assume that galaxies NGC 1241 and NGC 5248 have apparent magnitudes of m = 14 and m = 12, respectively. Calculate the distances to these two galaxies.

NGG 1241

m = 14

ΔV = 4200 - 3850 = 350 [km/s]

The diestance modulus can be expressed as:

$$m - M = 5 log_{10}(d) - 5$$

$$M = -5 log_{10}(d) + 5 + m$$

$$-5 log_{10}(d) + 5 + m = -4.43 - 6.15 log \Delta V$$

$$log_{10}(d) = \frac{-1}{5}(-4.43 - 6.15 log \Delta V - 5 - m)$$

$$d = 10^{\frac{-1}{5}(-4.43 - 6.15 log \Delta V - 5 - m)}$$

In [14]:

```
V = 350
m = 14
d1241 = 10**((-1/5)* (-4.43 - 6.15*np.log10(V) - 5 - m))
print(f'The distance is {d1241:.2e} [km].')
```

```
The distance is 6.53e+07 [km].
```

NGC 5248

m = 12

ΔV = 1250 - 1050 = 200 [km/s]

In [15]:

```
V = 200
m = 12
d5248 = 10**((-1/5)* (-4.43 - 6.15*np.log10(V) - 5 - m))
print(f'The distance is {d5248:.2e} [km].')
```

```
The distance is 1.31e+07 [km].
```

b) From these and their recessional velocities, calculate the value of the Hubble constant. For simplicity, also assume that the galaxies are seen edge-on (inclination angle i = 90), so that the central velocity corresponds to the maximum rotational velocity.

$$v_r = H_0 d$$

$$H_0 = \frac{v_r}{d}$$

NGC 1241

$v_r \approx$ 4050

In [16]:

```
H1241 = 4050/d1241
print(f'The value of the hubble constant for NGC 1241 is  {H1241:.2e} [1/s]')
```

```
The value of the hubble constant for NGC 1241 is  6.20e-05 [1/s]
```

NGC 5248

$v_r \approx$ 1150

In [17]:

```
H5248 = 4050/d5248
print(f'The value of the hubble constant for NGC 5248 is  {H5248:.2e} [1/s]')
```

```
The value of the hubble constant for NGC 5248 is  3.10e-04 [1/s]
```

Problem 2

The Fundamental Plane relationship says that the velocity dispersion, σ, of an elliptical galaxy is related to its size r e and surface brightness I e by:

$$r_e \alpha \sigma^{1.4} I_e^{-0.85}$$

a) Derive the Fundamental Plane relationship using arguments similar to those used to derive the Tully-Fisher relationship and the Faber-Jackson relationship in class. You should get an answer that is close, but not exactly the same as the equation above (i.e. the exponents on σ and I e will be slightly o□).

In calss, we started with:

$$F_g = F_c$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$M = \frac{v^2 R_{tot}}{G}$$

And then we added two assumptions:

1. ratio is some constant $\frac{M}{L} = C_{ml}$
2. Surface brightness is constant $\frac{L}{R^2} = C_{sb}$

We will follow about this same logic, with a few changes.

We can consider Luminosity as the integration of the surface brightness:

$$L = 2\pi \int_0^r I(r) r dr$$

We will also re-consider our kinetic energy term so that we can include the velocity dispersion term.

Typically, centripetal force is given by:

$$F_c = \frac{mv^2}{r}$$

We can simply change this v for the velocity dispersion if we remember that we are working in three dimensions. We just need to multiply the term by 3.

$$F_c = \frac{3m\sigma^2}{r}$$

With these differences, we can follow about the same logic as in class.

$$F_g = F_c$$

$$\frac{3m\sigma^2}{r} = \frac{GmM}{r^2}$$

$$M = \frac{3\sigma^2 R_{tot}}{G}$$

Just quickly solving the luminosity integral, we get:

$$L = \pi I(r) R^2$$

$$\frac{M}{L} = C_{ml} = \frac{3\sigma^2 R_{tot}}{GL} = \frac{3\sigma^2 R_{tot}}{G\pi I(r) R^2} = \frac{3\sigma^2}{G\pi I(r) R}$$

$$R = \frac{L}{M} \frac{3\sigma^2}{G\pi I(r)}$$

$$\frac{L}{R^2} = C_{sb}$$

$$R^2 = \frac{L}{C_{sb}}$$

$$\frac{L}{C_{sb}} = \frac{L^2}{M^2} [\frac{3\sigma^2}{G\pi I(r)}]^2$$

$$\frac{1}{C_{sb}} = \frac{L}{M^2} 9\sigma^4 \frac{1}{G^2\pi^2 I(r)^2}$$

$$\frac{1}{C_{sb}} = \frac{\pi I(r) R^2}{[\frac{3\sigma^2 R_{tot}}{G}]^2} 9\sigma^4 \frac{1}{G^2\pi^2 I(r)^2}$$

$$\frac{1}{C_{sb}} = \frac{9\sigma^4 \pi I(r) R^2}{G^2\pi^2 I(r)^2} \frac{G^2}{9\sigma^4 R}$$

$$\frac{1}{C_{sb}} = \frac{9\sigma^4 R}{I(r)\pi}$$

Dropping the constants:

$$I(r) \approx \sigma^4 R$$

b) The reason your answer for part (a) di□ers from the actual relationship is because the Mass-to-Light ratio for elliptical galaxies is not constant. Instead it varies with the mass of the galaxy. Determine how the M/L ratio must depend on mass to obtain the observed Fundamental Plane relationship. In other words, assuming M/L 9 M x , solve for x.

$$\frac{M}{L} \approx M^x$$

$$\frac{M}{L} = M^x = \frac{3\sigma^2 R_{tot}}{GL} = \frac{3\sigma^2 R_{tot}}{G\pi I(r) R^2} = \frac{3\sigma^2}{G\pi I(r) R}$$

$$R = \frac{1}{M^x} \frac{3\sigma^2}{G\pi I(r)}$$

$$R^2 = \frac{L}{C_{sb}}$$

$$\frac{L}{C_{sb}} = [\frac{1}{M^x} \frac{3\sigma^2}{G\pi I(r)}]^2$$

COME BACK TO THIS

In [ ]:

Problem 3

For this problem, you will generate a model galaxy spectrum for two idealized cases. The □rst is a burst of star formation which has just occurred (e.g., t=0, no stars have evolved o□ of the main sequence), while the second will be that same population, only viewed one billion years later.

a. Assume that a gas cloud with a mass of 10 9 M d collapses instantaneously (yes, this is unphysical, but it's an idealized case, so let's go with it), and stars are formed with masses distributed according to a Salpeter initial mass function (power law index =´2.35), with a minimum mass of 0.08 M d and a maximum of 100 M d . Calculate the number of stars of each spectral type, using the following de□nitions: O=20-100 M d ; B=3-20 M d ; A=1.7-3 M d ; F=1.1-1.7 M d ; G=0.8-1.1 M d ; K=0.6-0.8 M d ; M=0.08-0.6 M d . Write these values down in tabular form and show all work.

GRADER: This problem is fully in the coding notebook that I turned in. No need to grade it here.

$$M_{tot} = 10^9 M_\odot$$

index = -2.35

We start with the salpeter equation:

$$\xi(m)\Delta m = \xi_0 \left(\frac{m}{M_\odot}\right)^{-2.35} \left(\frac{\Delta m}{M_\odot}\right)$$

We first need to solve for the normalization constant.

$$M_{tot} = \int_{0.08M_{sun}}^{100M_{sun}} \xi_0 \frac{m^{-2.35}}{m} dm$$

$$M_{tot} = \xi_0 \frac{1}{M_\odot} [\frac{m^{-0.35}}{-0.35}] |_{0.08M_\odot}^{100M_\odot}$$

$$M_{tot} = \xi_0 \frac{1}{-0.35} [100 M_\odot^{-0.35} - 0.08 M_\odot^{-0.35}]$$

$$\xi_0 = \frac{M_{tot}}{\frac{1}{-0.35}[100 M_\odot^{-0.35} - 0.08 M_\odot^{-0.35}]}$$

In [40]:

```
msun = c.M_sun.value
mtot = 1e9*msun

#Value in kg
xi0 = mtot/( (1/(-0.35))*( ((100*msun)**(-0.35)) - ((0.08*msun)**(-0.35))  ))

print(f'The normalization constant is {xi0:.2e}.')
```

```
The normalization constant is 1.26e+49.
```

In [42]:

```
def ngal(lower_m, upper_m):
    """
    Takes a range of masses and computes the number of galaxies within the range.

    """

    return xi0*( (1/(-0.35))*( ((upper_m*msun)**(-0.35)) - ((lower_m*msun)**(-0.35))  ))
```

O type

In [50]:

```
no = ngal(20,100)
print(f'There are {no:.2e} O type stars.')
```

```
There are 1.35e+38 O type stars.
```

B type

In [51]:

```
nb = ngal(3,20)
print(f'There are {nb:.2e} B type stars.')
```

```
There are 2.96e+38 B type stars.
```

A type

In [55]:

```
na = ngal(1.7,3)
print(f'There are {na:.2e} A type stars.')
```

```
There are 1.34e+38 A type stars.
```

F type

In [56]:

```
nf = ngal(1.1,1.7)
print(f'There are {nf:.2e} F type stars.')
```

```
There are 1.22e+38 F type stars.
```

G type

In [57]:

```
ng = ngal(0.8,1.1)
print(f'There are {ng:.2e} G type stars.')
```

```
There are 1.02e+38 G type stars.
```

K type

In [58]:

```
nk = ngal(0.6,0.8)
print(f'There are {nk:.2e} K type stars.')
```

```
There are 1.03e+38 K type stars.
```

M type

In [59]:

```
nm = ngal(0.08,0.6)
print(f'There are {nm:.2e} M type stars.')
```

```
There are 1.10e+39 M type stars.
```

In [64]:

```
nstars = pd.DataFrame({'Types': ['O', 'B', 'A', 'F', 'G', 'K', 'M'],
                       'Number': [no,nb,na,nf,ng,nk,nm]})
nstars
```

Out[64]:

	Types	Number
0	O	1.351253e+38
1	B	2.957217e+38
2	A	1.340424e+38
3	F	1.223687e+38
4	G	1.020945e+38
5	K	1.025404e+38
6	M	1.096517e+39

Continuing the rest of this in the coding notebook...

In [ ]:

Problem 4

Active Galactic Nuclei. Galaxy interactions often result in gas being funneled to the center of a galaxy, which triggers a growth phase of the supermassive black hole. In this problem, you'll calculate the typical timescale over which this phase would be visible as an AGN. Suppose an elliptical galaxy undergoes a merger and the central black hole begins to accrete matter at the Eddington rate. If the total gas mass, M gas , funneled to the black hole is 1% of the overall mass of the galaxy's bulge, M bulge , estimate the maximum lifetime of the resulting growth phase. The measured stellar velocity dispersion of this elliptical galaxy is about 220 km/s. The publication McConnell & Ma 2013, ApJ, 764, 184M gives a comprehensive analysis of the scaling relationships between black holes and their host galaxies. The AGN luminosity is about 10 13 L d .

$$M_{gas} = 0.01 M_{buldge}$$

velocity dispersion:

$$\sigma = 220[km/s]$$

$$L_{AGN} = 10^{13} L_\odot$$

Scaling relation from the paper:

$$log_{10} M_* = \alpha + \beta log_{10} X$$

where

$$X = \frac{\sigma}{200 km/s}$$

According to the paper:

Our full sample of 72 galaxies yields an intercept α = 8.32 ± 0.05 and slope β = 5.64 ± 0.32. When upper limits are added, the sample of 164 galaxies yields α = 8.15 ± 0.05 and β = 5.58±0.30. The

so we will use

$$\alpha = 8.15$$

$$\beta = 5.64$$

Then, applying this to our values, we get:

$$log \frac{M_{BH}}{M_\odot} = 8.15 + 5.64 log_{10} \frac{200 km/s}{200 km/s}$$

This second term then disapears and we get:

$$M_{BH} = 10^{8.15} M_\odot$$

In [82]:

```
alpha = 8.15
beta = 5.64
mbh = c.M_sun* 10**(alpha)
print(f'The mass of the black hole is {mbh:.2e}, or {10**(8.15):.2e} solar masses.')
```

```
The mass of the black hole is 2.81e+38 kg, or 1.41e+08 solar masses.
```

(b) (3 pts) Using the appropriate scaling relationships given in McConnell & Ma, what is the estimated bulge mass of this Galaxy in units of M d ? Describe the scaling relationship that you chose and why.

$$log[\frac{M_{BH}}{M_\odot}] = \alpha + \beta log[\frac{M_{buldge}}{10^{11} M_{sun}}]$$

$$log[\frac{M_{buldge}}{10^{11} M_{sun}}] = \frac{1}{\beta}[log[\frac{M_{BH}}{M_\odot}] - \alpha]$$

$$M_{buldge} = 10^{11} M_{sun} 10^{\frac{1}{\beta}[log[\frac{M_{BH}}{M_\odot}] - \alpha]}$$

In [88]:

```
mbuldge = 1e11*c.M_sun*10**(np.log10((1/beta)* (mbh/c.M_sun)) - alpha)
print(f'The mass of the buldge is {mbuldge:.2e} or {1e11*10**(np.log10((1/beta)* (mbh/c.M_sun)) - alpha):.2e}:
```

```
The mass of the buldge is 3.53e+40 kg or 1.77e+10: solar masses.
```

(c) (1 pts) What is the total amount of gas mass funneled into the black hole in units of M d ?

In [102…

```
mgas = 0.01*mbuldge/c.M_sun
print(f'The total ammount of gas is {mgas:.2e} solar masses.')
```

```
The total ammount of gas is 1.77e+08 solar masses.
```

(d) (2 pts) The Eddington accretion rate is given by: m 9 “ □c L 2 where L is the AGN luminosity, □ is the e□ciency (typically 0.1 is assumed), and c is the speed of light. What is the Eddington accretion rate (in solar masses per year) for this growing black hole?

$$\dot{m} = \frac{L}{\epsilon c^2}$$

$$\epsilon = 0.1$$

and given

$$L_{AGN} = 10^{13} L_\odot$$

$$\dot{m} = \frac{10^{13} L_\odot}{0.1 c^2}$$

In [98]:

```
mdot = ((1e13*c.L_sun)/(0.1*(c.c**2))).decompose()
print(f'The acretion rate is {mdot:.2e}.')
```

```
The acretion rate is 4.26e+23 kg / s.
```

(e) (2 pts) Assuming a constant accretion rate with time, how long will it take for all the available gas to be accreted? Give your answer in Myr.

In [118…

```
t = ((mgas*u.Msun)/(mdot)).to(u.Myr)
print(f'It will take {t:.2e} for the available gas to be acreted.')
```

```
It will take 2.62e+01 Myr for the available gas to be acreted.
```

f (1 pts) How does this compare with the timescale for a typical merger, which is about 1 Gyr? Could this explain why only „ 10% of galaxies are observed to host an AGN at any given time?

In [115…

```
t_typical = 1e9*u.year

ratio = (t/t_typical).decompose()

print(f'The ratio of this timescale to the timescale of a typical merger is {ratio:.2e}.')
```

```
The ratio of this timescale to the timescale of a typical merger is 2.62e-02.
```

Since this gives such a small vale, it tells us that the merger timescale is significantly longer thatn the acretion time here, and does explain why only about 10% of galaxies might host an AGN at any given time.