

Homework 2: Written Problems

Kelcey Davis

Colaborators: rachel, london, bjorn

```
In [1]: import astropy.units as u
import astropy.constants as c
from astropy.coordinates import SkyCoord
from astropy.time import Time
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import glob
%matplotlib inline
```

```
In [2]: plt.rcParams['figure.figsize'] = (10, 10)
plt.rc('axes', labelsz=14)
plt.rc('axes', labelweight='bold')
plt.rc('axes', titlesz=16)
plt.rc('axes', titleweight='bold')
plt.rc('font', family='sans-serif')
```

Problem 1

Distance to and Mass of our Galaxy's SuperMassive Black Hole (SMBH).

Imagine that you have used an 8-m telescope to observe stars toward our Galaxy's Center (GC) regularly for more than 20 years. You happen to notice that one star moves back and forth across the sky in a straight line as it orbits the SMBH. This means that its orbit is directly edge-on. We are also seeing the longest extent of the orbit, meaning that when it appears to be farthest from the SMBH, it is indeed at its farthest point in all 3 dimensions (apocenter), and when it appears to be closest to the SMBH, it is in all 3 dimensions (pericenter). You measure the following properties of the star's orbit:

- The extent of the orbit, s , is: $s = 0.2482$

- The period of the orbit (P) is: $P = 15.24$ years

- The absolute value of the radial velocity of the orbit is: $v_{\text{peri}} = 7326 \text{ km s}^{-1}$ at pericenter and $v_{\text{apo}} = 473 \text{ km s}^{-1}$ at apocenter

Please answer the following questions and be sure to include a reality check for your answers.

a What is the eccentricity (e) of the star's orbit?

telescope aperture: 8m

observation time: 20 years

orbit length: 0.248"

period: $P = 15.24$ years

radial velocity at pericenter: $v_{\text{per}} = 7326 \text{ km/s}$

radial velocity at apocenter: $v_{\text{apo}} = 473 \text{ km/s}$

$$r = \frac{a(1-e^2)}{1+e\cos(\theta)}$$

$$r = \frac{a(1+e)(1-e)}{1+e\cos(0)}$$

$$r_p = a(1+e)$$

$$r_p = a(1-e)$$

we will assume that angular momentum is conserved:

$$L = rmv$$

$$r_p m v_p = r_a m v_a$$

$$\frac{r_p}{v_a} = \frac{r_a}{v_p}$$

$$r_p = \frac{r_a v_a}{v_p}$$

$$a(1+e) = \frac{a(1-e)v_a}{v_p}$$

$$1+e = \frac{v_a(1-e)}{v_p}$$

$$\frac{1+e}{1-e} = \frac{v_a}{v_p}$$

$$e = \frac{v_p - v_a}{v_a + v_p}$$

```
In [6]: e = (7326 - 473)/(473+7326)
print(f'The orbital eccentricity is e = {round(e,2)}.')
```

The orbital eccentricity is $e = 0.88$.

b What is the semi-major axis (a) in AU of the orbit?

$$r = \frac{a(1+e)(1-e)}{1+e}$$

$$e = 0.88$$

$$r = 0.2482 \text{ arcsec}$$

$$r = a \frac{(1+0.88)(1-0.88)}{1+0.88}$$

```
In [11]: frac = ((1+e)*(1-e))/(1+e)
print(f'r = {round(frac, 3)} a')
```

$$r = 0.121 \text{ a}$$

$$a = \frac{r}{0.121}$$

```
In [15]: a = 0.2482/frac
print(f'a = {round(a,1)} [arcsec]')
```

$$a = 2.0 \text{ [arcsec]}$$

c What is the mass of the black hole (MBH) in solar masses M_{\odot} ?

Total energy is conserved.

$$K_{BH} + U_{BH} = K_{Star} + U_{Star}$$

$$\frac{m_{BH}v_{BH}^2}{2} + \frac{Gm_{BH}m_{star}}{r_{BH}} = \frac{m_{Star}v_{Star}^2}{2} + \frac{Gm_{BH}m_{star}}{r_{BH}}$$

$$\frac{m_{BH}v_{BH}^2}{2} = \frac{m_{Star}v_{Star}^2}{2}$$

$$m_{BH} = \frac{m_{Star}v_{Star}^2}{v_{BH}^2}$$

d What is the distance to the Galactic Center in kpc?

Problem 2

Stegosaurus Stomp

a Estimate the approximate number of times that the Sun has circled the center of our Galaxy since our Sun's formation

We orbit the galactic center about once every 250 million years. Our sun is about 4.6 billion years old.

```
In [8]: orbits = 4600000000/250000000
print(f'We have orbited the galactic center about {round(orbits,1)} times.')
```

We have orbited the galactic center about 18.4 times.

b Where in the Galaxy was the Sun when the dinosaurs died off? Report your answer in phase angle (in units of degrees), where the angle is measured relative to the Galactic Center. Our current phase angle is 0° , and if we moved clockwise 1/4 of the way around the Galaxy, our phase angle would be 90° , on the complete opposite side of the Galaxy would be 180° , and so on.

One orbit around the galaxy is about 250 million years. the dinosaurs died off around 65 million years ago, or 26% of the time it would take the sun to orbit the galaxy.

If consider the full orbital path to be 360 degrees, we can think of the angle as 26% of 360 degrees , or **94 degrees**.

Problem 3

Just take the M5

The globular cluster M5 has an overall apparent visual magnitude of $m_V = 5.95$. Its overall absolute magnitude is $M_V = -9.92$. It is located about 7.5 kpc from Earth and is about 6.1 kpc above the Galactic midplane

a Estimate the amount of interstellar extinction between M5 and the Earth.

$$m_V = 5.95$$

$$M_V = -9.92$$

$$d = 7.5 \times 10^3 \text{ pc from Earth}$$

$$d = 6.1 \times 10^3 \text{ pc above galactic midplane}$$

using 12.1 from B.O.B.:

$$m_\lambda = M_\lambda + 5 \log_{10} d - 5 + A_\lambda$$

$$A_\lambda = m_\lambda - M_\lambda - 5 \log_{10} d + 5$$

```
In [17]: A = 5.95 + 9.92 - (5*np.log10(7.5e3)) + 5
print(f'The extinction is {round(A, 2)}.')
```

The extinction is 1.49.

b What is the amount of interstellar extinction per kpc?

```
In [23]: Apc = A/7.5e1
print(f'The extinction per kpc is {round(Apc, 4)}.')
```

The extinction per kpc is 0.0199.

Problem 4

It's a Brick. In space

Red Clump (RC) giant stars are good standard candles since they have a small luminosity (an absolute magnitude of about $K_s = -1.60$ mag, corrected for population synthesis modeling and translated from K to K_s) and color spread (an intrinsic (H- K_s) color of 0.07), trace the stellar density of the Galaxy quite well, and are easy to identify in a color magnitude diagram (they are highlighted in Figure 1 below). These stars were observed toward an extremely dense molecular cloud in the center of the Galaxy, called "the Brick" (or G0.253+0.012) and analyzed in Longmore et al. 2012, ApJ, 746, 117L. The apparent magnitude and color where the RC stars diminish (farther away / redder stars are blocked by the cloud) are used to determine the extinction and distance toward the Brick. Figure 1 (and the caption) describes the data in more detail. Using these data and the following relevant information

- The extinction law power-law α for the Galactic Center is about $\alpha \approx 2.21$
- The central wavelengths of the H and K_s bands are $1.677 \mu\text{m}$ and $2.168 \mu\text{m}$, respectively
- In this wavelength range, the extinction can be approximated as a power-law of the form $\lambda \lambda_0^\alpha$

a Determine the color / reddening at the location of the Brick and the apparent magnitude of the RCs in Figure 2.

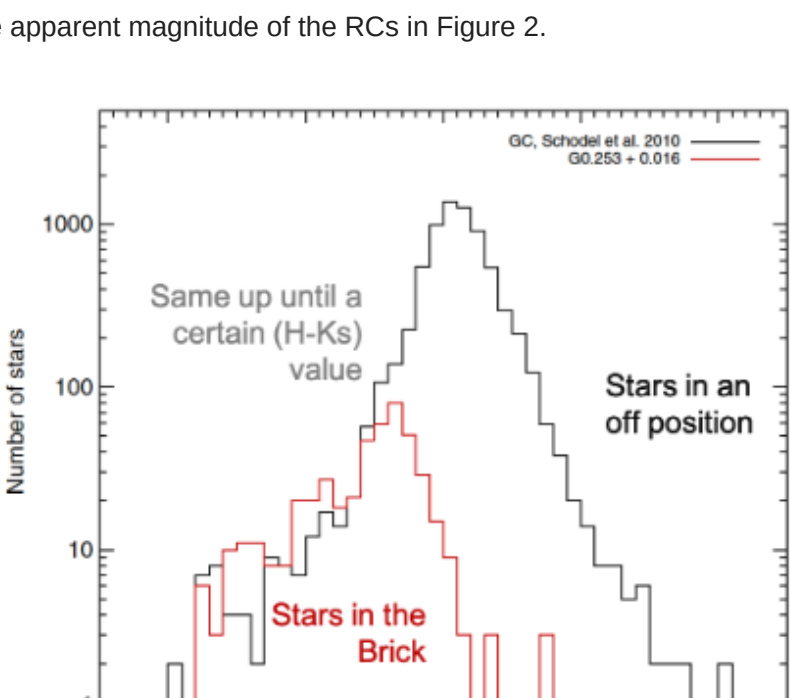
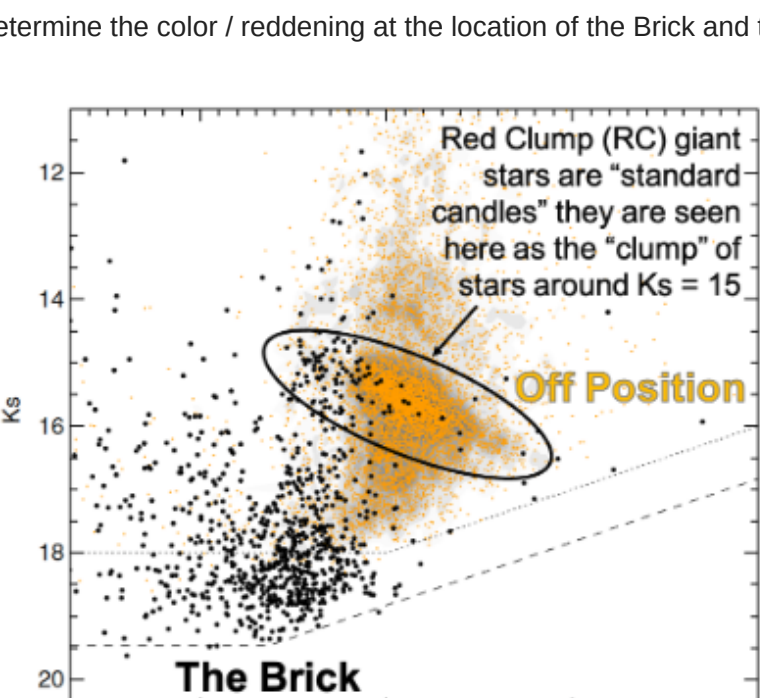


Figure 1: *Left:* Color Magnitude Diagrams from Longmore et al. (2012), comparing the apparent K_s -band magnitude on the y-axis and the (H- K_s) color on the x-axis. This plot shows stars on sightlines toward the Galactic Center and the source data is from the VLT/NACO (Schödel et al. 2010). This plot shows stars toward the cold, dense molecular cloud in the Galactic Center known as 'the Brick' in black, and toward an off position in orange and gray. *Right:* This plot compares the number of stars in each (H- K_s) bin for the Brick (red) and the off position (black) for the data from VLT/NACO, shown on the left.

The brick is centered around a color of $(H - K_s) \approx 2$ and an apparent magnitude of $K_s \approx 15$.

They span a range outlined in an oval in figure one around $1.3 < (H - K_s) < 2.9$ and $16.5 < K_s < 14.5$

b Determine the K_s band extinction (A_{K_s}) on the path to the Brick

$$E(X - Y) = A_x - A_y = (X - Y) - (X_0 - Y_0)$$

$$A_H - A_{K_s} = (H - K_s) - (H - K_s)_0$$

$$A_{K_s} = -2 + (1.677 \mu\text{m} - 2.168 \mu\text{m}) - A_H$$

$$A_{K_s} = -2.491 - A_H$$

$$A_H \approx \lambda_\alpha = 1.677^{2.21}$$

$$A_H \approx 3.13$$

$$A_{K_s} \approx -2.491 - 3.1 \approx -5.62$$

c Using these values, determine the distance to the Brick in kpc

$$A_\lambda = m_\lambda - M_\lambda - 5 \log_{10} d + 5$$

In [] :

In [] :