PHYS 4720/6720 - Galaxies and the ISM
Prof. Cara Battersby, Spring 2023

Problem Set #1

Due: Monday January 30 at 12:30pm.

Instructions:

- Please include your name, who you worked with, any resources you used on the first page of your submitted problem set.
- This problem set includes a coding problem set in Google Colab and this Written Problem Set. Please copy the colab file, save in your drive as "LastName_PS1," complete all cells in that colab notebook, and share with me on google (cara.battersby@uconn.edu). For the written portion, please either neatly handwrite or typeset the written problem set (if typeset save and submit as PDF). You can hand in your written problem set in person or through a link I will open at the start of class.
- The 'Welcome to Colaboratory' notebook posted is available for reference and I highly recommend you check it out, but you don't need to hand it in.

General advice and reminder on submitting professional assignments for this class.

1. Your written work must be **highly legible or typeset**.
2. UNITS! This isn't math, your answer is nothing without units. You can haz units! you must haz units. I can't emphasize this enough.
3. The expectation for this problem set is that you clearly demonstrate what you are calculating and why, and clearly outline the steps and process for solving the problem. The focus is on problem-solving skills, not getting the right numerical answer, and the clearer your work is, the easier it is to assign partial credit. You are expected to outline the problem (sometimes a diagram helps), present your strategy for solving it (including relevant equations, values, etc.), and solve it.
4. Finally, a "reality check" on your numerical answer is expected (does this answer make sense? Why or why not?).
5. Collaboration on problems is allowed (even encouraged!), but the final written work must be your own. ***A friendly reminder: Prof. Battersby has a zero-tolerance policy on cheating, including posting or looking up answers online, seeking or using previous solutions, copying, etc. Please just don't do it.***

Reminder: Problem Sets will be spot-graded, and overall completeness will be worth 5 pts.
Coding Problems: 5 problems for 25 pts total

Written Problems: 7 problems for 25 pts total
This problem set contains a number of problems that cover some basic astrophysics fundamentals we will use in this class (angular sizes, event horizons, magnitudes, flux and luminosity, etc.) that were covered in PHYS2701/2702. These should be somewhat straightforward for those of you who have taken these classes. For those of you who have not, please ask me questions, ask your classmates for help, consult Carroll & Ostlie, or the internet for help! You're not supposed to just 'know!'

1. **(2 pts) Angular Size.** A second of arc (or arcsecond) is 1/60 of a minute of arc (or arcminute), which in turn is 1/60 of a degree of arc on the sky. So an arcsecond is 1/3600 of a degree. A radian, as youll hopefully all remember, is the angle subtended at the center of a circle by a portion of the circle equal in length to the circles radius. A steradian is a measure of area on the sky in units of square radians.

   (a) From these definitions, write and evaluate an expression for the number of arcseconds in a radian. Give it to the nearest arcsecond.

   (b) The Hubble Space Telescope can detect 30,000 galaxies brighter than m=32 in a 10 arcmin x 10 arcmin region of sky. If we surveyed the entire sky (not just one hemisphere), how many galaxies would Hubble detect?

2. **(2 pts) Black Holes.** The observation that all galaxies have supermassive black holes in their centers begs the question: how were they created? The easiest theory is that small black holes were created in the early universe by the first stars (known as Population III stars)when they exploded as supernovae. These black holes would have masses of 10 $M_{\odot}$. These could then grow over time, to reach millions, or even billions, of Solar masses by today.

   However, we have observed galaxies at redshifts greater than six, only one billion years after the Big Bang, which have black holes with masses in excess of 10^9 $M_{\odot}$. Conventional methods of black hole growth would prevent a Population III black hole seed from growing so fast. Due to this difficulty, a number of theorists are now studying so-called 'direct-collapse black holes,' which is when a large cloud of gas, with M$\sim 10^5 M_{\odot}$ collapses directly into a black hole. Calculate the radius of the **event horizon** in these two cases (Pop III 'seeds' and direct-collapse), and discuss one object (for each case) in the physical world which is of a similar size.

3. **(4 pts) Galaxy Rotation.** One piece of evidence against the extragalactic nature of 'spiral nebulae' came came from Adriaan van Maanen, who in 1916 claimed to see the galaxy M101 rotating. The angular rotation rate he reported was 0.02 arcsec/year. The assumed diameter of M101 was 100 kpc.

   (a) Using van Maanens (flawed) observation and the assumed diameter of M101, estimate the speed of a point at the edge of the galaxy and compare it to the characteristic rotation speed of the Milky Way, 220 km/s.

   (b) The diameter of M101 is actually 45.4 kpc. Assuming it rotates at the same velocity as the Milky Way, estimate the angular rotation rate of stars at the edge of the galaxy. How long would it take for the galaxy to rotate through 1 arcsec? The maximum resolution observable from the ground today is roughly 0.5 arcsec. Could astronomers in the 1920s have detected the rotation of M101?

4. **(4 pts) When galaxies collide.** Two galaxies collide, each of which has an apparent magnitude of $m_{gal} = 12$. Assuming flux is conserved, what is the final magnitude of the merger product?

5. **(4 pts) Like the stars in the sky.** The Andromeda galaxy has a distance of 0.9 Mpc and an apparent magnitude $m_{gal} = 3.5$. Assuming each star has the same intrinsic brightness of the Sun (absolute magnitude of $M_{sun} = 4.74$ (was written as 5.48)), estimate how many stars are in Andromeda.

6. **(5 pts) Gravity.** Show that the total, gravitational potential energy of a uniform density sphere is

$$U = -\frac{3}{5}\frac{GM^2}{R} \tag{1}$$

where M is the total mass of the sphere and R is its radius. *Hint:* The equation for the gravitational potential energy of a point mass, dm_i at radius r, with M_{enc} mass enclosed within that shell:

$$dU = -G\frac{M_{enc}dm_i}{r} \tag{2}$$

7. **(4 pts) All together now.** You observe a *virialized* globular cluster of stars that contains 20,000 stars each with M = 0.5 $M_\odot$ on average and an average velocity dispersion of $\langle v \rangle = 10$ km s^{-1}. The observed angular diameter of the star cluster is $3'$, across.

   (a) What is the total mass of the cluster in units of $M_\odot$?

   (b) What are the expressions for the **total** kinetic (K) and potential (U) energies of the cluster?

   (c) What is the physical size of the cluster *(hint: use the virial theorem)*? Give your answer in pc.

   (d) How far away is the cluster (*Hint: $3'$ is still considered a small angle*)? Give your answer in kpc (kilo-parsec, 1 kpc = 1,000 pc).