Homework # 2 - Stellar Structure

Due: Thursday, February 9 at 2:00pm
Reading: Lecture Notes, Pols chapter 2-3.

Instructions: Homework must be highly legible, on white paper, and with multiple pages stapled. List at the top:

- Your name.
- Collaborators (if applicable).
- Estimated time spent to complete.

Homeworks are due at the beginning of class. Late homeworks will be marked down by 50% until the assignment is graded and returned, and will receive no credit after that. Clearly outline the process for solving the problem: partial credit will be given for presenting the correct steps to solve problem, even if you do not achieve the correct numerical answer.

Problems (3 questions, 20 total points):

1. In addition to eclipsing binaries, the radius of stars can be directly measured using interferometry. The CHARA array, run by Georgia State University, is currently the most powerful optical interferometer in the world, achieving a 1σ angular resolution of 200 μas ($2\times10^{-4\prime\prime}$). Consider the following types of stars: (**7 pts total**)

Table 1: Radii and space densities of stars.

Stellar Type	**Example**	**Radius**	**Density** (stars/pc^3)
Main Sequence	Sun	$\sim 1 R_\odot$	4×10^{-3}
K Giant	Arcturus	$\sim 25 R_\odot$	4×10^{-5}
M Supergiant	Betelgeuse	$\sim 1000 R_\odot$	5×10^{-8}
White Dwarf	Sirius B	$\sim 1 R_\oplus$	5×10^{-3}

   (a) What kind of star is most often measured with interferometry? (**3 pts**)
   (b) About how many stars can have radii measured with 10% precision (S/N=10) by CHARA? (**3 pts**)
   (c) Beyond the angular resolution uncertainty, what else might contribute uncertainty to the physical radius measurement? (**1 pt**)

2. The density profile of the Sun can be modeled with an empirical best-fit formula:

$$\rho(r) = [519(r/R)^4 - 1630(r/R)^3 + 1844(r/R)^2 - 889(r/R) + 155] \text{ g cm}^{-3}$$

at $r < 0.5R$, and $r \simeq 0.1$ g/cm^3 at $r > 0.5R$. (This fit comes mostly from astroseismology data, which we will discuss later in the course.) Here $R = 6.96\times10^{10}$ cm, the radius at the surface of the Sun. Plot the enclosed mass profile $m(r)$ of the Sun. Label the axes, and mark the average density $\bar{\rho}$ and radius where $\rho(r) = \bar{\rho}$. (**5 pts**)

3. Assume a star made solely of ionized hydrogen. (**8 pts total**)

   (a) Create (by sketch or computer) a plot of log density $\log(\rho)$ versus log temperature $\log(T)$ (where log is the base-10 logarithm). Mark the boundaries between regions dominated by ideal gas pressure, radiation pressure, and nonrelativistic electron degeneracy pressure (i.e., where $P_{\rm gas} = P_{\rm rad}$ and $P_{\rm gas} = P_{\rm e}$. (**5 pts total**)

   (b) Now consider the Sun's density profile given in Problem 2. At what radii do you expect ideal gas pressure, radiation pressure, and nonrelativistic electron degeneracy pressure to dominate? (**3 pts total**)

Extra Question for 4710 Honors and 6710 (1 question, 5 total points):

4. Plot a second set of boundary lines in the $\log(\rho) - \log(T)$ plot for a star made purely of ionized metals (i.e., elements heavier than helium). (**5 pts**)