Homework # 4 - Fusion (and Other Energy Sources)

Due: Thursday, March 2 at 3:30pm
Reading: Lecture Notes, Pols chapter 6, BOB chapter 10.3.

Instructions: Homework must be highly legible, on white paper, and with multiple pages stapled. List at the top:

- Your name.
- Collaborators (if applicable).
- Estimated time spent to complete.

Homeworks are due at the beginning of class. Late homeworks will be marked down by 50% until the assignment is graded and returned, and will receive no credit after that. Clearly outline the process for solving the problem: partial credit will be given for presenting the correct steps to solve problem, even if you do not achieve the correct numerical answer.

Problems (2 questions, 20 total points):

1. Assume that the p-p chain is responsible for all of the Sun's luminosity. (In your last homework, you showed that more of the solar luminosity comes from the p-p chain than the CNO cycle, and we will ignore the CNO fraction here.) At the core temperature of the Sun, the pp3 chain can be ignored (it happens $\ll$1% of the time). (**10 pts total**)
   (a) How many neutrinos are generated by the Sun each second? Write your answer in terms of the relative rates of the pp1 and pp2 chains. (**4 pts**)
   (b) What is the neutrino flux (number/time/area) incident on the Earth? (**2 pts**)
   (c) Assuming Poisson statistics, how many neutrinos would you need to measure the relative fractions of the pp1 and pp2 chains to a precision of 1%? (**4 pts**)

2. Let's compare the rate of energy released from mass accreting around a supermassive black hole to the energy released by fusion in stars. (**10 pts total**)
   (a) What is the gravitational potential energy change for a particle with mass m that starts at 100 pc and falls to the innermost stable circle orbit (ISCO) around a nonspinning black hole? (**3 pts total**)
   (b) Divide your answer from part (a) by the rest energy of the particle. We call this quantity the radiative efficiency η, where $L = \eta \dot{M} c^2$ and $\dot{M}$ is the mass infall rate. (**2 pts total**)
   (c) What is the radiative efficiency of thermonuclear fusion via the p-p chain? Consider the mass deficit per proton in the complete reaction ($4\text{p} \rightarrow {}^4_2\text{He} + 2\text{e}^+ + 2\nu_e$) and divide this by the rest energy of the proton. (**3 pts total**)
   (d) At what mass accretion rate will an AGN exceed the light from fusion of an entire galaxy of Sun-like stars? (**2 pts total**)

Extra Question for PHYS 6710 (1 question, 5 total points):

3. As we discussed in class, quantum tunneling plays a key role in nuclear fusion in the centers of stars. It is also crucial for the alpha decay of heavy elements like ${}^{235}_{92}\mathrm{U}$ and ${}^{239}_{94}\mathrm{Pu}$. Here you can think of the alpha particle as "trapped" within the nucleus by a potential like the Coulomb barrier, which it then tunnels through to escape and cause alpha decay. The rate of decay λ is proportional to this tunneling probability, and the half-life $\tau_{1/2} = \ln(10)/\lambda$. The alpha decay of ${}^{235}_{92}\mathrm{U}$ releases an energy of $E = 4.68$ MeV with a half-life $\tau_{1/2} = 7.1 \times 10^8$ yr. The energy released by alpha decay in ${}^{239}_{94}\mathrm{Pu}$ is 5.24 MeV: what is its half-life? (**5 pts**)