Homework 5

In [1]:
```
import astropy.units as u
import astropy.constants as c
from astropy.coordinates import SkyCoord
from astropy.time import Time
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import glob
%matplotlib inline
```

In [2]:
```
plt.rcParams['figure.figsize'] = (10, 10)
plt.rc('axes', labelsize=14)
plt.rc('axes', labelweight='bold')
plt.rc('axes', titlesize=16)
plt.rc('axes', titleweight='bold')
plt.rc('font', family='sans-serif')
```

Problem 1

Polaris is a supergiant star with a mass of 5.4M d , a radius of 37.5R d , and a luminosity of 1260L d . (10 pts total)

$M_{polaris} = 5.4 M_{sun}$

$R_{polaris} = 37.5 R_{sun}$

$L_{polaris} = 1260 L_{sun}$

(a) What is the eﬀective surface temperature of Polaris? (2 pts)

$L = 4\pi R^2 \sigma T^4$

$T = [\frac{L}{4\pi\sigma R^2}]^{1/4}$

In [82]:
```
sigma = c.sigma_sb.decompose(bases=u.cgs.bases)
M = 5.4*c.M_sun.decompose(bases=u.cgs.bases)
R = 37.5*c.R_sun.decompose(bases=u.cgs.bases)
L = 1260*c.L_sun.decompose(bases=u.cgs.bases)
T = ((L)/(4*np.pi*sigma*(R**2)))**(1/4)
print(f'The effective temperature is {T:.2e}.')
```

```
The effective temperature is 5.62e+03 K.
```

(b) Polaris is on the asymptotic giant branch: it has exhausted its hydrogen and has a luminosity powered almost entirely by helium fusion. Assume that Polaris has Y “ 0.9 (and X “ 0, Z “ 0.1) and half of the star is available for 3α reactions. How long will Polaris survive on the AGB? (Remember that 3 He nuclei are burned in each 3α fusion reaction.) (4 pts)

In [83]:
```
Y = 0.9
X = 0.0
Z = 0.1
#Assume half of the star is available for reactions
```

So half of the star is available for reactions. It is about 90% He and 10% Heavier metals. That means our available He mass is:

$\mu = \frac{1}{2X + \frac{3}{4}Y + \frac{1}{2}Z}$

In [84]:
```
mHe = (M*.9)/2
mHe
```

Out[84]: 4.831836×10^{33} g

We know that:

$E = L\Delta t$ (lecture 1/31)

so

$\Delta t = \frac{E}{L}$

We can get the time by finding the total energy due to the nuclear reactions and dividing it by the luminosity.

The energy from one trippe alpha process is:

7.275 MeV.

via https://en.wikipedia.org/wiki/Triple-alpha_process

In [69]:
```
tripa = 7.275*u.MeV.decompose(bases=u.cgs.bases)
tripa
```

Out[69]: $7.275\ 1.6021766 \times 10^{-6}\ \frac{\mathrm{cm^2\,g}}{\mathrm{s^2}}$

In [70]:
```
#mass of one He atom
m_one_He = 4.002602*u.u.decompose(bases=u.cgs.bases)
#Total ammount of He atoms
nHe = mHe/m_one_He
#Total energy will be the energy consumed by burning up all He in sets of 3:
Etot = nHe*tripa/3
```

In [80]:
```
time = Etot.value/L.value
time
```

Out[80]:
```
0.000606930614664507
```

In [81]:
```
print(f'It would take {time:.2e} seconds.')
```

```
It would take 6.07e-04 seconds.
```

(c) Like many massive (2 ă M ă 8M d) supergiant stars, Polaris is a Cepheid variable star. Cepheid variability is driven entirely by dynamical eﬀects, with no heat transfer. What is the period of variability for Polaris? (Hint: Look back at the Lecture 4 notes.) (4 pts)

From lecture 1/31:

$period \approx \tau_{ff}$

Again from the notes,

$\tau_{ff} = \frac{1}{\sqrt{G\bar{\rho}}}$

In [93]:
```
rho = M/((4/3)*np.pi*(R**2))
tau = 1/np.sqrt(G*rho).value
print(f'The pulsation period is approximately {round(tau,3)} [s].')
```

```
The pulsation period is approximately 0.199 [s].
```

Problem 2

The “Helium ﬂash” is tremendously luminous but very short lived, emitting L “ 1 ˆ 10 10 L d in about 3 s. By what factor does the radius of the core expand due to the Helium ﬂash? Assume that all of the Helium ﬂash energy goes into expanding the core, and the initial degenerate core can be approximated as a white dwarf. (Both assumptions are reasonably accurate.) (5 pts)

In [20]:
```
L = 1e10*c.L_sun.decompose(bases=u.cgs.bases)
t = 3*u.s.decompose(bases=u.cgs.bases)
G = c.G.decompose(bases=u.cgs.bases)
```

All the energy goes in to the core expansion, so we can consider that:

$E = \Delta U$

For a uniform sphere,

$E_{grav} = \frac{-3}{5}\frac{GM^2}{R}$

$E = \frac{-3}{5}\frac{GM_{final}^2}{R_{final}} - \frac{-3}{5}\frac{GM_{WD}^2}{R_{WD}}$

We can also assume that the energy is related to the Luminosity by:

$E = -L\Delta t$

So

$-L\Delta t = \frac{-3}{5}\frac{GM_{tot}^2}{R_{final}} - \frac{-3}{5}\frac{GM_{tot}^2}{R_{WD}}$

$L\Delta t = \frac{-3}{5}GM_{tot}^2\frac{1}{\Delta R}$

$\frac{1}{\Delta R} = \frac{-5L\Delta t}{3GM_{tot}^2}$

I will also make the assumption that the core is about as massive as the sun, since this is a fair approximation for a white-dwarf-like core.

In [27]:
```
M = u.M_sun.decompose(bases=u.cgs.bases)
dR = 1/((5*L*t)/(3*G*(M**2)))
rchange = dR.decompose(bases=u.cgs.bases)
print(f'The change in radius is {rchange:.2e}')
```

```
The change in radius is 1.38e+15 cm
```

Problem 3

The periodic table below is a great general visualization of where the elements come from, but it skips the details. Describe the following. (5 pts total)

The diﬀerent nuclear reactions in “dying low mass stars” that lead to carbon (C) as opposed to tin (Sn).

For this question, following notes and this source: https://sites.ualberta.ca/~pogosyan/teaching/ASTRO_122/lect17/lecture17.html

For dying low-mass stars, there is no iron-catastrophe leading to heavier elements. The star will fuse He through the tripple alpha process until the He flash occurs where a All the He is burned in a few seconds, creating C.

This process looks like:

$3^4He \rightarrow^{12}_{6} C$

This occurs for degenerate cores when

$M_{core} = 0.45 M_*$

The luminosity generated by this burning is about $10^{10} L_{sun}$, comprable to the luminosity of a galaxy. However, this does not escape the star at once, so the star itsself does not outshine its host galaxy.

The diﬀerent nuclear reactions in “exploding massive stars” that lead to elements lighter than iron (e.g., oxygen O, neon Ne, and magnesium Mg)

Mostly following class notes for this one.

Stars on the Asymptotic Giant Branch lose mass acoding to:

$\frac{dM}{dt} \approx 10^{-4}\frac{M_{sun}}{year}$

These stars undergo nucleosythesis vis the "s-process" that creates these elements following:

$^{13}_{6}C +^{4}_{2} He \rightarrow^{16}_{8} O + n$

and in stars heavier than two solar masses,

$^{22}_{10}Ne +^{4}_{2} He \rightarrow^{25}_{12} Mg + n$

The nuclear reactions in “merging neutron stars” that lead to elements like gold and uranium.

The fusion of these elements is driven by rapid neutron capture nucleosynthesis called the r-process.

Taking mostly from the wikipedia page:

https://en.wikipedia.org/wiki/R-process

These captures inherently must be rapid to prevent any decay as the elements are built up. This process directly contrasts the s-process, which is a slower neutron capture.