Final Exam (v2)

Due: Friday, April 28 at midnight

Instructions: Your assignments must be highly legible, on white paper, and with multiple pages stapled. List at the top:

- Your name.
- Estimated time spent to complete.

Please work alone and do not collaborate on this exam. Otherwise this is an open-book exam: you can use class notes, the textbook, Google/Wikipedia, or any other print or online resources. You can also ask your instructor questions before the exam is due. Good luck!

Problems (4 questions, 40 total points):

1. Let's start with some conceptual questions about the equations of stellar structure. (**10 pts total**)
   (a) What are the initial assumptions for the equations of stellar structure?
   (b) What is the difference between the Eulerian and Lagrangian frameworks for these equations?
   (c) What equation of state governs most regions of most stars?
   (d) What kinds of stars (or regions of stars) are governed by the other equations of state?
   (e) What energy transport mechanism dominates most regions of most stars?
   (f) How do the fusion processes differ for solar-mass stars and $M > 2M_\odot$ stars (e.g., Sirius) that are on the main sequence?
   (g) How do the fusion processes differ for solar-mass stars that are on the main sequence versus after the main sequence?
   (h) What kind of fusion happens in brown dwarf stars?
   (i) What phenomenon (or phenomena) produces fusion of elements heavier than iron?

2. How much He fusion must occur in the "He flash" to make the core non-degenerate for a red giant star with a ZAMS mass of $1M_\odot$? Assume that the core starts out moderately degenerate with equal pressure support from (non-relativistic) electron degeneracy and ideal gas, and becomes non-degenerate when ideal gas makes up 90% of the pressure support. Recall that the He flash occurs at a specific core mass and (initial) temperature, and assume that 2/3 of the energy output from fusion increases the kinetic energy and 1/3 increases the potential energy. (**10 pts**)

3. Lambda Tauri is an eclipsing close binary that is an "Algol variable." Star A is a main-sequence star with a mass of 7.2 $M_\odot$, and star B is a post-MS subgiant with a mass of 1.9 $M_\odot$. The binary has a period of 4.0 days and the zero-age main sequence age of both stars is 33 Myr. (**10 pts total**)

   (a) How long will A remain on the main sequence? (**2 pts**)

   (b) Now let's work backwards to infer the properties of A and B before mass transfer occurred. What was the minimum mass of star B before it left the main sequence and lost mass as a giant star? (**3 pts**)

   (c) Assuming the orbital separation and total mass of the system has remained constant, and using the minimum mass of star B in part (b), what was the radius of B when mass transfer started? (**5 pts**)

4. The first generation of stars in the early Universe ("Population III") have no metals, just 75% hydrogen and 25% helium. (**10 pts total**)

   (a) How does the lack of metals ($Z = 0$) make a Population III star different from a subsequent, metal-enriched "Population I" star like the Sun? Make at least two calculations to quantitatively support your answer (for example, you might compare ratios of the equations for stellar structure). (**5 pts**)

   (b) It may be possible for collapsing primordial molecular clouds to continue accreting mass beyond the Jeans mass, collapsing so quickly that they form a seed black hole "quasi-star." What is the maximum opacity of a $10^5 M_\odot$ cloud with $T = 10^4$ K for it to remain below the Eddington limit and optically thin as it collapses? (**5 pts**)