Homework # 3 - Stellar Energy Transport and Generation

Due: Tuesday, February 21 at 2:00pm
Reading: Lecture Notes, Pols chapter 5, BOB chapter 10.

Instructions: Homework must be highly legible, on white paper, and with multiple pages stapled. List at the top:

- Your name.
- Collaborators (if applicable).
- Estimated time spent to complete.

Homeworks are due at the beginning of class. Late homeworks will be marked down by 50% until the assignment is graded and returned, and will receive no credit after that. Clearly outline the process for solving the problem: partial credit will be given for presenting the correct steps to solve problem, even if you do not achieve the correct numerical answer.

Problems (4 questions, 20 total points):

1. Most main-sequence stars are in hydrostatic equilibrium, have an equation of state dominated by the ideal gas law, and have energy transport dominated by radiation. (**8 pts total**)
   (a) Simplify the energy transport equation as $dT/dr \sim -T_c/R$ and with $T_{\text{eff}} \propto T_c$. Using the appropriate opacity for a hot ionized star, what is the proportionality of the central temperature T_c with total mass M, luminosity L, and radius R? (**2 pts**)
   (b) Now derive the proportionality of T_c with M and R using the hydrostatic equilibrium equation and the ideal gas law. Again, you can simplify by assuming that $dP/dr \sim -P_c/R$ and that the central density $\rho_c \propto \bar{\rho}$. (**2 pts**)
   (c) Combine the two equations above to derive the proportionality of L with M and R for a typical main sequence star. (**2 pts**)
   (d) The observed luminosity-mass relation is actually $L \propto M^{3.8}$ (see the notes from Lecture 1). What is the maximum mass of a star with Solar abundance ($X = 0.7$) before it exceeds the Eddington limit and blows off its outer layer? (**2 pts**)

2. Energy generated by fusion in the center of a star travels slowly outward via random-walking photons. What is the average time for energy to reach the surface of the following stars? You can assume that the stars have Solar composition, are almost entirely hot and ionized, and have uniform opacity and density at all radii. (Uniform opacity and density is a bad assumption, and is why your calculations will underestimate the actual energy escape time by a factor of $\gtrsim$100!) (**6 pts total**)
   (a) the Sun (**2 pts**)
   (b) Sirius A: $M = 2M_\odot$, $R = 1.7R_\odot$ (**2 pts**)
   (c) Betelgeuse: $M \simeq 20M_\odot$, $R \simeq 100R_\odot$ (**2 pts**)

3. Main sequence stars fuse hydrogen into helium by one of two pathways: the p-p chain and the CNO cycle. These reactions can be approximated as:

$$\epsilon_{pp} \simeq \epsilon'_{0,pp} \rho X^2 T_6^4 \tag{1}$$

$$\epsilon_{CNO} \simeq \epsilon'_{0,CNO} \rho X X_{CNO} T_6^{19.9} \tag{2}$$

These equations neglect electron screening, alternate reaction chains, and higher-order terms. The variables are density ρ, temperature $T_6 = T/(10^6\mathrm{K})$, hydrogen abundance $X = 0.7$, abundance of C+N+O $X_{CNO} = 0.0081$, $\epsilon'_{0,pp} = 1.08 \times 10^{-7}$ erg s^{-1} cm^3 g^{-2}, and $\epsilon'_{0,CNO} = 8.24 \times 10^{-26}$ erg s^{-1} cm^3 g^{-2}. What is the fraction of energy generated by each process in the following stars? (**6 pts total**)

   (a) the Sun with: $T_c = 15.7 \times 10^6$ K, $\rho_c = 150$ g/cm^3. (**2 pts**)

   (b) Sirius A, with $M = 2.06 M_\odot$ and $R = 1.71 R_\odot$. Use this mass and radius to estimate T_c from the relation derived in problem 1b, and similarly scale central density from the Solar value using the ratio between the mean density of Sirius and the Sun. (**3 pts**)

   (c) Assuming that both stars have the same fraction of mass available for hydrogen burning (i.e., Sirius A has 2.06 times more mass available for hydrogen fusion), what is the ratio between the total H-burning fusion energy output of Sirius compared to the Sun? Why is this different from the ratio of the surface luminosities observed as $L \propto M^{3.8}$? (**1 pt**)

Extra Question for 6710 (1 question, 5 total points):

4. Massive ($\gtrsim 100 M_\odot$) stars have an equation of state dominated by radiation pressure rather than the ideal gas law. How is L proportional to M and R for massive stars? (**5 pts**)