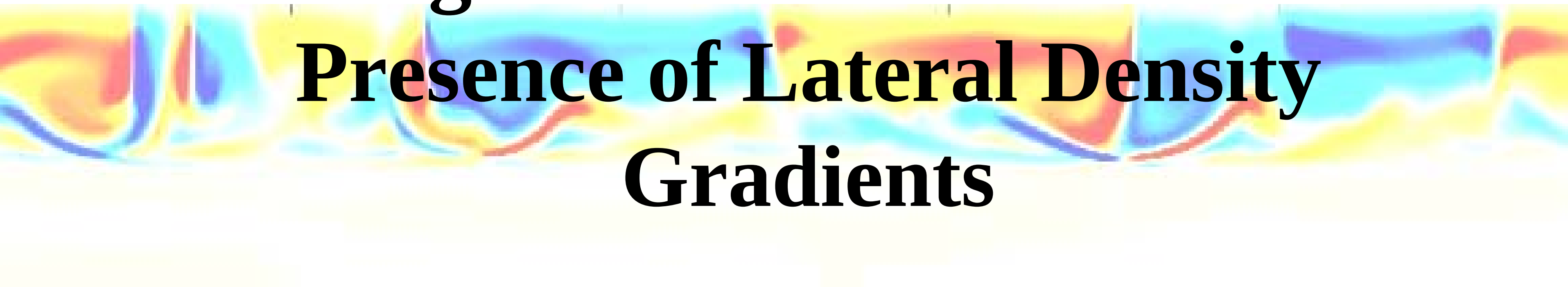
Langmuir Circulation in the Presence of Lateral Density Gradients



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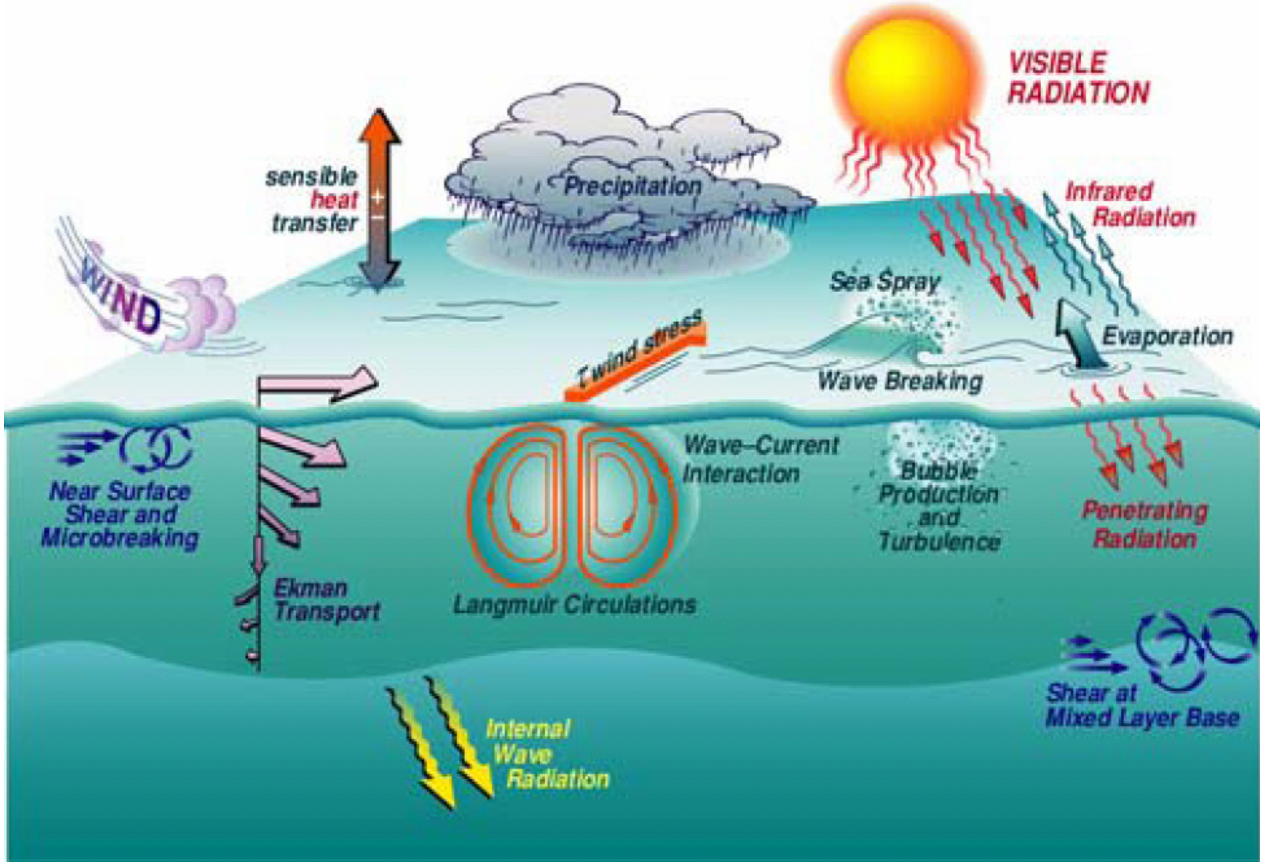


(Bob Weller, WHOI)

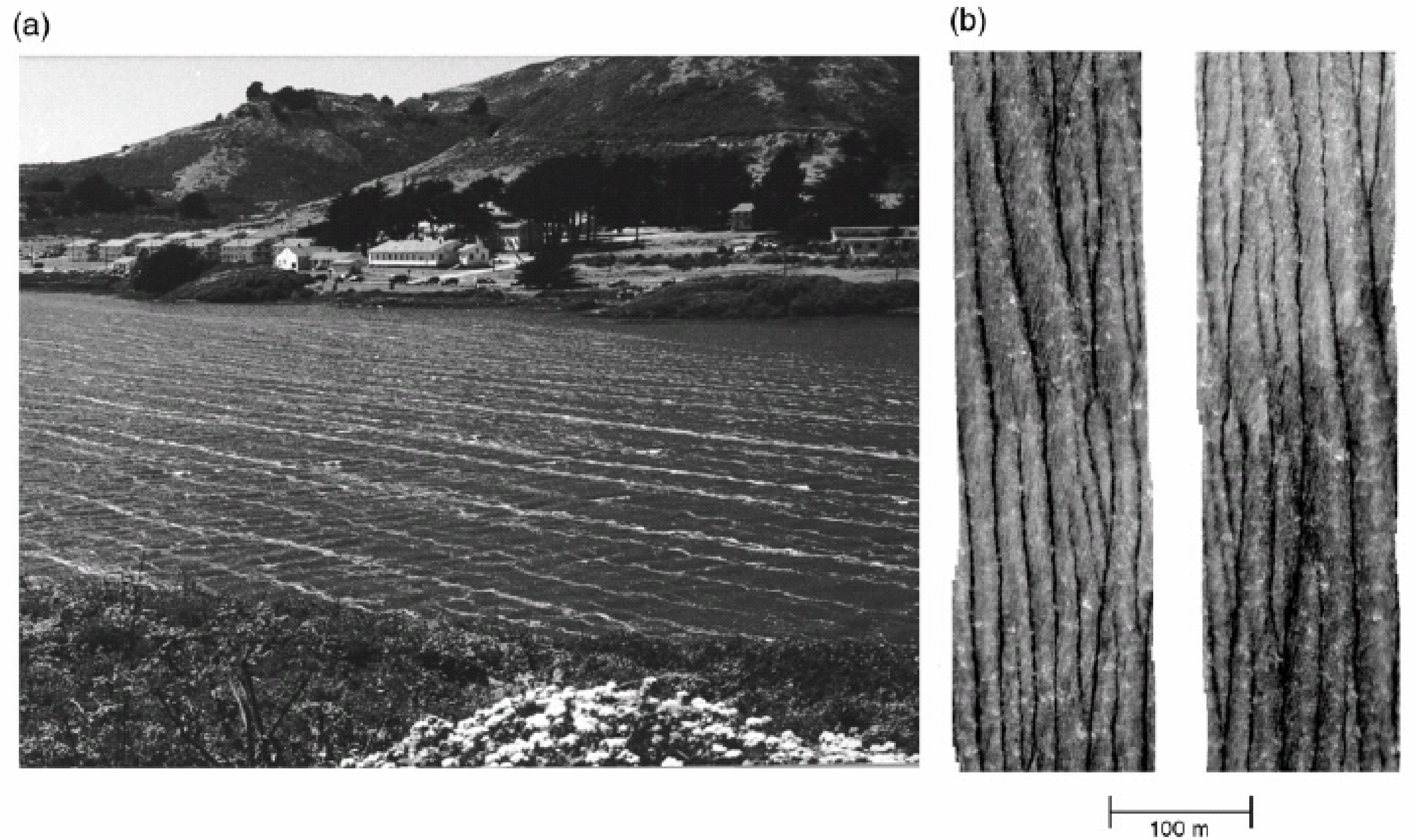
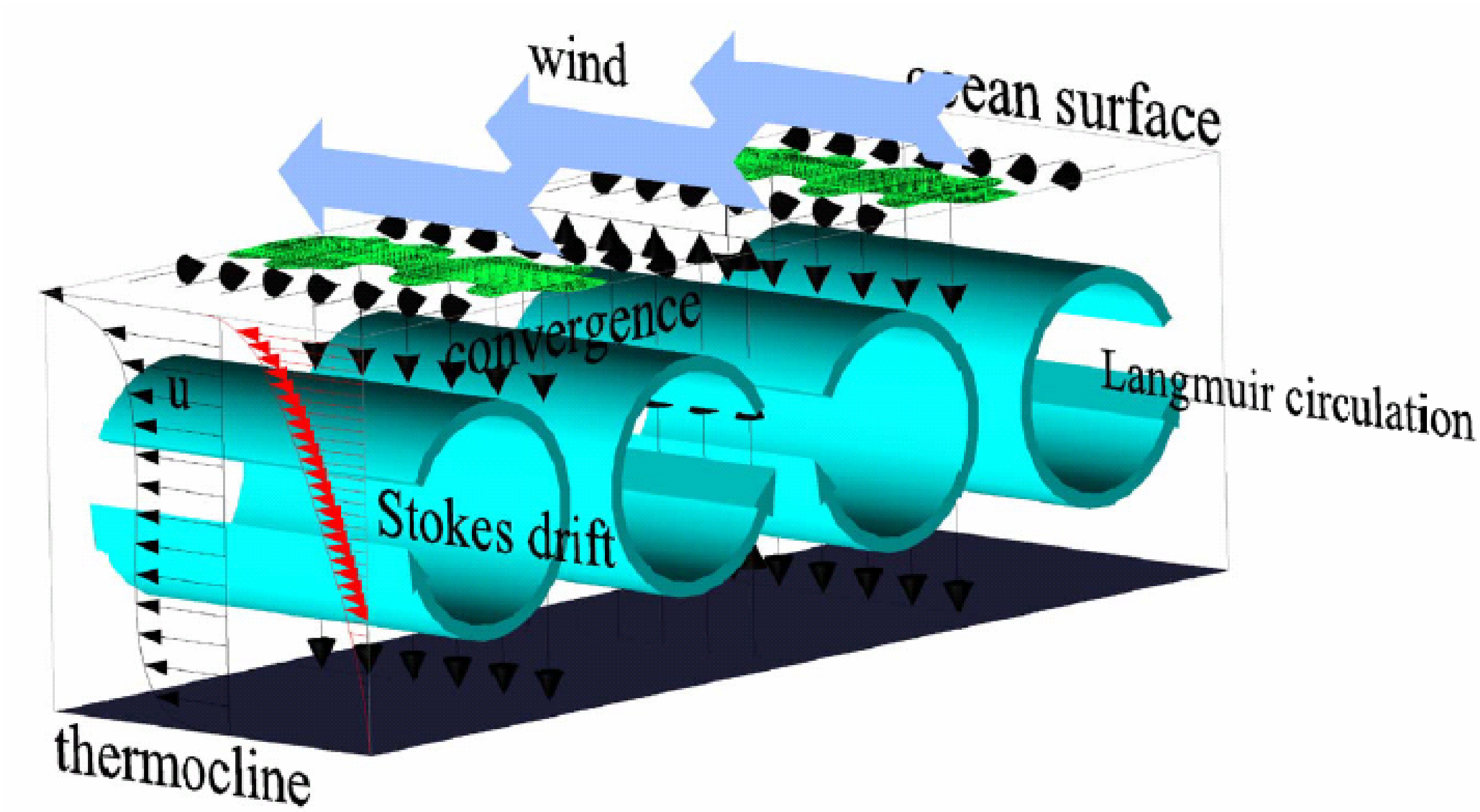
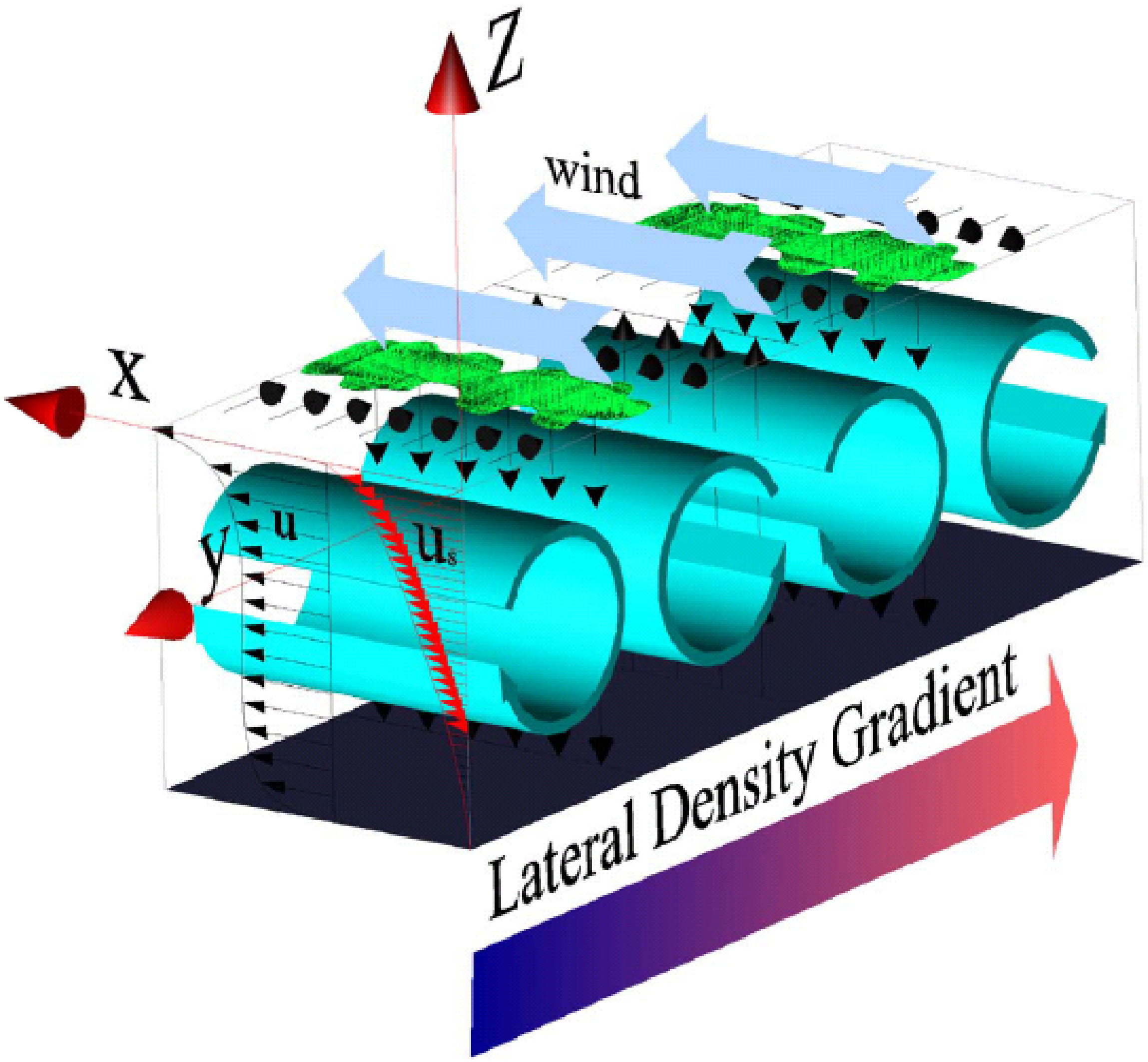


Figure1. Images of LC windrows: (left) a photograph of Rodeo Lagoon in CA (Szeri 1996), (right) an infrared image of the surface of Tampa Bay (Marmorino et al. 2005)


wind
ocean surface
convergence
u
Langmuir circulation
Stokes drift
thermocline

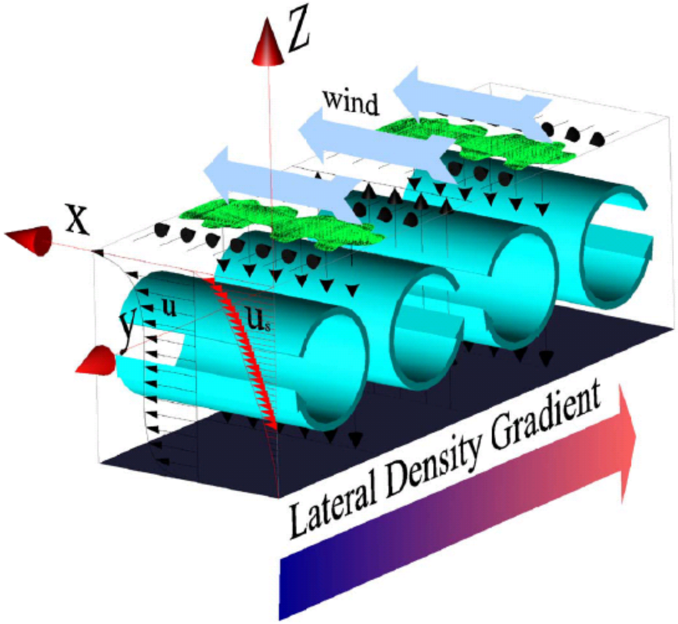
Process Study



Physical Ingredients

- Prescribed surface-wave Stokes drift $u_s(z)$
- Craik-Leibovich parameterization of surface wave effects though CL vortex force: $\vec{u}_s \times \vec{\omega}$

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho_0}\vec{\nabla}P - \frac{\rho}{\rho_0}g\vec{k} + \boxed{\vec{u}_s \times (\nabla \times \vec{u})} + \nu_e \nabla^2 \vec{u}$$



Physical Ingredients

- Imposed lateral stratification
- Non-hydrostatic dynamics
- No along-wind variation
- Coriolis acceleration not included

Scales

- Length (H) – mixed layer depth ~ 100m
- Velocity (U, V) –
  - x-velocity (u): $U = U_* \cdot R_*, \qquad U_* = \sqrt{\frac{\tau_w}{\rho}} \qquad R_* = \frac{U_* H}{\nu_e}$
  - cellular velocities (v, w): $V = \sqrt{U_{s_0} \cdot U_*}$
- Time ([t]) – convective scale: $T = H/V$
- Temperature (ΔT) – horizontal difference along one cell length scale

Mathematical Problem Formulation

Non-dimensional Equations

$$u_t - L_a(u_{yy} + u_{zz}) = -(\Psi_z u_y - \Psi_y u_z)$$

$$\Omega_t - L_a(\Omega_{yy} + \Omega_{zz}) = -(\Psi_z \Omega_y - \Psi_y \Omega_z) - Q_J \theta_y + u_s'(z) \cdot u_y$$

$$\theta_t - P_e^{-1}(\theta_{yy} + \theta_{zz}) = -(\Psi_z \theta_y - \Psi_y \theta_z) - A_0 \Psi_z$$

$$T = A_0 y + \theta$$

$$\Omega = \Psi_{yy} + \Psi_{zz}$$

Boundary Conditions:

$$\frac{\partial u}{\partial z}\Big|_{z=0,-1} = 1 \qquad \Omega|_{z=0,-1} = 0$$

$$\Psi|_{z=0,-1} = 0 \qquad \frac{\partial T}{\partial z}\Big|_{z=0,-1} = 0$$

Non-dimensional Parameters

- Laminar Langmuir Number: $La \equiv \frac{\nu_e}{VH} \Rightarrow \frac{1}{Re}$
- Peclet Number: $Pe \equiv \frac{VH}{\alpha_e} \Rightarrow \frac{\text{thermal diffusive time scale}}{\text{convective time scale}}$
- Grashof Number: $Gr \equiv \frac{\beta g H^3 \Delta T}{\nu_e^2} \Rightarrow \frac{\text{buoyancy}}{\text{viscous forcing}}$ $\qquad Q_J \equiv Gr La^2$
- Box width: L

Numerical Method
--- Semi-implicit pseudo-spectral method

1. FFT cross-wind : $u(y,z,t) = \sum_{n=-N_y/2+1}^{N_y/2} \hat{u}_n(z,t) e^{inky}$

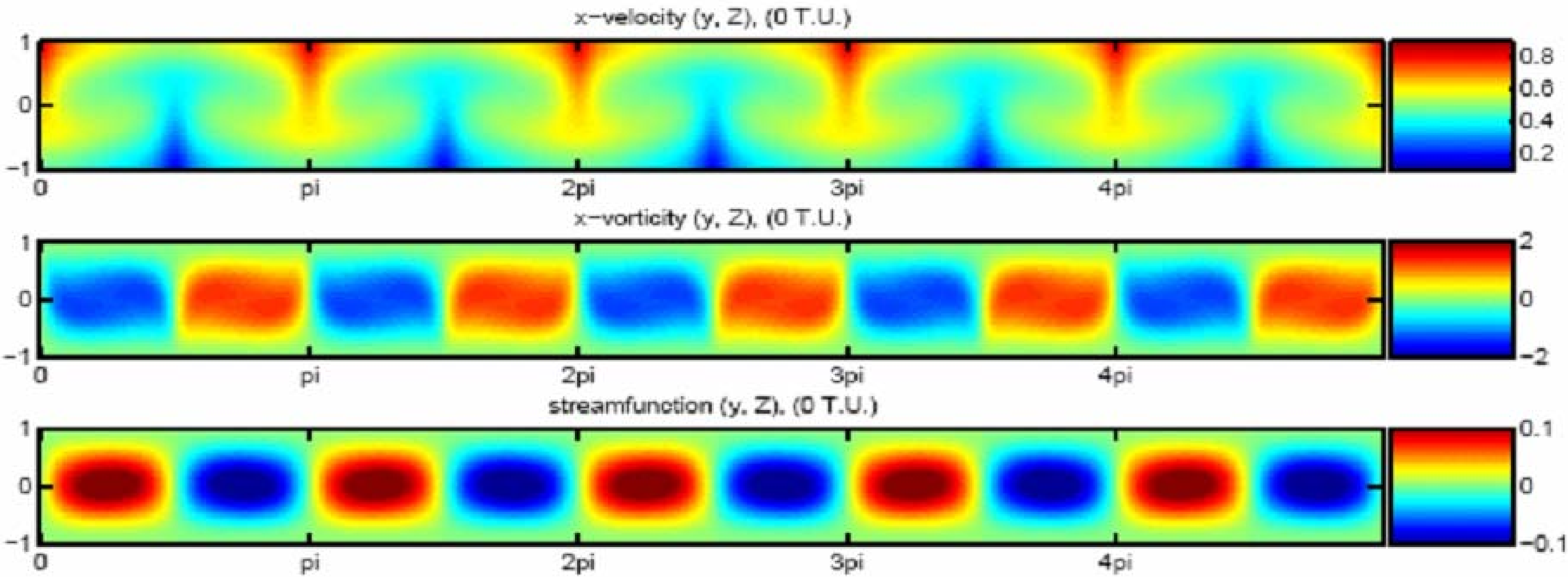
2. Semi-implicit Temporal Discretization:
   i) Linear Terms (LT) – Crank-Nicolson (CN)
   ii) Nonlinear Terms (NT) – Adams-Bashforth (AB)

$$\frac{U^{(s+1)} - U^{(s)}}{\Delta t} = \underbrace{\frac{LT^{(s+1)} + LT^{(s)}}{2}}_{CN} + \underbrace{\frac{1}{2}\left[3NT^{(s)} - NT^{(s-1)}\right]}_{AB};$$

3. Helmholtz Equation:
   Clenshaw-Curtis Algorithm (Chebyshev transform)

$$\hat{u}_{zz} - \lambda_n \hat{u} = \hat{G}_n$$

Homogeneous Steady State



+

$La = 0.01 \qquad Pe = 4000$

$L = 5\pi \qquad du_s/dz = 1 \qquad A_0 = 0$

Lateral Density Stratification

$Q_J \cdot A_0 = 4.0$

Video case11

Physics

$$\Omega_t - L_a(\Omega_{yy} + \Omega_{zz}) = -(\Psi_z\Omega_y - \Psi_y\Omega_z) - Q_J\theta_y + u_s'(z)\cdot u_y$$

- 0th order – Couette flow: $U_B = z$

  Gravity perpendicular with background density gradient.

  No energy released to create cells
- Perturbed system:

  x-vorticity ← C-L vortex force

  ← perturbation buoyancy
- 1st order –Tilt, preferred length scale and tilt slope

Numerical Linear Stability Analysis

Background: x-direction Couette flow along the wind direction.

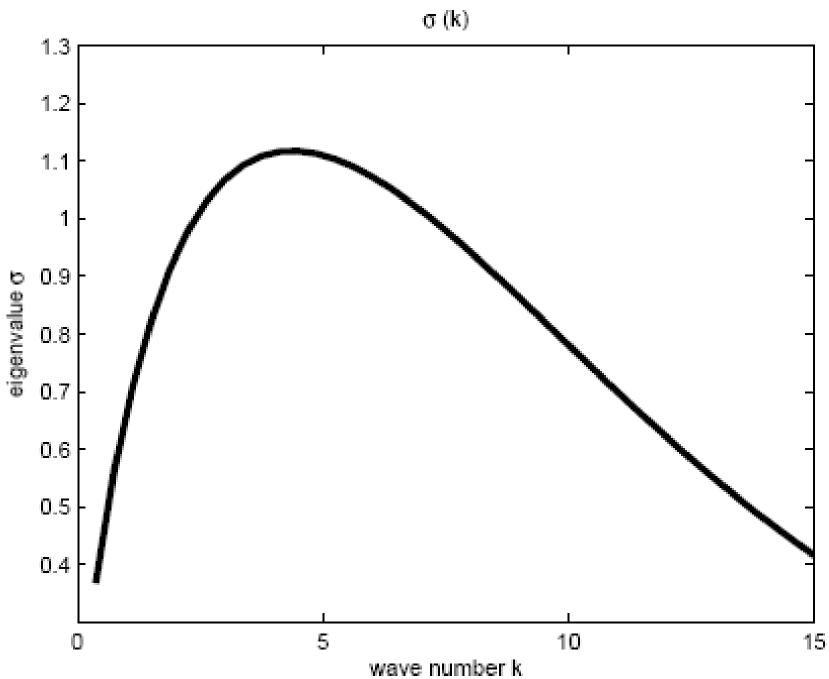
$$u = U_B(z) + u'(y,z,t) = z + u'(y,z,t)$$

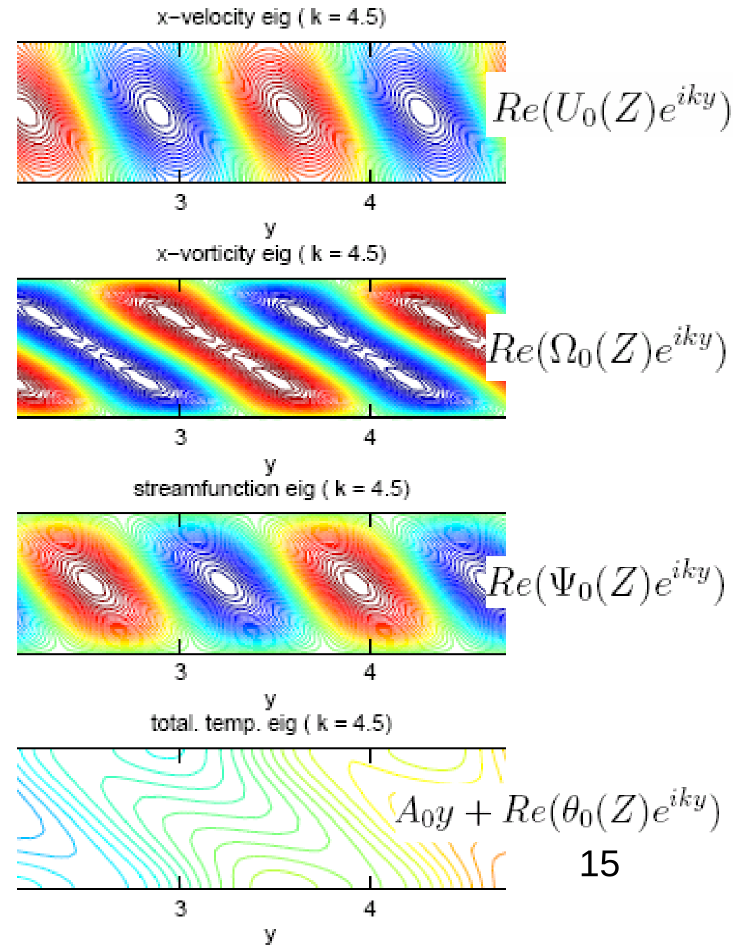
$$\Omega = \Omega'(y,z,t) \qquad \Psi = \Psi'(y,z,t) \qquad \theta = \theta'(y,z,t)$$

Perturbation form:

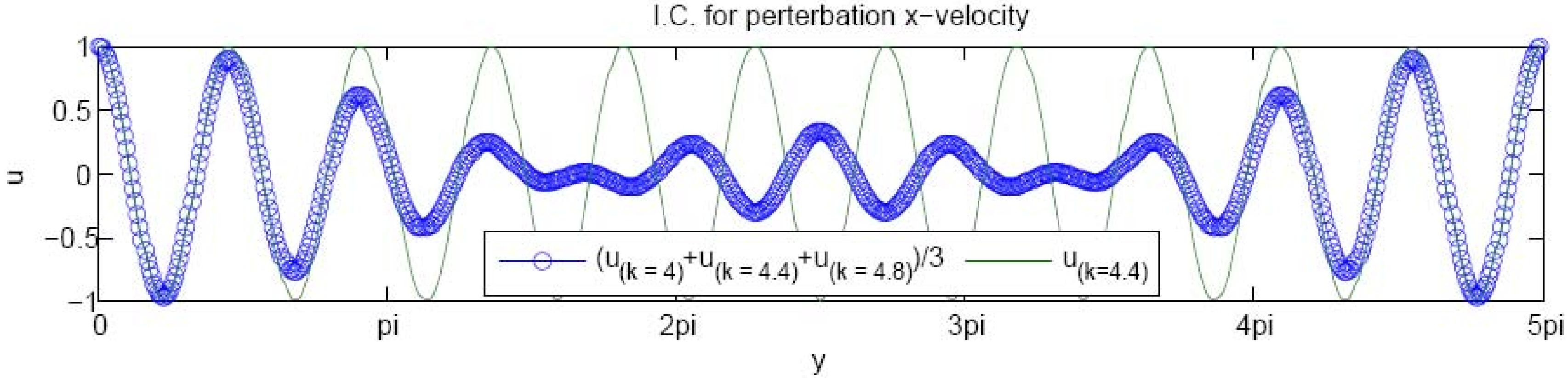
$$u = u_0(z)e^{iky}e^{\sigma t} \qquad \Omega = \Omega_0(z)e^{iky}e^{\sigma t}$$

$$\Psi = \Psi_0(z)e^{iky}e^{\sigma t} \qquad \theta = \theta_0(z)e^{iky}e^{\sigma t}$$





Video case13

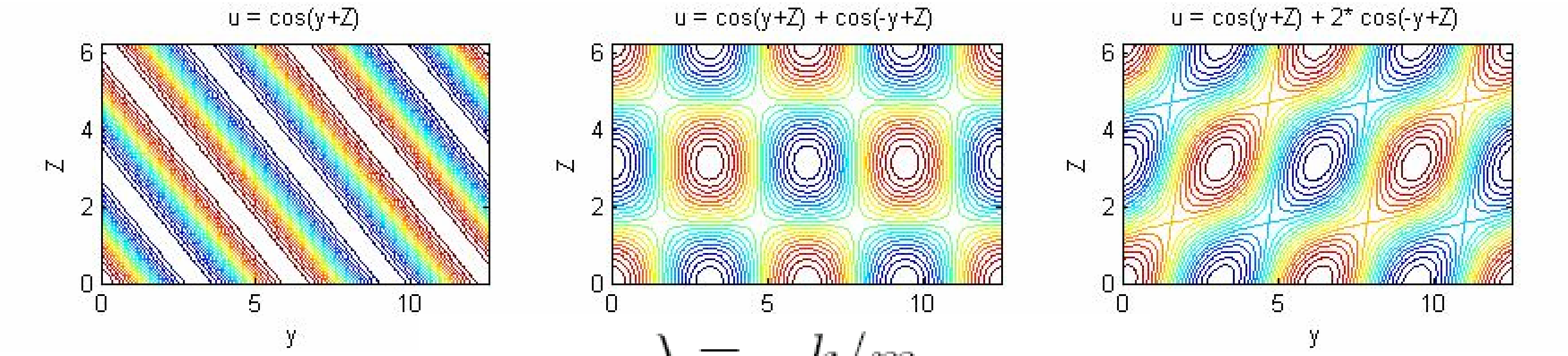


Analytical Linear Stability Analysis

- Perturbation form: $\begin{pmatrix} u \\ \Omega \\ \Psi \\ \theta \end{pmatrix} = \begin{pmatrix} U_0 \\ \Omega_0 \\ \Psi_0 \\ \Theta_0 \end{pmatrix} e^{i(ky+mz)} e^{\sigma t}$

- Explore single mode

$$a \cdot \cos(ky + mz) + b \cdot \cos(ky - mz)$$



$$\lambda \equiv -k/m$$

defining slope of single mode

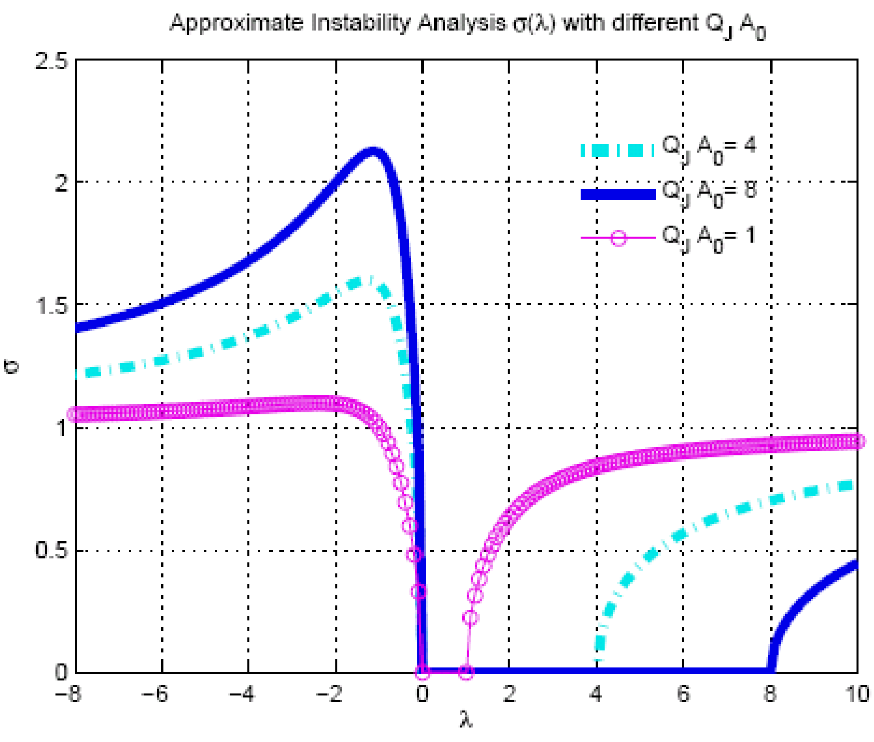
$$\cos(ky + mz)$$

Neglect diffusivity…

$$\sigma^2(k^2 + m^2) = k^2 + Q_J A_0 km$$

- Instability case $\sigma^2 > 0$

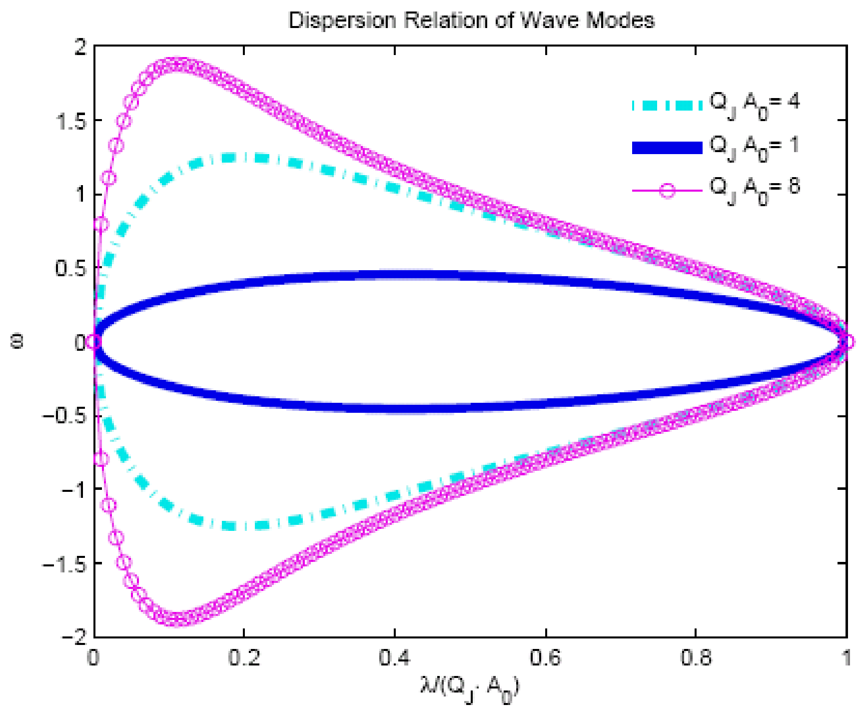
$$\sigma = \sqrt{\frac{\lambda^2 - Q_J A_0 \lambda}{\lambda^2 + 1}}$$



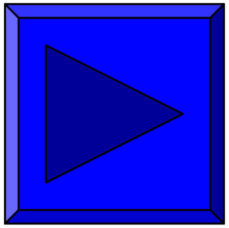
- Wave case $\sigma^2 < 0$

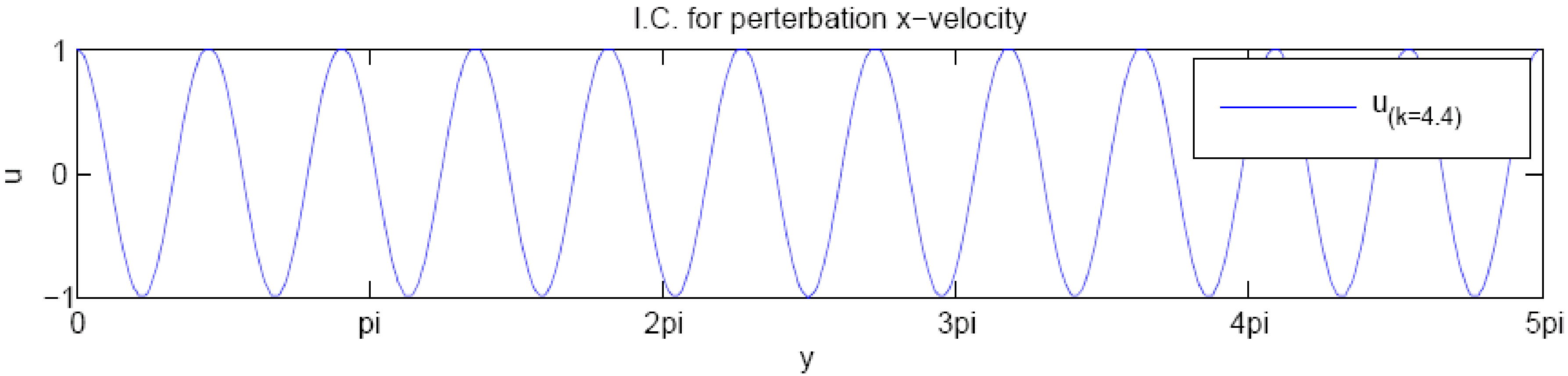
$$\sigma^2 = -\omega^2 = \frac{\lambda^2 - Q_J A_0 \lambda}{\lambda^2 + 1}$$

$$\omega = \pm\sqrt{\frac{Q_J A_0 \lambda - \lambda^2}{\lambda^2 + 1}}$$



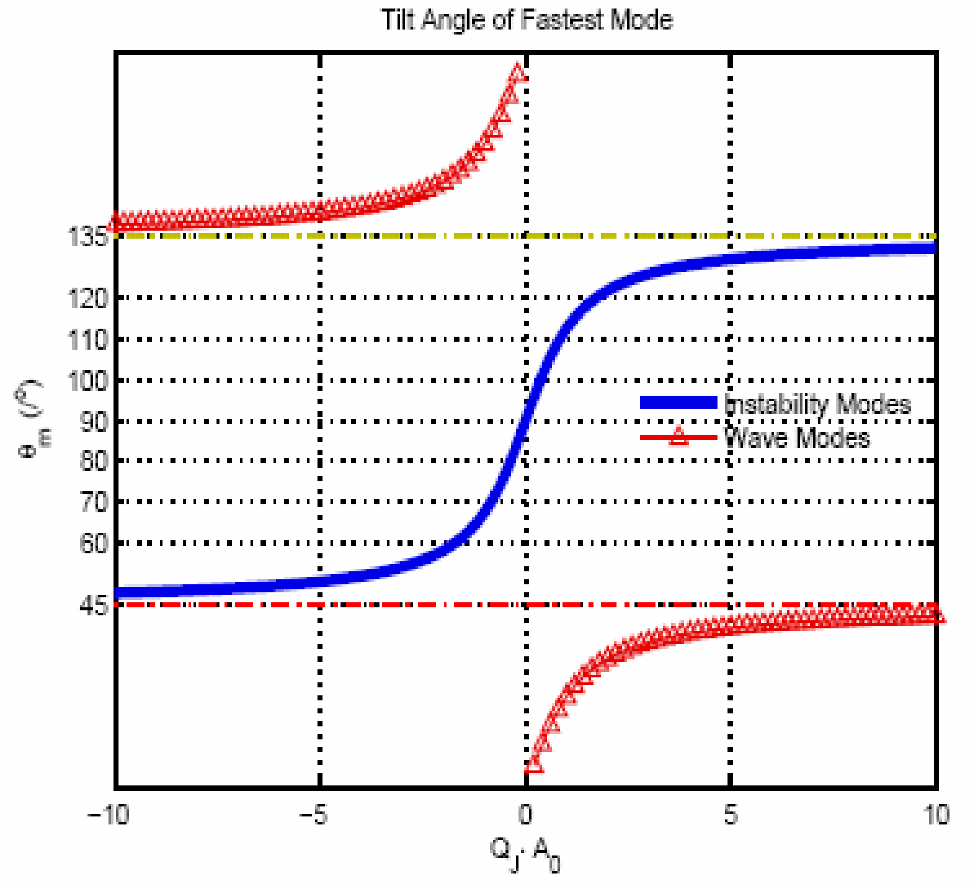
Video case12

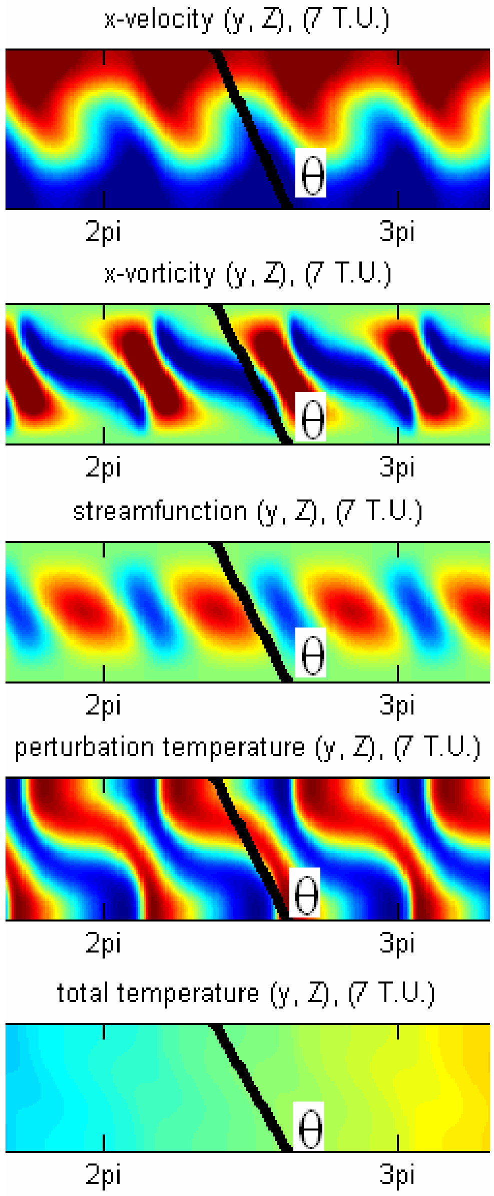




Angle Selection

- Plot: the fastest developing mode's tilt angle.
- Fastest wave modes propagate perpendicular to most unstable modes (cells tilt direction)

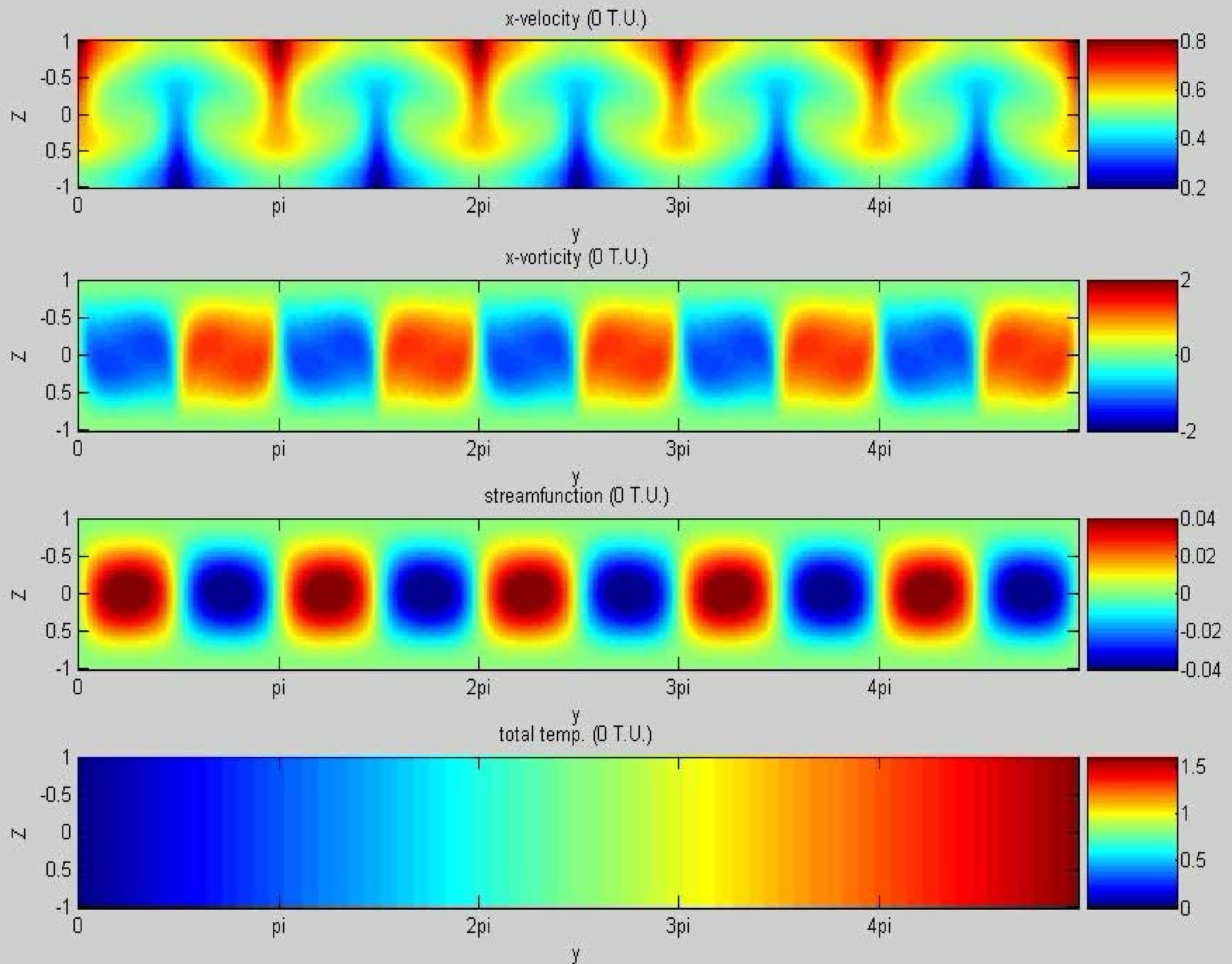




Future Work

- Secondary instability analysis
- Analyze the energetic budget for numerical simulation
- Add Coriolis parameters creating lateral density gradient by Thermal Wind balance

Thank you!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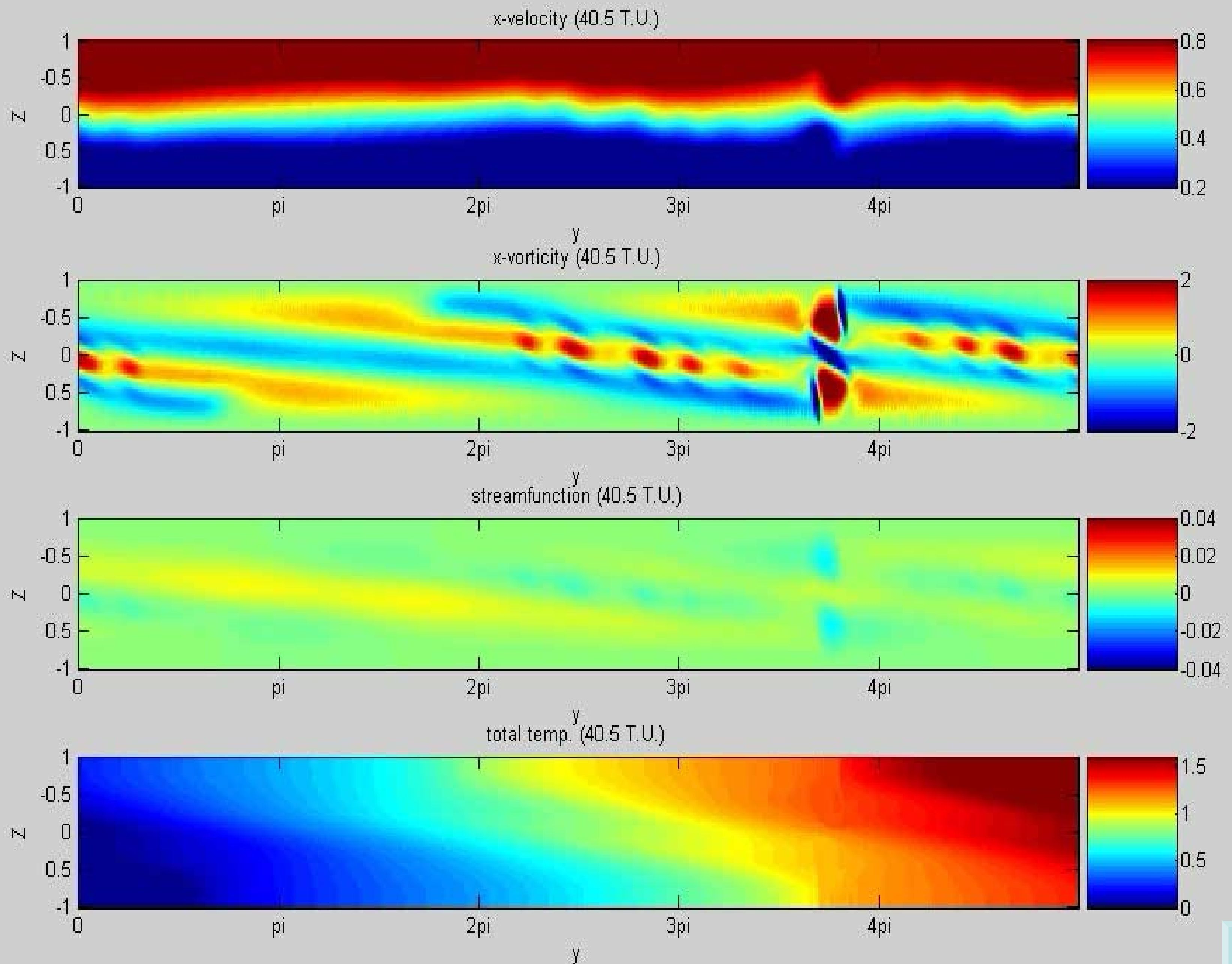
x-vorticity (0 T.U.)
streamfunction (0 T.U.)
total temp. (0 T.U.)
z
y
0
pi
2pi
3pi
4pi
1
-0.5
0
0.5
-1
0.8
0.6
0.4
0.2
2
0
-2
0.04
0.02
0
-0.02
-0.04
1.5
1
0.5
0


x-velocity (40.5 T.U.)
x-vorticity (40.5 T.U.)
streamfunction (40.5 T.U.)
total temp. (40.5 T.U.)
z
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3pi
4pi
1
-0.5
0
0.5
-1
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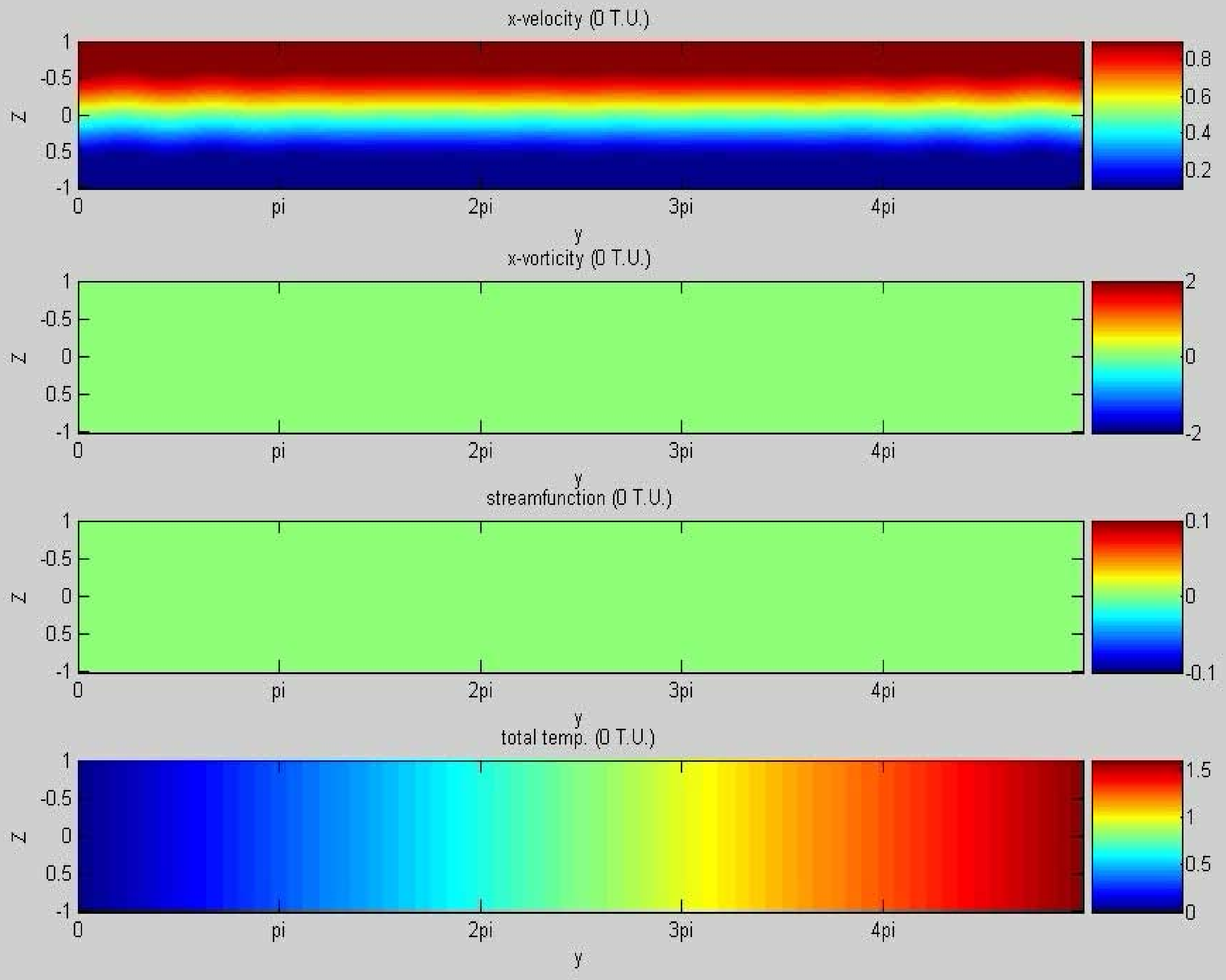
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x-vorticity (0 T.U.)
streamfunction (0 T.U.)
total temp. (0 T.U.)
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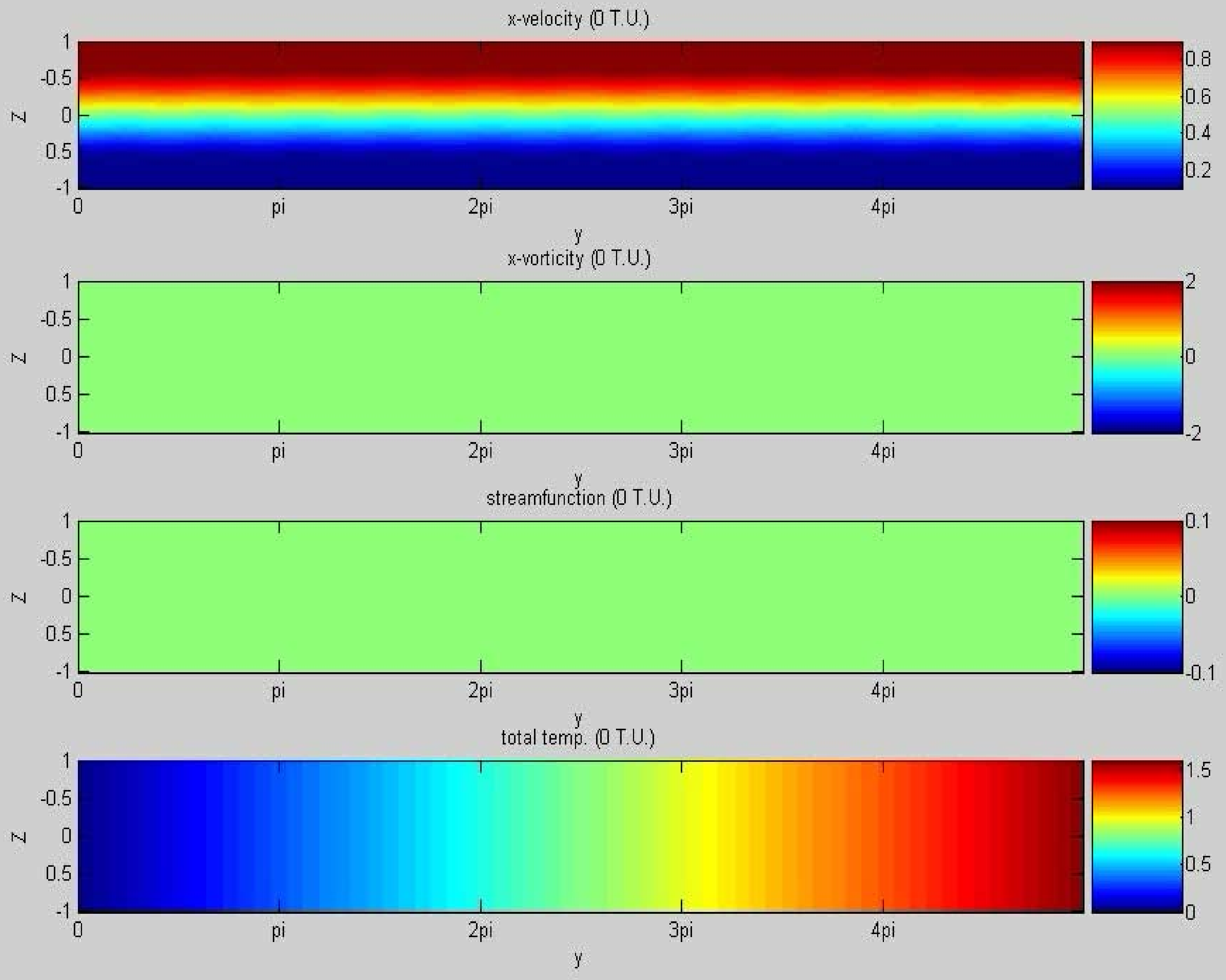
-0.1
1.5



x-velocity (0 T.U.)
x-vorticity (0 T.U.)
streamfunction (0 T.U.)
total temp. (0 T.U.)
z
y
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2pi
3pi
4pi
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-2
0.1
-0.1
1.5