Langmuir Circulation in the Presence of Lateral Density Gradients

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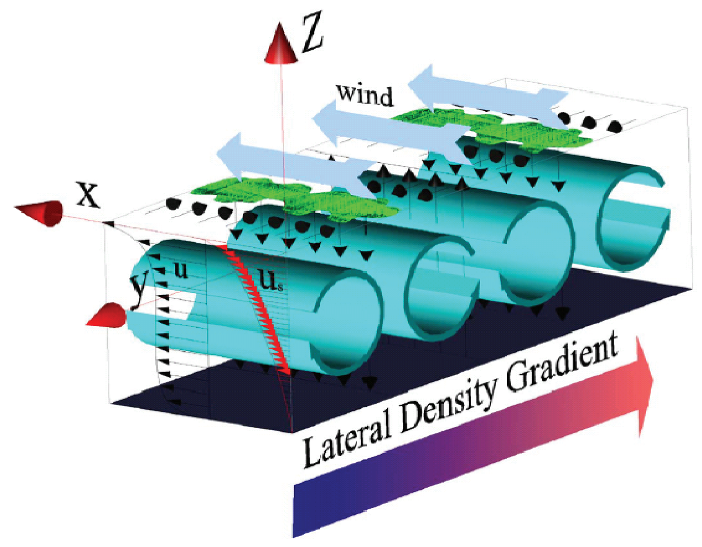
1. Motivation

- Langmuir circulation (LC): a wind and surface-wave driven vertical motion within the ocean surface mixed layer.
- How do lateral density gradients (associated with, e.g., submesoscale fronts) impact LC ?



2. Physical Ingredients

Framework:
- Non-hydrostatic dynamics
- No along-wind variation
- Coriolis acceleration not included

Scales:
- Length (H) : mixed layer depth - 10m~100m
- x-velocity (U) : along-wind velocity scale, $U = U_* \cdot R_*$, where $U_* \equiv \sqrt{\tau_w/\rho}$ $\quad R_* = U_* H/\nu_e$
- (y, z)-velocity (V) : cross-wind cellular velocity scale, $V = \sqrt{U_{so} \cdot U_*}$
- Time (t) : convection time scale $T = H/V$
- Temperature (ΔT) : horizontal difference across a single cell

Non-Dimensionalized Governing Equations:

$$U_t + \Psi_z U_y - \Psi_y U_z = La\nabla^2 U,$$

$$\Omega_t + \Psi_z\Omega_y - \Psi_y\Omega_z = \frac{\mathrm{d}U_s}{\mathrm{d}z}U_y - Ri_h\Theta_y - \chi Ri_h + La\nabla^2\Omega,$$

$$\nabla^2\Psi = \Omega,$$

$$\Theta_t + \Psi_z\Theta_y - \Psi_y\Theta_z = -\Psi_z + Pe^{-1}\nabla^2\Theta.$$

* Switching term: on, "free" case; off, "frozen" case.

Parameters:
- Laminar Langmuir number : $La \equiv \frac{\nu_e}{VH} \Rightarrow \frac{\text{convective time scale}}{\text{momentum diffusion time scale}}$
- Peclet number : $Pe \equiv \frac{VH}{\alpha_e} \Rightarrow \frac{\text{thermal diffussive time scale}}{\text{convective time scale}}$
- Horizontal Richardson number: $Ri_h \equiv \frac{M^2}{(W_*/H)^2} \Rightarrow \frac{\text{horizontal Brunt frequency}}{\text{vertical convection}}$

3. Methodology

Linear Stability Analysis :
- Differential eigenvalue problem
- $\phi = \phi_0(z)e^{iky}e^{\sigma t}$
- Chebyshev collocation method

DNS --- Semi-implicit pseudo-spectral method

- Fourier--Chebyshev spatial discretization
- Nonlinear terms are time-advanced with Adams-Bashforth scheme, and linear terms with Crank-Nicholson scheme
- Resulting Helmholtz equation (in z) is solved using Clenshaw-Curtis algorithm (with Chebyshev transform) in O(N) operations per horizontal wavenumber, where N+1 = # of Chebyshev modes

4. Frozen Background

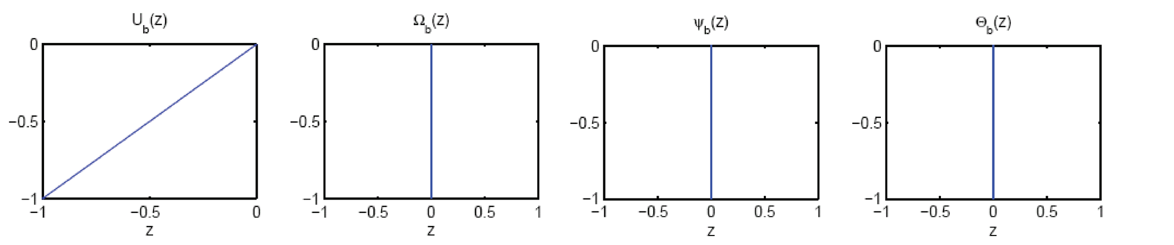


Figure 1. LC basic state with frozen lateral stratification: x-velocity, x-vorticity, streamfunction, and vertical temperature profiles

- In the linear case, cells are tilted toward the heavy fluid (at left end), creating vertical stratification
- Nonlinear evolution reveals that the cells shear apart owing to a complex interplay between buoyancy driven flow and pre-existing (tilted) Langmuir cells

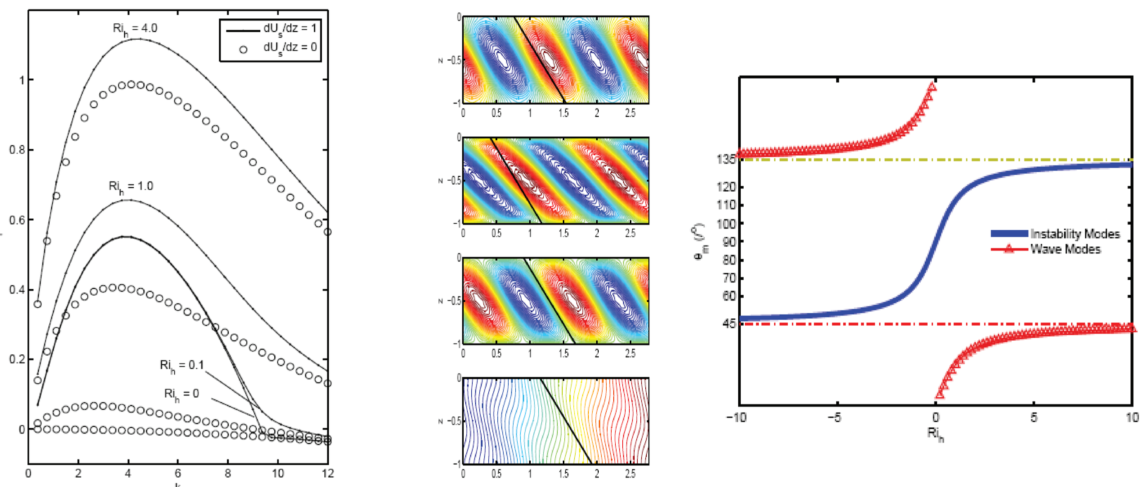


Figure 2. Linear stability results obtained from 2D system with frozen lateral stratification at La = 0.01, and Pe = 4000: (left) real part of eigenvalues, with Rih = 4, 1, 0.1, 0. from top to bottom, Solid lines - with Stokes drift, circles - buoyancy driven instability; (middle) stationary mode eigenfunctions with Stokes drift; (right) maximum tilte angle with Rih.

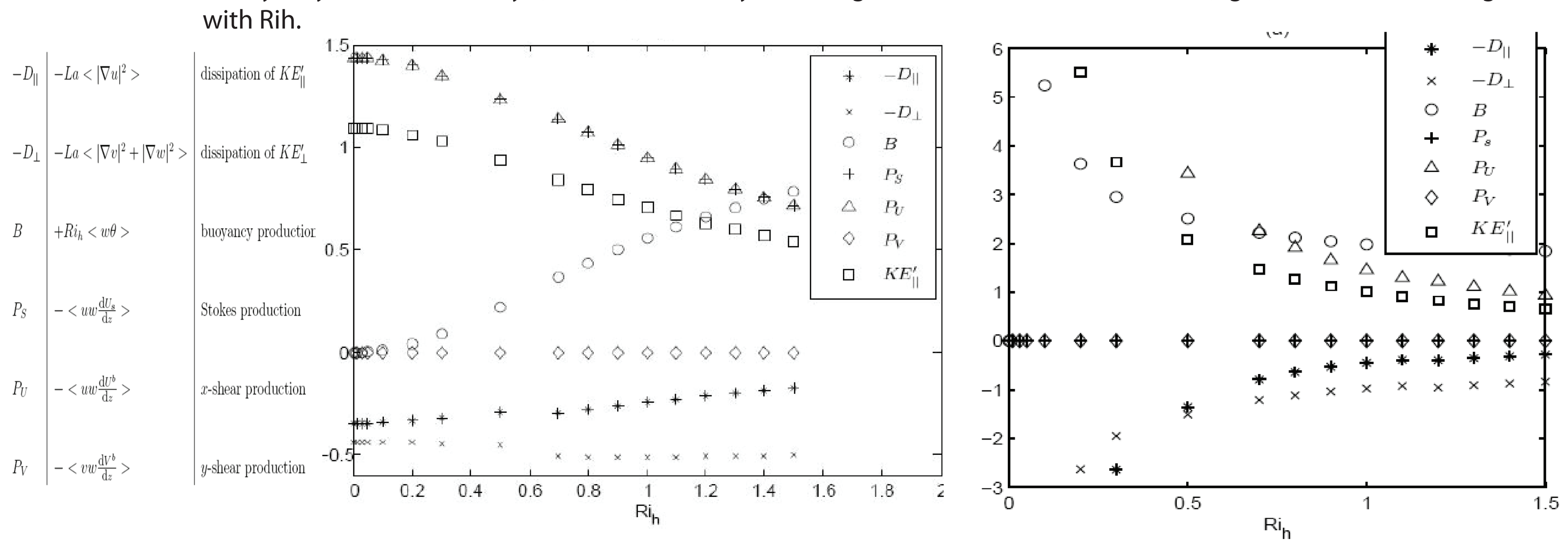


Figure 3. Energy budget from linear stability results. x = 0, La = 0.01, and Pe = 4000: (left) dUs/dz = 1 (right) dUs/dz = 0

- LC and buoyancy-driven instability cause vertical mixing together in linear regime, but counteract in second order
- Buoyancy-driven instability overtakes LC at Rih >> O(1.4), in contrast of typical submesoscale fronts: Rih = O(0.1)

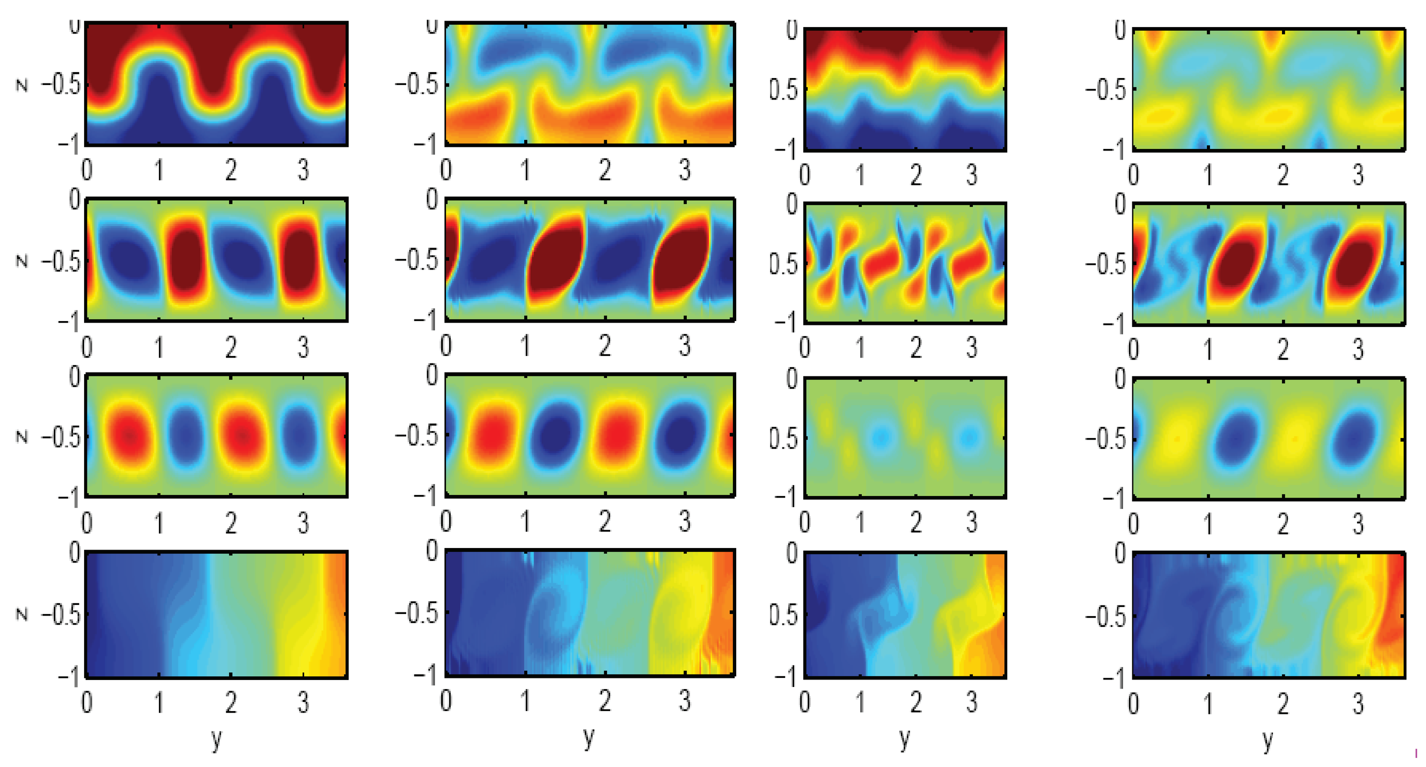


Figure 4. DNS, Rih = 0.5, La = 0.01, Pe = 4000, dUs/dz = 1.0: (left two) early time, cells slightly tilted (middle to rght) nonlinear evolution, periodic shearing of cells (right) stick-slip oscillation

5. Free Background

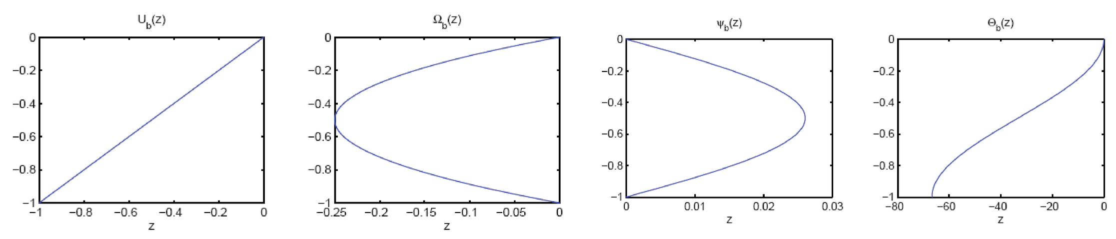


Figure 5. LC basic state with unfrozen lateral stratification: x-velocity, x-vorticity, streamfunction, and vertical temperature profiles

- For smaller k instability mode (wider cells), the bifurcation is stationary and LC preserves lateral stratification but mixes horizontal stratification
- For larger k instability mode (narrower cells), the bifurcation leads to traveling LC; the subsequent nonlinear evolution involves cell shearing reminiscent of the secondary instability in the frozen case

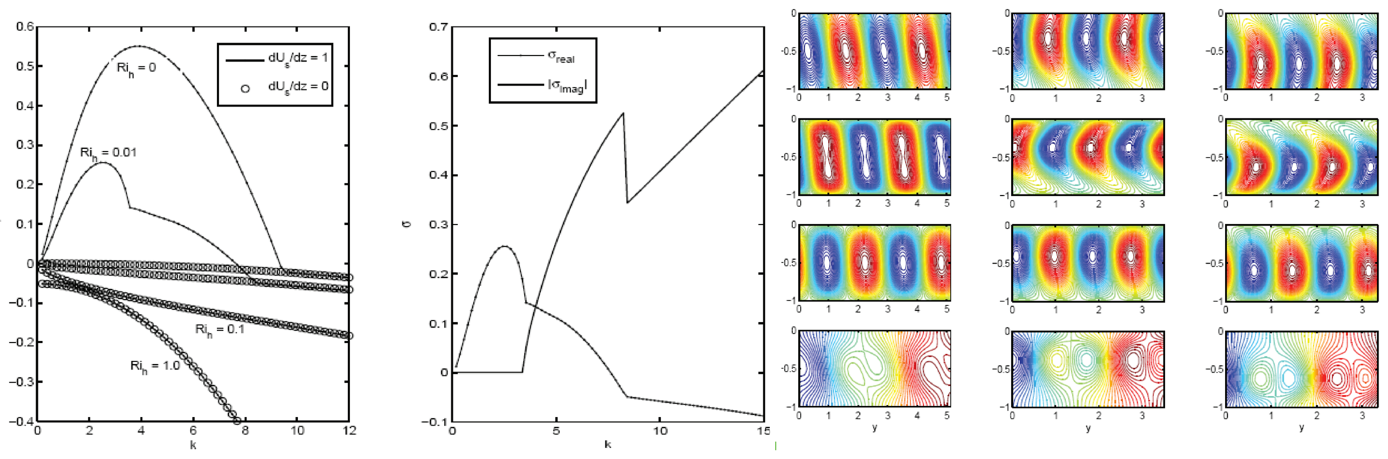


Figure 6. Linear stability results obtained from 2D system with free lateral stratification at La = 0.01, Pe = 4000. (left) real part of eigenvalue with Rih = 0, 0.01, 0.1, 1. (middle) imaginary and real parts of rigenvalue at Rih = 0.01. (right) eigen functions of stationary mode and two traveling modes.

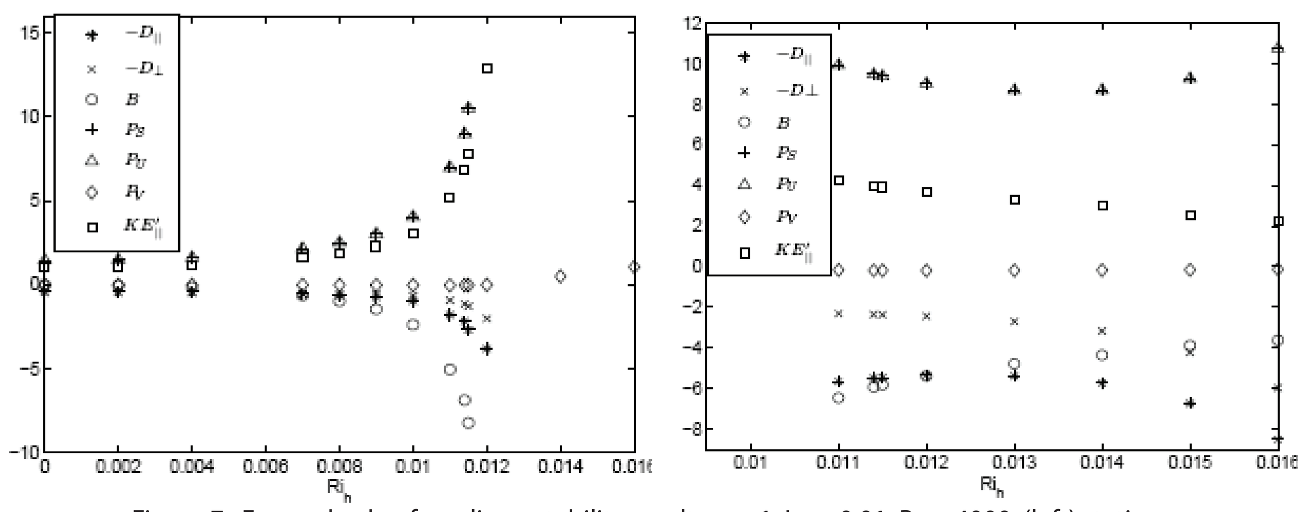


Figure 7. Energy budge from linearstability results. x = 1, La = 0.01, Pe = 4000. (left) stationary mode; (right) traveling mode.

- Stationary mode: 0.008< Rih < 0.012, LC eradicates vertical stratification
- Traveling mode: Rih >> O(0.01), horizontal shear flow shears LC apart, mixed layer vertically stratified

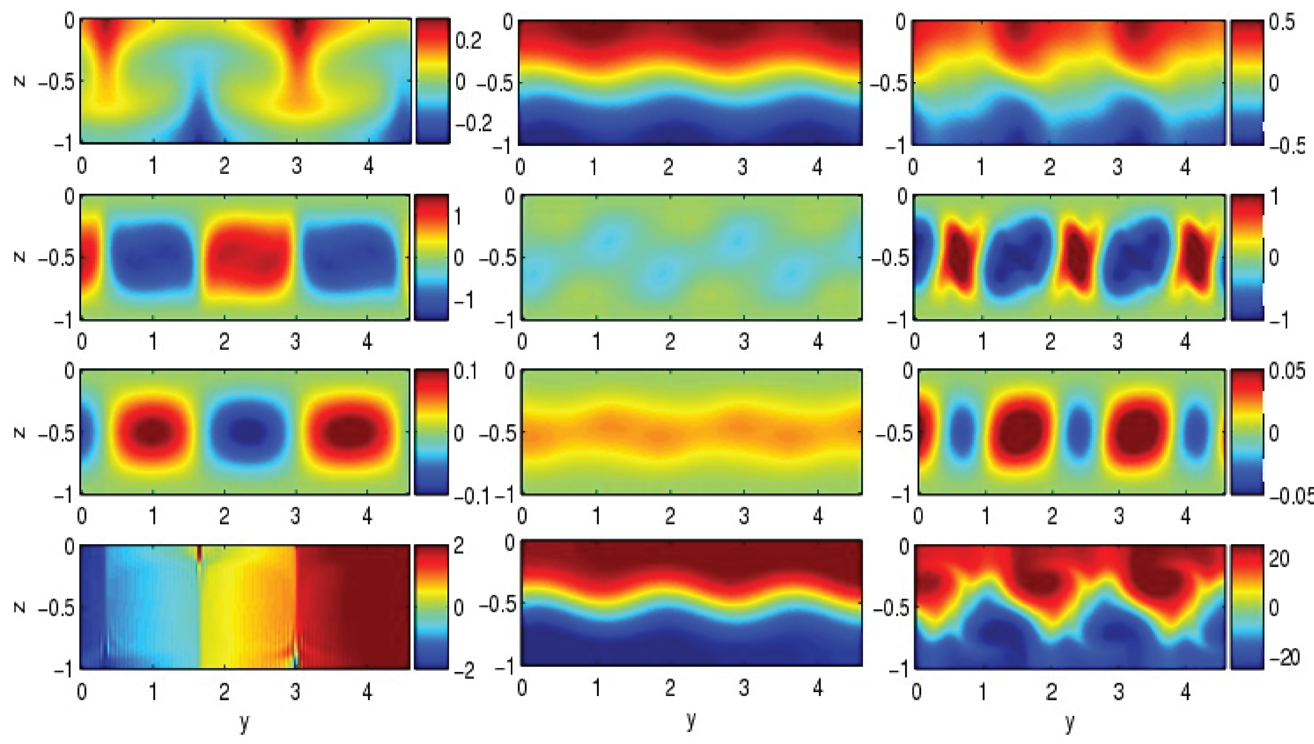


Figure 8. DNS, (left) k = 2.55, Rih = 0.01, La = 0.01, Us'(z)= 1, and Pe = 4000; (middle and right) k = 3.75, Rih = 0.016, La = 0.01, Us'(z) = 1, and Pe = 4000

Future Work

- Perform secondary stability analysis
- Add Coriolis effects to allow a lateral density gradient to be in thermal wind balance

References

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- The role of secondary shear instabilities in the equilibration of symmetric instability. Taylor, J. and Ferrari, R. (2009). J. Fluid Mech., Vol. 622, pp. 103-113

This work was funded by NSF CAREER award 0348981administered by the Physical Oceanography program.