

Задача №2

$$a) A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} \rightarrow \det |A - \lambda E| = 0$$

$$\Downarrow (2-\lambda)(3-\lambda) - 2 = 0$$

$$6 - 5\lambda + \lambda^2 - 2 = 0$$

$$\lambda_1 = 1$$

$$\lambda_2 = 4$$

$$a) \lambda_1 = 1$$

$$b) \lambda_2 = 4$$

$$\begin{cases} x + 2y = 0 \\ x + 2y = 0 \end{cases}$$

$$\Downarrow x = -2y$$

$$\bar{u}_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$\begin{cases} -2x + 2y = 0 \\ x - y = 0 \end{cases}$$

$$x = y$$

$$\bar{u}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$b) A = \begin{bmatrix} 4 & 1 & -1 \\ 1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix} \rightarrow \det |A - \lambda E| = 0$$

$$\Downarrow (4-\lambda)[(4-\lambda)^2 - 1] - [(4-\lambda) - 1] - [-1 + (4-\lambda)] = 0$$

$$(4-\lambda)[(4-\lambda)^2 - 1] - 2(4-\lambda) + 2 = 0$$

$$(4-\lambda)[16 - 8\lambda + \lambda^2 - 1] - 6 + 2\lambda = 0$$

$$-\lambda^3 + 12\lambda^2 - 45\lambda + 54 = 0$$

$$(\lambda - 3)(-\lambda^2 + 9\lambda - 18) = 0$$

$$\lambda_1 = 3 \quad \lambda_2 = 3 \quad \lambda_3 = 6$$

Собственные вектора:

$$\lambda_1 = 3$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} \Rightarrow x+y=z \Rightarrow \begin{cases} x = -y+z \\ y = z \\ z = z \end{cases}$$

$$\text{осп: } \left\{ y \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\} \quad \begin{cases} a) z=0, y=1 \\ b) z=1, y=0 \end{cases}$$

$$\bar{u}_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\bar{u}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda_2 = 6$$

$$\begin{pmatrix} -2 & 1 & -1 \\ 1 & -2 & -1 \\ -1 & -1 & -2 \end{pmatrix} \sim \begin{pmatrix} -2 & 1 & -1 \\ 0 & -3 & -3 \\ 0 & -3 & -3 \end{pmatrix} \Rightarrow \begin{cases} y = -z \\ x = z \end{cases}$$

$$\bar{u}_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Задача №3

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

$$\det |A - \lambda E| = 0$$

$$(1-\lambda)(3-\lambda) - 4 = 0$$

$$\lambda^2 - 4\lambda - 1 = 0$$

$$D = 16 + 4 = 20$$

$$\lambda_{1/2} = \frac{4 \pm \sqrt{20}}{2} \Rightarrow \lambda_1 = -0,236$$

$$\lambda_2 = 4,236$$

$$\lambda_1 = -0,236$$

$$\lambda_2 = 4,236$$

$$\begin{aligned} 1,236x + 2y &= 0 \\ 2x + 3,236y &= 0 \end{aligned}$$

$$\begin{aligned} -3,236x + 2y &= 0 \\ 2x - 1,236y &= 0 \end{aligned}$$

\Downarrow

$$\begin{aligned} 1,236x &= -2y \\ 2x &= -3,236y \Rightarrow x = -\frac{2y}{1,236} \\ x &= -\frac{3,236}{2}y \end{aligned}$$

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$$\begin{aligned} 2y &= 3,236x \Rightarrow y = \frac{3,236x}{2} \\ 1,236y &= 2x \Rightarrow y = \frac{2x}{1,236} \end{aligned}$$

$$x = -1,618y$$

$$\bar{u}_1 = \begin{pmatrix} -1,618 \\ 1 \end{pmatrix}$$

$$y = 1,618x$$

$$\bar{u}_2 = \begin{pmatrix} 1 \\ 1,618 \end{pmatrix}$$

$$U = \begin{pmatrix} -1,618 & 1 \\ 1 & 1,618 \end{pmatrix}$$

$$U^{-1} = \frac{1}{-3,617} \begin{pmatrix} 1,618 & -1 \\ -1 & -1,618 \end{pmatrix}$$

$$A = U \Lambda U^{-1}$$

$$a) \begin{pmatrix} -1,618 & 1 \\ 1 & 1,618 \end{pmatrix} \begin{pmatrix} -0,236 & 0 \\ 0 & 4,236 \end{pmatrix} = \begin{pmatrix} 0,381 & 4,236 \\ -0,236 & 6,853 \end{pmatrix}$$

$$b) -\frac{1}{3,617} \begin{pmatrix} 0,381 & 4,236 \\ -0,236 & 6,853 \end{pmatrix} \begin{pmatrix} 1,618 & -1 \\ -1 & -1,618 \end{pmatrix} = \begin{pmatrix} -3,619 & -7,234 \\ -7,234 & -10,852 \end{pmatrix}$$

$$b) \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$