Clinical Trial Data Analysis

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Data Cleaning

We preprocessed the data from its original format in the file "data/clinical_cleaned.R" so that it would be sufficiently anonymized. We took the following steps:

- Initially, there were 188 subjects. 52 of these were never assigned to a treatment group and were only observed at baseline. This ensures that censoring was independent of treatment, so removing them from the analysis does not introduce any selection bias. We are left with 66 subjects in group A and 70 subjects in group B.
- We renamed treatments, subject IDs, and site IDs to remove identifying information.
- Each individual was observed for a total of 14 days, 7 during the baseline week and 7 after receiving treatment. We averaged the 7 measurements taken during each of these time periods, leaving us with 2 measurements per individual.

Exploratory Data Analysis

We notice several things patterns from Figure 1. Some of the sites have many more subjects than others: these are sites 1, 3, and 6, and 7. Sites 2, 5, and 8 have very few subjects, so we will have low power at these sites. For example, after removing the missing values, site 8 only has one person in group B and two in group A, so the site will only have 3 unique permutation statistics.

```
clinical %>% filter(VISITNUM == 1) %>% ggplot(aes(x = factor(SITEID))) +
    geom_bar(aes(fill = tr), position = "dodge") + xlab("Site ID") +
    ylab("Count") + ggtitle("Number of Individuals per Treatment") +
    labs(fill = "Treatment") + theme_bw()
```

The data are not in the right shape to analyze a single variable. Below, we include a function to take the raw data and reshape it to analyze one variable of interest.

Figure ?? shows the distribution of the final outcome measures between the two treatment groups. Based on these distributions, we would expect to detect a difference in outcomes between the treatments for the daily_heart, daily_hrdq, and heart_freq measures.

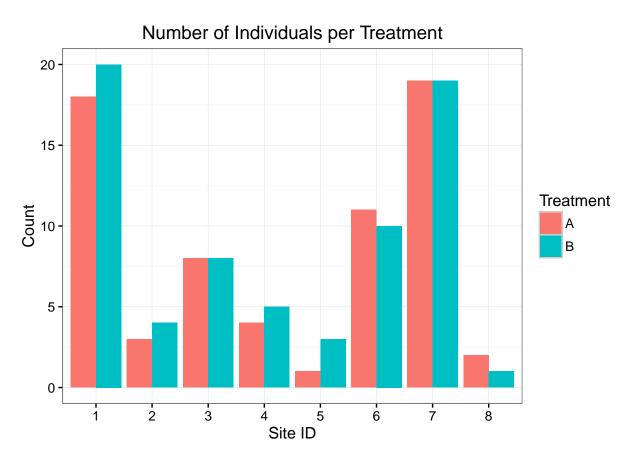


Figure 1: Number of subjects in each treatment group, by site ID.

```
plot_distrs <- lapply(continuous_vars, function(variable) {</pre>
    p <- reshape_data(variable) %>% mutate(tr = factor(tr)) %>%
         ggplot(aes(Outcome)) + geom_density(alpha = 0.6, aes(fill = tr)) +
         # geom_histogram(binwidth = 0.5, position = 'dodge', aes(fill
    \# = tr)) +
    # scale_fill_discrete(name='Experimental\nCondition',
    # breaks=c('0', '1'), labels=c('A', 'B')) +
    labs(x = variable, fill = "Treatment") + theme bw() + theme(axis.text.x = element text(size = 12),
         axis.text.y = element_text(size = 12), axis.title = element_text(size = 16),
         title = element_text(size = 16), legend.title = element_text(size = 12),
         legend.text = element_text(size = 14), strip.text.x = element_text(size = 12))
    return(p)
})
tmp <- ggplot_gtable(ggplot_build(plot_distrs[[1]]))</pre>
leg <- which(sapply(tmp$grobs, function(x) x$name) == "guide-box")</pre>
legend <- tmp$grobs[[leg]]</pre>
plot_distrs <- lapply(plot_distrs, function(x) x + theme(legend.position = "none"))</pre>
plot_distrs[[length(plot_distrs) + 1]] <- legend</pre>
pdf("../ms/fig/clinical_distr.pdf", width = 8, height = 4)
do.call(grid.arrange, c(plot_distrs, nrow = 2))
dev.off()
## pdf
##
do.call(grid.arrange, c(plot_distrs, nrow = 2))
                                                      1.25
  1.5
                                                      1.00
                                                                                0.75
density
0.5
                          density
0.5
                                                    density
0.50
                                                                              density
0.50
                                                                                0.25
                                                      0.25
 0.0
                            0.0
                                                      0.00
                                                                                0.00
         .5 1.0 1.5
daily_heart
    0.0
        0.5
                              0.0
                                       1.0
                                                         0.0 0.5
                                                                 1.0 1.5
                                   daily_regurg
                                                              daily_dysp
                                                                                         daily_hrdq
                            0.3
  0.3
                          density
0.2
0.1
                                                      0.2
density
0.2
                                                    density
0.1
                                                                                       Treatment
  0.1
  0.0
                            0.0
                                                      0.0
                                                        0.0
             5.0
                 7.5
                                                                 5.0
    0.0
         2.5
                      10.0
                                                             2.5
                                                                      7.5
          heart_freq
                                    regurg_freq
                                                              dysp_freq
```

Figure 2: Distribution of continuous outcomes

Analysis - Primary endpoint

To illustrate the different methods, let's first restrict attention to the primary endpoint, daily_hrdq. Throughout, we compare two models: the first model considers the final outcome measure, adjusting for site ID and perhaps also the baseline score. The second model considers the difference between outcome and baseline score as its dependent variable, adjusting for site ID.

We compare four testing methods: the usual parametric ANCOVA, a permutation test considering the difference in means as test statistic, stratifying by site, the stratified permutation test, using the t-statistic from the linear regression instead, and a permutation test in the style of Freedman and Lane (1983), using a linear regression to adjust for covariates.

	В	A
Min.	-2.79	-2.81
1st Qu.	-0.90	-1.10
Median	-0.60	-0.69
Mean	-0.57	-0.74
3rd Qu.	-0.16	-0.37
Max.	1.61	0.91

Table 1: Summary statistics for change in primary endpoint (daily hrdq) for the two treatment groups.

Parametric ANCOVA

First, we do a standard parametric ANCOVA using the two models. Under model 1, using the outcome as response and adjusting for the baseline value, treatment has a significant effect. Under model 2, the significance goes away. (Note that treatment is still coded as A and B, which are characters. The 1m function is smart enough to know that it should be coded as a factor, where we compare the effect of B relative to A. In general, be very careful when you're doing this!)

```
# method 0: parametric ANCOVA
lm1 <- lm(Outcome ~ Baseline + tr + factor(SITEID), data = daily_hrdq)
print(xtable(summary(aov(lm1))), include.rownames = TRUE)</pre>
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Baseline	1	27.00	27.00	65.16	0.0000
tr	1	1.73	1.73	4.17	0.0432
factor(SITEID)	7	4.50	0.64	1.55	0.1563
Residuals	126	52.20	0.41		

```
daily_hrdq$lm1_resid <- residuals(lm1)

lm2 <- lm(difference ~ tr + factor(SITEID), data = daily_hrdq)
print(xtable(summary(aov(lm2))), include.rownames = TRUE)

daily_hrdq$lm2_resid <- residuals(lm2)</pre>
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
tr	1	0.92	0.92	1.71	0.1936
factor(SITEID)	7	5.48	0.78	1.46	0.1876
Residuals	127	68.13	0.54		

Stratified permutation test

Suppose each individual, i = 1, ..., N has two potential outcomes $(Y_i(A), Y_i(B))$ that indicate their response to treatments A and B, respectively. We randomly assign treatment to individuals; treatment determines which of $Y_i(A)$ and $Y_i(B)$ we observe. We are unable to observe both. Suppose we wish to test the null hypothesis that individual by individual, treatment has no effect. This is referred to as the "sharp null" hypothesis:

$$H_0: Y_i(A) = Y_i(B), i = 1, \dots, N.$$

Then, whether individual i received A or B amounts to an arbitrary label. Once we observe their response under one treatment, we know what it would have been under the other; namely, it would be the same.

Treatment was completely randomized at each site, independently across sites. Assuming that drop-out from the study was independent of treatment assignment (which is true if treatment was blinded and assigned at random), then we may condition on the number of individuals who received A and B at each site. Any assignment of treatments which preserves the number of treated and controls at each site is valid and has equal probability of occurring. Therefore, by using this principle of equal probabilities and by imputing the unobserved potential outcomes assuming that the null hypothesis is true, we may obtain the permutation distribution of any statistic under the null hypothesis.

We compare two test statistics: the usual difference in means between group B and group A, and the difference in means within each site, aggregated by taking the sum of their absolute values over sites. The first statistic is more comparable to what one would obtain from an ANCOVA and it is readily interpretable. It does not directly account for the stratification by site, so variation between sites may be hidden. The second statistic is useful for testing the two-sided alternative hypothesis that treatment has some effect. It may be more powerful than the simple difference in means if the effect of the drug varies across sites. For instance, if the drug has a positive effect at one site but a negative effect at another site, this test statistic would be large; if we used the simple difference in means without accounting for sites, then the positive effect and negative effect may cancel each other out and appear as no effect.

Figure ?? shows the permutation distributions of the simple difference in means. Figure ?? shows the permutation distribution of the absolute value of the difference of means within strata, summed across strata. There are several points to notice: * The second test is far less powerful than the first. This is likely because some sites had very few patients. These sites have correspondingly few unique permutations, and so they often contribute the same amount to the summed difference in means statistic. It is impossible to detect a difference at small significance levels when there are only 3 patients. * The summed difference of means only makes sense for testing against a two-sided alternative, since taking the absolute value removes the sign of the effect. The permutation distribution is strictly non-negative and skewed to the right. * This statistic is less interpretable than the difference in means because it doesn't correspond directly to any feature of the distribution of outcomes.

Since the summed difference in means statistic has such lower power, we abandon it for the rest of the analyses.

```
# method 1 : do permutation of differences
observed_diff_means1 <- mean(daily_hrdq[daily_hrdq$tr == "B",
        ]$Outcome) - mean(daily_hrdq[daily_hrdq$tr == "A", ]$Outcome)
diff_means_distr1 <- stratified_two_sample(group = daily_hrdq$tr,
        response = daily_hrdq$Outcome, stratum = daily_hrdq$SITEID,</pre>
```

```
reps = 10000)
diff_means_pvalue1 <- t2p(observed_diff_means1, diff_means distr1,</pre>
    alternative = "two-sided")
observed_diff_means2 <- mean(daily_hrdq[daily_hrdq$tr == "B",</pre>
    ]$difference) - mean(daily_hrdq[daily_hrdq$tr == "A", ]$difference)
diff_means_distr2 <- stratified_two_sample(group = daily_hrdq$tr,</pre>
    response = daily hrdg$difference, stratum = daily hrdg$SITEID,
    reps = 10000)
diff_means_pvalue2 <- t2p(observed_diff_means2, diff_means_distr2,</pre>
    alternative = "two-sided")
data.frame(perm = c(diff_means_distr1, diff_means_distr2), model = c(rep("Outcome",
    length(diff_means_distr1)), rep("Differenced", length(diff_means_distr2))),
    xintercept = c(rep(observed_diff_means1, length(diff_means_distr1)),
        rep(observed_diff_means2, length(diff_means_distr2)))) %>%
    ggplot(aes(x = perm, fill = model)) + geom_histogram() +
    facet_grid(~model) + geom_vline(aes(xintercept = xintercept)) +
    theme_bw()
```

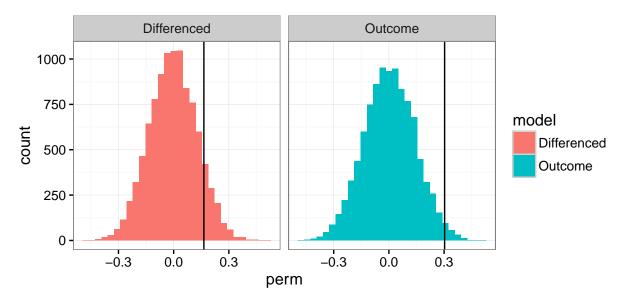


Figure 3: Permutation distribution of the difference of means for daily_hrdq

```
response = daily_hrdq$difference, stratum = daily_hrdq$SITEID,
    groups = unique(daily_hrdq$tr), strata = unique(daily_hrdq$SITEID)))
diff_means_distr_bystrata2 <- stratified_two_sample(group = daily_hrdq$tr,
    response = daily_hrdq$difference, stratum = daily_hrdq$SITEID,
    stat = "mean_within_strata", reps = 10000)
diff_means_bystrata_pvalue2 <- t2p(obs_diff_means_bystrata2,
    diff_means_distr_bystrata2, alternative = "two-sided")

data.frame(perm = c(diff_means_distr_bystrata1, diff_means_distr_bystrata2),
    model = c(rep("Outcome", length(diff_means_distr_bystrata1)),
        rep("Differenced", length(diff_means_distr_bystrata2))),
    xintercept = c(rep(obs_diff_means_bystrata1, length(diff_means_distr_bystrata1)),
        rep(obs_diff_means_bystrata2, length(diff_means_distr_bystrata2)))) %>%
    ggplot(aes(x = perm, fill = model)) + geom_histogram() +
    facet_grid(~model) + geom_vline(aes(xintercept = xintercept)) +
    theme_bw()
```

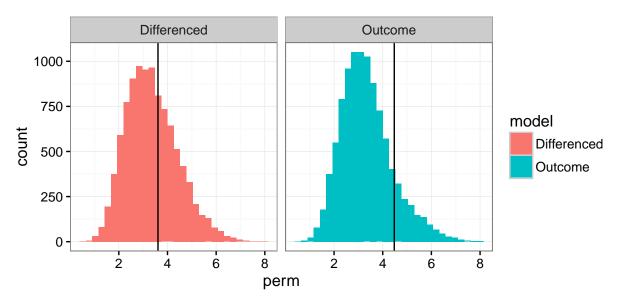


Figure 4: Permutation distribution of the difference of means within strata, summed across strata, for daily hrdq

Covariate-adjusted permutation test

We would like to test for a difference in outcomes between the treatment groups group, but control for other covariates. In particular, when the outcome is the average response during the second week of follow-up, we would like to control for the average response during the first week of follow-up and the location. When the outcome is the difference in responses between the second and first weeks, we would just like to control for location. We will use the approximate permutation test derived by Freedman and Lane (1983) to do so.

Let Y denote the response, Z denote the treatment indicator (1 if the person gets treatment B, 0 if they get A), and X denote a covariate which may be correlated with Y. We may write the following equation:

$$Y = \beta_0 + \beta_1 X + \beta_2 Z + \varepsilon$$

Under the sharp null H_0 , Z has no effect on Y. In other words, the null hypothesis is that $\beta_2 = 0$. In a randomized experiment such as this, there are two ways we may test this null hypothesis.

Standard linear model

First, we may permute the treatment assignments Z directly. Treatment was assigned at random within each site. This ensures that Z and ε are statistically independent, conditional on site. Therefore, we may conduct a test by permuting treatment assignments Z within site, independently across sites, and calculating a test statistic for each such permutation. We choose to use the t statistic from the linear model as the test statistic.

```
# method 2 : linear model
observed t1 <- summary(lm1)[["coefficients"]]["trB", "t value"]
lm1 t distr <- replicate(10000, {</pre>
    daily_hrdq$tr_perm <- permute_within_groups(daily_hrdq$tr,</pre>
        daily_hrdq$SITEID)
    lm1_perm <- lm(Outcome ~ Baseline + tr_perm + factor(SITEID),</pre>
        data = daily_hrdq)
    summary(lm1_perm)[["coefficients"]]["tr_permB", "t value"]
})
lm_pvalue_1 <- t2p(observed_t1, lm1_t_distr, alternative = "two-sided")</pre>
observed_t2 <- summary(lm2)[["coefficients"]]["trB", "t value"]</pre>
lm2 t distr <- replicate(10000, {</pre>
    daily_hrdq$tr_perm <- permute_within_groups(daily_hrdq$tr,</pre>
        daily_hrdq$SITEID)
    lm2_perm <- lm(difference ~ tr_perm + factor(SITEID), data = daily_hrdq)</pre>
    summary(lm2 perm)[["coefficients"]]["tr permB", "t value"]
})
lm_pvalue_2 <- t2p(observed_t2, lm2_t_distr, alternative = "two-sided")</pre>
data.frame(perm = c(lm1_t_distr, lm2_t_distr), model = c(rep("Outcome",
    length(lm1_t_distr)), rep("Differenced", length(lm2_t_distr))),
    xintercept = c(rep(observed_t1, length(lm1_t_distr)), rep(observed_t2,
        length(lm2_t_distr)))) %>% ggplot(aes(x = perm, fill = model)) +
    geom_histogram() + facet_grid(~model) + geom_vline(aes(xintercept = xintercept)) +
    xlab("t Statistic") + ylab("Frequency") + ggtitle("Permutation Distributions") +
    theme_bw()
```

Linear model residuals

Let's take an alternative view of the problem. We still write $Y = \beta_0 + \beta_1 X + \beta_2 Z + \varepsilon$. However, now, we do not treat the ε as random. They are simply defined to be the difference between Y and the data's linear projection onto the plane $\beta_0 + \beta_1 X + \beta_2 Z$.

If the null hypothesis is true, then $\varepsilon = Y - \beta_0 - \beta_1 X$. Therefore, we may estimate the errors $\hat{\varepsilon}$ by $Y - \hat{Y}$, where \hat{Y} is obtained by regressing Y on X but not Z. They approximate the true errors ε from the equation $Y = \beta_0 + \beta_1 X + \beta_2 Z + \varepsilon$. The ε are orthogonal to X and Z. Therefore, within sites, these estimated $\hat{\varepsilon}$ are approximately exchangeable.

Permutation Distributions

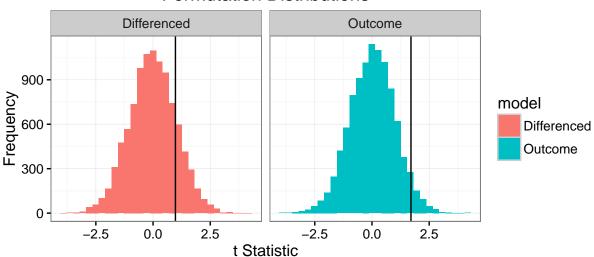


Figure 5: Permutation distribution of the stratified permutation test between daily_hrdq and treatment after controlling for covariates in a linear model.

We construct a permutation distribution using several steps:

- 1. Estimate $\hat{\varepsilon}$ by $Y \hat{\beta}_0 \hat{\beta}_1 X$, where $\hat{\beta}_0$ and $\hat{\beta}_1$ are obtained by regressing Y on X.
- 2. Construct permuted errors $\hat{\varepsilon}^{\pi}$ by permuting the $\hat{\varepsilon}$ within sites.
- 3. Construct permuted responses $Y^{\pi} = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\varepsilon}$.
- 4. Regress Y^{π} on X and Z. The test statistic is the t-statistic for the coefficient of Z.

For this dataset, we are guaranteed that treatment Z is independent of ε , as it is randomized within site. We should also check that site ID is uncorrelated with vare psilon. If not, then the $\hat{\varepsilon}$ are not exchangeable across sites. However, our permutations are done within sites, so even if this is violated, there is not a big issue. We check these associations using residual plots in Figure ??. Indeed, the distribution of residuals looks nearly equal between treatment groups in both models. The distribution of residuals varies a bit across sites, but they all appear roughly centered around 0. Sites 4 and 5 have larger variance in the differenced model. As mentioned, this should not be an issue since we permute treatments within sites, independently across sites. (In general, when treatment is not randomly assigned, this condition is necessary for the permutation test to be valid.)

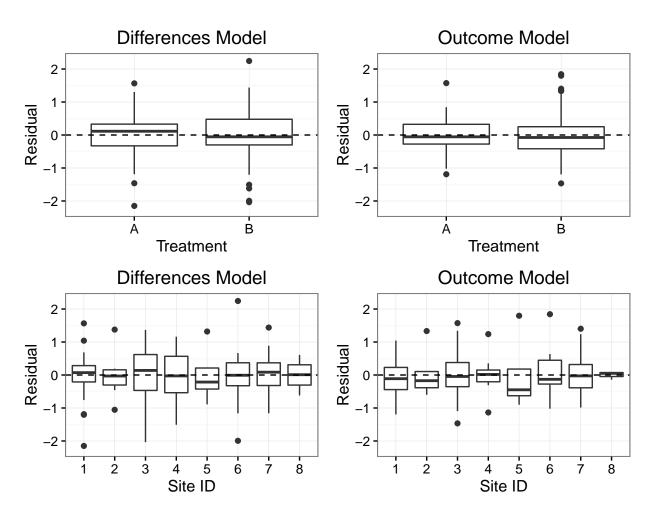


Figure 6: Residual plots of the linear regression of daily_hrdq on treatment and covariates.

```
lm2_no_tr <- lm(difference ~ factor(SITEID), data = daily_hrdq)</pre>
lm2_resid <- residuals(lm2_no_tr)</pre>
lm2 yhat <- fitted(lm2 no tr)</pre>
observed_t2 <- summary(lm2)[["coefficients"]]["trB", "t value"]</pre>
lm2 t distr <- replicate(10000, {</pre>
    lm2_resid_perm <- permute_within_groups(lm2_resid, daily_hrdq$SITEID)</pre>
    daily hrdg$response fl <- lm2 yhat + lm2 resid perm
    lm2 perm <- lm(response fl ~ tr + factor(SITEID), data = daily hrdq)</pre>
    summary(lm2_perm)[["coefficients"]]["trB", "t value"]
})
fl_pvalue_2 <- t2p(observed_t2, lm2_t_distr, alternative = "two-sided")</pre>
data.frame(perm = c(lm1_t_distr, lm2_t_distr), model = c(rep("Outcome",
    length(lm1_t_distr)), rep("Differenced", length(lm2_t_distr))),
    xintercept = c(rep(observed_t1, length(lm1_t_distr)), rep(observed_t2,
        length(lm2_t_distr)))) %>% ggplot(aes(x = perm, fill = model)) +
    geom histogram() + facet grid(~model) + geom vline(aes(xintercept = xintercept)) +
    theme bw()
```

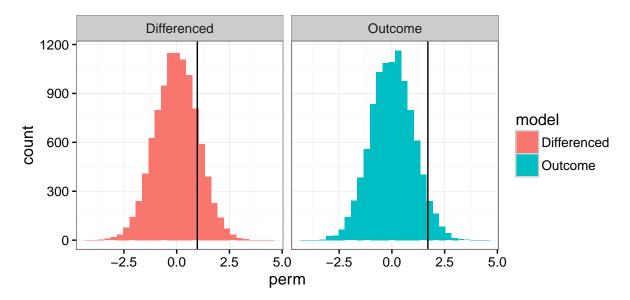


Figure 7: Permutation distribution of the Freedman-Lane tests of correlation between daily_hrdq and treatment after controlling for covariates.

Table ?? shows the p-values for each of the four methods, using the two models (treatment controlling for baseline, and the difference of treatment minus baseline). For the models using the treatment measure as the dependent variable, the effect of daily_hrdq is significant at the 0.05 level in two of the five models and is significant at the 0.1 level in four of the five models. It is never significant in the models using the difference from baseline to treatment. As expected, the stratified permutation test using the sum of differences in means across sites has low power. We restrict our attention to the model using treatment measurement and controlling for the baseline, as this is what RB's original analysis does. Now, we run this procedure for all continuous variables. Table ?? shows the results.

```
outcome_pvalues <- c(summary(aov(lm1))[[1]]["trB", "Pr(>F)"],
    diff_means_pvalue1, diff_means_bystrata_pvalue1, lm_pvalue_1,
    fl_pvalue_1)
differenced_pvalues <- c(summary(aov(lm2))[[1]]["trB", "Pr(>F)"],
    diff_means_pvalue2, diff_means_bystrata_pvalue2, lm_pvalue_2,
    fl_pvalue_2)
pvalues_table <- cbind(differenced_pvalues, outcome_pvalues)

tests <- c("Parametric ANCOVA", "Unadjusted permutation", "Unadjusted permutation (summed across strata "Linear regression permutation", "Residual permutation")
rownames(pvalues_table) <- tests
colnames(pvalues_table) <- c("Differences", "Outcome")

print(xtable(pvalues_table, digits = 3, caption = "Comparison of p-values for two measures (average out include.rownames = TRUE)</pre>
```

	Differences	Outcome
Parametric ANCOVA		
Unadjusted permutation	0.191	0.031
Unadjusted permutation (summed across strata)	0.750	0.279
Linear regression permutation	0.333	0.085
Residual permutation	0.326	0.098

Table 2: Comparison of p-values for two measures (average outcome during treatment vs. difference of average outcome and average baseline) of the primary endpoint.

```
set.seed(919547773) # Generated on Random.org Timestamp: 2016-11-09 15:39:29 UTC
continuous_vars <- c("heart_freq", "regurg_freq", "dysp_freq",</pre>
    "daily_heart", "daily_regurg", "daily_hrdq", "daily_dysp")
tests <- c("Parametric ANCOVA", "Stratified permutation", "LM permutation",
    "Residual permutation")
pvalues_table_contin <- as.data.frame(matrix(NA, nrow = length(continuous_vars),</pre>
    ncol = 4))
i <- 0
for (col in continuous_vars) {
    i < -i + 1
    tmpdata <- reshape data(col)</pre>
    lm1 <- lm(Outcome ~ Baseline + tr + factor(SITEID), data = tmpdata)</pre>
    pvalues_table_contin[i, 1] <- summary(aov(lm1))[[1]]["tr",</pre>
        "Pr(>F)"]
    observed_diff_means <- mean(tmpdata[tmpdata$tr == "B", ]$Outcome) -</pre>
        mean(tmpdata[tmpdata$tr == "A", ]$Outcome)
    diff_means_distr <- stratified_two_sample(group = tmpdata$tr,</pre>
        response = tmpdata$Outcome, stratum = tmpdata$SITEID,
        reps = 1000)
    pvalues_table_contin[i, 2] <- t2p(observed_diff_means, diff_means_distr,</pre>
        alternative = "two-sided")
    observed_t <- summary(lm1)[["coefficients"]]["trB", "t value"]</pre>
    lm1_t_distr <- replicate(1000, {</pre>
```

```
tmpdata$tr_perm <- permute_within_groups(tmpdata$tr,</pre>
             tmpdata$SITEID)
        lm1_perm <- lm(Outcome ~ Baseline + tr_perm + factor(SITEID),</pre>
            data = tmpdata)
        summary(lm1_perm)[["coefficients"]]["tr_permB", "t value"]
    pvalues_table_contin[i, 3] <- t2p(observed_t, lm1_t_distr,</pre>
        alternative = "two-sided")
    lm_no_tr <- lm(Outcome ~ Baseline + factor(SITEID), data = tmpdata)</pre>
    lm_resid <- residuals(lm_no_tr)</pre>
    lm_yhat <- fitted(lm_no_tr)</pre>
    observed_t <- summary(lm1)[["coefficients"]]["trB", "t value"]</pre>
    lm_t_distr <- replicate(1000, {</pre>
        lm_resid_perm <- permute_within_groups(lm_resid, tmpdata$SITEID)</pre>
        tmpdata$response_fl <- lm_yhat + lm_resid_perm</pre>
        lm_perm <- lm(response_fl ~ Baseline + tr + factor(SITEID),</pre>
            data = tmpdata)
        summary(lm_perm)[["coefficients"]]["trB", "t value"]
    })
    pvalues_table_contin[i, 4] <- t2p(observed_t, lm_t_distr,</pre>
        alternative = "two-sided")
rownames(pvalues_table_contin) <- continuous_vars</pre>
colnames(pvalues_table_contin) <- tests</pre>
summarytab <- xtable(pvalues_table_contin, digits = 3, align = paste0(c("r|",</pre>
    rep("p{1.2in}", ncol(pvalues_table_contin))), collapse = ""),
    caption = "Comparison of p-values from four tests, for each continuous endpoint.",
    label = "tab:clinical_pvalues")
print(summarytab, include.rownames = TRUE)
```

	Parametric	AN-	Stratified permuta-	LM permutation	Residual permuta-
	COVA		tion	•	tion
heart_freq	0.035		0.006	0.080	0.082
$regurg_freq$	0.136		0.118	0.280	0.220
$dysp_freq$	0.565		0.948	0.616	0.592
daily_heart	0.032		0.004	0.056	0.068
$daily_regurg$	0.142		0.174	0.286	0.246
$daily_hrdq$	0.043		0.012	0.088	0.098
$daily_dysp$	0.582		0.810	0.756	0.722

Table 3: Comparison of p-values from four tests, for each continuous endpoint.

```
print(summarytab, include.rownames = TRUE, type = "latex", file = "../ms/fig/results_pvalue_summary.tex
```